

# Geometric biases in power-spectrum measurements

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## ABSTRACT

The observed distribution of galaxies has local transverse isotropy around the line of sight (LOS) with respect to the observer. The difference in the statistical clustering signal along and across the LOS encodes important information about the geometry of the Universe, its expansion rate and the rate of growth of structure within it. Because the LOS varies across a survey, the standard fast Fourier transform (FFT) based methods of measuring the anisotropic power spectrum (APS) cannot be used for surveys with wide observational footprint, other than to measure the monopole moment. We derive a simple analytic formula to quantify the bias for higher order Legendre moments, and we demonstrate that it is scale independent for a simple survey model, and depends only on the observed area. We derive a similar numerical correction formula for recently proposed alternative estimators of the APS that are based on summing over galaxies rather than using an FFT, and can therefore incorporate a varying LOS. We demonstrate that their bias depends on scale but not on the observed area. For a quadrupole the bias is always less than 1 per cent for  $k > 0.01 h \text{ Mpc}^{-1}$  at  $z > 0.32$ . For a hexadecapole the bias is below 5 per cent for  $k > 0.05 h \text{ Mpc}^{-1}$  at  $z > 0.32$ .

**Key words:** methods: data analysis – methods: numerical – galaxies: statistics – dark energy – distance scale – large-scale structure of Universe.

## 1 INTRODUCTION

The three-dimensional power spectrum (PS) of galaxies is one of the most important measurements that can be made from galaxy surveys. The baryon acoustic oscillation feature in the PS can be used to obtain sub-per cent constraints on the expansion history of the Universe. The redshift-space distortions (RSD; Kaiser 1987) allow precise measurements of the growth rate of structure; the Alcock–Paczynski (AP; Alcock & Paczynski 1979) effect constraints very tightly the geometry of the Universe (for the most recent measurements, see e.g. Anderson et al. 2014; Samushia et al. 2014).

Both RSD and AP are imprinted into galaxy distribution as a signature along the line of sight (LOS) from the observer. To extract these signals in an unbiased way it is important that we analyse the data using the correct LOS that varies from one galaxy pair to another. It is geometrically impossible to define a Cartesian coordinate grid in such a way that the  $\hat{z}$ -axis is everywhere aligned with the LOS direction. Thus, the anisotropic power spectrum (APS) cannot

be measured in Cartesian coordinates with one of the directions in dual Fourier space identified with the LOS.

Because the PS is a two-point statistic, it relies on the properties of pairs of overdensities, although estimation methods can instead be based on pairs of galaxies. The varying LOS means that, for pairs of galaxies separated by wide angles, the RSD for the galaxies in a pair will not be parallel. The resulting clustering signal including these wide angle (WA) effects can be accurately modelled (Szalay, Matsubara & Landy 1998; Szapudi 2004), but the difference beyond assuming a single LOS to the mid-point between the pair of galaxies is small (Beutler et al. 2012; Samushia, Percival & Raccanelli 2012; Yoo & Seljak 2015). In this paper, we will concentrate on quantifying the effect of different methods to allow for the varying LOS between different pairs of galaxies. We will assume that the WA effects are small.

For distant surveys covering a small spatial region, the LOS will not vary significantly across the survey. In order to see where the approximation of a single-LOS breaks down, suppose that we consider making a general FT of the overdensity field,

$$\tilde{\delta}(\mathbf{k}) = \int d^3r \delta(\mathbf{r}) e^{-i\mathbf{k}\cdot\mathbf{r}}, \quad (1)$$

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using a fast Fourier transform (FFT) algorithm, with  $\hat{z}$ -axis pointed towards the middle of the survey. The APS could then be computed by averaging  $\tilde{\delta}(\mathbf{k})$  in wavenumber bins,

$$P(\mathbf{k}) = \frac{1}{V} \langle \tilde{\delta}(\mathbf{k}) \tilde{\delta}^*(\mathbf{k}) \rangle, \quad (2)$$

where  $V$  is the volume of the survey. This method has been successfully used in the past to measure the monopole moment of the PS, where the LOS direction is irrelevant, and we will now contrast it with recent methods for measuring higher order moments. We will refer to this method as a *single-LOS* method.

What we actually want is a method to compute the APS, or moments of it, as if there were no LOS variations, i.e. automatically correct for LOS variations in the computation of the APS, allowing the fast comparison with models and the retention of all information. The most natural way of doing this, allowing for a radially orientated LOS is to perform an FT in spherical coordinates with the origin at the observers position. Even though approach has been applied to previous galaxy surveys (see e.g. Tadros et al. 1999; Percival et al. 2004; Leistedt et al. 2012), FFTs have an advantage of being faster and more efficient in terms of computing both the PS estimators and the subsequent likelihood analysis of the measurements.

In principle, it is also possible to measure the corrected APS on a Cartesian grid by replacing the FFT with a sum over galaxy pairs (Yamamoto et al. 2006). This allows an LOS to be defined for each galaxy pair in the sample (effectively having a separate coordinate grid, or  $\hat{z}$ -axis for each galaxy pair). It is however impractical for modern large galaxy surveys, as it would require prohibitively large computing times. We will refer to this method as a *pairwise-LOS* method (see Section 2.1). As discussed above, in this paper we will ignore WA effects, and thus consider that the pairwise-LOS method gives an exact result.

A practical approximation for the pairwise-LOS method is to define an LOS for a chosen galaxy in each pair. This allows the estimation method to be reduced from a sum over pairs to a sum over galaxies, which is computationally faster (Blake et al. 2011). This approximation will break down for galaxy pairs with very large angular separation (it is effectively another WA effect) but will become increasingly accurate for smaller scale measurements. The algorithm can be reduced to series of FFTs (Bianchi et al. 2015; Scoccimarro 2015) and is significantly faster than the pairwise-LOS algorithms, which makes it feasible for the analysis of galaxy surveys. This method has been applied to WiggleZ data (Blake et al. 2011) and BOSS data (Beutler et al. 2014) and the results suggest that there is not an appreciable bias with respect to the pairwise-LOS method. We will refer to this method as a *moving-LOS* method (see Section 2.3).

In this paper, we aim to quantify the biases in the APS induced by single-LOS and moving-LOS methods with respect to the pairwise-LOS method. To make the discussion clearer we make certain simplifying assumptions: we ignore the effects of mask and selection function (window effects). Correcting for the mask and selection effects is not a trivial task, but this problem is almost independent from the issue that we want to address in this work, so we will assume that the window effects have been properly dealt with to required accuracy. We also ignore the discrete nature of galaxy survey data and will write all equations as integrals over overdensity field rather than sums over galaxies. These assumptions help to keep the discussion clearer and the equations compact and do not affect any of our main conclusions. The approximation is further justified by the fact that for large separations (where the geometric bias is larger) the effects of discreteness of the galaxy field are negligible.

We denote vectors by bold symbols ( $\mathbf{r}$ ), unit vectors by bold symbols with a hat ( $\hat{\mathbf{r}} = \mathbf{r}/|\mathbf{r}|$ ), and the modulus of a vector with italic symbols ( $r = |\mathbf{r}|$ ). A scalar product is assumed between two sequential (unit) vectors.

## 2 ANISOTROPIC POWER SPECTRUM

We start with the basic premise of a correlated galaxy overdensity field, with the correlation function (CF) between two galaxies at positions  $\mathbf{r}_1$  and  $\mathbf{r}_2$ ,  $\xi(\mathbf{r}_1, \mathbf{r}_2) \equiv \langle \delta(\mathbf{r}_1) \delta(\mathbf{r}_2) \rangle$ . For convenience, we define two vectors

$$\mathbf{r}_- \equiv \mathbf{r}_2 - \mathbf{r}_1, \quad (3)$$

$$\mathbf{r}_+ \equiv (\mathbf{r}_2 + \mathbf{r}_1)/2, \quad (4)$$

where  $\mathbf{r}_-$  connects the two galaxies and  $\mathbf{r}_+$  is the vector from the observer to their mid-point, which we will identify with the LOS of the galaxy pair. Because of local transverse isotropy around the LOS, and our assumption of no WA effects, the CF will only be a function of the distance between the galaxies and the angle they make with respect to the LOS.  $\xi(\mathbf{r}_1, \mathbf{r}_2) = \xi(r_-, \hat{\mathbf{r}}_- \hat{\mathbf{r}}_+)$ .

The angular dependence of the CF is usually expanded into Legendre polynomials

$$\xi(r_-, \hat{\mathbf{r}}_- \hat{\mathbf{r}}_+) = \sum_{\ell} \xi_{\ell}(r_-) \mathcal{L}_{\ell}(\hat{\mathbf{r}}_- \hat{\mathbf{r}}_+), \quad (5)$$

with most of the useful information in first three even multipoles (Taruya, Saito & Nishimichi 2011; Kazin, Sánchez & Blanton 2012).

The APS is defined as an FT of CF and can also be decomposed into Legendre polynomials with respect to LOS

$$P(\mathbf{k}) \equiv \int d^3 r_- \xi(r_-, \hat{\mathbf{r}}_- \hat{\mathbf{r}}_+) e^{-i\mathbf{k} \cdot \mathbf{r}_-} = \sum_{\ell} P_{\ell}(k) \mathcal{L}_{\ell}(\hat{\mathbf{k}} \hat{\mathbf{r}}_+). \quad (6)$$

This is a standard definition and theoretical predictions of APS are computed for this quantity (see e.g. Reimberg, Bernardeau & Pitrou 2015, and references therein). The PS multipoles are related to the CF multipoles by

$$P_{\ell}(k) = 4\pi i^{\ell} \int dr_- \xi_{\ell}(r_-) j_{\ell}(kr_-) r_-^2. \quad (7)$$

### 2.1 Pairwise-LOS method

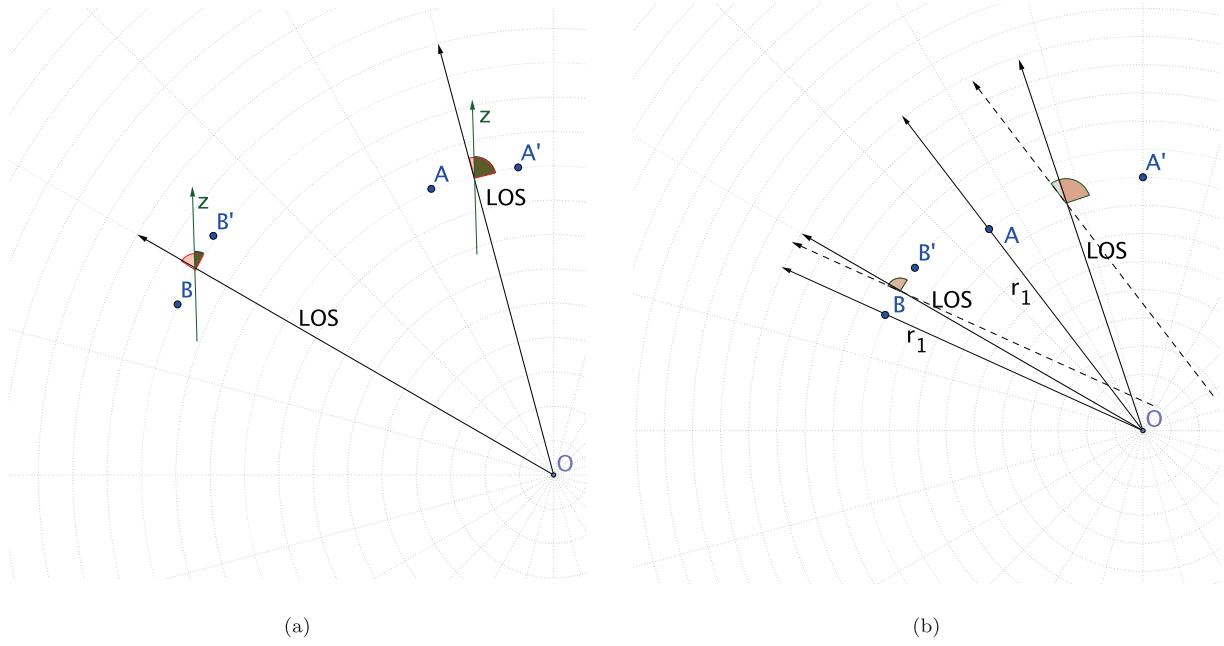
In the pairwise-LOS method, one would correct for the varying LOS, by computing the integral over the overdensity field simultaneously assigning correct LOS direction to all galaxy pairs. In other words, one would compute the multidimensional integral

$$P_{\ell}(k) = \frac{2\ell + 1}{4\pi V} \int d\hat{\mathbf{k}} d^3 r_1 d^3 r_2 \delta(\mathbf{r}_1) \delta(\mathbf{r}_2) e^{-i\mathbf{k} \cdot \mathbf{r}_1} e^{i\mathbf{k} \cdot \mathbf{r}_2} \mathcal{L}_{\ell}(\hat{\mathbf{k}} \hat{\mathbf{r}}_+). \quad (8)$$

The expectation value of this integral is

$$P_{\ell}(k) = \frac{2\ell + 1}{4\pi V} \int d\hat{\mathbf{k}} d^3 r_- d^3 r_+ \xi(r_-, \hat{\mathbf{r}}_- \hat{\mathbf{r}}_+) e^{-i\mathbf{k} \cdot \mathbf{r}_-} \mathcal{L}_{\ell}(\hat{\mathbf{k}} \hat{\mathbf{r}}_+) \quad (9)$$

and the APS computed in such way would coincide with the definitions of equations (6) and (7) and would therefore be exact. Computing this multidimensional integral over large galaxy sample



**Figure 1.** (a) Angles between the pair separation vector and the LOS assumed in the pairwise-LOS method (red) and the single-LOS method (green). (b) Angles between the pair separation vector and the LOS assumed in the pairwise-LOS method (red) and the moving-LOS method (green) for two example pairs of galaxies AA' and BB'. The moving-LOS method fails (i.e. red and green angles are significantly different) for pairs with large separations, while the single-LOS method fails for pairs whose true LOS is different from the fixed LOS assumed in the single-LOS method. The dashed arrows on panel (b) are parallel to lines OB and OA.

however demands prohibitively large CPU time and is not currently viable.

## 2.2 Single-LOS method

In the single-LOS method the  $\hat{z}$ -axis is pointed towards the middle of the survey footprint and it is assumed that  $\hat{r}_+ \sim \hat{z}$  within the survey volume (See Fig. 1a). This approximation allows us to rewrite equation (8) as

$$P_\ell(k) = \frac{2\ell + 1}{4\pi V} \int d\hat{k} \mathcal{L}_\ell(\hat{k}\hat{z}) \left| \int d^3r \delta(r) e^{-ikr} \right|^2. \quad (10)$$

The expectation value of the integral is

$$P_\ell(k) = \frac{2\ell + 1}{2} \frac{1}{4\pi} \int d\hat{k} P(k) \mathcal{L}_\ell(\hat{k}\hat{z}), \quad (11)$$

where

$$P(k) = \frac{1}{V} \iint d^3r_- d^3r_+ \xi(r_-, \hat{r}_- \hat{r}_+) e^{-ikr_-}, \quad (12)$$

as before. After integrating over  $r_-$  and  $\hat{k}$  this expression reduces to

$$P_\ell(k) = P_\ell^i(k) \frac{1}{V} \int d^3r_+ \mathcal{L}_\ell(\hat{z}\hat{r}_+), \quad (13)$$

where  $P^i$  is the true PS multipole defined by equations (6) and (7).

For  $\ell = 0$  the single-LOS APS reduces to the true APS as  $\mathcal{L}_0(x) = 1$ . The monopole of the APS is unbiased even when measured with the single-LOS method. For higher order multipoles the bias is always present, but could be small in some limits. For example, if the survey area is small and the  $\hat{z}$ -axis is pointed towards the centre of the survey, LOS directions of all pairs will be very close to  $\hat{z}$ . In this case,  $\hat{r}_+ \sim \hat{z}$  and  $\mathcal{L}_\ell(\hat{z}\hat{r}_+) \sim 1$ , resulting in small bias.

## 2.3 Moving-LOS method

The moving-LOS method is an approximation of equation (9) (See Fig. 1b). The overdensity field is transformed as

$$\delta_\ell(\mathbf{k}) = \int d^3r \delta(r) \mathcal{L}_\ell(\hat{\mathbf{k}}\hat{\mathbf{r}}) e^{-ikr} \quad (14)$$

and the APS multipoles are computed as

$$P_\ell(k) = \frac{2\ell + 1}{4\pi V} \int d\hat{\mathbf{k}} \delta_0(\hat{\mathbf{k}}) \delta_\ell(\mathbf{k}). \quad (15)$$

The APS computed in using this expression is equivalent to

$$P_\ell(k) = \frac{2\ell + 1}{4\pi V} \int d^3r_1 d^3r_2 d\hat{\mathbf{k}} \delta(r_1) \delta(r_2) e^{-ikr_1} e^{ikr_2} \mathcal{L}_\ell(\hat{\mathbf{k}}\hat{\mathbf{r}}_1) \quad (16)$$

and the expectation value of that expression is

$$P_\ell(k) = \frac{2\ell + 1}{4\pi V} \int d^3r_- d^3r_+ d\hat{\mathbf{k}} \xi(r_-, \hat{r}_- \hat{r}_+) e^{-ikr_-} \mathcal{L}_\ell(\hat{\mathbf{k}}\hat{\mathbf{r}}_1). \quad (17)$$

Using properties of spherical harmonics this can be further reduced to

$$P_\ell(k) = \int dk' \sum_{\ell'} P_{\ell'}^i(k') W_{\ell\ell'}(k, k'), \quad (18)$$

where

$$W_{\ell,\ell'}(k, k') = \frac{2k'^2}{\pi} \int dr_- r_-^2 j_\ell(kr_-) j_{\ell'}(k'r_-) \mathcal{F}_{\ell\ell'}(r_-) \quad (19)$$

and

$$\mathcal{F}_{\ell,\ell'}(r_-) \equiv \frac{4\pi i^{\ell-\ell'}}{V(2\ell'+1)} \int d^3 r_+ d\hat{r}_- \sum_{mm'} Y_{\ell'm'}(\hat{r}_-) Y_{\ell'm'}^*(\hat{r}_+) Y_{\ell m}^*(\hat{r}_-) Y_{\ell m}(\hat{r}_+) \quad (20)$$

(see Appendix for details).

This expression will reduce to the true APS only if  $W_{\ell\ell'}(k, k') = \delta_{\ell\ell'}\delta(k - k')$ . In the limit of very small separations  $\hat{r}_+ \sim \hat{r}_1$ , the  $\mathcal{F}$  tends to  $\delta_{\ell\ell'}$  by the virtue of the closure relation for the spherical harmonics, making  $W_{\ell\ell'}(k, k')$  converge to  $\delta_{\ell\ell'}\delta(k - k')$  in the mean.<sup>1</sup> Properties of spherical harmonics also enforce the condition  $W_{0\ell'} \propto \delta_{0\ell'}$  for all  $k$  and  $k'$ , which means that, as with the single-LOS method, there is no bias for  $\ell = 0$ . Unlike the single-LOS method, the bias cannot be expressed as a simple scale-independent scaling of the APS.

### 3 APS BIAS AS A FUNCTION OF SKY AREA, REDSHIFT AND SCALE

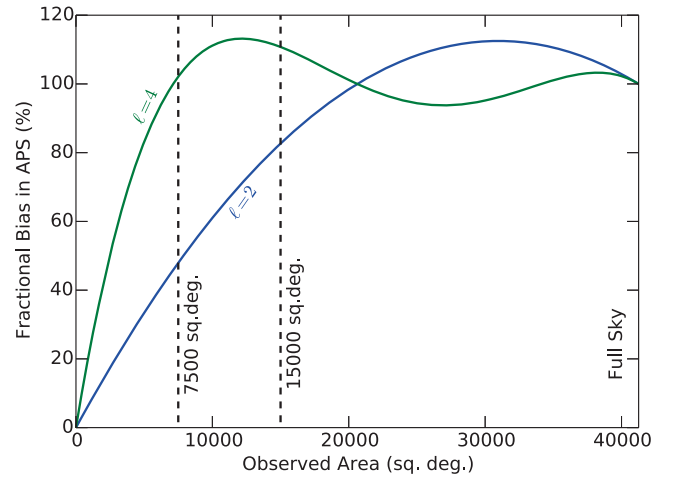
In this section, we will quantify the biases in the single-LOS and moving-LOS methods, using the pairwise-LOS method as the reference. To compute the biases we need to specify the geometry of the observed volume as the biases will depend on the distribution of pair separations. For simplicity, we will assume that the observed patch of the sky is circularly symmetric, the  $\hat{z}$ -axis is pointing to the centre of the observed area (this choice results in the least bias for the single-LOS method with the assumed LOS along this direction) and the mask and selection functions are uniform. We will also assume that the width of redshift bin is small compared to the distance of the sample from the observer. This simple model clearly lacks the detailed window of an actual survey, but the angular distribution of pair separations will be roughly correct, and the radial thinness is a conservative choice as it forces pair separations of the same physical separation to wider angular separations.

For this geometry, using a single-LOS analysis the bias, given by equation (13), reduces to

$$P_\ell(k) = P_\ell^i(k) \frac{\int_0^{\vartheta_{\max}} d\vartheta \sin(\vartheta) \mathcal{L}_\ell[\cos(\vartheta)]}{\int_0^{\vartheta_{\max}} d\vartheta \sin(\vartheta)}. \quad (21)$$

The bias in the APS is independent of the wavenumber and redshift and only depends on the angular extent of the observed area. We plot this bias for  $\ell = 2$  and  $\ell = 4$  in Fig. 2. We see a gradual increase in the biases with area for small surveys, with biases that are already larger than 1 per cent when the footprint is only 1000 deg<sup>2</sup>. For hemispherical or full-sky surveys with a single-LOS, both the quadrupole ( $\ell = 2$ ) and the hexadecapole ( $\ell = 4$ ) are zero (the fractional error is 1): here any increase in the clustering strength caused by RSD along the single-LOS is matched by an increase perpendicular to the single-LOS around the edges of each hemisphere.

For the moving-LOS method, the APS bias given by equation (18) cannot be expressed as a simple ratio of true and measured power spectra. For the simple – ‘thin spherical cap’ – geometry we have assumed, we can use the properties of spherical harmonics to reduce



**Figure 2.** Fractional error in the measured APS as a function of observed area for the single-LOS method.

the five-dimensional integral in equation (20) to a one-dimensional integral

$$\mathcal{F}_{\ell\ell'}(\eta) = i^{\ell-\ell'} \frac{2\ell+1}{2} \int_{-1}^1 d\mu \mathcal{P}_\ell^0(\mu) \mathcal{P}_{\ell'}^0(\mu) \mathcal{P}_\ell^0\left(\frac{1-\eta\mu/2}{\sqrt{1-\eta\mu+\eta^2/4}}\right) \quad (22)$$

(see Appendix), where  $\mathcal{P}$  denote associated Legendre polynomials and  $\eta \equiv r_-/r_+$ . Examining equation (22) we see that the expression tends to  $\delta_{\ell\ell'}$  when  $\eta$  tends to zero, suggesting that, for a fixed scale, the approximation works better the further away that galaxies are from the observer, as expected. Equation (22) also shows that the bias depends only on the wavenumber and does not depend on the observed sky coverage: given that the bias is only related to how each pair of galaxies is treated rather than the distribution of pairs, this is also expected for scales unaffected by the window.

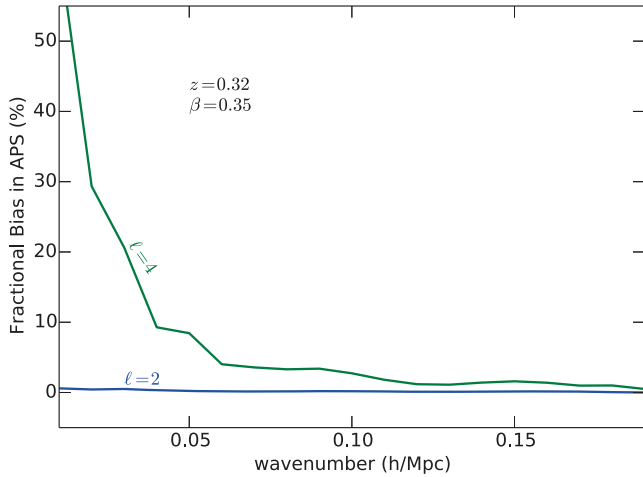
Fig. 3 shows the fractional bias in the APS at  $z = 0.32$  for  $\beta = 0.35$ .<sup>2</sup> The bias in the quadrupole lies below 1 per cent at wavenumbers above 0.01  $h/\text{Mpc}$ . The bias in the hexadecapole is larger and reaches a sub-per cent level only for wavenumbers above 0.1  $h \text{ Mpc}^{-1}$ . At higher redshifts the biases are reduced even further.

### 4 SUMMARY AND COMPARISON WITH PREVIOUS WORK

In this work, we quantify biases on the APS measured by various computational methods, focusing on methods correcting for the varying LOS. We have ignored WA effects that arise because the peculiar velocity shifts in galaxies in a pair are not parallel. At  $200h^{-1} \text{ Mpc}$  this effect is less than a percent already at  $z = 0.2$ , and decreases for the smaller scales usually of interest in analyses (see e.g. Raccanelli, Samushia & Percival 2010; Samushia et al. 2012). Instead, the approximations discussed in this work are related to the way of measuring the APS, and we

<sup>1</sup> In this limit the moving-LOS method basically reduces to the pairwise-LOS, since the pairwise-LOS method will have the same expansion as equation (20) but with  $r_1$  replaced by  $r_+$ .

<sup>2</sup>  $z = 0.32$  is an effective redshift of BOSS CMASS sample.  $\beta$  parameter describes the amount of anisotropy in APS (see e.g. Hamilton 1998, for a proper definition).



**Figure 3.** Fractional error in the measured APS as a function of observed area for the moving-LOS method for  $z = 0.32$  and  $\beta = 0.35$ .

have compared two methods - one known to be wrong (single-LOS), and one where small biases have previously been assumed (Blake et al. 2011; Beutler et al. 2014) but that we now quantify (moving-LOS).

To provide a baseline, we have considered ignoring the variation in the LOS and assuming a single-LOS: here we have shown that we are left with a significant bias in measurements other than the monopole moment of the PS, that is independent of scale and only depends on the area of the angular footprint. This has been known for a long time, and consequently this method has not been widely used to measure the APS, other than the monopole. We have shown through an analytic formula for the induced bias of APS multipoles (equation 13) that the bias on the quadrupole is large even for surveys as small as  $1000 \text{ deg}^2$ . Our results can be used to correct this effect to leading order for analysis methods in which the observed area is subdivided into smaller regions to make the bias smaller (see e.g. Cole, Fisher & Weinberg 1994; Hemantha, Wang & Chuang 2014).

We have also considered a measurement method proposed more recently that performs the transform summing over galaxies in a way that allows varying LOS to be factored into the measurement. Ideally, we would want to sum over pairs rather than galaxies, but this is impractical, and this revision leads to a small bias in the anisotropic measurements. We have derived an analytic formula for this bias that, in the limit of narrow redshift bin, can be reduced to a one-dimensional integral over the true PS (equation 18). This shows that the bias is a function of scale (but not the area) and only depends on the ratio of pair separation to the distance of pair from the observer  $r_-/r_+$ . While small, this correction can easily be calculated and measurements corrected for this effect.

The new method calculating the APS using a sum over galaxies (moving-LOS) remains accurate for the quadrupole APS at scales above  $k = 0.01 h^{-1} \text{ Mpc}$  even at small redshifts. The bias in the hexadecapole moment is larger, but also decays at high redshifts. Our correction formulas in equations (13) and (18) can be used to correct the biases to leading order.

Our analysis of the significance of making various LOS approximations makes a number of simplifying assumptions about the survey for the sake of analytical clarity. We assume the thin-shell approximation and ignore boundary effects, the effects of mask and the redshift dependence of the mean galaxy number density. While

the primary effects of these assumptions can be corrected when making clustering measurements, it is likely that they couple with the geometric biases considered here. Indeed, these approximations are likely to change the corrections due to LOS assumptions at a comparable order to the corrections presented here for a simplified survey (although when the single-LOS method is used on data with wide footprint the geometric effects are likely to be dominant). A more precise correction would require a detailed study of how the mask effects couple with the geometric biases.

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## REFERENCES

- Alcock C., Paczynski B., 1979, *Nature*, 281, 358  
 Anderson L. et al., 2014, *MNRAS*, 441, 24  
 Beutler F. et al., 2012, *MNRAS*, 423, 3430  
 Beutler F. et al., 2014, *MNRAS*, 443, 1065  
 Bianchi D., Gil-Marín H., Ruggeri R., Percival W. J., 2015, preprint (arXiv:1505.05341)  
 Blake C. et al., 2011, *MNRAS*, 415, 2876  
 Cole S., Fisher K. B., Weinberg D. H., 1994, *MNRAS*, 267, 785  
 Hamilton A. J. S., 1998, *Astrophys. Space Sci. Libr.*, 231, 185  
 Hemantha M. D. P., Wang Y., Chuang C.-H., 2014, *MNRAS*, 445, 3737  
 Kaiser N., 1987, *MNRAS*, 227, 1  
 Kazin E. A., Sánchez A. G., Blanton M. R., 2012, *MNRAS*, 419, 3223  
 Leistedt B., Rassat A., Réfrégier A., Starck J.-L., 2012, *A&A*, 540, A60  
 Percival W. J. et al., 2004, *MNRAS*, 353, 1201  
 Raccanelli A., Samushia L., Percival W. J., 2010, *MNRAS*, 409, 1525  
 Reimberg P. H. F., Bernardeau F., Pitrou C., 2015, preprint (arXiv:1506.06596)  
 Samushia L., Percival W. J., Raccanelli A., 2012, *MNRAS*, 420, 2102  
 Samushia L. et al., 2014, *MNRAS*, 439, 3504  
 Scoccimarro R., 2015, preprint (arXiv:1506.02729)  
 Szalay A. S., Matsubara T., Landy S. D., 1998, *ApJ*, 498, L1  
 Szapudi I., 2004, *ApJ*, 614, 51  
 Tadros H. et al., 1999, *MNRAS*, 305, 527  
 Taruya A., Saito S., Nishimichi T., 2011, *Phys. Rev. D*, 83, 103527  
 Yamamoto K., Nakamichi M., Kamino A., Bassett B. A., Nishioka H., 2006, *PASJ*, 58, 93  
 Yoo J., Seljak U., 2015, *MNRAS*, 447, 1789

## APPENDIX: BIAS IN MOVING-LOS METHOD

We will rewrite equation (17) using the plane wave expansion,

$$e^{ikr} = \sum_{\ell m} i^\ell 4\pi j_\ell(kr) Y_{\ell m}(\hat{\mathbf{k}}) Y_{\ell m}^*(\hat{\mathbf{r}}), \quad (\text{A1})$$

and the addition theorem for Legendre polynomials,

$$\mathcal{L}(\hat{\mathbf{k}}\hat{\mathbf{r}}) = \frac{4\pi}{2\ell + 1} \sum_m Y_{\ell m}(\hat{\mathbf{k}}) Y_{\ell m}^*(\hat{\mathbf{r}}). \quad (\text{A2})$$

This results in

$$\begin{aligned}
 P_\ell(k) &= \frac{2\ell+1}{4\pi V} \sum_{\ell'm'} \sum_{\ell''m''} \sum_m \int d^3r_- d^3r_+ d\hat{\mathbf{k}} \\
 &\quad \xi_{\ell'}(r_-) \frac{4\pi}{2\ell'+1} Y_{\ell'm'}(\hat{\mathbf{r}}_-) Y_{\ell'm'}^*(\hat{\mathbf{r}}_+) \\
 &\quad i^{\ell''} 4\pi j_{\ell''}(kr) Y_{\ell''m''}^*(\hat{\mathbf{k}}) Y_{\ell''m''}(\hat{\mathbf{r}}_-) \\
 &\quad \frac{4\pi}{2\ell+1} Y_{\ell m}(\hat{\mathbf{k}}) Y_{\ell m}^*(\hat{\mathbf{r}}_1). \tag{A3}
 \end{aligned}$$

Integrating over  $\hat{\mathbf{k}}$ , by virtue of orthogonality of spherical harmonics, results in equations (18)–(20).

The expression in equation (20) is invariant under the rotation of coordinate system. This symmetry can be used to align the direction of  $\hat{\mathbf{z}}$  axis with  $\hat{\mathbf{r}}_+$  as we integrate over  $d\hat{\mathbf{r}}_+$ . Ignoring the boundary effects and assuming that the redshift shell is thin, this results in

$$\begin{aligned}
 \mathcal{F}_{\ell\ell'}(r_-) &= i^{\ell-\ell'} \sqrt{\frac{4\pi}{(2\ell'+1)}} \\
 &\quad \times \int d\hat{\mathbf{r}}_- Y_{\ell'0}(\hat{\mathbf{r}}_-) \sum_m Y_{\ell m}^*(\hat{\mathbf{r}}_-) Y_{\ell m}(\hat{\mathbf{r}}_1), \tag{A4}
 \end{aligned}$$

as the integral over  $d^3r_+$  is equal to volume and only terms with  $m' = 0$  because  $\mathbf{r}_+$  is pointing in  $\hat{\mathbf{z}}$  direction. This expression is again invariant with respect to rotations in azimuthal angle (ignoring boundary effects) and can be reduced to

$$\begin{aligned}
 \mathcal{F}_{\ell\ell'}(r_-) &= 2\pi i^{\ell-\ell'} \sqrt{\frac{4\pi}{(2\ell'+1)}} \\
 &\quad \times \int d\mu_- Y_{\ell'0}(\mu_-) Y_{\ell'0}^*(\mu_-) Y_{\ell 0}(\mu_1), \tag{A5}
 \end{aligned}$$

where the  $m' \neq 0$  terms are killed by the azimuthal integral and  $\mu_-$  and  $\mu_1$  are the cosines of respective polar angles. Since  $\mathbf{r}_1 = \mathbf{r}_+ - \mathbf{r}_-/2$  we have  $\mu_1 = (1 - \eta\mu_-/2)/\sqrt{1 - \eta\mu_- + \eta^2/4}$ , which will result in equation (22).

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