

This is the author's final, peer-reviewed manuscript as accepted for publication. The publisher-formatted version may be available through the publisher's web site or your institution's library.

In-store referrals on the internet

Gangshu (George) Cai, Ying-Ju Chen

### How to cite this manuscript

If you make reference to this version of the manuscript, use the following information:

Cai, G., & Chen, Y. (2011). In-store referrals on the internet. Retrieved from <http://krex.ksu.edu>

### Published Version Information

**Citation:** Cai, G., & Chen, Y. (2011). In-store referrals on the internet. *Journal of Retailing*, 87(4), 563-578.

**Copyright:** © 2011 New York University. Published by Elsevier, Inc.

**Digital Object Identifier (DOI):** doi:10.1016/j.jretai.2011.09.005

**Publisher's Link:** <http://www.sciencedirect.com/science/article/pii/S0022435911000856>

This item was retrieved from the K-State Research Exchange (K-REx), the institutional repository of Kansas State University. K-REx is available at <http://krex.ksu.edu>

# In-Store Referrals on the Internet\*

May 23, 2011

## Abstract

In the contemporary e-business, a retailer may display the links to the competing retailers directly (direct referral), or display the referral link provided by a third-party advertising agency (third-party referral), and these referrals may be either one-way or two-way. In this paper, we show that the referrals may align the retailers' incentives and facilitate implicit collusion, and one-way referral may result in a win-win situation, thereby providing an economic rationale for these seemingly puzzling phenomena. Using third-party referrals may enhance the retailers' collusion despite the potential disutility and revenue leakage, and referral services may be detrimental for the consumer welfare.

*Keywords: retailer referral; third-party referral, channel competition; game theory*

---

\*We thank the review team for their detailed comments and many valuable suggestions that have significantly improved the quality of the paper. All the remaining errors are our own.

# 1 Introduction

In recent years, there has been a revolution in the Internet retail industry. Pioneered by Amazon.com in 2006, Sears, Target, and Walmart, among other leading retailers, have started to display (and sell) the products of their competitors through their own websites. As an example, in the “Sell On Sears” program, other retailers (competitors) are allowed to show their products directly on Sears.com and consumers that browse Sears.com can purchase these products through Sears’ secure checkout process. After the transactions are completed by consumers, Sears forwards the payments directly to the corresponding retailers. In return, Sears charges a modest monthly fee (\$39.99) and requests a pre-specified commission rate (ranging from 7% to 25%) for every transaction that takes place through this Sell On Sears program.<sup>1</sup> Likewise, Amazon.com and Walmart marketplace also offer competing retailers to sell similar products via their websites. In contrast with these referrals offered and controlled directly by the retailers, some retailers resort to third parties to provide these referral services. For example, Buy.com, JC Penny, Macy’s, Target, and Walmart all participate in the sponsored advertisement programs by Google or Yahoo!. Upon a consumer’s search for some specific product, the websites return a list of links of (Google) sponsored advertisement on their websites typically displayed at either the sidebars or the bottom of the search results.

These referral services, termed as “*in-store referrals*” in our paper, have been implemented in the top 20 online retailers within one year of time after Amazon first introduced it and can be seen on the websites of numerous major online retailers nowadays.<sup>2</sup> Despite the common purpose of sharing more information to assist the consumer search, these referral services also differ in some other aspects. As aforementioned, there are at least two forms of in-store referrals: the “*direct referral*” in which a retailer displays other retailers’ items directly on her own website, and the “*third-party referral*” in which a retailer displays the referral link provided by a third-party advertising agency to other retailers. Furthermore, these referrals may be either one-way or two-way. For example, the Sell On Sears program is apparently a one-way direct referral, as those retailers that subscribe in this program may not “return the favor” by showing the advertisements of Sears on their own websites. Another example is the relationship between Macy’s and Overstock.com. Specifically, Overstock.com lists the advertisement of Macy’s via third-party referrals, but on the Macy’s website there is no indication for Overstock.com. On the contrary, Amazon and ANTonline.com adopt the

---

<sup>1</sup>This is different from “Advertise On Sears,” in which the advertisers pay Sears for displaying their advertisements on a cost-per-click basis; in this case, no actual transactions take place at Sears.com. See the detailed descriptions of these programs at <https://seller.marketplace.sears.com/>.

<sup>2</sup>See more details at <http://www.amazonstrategies.com/2009/08/walmart-launches-walmart-marketplace.html>.

two-way direct referral policy, because the majority of retailers in their marketplaces also display the links of Amazon and ANTONline.com, which allow the consumers to return directly to these host retailers. There are even some retailers that adopt a mixed business model: Amazon and Walmart use both direct referrals and third-party referrals for some specific categories of items but not others.<sup>3</sup>

These pervasive and apparently different referral strategies give rise to a number of intriguing research questions. When should the retailers adopt referrals? How does the direct referral fare compared to the third-party referral? How does the referral affect the retailers' competition and profits? Is the seemingly unfair one-way referral ever profitable? Note that in practice these transactions and contracting between online retailers and third-party referral agencies may be fairly complicated, and the categorization may change over time. While we acknowledge that in reality the referral business is more of a hybrid system and various logistic and practical concerns may be influential, our goal in this paper is to identify the fundamental economic drivers of these different forms of in-store referrals in the most parsimonious setting. Through this simple and easily extendable framework, we hope to highlight the strategic interactions between the competing retailers, the consumers' behaviors, and the market equilibria under these referral services. In compliance with this objective, we will also abstract away the tedious issues regarding distribution policies, consumer returns, and cash deposit or direct wire transfers between the referring and referred retailers.<sup>4</sup>

To address our research questions, we construct a model in which two online retailers sell horizontally differentiated products to the consumers. Ex ante, some consumers may be unaware of either product, and may visit the website of only one retailer. This consumer unawareness creates an incentive for the retailers to advertise on the other retailer's website, and it brings up the incentive issues of whether a retailer should accept such a request. As aforementioned, we consider various in-store referrals, including both one-way and two-way, and both direct and third-party referrals. In our model, consumers are price sensitive, possess heterogeneous preferences/tastes, and ex ante may

---

<sup>3</sup>The above observations are made through browsing the official websites of these online retailers (data collected in April 2011).

<sup>4</sup>It is worth mentioning that these in-store referrals are somewhat different from other traditional brick-and-mortar business models, even though similar transaction schemes are used (e.g., the "Sell Through Sears" program). However, in the offline channel, storage fees and inventory holding costs become primary concerns. On the contrary, the referral links in the online channels do not occupy any physical "shelf space," which seems to suggest why Internet retailers are more willing to display the products for their competing retailers. This is the primary difference between the retailers' co-location phenomenon in the offline world and our online retailer referrals.

have different levels of awareness. To explicitly model the strategic effects of in-store referral, we classify the consumers into three groups: two partially informed segments that originally are aware of only one retailer, and a group of fully informed consumers who are aware of both retailers (i.e., the “comparison shopping segment” in the terminology of Chen et al. (2002)). Within each segment, consumers possess heterogeneous preferences, and we model this heterogeneity via the Hotelling model. If a retailer provides the in-store referral, (some of) her partially informed consumers will be informed of the other retailer and become fully informed consumers.

In our model, the referred retailer commits to share a fixed proportion of the revenue through those referral links with the referring retailer. In the case with third-party referral, the third party would keep a portion of the shared revenue (from the referred retailer). These proportions represent the relative bargaining powers of the referred retailers and the third party and are sometimes specified explicitly in public. For example, in the Sell On Sears program, these commission rates are pre-specified and vary across different categories of items: 18% for appliances, 15% for baby products, 7% for computers & electronics, 15% for fitness & sports, health & wellness and home appliances, 20% for jewelry, and so on. Likewise, third-party referral infomediaries typically charge retailers in a similar manner; see the sponsored advertisement policies by Google at [Google.com](http://Google.com).<sup>5</sup> In addition to the different revenue sharing rules, we also assume that under third-party referral, a disutility is applied to those consumers that purchase from the referred retailer. This captures the phenomenon that consumers shopping through third-party referral may find it inconvenient to complete the purchase.<sup>6</sup>

We find that, upon switching from the non-referral scenario to the one-way direct referral, the referring retailer may be better off, even though she inevitably forgoes (part of) her partially informed consumers who are now aware of both products and indeed purchase from her competitor eventually. This may sound a bit unintended, as conventional wisdom may lead one to believe that

---

<sup>5</sup>Incidentally, a modest monthly fee may be charged (e.g., \$39.99 will be assessed at the beginning of each pay period), which essentially covers hosting and maintenance costs associated with the retailers’ accounts. Since these monthly fees are relatively small compared to the shared profits (e.g., 20% for a coffee maker listed at \$35 with over hundreds of sales per month) and fixed across categories, we omit them in our model.

<sup>6</sup>This assumption is motivated and supported by the contemporary third-party sponsored advertisements, because in most cases these referral links by Google or Yahoo! do not lead the consumers directly to the webpages where they can check out their targeting products. In contrast, under the direct referrals, consumers can check out immediately through the referral links. This convenience level largely reduces the consumers’ disutility in purchasing the products. Incidentally, this disutility may also arise from the trust issue, as retailers are forced to check out through the retailer websites that they are not familiar with under third-party referrals.

the retailers are better off retaining their (local) monopoly power and should avoid the information disclosure that intensifies competition. The primary channel to facilitate this one-way referral is the alignment of retailers' incentives. On one hand, displaying direct referrals creates more fully informed consumers and thus more consumers make their purchase decisions by comparing prices. On the other hand, the associated revenue sharing allows the retailers to align their objectives better; as the referring retailer also collects some revenue from the referred consumers, her intention to "steal the demand" from the competitor becomes moderate. These two conflicting economic forces drive the prices in opposite directions. When the revenue sharing is significant (i.e., the referring retailer obtains a reasonable proportion of the revenue created through the referrals), the second force is so overwhelming that the referring retailer also benefits through this seemingly unfair one-way referral. This mutual benefit also suggests that if the revenue sharing portion is *endogenously* determined through the negotiation, any sufficiently large sharing portion may emerge as an equilibrium outcome.

Our results also suggest that in-store referrals may actually push the selling prices up due to this salient collusion between the retailers. This result is quite ironical, as in-store referrals are commonly perceived as an additional service that provides consumers more information for price comparisons. For example, a Walmart spokesman notes that "[o]ur focus is always on our customers, and by adding nearly one million new items we give them even more reasons to shop Walmart.com." As another example, "[t]he Sears marketplace platform provides new opportunities for us to deliver more shopping choices to our customers," according to a Sears spokesman.<sup>7</sup> While our results confirm their claims that some consumers indeed get exposed to and consequently purchase new products through these informative referral links, the collusion effect due to the incentive alignment may drive up the selling prices and ultimately hurt the consumers. We also find that this collusion effect is more pronounced under two-way direct referral. Compared with the one-way referral, however, the originally referred retailer may resist switching to the two-way referral even though the joint profit may be unambiguously higher under two-way referral. Similar results are obtained under third-party referrals.

We can also articulate the comparison between direct referrals and third-party referrals. While varying the revenue sharing between the retailers, the retailers' preferences over one-way direct and third-party referrals are exactly the opposite. Recall that under third-party referral, a portion of revenue goes to the third party; consequently, the change of revenue sharing proportion has a less significant impact under third-party referral than under direct referral. Moreover, in principle an increase in revenue sharing rate gives rise to a higher payoff for the referring retailer but a lower

---

<sup>7</sup>See more details in "Two big chains open their sites to other retailers" at <http://www.internetretailer.com>.

payoff for the referred retailer. Thus, as the revenue sharing becomes more significant, the more direct benefit under direct referral makes the referring retailer prefer direct referral to third-party referral, whereas the referred retailer suffers less under third-party referral. This gives rise to a preference reversal between the retailers. We find that consumer welfare crucially depends on the battle between the collusion effect and information effect, and consumers may prefer either channel structures under different market conditions. We also investigate several variants of our model characteristics regarding the symmetry of market bases and the proportion of partially informed consumers that become fully informed consumers upon seeing the referrals. Overall, our qualitative insights are not altered by these modifications.

As aforementioned, our paper is related to a small but influential literature on referral services. Chen et al. (2002) consider a setting with two retailers and one third-party referral infomediary that owns her own market base. Their primary goal is to evaluate whether the third-party infomediary should refer her consumers exclusively to only one retailer, or to both retailers to facilitate competition. In contrast, we study the direct referrals initiated by the competitive retailers themselves. Furthermore, in our third-party referral scenario, we eliminate the third-party's market base; yet, the retailers may still benefit from the referrals due to the collusion effect. Ghose et al. (2007) consider a supply chain setting in which the manufacturer is able to refer consumers to some of the retailers that sell for the manufacturer. They identify an intriguing tradeoff between the online referral channel and offline physical channel. As consumers incur heterogeneous discovery costs while purchasing offline, the retailers are able to price discriminate them by elaborating this heterogeneity. On the other hand, in the online channel, the absence of discovery costs makes consumers rather homogeneous; thus, the price discrimination becomes inactive but the retailers can then charge consumers more due to the discovery cost saving. While in our model consumers are also heterogeneous with regard to their awareness of the retailers, within the same segment they have the same discovery (search) costs and thus this price discrimination effect is absent. The benefit of referral services (from the retailers' perspective) primarily comes from the incentive alignment between the retailers. See also Iyer and Pazgal (2003), Libai et al. (2003), Viswanathan et al. (2007), and Xue et al. (2004) along the same line.

There are also numerous papers that investigate the consumer referrals, i.e., how manufacturers or retailers should reward the existing consumers for bringing in new consumers, see, e.g., Bialogorsky et al. (2001), Brown and Reingen (1987), and Ryu and Feick (2007). Unlike the manufacturer referral and consumer referral, retailer referral gives rise to a novel issue as the parties that offer referrals compete directly with those beneficiaries. To the best of our knowledge, this recent phenomenon has never been investigated in the academic literature. Our paper is also re-

lated to the vast literature on retail pricing (e.g., Hall et al. (2010), Kopalle et al. (2009), and Levy et al. (2004)), retailers' price competition (e.g., Choi, 1991; Desai, 2001; Hess et al., 1994; Iyer et al., 2005; Kopalle et al., 2009; Padmanabhan and Png, 1997) and channel conflict and coordination (see Brown, 1981; Brown et al., 1983; Cai, 2010; Chiang et al., 2003; Ingene and Parry, 1995; Lusch, 1976; Tsay, 2002; Yuan and Krishna, 2008). In contrast with the aforementioned papers, our model includes the direct referrals and revenue sharing that are not addressed. Thus, we present a simple yet practical scenario in which the retailers cooperate through the referrals and then compete in setting prices. The one-way and two-way referrals investigated in our paper are also related to the single and bi-directional linking in a large emerging literature in social networking; see Jackson (2008) for a comprehensive survey. Our work contributes to this literature by introducing the tactical (pricing) decisions of economic agents (retailers) after the network structure is determined.

The rest of this paper is organized as follows. Section 2 introduces the model setup, including the direct and third-party referrals. Sections 3 and 4 investigate the equilibrium behaviors under the direct referral and the third-party referral, respectively, and Section 5 compares these two referral policies and their impacts on the market equilibria. In Section 6, we extend our model setting to accommodate asymmetry between the retailers and examine other variants. Section 7 provides some concluding remarks and possible directions for future work. All the proofs are relegated to the appendix.

## 2 The model

We consider a distribution channel in which two online retailers, denoted by  $i = 1, 2$ , sell horizontally differentiated products to the consumers.

**Channel structure and referrals.** Ex ante, some consumers may be unaware of either product, and may visit the website of only one retailer. This consumer unawareness creates an incentive for the retailers to advertise on the other retailer's website, and we refer to this phenomenon as the in-store referral. Accordingly, it brings up the incentive issues of whether a retailer should accept such a request. As the primary goal of our paper is to investigate the referral strategies, we consider two types of in-store referrals. In the first category, a retailer would display the other retailer's items directly on her own website. This scenario is labeled as the "retailer direct referral" (DR). In the second category, labeled as the "third-party referral" (TR), a retailer would display the referral link provided by a third-party advertising agency to the other retailer. In each cate-



gory, we further distinguish between two scenarios, depending on whether only one retailer would display DR or TR for the other retailer (labeled as one-way referral) or both retailers display DR or TR (two-way referral). In the case of one-way referral, without loss of generality, we assume that Retailer 1 provides the link for Retailer 2 on her own website. Furthermore, although for ease of exposition we only investigate the exclusive DR or TR scenarios, our analysis can be applied to the mixed model in which one retailer offers the retailer direct referral whereas the other retailer provides a third-party sponsored link in return. This mixed scenario is briefly discussed in Section 7. The variable cost of each retailer is normalized to zero, and for simplicity we assume that the retailers have unlimited production capacity.

**Consumer segments.** In our model, consumers are price sensitive, possess heterogeneous preferences/tastes, and ex ante may have different levels of awareness. To explicitly model the strategic effects of in-store referral, we classify the consumers into three groups,  $G_1$ ,  $G_2$ , and  $G_c$ , based on their ex ante awareness/knowledge of the retailers. The group  $G_i$  ( $i = 1, 2$ ) comprises consumers that are familiar with (aware of) Retailer  $i$  but are unaware of Retailer  $j$ . We label these groups of consumers the “*partially informed consumers*” of these two retailers, respectively. On the contrary,  $G_c$  comprises consumers who are aware of both retailers and thus are called “*fully informed consumers*,” or, in the terminology of Chen et al. (2002), the “comparison shopping segment.” Within each segment, consumers possess heterogeneous preferences, and we model this heterogeneity via the Hotelling model. Specifically, the heterogeneous preference of each consumer is represented by an ideal point, denoted by  $x$ , that lies on a line segment. We normalize the line segment to the unit interval,  $[0, 1]$ , and consumers reside uniformly on this interval. The two retailers are located at the endpoints of this line segment: 0 for Retailer 1 and 1 for Retailer 2. With some abuse of notation, we denote product  $i$  as the product provided by retailer  $i$ , where  $i = 1, 2$ .

**Consumer preferences.** Upon purchasing the product, a consumer obtains a (gross) valuation  $v$ , irrespective of the product identity ( $i$ ). This common valuation assumption is typically adopted in the Hotelling model. In reality, it is possible that the consumers differ not only in their ideal points but also in their inherent valuations. While our main results are qualitatively similar even with heterogeneous valuations, we omit the details to avoid distraction. In addition, since the ideal point  $x$  may differ from the retailers’ locations, the consumer incurs a “transportation cost,” denoted by  $t$ . This transportation cost captures the negative utility arising from the discrepancy between her ideal point and the product position. By our construction, if a consumer located at  $x$  purchases product 1, her net utility is  $v - tx - p_1$ , where  $p_1$  is the price charged by retailer 1; likewise, her utility is  $v - t(1 - x) - p_2$  if she purchases from retailer 2. From the above descriptions, we

have assumed that the retailers are horizontally differentiated. Thus, our analysis is not applicable to the case with homogeneous retailers that sell identical products. Furthermore, this horizontal differentiation setting also cannot accommodate the case in which products can be unambiguously ranked by all the consumers. In this case, a vertical differentiation setting with different product qualities is more appropriate. These alternative scenarios are discussed in Section 7.

To elaborate on the difference among the three consumer segments, let us first cast aside the effect of in-store referral. In the absence of in-store referrals, partially informed consumers are aware of only their respective retailers. We assume that  $G_i$  has a uniformly distributed population density of  $\alpha_i, i = 1, 2, c$ , where  $\alpha_c$  is normalized to 1 without loss of generality. In the base model, we let  $\alpha_i = 1, i = 1, 2$  to highlight the main findings; in Section 6, we relax this assumption. Given the prices  $p_i$  and  $p_j$  of Retailers  $i$  and  $j$ , consumers evaluate their payoffs upon purchasing from either retailer or staying empty-handed and determine their purchasing behaviors accordingly. In the segments  $G_1$  and  $G_2$ , a consumer's utility is given by  $u_1 = v - tx - p_1$  and  $u_2 = v - t(1 - x) - p_2$ . For  $G_c$ , the utility of a consumer located at  $y$  is given by

$$u^c = \begin{cases} v - tx - p_1, & \text{if purchasing from Retailer 1;} \\ v - t(1 - x) - p_2, & \text{if purchasing from Retailer 2.} \end{cases} \quad (1)$$

To exclude some trivial results of the retailers' referral strategies, we assume that  $G_1$  and  $G_2$  are never fully covered (i.e., partially covered) without referral, while  $G_c$  is always fully covered. Similar assumptions on the market coverage are adopted in various papers, e.g., Desai (2001) and Liu et al. (2006). If there is no full coverage for  $G_c$  consumers, our problem will degenerate to a "competition" between two local monopoly retailers. This essentially implies that the retailers can set the prices, assuming that the other retailer has no actual impact on her own consumers. Furthermore, if full coverage occurs for the partially informed consumers  $G_1$  and  $G_2$ , the purchasing decisions of these consumers become insensitive to prices, which is unreasonable in practice. The specific conditions required to validate this assumption will be given as we analyze the equilibrium behaviors of the consumers and retailers. It is also worth mentioning, however, that even with local monopolies, the referrals may still benefit the retailers, because now some partially informed consumers become fully informed and therefore may choose to purchase the (alternative) products rather than walking away. Thus, the referrals simply increase the effective demands of the retailers without adjusting the competition intensity. Moreover, to facilitate simple closed-form expressions and focus exclusively on the strategic interactions regarding the referrals, we assume that the transportation parameter  $t$  is  $1/2$  and normalize the valuation  $v$  to 1. Most of our results are qualitatively similar if we were to use a different combination of parameters.

**Retailers' referral strategies.** Now we introduce the retailers' referral strategies. If Retailer  $i$  decides to provide the in-store referral, (some of) the consumers in  $G_i$  will become informed of the other retailer; consequently, they join group  $G_c$  to become fully informed consumers. In our basic framework, we assume that every consumer in  $G_i$  becomes aware once the retailer provides the referrals. In Section 6, we extend our analysis to incorporate the possibility that only a portion of consumers turn to group  $G_c$ . Under the retailer's referral, the referred retailer commits to share a fixed proportion  $\rho$  of the revenue through those links with the referring retailer,  $i$ . This is made possible as nowadays cookies and traces of online transactions can be easily recorded.

In the case with third-party referral, the third party would keep  $1 - \gamma$  percent of the shared revenue (from the referred retailer) and the referring retailer collects  $\gamma$  portion of the revenue that results from these referrals. These proportions represent the relative bargaining powers of the referred retailers and the third party, and they are assumed to be independent of the referral strategies for ease of exposition. Since we specify the revenue sharing rule in the third-party referral, our setting is in essence the cost-per-action/ cost-per-conversion model. This is in line with the recent trend in the online advertising, as there have been some discussions on how cost-per-action model outperforms the contemporary cost-per-click model (Agarwal et al. (2009), Helft (2007), Zhu and Wilbur (2011), and the references therein).<sup>8</sup> In Section 7, we discuss an alternative cost-per-click model. It is worth mentioning that an alternative two-part tariff is adopted in both Chen et al. (2001) and Ghose et al. (2007). However, as the fixed transfers do not affect the retailers' pricing strategies, our results can be applied directly to this alternative setting in which the proportion  $\rho$  is interpreted as the commission rate of the two-part tariff. In addition to the different revenue sharing rules, we also assume that under third-party referral, a disutility  $\delta$  is applied to those consumers that purchase from the referred retailer, which captures the phenomenon that consumers shopping through third-party referral may find it inconvenient to complete the purchase (see Footnote 6).

Since the game involves multiple stages of strategic interactions, we adopt subgame perfect Nash equilibrium as our solution concept (Fudenberg and Tirole, 1991). In the following, we first present the benchmark case in which no referral is adopted. Following this, we then investigate the impacts of the direct and third-party referrals and compare our results across scenarios.

---

<sup>8</sup>See also "What is pay-per-action advertising?" at Google;  
<http://adwords.google.com/support/bin/answer.py?answer=61449&topic=11637>.

## 2.1 A benchmark case without referrals

When no referrals are adopted, the effective demands seen by the retailers are derived as follows. Suppose that the retailers have selected the selling prices  $p_1$  and  $p_2$ . Given that the partially informed consumers are partially covered, the marginal consumer who is indifferent between purchasing the product from Retailer  $i$  and staying empty-handed is determined by  $u_i = v - tx - p_i = 0$ ; that is,  $x = 2(1 - p_i)$  after substituting  $v$  by 1 and  $t$  by  $1/2$ . Accordingly, this also determines the “demand curve” that Retailer  $i$  sees from the partially informed consumers. Likewise, in the segment of fully informed consumers  $G_c$ , the marginal consumer who is indifferent between purchasing from either retailer is obtained by solving  $v - tx - p_1 = v - t(1 - x) - p_2$ , which yields  $x = \frac{1}{2} - p_1 + p_2$ . Thus, the effective demands to Retailers 1 and 2 are respectively

$$D_1^n = \frac{1}{2} + 2(1 - p_1) - p_1 + p_2, \text{ and } D_2^n = \frac{1}{2} + p_1 + 2(1 - p_2) - p_2,$$

where the superscript  $n$  stands for “no referral.” The corresponding retailers’ profits are then

$$\Pi_1^n = p_1 D_1^n, \text{ and } \Pi_2^n = p_2 D_2^n.$$

Straightforward algebra shows that in the unique equilibrium, the retailers will set prices  $p_1^n = p_2^n = 1/2$ , and the corresponding maximum profits for the retailers are  $\Pi_1^n = \Pi_2^n = 3/4$ . It can be verified ex post that given  $v = 1$  and  $t = 1/2$ , partially informed consumers are indeed partially covered whereas the fully informed consumers are fully covered.

## 3 Direct referrals

In this section, we focus on the case with retailer direct referrals (DR) and discuss two scenarios: one-way and two-way referrals.

### 3.1 One-way direct referral

In the one-way referral, Retailer 2 lists her item on Retailer 1’s website. Given the presence of direct referral, the partially informed consumers in  $G_1$  become informed about Retailer 2 and thus join  $G_c$  as fully informed consumers, whereas the consumers in  $G_2$  stay partially informed. Consequently, the (modified) effective demands for the retailers are given by

$$D_1^{d1} = 1 - 2p_1 + 2p_2, \text{ and } D_2^{d1} = 3 + 2p_1 - 4p_2,$$

where the superscript  $d1$  indicates the “one-way direct referral.” Note that these effective demands come directly from the characterization of marginal consumers in  $G_2$  and  $G_c$ . Because the procedure is exactly the same as that in the benchmark case without referrals, we omit the details for brevity. Given the effective demands, we can recall that Retailer 2 shares  $\rho$  portion of her revenue with Retailer 1 from selling through Retailer 1’s website and write down the retailers’ profits as follows:

$$\Pi_1^{d1} = p_1 D_1^{d1} + \rho p_2 \left(\frac{1}{2} - p_2 + p_1\right), \text{ and } \Pi_2^{d1} = p_2 D_2^{d1} - \rho p_2 \left(\frac{1}{2} - p_2 + p_1\right).$$

We then derive the equilibrium prices based on the above formulations, as stated in the following lemma.

**Lemma 1.** *In the one-way DR scenario, the equilibrium prices are*

$$p_1^{d1} = \frac{28 - \rho^2}{2(28 - 8\rho + \rho^2)} \text{ and } p_2^{d1} = \frac{14 - 3\rho}{28 - 8\rho + \rho^2}, \quad (2)$$

and the corresponding retailers’ profits are

$$\Pi_1^{d1} = \frac{392 - 42\rho^2 + 10\rho^3 - \rho^4}{(28 - 8\rho + \rho^2)^2} \text{ and } \Pi_2^{d1} = \frac{(14 - 3\rho)^2(4 - \rho)}{(28 - 8\rho + \rho^2)^2}.$$

Lemma 1 characterizes the equilibrium prices and the corresponding retailers’ profits in the one-way direct referral scenario. From (2), we observe that the selling prices get higher as the revenue sharing becomes more significant (i.e.,  $\rho$  is higher). This is because the revenue sharing scheme makes the retailers’ incentives more aligned, which mitigates the intense competition between retailers. We also observe that Retailer 1 (the referring retailer) benefits from this revenue sharing more as the portion  $\rho$  becomes higher, because she is entitled to keep a higher share of Retailer 2’s proceeds through referrals. On the other hand, Retailer 2 collects a lower profit overall if this revenue sharing portion is higher.

The above discussions indicate the economic consequence of this “collusive” revenue sharing. Based on these findings, we can now compare the equilibrium outcomes in the one-way direct referral scenario with the benchmark case without referrals. We find that

**Proposition 1.** *Compared with the non-referral case, in the one-way DR scenario, Retailer 2 is strictly better off while Retailer 1 is better off if and only if  $\rho$  is sufficiently large.*

Proposition 1 shows that the direct referral unambiguously benefits Retailer 2, because she is now able to serve more consumers (through the link put on Retailer 1’s website). On the other hand, Retailer 1 faces a trade-off. If she can retain a large proportion of the revenue generated

from these referrals ( $\rho$  is high), the direct referral increases her profit; otherwise, she forgoes too much demand to the competing retailer and would be better off to abandon it. On a related note, the prices set by both retailers are uniformly higher with referrals, which may yield a testable implication. Conventional wisdom may lead one to believe that the retailers are better off retaining their (local) monopoly power and should avoid, to the extent possible, the information disclosure that intensifies competition. However, our results complement this by offering a new rationale, and this collusion effect may even be mutually beneficial to both retailers even in the case of seemingly unfair one-way referral.

Proposition 1 also shows that switching to one-way direct referral may be mutually beneficial for both retailers. This strategic move is Pareto improving (and thus will be approved by both parties) when the revenue sharing is significant. On a related note, if the revenue sharing portion is endogenously determined through the negotiation between these retailers, the retailers should settle down with a sufficiently large sharing portion that reflects the referring retailer's incentive to comply and their relative bargaining powers. It is rather straightforward to add an additional stage in which the retailers determine the revenue sharing proportion through a Nash bargaining framework, and we omit the details for brevity.

### 3.2 Two-way direct referral

Let us now switch to the two-way direct referral scenario. In this case, both retailers list each other's item on their own websites; consequently, all the consumers in  $G_1$  and  $G_2$  become fully informed consumers. The corresponding demands for the retailers become

$$D_1^{d2} = 3 \left( \frac{1}{2} - p_1 + p_2 \right), \text{ and } D_2^{d2} = 3 \left( \frac{1}{2} + p_1 - p_2 \right).$$

As each retailer shares with the other retailer  $\rho$  portion of their additional revenues earning from the other retailer's website ( $\rho \leq 0.75$  is required for full coverage in  $G_c$ ), the retailers' profits are respectively

$$\begin{aligned} \Pi_1^{d2} &= p_1 D_1^{d2} + \rho p_2 \left( \frac{1}{2} - p_2 + p_1 \right) - \rho p_1 \left( \frac{1}{2} - p_1 + p_2 \right), \\ \Pi_2^{d2} &= p_2 D_2^{d2} - \rho p_2 \left( \frac{1}{2} - p_2 + p_1 \right) + \rho p_1 \left( \frac{1}{2} - p_1 + p_2 \right). \end{aligned} \quad (3)$$

The equilibrium outcomes are characterized in the following lemma.

**Lemma 2.** *In the two-way DR scenario, the equilibrium prices and retailers' profits are*

$$p_1^{d2} = p_2^{d2} = \frac{3 - \rho}{6 - 4\rho}, \text{ and } \Pi_1^{d2} = \Pi_2^{d2} = \frac{3(3 - \rho)}{12 - 8\rho}.$$

It can be verified that the prices and profits stated in the lemma are all increasing with the revenue sharing proportion  $\rho$ . This is because the higher the proportion is, the stronger the collusion effect would be. Thus, the incentive alignment allows the retailers to extract more revenue from consumers by forming an ally. We can also compare the two-way referrals with other scenarios as in the following proposition.

**Proposition 2.** *Both retailers prefer two-way DR to the case without referrals. Moreover, compared with one-way DR, Retailer 1 always prefers two-way DR, whereas Retailer 2 prefers two-way DR if and only if  $\rho > 0.49$ .*

Proposition 2 shows that referring to the opponent's website is mutually beneficial compared to the case without referrals. This is because it facilitates a better incentive alignment and the (tacit) collusion allows both retailers to raise the selling prices. As the additional revenue extracted from the consumers is fairly shared between the retailers, both retailers benefit. On the other hand, compared to the one-way direct referral, Retailer 1 is also able to expand her market through the link on Retailer 2's website. However, whether Retailer 2 benefits from referring the consumers depends on the revenue sharing. As anticipated, two economic forces are in tension, namely the collusion effect and the inevitable give-away regarding the market share. When the revenue sharing is not significant, the second effect dominates and Retailer 2 would be better off not referring consumers to Retailer 1.

## 4 Third-party referrals

In this section, we turn to the third-party referral (TR) scenario. We investigate two scenarios: one-way and two-way referrals and then compare the equilibrium outcomes in the end.

### 4.1 One-way third-party referral

In this scenario, suppose that Retailer 1 provides the sponsored link by the third party. Given this third-party referral, the fully informed consumers in the original segment  $G_c$  obtain their utilities as in (1). However, if an originally partially informed consumer for Retailer 1 observes the third-party sponsored link, he would incur an additional cost ( $\delta$ ) if purchasing from Retailer 2 instead. Thus,

his utility upon these two purchasing decisions becomes:

$$u^{t1} = \begin{cases} v - tx - p_1, & \text{if purchasing from Retailer 1;} \\ v - \delta - t(1 - x) - p_2, & \text{if purchasing from Retailer 2.} \end{cases}$$

Thus, he would purchase from Retailer 1 if and only if  $v - tx - p_1 \geq v - \delta - t(1 - x) - p_2$ . Define the marginal consumer as the one that is indifferent between purchasing from either retailer. Accordingly, the effective demands for the two retailers are

$$D_1^{t1} = 1 + \delta - 2p_1 + 2p_2, \text{ and } D_2^{t1} = 1 - \delta + 2p_1 - 2p_2 + 2(1 - p_2),$$

where the superscript  $t1$  indicates the ‘‘one-way third-party referral.’’ Accounting for the profit-sharing between the two retailers and the third party, the retailers’ profits can be written as follows:

$$\Pi_1^{t1} = p_1 D_1^{t1} + \gamma \rho p_2 \left( \frac{1}{2} - \delta - p_2 + p_1 \right), \text{ and } \Pi_2^{t1} = p_2 D_2^{t1} - \rho p_2 \left( \frac{1}{2} - \delta - p_2 + p_1 \right). \quad (4)$$

To ensure that the partially informed consumers are partially covered and the fully informed consumers are fully covered, we adopt the following assumption:

**Assumption 1.**

$$0 \leq \delta < \bar{\delta}_{t1} \equiv \min \left\{ \bar{\delta}_1, \frac{14 - 3(1 + 2\gamma)\rho + 2\gamma\rho^2}{32 - 3(1 + \gamma)\rho + 2\gamma\rho^2}, \frac{14 - (3 - 2\gamma)\rho}{20 - (3 + \gamma)\rho} \right\},$$

where

$$\bar{\delta}_1 = \begin{cases} \frac{\gamma(2-\rho)\rho}{4-6\rho}, & \text{if } \rho < 4/6; \\ \infty, & \text{otherwise.} \end{cases}$$

The third term of this assumption ensures that the number of fully informed consumers is nonnegative. Solving the optimal solution from (4), we compare the optimal profits with those in the non-referral case and then obtain the following result.

**Proposition 3.** *Compared with non-referral, Retailer 2 strictly prefers one-way TR. On the other hand, there exists a cutoff  $\delta^{t1}$  such that Retailer 1 prefers one-way TR to no referral if and only if  $\delta^{t1} < \delta < \bar{\delta}_{t1}$  (see Figure 1).*

Proposition 3 shows that the referred retailer always benefits from the referral, even if she has to share some profit with her opponent. This comes from two effects. First, the referral effectively aligns the retailers’ incentives and mitigates the competition, thereby allowing them to set selling prices to extract more from consumers. Second, the referred retailer faces additional demands



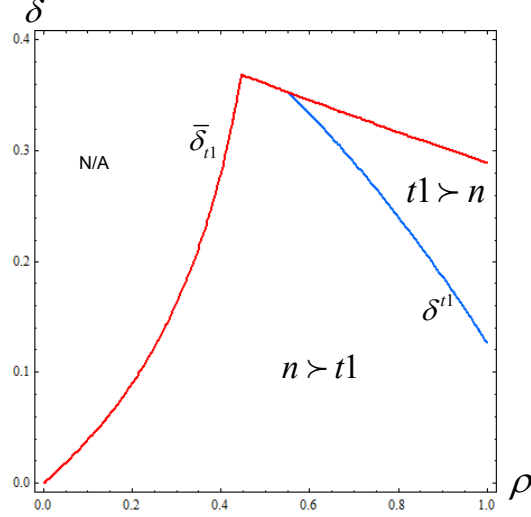


Figure 1: Retailer 1's preference between one-way TR and no referral scenarios ( $\gamma = 0.7$ ).

by way of the sponsored link. Even if the consumers incur a disutility and the third party takes away some revenue, this demand expansion still unambiguously benefits the referred retailer. On the contrary, whether the referring retailer (Retailer 1) benefits critically depends on the disutility incurred by the consumers, and this leads to an ambiguous consequence. On one hand, when the consumers suffer seriously from the disutility, Retailer 1 benefits from tacit collusion while retaining a larger number of consumers. On the other hand, when the disutility is negligible, too many consumers are directed to the opponent, thereby giving rise to a net loss for the referring retailer.

## 4.2 Two-way third-party referral

Now we switch to the two-way referral scenario. In this case, both retailers provide a link sponsored by the third party. The original fully informed consumers again obtain utilities through (1). However, an originally partially informed consumer now is aware of the other retailer and incurs a disutility if he purchases from this alternative retailer. Thus, his utility can be expressed as

$$u^{t2} = \begin{cases} v - \delta - tx - p_1, & \text{if purchasing from Retailer 1;} \\ v - t(1-x) - p_2, & \text{if purchasing from Retailer 2.} \end{cases}$$

We can again specify the marginal consumers and derive the effective demands as follows:

$$D_1^{t2} = \frac{3}{2} - 3p_1 + 3p_2, \text{ and } D_2^{t2} = \frac{3}{2} + 3p_1 - 3p_2.$$

Since Retailer  $i$  shares  $\rho$  percent of its revenue of selling through the third party with Retailer  $3-i$  (a percentage of  $\gamma$ ) and the third party  $(1-\gamma)$ , the retailers' profits become

$$\begin{aligned}\Pi_1^{t2} &= p_1 D_1^{t2} + \gamma \rho p_2 \left(\frac{1}{2} - \delta - p_2 + p_1\right) - \rho p_1 \left(\frac{1}{2} - \delta - p_1 + p_2\right), \\ \Pi_2^{t2} &= p_2 D_2^{t2} - \rho p_2 \left(\frac{1}{2} - \delta - p_2 + p_1\right) + \gamma \rho p_1 \left(\frac{1}{2} - \delta - p_1 + p_2\right).\end{aligned}\tag{5}$$

To ensure full coverage condition in the two-way referral scenario, we assume that

**Assumption 2.**

$$0 \leq \delta < \bar{\delta}_{t2} \equiv \min\left\{\frac{3-\rho-3\gamma\rho}{4\rho}, \frac{3-\rho-3\gamma\rho}{6+2(1-\gamma)\rho}\right\}.$$

We can derive the equilibrium outcomes and compare them with those in other aforementioned scenarios. The results are summarized in the next proposition.

**Proposition 4.** *Compared with the benchmark case without referrals, both retailers prefer two-way TR if and only if  $\gamma > \gamma^{t2} \equiv \frac{3-4\delta(3-\rho)-\rho-4\delta^2\rho}{6-\delta(6-4\rho)+\rho-4\delta^2\rho}$ .*

As anticipated, the two-way referral is mutually beneficial to both retailers if the revenue sharing is significant, as it allows the retailers to sustain a stronger notion of collusion. A subtle point here is that this collusion effect is more pronounced when the consumers incur a higher disutility through the sponsored links (as the cutoff  $\gamma^{t2}$  decreases in  $\delta$ ). To understand this result, recall that a higher disutility creates more intrinsic differentiation and alleviates the competition between the retailers; as a result, an increase of disutility actually facilitates a better alignment of retailers' incentives.

We can also compare the retailers' profits across different scenarios.

**Proposition 5.** *Compared with the one-way TR, Retailer 1 always prefers two-way TR to one-way TR; whereas there exists a threshold  $\delta^{t2}(\rho)$  such that Retailer 2 prefers two-way TR if and only if  $\delta^{t2} < \delta < \bar{\delta}_{t2}$  (see Figure 2).*

The reason that Retailer 1 strictly prefers two-way TR to one-way TR is similar to that of Proposition 2 in that Retailer 1 gains additional demand through the link from Retailer 2's website. Intuitively, Retailer 1's advantage improves as the revenue sharing rate from the third party (i.e.,  $\gamma$ ) increases. Moreover, a higher value of  $\delta$  or  $\rho$  also alleviates the competition and facilitates a higher level of collusion. For Retailer 2 to be better off, the benefit from a tighter collusion must outweigh the loss of giving away her monopoly position. This only occurs when the revenue

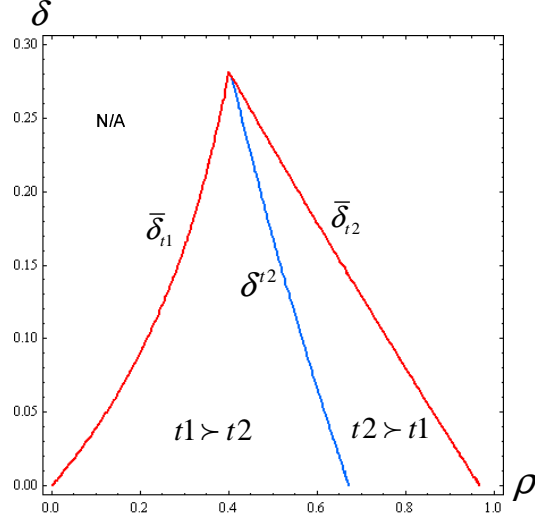


Figure 2: Retailer 2's preference between one-way and two-way TR scenarios ( $\gamma = 0.7$ ).

sharing is intense and the consumers incur a high disutility through referrals; see Figure 2 for a graphical illustration. It is also worth mentioning that Proposition 5 indicates that in the TR scenario, two-way TR can emerge as a market equilibrium when the consumers incur a sufficiently high disutility ( $\delta > \delta^{t2}$ ). This may explain why in the presence of third-party referrals (e.g., Google sponsored advertisement), more and more retailers are adopting third-party referrals on their websites concurrently despite their competitive positions.

## 5 Comparisons between direct referrals and third-party referrals

We can now articulate the comparison between direct referrals and third-party referrals. Let us start with the one-way referrals. Our findings are demonstrated via Figures 3 and 4 and summarized in Proposition 6.

**Proposition 6.** *Compared with the one-way TR, there exist two thresholds  $\delta_a^{td}$  and  $\delta_b^{td}$  such that*

- *Retailer 1 prefers one-way DR to one-way TR if and only if  $\delta < \delta_a^{td}$ , while Retailer 2 prefers one-way DR to one-way TR if and only if  $\delta < \delta_b^{td}$  in the feasible domain (i.e.,  $\delta < \bar{\delta}_{t1}$ ).*
- *Retailer 1 prefers one-way DR to one-way TR if the revenue sharing is more significant, whereas Retailer 2's preference is the opposite.*

Proposition 6 shows that both retailers prefer one-way TR to one-way DR if and only if the consumers incur a sufficiently high disutility  $\delta$  (provided that the full coverage assumption is maintained). This result is rather intuitive, as a higher disutility mitigates the competition between the retailers but the collusion effect through referrals remains strong under the third-party referral. A more interesting result is that while varying the revenue sharing between the retailers, the retailers' preferences over direct and third-party referrals are exactly the opposite (see for example when  $\delta = 0.2$  in Figures 3 and 4). To understand this result, recall that under TR, a portion of revenue ( $\gamma$ ) goes to the third party; consequently, the change of the revenue sharing proportion  $\rho$  has a less significant impact under TR than under DR. Moreover, in principle an increase in  $\rho$  gives rise to a higher payoff for the referring retailer (Retailer 1) but a lower payoff for the referred retailer (Retailer 2). Thus, as the revenue sharing becomes more significant ( $\rho$  is larger), the more direct benefit under DR makes Retailer 1 prefer DR to TR, whereas Retailer 2 suffers less under TR. This gives rise to a preference reversal between the retailers, as illustrated in Figures 3 and 4.

It is worth mentioning that the full-coverage equilibrium can be sustained only when  $\delta$  is relatively small. Specifically, in order to induce fully informed consumers to purchase the product via the referrals, the disutility cannot be too significant. This is because the large disutility may discourage the consumers from purchasing through the referral links. In such a scenario, the full-coverage equilibrium breaks down, and the retailers may lose some fully informed consumers when they increase the value of  $\delta$ . In addition, the collusion effect is also mitigated, because now the referrals become less effective.

Now we switch to the two-way referrals. Likewise, we observe that TR is mutually beneficial for both retailers (compared with DR) whenever the consumers incur a moderate disutility for the third-party sponsored referrals.

**Proposition 7.** *There exists a threshold  $\delta_c^{td}$  such that both retailers prefer two-way DR to two-way TR if and only if  $\delta < \delta_c^{td}$ .*

Similar to Proposition 6, two-way TR benefits from a higher collusion level rendered by a higher disutility. Intuitively, a higher revenue sharing from the third-party will improve two-way TR against two-way DR. Having identified the driving forces and their economic consequences for the retailer and third-party referrals, a natural question is whether these results are prone to the specific model characteristics adopted in our basic model. To address this issue, in the next section we extend our setup to evaluate the impacts of our model characteristics.

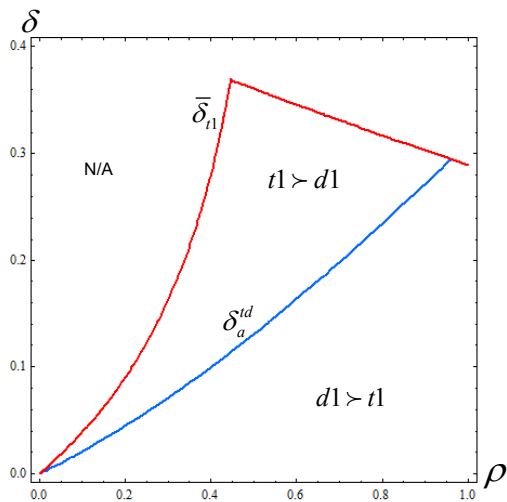


Figure 3: Retailer 1's preference between one-way DR and TR ( $\gamma = 0.7$ ).

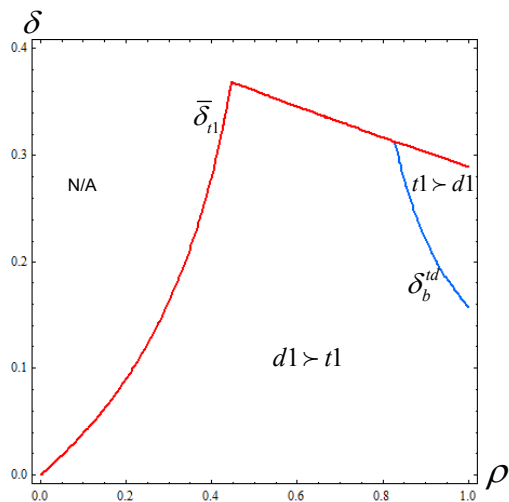


Figure 4: Retailer 2's preference between one-way DR and TR ( $\gamma = 0.7$ ).

## 6 Extensions

In this section, we investigate the robustness of our results and the impacts of key components of our model setting. To highlight the “sensitivities” of our results to these components, we shall focus on the scenarios in which referrals are preferable and thus are adopted by the retailers. As the analysis is analogous to that for the basic framework, we omit the tedious algebraic derivations and simply present the findings.

### 6.1 Consumer welfare

Insofar we focus on the retailers' perspectives and compare their profits under different scenarios. However, an important aspect of in-store referrals is how they affect the consumer welfare.<sup>9</sup> To this end, we in this subsection examine the welfare implications based on the equilibrium characterization of optimal prices. In the case of non-referral and direct referral (DR), the surplus of an individual fully informed consumer located at  $x$  is given by

$$CW_s(x) = \begin{cases} v - tx - p_1^*, & \text{if } x \leq \frac{1}{2} - p_1^* + p_2^*; \\ v - t(1-x) - p_2^*, & \text{if } \frac{1}{2} - p_1^* + p_2^* < x \leq 1. \end{cases}$$

<sup>9</sup>We thank an anonymous reviewer for this valuable suggestion.

In the case of third-party referral (TR), for an originally partially informed consumer for Retailer 1 located at  $x$  who now becomes informed of Retailer 2, his/her surplus is given by

$$CW_s^{t1}(x) = \begin{cases} v - tx - p_1^*, & \text{if } x \leq \frac{1}{2} + \delta - p_1^* + p_2^*; \\ v - \delta - t(1-x) - p_2^*, & \text{if } \frac{1}{2} + \delta - p_1^* + p_2^* < x \leq 1. \end{cases}$$

For a partially informed consumer for Retailer  $i$  located at  $y$  is given by

$$CW_i(x) = v - ty - p_i^*, \quad 0 \leq y \leq \frac{v - p_i^*}{t}.$$

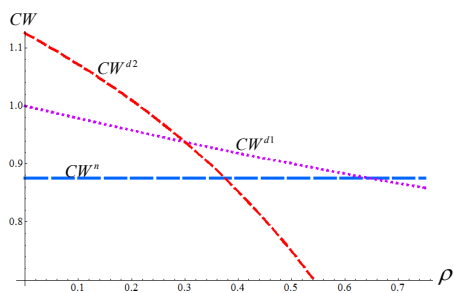


Figure 5: Consumer welfare comparison among non-referral, one-way DR, and two-way DR w.r.t.  $\rho$ .

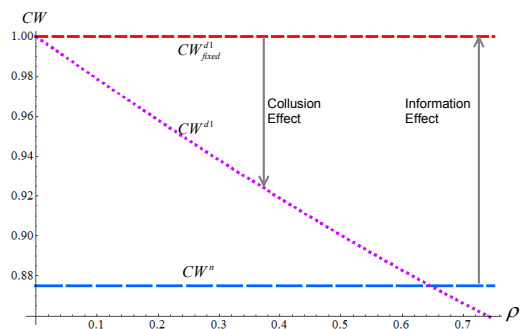


Figure 6: Impact of collusion and information on consumer welfare in one-way DR w.r.t.  $\rho$ .

Following the above logic, we compute the consumer welfare for all scenarios and include them in the appendix. We first compare consumer welfare in the non-referral, one-way referral, and two-way referral scenarios, as illustrated in Figure 5. Figure 5 shows that, when the revenue sharing rate ( $\rho$ ) is low, two-way DR has the highest consumer welfare, followed by one-way DR and non-referral in descending order. However, non-referral gains its momentum while two-way referral fades away as  $\rho$  grows. This is because, as  $\rho$  grows, the *collusion effect* between retailers (through the price increase) consequently becomes more significant, which subsequently erodes consumer welfare. As illustrated in Figure 6, originally partially informed consumers benefit from knowing the existence of the other retailer; this gives rise to the *information effect* as the information revelation creates potential gain for the consumers. However, a higher revenue sharing rate enables retailer forge stronger alliance and charge a higher price. Note that the information effect is more significant in the two-referral scenario, whereas the collusion effect becomes higher when there are more referred consumers. We also observe a similar phenomenon in Scenario TR for any given  $\gamma$  and  $\delta$ . Since social welfare includes both consumer welfare and retailers' profits, it is easy to infer that the entire

society benefits from more information while the collusion effect cancels out as the prices are net transfers between the consumers and retailers. In other words, for both DR and TR, two-way referral outperforms one-way referral, which subsequently outperforms the non-referral scenario from the social welfare perspective.

However, consumers might prefer one, either DR or TR, over the other depending on the values of  $\rho$ ,  $\gamma$ , and  $\delta$ . As shown in Figures 7 and 8, consumers prefer DR to TR if  $\delta$  is sufficiently large. This occurs because a larger  $\delta$  leads to a higher collusion effect in TR, regardless of either one-way referral or two-way referral. We also find that the dominant area of TR in two-way referral is larger than that in one-way referral. This is because the overall price disparity between TR and DR is higher in one-way referral when  $\delta$  is small but the price disparity becomes less significant as  $\delta$  grows. The dominant area of TR decreases as  $\gamma$  increases in both one-way and two-way referral.

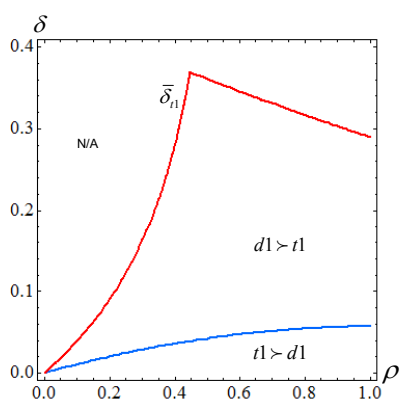


Figure 7: Consumer welfare comparison between one-way DR ( $d1$ ) and one-way TR ( $t1$ ) given  $\gamma = 0.7$ .

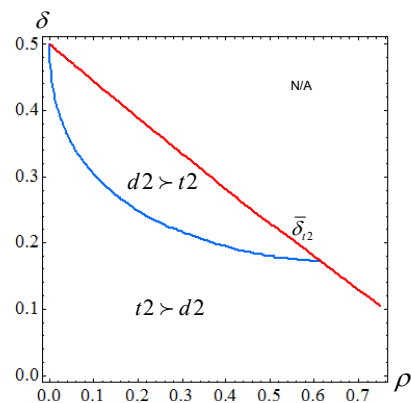


Figure 8: Consumer welfare comparison between two-way DR ( $d2$ ) and two-way TR ( $t2$ ) given  $\gamma = 0.7$ .

## 6.2 Market base

Let us now relax the assumption of symmetric consumer base and introduce asymmetric  $\{\alpha_i\}$ 's. From Figures 5 and 6, we observe that in the one-way referral scenario, Retailer 2's profit also increases as  $\alpha_1$  becomes larger because of the additional demand through the referral. While Retailer 1's profits in the two-way referral (DR or TR) scenarios increases in  $\alpha_2$  for a similar reason, her profit in one-way referrals decreases as  $\alpha_2$  grows. This is because when Retailer 2 is endowed with a larger loyal segment, she is relatively advantageous in competing against Retailer

1. Moreover, Retailer 1 always benefits while switching from one-way DR to two-way DR; on the other hand, Retailer 2 prefers two-way DR only when she has a relatively small initial market base. Note that when Retailer 2 is endowed with more partially informed consumers, she is less inclined to share with her competitor and thus prefers to stay with one-way DR. These observations are consistent with Propositions 2, 3, 4, and 5.

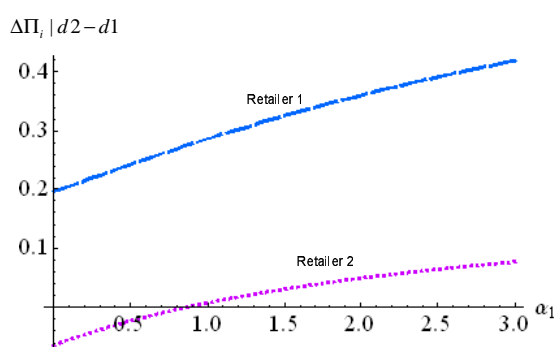


Figure 9: Sensitivity of retailers' profit difference in DR with respect to  $\alpha_1$  ( $\alpha_2 = 1$ ,  $\rho = 0.5$  and  $\beta = 1$ ).

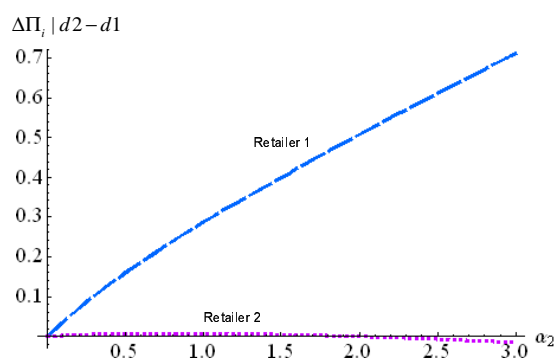


Figure 10: Sensitivity of retailers' profit difference in DR with respect to  $\alpha_2$  ( $\alpha_1 = 1$ ,  $\rho = 0.5$  and  $\beta = 1$ ).

We further find that the one-way DR is not a stable equilibrium. Even though switching from no referral scenario to one-way DR is a win-win strategy, both retailers may be better off switching from one-way DR to two-way DR. This is supported by the fact that after Amazon first introduced the in-store referral in 2006, it has been implemented in the top 20 online retailers within one year and is by now a common practice for the majority of online retailers. In light of our analysis, the one-way referral may be a transient market phenomenon as it leads to a win-win situation from no referrals, and it seems typical that the retailer who initiated the contact should offer to refer her competitor on her own website first.

According to Propositions 6 and 7, third-party referral could outplay direct referral for both players. Based on that, it is easy to understand that a bigger difference between the two initial base demands will amplify the difference between the third-party referral and direct referral. To showcase that, we demonstrate through a case where both retailers prefer third-party referral to direct referral when  $\alpha_i$  are asymmetric. As illustrated by Figure 11, there exists an area where both retailers obtain higher profits under two-way TR than two-way DR (when  $\alpha_1$  is close to  $\alpha_2$ ). This



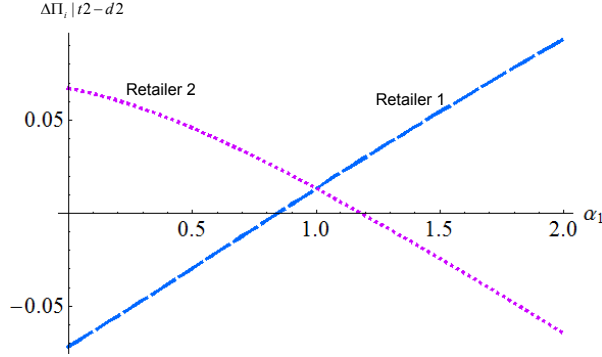


Figure 11: Sensitivity of retailers' profit difference of two-way referrals with respect to  $\alpha_1$  ( $\alpha_2 = 1$ ,  $\rho = 0.3$ ,  $\delta = 0.3$ ,  $\gamma = 0.7$ , and  $\beta = 1$ ).

is consistent with Proposition 7 and it suggests that third-party referral may be adopted more likely when the retailers are rather equally competitive. However, as  $\alpha_1$  grows, Retailer 2 becomes less optimistic about the third-party referral due to its resistance of a higher level of collusion between them, whereas Retailer 1 becomes more positive about third-party referral. As  $\rho$  becomes smaller, the win-win area of third-party referral for both retailers shrinks.

### 6.3 Switching rate

We now suppose that upon seeing the referral links, only a proportion  $\beta < 1$  of partially informed consumers will compare the prices of both retailers before purchase. Thus, our basic framework corresponds to the special case where  $\beta = 1$ . The results on the third-party referrals are qualitatively similar and thus are omitted to avoid redundancy. From Figure 12, we observe that Retailer 1 prefers two-way referral to no referral but prefers no referral to one-way (because  $\rho$  is not sufficiently large) when  $\beta = 1$ . As  $\beta$  decreases, both one-way and two-way direct referrals become less preferred at first, whereas when  $\beta$  is sufficiently small, Retailer 1's profit under the one-way referral increases as  $\beta$  continues to decrease (see Figure 12). In contrast, Retailer 2's profits in both one-way and two-way referrals consistently decrease as the partially informed consumers switch less often, because Retailer 2's encroachment into Retailer 1's market becomes less significant (see Figure 13).

We can also articulate the difference between the direct referral and third-party referral. Through extensive numerical experiments, we observe a mixed result regarding the comparative statics and the retailers' preferences. Instead of presenting various numerical examples, we in the following discuss some reasons for these ambiguous findings. Specifically, an increase of switching rate gives rise to several effects. First, as the consumers search and compare prices more often/likely

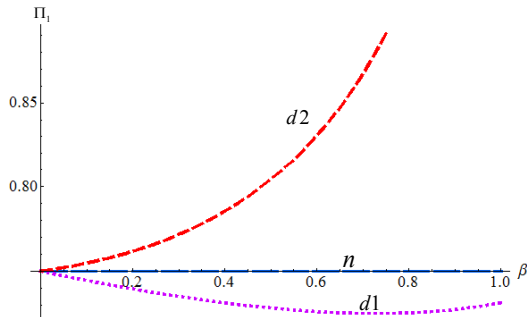


Figure 12: Sensitivity of Retailer 1' profits with respect to  $\beta$  ( $\alpha_1 = 1$ ,  $\alpha_2 = 1$ , and  $\rho = 0.75$ ).

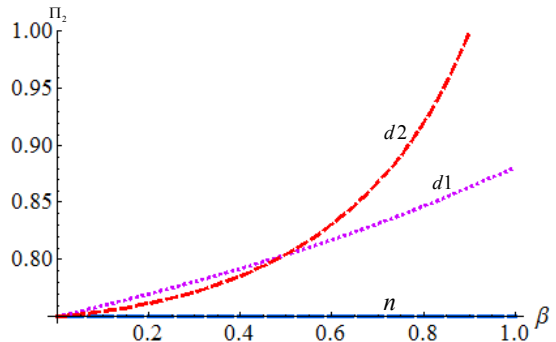


Figure 13: Sensitivity of Retailer 2' profits with respect to  $\beta$  ( $\alpha_1 = 1$ ,  $\alpha_2 = 1$ , and  $\rho = 0.75$ ).

( $\beta$  increases), the intrinsic differentiation between the retailers is less pronounced. On the other hand, upon offering the referrals to the consumers, the retailers' incentives are more aligned with more switching and revenue sharing; this induces the retailers to raise the prices and the collusion becomes tighter. Moreover, under the third-party referral more switching may lead to a higher portion of disutility, and the retailers ultimately have to compensate the switching consumers implicitly by lowering the prices. These three effects are somewhat conflicting and/or complementary, and drive all possible results in our numerical investigations.

## 7 Conclusions

In this paper, we investigate the strategic effects of in-store referrals. We find that upon switching from the non-referral scenario to the one-way direct referral, the referred retailer is always better off, whereas the referring retailer benefits only when she retains a sufficiently large proportion of the revenue generated from these referrals. Our results offer a new rationale for information disclosure, as it enhances the implicit collusion that may be mutually beneficial to both retailers even in the case of seemingly unfair one-way referral. Upon offering referral services, both retailers may raise the prices accordingly, thereby hurting the consumer welfare. We also find that both retailers prefer third-party referral to direct referral when the consumers incur a sufficiently high disutility. This is because a higher disutility mitigates the competition between the retailers but the collusion effect through referrals remains strong under the third-party referral. Furthermore, while varying the revenue sharing between the retailers, the retailers' preferences over one-way direct and third-party referrals are exactly the opposite. Consumer welfare crucially depends on the battle between the

collusion effect and information effect, and consumers may prefer either channel structures under different market conditions. Our qualitative results are not prone to the symmetry of market bases and the proportion of partially informed consumers that become fully informed consumers upon seeing the referrals.

Our model can be extended in a number of ways. For example, our analysis does not apply to the case with homogeneous retailers that sell identical products, as exemplified by Amazon.com which displays referrals to other retailers selling the same books often at lower prices. In this alternative setting, a critical technical challenge is the possibility of mixed-strategy equilibrium because each retailer has a strong incentive to undercut the opponent's price. On a related note, online retailers typically sell products that are vertically differentiated (i.e., all consumers can commonly rank the products by their qualities). This requires a different setting and a separate analysis.

As another extension, it would be intriguing to consider the alternative cost-per-click model in the third-party referrals. Although this requires a separate analysis, we believe that qualitatively our results should hold for the following reasons. In the third-party referral, the important feature is that some portion of the benefit through the referrals will be taken away by the third party. Thus, although this channel facilitates the tacit collusion between the retailers, it inevitably creates some revenue leakage to the outside third party. This is the primary trade-off in using the third-party referral. Viewed in this way, it seems that this result is not prone to how the third party and the referring retailer splits the pie, as long as the leakage is present.

In addition, it is possible to allow the retailers to adopt different referral strategies (e.g., Retailer 1 provides the referral link directly to Retailer 2, but Retailer 2 refers the consumers to Retailer 1 through the third-party sponsored link). We have conducted analytical as well as numerical investigations of this mixed model, and find that it shares many similar insights with our results. Specifically, as the revenue sharing is more significant, the collusion effect is stronger; on the other hand, when the disutility is higher, the retailers compete less intensively. As these qualitative findings largely coincide our results from the separate settings, we omit the details to avoid redundancy. As another possible extension, one could also consider the competition among multiple retailers. In such a scenario, it would be intriguing to see how referrals between a pair or among a group of retailers affect the strategies of other retailers that are not involved in the referrals. Moreover, it would be intriguing to introduce the active roles of manufacturers and see how the retailer referrals affect their competitive strategies. The industry equilibrium regarding the referrals is non-trivial, and it remains a research priority.

Finally, there are certainly other benefits and concerns for offering the referrals. As a notable example, Jeff Bezos at Amazon has stated that “[o]ur strategy is to become an electronic commerce destination. When somebody thinks about buying something online, even if it is something we do not carry, we want them to come to us. We would like to make it easier for people online to find and discover the things they might want to buy online, even if we are not the ones selling them.” This implies that web sites that place links on other sites want to attract consumers who will buy other products and come back in the future. Additionally, some leading online retailers (such as Amazon) may have the power to set the market price due to their brand names and market scales. In such a scenario, other retailers that sell through the leading retailers’ websites can only undercut the prices in order to attract consumers. Coupled with the contemporary one-shipment practice, these reputation and reliability concerns are particularly important for understanding the consumers’ online shopping behaviors. While these issues are not captured in our model, it would be very interesting to disentangle the relative impacts of these various motivations on the referral phenomenon.

## Appendix

**Proof of Lemma 1.** To characterize the equilibrium, we obtain the following first-order conditions as follows:

$$\frac{\partial \Pi_1^{d1}}{\partial p_1} = 1 - 4p_1 + (2 + \rho)p_2, \text{ and } \frac{\partial \Pi_2^{d1}}{\partial p_2} = 3 - \frac{\rho}{2} + (2 - \rho)p_1 - 2(4 - \rho)p_2.$$

Combining the above two equations and solving for the price pair, we obtain the optimal solution given in the lemma. Note that in the above derivation, we have assumed that the partially informed consumers are partially covered and the fully informed consumers are fully covered. To this end, we have to verify that no retailer can benefit from unilaterally deviating from the equilibrium. As the above first-order conditions show the joint concavity of the profit functions, it suffices to check that each retailer has no incentive to raise the price high enough so that the fully informed consumers in  $G_c$  are not fully covered. There are two possibilities, depending on whether the deviating retailer is Retailer 1 or 2.

Let us first focus on Retailer 1’s incentive. If she increases the price  $p_1$  sufficiently high to make the fully informed consumers partially covered, her effective demand becomes

$$D_1^{d1} = 2 \frac{v - p_1}{t} = 4 - 4p_1,$$

which arise from the two partially covered groups  $G_1$  and  $G_c$ . Note that this becomes independent of Retailer 2's price. Accordingly, Retailer 1's profit becomes

$$\Pi_1^{d1} = p_1 D_1^{d1} + \rho p_2 (v - p_2^{d1}) = \frac{\rho(14 - 3\rho)(14 - 5\rho + \rho^2)}{(28 - 8\rho + \rho^2)^2} + 4(1 - p_1)p_1.$$

Applying the first-order condition, we obtain that  $\partial\Pi_1/\partial p_1 = 4 - 8p_1$ , and the second-order condition is  $\partial^2\Pi_1^{d1}/\partial p_1^2 = -8 < 0$ . Thus,  $\Pi_1^{d1}$  is convex in  $p_1$  and reaches its maximum at  $p_1 = 1/2$ . This price is smaller than  $p_1^{d1}$  for any  $\rho \in [0, 1]$ ; thus, it is infeasible. The maximum profit Retailer 1 can attain by such a deviation is through setting the boundary price  $\frac{42 - 13\rho + 2\rho^2}{28 - 8\rho + \rho^2}$ , which renders a profit of

$$\frac{-2352 + 1764\rho - 652\rho^2 + 121\rho^3 - 11\rho^4}{(28 - 8\rho + \rho^2)^2}.$$

It can then be verified that the above profit is always lower than  $\Pi_1^{d1}$ . Thus, there is no profitable deviation for Retailer 1. Using a similar approach, we can also verify that for Retailer 2,  $\partial\Pi_2/\partial p_2 = (6 - \rho)(1 - 2p_2)$ , and  $\partial^2\Pi_2^{d1}/\partial p_2^2 = -2(6 - \rho) < 0$ . If Retailer 2 would deviate by adopting the boundary price  $\frac{2(14 - 6\rho + \rho^2)}{28 - 8\rho + \rho^2}$ , her new profit is

$$\frac{2\rho(336 - 284\rho + 98\rho^2 - 16\rho^3 + \rho^4)}{(28 - 8\rho + \rho^2)^2},$$

which is always lower than  $\Pi_2^{d1}$ . Therefore, no retailer would deviate from the above equilibrium unilaterally.  $\square$

**Proof of Proposition 1.** Comparing the retailers' profits, we obtain that

$$\begin{aligned}\Pi_1^{d1} - \Pi_1^n &= \frac{392 - 42\rho^2 + 10\rho^3 - \rho^4}{(28 - 8\rho + \rho^2)^2} - \frac{3}{4}, \\ \Pi_2^{d1} - \Pi_2^n &= \frac{(14 - 3\rho)^2(4 - \rho)}{(28 - 8\rho + \rho^2)^2} - \frac{3}{4} > 0,\end{aligned}$$

given  $\rho \in [0, 1]$ . It can then be verified after straightforward algebra that  $\Pi_1^{d1} - \Pi_1^n$  strictly increases in  $\rho$  and has a single crossing point at  $\rho = 0.807$ . Thus, Retailer 1 is better off if and only if  $\rho$  is sufficiently large (i.e.,  $\rho > 0.807$ ).<sup>10</sup> The price differences are as follows:

$$p_1^{d1} - p_1^n = \frac{(4 - \rho)\rho}{28 - 8\rho + \rho^2}, \quad \text{and} \quad p_2^{d1} - p_2^n = \frac{(2 - \rho)\rho}{2(28 - 8\rho + \rho^2)},$$

---

<sup>10</sup>Note that this value does not represent the actual revenue sharing rate used in practice. It should be understood as a qualitative cutoff level above which the profit sharing from the referrals is significant. Naturally, in practice the revenue sharing depends on many other factors, and the retailers need not be symmetric. These issues will all affect the cutoff level, but it is our hope that this cutoff structure sheds some light to the in-store referrals phenomenon.

which are apparently positive.  $\square$

**Proof of Lemma 2.** From the first-order conditions:

$$\frac{\partial \Pi_1^{d2}}{\partial p_1} = \frac{1}{2}(3 - \rho - 4(3 - \rho)p_1 + 6p_2) = 0, \text{ and } \frac{\partial \Pi_2^{d2}}{\partial p_2} = 3p_1 + \frac{1}{2}(3 - \rho)(1 - 4p_2) = 0,$$

we can obtain the results directly as stated in the lemma. It can then be verified that no profitable deviation is possible. As this requires only tedious algebraic operations and is similar to the proof of Lemma 1, we omit the details.  $\square$

**Proof of Proposition 2.** Comparing the retailers' profits, we obtain the following:

$$\Pi_1^{d2} - \Pi_1^n = \Pi_2^{d2} - \Pi_2^n = \frac{3\rho}{12 - 8\rho} \geq 0,$$

given  $\rho \in [0, 0.75]$ . Note that  $\rho \leq 0.75$  is required for full coverage in  $G_c$  under two-way direct referral. Thus, for both retailers, two-way direct referral outperforms the case without referrals. Now we compare the retailers' profits under one-way and two-way direct referrals. We find that

$$\begin{aligned} \Pi_1^{d2} - \Pi_1^{d1} &= \frac{3(3 - \rho)}{12 - 8\rho} - \frac{392 - 42\rho^2 + 10\rho^3 - \rho^4}{(28 - 8\rho + \rho^2)^2}, \\ \Pi_2^{d2} - \Pi_2^{d1} &= \frac{3(3 - \rho)}{12 - 8\rho} - \frac{(14 - 3\rho)^2(4 - \rho)}{(28 - 8\rho + \rho^2)^2}. \end{aligned}$$

Given  $\rho \in [0, 0.75]$ , we can show that  $\Pi_1^{d2} - \Pi_1^{d1} \geq 0$ ; furthermore, we find that  $\Pi_2^{d2} - \Pi_2^{d1} \geq 0$  if and only if  $\rho > 0.49$ .  $\square$

**Proof of Proposition 3.** Applying first-order conditions:

$$\frac{\partial \Pi_1^{t1}}{\partial p_1} = 1 + \delta - 4p_1 + (2 + \gamma\rho)p_2, \text{ and } \frac{\partial \Pi_2^{t1}}{\partial p_2} = 3 - \delta(1 - \rho) - \frac{\rho}{2} + (2 - \rho)p_1 - 2(4 - \rho)p_2,$$

we obtain the following equilibrium prices:

$$\begin{aligned} p_1^{t1} &= \frac{28 - 6(1 - \gamma)\rho - \gamma\rho^2 + 2\delta(6 - \gamma(1 - \rho)\rho)}{2(28 - 2(3 + \gamma)\rho + \gamma\rho^2)}, \\ p_2^{t1} &= \frac{14 - 3\rho - \delta(2 - 3\rho)}{28 - 2(3 + \gamma)\rho + \gamma\rho^2}. \end{aligned}$$

The corresponding retailers' profits are given by

$$\Pi_1^{t1} = \frac{T_1 + T_2}{2(28 - 2(3 + \gamma)\rho + \gamma\rho^2)^2}, \text{ and } \Pi_2^{t1} = \frac{(4 - \rho)(14 - 3\rho + \delta(-2 + 3\rho))^2}{(28 - 2(3 + \gamma)\rho + \gamma\rho^2)^2}.$$

where

$$\begin{aligned} T_1 &= 784 + 336(-1 + \gamma)\rho + 4(9 - 32\gamma + 2\gamma^2)\rho^2 \\ &\quad - 2\delta^2(-72 + \gamma\rho(-28 + 60\rho - 9\rho^2) + \gamma^2\rho^2(2 - 4\rho + \rho^2)), \\ T_2 &= 4\gamma(3 + 2\gamma)\rho^3 - 2\gamma^2\rho^4 \\ &\quad + \delta(672 - 24(6 + 29\gamma)\rho + 4\gamma(88 + 9\gamma)\rho^2 - 2\gamma(21 + 10\gamma)\rho^3 + 5\gamma^2\rho^4). \end{aligned} \tag{6}$$

It can then be verified that under Assumption 1, no profitable deviation is possible. As this requires only tedious algebraic operations, we omit the details for brevity.

We now compare the retailers' profits under one-way third-party referral with those without referral. For Retailer 2, we obtain that

$$\Pi_2^{t1} - \Pi_2^n = \frac{(4 - \rho)(14 - 3\rho - \delta(2 - 3\rho))^2}{(28 - 2(3 + \gamma)\rho + \gamma\rho^2)^2} - \frac{3}{4}, \quad (7)$$

Let us now divide our analysis into cases. First, if  $2 - 3\rho = 0$ ,  $\Pi_1^{t1} - \Pi_1^n$  is independent of  $\delta$ . Furthermore, after straightforward algebra, we can show that  $\frac{(4-2/3)(14-3 \times 2/3)^2}{(28-2(3+\gamma) \times 2/3 + \gamma \times (2/3)^2)^2} > \frac{3}{4}$ , i.e.,  $\Pi_2^{t1} - \Pi_2^n$  is strictly positive. If  $2 - 3\rho \neq 0$ , as the right-hand side of (7) is a second-order function of  $\delta$ , there are two roots  $\delta$  for  $\Pi_2^{t1} - \Pi_2^n = 0$  as follows:

$$\begin{aligned} \delta_1 &= \frac{224 + \sqrt{3}\sqrt{(2 - 3\rho)^2(4 - \rho)(28 - (6 + \gamma(2 - \rho))\rho)^2} - 2\rho(220 - 84\rho + 9\rho^2)}{2(2 - 3\rho)^2(4 - \rho)}, \\ \delta_2 &= \frac{224 - \sqrt{3}\sqrt{(2 - 3\rho)^2(4 - \rho)(28 - (6 + \gamma(2 - \rho))\rho)^2} + 2\rho(220 - 84\rho + 9\rho^2)}{2(2 - 3\rho)^2(4 - \rho)}, \end{aligned}$$

and  $\Pi_2^{t1} - \Pi_2^n > 0$  if and only if  $\delta < \delta_1$  or  $\delta > \delta_2$  (it is verifiable that  $\delta_1 < \delta_2$ ). Note that the effective demand of Retailer 2 from the originally partially informed consumers in  $G_1$  is

$$\frac{1}{2} - \delta - \frac{14 - 3\rho + \delta(-2 + 3\rho)}{28 - 2(3 + \gamma)\rho + \gamma\rho^2} + \frac{28 + 6(-1 + \gamma)\rho - \gamma\rho^2 + 2\delta(6 + \gamma(-1 + \rho)\rho)}{2(28 - 2(3 + \gamma)\rho + \gamma\rho^2)},$$

which must exceed zero; as a result, the following condition must hold:  $\delta < \frac{14 - (3 - 2\gamma)\rho}{20 - (3 + \gamma)\rho} \equiv \delta_3$  where  $\delta_3$  is the second term of  $\bar{\delta}_{t1}$  as defined in Assumption 1. Since  $\frac{14 - (3 - 2\gamma)\rho}{20 - (3 + \gamma)\rho}$  is less than  $\delta_1$ , we conclude that  $\Pi_2^{t1} - \Pi_2^n > 0$  for any  $\rho \in [0, 1]$  and  $\gamma \in [0, 1]$ .

Next, we compare the profits for Retailer 1. The profit difference in the two scenarios is

$$\Pi_1^{t1} - \Pi_1^n = \frac{T_1 + T_2}{2(28 - 2(3 + \gamma)\rho + \gamma\rho^2)^2} - \frac{3}{4},$$

where  $T_1$  and  $T_2$  are given in (6). The right-hand side of the above equation is of order two with respect to  $\delta$ . Thus, the two roots of  $\delta$  for  $\Pi_1^{t1} - \Pi_1^n = 0$  are as follows:

$$\begin{aligned} \delta_4 &= -\frac{T_3 + \sqrt{T_4}}{4(72 + \gamma\rho(28 - 60\rho + 9\rho^2) - \gamma^2\rho^2(2 - 4\rho + \rho^2))}, \\ \delta^{t1} &= -\frac{T_3 - \sqrt{T_4}}{4(72 + \gamma\rho(28 - 60\rho + 9\rho^2) - \gamma^2\rho^2(2 - 4\rho + \rho^2))}. \end{aligned}$$

where

$$\begin{aligned} T_3 &= 672 - 24(6 + 29\gamma)\rho + 4\gamma(88 + 9\gamma)\rho^2 - 2\gamma(21 + 10\gamma)\rho^3 + 5\gamma^2\rho^4, \\ T_4 &= (28 - 2(3 + \gamma)\rho + \gamma\rho^2)^2 [864 + \gamma^2\rho^2(332 + 12\rho - 3\rho^2) - 4\gamma\rho(332 - 24\rho - 9\rho^2)]. \end{aligned}$$

To make  $\Pi_1^{t1} - \Pi_1^n > 0$ , we must have  $\delta < \delta_4$  or  $\delta > \delta^{t1}$ . We can algebraically verify that  $\delta_4 < 0$ , which rules out the possibility of  $\delta < \delta_4$ . Assumption 1 requires that  $0 < \delta_1^{t1} < \bar{\delta}_{t1}$  given  $\rho \in [0, 1]$ ,  $\delta \in [0, 1]$ , and  $\gamma \in [0, 1]$ . Thus, we conclude that Retailer 1 prefers one-way third-party referral to no referral if and only if  $\delta_5 < \delta < \bar{\delta}_{t1}$ . In the proposition we simply relabel these thresholds.  $\square$

**Proof of Proposition 4.** Applying the first-order conditions:

$$\begin{aligned}\frac{\partial \Pi_1^{t2}}{\partial p_1} &= \frac{3}{2} - \frac{\rho}{2} + \delta\rho - 2(3 - \rho)p_1 + (3 - (1 - \gamma)\rho)p_2, \\ \frac{\partial \Pi_2^{t2}}{\partial p_2} &= \frac{3}{2} - \frac{\rho}{2} + \delta\rho + (3 - (1 - \gamma)\rho)p_1 - 2(3 - \rho)p_2,\end{aligned}$$

we can obtain the optimal prices as follows:

$$p_1^{t2} = p_2^{t2} = \frac{3 - \rho(1 - 2\delta)}{2[3 - \rho(1 + \gamma)]},$$

and the optimal profits are given by

$$\Pi_1^{t2} = \Pi_2^{t2} = \frac{[3 - (1 - 2\delta)\rho][3 - (1 - \gamma)(1 - 2\delta)\rho]}{4[3 - \rho(1 + \gamma)]}.$$

It is verifiable that no profitable deviation is possible. We omit the details for brevity.

We can then compare the retailers' profits in the two-way third-party referral with those without referrals. We obtain that

$$\Pi_1^{t2} - \Pi_1^n = \frac{1}{4} \left[ \frac{(3 - (1 - 2\delta)\rho)(3 - (1 - \gamma)(1 - 2\delta)\rho)}{3 - \rho(1 + \gamma)} - 3 \right].$$

Using the contour plot, we can graphically show that  $\Pi_1^{t2} - \Pi_1^n$  increases in  $\gamma \in [0, 1]$  for any  $\rho \in [0, 1]$  and  $\delta \in [0, 1]$ . Solving  $\Pi_1^{t2} - \Pi_1^n = 0$  yields the unique boundary value as specified in the proposition.  $\square$

**Proof of Proposition 5.** For Retailer 1, we obtain the profit difference as:

$$\Pi_1^{t2} - \Pi_1^{t1} = \frac{(3 - (1 - 2\delta)\rho)(3 - (1 - \gamma)(1 - 2\delta)\rho)}{4(3 - \rho(1 + \gamma))} - \frac{T_1 + T_2}{2(28 - 2(3 + \gamma)\rho + \gamma\rho^2)^2},$$

where the values of  $T_1$  and  $T_2$  are given in (6). We can then obtain the two roots of  $\delta$  for  $\Pi_1^{t2} - \Pi_1^{t1} = 0$  and verify that both roots are outside the feasible area as defined by Assumptions 1 and 2. Since  $\Pi_1^{t2} - \Pi_1^{t1}$  is convex in  $\delta$ , Retailer 1 always prefers two-way TR to one-way TR. Likewise, the profit difference for Retailer 2 is

$$\Pi_2^{t2} - \Pi_2^{t1} = \frac{(3 - (1 - 2\delta)\rho)(3 - (1 - \gamma)(1 - 2\delta)\rho)}{4(3 - \rho(1 + \gamma))} - \frac{(4 - \rho)(14 - 3\rho - \delta(2 - 3\rho))^2}{(28 - 2(3 + \gamma)\rho + \gamma\rho^2)^2},$$



and straightforward algebra characterizes the two roots for  $\Pi_1^{t2} - \Pi_1^{t1} = 0$ . Since the larger root could be less than  $\min\{\hat{\delta}_{t1}, \hat{\delta}_{t1}\}$  such that Retailer 2 prefers two-way TR if and only if  $\delta > \delta^{t2}$ . Although closed-form expressions are available, we omit them here as the specific values are not directly useful for our results.  $\square$

**Proofs of Propositions 6 and 7.** These two propositions follow directly from comparing the profit differences across scenarios, and we omit the detailed derivations for conciseness.  $\square$

**Derivations of consumer welfare in Section 6.1.** Following the equations of consumer welfare provided in Section 6.1, we integrate each individual consumer welfare along the Hotelling lines and then obtain the overall consumer welfare in non-referral, one-way DR, and two-way DR, respectively, as follows:

$$\begin{aligned} CW^n &= \frac{7}{8}; \\ CW^{d1} &= \frac{1568 - 1232\rho + 468\rho^2 - 84\rho^3 + 7\rho^4}{2(28 - 8\rho + \rho^2)^2}; \\ CW^{d2} &= \frac{27 - 30\rho}{24 - 16\rho}. \end{aligned}$$

To single out the collusion effect and information effect, we fix the retail prices at the level of non-referral and recompute the consumer welfare in one-way and two-way DR, respectively,

$$CW_{fixed}^{d1} = 1, \text{ and } CW_{fixed}^{d2} = \frac{9}{8}.$$

The collusion effect is computed as  $CW^{d1} - CW_{fixed}^{d1}$  in  $d1$ , and the information effect is computed as  $CW_{fixed}^{d1} - CW^n$ ; likewise, we can compute them for other scenarios. The collusion effect is typically negative whereas the information effect is positive. We then can uniquely depict the consumer welfare comparison with respect to  $\rho$ , since  $\rho$  is the sole parameter. The comparison results are shown in Figures 5 and 6 in Section 6.1.

We now consider TR. Similarly, we obtain

$$\begin{aligned} CW^{t1} &= \frac{8(196 - \delta(112 - 59\delta)) - 8(84 + 3\delta(8\delta - 1) + \gamma(70 + \delta(7\delta - 12)))\rho}{2(28 - (6 + \gamma(2 - \rho))\rho)^2} \\ &\quad + 2(\gamma^2(26 - (4 - \delta)\delta) + 18(2 + \delta + \delta^2) + 2\gamma(86 - \delta(25 - 7\delta)))\rho^2 \\ &\quad - 2\gamma(\gamma(18 - (8 - \delta)\delta) + 3(8 - (1 - \delta)\delta))\rho^3 + \gamma^2(7 - (4 - \delta)\delta)\rho^4; \\ CW^{t2} &= \frac{99 - 72(1 - \delta)\delta - 6(11 + 19\gamma)\rho - 48\delta(1 - \gamma(1 - \delta) + \delta)\rho}{8(3 - \rho - \gamma\rho)^2} \\ &\quad + (11 + \gamma(38 + 29\gamma) + 24\delta + 8(3 - \gamma)\gamma\delta + 8(2 + \gamma(2 + \gamma))\delta^2)\rho^2. \end{aligned}$$

We fix the retail prices at the level of non-referral and recompute the consumer welfare in one-way and two-way TR, respectively:

$$CW_{fixed}^{t1} = \frac{1}{8} (11 - 4\delta + 4\delta^2), \text{ and } CW_{fixed}^{t2} = \frac{7}{4} - \frac{3\delta}{2} + \delta^2.$$

We can compare  $t1$  and  $t2$  and obtain similar results like in Figures 5 and 6. We omit listing them here for brevity. Given these, we can then compare TR and DR in both one-way and two-way scenarios and obtain Figures 7 and 8 as shown in in Section 6.1.  $\square$

## References

- Agarwal, N., S. Athey, and D. Yang (2009). Skewed bidding in pay-per-action auctions for online advertising. *American Economic Review* 99(2), 441–47.
- Biyalogorsky, E., E. Gerstner, and B. Libai (2001). Customer referral management: Optimal reward programs. *Marketing Science* 20(1), 82–95.
- Brown, J. (1981). A cross-channel comparison of supplier-retailer relations. *Journal of Retailing* 57(4), 3–18.
- Brown, J. and P. Reingen (1987). Social ties and word-of-mouth referral behavior. *Journal of Consumer Research* 14(3), 350–362.
- Brown, J. R., R. F. Lusch, and D. D. Muehling (1983). Conflict and power-dependence relations in retailer-supplier channels. *Journal of Retailing* 59(4), 53–80.
- Cai, G. (2010). Channel selection and coordination in dual-channel supply chains. *Journal of Retailing* 86(1), 22–36.
- Chen, Y., G. Iyer, and V. Padmanabhan (2002). Referral infomediaries. *Marketing Science* 21, 412–434.
- Chen, Y., C. Narasimhan, and Z. Zhang (2001). Individual marketing with imperfect targetability. *Marketing Science* 20, 23–41.
- Chiang, W. K., D. Chhajed, , and J. D. Hess (2003). Direct marketing, indirect profits: A strategic analysis of dual-channel supply chain design. *Management Science* 49(1), 1–20.
- Choi, S. (1991). Price competition in a channel structure with a common retailer. *Marketing Science* 10, 271–296.

- Desai, P. (2001). Quality segmentation in spatial markets: When does cannibalization affect product line design? *Marketing Science* 20(3), 265–283.
- Fudenberg, D. and J. Tirole (1991). *Game Theory*. Cambridge, Massachusetts: The MIT Press.
- Ghose, A., T. Mukhopadhyay, and U. Rajan (2007). The Impact of Internet Referral Services on a Supply Chain. *Information Systems Research* 18(3), 300–319.
- Hall, J., P. Kopalle, and A. Krishna (2010). Retailer dynamic pricing and ordering decisions: category management versus brand-by-brand approaches. *Journal of Retailing* 86(2), 172–183.
- Helft, M. (2007). Google Tests an Ad Idea: Pay Only for Results. *New York Times* 21, 21 March 2007.
- Hess, J., E. Gerstner, and D. Holthausen (1994). Price discrimination through a distribution channel: Theory and evidence. *American Economic Review* 84(5), 1437–1445.
- Ingene, C. and M. Parry (1995). Coordination and manufacturer profit maximization: The multiple retailer channel. *Journal of Retailing* 71 (Summer), 129–151.
- Iyer, G. and A. Pazgal (2003). Internet shopping agents: Virtual co-location and competition. *Marketing Science* 22(1), 85–106.
- Iyer, G., D. Soberman, and J. Villas-Boas (2005). The targeting of advertising. *Marketing Science* 24, 461–476.
- Jackson, M. (2008). *Social and economic networks*. Princeton University Press.
- Kopalle, P., D. Biswas, P. Chintagunta, J. Fan, K. Pauwels, B. Ratchford, and J. Sills (2009). Retailer pricing and competitive effects. *Journal of Retailing* 85(1), 56–70.
- Levy, M., D. Grewal, P. Kopalle, and J. Hess (2004). Emerging trends in retail pricing practice: implications for research. *Journal of Retailing* 80(3), xiii–xxi.
- Libai, B., E. Biyalogorsky, and E. Gerstner (2003). Setting referral fees in affiliate marketing. *Journal of Service Research* 5(4), 303–315.
- Liu, Y., S. Gupta, and Z. Zhang (2006). Note on self-restraint as an online entry-deterrence strategy. *Management Science* 52(11), 1799–1809.
- Lusch, R. F. (1976). Channel conflict - its impact on retailer operating performance. *Journal of Retailing* 52(2), 3–12.

- Padmanabhan, V. and I. Png (1997). Manufacturer's returns policies and retail competition. *Marketing Science* 16(1), 81–94.
- Ryu, G. and L. Feick (2007). A penny for your thoughts: Referral reward programs and referral likelihood. *Journal of Marketing* 71(1), 84–94.
- Tsay, A. (2002). Risk sensitivity in distribution channel partnerships: implications for manufacturer return policies. *Journal of Retailing* 78(2), 147–160.
- Viswanathan, S., J. Kuruzovich, S. Gosain, and R. Agarwal (2007). Online infomediaries and price discrimination: Evidence from the automotive retailing sector. *Journal of Marketing* 71(3), 89–107.
- Xue, L., G. Ray, and A. Whinston (2004). The Impact of an Internet Shopping Infomediary on Channel Competition with Multiple Brands. *ICIS 2004 Proceedings*, 967–978.
- Yuan, H. and A. Krishna (2008). Pricing of mall services in the presence of sales leakage. *Journal of Retailing* 84, 95–117.
- Zhu, Y. and K. Wilbur (2011). Hybrid Advertising Auctions. *Marketing Science* 30(2), 249–273.