

APPLICATION OF ZERO-ONE PROGRAMMING
TO THE MAKE-OR-BUY DECISION

by 587

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CHAPTER I

INTRODUCTION

With the advent of technological changes, the scope of problems encountered in industrial firms has broadened and correspondingly presented management with more decisions of a higher degree of complexity to be made. In this report a specific type of decision is studied. Usually when a firm enters into a contract with the government a tight money situation exists. That is, profits received tend to be low from the contracts for the performance of certain services for the government. It also implies that fiscal year budgeting is a difficult task. Typically the budgeting is established for the current budget period but is forecasted for future budget periods. Thus when making a decision on the feasibility of producing a particular product it is necessary to evaluate what resources required to manufacture the product need to be budgeted for. Also it is necessary to note what effect the resource allocation to one product has on the total system. Perhaps there are one or two resources when considering the total system, which are critical resources, and, as a result, exert much influence on the total system.

Sometimes because of critical internal limitations of a particular resource it is necessary to make a decision to have a product manufactured at an external source. Thus in addition to budgeting for the allocation of resources required internally the situation of external buying can influence tremendously the decision making process and correspondingly the total system.

During the last few years two factors have effected significantly the operations of most industrial firms, including to some extent, the firm mentioned above. The two factors are: (1) a rapid technological change, and (2) an increase in competition between firms. As a result, each year considerable expenditures are made by firms on investments (for the manufacturing of products), each of which probably has some expected future rate of return. Because of the amount expended and the long period of time before a return on the investment is realized, it is obvious that planning and controlling the expenditures on such investments is necessary to attain goals set by the firm in accordance with the two factors mentioned above. The planning and controlling of these expenditures on investments is the capital budgeting problem. Capital allocation is a direct result of capital budgeting. In this report each investment represents the planning and production for a particular product. The expenditures for an investment for the capital budgeting problem is the amounts required (or the cost of capital) if the product is to be produced.

It is necessary for the acceptance or rejection decision of an investment to have available some method or criteria for making that decision. A final decision of accepting or rejecting a proposed investment obtained through the decision making process signifies whether a product should be invested in. Usually when considering the capital budgeting problem past analysis and methods of obtaining the decision have been restricted to decision making for internal operations of a firm.

When considering an acceptance or rejection criteria an additional

problem exists at some industrial firms. This problem consists of obtaining the make-or-buy decision. The make decision represents the product(s) invested in to be made by internal operations of a firm, whereas the buy decision represents the investments on product(s) that are made external to the operations of the firm. The products bought from an external source are then resold to a customer. The make-or-buy decision for the firm is usually affected or influenced by some internally required resource which is constrained to its limit. Theoretically for the make-or-buy decision three solutions to the problem are available: The three solutions are: (1) make all of a product internally, (2) buy all of a product externally, or (3) make some and buy some. The fact that a product is new, modified, or a current product is not taken into consideration in this report.

The problem studied in this report thus is stated as follows. Given the proposed investments or a priority value assigned to each investment for n products and m parts in each product to be manufactured, consider the combination of a capital budgeting constraint and constraints on other resources required for make-or-buy decision making to maximize the present value (or priority value) of the parts and thus find the optimum part-mix and product-mix for purposes of obtaining a make-or-buy decision. The part (or product)-mix represents the mix of parts which should be bought and those which should be made. Consideration is given to single and multiple product cases where each product has m parts. In addition single and multiple budget periods are evaluated.

It should be noted that the capital budgeting problem is essentially a subproblem of the make-or-buy problem. That is, capital budgeting is

only one factor among many which could influence the final make-or-buy decision.

1.1 Literature Survey

For this report three areas of literature have been searched. The three areas included articles pertaining to capital budgeting or capital allocation problems, make-or-buy decision problems, and zero-one integer programming. In recent years, various individuals [17,66,69] have applied zero-one programming to the capital budgeting and capital allocation problem. Zero-one programming presents a method of reaching the acceptance-rejection decision for each investment. To date literature pertaining to the make-or-buy decision have primarily used cost accounting as a means of solving the problem. One author [12] attempted to compare cost accounting procedures and linear programming procedures as methods of obtaining a solution to the make-or-buy problem.

The subject of capital budgeting has been the topic of many economic and technical papers in the last few years. In this report it serves as one of several factors for the mathematical formulations to be discussed. In his presentation of capital budgeting Dean [22] has attempted to remove deficiencies in procedures presented in previous literature. Emphasis has been placed on the economic viewpoint of investing. It has been recommended that an internal rate of return to be determined for each investment being considered by a firm. A procedure of ranking the rate of returns was proposed. This approach guarantees an optimal selection of investments when the following assumptions of economic theory were satisfied. The assumptions were: (1) perfect certainty, (2) perfect

capital market, (3) projections of the rate of returns, and (4) independent investments of products. Lorie and Savage, and Weingartner [47, 69] have illustrated that Dean's approach [22] has its shortcomings because it is the usual case in business that one or more of the above assumptions cannot be met.

Lorie and Savage [47] have illustrated why and how the rate of return approach failed in certain cases. Dean's procedures [22] break down when the projects are not independent or when expenditures for an investment are budgeted for more than one budget period. Lorie and Savage then proposed that the present value of the investments being considered be maximized. In addition this approach assumed that cost of capital has been given and that the investments have been considered independent. The method of obtaining a solution to the problem has been first to rank the ratios of the present value to expenditures of a project. Then proceed to select investments with the highest ratios until the total capital budget funds have been exhausted. This approach has been applied to three problems of capital rationing: (1) one budget period case, (2) multiperiod budget case, and (3) the independency of investments. This method of approach of capital budgeting has its deficiencies also.

Weingartner [69] has treated Lorie and Savage's approach in a different and more efficient manner for the multiproject problem with either a single or multiple period budget. Initially a linear programming approach has been utilized to replace the Lorie and Savage approach. It has been formulated as follows.

Maximize

$$Z = c_j x_j, \quad j = 1, 2, \dots, n,$$

subject to

$$a_{tj}x_j \leq b_t, \quad t = 1, 2, \dots,$$

and

$$0 \leq x_j \leq 1,$$

where c_j is the present value of revenue and costs associated with individual projects. The coefficient, a_{tj} , is the cost of capital of project j in budget year t and b_t represents the budget ceiling for year t . The variable, x_j represents the fraction of project j undertaken. The problem of observing a combination of projects instead of just one project at a time is solved using the above formulation.

Weingartner also has obtained an exact solution using a zero-one programming algorithm. An important assumption of this approach was that x_j was expressed as a zero or one. This implied either a complete acceptance or rejection of a project. A general approach to capital budgeting has been dealt with by Weingartner in relation to that presented by Charnes, Cooper, and Miller [17]. Related problems discussed by Weingartner have been budget deferrals, multiple budgets, parametric methods, horizon models, mutually exclusive projects contingent projects, and imperfect capital with or without being interdependent.

Birnberg, Pondy, and Davis [8] have conducted a study on voting rules which have established a capital budgeting committee for the allocation of resources. It has represented a behavioral approach to resource allocation. Fleischer [27] has discussed the two major issues associated with the rate of return method for capital allocation: (1) the ranking error, and (2) the preliminary selection. The ranking problem has excluded irreducibles or qualitative factors, thus the ranking error problem. The preliminary selection problem has represented

the selection of alternatives preliminary to the preparation of the final capital budget. Fleischer has discussed further the proper approach to solving the problem. In addition Fleischer [26] has proposed a technique to be used to consider 'irreducibles' and monetary data together in determining mutually exclusive alternative investment proposals. The logic used has been discussed and a formal structure of a solution was presented.

Unger [66] has assumed that investment opportunities are indivisible in nature. He has proposed a model which maximized the discounted sum of dividends paid to its shareholders. Balas's zero-one algorithm [3] and partitioning procedures have been combined for the allocation of a limited amount of capital among a specified set of investment opportunities.

Various algorithms have been proposed for the solution of an integer linear programming problem. Gomory [30] has initiated the integer programming proposals. An excellent exposition of integer programming has been introduced by Balinski [5] and Beale [7]. An enumerative approach has been developed by Hammer and Rudeanu [35] using branching and bounding processes subject to a set of rules. The rules are attributed to pseudo-Boolean functions. This approach has been coded in FORTRAN IV for the 360/50 computer by Char [14]. Salkin and Speilberg [62] and Lemke and Speilberg [63] have developed an Adaptive Binary Program. The computer program solves linear programming problems subject to an additional constraint that a variable must be either zero or one. In his thesis, Char [14] has compared both Hammer and Rudeanu's, and Salkin and Speilberg's algorithms using shop scheduling, line balancing, delivery, capital budgeting, traveling salesman, and fixed-charge problems. Because of the overall superiority of the Hammer and Rudeanu's program, it has been

selected to be used in the solution of the make-or-buy problem discussed in this report.

The problem of make-or-buy has been discussed briefly in technical literature. Most literature on make-or-buy has been in the business and management fields. Culliton [20] and Gross [31] have been prominent in attempting to resolve the problem of make-or-buy.

Culliton [20] has proposed a group of principles which would aid management in the decision making process of the make-or-buy problem. In general, the following points are regarded as important: (1) determine the quality of product needed, (2) estimate quantity required, (3) compare the cost of making with the cost of buying, and (4) compare the cost to the firm as a whole. However, special emphasis has also been given to the following factors: (1) establish a time table of product duration, (2) select a method of estimating cost which will be most accurate, and (3) evaluate all factors affecting the make-or-buy decision when considering cost to the firm. The approach taken by Culliton has been to consider the theories and techniques of purchasing and the source of supply. He also has discussed in detail, cost, quality, and quantity of the product, production at the right time, and external factors on individual make-or-buy problems.

The evaluation of relevant data to the make-or-buy problem has been considered essential to the decision making process by Gross [31]. He has emphasized the development of principles for quantitative factors. For qualitative factors; however, it has been necessary that further evaluation by management take place. As in [20], Gross [31] also has attempted to develop principles to serve as a base for making the make-

or-buy decision by management. He has listed the following principles as being important:

1. quality requirements for a product involved in a make-or-buy decision be established,
2. the quantity of the product required should be continually updated,
3. the cost should be compared for each of the alternatives
4. factors from financing related to cost need to be considered, and
5. whether the decision conform with circumstances and company policy.

It should be noted that the above five principles are similar to those discussed by Culliton [20].

An attempt has been made by Burton and Holzer [12] to compare cost accounting and a proposed linear programming approach for the make-or-buy problem. It has been shown that for more than two products, cost accounting technique of decision making becomes laborious. In addition it has been stated that linear programming presents a much easier and faster approach for decision making purposes. The possibility of considering capital equipment has not been considered, thus capital budgeting did not enter into the mathematical formulation. As a result the problem has been only to minimize total cost per unit subject to demand requirements and production constraints. The decision of making all, buying all, and making some and buying some has been considered.

Others which have contributed somewhat to make-or-buy theory are Fabrychy [24], Higgins [38], and Hubler [40]. Fabrychy has approached

the make-or-buy problem for a procurement and inventory study. Other literature has presented no technical and mathematical approaches other than cost accounting.

A similar type of problem considered quite frequent in industry is the lease-buy problem which has been discussed by Weekes, Chambers, and Mallick [68]. They have primarily discussed a method for deriving optimal implementation schedules and cost evaluations for such a problem. Basic characteristics of lease-buy situations have been introduced.

1.2 Proposed Research

This report studies the make-or-buy problem with capital budgeting as one of the influential factors. The approach used for obtaining the make-or-buy decision is the zero-one programming algorithm developed by Hammer and Rudeanu [34] and coded by Char [14]. A mathematical model is developed with a certain objective function and restrictions which pertain to an industrial firm.

The objective function introduced utilizes the assignment of priority values for purposes of influencing the make or buy decision. In particular, the priority values method is proposed to provide a method of including and evaluating both quantitative and qualitative factors.

The constraints proposed in the model are established with special consideration of an industrial firm. Five stages of product flow are also included in the mathematical development. The fundamental concepts of capital budgeting and make-or-buy are discussed in Chapter II. The problem is then developed in mathematical terms for solution by zero-one programming. A sample problem is presented which illustrates the mathematical development for the make-or-buy problem. This sample problem

is set up for a single product, multiple parts, and single budget period model.

Computational results for problems of various formulations which pertain to the industrial firm mentioned above are discussed in Chapter III. The first problem discussed is a multiple product, multiple parts, and single budget period problem where all parts are independent. In the second problem, two parts in different products are assumed to be identical for a multiple product, multiple parts, and single budget period. The third problem takes into consideration a multiple budget period in addition to multiple products, and multiple parts. This problem assumes that all parts are independent. The fourth problem is identical to the third one except that a part which is used in different products is considered. Conclusions and Summary on the report are stated in Chapter IV.

CHAPTER II

DEVELOPMENT OF A MAKE-BUY DECISION MODEL

In this chapter, four sections are included. Section 2.1 discusses the nature of the make-or-buy problem. Briefly, the industrial firm requiring a solution to such a problem is discussed. Section 2.2 analyzes the make-or-buy decision. Factors which have some influence on the final decision are mentioned. In addition, the capital budgeting problem is discussed to serve as a base for the mathematical formulation. The mathematical formulation is developed in Section 2.3 in the form of zero-one integer programming. Three variations of the objective function are proposed. The constraints are set up for five stages of product flow. Stage 1 represents the concepts stage. Development of the production processes and fabrication of a prototype consist of stage 2. The initial production is established in stage 3. Stage 4 includes full production activities. Inventory storage or shipping to the customer represents stage 5. A sample problem is presented for the illustration of the mathematical development in Section 2.4.

2.1 Nature of the Problem

In discussing the make-or-buy problem it is desirable to limit the type of organizations to which the discussion pertains. This report intends to discuss only those manufacturing firms which produce products for government utilization. It is a type of manufacturing concern which stresses the production of high quality products. The products to be manufactured, usually require special labor skills, sometimes

state-of-the-art technological know-how, and facilities with an extensive range on the types of machinery. The products manufactured represent a diversified range from those machined, to rubber and plastics, and to electrical and electronic devices.

At such manufacturing firms whenever an external source is contracted to fabricate a product, that product is being bought. Correspondingly, whenever a product is scheduled to be fabricated internally, that product is being made. Hence, when a product is bought externally, a decision to buy and not to make has been established. In addition, when an internal schedule has been established for a product, the decision has been to make and not to buy. This type of decision making is an end result of studying, weighing, and evaluating many internal and external factors which could effect the final decision. It means that before the final decision to make or buy is made, some method or process of interrelating factors is required. Even before this stage, it is necessary to collect the proper information and data. In addition, it is helpful if guidelines are available for use by managers making the final decisions.

The information on factors leading to a final decision are supplied by many divisions within the industrial firm. These divisions typically perform a function which helps attain the firms' goals. The decisions, however, cannot be the responsibility of just one division, except when the final decision is being considered. The divisions are Accounting, Computer Science, Engineering, Manufacturing, Planning, and Quality Control.

The concept of the make-or-buy decision making has continually produced problems for most managers where this type of decision making is necessary. The most prevalent problem is how the different factors should be related, and what influence each factor should have on the final decision. Decision making for one product at a time has typically been carried on at the industrial firm. Even when one product is considered, the decision making process is difficult. When it is decided to expand the decision making process to more than one product simultaneously the complexity of the problem increases: especially when the same resource is allocated to more than one product. It is usually difficult to make the decision, because of the many quantitative and qualitative factors that could enter the decision making process. Based on a broad view, the main factors are the three resources: land, labor, and capital. However, it is the usual case that factors are in much finer detail than the three main resources. In addition, the qualitative factors are not considered resources. They could be factors such as design stability, quality level or technological know-how. To evaluate these factors, it is necessary that a method of analysis provide a path for a more efficient decision-making process to reach the optimal product-mix. It is necessary to eliminate or make assumptions about these factors which cannot be measured in quantitative terms. An end result is the possible improvement of the performance of a firm and the optimal allocation of resources among investments.

When considering the make-or-buy problem the manager has at his discretion three possible solutions: (1) make all of a product internally, (2) buy all of a product externally, and (3) make some internally and buy some externally.

2.2 Analysis of the Problem

Extensive studies of the capital budgeting problem influences further research of the make-or-buy problem. In addition, an assumption by Burton and Holzer [12] that the capital budgeting problem be ignored in their formulation of the make-or-buy problem created the fact that one of the more important influencing factors was ignored. It should be emphasized that capital budgeting is only one of many factors in the make-or-buy decision. However, because of the emphasis in past years on this subject, it serves as an excellent base for establishing a make-or-buy model.

Capital budgeting consists of the planning and control of expenditures for assets with an expected resulting return. This return need not be in terms of monetary value. At least four elements are generally required when establishing a capital budget for expenditures: (1) a demand for capital should exist, (2) a supply of capital must be available, (3) the timing or the use of the cost of capital to obtain demanded capital is important, and (4) a method of ranking, appraising or selecting investments is required.

The concept of the cost of capital has been subject to much controversy in literature. It is defined as the cost of capital equipment which is required as a condition for undertaking the manufacturing of a product with a minimum rate of return on the investment. This cost can also be considered an opportunity cost, since there is the opportunity to demand a lower cost for capital equipment obtained, and consequently to produce a higher rate of return and additional revenue intake.

Thus, to have a capital budget there should be a demand for something

which is being invested in. The demand must be determined using some analysis or marketing procedure. Corresponding to the demand there has to be sources for the supply of what is demanded. In order to determine if supply is available to satisfy demand and what the sources of supply are, some analytical or premarketing procedure must be established.

It is obvious that when more than one investment is considered in a capital budget, the problem of selecting the investments which generate the higher rate of returns increases in complexity. As a result, the problem of satisfying the demand becomes quite complex when more than one investment is considered at the same time.

When the capital budget is established for a firm in the manufacturing industry, using the generally required information mentioned above, additional elements must be considered. The first element is the generation or introduction of a possible new investment or modification of an existing investment. Included in this stage are the initial plans, sketches or prints of the product which is being invested in. Also, it is necessary to determine the demand of capital in case it is decided that an investment should be made on the product. The second element is the evaluation of the product. Measures should be established for the estimation of benefits and costs of the product. After this is completed, a determination is made if the benefits of investment and cost are of a reasonable value. The third element represents the selection of those investments which benefit the manufacturing firm the most. This stage is the stage of critical decision-making for making or not making the investment in a product. Important to this stage is the manufacturing, if necessary, of a prototype of the product in demand, to further justify the feasibility of investing in the

product. The last element is the execution stage or that stage which provides manufacturing facilities, or replacement of existing internal facilities. It is also possible that, because the demand for a product and the overloading of internal facilities, the product must be produced at an external source.

In this report, it is assumed that the capital budgeting is utilized to control (or allocate) the expenditures on investments. In order to control the expenditures and to determine optimally what products should be invested in, various mathematical approaches have been established [17, 53, 69]. A mathematical formulation is established below for the general capital budgeting problem described previously. It is assumed that the total demand for each investment is known, regardless of the way it is determined, that is, by a marketing survey, contract, or estimation. Further, it is assumed that the availability of supply from various sources is known and that the supply satisfies the demand.

The capital budgeting problem for a firm in the manufacturing industry is generally stated as follows. Consider simultaneously from a multiple number of products the demand desired, the supply availability, the cost of capital during specific time periods, and the present value or the rate of return. It is desired to maximize the present value or the rate of return when the total cost of capital available for all investments (or products) is known. It is possible to make either the product internally or not to make it at all.

The general problem is initially stated mathematically as a linear programming problem to obtain an approximate solution for one budget period.

Maximize

$$Z = CX \quad (1)$$

subject to

$$AX \leq P_0 \quad (2)$$

and

$$0 \leq X \leq 1 \quad (3)$$

where

C vector representing the present value

X vector representing the amount invested

A matrix representing the cost of capital

P_0 vector representing total capital dollars available

An extension to the above formulation is the restriction that each components in the vector X is either 0 or 1. This implies that an exact solution of either investing or not investing is obtained. This type of restriction has been utilized in studies by Weingartner [69] and Petersen [53]. In addition, a formulation for more than one budget period has been considered. It is stated as follows.

Maximize

$$Z = CX \quad (4)$$

subject to

$$AX \leq P_0 \quad (5)$$

and

$$X = 0 \text{ or } 1 \quad (6)$$

where all notation is defined as above.

The capital budgeting mathematical formulations to date have been restricted to the option of either investing or not investing internally

in a product. Because of competition between firms and technological advances, a third option is becoming important to the manufacturing industry. The third option is that of buying a product externally for resale. This creates the make-or-buy decision. The decision for make-or-buy is important for purposes of cost reduction, profit increase, product-quality improvement, and optimal allocation of resources.

To reiterate, the decision to manufacture or make a product in the firm's plant, which is the internal source, is the make decision. Whereas, the decision to obtain a product from an external source is the buy decision. A possible third decision is to make some internally and buy some externally. In their article [12], Burton and Holzer has studied this possibility using linear programming. The following solutions are considered in this report: (1) Make all of a product internally, and (2) buy all of a product externally. This restriction is established because the products being fabricated required, in most cases, special technical know-how and specified types of capital equipment. It is possible that a product is neither made nor bought by using special mathematical restrictions. This avenue of solution is mentioned in the formulation of the problem.

When studying the make-or-buy decision, it is necessary to establish what factors effect such a decision. If only one product and one budget period is being considered, the decision is usually easier to reach because of no interrelationships with factors of another product. This is similiar to the one product capital budget problem, except for additional constraints that are included. When more than one product and more than one budget period are considered, the complexity of the decision-making task increases quite rapidly because of interrelationships between