

A STUDY OF FFT PRUNING AND ITS APPLICATIONS

by

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## CHAPTER I

### INTRODUCTION

Since the development of the fast fourier transform (FFT) by Cooley and Tukey [5], considerable attention has been devoted to its modification to secure increased speed for computational purposes. Basically there are four modifications to increase the computational efficiency. These are: (1) innerloop nesting, (2) change in radix, (3) data shuffling and unscrambling when the input data is real, and (4) eliminating operations on zeros when the number of nonzero input data points is considerably smaller than the desired number of output points; or the desired number of transform points is considerably smaller than the number of input points.

The first modification is used in decimation in frequency and decimation in time algorithms, some aspects of which are discussed in Chapter II. The second and third modifications are discussed in [3], [8] and [16] respectively.

This report is primarily concerned with the fourth modification which is referred to as FFT pruning. FFT pruning eliminates operations that do not contribute to the final output. It can be applied to both discrete time and frequency domains, and saves considerable time. Applications of FFT pruning include speech processing, estimation of autocorrelation functions, and computing narrow band Fourier spectra with increased frequency resolution.

FFT pruning concepts are introduced in Chapter III, while experimental results pertaining to some applications are considered in Chapter IV. Conclusions and recommendations for future work are presented in Chapter V.

## CHAPTER II

## DECIMATION IN TIME AND FREQUENCY

## 2.1 Discrete Fourier Transform

The Fourier transform pair for continuous signals can be written in the form

$$F_x(f) = \int_{-\infty}^{\infty} x(t) e^{-i2\pi ft} dt$$

$$x(t) = \int_{-\infty}^{\infty} F_x(f) e^{i2\pi ft} df$$

for  $-\infty < f < \infty$ ,  $-\infty < t < \infty$ , and  $i = \sqrt{-1}$ .  $F_x(f)$  represents the frequency domain function corresponding to the time domain function  $x(t)$ . Analogous to the Fourier transform, the discrete Fourier transform (DFT) is a transform that is used for the Fourier analysis of data sequences. Thus, if  $\{X(m)\}$  denotes a sequence  $X(m)$ ,  $m=0, 1, \dots, (N-1)$  of  $N$  finite valued real or complex numbers, then its DFT is defined as

$$C_x(k) = \frac{1}{N} \sum_{m=0}^{N-1} X(m) W^{km}, \quad k = 0, 1, \dots, (N-1) \quad (2.1)$$

where  $W = e^{\frac{-i2\pi}{N}}$ ,  $i = \sqrt{-1}$ .

Again, the corresponding inverse discrete Fourier transform (IDFT) is defined as

$$X(m) = \sum_{k=0}^{N-1} C_x(k) W^{-km}, \quad m = 0, 1, \dots, (N-1) \quad (2.2)$$

Equations (2.1) and (2.2) constitute the DFT pair.