PROBABILISTIC MODELS PERTAINING TO MODERN CONVEYOR THEORY

by

MORRIS HENRY SCHNEIDER

B. S., University of Nebraska, 1959

A THESIS

submitted in partial fulfillment of the

requirements for the degree

MASTER OF SCIENCE

Department of Industrial Engineering

KANSAS STATE UNIVERSITY
MANHATTAN, KANSAS

1961

TABLE OF CONTENTS

INTRODUCTION

A conveyor is defined as a device for moving material from one point to another at the same or a different elevation, with continuous or intermittent forward movement and continuous drive. There are many different types and classes, such as: belt, elevating, carrier chain and cable, haulage, roller, hook, screw, pipeline, and special conveyors and auxiliary equipment which includes, feeders, screens, hoppers, chutes, troughs, spouts, and shakers.

Conveyors are widely used in industry today. Sometimes they are used for delivering items such as materials, parts, or finished goods from one area to another. Sometimes they are also used for storing these items. Perhaps more frequently than either of these two purposes individually, they are used for the combined purpose of both delivery and storage.

In modern Industrial Engineering and Management, the use of mathematical analysis has become very important. Applications of mathematics are being made in a wide variety of decision-making and design areas which were formerly handled, for the most part, by empirical methods. This newer, more quantitative approach has been marked by a great deal of interest in more adequate theoretical descriptions of physical facilities and activities.

The area of movement of materials within a system has been neglected, in comparison with most of the related activities, until very recently. While much attention has been paid to the problems which deal with materials, materials procurement, materials use, and materials transformation, not much has been done with regard to theoretical treatment of the movement of these materials within a system. In particular, the development of a conceptual model, or mathematical description, of conveyors, has been slow in appearing.

It is the purpose of this thesis to consider the question of a theoretical model for conveyors and to develop a workable method for applying the theoretical considerations to practical problems in industrial conveyor design.

Statement of The Problem

As American industry becomes more automated, conveyors are finding their place in many industries. The company that desires to put conveyors to use is finding that it must make decisions about a conveyor system on some basis other than the rule of thumb. For years, there has been no real effort to face the many problems that a conveyor system poses. The rule of thumb method commonly used in the past, was to make the system as large as management thought they needed or could afford, without adequate knowledge of the actual results that would be produced.

In this thesis an attempt was made to arrive at a method for determining a specific size for the conveyor, depending upon the circumstances that apply to a particular system. The method used to attack the problem was based on the development of a mathematical model that would apply to general conveyor systems.

Review of Literature

In 1958 Kwo[1] presented a paper to demonstrate that simple loop conveyors can be operated smoothly without any difficulty provided only a few simple principles, applicable to practically all types of conveyors, are observed. The fundamental message was that a conveyor is part of a system. Therefore, it should not be considered as an isolated object independent of the areas which it is linking together. In his presentation, a simulation method for the analysis of conveyor operations was presented, and expressions for the design of a conveyor were developed and explained. The fundamental principles upon which the theory was developed are as follows:

1. The Speed Rule: The speed of the conveyor, in terms of the number of carriers per unit time, must be within the permissible range.

---

[1] T. T. Kwo, "A Theory of Conveyors." Management Science, Vol. 5, No. 1, October 1958.

2. The Capacity Constraint: The conveyor must have enough capacity, in terms of the number of racks, to accommodate the accumulated items, the intentional reserve stock, and the temporary requirements at the loading and unloading points due to the fact that these two points are geographically separated.
3. The Uniformity Principle: The conveyor must be loaded as well as unloaded uniformly throughout its entire length.

Kwo's method of handling these data requires a lengthy and detailed simulation for each and every case. In essence, the simulation involves the construction of a table which indicates the distribution of items on the conveyor from one instant to the next. It is simple to use but often involves a tremendous amount of computation. The simulation need not necessarily be made for every instant, but might be established for other time intervals such as every minute.

In 1960 Mayer[1] used a theoretical approach with a mathematical model. The paper was limited as follows: The theory was developed for a conveyor which carries discrete units of production away from independent, multiple loading points; the theory disregards economics and concerns itself only with the functioning of the physical system.

---

[1] Hugo E. Mayer, "An Introduction to Conveyor Theory." The Western Electric Engineer, January 1960.

Since chance plays a major role in determining how the conveyor is loaded, Mayer developed a model which describes the system based upon probability theory.

Given the same constant cycle time at each work station, the number of hooks, h, which pass a work station during a work cycle will remain constant. Then the probability, q, that an attempt to load a hook is not made is 1-p, where p represents probability of success. As a hook passes the work stations, it experiences a sequence of "Bernoulli" trials. Repeated independent trials are called "Bernoulli" trials if there are only two possible outcomes for each trial and their probabilities remain constant. Under these conditions, the following theorem applies:

Let $b(k; n,p)$ be the probability that n "Bernoulli" trials with probabilities p for success and $q = 1-p$ for failure result in k successes and $n-k$ failures $(0 \leq k \leq n)$ then $b(k; n,p) = \binom{n}{k} p^k q^{n-k}$ where

$b(k; n,p)$ = the probability of k successes in n repeated independent trials.

k = the number of successes. A success occurs each time a worker attempts to place a product on the hook. k assumes all integer values from 0 to n.

n = the number of work stations through which the hook passes.

p = the probability (of success) that an attempt to load a hook occurs at a work station.

q = the probability (of failure) that no attempt is made to load a hook at a work station. Because it is certain that an attempt at loading either is or is not made, we have $p + q = 1$.

$\binom{n}{k}$ = the number of ways that k successes may be obtained in n trials. The numerical value is given by

$$\frac{n!}{k!(n-k)!}$$

This equation is the mathematical model which describes how work is offered to the hooks.

This model provides a method of obtaining the probability of successfully hooking on (or of not being successful) but requires that the mathematics of each individual case be carried out.

NATURE OF THE PROBLEM

The object of this thesis is to provide for any user of conveyors a working tool that will help him design a conveyor using probability theory. It was decided to develop the working tools for conveyors up to and including 100 work stations because it was the belief that this would be large enough to fulfill the needs of the medium and small industries. The industries with more work stations can be grouped into a class where a separate study is needed. From this point on, the number of work stations will be referred to as n. The binomial distribution as used by Mayer was used as the mathematical model, because sample calculations were completed using the model and found to be satisfactory. Tables or graphs are to be made using this model for a variety of k's for each n and p values from 0.1 to 0.9 for each k. The basic equation would remain the same, that is

$$b(k;\ n,p) = \binom{n}{k} p^k (1-p)^{n-k} \text{ where}$$

k = the number wanting to load the conveyor out of n.

n = the number of work stations or the maximum possible who could be interested in loading the conveyor.

p = the probability that any given station will want to load any given hook.

b = the probability that actually k out of n will want to load the hook under consideration.

A program for the IBM 650 was developed so the many thousand calculations could be made more rapidly. Values of n from 5 to 100 by fives were selected and for each n, values of k from 2 to n by twos were used. For each k value, the p value from 0.1 to 0.9 by 0.1's was used. These values were selected because, since the final information was placed in graphs, it did not seem necessary that a value for every n and k would be needed. The values of 0.0 and 1.0 for p were not used because of the mathematics involved in the basic equation. It was soon evident that the numbers were becoming so small that the 50 digit capacity of the 650 was exceeded, so the basic equation was broken down into calculable parts. The factorial term was handled separately from the balance of the equation. All of the values for the separate parts of the equation are on file in the Department of Industrial Engineering, Kansas State University.

During the attempt to put this material in usable form, it was discovered that the cumulative values found in standard Poisson and Normal Distribution tables were checked and found to be accurate enough for all practical purposes. Rather than taking the previous answer and adding a number each time, standard tables were used in plotting the graphs.

It was decided to plot, for each n, values of R vs P for representative values of k. R is defined as the "risk" that k or more stations will want to hook onto the conveyor at any given time. For $np > 5$ the tables for the normal approximation to the binomial were used. Such a table is like the one found in Dixon and Massey[1]. For values of $np < 5$

---

[1] Wilfrid J. Dixon and Frank J. Massey Jr., Introduction to Statistical Analysis. New York, McGraw Hill Book Co. Inc., 1957, p 382.

Molina's[1] Poisson Table was used. According to Duncan[2], the use of the tables as just described is acceptable practice from a statistical viewpoint.

CONVEYOR THEORY AS A PRACTICAL TOOL

The matter of placing the data into a form where it would be practical for users proved to be somewhat of a problem. It was evident from the start that the material would have to be placed either in chart or graph form. Some consideration was given as to how these data might be placed on a device similar to a slide rule. An examination of the curves shows that they bear enough resemblance to each other, especially if all normal values are used, that this might be a distinct possibility.

After considerable thought it was decided, for the purpose of this thesis, that graphs would be the most practical. This may not be the only way the material could be presented, but it appears to be a useful approach.

The plotting of R vs P for each n value makes it possible for the user to have all the information about any n value on a single page.

---

[1] E. C. Molina, Poisson's Exponential Binomial Limit. New York, D. VanNostrand Company, Inc. 1942.

[2] Acheson J. Duncan, Quality Control and Industrial Statistics. Homewood, Illinois, Richard D. Irwin, Inc. 1959, p 88.

The upper and lower portion of the curves have been eliminated from the working graphs for practicality. The curves were terminated at 0.9 because it seems doubtful that anyone would not be willing to take a risk of at least 10 percent for some value of k. Similarly, the region from 0.0 to 0.05 is not one which would commonly be used in comparison with the remainder of the curves. It might be noted, however, that this latter region is one which is capable of development in great detail; a separate study aimed at development and presentation of this set of values should yield some interesting and potentially useful results. Included in the appendix is one set of detailed curves showing what happens in the areas above $R = 0.9$ and below $R = 0.05$.

Before applying the charts to a specific problem, a relationship between P and a conveyor system has to be developed. Any industrial firm desiring to use conveyors must know enough about their own process to determine the intervals between attempts to load the conveyor at each work station. In many processes, these intervals will be constant, or so nearly constant that they may be treated as such. In others, there will be a considerable variation from time to time and/or from station to station. In those instances where this variation occurs in an important location, it may be necessary to adjust the various design factors to take this into account. However, in most industrial installations of the type which will primarily be interested in utilizing a single conveyor, it is practical to use a single value to represent these intervals. This value, called m in this thesis, may be taken as either a mean value or as (in the case of some very important variation as noted above) a limiting value. So long as a user is consistent in his choices, it does not matter whether means or limiting values are selected. Limiting values may be required because of such things as job difficulty, fragile or easily spoiled materials, complex timing of feeder lines, or many other factors which are functions of the situation.

The other element which it is necessary to identify is the time between hooks, denoted here as t. The time can be set by the number of hooks on a hook type conveyor or by the speed or by a combination of these.

Considering any station,

$$\frac{\text{the number of hooks}}{\text{interval, each station}}$$

$$= t/m$$

$$= \frac{\text{time/hook}}{\text{time/job}}$$

$$= \text{job/hook}$$

$$= P$$

This yields the relationship between the probability value P and the conveyor system. Applying this to an example for further clarification, let us consider a conveyor system where there are 50 work stations using the Risk value of 0.7. What would be the P value when 36 stations are attempting to load the conveyor? On the left hand ordinate of Plate No. X go to the R value of 0.7 and then across to the $k = 36$ line and drop down to the P coordinate. This gives a reading of $P = 0.75$ which is the value used to design the conveyor. Using this P value along with the appropriate m value, as determined by the conveyor user, the conveyor may be designed to meet the t set down by the assumptions and the mathematical relationship.

Thus, a user must pre-determine m, select a value of R for a specific k, and find P from the chart appropriate for the number of stations (n) present. Knowing P, he may then design the number of hooks and the speed of motion to obtain a t which fits the value $t = mP$.

EXPLANATION OF PLATES I THROUGH XX

Plates I through XX are the working tools developed to aid users of conveyor systems in their design.

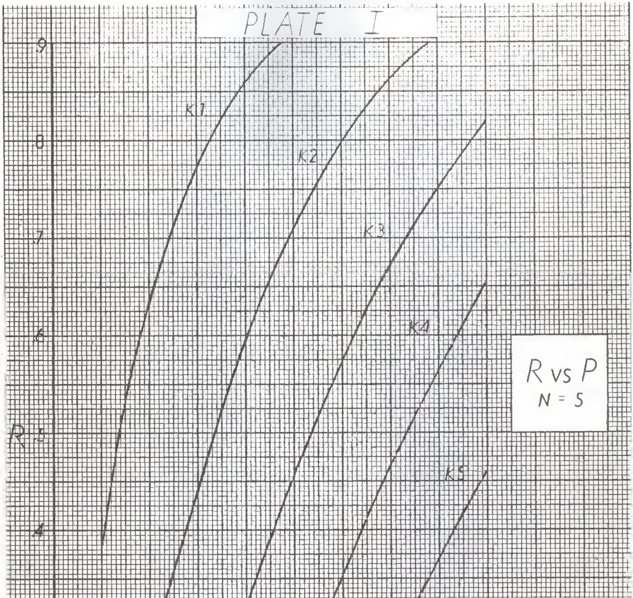


PLATE I
R vs P
N = 5
K1
K2
K3
K4
K5
R
P
.9
.8
.7
.6
.5
.4
.3
.2
.1
0
.1
.2
.3
.4
.5
.6
.7
.8
.9

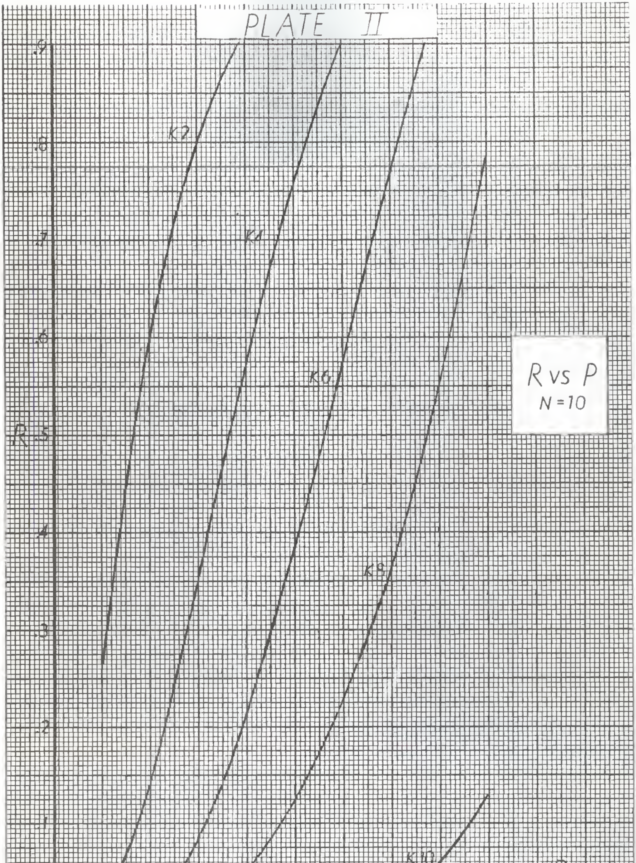


PLATE II
R vs P
N = 10
K2
K4
K6
K9
K10
R
P
.1
.2
.3
.4
.5
.6
.7
.8
.9
0
1
2
3
4
5
6
7
8
9

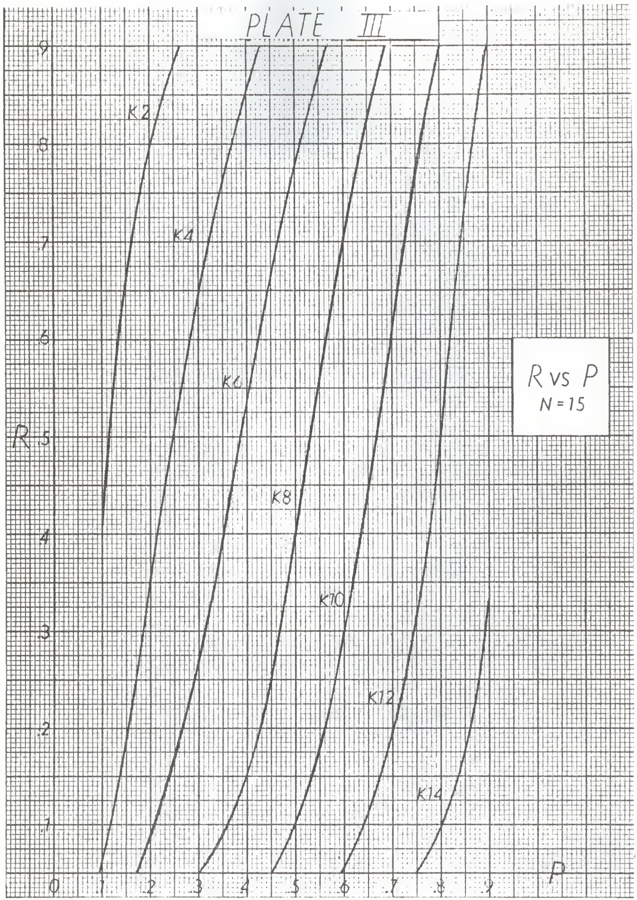


PLATE III
R vs P
N = 15
R
P
K2
K4
K6
K8
K10
K12
K14
0
.1
.2
.3
.4
.5
.6
.7
.8
.9

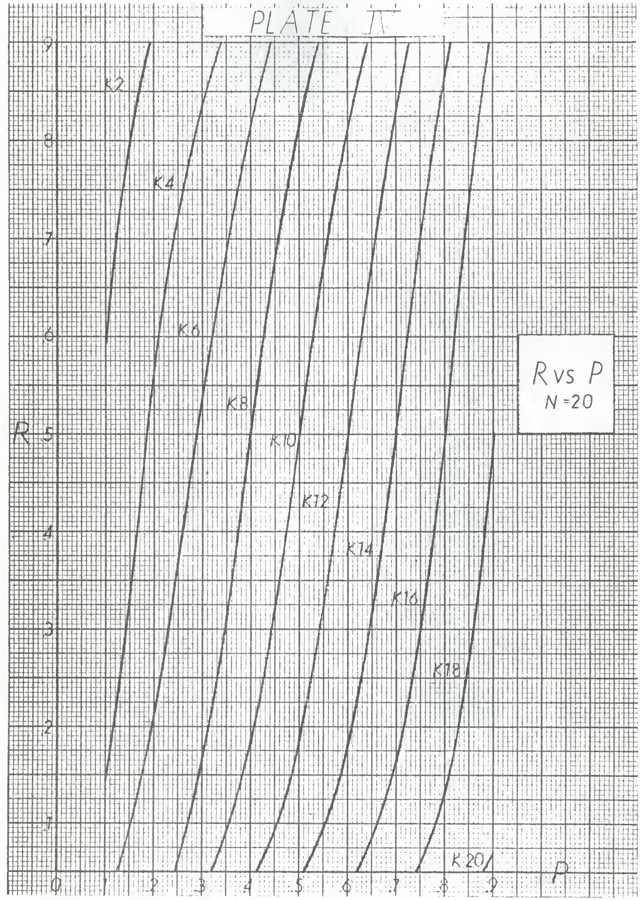


PLATE IV
R vs P
N =20
R
P
K2
K4
K6
K8
K10
K12
K14
K16
K18
K20
0
.1
.2
.3
.4
.5
.6
.7
.8
.9

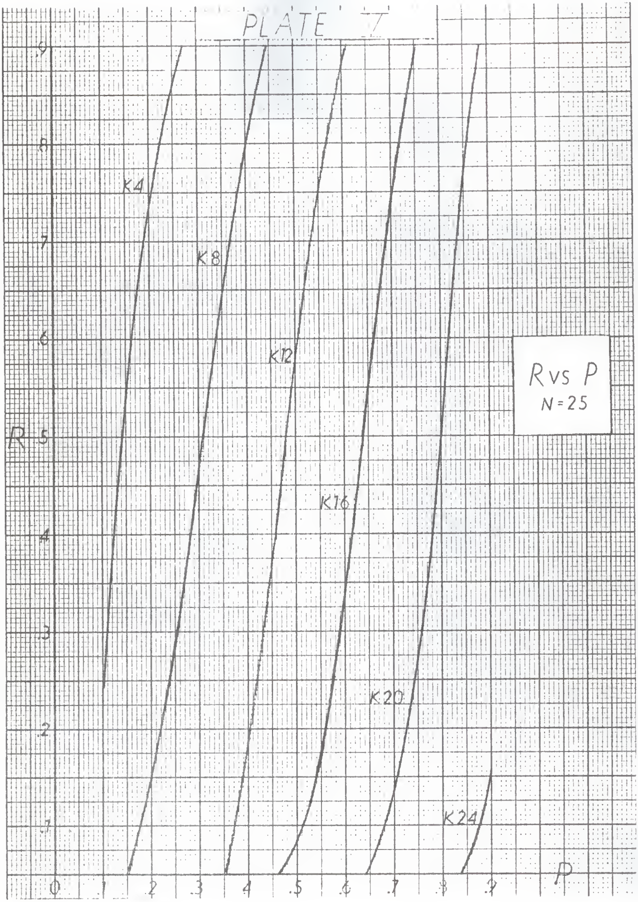


PLATE
R vs P
N=25
K4
K8
K12
K16
K20
K24
R
P
0
.1
.2
.3
.4
.5
.6
.7
.8
.9

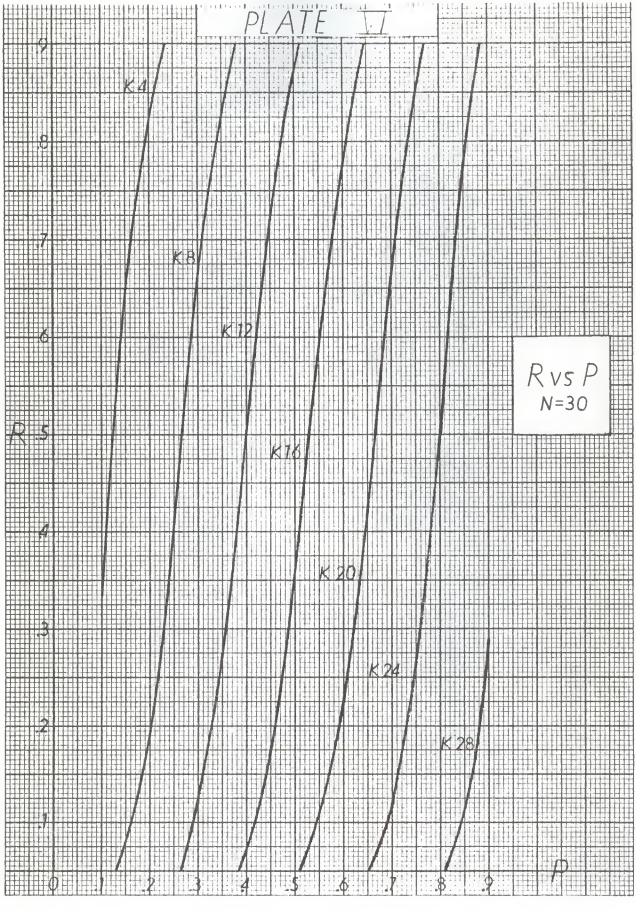


PLATE VI
R vs P
N=30
K4
K8
K12
K16
K20
K24
K28
R
P
0
.1
.2
.3
.4
.5
.6
.7
.8
.9

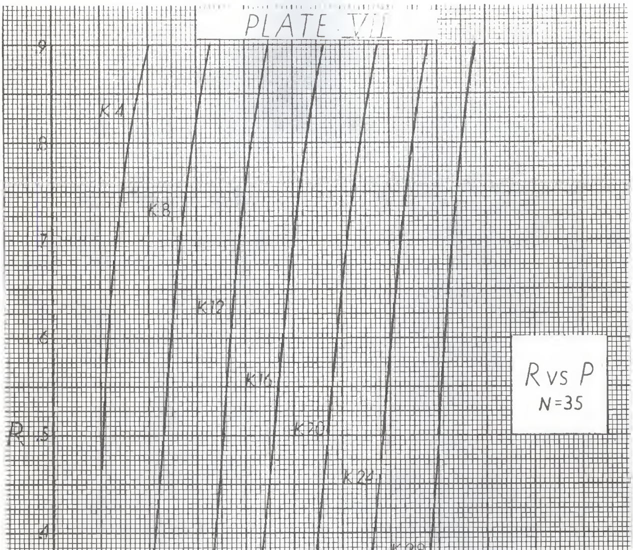
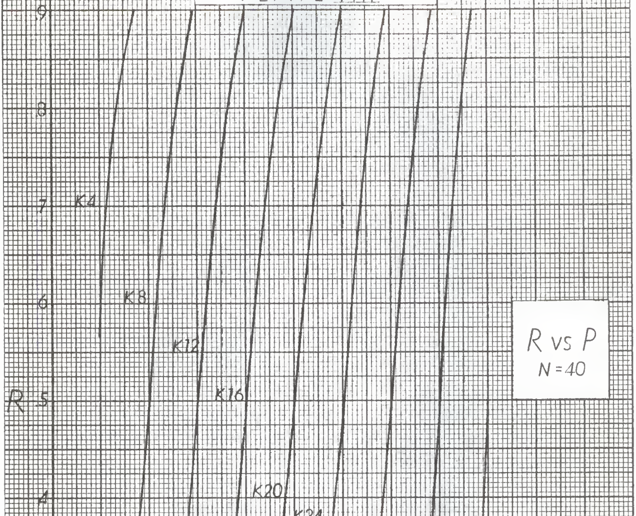


PLATE VII
R vs P
N=35
K4
K8
K12
K16
K20
K24
K28
K32
R
P
0
.1
.2
.3
.4
.5
.6
.7
.8
.9

PLATE VIII



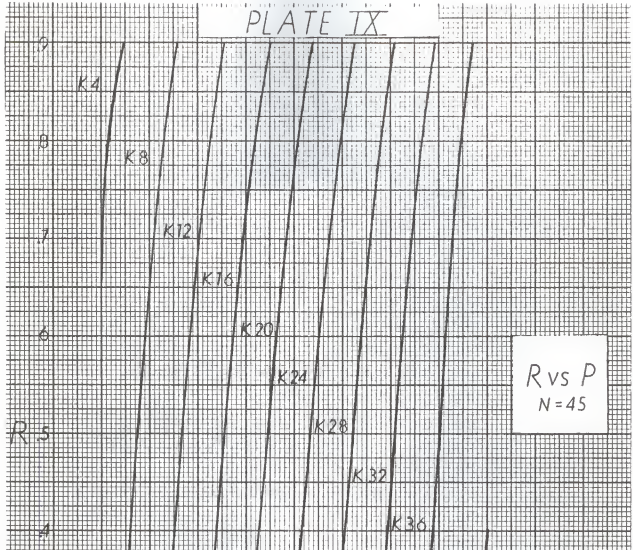


PLATE IX
R vs P
N = 45
K4
K8
K12
K16
K20
K24
K28
K32
K36
K40
R
P
0
.1
.2
.3
.4
.5
.6
.7
.8
.9

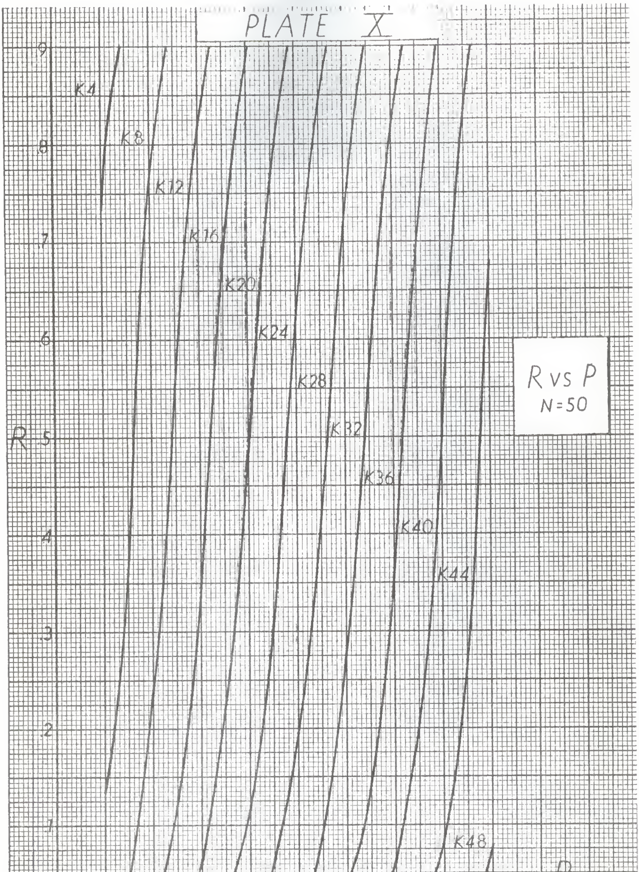


PLATE X
R vs P
N=50
K4
K8
K12
K16
K20
K24
K28
K32
K36
K40
K44
K48
R
P
0
1
2
3
4
5
6
7
8
9

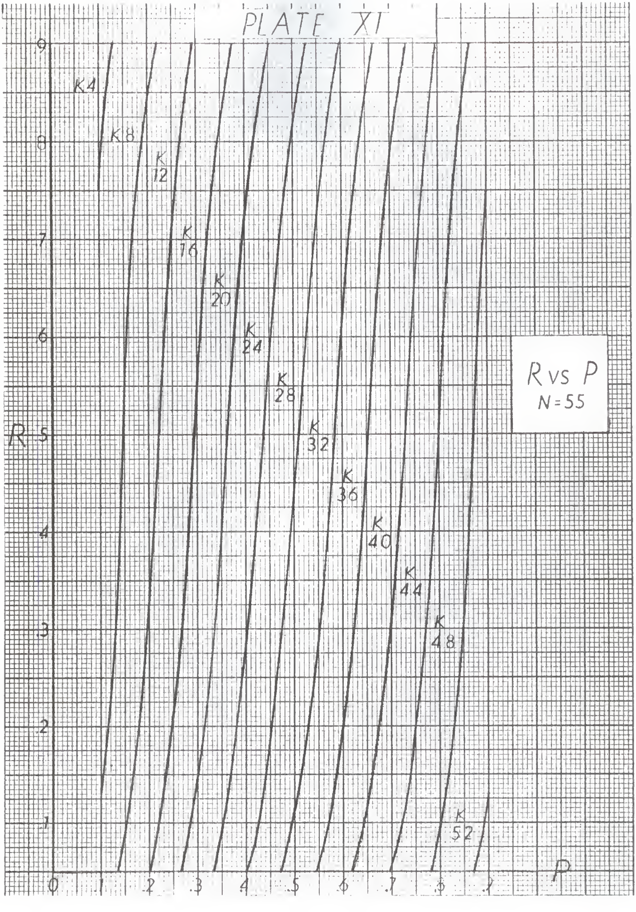


PLATE XI
R vs P
N=55
R
P
K4
K8
K 12
K 16
K 20
K 24
K 28
K 32
K 36
K 40
K 44
K 48
K 52
0
.1
.2
.3
.4
.5
.6
.7
.8
.9
1
2
3
4
5
6
7
8
9

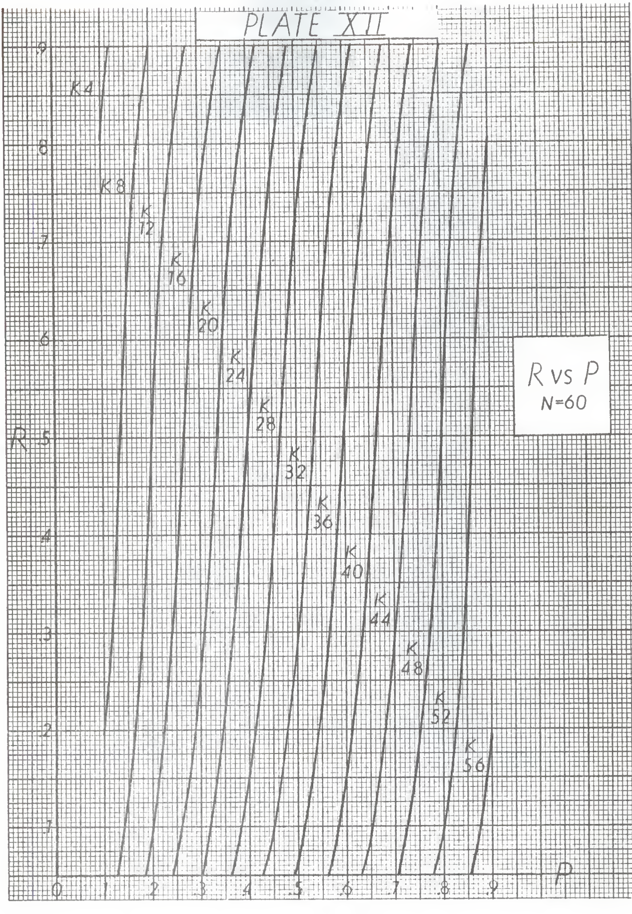


PLATE XII
R vs P
N=60
R
P
K4
K8
K 12
K 16
K 20
K 24
K 28
K 32
K 36
K 40
K 44
K 48
K 52
K 56
0
.1
.2
.3
.4
.5
.6
.7
.8
.9
.1
.2
.3
.4
.5
.6
.7
.8
.9

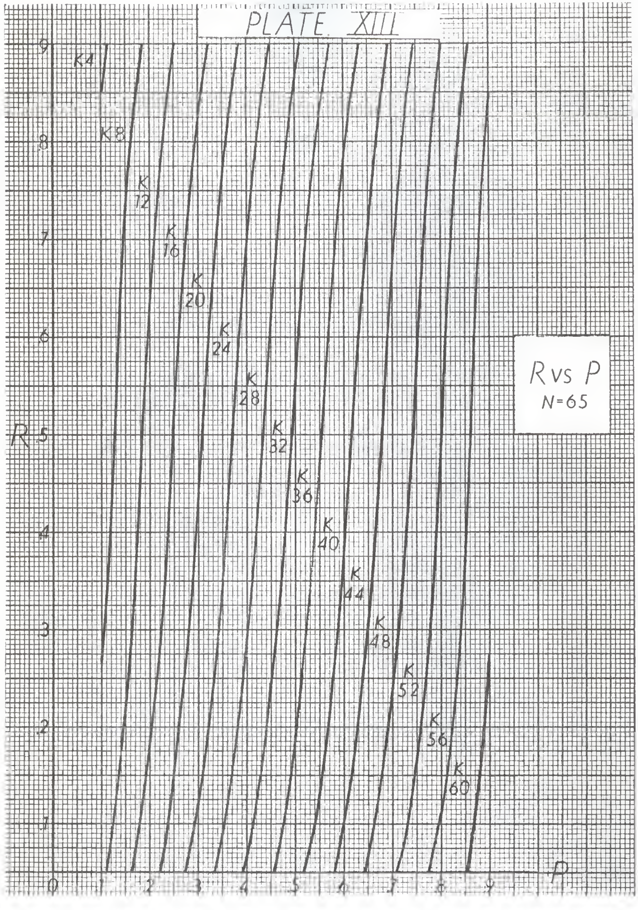
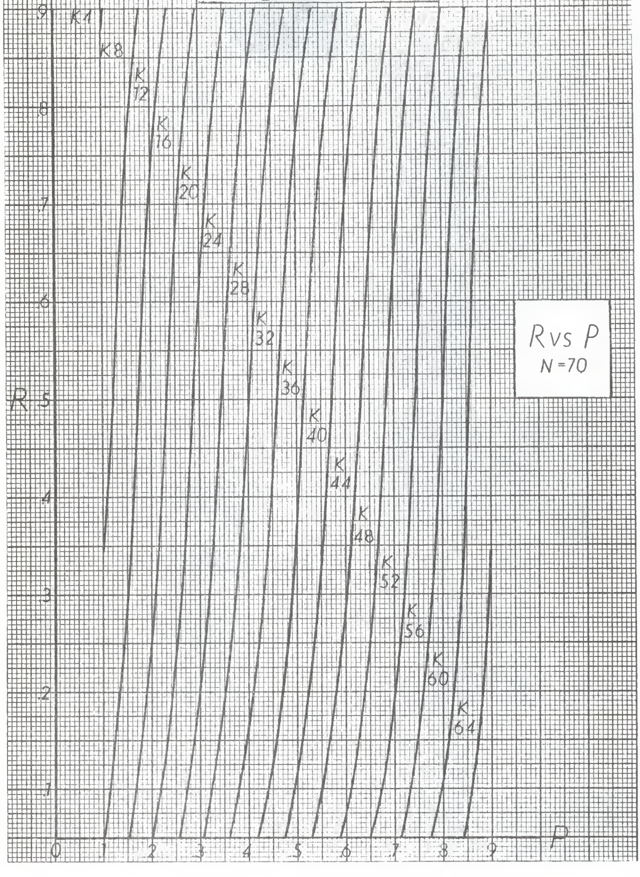


PLATE XIII
R vs P
N=65
K4
K8
K 12
K 16
K 20
K 24
K 28
K 32
K 36
K 40
K 44
K 48
K 52
K 56
K 60
R
P
0
1
2
3
4
5
6
7
8
9

PLATE XIV


R vs P
N =70
R
P
K 4
K 8
K 12
K 16
K 20
K 24
K 28
K 32
K 36
K 40
K 44
K 48
K 52
K 56
K 60
K 64
0
1
2
3
4
5
6
7
8
9

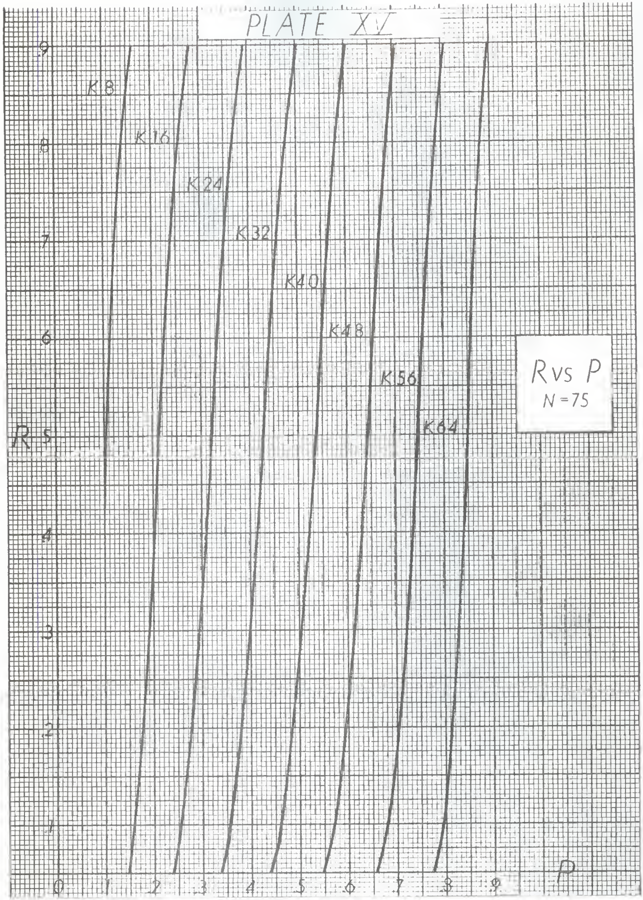


PLATE XV
K8
K16
K24
K32
K40
K48
K56
K64
R vs P
N = 75
R
P
0
1
2
3
4
5
6
7
8
9

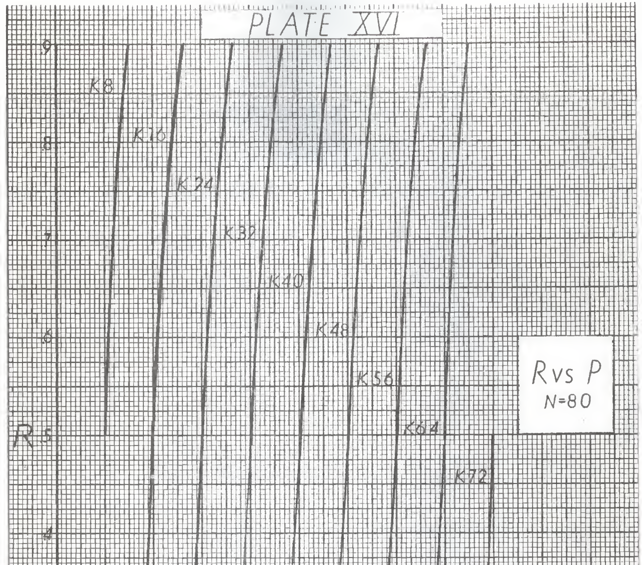


PLATE XVI
R vs P
N=80
K8
K16
K24
K32
K40
K48
K56
K64
K72
R
P
0
.1
.2
.3
.4
.5
.6
.7
.8
.9

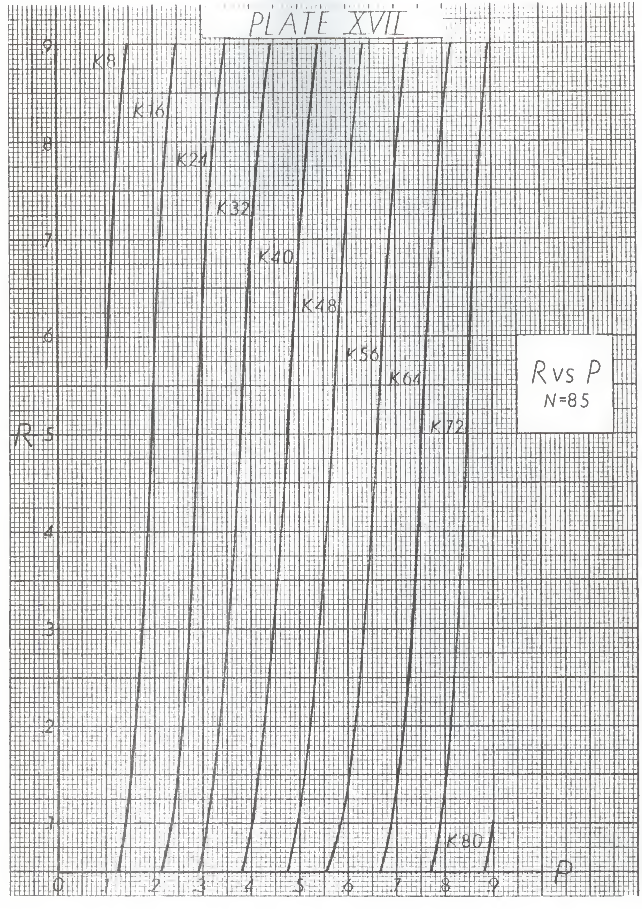


PLATE XVII
R vs P
N=85
K8
K16
K24
K32
K40
K48
K56
K64
K72
K80
R
P
0
1
2
3
4
5
6
7
8
9

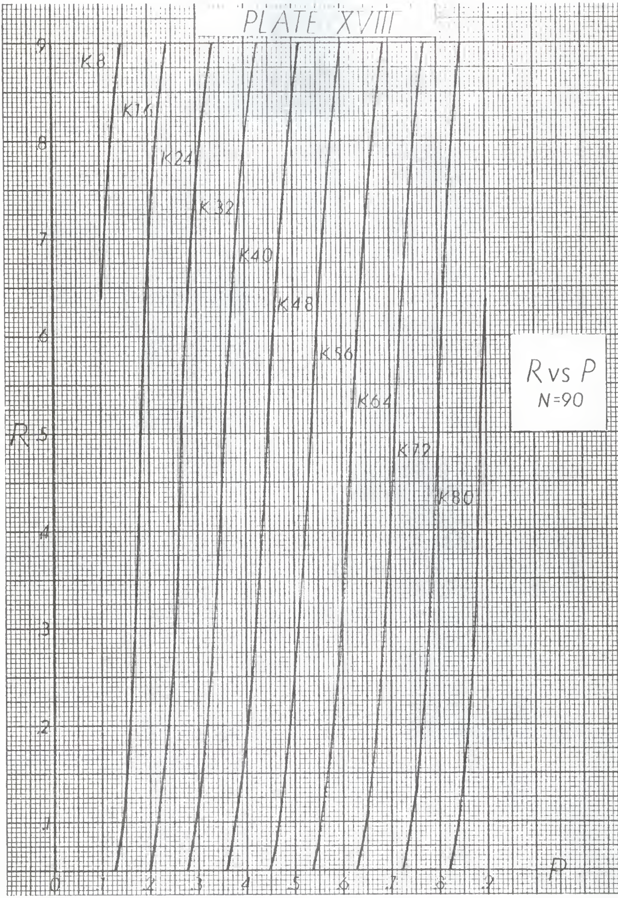


PLATE XVIII
R vs P
N=90
K8
K16
K24
K32
K40
K48
K56
K64
K72
K80
R
P
0
.1
.2
.3
.4
.5
.6
.7
.8
.9
.1
.2
.3
.4
.5
.6
.7
.8
.9

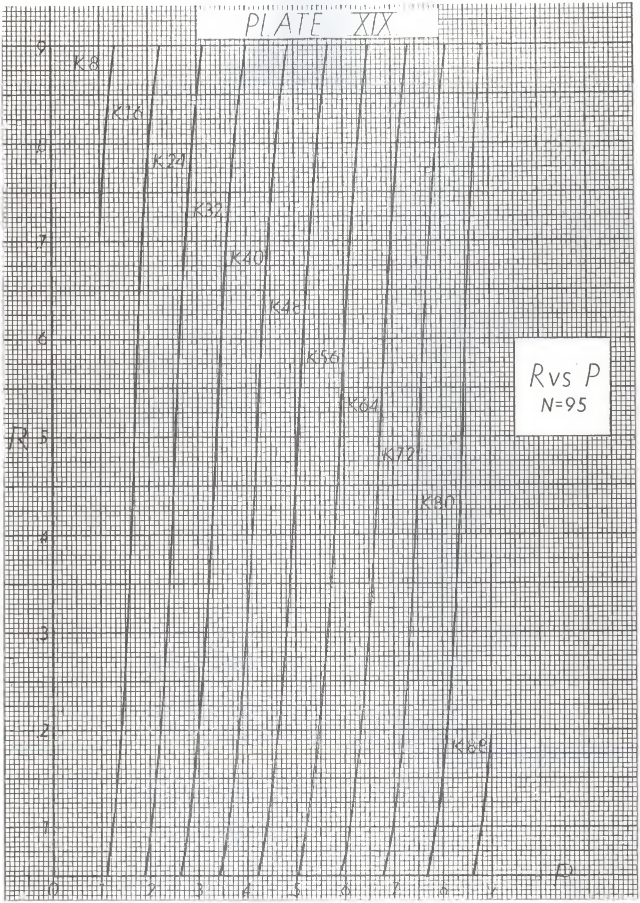


PLATE XIX
Rvs P
N=95
K8
K16
K24
K32
K40
K48
K56
K64
K72
K80
K88
R
P
0
1
2
3
4
5
6
7
8
9

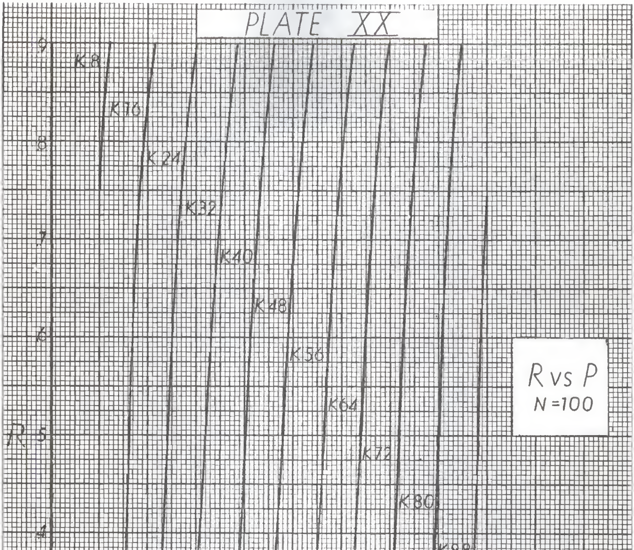


PLATE XX
R vs P
N =100
K8
K16
K24
K32
K40
K48
K56
K64
K72
K80
K88
R
P
0
1
2
3
4
5
6
7
8
9

SUMMARY AND CONCLUSIONS

The charts shown in Plates I through XX provide a relatively simple, workable approach to the design of conveyor systems, using one of the newest theoretical models.

A step-by-step summary of the use of the plates follows;

1. Decide what risk value the company is willing to take.
2. Select a plate which is representative of the number of work stations in the system.
3. Determine the number of stations which will be attempting to use the conveyor.
4. Using the value on the R ordinate cross to the proper k curve and drop to the coordinate P axis.
5. Using this P value design the conveyor using the relationship $t = mP$.

Obviously this is not going to answer all the problems that conveyor users are going to face. The users must have enough knowledge about their process to determine the value of m as used in this thesis. This may or may not be a simple problem.

The parameters considered here are not the only ones that would apply to conveyor use. These were used in an effort to study the typical case and cannot possibly include all that would apply to specific problems.

One of the things which the writer believes this thesis does point out is the need for additional work in this area.

In the determination of the m values used it is believed that some consideration should be given as to whether to use average or limiting values. This could mean some difference in the conveyor design.

There is also the possibility that the hypergeometric distribution could be used instead of the binomial distribution. This might be used to describe what happens in a hook type conveyor when a hook is taken out of the system by being put to use. Such would be the case where the conveyor was being used for storage as well as the movement of parts or material.

This thesis has not considered the economic aspects of conveyor theory. Undoubtedly this should be investigated thoroughly to see what considerations this would bring about.

As previously mentioned the portion of the graphs used in this thesis for values of R from 0.0 to .05 might profitably be investigated in detail in a separate study.

ACKNOWLEDGMENTS

The writer gratefully acknowledges the assistance rendered by faculty and staff members in the Department of Industrial Engineering. In particular I would like to thank my major professors, Dr. Irwin L. Reis, Head of the Department of Industrial Engineering and Dr. Samy E. G. Elias, for their suggestions and direction in pursuing the work presented in this thesis.

REFERENCES

(1) Dixon, Wilfred J. and Frank J. Massey, Jr. Introduction to Statistical Analysis. New York: McGraw Hill Book Co., 1957.

(2) Duncan, Acheson J. Quality Control and Industrial Statistics. Homewood, Illinois: Richard D. Irwin, Inc., 1959.

(3) Feller, W. An Introduction to Probability Theory and Its Applications. New York: John Wiley, 1950.

(4) Kwo, T. T. "A Theory of Conveyors." Management Science, Vol. 5, No. 1.

(5) Mayer, Hugo E. "An Introduction to Conveyor Theory." The Western Electric Engineer, January 1960.

(6) Molina, E. C. Poisson's Exponential Binomial Limit. New York: D. VanNostrand Company, Inc., 1942.

(7) Morse, P. E. and G. E. Kimball. Methods of Operations Research. New York: John Wiley, 1950.

(8) Munroe, M. E. The Theory of Probability. New York: McGraw Hill Book Co., 1951.

APPENDIX A

Appendix A is a detailed discussion of the computer program used to verify the normal and Poisson values used in making the plates.

Because the basic equation was one of simple mathematics it was decided to use the new for transit method of programming for the IBM 650 computer.

The original basic equation was broken into two parts as follows: the factorial value was taken separately from the balance of the equation, because values for the factorial were readily obtained from standard mathematical tables. The balance of the equation which shall be referred to as sub-b was programmed for the computer. The original thinking was to program sub-b = $p^k(1-p)^{n-k}$ for values of:

N = 0, 100, 5

K = 0, N, 2

P = 0, 0.5, .1

It was soon discovered that the value of p^k at the $p = 0$, $k = 0$ was indeterminant and the computer did not know what to do so it would stop. The values were then changed as follows:

N = 5, 100, 5

K = 2, N, 2

P = .1, 0.5, .1

It was believed that the p values would be a mirror image above $p = .5$ so p was stopped at this point.

This assumption was quickly proven wrong so the p value had to be rewritten as:

P = .1,0.9,.1

with these values a program was written as follows:

```
00010B=.1
000020D05N=5,100,5
000030D05K=2,N,2
000040A=(B**K)*
000001((1-B)**(N-K))
000050PUNCH,A,B,N,K
000060IF(B-.9)7,9,9
000070B=B+.1
000080GOTO1
000090END
```

This program worked until the capacity of the computer was exceeded. It was then decided to rewrite the program for each p value and let the computer determine the limit needed for each of these values. The program was then rewritten as follows:

```
00010B=.1
000020D05N=5,100,5
000030D05K=2,N,2
000040A=(B**K)*
000001((1-B)**(N-K))
```

```
000050PUNCH,A,B,N,K
000060CONTINUE
000070END
```

This program produced the desired results for the sub-b values. A total of nine programs were needed. They were the same as the last program listed except the value for B was varied from 0.1 to 0.9 by 0.1's.

APPENDIX B

Appendix B discusses the portion of the curve where the R values were terminated. Figure 1 shows the upper and lower portion of the curves for N = 20.

Inspection of this set of curves indicates that the upper values all approach the value one. The lower portion of the curves all approach the value zero. For Fig. 1 the values below $P = 0.25$ are Poisson while the values to the right of $P = 0.25$ are from the normal tables. The values of k2 and k4 definitely show a shape in the upper portion of the curve different from the other k values.

Figure 2 shows two curves for $k = 8$ and $n = 20$. The solid curve is plotted from the normal values while the dotted curve is representative of the Poisson values. It can also be seen in the upper and lower portion of these two curves where the differences occur. This is further reason for using the Poisson at the left portion while the normal values are used for the larger np values.

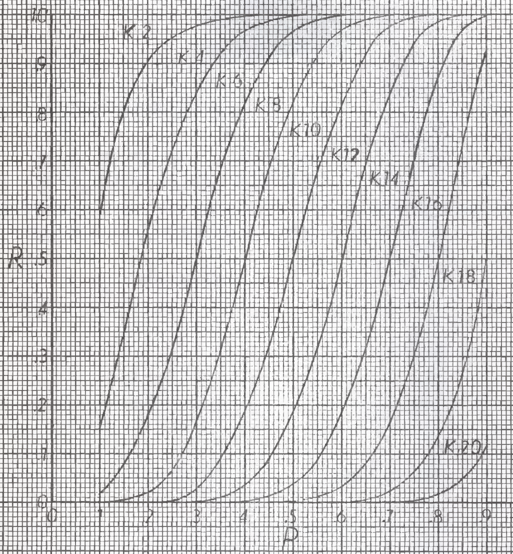


FIG. 1

R VS P FOR N=20 SHOWING THE UPPER AND LOWER PORTION OF THE CURVES.

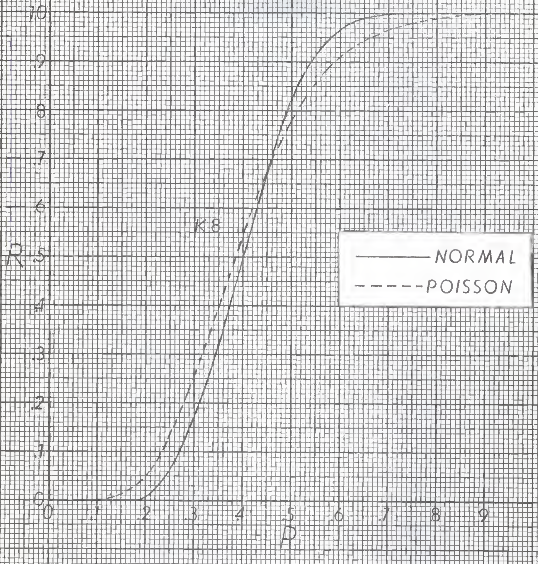


FIG. 2

R VS P FOR N=20 K8 SHOWING THE COMPARISON OF THE NORMAL CURVE WITH POISSON CURVE.

PROBABILISTIC MODELS PERTAINING TO
MODERN CONVEYOR THEORY

by

MORRIS HENRY SCHNEIDER

B. S., University of Nebraska, 1959

AN ABSTRACT OF A THESIS

submitted in partial fulfillment of the

requirements for the degree

MASTER OF SCIENCE

Department of Industrial Engineering

KANSAS STATE UNIVERSITY
MANHATTAN, KANSAS

1961

As industry increasingly faces the problem of accurately determining the best way to solve materials handling problems, the importance and use of a modern conveyor theory will increase.

This thesis presents a method based on modern, mathematical conveyor theory rather than on outdated "rules of thumb."

The approach used was to develop a tool that could be used in conveyor design by using the binomial probability distribution as the basic mathematical model. The expansion of the model was done with the help of the IBM 650 computer. During the time spent trying to determine how best to present the material it was discovered that both the normal approximation to the binomial and the Poisson distribution could satisfactorily be used in certain regions important to developing the tool. Using the 650 as a cross reference on the two standard tables, the graphs for use in conveyor design were made.

The final graphs are so designed that a user of conveyor systems needs to know a minimum about his own system in order to use the developed tool. The user needs to know only enough about his own process to be able to establish the cycle time at any work station and to select an appropriate risk. He needs no special mathematical training. The graphs are so designed that the user can, knowing this cycle time and this desired risk value, design the conveyor system which will best fit the conditions he has placed on it.

The writer does not believe that this is the ultimate tool or that it will be a panacea for all conveyor problems. It is felt however, that it does present a better starting place than has been established by previous work in this field.