APPLICATIONS OF FOURIER AND BIFORE ANALYSES TO DISCRETE SIGNALS

by

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CHAPTER I

INTRODUCTION

1.1 With the increased use of sampled-data systems, the discrete Fourier transform and the BIFORE (Binary Fourier Representation) transform have found a variety of applications which include: (1) image coding [12, 14], (2) spectral analysis of digital systems [3], (3) signal representation and classification [17], and, (4) speech processing [11, 13].

This report has three main objectives: (1) to acquaint the reader with brief introductions to the discrete Fourier and BIFORE transforms, (2) to illustrate some applications of these transforms by means of numerical examples, and, (3) to provide a documentation of the digital computer programs used in (2).

In Chapter II, the discrete Fourier transform is introduced. The corresponding algorithm which enables rapid evaluation of the discrete Fourier transform is discussed in Chapter III. This algorithm is the so-called fast Fourier transform. Applications of the fast Fourier transform for rapid evaluation of the Fourier power and phase spectra, crosscorrelation and convolution are also included.

Chapter IV introduces the BIFORE transform with its associated power and phase spectra. A computer program which facilitates rapid computation of the BIFORE transform and spectra is included. In conclusion, some aspects of the relation between the BIFORE and discrete Fourier spectra are considered.

CHAPTER II

THE DISCRETE FOURIER TRANSFORM

2.1 Definition of the Discrete Fourier Transform (DFT)

Let $\{X(m)\}$ denote a sequence $X(m)$, $m = 0, 1, \ldots, (N-1)$ of N finite valued real or complex numbers. The discrete or finite Fourier transform is defined as

$$C_x(k) = \frac{1}{N} \sum_{m=0}^{N-1} X(m) W^{km}, \quad k = 0, 1, \ldots (N-1)$$

where $W = e^{-i \frac{2\pi}{N}}$ and $i = \sqrt{-1}$. (2 - 1)

The exponential functions W^{km} in (2-1) are orthogonal; that is,

$$\sum_{m=0}^{N-1} W^{km} W^{-\ell m} = \begin{cases} N & \text{if } \frac{k-\ell}{N} \text{ is an integer} \\ 0 & \text{otherwise.} \end{cases} \qquad (2 - 2)$$

Thus, using (2-1) and (2-2), $\{X(m)\}$ can be expressed as the inverse discrete Fourier transform (IDFT)

$$X(m) = \sum_{k=0}^{N-1} C_x(k)\, W^{-km}, \quad m = 0, 1, \ldots, (N-1). \qquad (2 - 3)$$

2.2 Some Properties of the DFT

1. Since the exponential functions W^{km} are N - periodic, it follows that the sequences $\{C_x(k)\}$ and $\{X(m)\}$ in (2-1) and (2-3), respectively, are also N - periodic. That is, the sequences $\{X(m)\}$ and $\{C_x(k)\}$ satisfy the following conditions:

$$X(\pm m) = X(sN \pm m)$$

$$C_x(\pm k) = C_x(sN \pm k) \qquad s = 0, \pm 1, \pm 2, \ldots \qquad (2 - 4)$$

2. If the sequence $\{X(m)\}$ in (2-1) is real, then the DFT coefficients $C_x(k)$ in (2-3) are such that

$$C_x(N/2 + \ell) = \bar{C}_x(N/2 - \ell), \quad \ell = 1, 2, \ldots, (N/2 - 1) \qquad (2 - 5)$$

where $\bar{C}_x(k)$ is the complex conjugate of $C_x(k)$, $k = 1, 2, \ldots, (N - 1)$.

3. $C_x(0)$ and $C_x(N/2)$ are real if the input sequence $\{X(m)\}$ is real. This follows directly from (2-1).

2.3 Computation of the DFT Coefficients Using Matrix Multiplication

For the purpose of discussion, consider the case N = 8. Then (2-1) yields the matrix equation

$$\begin{bmatrix} C_x(0) \\ C_x(1) \\ C_x(2) \\ C_x(3) \\ C_x(4) \\ C_x(5) \\ C_x(6) \\ C_x(7) \end{bmatrix} = \frac{1}{N} \begin{bmatrix} 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 \\ 1 & W & W^2 & W^3 & W^4 & W^5 & W^6 & W^7 \\ 1 & W^2 & W^4 & W^6 & W^8 & W^{10} & W^{12} & W^{14} \\ 1 & W^3 & W^6 & W^9 & W^{12} & W^{15} & W^{18} & W^{21} \\ 1 & W^4 & W^8 & W^{12} & W^{16} & W^{20} & W^{24} & W^{28} \\ 1 & W^5 & W^{10} & W^{15} & W^{20} & W^{25} & W^{30} & W^{35} \\ 1 & W^6 & W^{12} & W^{18} & W^{24} & W^{30} & W^{36} & W^{42} \\ 1 & W^7 & W^{14} & W^{21} & W^{28} & W^{35} & W^{42} & W^{49} \end{bmatrix} \begin{bmatrix} X(0) \\ X(1) \\ X(2) \\ X(3) \\ X(4) \\ X(5) \\ X(6) \\ X(7) \end{bmatrix}. \qquad (2 - 6)$$

In particular, consider the case when $\{X(m)\}$ is real. Then, using the fact that the function W is N - periodic and applying (2-5) to (2-6), there results

$$\begin{bmatrix} C_x(0) \\ C_x(1) \\ C_x(2) \\ C_x(3) \\ C_x(4) \end{bmatrix} = \frac{1}{8} \begin{bmatrix} 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 \\ 1 & W^1 & W^2 & W^3 & W^4 & W^5 & W^6 & W^7 \\ 1 & W^2 & W^4 & W^6 & W^0 & W^2 & W^4 & W^6 \\ 1 & W^3 & W^6 & W^1 & W^4 & W^7 & W^2 & W^5 \\ 1 & W^4 & W^0 & W^4 & W^0 & W^4 & W^0 & W^4 \end{bmatrix} \begin{bmatrix} X(0) \\ X(1) \\ X(2) \\ X(3) \\ X(4) \\ X(5) \\ X(6) \\ X(7) \end{bmatrix} \qquad (2-7)$$

where

$$W = e^{-i\frac{2\pi}{8}} = \frac{\sqrt{2}}{2} - i\frac{\sqrt{2}}{2}$$

$$W^2 = -i$$

$$W^3 = -\frac{\sqrt{2}}{2} - i\frac{\sqrt{2}}{2}$$

$$W^4 = -1$$

$$W^5 = -\frac{\sqrt{2}}{2} + i\frac{\sqrt{2}}{2}$$

$$W^6 = +i$$

and

$$W^7 = \frac{\sqrt{2}}{2} + i\frac{\sqrt{2}}{2}.$$

In sequel, a numerical example is considered.

<u>Example 2-1</u>. Given the data sequence $X(0) = 1$, $X(1) = 2$, $X(2) = 1$, $X(3) = 1$, $X(4) = 3$, $X(5) = 2$, $X(6) = 1$, $X(7) = 2$, compute the DFT coefficients $\{C_x(k)\}$ by evaluating (2-7). From (2-7) it follows that

$$C_x(0) = (1/8)\left[X(0) + X(1) + X(2) + X(3) + X(4) + X(5) + X(6) + X(7)\right].$$