

DETERMINATION OF THE FLOW OVER A BLUNT BODY IN A
SUPERSONIC STREAM BY THE METHOD OF
UCHIDA AND YASUHARA

by 593

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NOMENCLATURE

English alphabets:

x, y, z	Cartesian coordinates
s, n, t	Arc length of α , β , and γ axes, respectively
$h_\alpha, h_\beta, h_\gamma$	Parameters in curvilinear coordinates.
P	Pressure
T	Temperature (absolute)
S	Entropy, energy per degree per unit mass
i	Enthalpy, energy per degree per unit mass
R	Gas constant, length per degree
C_p, C_v	Specific heat at constant pressure and volume, respectively
q	Velocity
$q_\alpha, q_\beta, q_\gamma$	α , β , and γ components of q , respectively
u, v, w	x , y , and z components of q , respectively
q_n, q_t	Normal and tangential velocity component on a curved shock wave, respectively
a	Velocity of sound
E	Total energy, energy per unit mass
w	Angle between velocity vector and tangent to shock wave
K	Connection factor for boundary condition at shock wave
d	Diameter of sphere
b	Detachment distance of bow shock wave from nose of body
P_0	Total pressure in uniform upstream

Greek letters:

α, β, γ	Orthogonal curvilinear coordinates
ρ	Density
γ	$C_p/C_v =$ ratio of specific heat
ψ	Stream function
ξ	γ component of rot q
ϵ	Constant ($\epsilon=0$ in two-dimensional flow and $\epsilon=1$ in axisymmetric flow)
δ	Deflection angle of streamline through shock wave
θ	Angle of flow direction
l	Reference length

Subscripts:

0	Quantities in state at rest of undisturbed flow
*	Quantities at state of locally sonic
b	Value along a certain boundary, which will be replaced by s in the present condition
a	Quantities along the fixed surface
s	Quantities immediately behind the shock wave
1, 2	Quantities before and behind the shock wave, respectively
m	Grade of approximation

CHAPTER I

INTRODUCTION

The problem of the high-speed flow past a blunt-nosed body has been a subject of considerable interest. In this problem, the bow shock wave is detached from the body surface. The difficulty encountered in attempting to analyze the entire flow field seems to be caused by an inability to find proper means of quasi-linearization. By the fact that the flow around the body is a non-linear, mixed subsonic-supersonic flow and the location of the bow shock wave in front of the body is not known in advance, neither the small-disturbance theory nor the analytical method of the hodograph plane can be applied in the general case of a detached, curved shock wave.

Except for a wedge or cone of small apex angle, the effects of rotation (i.e. vorticity) cannot be neglected. In order to retain the non-linearity of the flow pattern as well as the effects of rotationality, numerical methods have been studied by Uchida and Yasuhara (3) and several other authors (5, 8, 10). Many of these authors treated the present special problem by the relaxation method, starting with the observed form of shock wave or requiring the pressure distribution on the entire body and the shock shape. As a result, these methods require some definite, prior knowledge of the flow and, thus, do not represent complete solutions to the problem. On the other hand, although the method of Uchida and Yasuhara requires iterating both the shock shape and streamline pattern, it requires no fixed assumptions regarding values of the flow variables, either at the surface or in