

LARGE AMPLITUDE FREE VIBRATIONS OF CIRCULAR  
PLATE WITH STEPWISE THICKNESS

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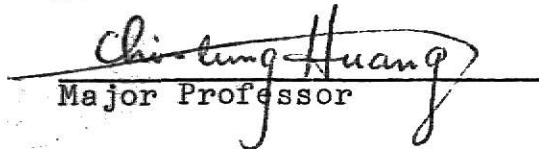
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## NOMENCLATURE

|                                   |   |
|-----------------------------------|---|
| $r, \theta, z$                    | Cylindrical coordinates used to describe the initial configuration of the plate |
| $h(r), a$                         | Thickness function and radius of the plate                                      |
| $h_0$                             | Minimum thickness   |
| $t$                               | Time variable   |
| $u, w$                            | Radial and transverse displacements of the middle plane                         |
| $\epsilon_r^0, \epsilon_\theta^0$ | Radial and circumferential strains of the middle plane                          |
| $\epsilon_r, \epsilon_\theta$     | Radial and circumferential strains  |
| $\sigma_r, \sigma_\theta$         | Radial and circumferential stresses   |
| $a_{11}, a_{12}, a_{22}$          | Stress-strain relation coefficients   |
| $N_r, N_\theta$                   | Middle plane forces per unit length   |
| $M_r, M_\theta$                   | Bending moments per unit length   |
| $Q$                               | Transverse shearing force per unit length                                       |
| $P(r, t)$                         | Loading intensity   |
| $\nu = -a_{12}/a_{22}$            | Poisson's ratio.  |
| $c = a_{11}/a_{22}$               |   |
| $D(r)$                            | Flexural rigidity   |
| $T, V$                            | Kinetic and potential energy  |
| $U$                               | Potential energy due to loading   |
| $V_1, V_2$                        | Potential energy due to stretching and bending of the plate                     |
| $\xi, \tau$                       | Dimensionless space and time variables  |
| $h(\xi)$                          | Dimensionless thickness function  |

|  |                                       |
|--|---------------------------------------|
| $A, \alpha$  | Amplitude parameters                  |
| $g, F$   | Dimensionless shape functions         |
| $\Psi$   | Stress function                       |
| $\omega$   | Dimensionless angular frequency       |
| $\lambda$  | Nonlinear eigenvalue                  |
| $\bar{Y}, \bar{H}, \bar{U}, \bar{V},$<br>$\bar{Z}, \bar{W}, \bar{X}$ | (6x1) vector functions                |
| $\bar{0}$  | (6x1) null vector                     |
| $[A], [B],$<br>$[C], [D]$  | Coefficient matrices                  |
| $C, S, M, V, N$  | Initial values                        |
| $T_0, T_1$   | Linear and nonlinear period           |
| $W_0$  | Deflection at the center of the plate |

## I. INTRODUCTION

The linear theory for the motion of elastic plates is based on the assumption that the deflections are small in comparison with the thickness of plates. However, light weight structures of thin plates may be required to withstand large amplitudes of vibration when subjected to severe dynamic loading conditions. If the amplitude of motion is of the same order of magnitude as the thickness of the plate, it is necessary to modify the mathematical model from linear plate theory to include deformation of the middle plane.

In 1956, Chu and Herrmann (1) studied the large amplitude free vibrations of a rectangular plate. By applying a perturbation method, they showed that the in-plane inertial force and buoyancy terms can be neglected and obtained equations which are dynamic analogs of the von Karman (2) equations of static equilibrium. With appropriate choices for the displacement functions the space variables were eliminated, and the remaining ordinary differential equation in terms of the time variable was solved in the form of an elliptic cosine function.

Nowinski (3) utilized von Karman's dynamic equations in an investigation of the free nonlinear vibrations of a circular plate built in at the boundary. He represented the deflection as a series of separable terms and used an orthogonalization procedure to eliminate the space variable. By confining the study to one term of the series, the solution in the time variable was found in the form of elliptic functions.

In 1970, Huang and Sandman (4) utilized von Karman's dynamic equations to describe the large amplitude axisymmetric oscillations of a circular plate with a clamped and immovable boundary. They used a different approach from the previous investigators. Harmonic vibrations were assumed and the time variable was eliminated by applying a Kantorovich averaging method (10). The remaining nonlinear eigenvalue problem was solved numerically by considering the related initial value problem (5, 6).

In this report the general governing partial differential equations of large amplitude axisymmetric oscillations of a variable thickness circular plate are derived. Following the same approach as Huang and Sandman, harmonic vibrations are assumed and the time variable is eliminated by applying a Kantorovich averaging method. For simplicity only examples of stepwise thickness plates under free vibration are investigated. This is a nonlinear eigenvalue problem solved numerically by considering the related initial value problems. The influence of amplitude on the shape function of vibration is illustrated. The induced stresses for different amplitude conditions are calculated and the response curves are investigated.