

BAYESIAN APPROACH TO QUALITY CONTROL

by

PRAKASH VAMAN JOSHI

B.E. (Electrical)

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## CHAPTER 1

### INTRODUCTION

#### 1.1 Review:

This report is primarily concerned with the Bayesian approach to various aspects of quality control. By way of introduction, we will briefly discuss both quality control and Bayesian theory.

First, acceptance sampling plans will be discussed. Inspection is done at various steps in manufacturing. It can be carried out for the raw material or at various stages while manufacturing, as well as for the final product. The final product may be inspected by the manufacturer himself or the customer may inspect the product at the time of purchase. This inspection is generally carried out on the sampling basis. Sampling inspection is done for various reasons. A product may be destroyed while testing or the cost of 100% inspection may be excessive. Hence all acceptance tests are done on sampling basis. Due to modern acceptance sampling methods, it is possible to carry out better quality improvement than that which might be possible by 100% inspection. If a large percentage of defective items are found in a batch, it is proper to reject the whole batch instead of rejecting individual items. By this method striking quality improvement can be carried out [Grant & Leavenworth].

The following symbols are generally used in relation with sampling acceptance plans -

$N$  = number of items in a given lot.

$n$  = number of items in a sample.

$M$  = number of defective items in a given lot of size  $N$ .

$m$  = number of defective items in a given sample of size  $n$ .