ASSESSING COLLEGE STUDENTS' RETENTION AND TRANSFER FROM CALCULUS TO PHYSICS

by

LILI CUI

B.S., Xuzhou Normal University, China, 2000

AN ABSTRACT OF A DISSERTATION

Submitted in partial fulfillment of the requirements for the degree

DOCTOR OF PHILOSOPHY

Department of Physics
College of Arts and Sciences

KANSAS STATE UNIVERSITY
Manhattan, Kansas

2006

Abstract

Many introductory calculus-based physics students have difficulties when solving physics problems involving calculus. This study investigates students' retention and transfer from calculus to physics. While retention is the ability to recall your knowledge at a later point in time, transfer of learning is defined as the ability to apply what one has learned in one situation to a different situation.

In this dissertation we propose a theoretical framework to assess students' transfer of learning in the context of problem solving. We define two kinds of transfer – horizontal transfer and vertical transfer. Horizontal transfer involves applying previously learned ideas in a problem. Vertical transfer involves constructing new ideas to solve the problem. Students need to employ both horizontal and vertical transfer when they solve any problem. This framework evolves through this research and provides a lens that enables us to examine horizontal and vertical transfer. Additionally, this proposed framework offers researchers a vocabulary to describe and assess transfer of learning in any problem solving context.

We use a combination of qualitative and quantitative methods to examine transfer in the context of problem solving. The participants in this study were students enrolled in a second-semester physics course taken by future engineers and physicists, calculus instructors and physics instructors. A total of 416 students' exam sheets were collected and reviewed. Statistical methods were used to analyze the quantitative data. A total of 28 students and nine instructors were interviewed. The video and audio recordings were transcribed and analyzed in light of the aforementioned theoretical framework.

A major finding from this study is that a majority of students possess the requisite calculus skills, yet have several difficulties in applying them in the context of physics. These difficulties included: deciding the appropriate variable and limits of integration; not being clear about the criteria to determine whether calculus is applicable in a given physics problem, and others. This study also provides a detailed understanding of students' difficulties in terms of our theoretical framework. Instructional strategies are suggested at the end to facilitate the transfer from calculus to physics.

ASSESSING COLLEGE STUDENTS' RETENTION AND TRANSFER FROM CALCULUS TO PHYSICS

by

LILI CUI

B.S., Xuzhou Normal University, China, 2000

A DISSERTATION

Submitted in partial fulfillment of the requirements for the degree

DOCTOR OF PHILOSOPHY

Department of Physics
College of Arts and Sciences

KANSAS STATE UNIVERSITY
Manhattan, Kansas

2006

Approved by:

Major Professor
N. Sanjay Rebello

Copyright

Abstract

Many introductory calculus-based physics students have difficulties when solving physics problems involving calculus. This study investigates students' retention and transfer from calculus to physics. While retention is the ability to recall your knowledge at a later point in time, transfer of learning is defined as the ability to apply what one has learned in one situation to a different situation.

In this dissertation we propose a theoretical framework to assess students' transfer of learning in the context of problem solving. We define two kinds of transfer – horizontal transfer and vertical transfer. Horizontal transfer involves applying previously learned ideas in a problem. Vertical transfer involves constructing new ideas to solve the problem. Students need to employ both horizontal and vertical transfer when they solve any problem. This framework evolves through this research and provides a lens that enables us to examine horizontal and vertical transfer. Additionally, this proposed framework offers researchers a vocabulary to describe and assess transfer of learning in any problem solving context.

We use a combination of qualitative and quantitative methods to examine transfer in the context of problem solving. The participants in this study were students enrolled in a second-semester physics course taken by future engineers and physicists, calculus instructors and physics instructors. A total of 416 students' exam sheets were collected and reviewed. Statistical methods were used to analyze the quantitative data. A total of 28 students and nine instructors were interviewed. The video and audio recordings were transcribed and analyzed in light of the aforementioned theoretical framework.

A major finding from this study is that a majority of students possess the requisite calculus skills, yet have several difficulties in applying them in the context of physics. These difficulties included: deciding the appropriate variable and limits of integration; not being clear about the criteria to determine whether calculus is applicable in a given physics problem, and others. This study also provides a detailed understanding of students' difficulties in terms of our theoretical framework. Instructional strategies are suggested at the end to facilitate the transfer from calculus to physics.

Table of Contents

List of Figures

List of Tables

Acknowledgements

In sincere gratitude I would like to thank:

- Dean Zollman, my former advisor, for giving me the opportunity to study in the U.S., willingness to be flexible in choosing and changing my research topic, and all the help and insights. I would not be here without him.
- Sanjay Rebello, my advisor, for infinite hours of discussion and advice. He is always willing to offer help regarding research, teaching and other aspects of my life. He has made a difference in my experience at K-State.
- Kim Coy, administrator for the Physics Education Research Group, for help of understanding American customs, culture, and language.
- Zdeslav Hrepic, former graduate student, he was the one urged me to accept the offer and come to K-State. I have never regretted this decision.
- Andrew Bennett for his guidance and support on mathematics education, and the wonderful teaching opportunity at Dodge City Community College. I appreciate his confidence in me.
- Peter Fletcher for learning things from another perspective. He came at the right time.
- Amit Chakrabarti, for serving on my dissertation committee and for all the help from my first semester here. He showed me how to be a wonderful teacher.
- All of the members of the Physics Education Research Group, for all their support and wisdom.
- All of the students and faculty participants in this research. Their willingness and efforts made this study possible.
- My parents and all my friends.

Dedication

I dedicate this dissertation to my parents, Weiguo Cui & Fusheng Gu.

CHAPTER 1 - INTRODUCTION

1.1 Historical Background

Isaac Newton is no doubt considered one of the most important physicists of all times. He was officially a professor of mathematics. Newton is considered by some as first to develop calculus. His book *Philosohiae Naturalis Principia Mathematic* introduced calculus as a way to solve problems in physics. The development of calculus and physics intertwines with each other. Although the idea that Newton developed calculus to solve problems in physics, has been challenged recently, it is clearly established that calculus was used to solve physics problems in the eighteenth century, by Laplace, Lagrange, Green, Gauss and other famous physicists. Thus the connections between calculus and physics cannot be overemphasized. It is under this historical backdrop that we conduct this study.

Today, the fundamentals of calculus and classical physics are taught at the high school or introductory college level throughout the world, often in separate courses, taught in separate departments. Yet these two subjects are so closely intertwined that it would be meaningful both from a pragmatic as well as a philosophical point of view to investigate how contemporary students see the connection between calculus and physics. Because these classes are typically taught sequentially, calculus followed by physics, it is also relevant to investigate, how students apply or transfer the knowledge they learned in their calculus courses into physics courses. The research described in this dissertation explores these issues in the context of introductory undergraduate physics courses taught at Kansas State University.

1.2 Motivation for This Study

Typically there are three kinds of introductory physics courses offered in most U.S. universities: conceptual-based physics, algebra-based physics and calculus-based physics. Most science and engineering majors are required to take calculus-based physics. Students are usually required to concurrently take both their first calculus and physics

courses, or take at least one calculus course prior to taking physics. While a few integrated curricula (e.g. Dunn and Barbanel, 2000; Yeatts and Hundhausen, 1992) have been developed and have been found useful in teaching calculus and physics, in most universities, calculus and physics are taught as two separate subjects in their respective departments.

The connection between calculus and calculus-based physics is obvious both from the historical view and practical perspectives. Anecdotally I have often found that some physics teachers claim that their students do not have the pre-requisite calculus knowledge to help them master physics. Is this the case? There has been no significant research on transfer of learning from calculus to physics. Therefore, assessing transfer of learning from calculus to physics is the central focus of this study.

1.3 Transfer of Learning

To understand how students apply what they have learned in a calculus course to a physics course, we investigate an issue that has long interested educators: transfer of learning. Transfer of learning is often defined as the ability to apply what has been learned in one context to a new context (e.g. Byrnes, 1996). Transfer of learning has often been referred to as the ultimate goal of education. Educators hope students can transfer the knowledge they have learned in one context to a new context, for example, from one problem to another problem, or, from one course to another, and most importantly, from school to the real world.

Transfer of learning has often been an ambitious goal for educators. Researchers have found transfer of learning to be difficult to identify, let alone measure. In the past most researchers who sought to answer the question, "Does X transfer from A to B?" where X was a particular concept or skill, and A and B were the learning and target contexts respectively, found that in fact transfer was extremely rare. In most cases, students were unable to apply a principle or schema extracted from a particular learning situation to a new target situation. Most researchers realized that this experimental evidence of lack of transfer was almost in direct contradiction to everyday experiences in which people are often able to perform successfully in new situations, indicating that they

have productively transferred what they have learned in previous situations. More recently, several researchers have sought to bridge the contradiction between the lack of experimental evidence of transfer and its apparent ubiquity in everyday life. These researchers view transfer as a dynamic process in which the learner constructs knowledge in the new situation. We have found this perspective to be useful in our research on transfer of learning from calculus to physics.

1.4 Research Questions and Strategies

To assess the transfer from calculus to physics, the following three research questions naturally come to our mind:

> Research Question #1: To what extent do students retain and transfer their calculus knowledge when solving problems in introductory physics?
>
> Research Question #2: What mental processes are involved as students transfer what they have learned in calculus to introductory physics?
>
> Research Question #3: What strategies may facilitate students transfer from calculus to physics?

A grounded theory approach was used at the beginning of this study to cast a wide net and collect data from a wide range of sources – both qualitative and quantitative. Based on an analysis of these data we constructed a theoretical framework that was deemed to be useful in examining the research questions.

We examine the aforementioned questions, especially #2 and #3 from the perspective of the learners and educators. Therefore, we adopted a phenomenological standpoint. Phenomenology is the primary philosophical standpoint for this research, because it explores the lived experience of people—students and teachers in this study. We employed clinical interviews to explore the variations in the ways in which students described their learning experiences in calculus and physics and utilized a phenomenographic approach to ascertain these variations.

1.5 Broader Impacts

The result of this study will help researchers and teachers understand the process that students use to transfer their calculus knowledge to physics courses. This research will identify common difficulties students have and propose instructional strategies to facilitate the transfer process, to help students learn physics.

In a broader sense, the general research results and proposed instructional strategies emerging from this research to facilitate the transfer of learning from calculus to physics can be used for any other two subjects, such as from physics courses to engineering courses. In an even broader sense, this research study will also provide insights into how students engage in transfer learning how to solve abstract, well structured problems to solving more concrete, situated and ill-structured problems, similar to those that they are likely to face in their everyday lives.

1.6 Road Map of Dissertation

The dissertation comprises three major parts: the first concerns itself with the theoretical framework; the second part describes the design of the research and data collection methods; and the third part reports the results and discusses the overall research findings.

In Chapter 2 we provide a comprehensive review of related research covering a model for transfer of learning, traditional and contemporary views of transfer, and research on calculus education and research on problem solving in physics. In Chapter 3 we describe the research framework that provides us a lens through which to view and reframe the research questions described earlier in this chapter. It also lends us a theoretical viewpoint from which the research was conducted. In Chapter 4 we describe the research design based on the theoretical framework, the research setting, as well as data collection and analysis methods. The selected research methodologies are briefly discussed. Chapter 5 presents the key findings of this study in terms of responses to each of the aforementioned research questions and finally. Chapter 6 discusses implications of the research and mapping of the research results onto a common framework.

Recommendations for teachers and curriculum developers are also summarized in this chapter.

CHAPTER 2 - LITERATURE REVIEW

2.1 Chapter Overview

The literature summary in this chapter is presented in three sections: Research on mathematics education, especially on calculus learning, research on physics learning and problem solving in physics, and research on the transfer of learning. The mathematics education and physics education articles have been discussed separately to represent the different ideas from the two subjects.

2.2 Research on Mathematics Education

2.2.1. Overview of Mathematics Education

Mathematics education research has been growing rapidly over the past three decades (Kilpatrick, 1992). Research has focused on understanding the nature of mathematical thinking, teaching, and learning; and has been applied to improve mathematics instruction. In the last two decades, there has been extensive research in the teaching and learning of undergraduate mathematics, topics covered such as functions (e.g. Breidenbach, 1992; Carlson, 1998; Even, 1998), topics from calculus (e.g. Asiala, 1997; Clark, 1997; Frid, 1994; Williams, 1991), and topics from post-calculus(e.g. Gibson, 1998; Harel, 1998; Zazkis, 1996). It is beyond the scope of this dissertation to provide a comprehensive overview of the vast field of mathematics education research. In the context of this study, research on calculus learning has been reviewed below.

2.2.2. Calculus Learning and Calculus Reform

> "Calculus is central to the mathematical sciences, is fundamental to the study of all sciences and engineering, and belongs in the core undergraduate mathematics curriculum for all students."
>
> Douglas, 1986

For decades, calculus has been the introductory mathematics course for most science and engineering majors in the college level. It is generally considered as the foundation of college mathematics. Calculus is the "language of change". The first two major goals of calculus instruction were described as Davis (1985):

- Develop students' understanding of concepts as well as their ability to use the relevant procedures
- Expose students to a broad range of problems and problem situations

However, in the early 1980s, many mathematics professors became to dissatisfied with undergraduate calculus education because of students' weak conceptual understanding and high failure rates (e.g. Douglas, 1986; Selden, 1994). An investigation of final examination questions in collegiate calculus courses (Steen, 1987) revealed that 90 percent of the items focused on calculation and only 10 percent on higher order challenges. Calculus reform took place to address this need. Ronald Douglas is considered as the "father" of Calculus Reform because he organized the Tulane Conference on *Developing Curriculum and Teaching Methods for Calculus at the College Level*. The focus of the conference was overhauling both the content and pedagogy of calculus. The report of the conference -- *"Toward a Lean and Lively Calculus"* (Douglas, 1986) has been cited in numerous papers. In the report, many suggestions for teaching calculus were proposed:

- Use complex problems from the "real world" as a context for doing calculus.
- Use elementary theoretical problems.
- Use occasional non-standard, context-free problems.
- Ask students to construct examples.
- Assign multi-steps problems, and problems that go beyond "plug into the technique we just studied."
- Give mathematics reading assignments so that students need to work through problems on those readings

Tucker (1995) believed that "the hallmarks of calculus reform [are] changes in modes of instruction and use of technology, along with an increased focus on conceptual understanding and decreased attention on symbol manipulation". As the result of calculus reform, several new textbooks have been written that claim "a fresh, new approach to the concepts of calculus" (LaTorre, 1998) with a goal "to provide students with a clear understanding of the ideas of calculus" (Hughes-Hallett, 1998).

The Mathematical Association of America (MAA) report *Assessing Calculus Reform Efforts: A Report to the Community* (Tucker, 1995) suggested "large numbers of reform instructors report that new instructional methods are having positive effects on students' conceptual understanding, mathematical reasoning, and problem solving abilities". Many empirical studies confirm that the reform efforts are having positive effects on students' learning of calculus (e.g. Bookman, 1994; Meel, 1998; Park, 1996; Schwingendorf, 2000). The proponents of calculus reform believe the new approach helps students develop a deeper understanding of the concepts and uses of calculus, in part by shifting the burden of lengthy calculations to computers. However, not all mathematicians are in favor of calculus reform (e.g Askey, 1997; Klein and Rosen, 1997; Wilson, 1997; Wu, 1996). Wilson (1997) questioned whether the calculus reform was a good idea because the reformed calculus courses and textbooks do not give students enough background in solving complicated mathematics problems. Wu (1996) believed the calculus reform did not improve what was unsatisfactory in the traditional curriculum since the basic questions in calculus education -- why calculus is true and calculus is important -- still remained unanswered. While the calculus reform movement or its impact is not the focus of this dissertation, it provides a useful backdrop as we examine the kinds of calculus knowledge and skills that we can expect students to bring into a physics classroom.

2.2.3. Assessment of Calculus Learning

Assessment has always been an important topic in education since it shapes students' notions of what is important. How does one assess students' learning in calculus? Schoenfeld (1997) in his NSF report *Student Assessment in Calculus* concluded that various pencil-and-paper assessment tasks are still the most widely used assessment

techniques in mathematics education, not limited to calculus learning. Broadly speaking, there are two kinds of pencil-and paper assessment tasks: well-structured tasks and ill-structured tasks.

Typical well-structured assessment tasks include multiple-choice items and short-answer items. Because of the large enrollment of most calculus courses, these are the most widely used assessment tools. Multiple-choice and short-answer problems are objective, efficient and reliable. However, they usually only require "a computation procedure" and so do not focus on conceptual understanding. Dunbinksy and Ralston (1992) found 65% - 75% of the first semester calculus items are symbolic manipulations and require little understanding. The percentages are even higher for the second and third semester of calculus exams. Schoenfeld argued that multiple-choice and short-answer problems covey the idea that mathematics is "made up of unrelated bits and pieces and that learning mathematics is memorizing rules and procedures or requiring a bag of tricks", and rarely assess students' ability to "solve problems, synthesize ideas, create new knowledge or communicate observations." However, these types of tasks are still most widely assessments of student learning in calculus.

Ill-structured tasks include open-ended items and student-constructed tests. These problems usually have more than one correct answer or several paths to get the correct answer. Students need to show their reasoning and explain how they got their answer or the method they chose. Schoenfeld believed these tasks "call for qualitative interpretations, modeling, and other deep mathematical skills". However, grading these assessment tasks can be time consuming and therefore most calculus instructors who teach large enrollment classes typically refrain from using them.

2.3 Research on Physics Education

2.3.1. Overview of Physics Education

Research in physics education has been growing rapidly over the past three decades. Physics education research is motivated by physics professors' dissatisfaction with students' weak conceptual understanding and problem solving skills in physics. Broadly speaking physics education research has focused on the following areas:

- Identifying students' misconceptions and difficulties in various physics topics (e.g. McDermott, 1984; McDermott, Rosenquist et al., 1987)
- Developing conceptual inventories to assess student conceptual learning in physics (e.g. Hestenes, Wells et al., 1992; Beichner, 1994; Engelhardt and Beichner, 1996; Maloney, O'Kuma et al., 2001)
- Studying the problem-solving strategies used by students physics (e.g. Maloney, 1993; Heller, Keith et al., 1992)
- Development of new instructional techniques, such as Studio Physics, Workshop Physics (e.g. Wilson, 1994; Laws, 1991), and developing teaching materials (e.g. Zollman, 1995; Zollman, Rebello et al., 2002; McDermott, 1996)
- Identifying students' beliefs and attitude towards physics learning (e.g. Redish, 1994; Redish, Saul et al., 1998; Hammer, 1995; Hammer and Elby, 2002)
- Understanding students' mental models in physics (e.g. Bao and Redish, 1999; Bao, Hogg et al., 2002; Rebello, Itza-Ortiz et al., 2003)
- Modeling students understanding by using insights from research in psychology and cognitive science (e.g. diSessa, 1988; Redish, 1994; Mestre, 1994; Rebello, Zollman et al., 2005)

Numerous studies have been focused on the learning and teaching of physics, primarily at the university level in each of the areas above. Clearly, it is beyond the scope of this dissertation to provide a comprehensive overview of the vast field of physics education research and curriculum development. In the context of this study, research on problem solving has been reviewed next.

2.3.2. Physics Problem Solving

2.3.2.1. Definition of a Physics Problem

Any investigation of problem solving needs a clear definition of what constitutes a problem. According to Newell and Simon (1972), a problem is defined as a situation when an individual "wants something and does not know immediately what series of actions he or she can perform to get it". For many physics instructors and students, the term problem refers to end-of-chapter tasks often found in introductory college physics textbooks (Maloney, 1993), it "represents a situation in which certain information is given, most often as numerical values for variables in the situation, and the value of one of the other possible variables is to be determined." This description of a physics problem is the operative definition for this dissertation. Problem solving is the process that an individual goes through to obtain the answer to a problem i.e. find an unknown quantity requested in the problem statement.

2.3.2.2. Research on Problem Solving in Physics

Problem solving has always been a popular area in physics education research. Physics teachers typically want their students acquire the ability to solve physics problems. The comparison of expert and novice problem solving in physics have provided a useful lens to help identify the key features of novice and expert behaviors. (e.g. Larkin, 1980; Schultz,1991; Sweller, 1988). Reif and Heller (1982) found that novices typically tend to grab an equation and plug in numbers when solving a physics problem. Chi (1981) found that experts categorized problems according to "deep structure," while novices tended to categorize according to "surface features". According to Schultz (1991), the four abilities for successful problem solving in physics are:

1) organize quantitative calculation though qualitative understandings;
2) represent a problem situation via multi-representations, like diagrams or drawings;
3) organize one's knowledge; and
4) evaluate the answers.

Sweller (1988) proposed that novices' use of means-ends analysis on standard textbook problems was counter productive for learning the physics concepts that underlie problem solving with understanding. When the students focus on the goal of finding a specific numerical answer, this focus will direct their attention to the manipulation of equations and they consequently expend little effort on carrying out a qualitative analysis involving other representations. Also, applying the means-ends heuristic requires a significant part of the cognitive resources of the problem solver, so very few resources are available to consider the concepts and principles and how they apply. Researchers agreed that traditional ends-of-chapter problems did not help students to develop conceptual understanding of physics, nor to be a successful problem solver.

The research methods for assessing problem solving ability in the aforementioned studies were very similar. Researchers first developed some problems, and then asked research subjects to solve those problems in an interview situation. Based on this research many strategies have been developed to investigate and facilitate the problem solving process.

The five-step problem solving strategy was developed by the physics education research group at University of Minnesota (Heller, Keith et al., 1992). Heller believed it represented an effective way to organize thinking and produce a solution based on the provided information. However, the quality of the solution still depended on the physics knowledge that students used in obtaining the solution. The five-step strategy also made it easier to look back through one's solution to check for incorrect knowledge and assumptions, which was an important tool for learning physics. Heller argued that if students learned to use this strategy effectively, they would find it a valuable tool to use for solving new and complex problems. The five steps are:

1) comprehend the problem situation;
2) represent the problem using formal terms;
3) plan a solution;
4) execute the plan; and
5) interpret and evaluate the solution.

The Physics Education group at University of Minnesota (Heller, Keith et al., 1992; Heller and Hollabaugh, 1992) also proposed the idea of cooperative group problem solving using context-rich problems. Context rich problems place the student in an authentic real-life context. They often do not provide all of the information and do not necessarily have only one correct answer. The level of difficulty of these problems requires students to work in cooperative groups. These are clearly non-traditional problems. Yet another example is Physics Jeopardy problems. Physics Jeopardy was a new format for physics problems proposed by Van Heuvelen (1999). In Physics Jeopardy, the problem starts with a mathematical equation, a graph or a diagram that describes a physical process. The problems solver needs to construct other representations of the problem which are consistent with the given situation. Van Heuvelen suggested that Jeopardy problems had several strengths to promote problem solving with understanding. The strengths included: students giving meanings to the symbols in the equations, preventing students from relying on mathematical formula, helping students to learn to translate between different representations and Jeopardy problems were easy to design. Van Heuvelen also pointed out that since Jeopardy problems were new, students needed to practice with easy examples before put them on tests. Other problems types, included Active Learning Problems Sheets (ALPS), problem posing and ranking tasks (e.g. Van Heuvelen, 1991; Maloney, 1987; Mestre, 2002) were proposed and proven to improve students' problem solving abilities. Curricula (e.g. Bascones and Novak, 1985; Van Heuvelen, 1991) have also been modified and claimed to help students develop problem solving skills.

There has also been extensive research on problem solving outside the field of physics education. Many of these efforts could potentially inform the study of problem solving in physics. Research has been conducted from the cognitive science perspective. Cognitive load during problem solving was attributed as one of the reasons why students choose to use means-end analysis during problem solving (Sweller, 1988). Ashcraft and Kirk (2001) suggested that mathematics anxiety could decrease the number of working memory slots available to a person solving a math problem, even when that person possessed the math skills necessary for solving that problem. There might be similar implications for people with physics anxiety attempting to solve physics problems.

Jonassen (2000) and other researchers who were studying generic (not necessarily physics) problem solving suggested that the ill-structured problems can help student develop the generic problem solving skills compared with the well-structured problem. The context-rich and Jeopardy problems discussed above will fall in the category of ill-structured problems. Another form of non-traditional problems is proposed by Bransford (1989) and co-workers. They have suggested the use of contrasting cases to help students look past the surface features in a problem and focus instead on deeper structure.

As a summary, numerous studies found that experts and novices used different procedures to solve problems (e.g. Larkin, 1980; Schultz,1991; Sweller, 1988). Different knowledge structure could be one possible reason why experts and novice used different approaches. Students tent to use means-ends analysis on standard textbook problems. Many instructional strategies have been proposed to help students to become better problem solvers. Researchers have expanded their repertoire of problems used to investigate problem solving skills and develop these skills in students.

A vast majority of problems that students encounter in introductory physics continue to be traditional end-of-chapter problems. Therefore the initial stages of the research described in this dissertation focuses on solving traditional end-of-chapter problems. However, as the research progressed, other types of problems such as Jeopardy problems and contrasting cases were used to investigate students problem solving and transfer from calculus to physics.

2.4 Transfer of Learning

2.4.1. Transfer of Learning and Problem Solving

Transfer of learning is often (e.g. Reed, 1993; Singley and Anderson, 1989) defined as the ability to apply what one has learned in one situation to a different situation. Several researchers (e.g. McKeough, Lupart et al., 1995) have described transfer of learning as the ultimate goal of education. Problem solving in physics is tightly related to transfer of learning. To solve a problem, individuals need to successfully transfer their

knowledge from the context in which it was first learned to the context of the particular problem – which in physics often means applying their knowledge from an abstract, idealized context to a more concrete context. As mentioned before, the ultimate goal of schooling is to prepare students for the new problems after they finish their school lives. We can not teach everything to students in school. However, if we help students learn the ability to transfer of knowledge to a new problem situation, we can contribute to their development as life-long learners.

2.4.2. Factors Influencing Transfer

"How People Learn" (Bransford, Brown et al., 1999) provides a summary of the factors that influencing peoples' ability to transfer their learning from one context to another. These include:

- The amount and type of initial knowledge are considered to be key determinant factors in transfer. For instance, the knowledge students learned in their calculus courses can influence how much they can transfer to a physics course.
- Time spent learning for understanding is another factor. Students with deeper understanding of a concept are more likely to be able to transfer that concept to other situations..
- Multiple learning contexts can be crucial. If student learns the concept in multiple situations, they would be more likely to construct abstract representations of their knowledge, and transfer these abstract representations to other problems.
- Frequent feedback can also facilitate transfer. A type of feedback that has been utilized in education research is the use of contrasting cases. Providing cases for students that contrast to previous learning may help them become aware of features that may not have noticed in the old situation, feature they may not have brought forth in their mind when presented with the new situation. Understanding when, where, and why to use new knowledge may be enhanced through the use of contrasting cases.

- Metacognitive approaches to teaching can also increase transfer by help students better understand themselves as learner. By asking students to reflect on their own learning and think about what helped and hindered their process of learning can also help students transfer their knowledge to new situations.

2.4.3. Traditional and Contemporary Views of Transfer

Rebello (2005) has reviewed the differences between traditional and contemporary views of transfer of learning. Traditional models (Bassok, 1990; Chen and Daehler, 1989; Adams, Kasserman et al., 1988; Brown and Kane, 1988; Novick, 1988; Nisbett, Fong et al., 1987; Perfetto, 1983; Reed, Ernst et al., 1974; Wertheimer, 1959; Throndike and Woodworth, 1901) view transfer from a pre-defined researcher's point of view. These approaches view transfer as a passive, static process where students apply their prior knowledge of the initial learning situation to the new situation. Contemporary models (Lobato, 2003; Lobato, 1996; Bransford and Schwartz, 1999; Greeno, Moore et al., 1993) view transfer from the students' point of view and as an active, dynamic process where students construct a knowledge structure in the new situation.

Greeno and his colleagues (1993) focus on the socio-cultural aspects of transfer by examining activities that the learner performs in the learning context. They view transfer in terms of affordances and constraints of activity. They are interested on the extent to which participating in an activity while being attuned to the affordances and constraints in one situation influences the learners' ability to participate in a different situation.

Lobato's (2003) Actor-Oriented Transfer (AOT) model views transfer as the personal construction of similarities between the two contexts. She focuses on how the "actors" (or learners) see the two contexts as similar. Lobato suggests that students may transfer both productively and unproductively, which researchers may not have previously considered. She argues that researchers should not decide a priori what students should transfer but rather adopt a student-centered perspective to find out what students do transfer and investigate the mediating factors.

Bransford and Schwartz (1999) view transfer in terms of Preparation for Future Learning (PFL). Rather than focus on Sequestered Problem Solving (SPS), in which student are expected to solve a problem "cold" to assess whether the can transfer their learning, their PFL approach focuses on whether students can learn to problem-solve in a new context. Bransford and Schwartz believe transfer is more likely to be observed if students are given the opportunity to reconstruct their learning in the transfer context in the same way as they did in the learning context.

Table 2-1 summarizes the comparison of traditional and contemporary views of transfer of learning along several dimensions.

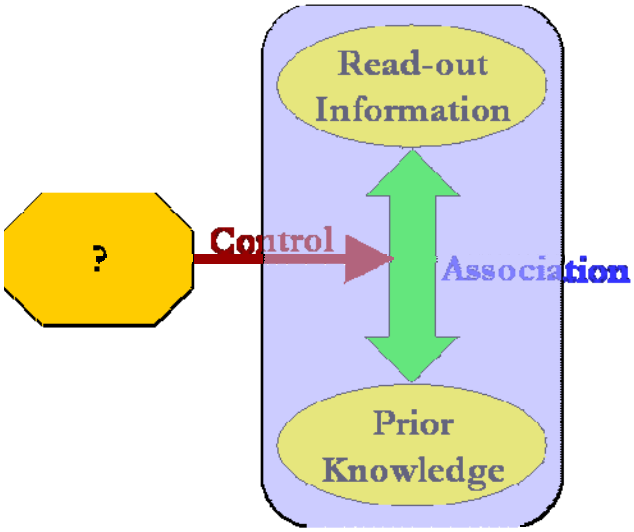
Table 2-1: Research in Transfer of Learning

	Traditional Perspective	Contemporary Perspective
Research Questions	Can learners successfully apply knowledge previously acquired in the learning task to transfer task?	How do learners actively construct knowledge in the transfer task based on experiences in the learning task?
Typical Expectations	Few students are able to transfer what they have learned in learning context to the transfer context.	Transfer is ubiquitous and it is our tools that are blunt and unable to detect it.
Assessment	Assessment tests whether learners can successfully problem-solve in a transfer context.	Assessment based on whether learners can *learn* to problem-solve in a transfer scenario.
Researcher's Role	The researcher pre-defines the structural similarities between the learning and transfer context.	The researcher investigates what the learner sees as similar between the two scenarios.
Dynamism	Transfer is a static construct, i.e. students can either apply their knowledge in a transfer context or they cannot.	Transfer is dynamic, i.e. students can learn in the transfer context based on their prior experiences.
Domain	Attention paid mostly to the cognitive and psychological aspects of transfer.	Attention also paid to the motivational and socio-cultural factors that affect transfer.

2.4.4. Transfer Framework

Based on the abovementioned contemporary views of transfer, the KSU Physics Education Group (Rebello, Zollman et al., 2005) developed an analytical framework by considering transfer as a dynamic process. The framework is based on a two-level framework presented by Redish (2003) as shown in Figure 2-1. The first level refers to associations between knowledge elements, while the second level refers to factors that control these associations.

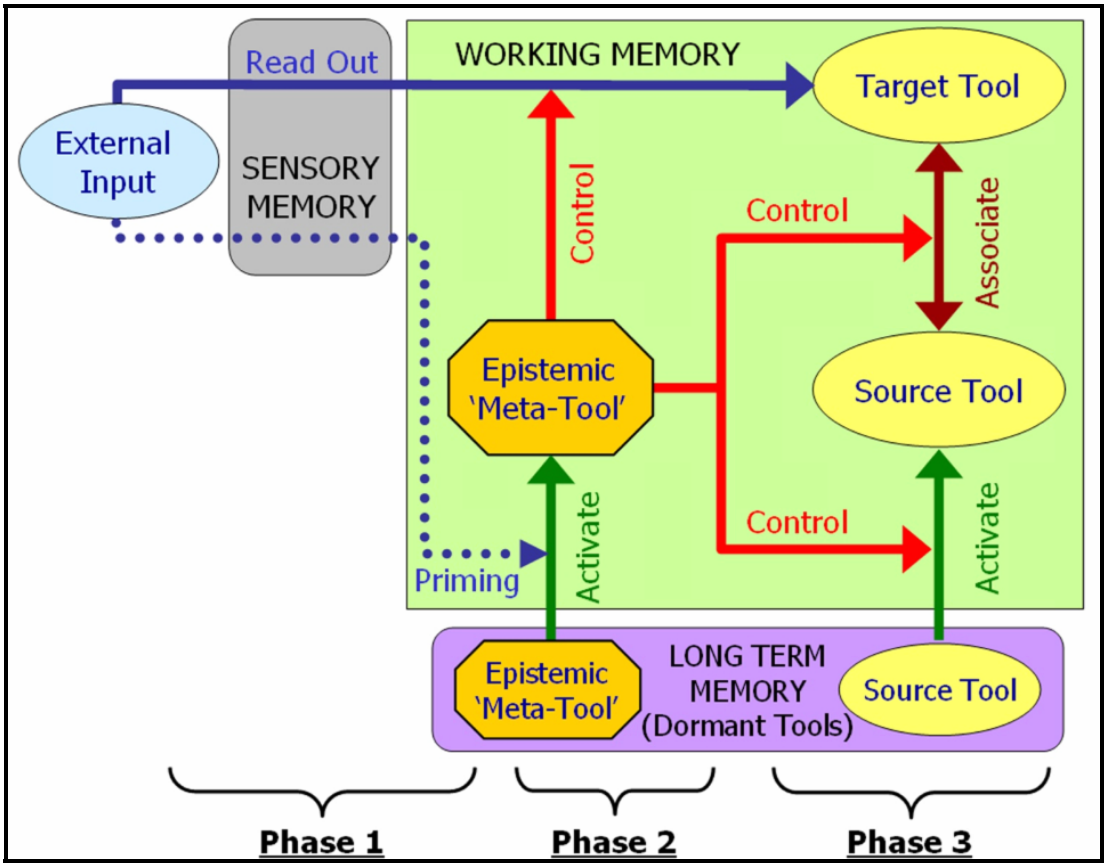
Figure 2-1: Two-level Framework for Transfer



In our model, transfer is the dynamic creation of associations between a learner's prior knowledge and information that is read-out by the learner from a new situation (e.g., a given physics problem). The learner's epistemic mode[1] controls read-out of information as well as activation of prior knowledge. According to Rebello (2005), this transfer model "does not make distinctions between productive and unproductive associations that a learner might make in a given situation, rather it examines all possible associations that a learner might make in a given situation." This transfer model that describes the dynamics of the process of knowledge construction in a new situation is shown in Figure 2-2.

[1] Epistemic mode is often referred as epistemic resources by other researchers.

Figure 2-2: Model for Transfer of Learning



This framework consists of four elements. First are the external inputs provided by the interviewer and interview materials. Tools can be acquired in a prior learning (source) context or in the present transfer (target) context: Source tools are the prior knowledge or experiences including those gained from earlier situation in the interview. Target tools include information about the new context that the learner wants to get. The third element in the framework is the workbench which includes dynamic mental processes that help the learner associate the source and target tools. The fourth element is the answer (not shown in the figure above) which is either an intermediate stopping point or a final conclusion of the reasoning process and sometimes a starting point of metacognition. Often it is the created association between the target tool and the source tool. When these two elements are tightly associated in the answer, we can expect that in future problem scenarios they will be both activated together i.e. they are inseparable so that if one is activated the other one is activated with it.

In this research, we focus only on the association between the target and source tool, which we consider as transfer. Although we recognize that other elements in the abovementioned framework are important, they are beyond the scope of this study and could be the interests for future research.

In summary, we have defined transfer as the creation of associations between read-out information and prior knowledge as shown in Figure 2-1. The association is controlled by other factors e.g. learners' epistemology, motivation etc. The way in which the association is created is shown in Figure 2-2. In this research project we focus on the created association.

2.5 Assessment of Transfer of learning

2.5.1. Methods to Assess Transfer of Learning

Broadly speaking there are two kinds of techniques typically used to assess transfer of learning: "one-shot" assessment and "graduated prompting". Both methods have been used in this study.

From the perspective of transfer as a static process, one-shot assessment techniques typically ask students to solve a particular problem that apply concepts the students are supposed to have learned during the initial learning situation. Typical end-of-chapter problems similar to those asked on most exams are examples of one-shot assessments of transfer. Studies (Brown, 1983; Bruer, 1993; Bransford and Schwartz, 1999) found that one-shot assessments often underestimate transfer of learning because they focus on whether or not students are able to correctly solve the problem rather than what knowledge and skills they bring to bear in the process of constructing the solution. A more accurate measure of transfer may benefit the researcher by providing insights into the ease with which students are able to learn how to solve the new set of problems as opposed to whether or not they could solve the problems in a one-shot test (Singley and Anderson, 1989).

From the contemporary view of transfer as a dynamic process, graduated prompting strategy has been developed and used during assessment and feedback situations. Researchers (e.g. Campione, 1987; Newmann, 1989) used prompting to assess the ease with which students were able to transfer their knowledge from one situation to another. The technique of graduated prompting is usually used in an individual interview situation. An example of a general prompt is "Can you think of something that you did earlier that may help you solve the question?" The technique of graduated prompting provides a more valuable assessment strategy on transfer of learning than simple one-shot assessments.

2.5.2. Assessing Transfer from Algebra and Physics

Bassok (Bassok and Holyoak, 1989; Bassok, 1990) investigated transfer between algebra and physics by noticing that these two subjects have an "extremely close formal relationship" and are usually taught at the same time in high school. An interesting "transfer asymmetry" was found. Most students who learned algebra could apply their algebra knowledge to isomorphic physics problems, however very few of the students who learned physics could apply their knowledge to the isomorphic algebra problems. The authors believed this asymmetry was because algebra instruction emphasizes the abstract nature whereas physics instruction emphasizes the physical concepts. Algebra is more context-free compared to physics. So it was not surprising that students were more successful in transferring algebra knowledge to physics than vice versa.

More recently, Tuminaro (2004) examined why algebra-based physics students perform poorly on mathematical problem solving tasks in physics. He believed that instead of the lack of algebra knowledge, students did not know how to apply the mathematical skills to particular problem situations in physics. In his dissertation, Tuminaro proposed a cognitive framework to analyze introductory students' use of mathematics in physics (Tuminaro, 2004). Tuminaro's framework introduced the relevant cognitive structures, which he calls mathematical resources, and the relationship between these structures for describing and analyzing mathematical thinking and problem solving. He also used his framework to explain why students made mathematics errors when solving physics problems. The reasons cited by Tuminaro can be summarized as next:

1) Using an inappropriate resource: Mathematical resources are knowledge elements that are activated in problem solving. Resources are neither right nor wrong. In this type of error, a resource that is activated cannot be mapped into a useful facet for the particular problem situation. An example that is relevant to our research would be a student attempting to use summation when the problem requires the use of integration.

2) Using an appropriate resource, but mapping it inappropriately: This error occurs when an appropriate resource is activated, but it is inappropriately mapped into a particular problem situation. An example that is relevant to our research would be a student using integration but not integrating over the correct variable or limits of integration.

3) Appropriate epistemic game, but wrong move within that game: An epistemic game is a pattern of activities that use particular kinds of knowledge to create new knowledge or solve a problem. Students can play an epistemic game that is appropriate for solving a particular problem, but use an inappropriate interpretive device (i.e. make an inappropriate move within an epistemic game) to cause a process error. This error would be following an appropriate problem solving procedure, but incorrectly completing one of the steps in the procedure.

4) Inappropriate framing leading to an inappropriate epistemic game: Frames are expectations that determine how individuals interpret situations or events. If the student inappropriately frames the problem situation, then it can lead him to play an inappropriate epistemic game and cause error. This situation would be misinterpreting the problem and therefore using an incorrect problem solving schema. An example that is relevant to our research is a student who expects to find the electric field E whenever she/he sees the constant k, regardless of the fact that the problem actually asks for electric potential V.

In this research, we examine the problem solving by students from the perspective of the kinds of errors enumerated by Tuminaro. However, since he only used traditional physics problems in his research, it would be important to look beyond his framework. We used both traditional and non-traditional problems in our research.

2.5.3. Assessing Transfer from Trigonometry to Physics

Ozimek (2004) examined the retention and transfer from trigonometry to physics at the introductory college level. From the traditional view of transfer, he found no evidence of transfer based on the correlation between performance on online trigonometry problems and physics problems that utilized the same trigonometry concept. However, from the contemporary view, he found students do transfer what they learned in their trigonometry class to their physics class, both from the perspectives of Preparation for Future Learning (Bransford and Schwartz, 1999) and Actor-Oriented Transfer (Lobato, 2003). Overall, Ozimek's research clearly demonstrated the limitation of one-shot measurements in detecting transfer. Furthermore Ozimek showed that transfer was detectable when viewed from a more contemporary perspective. Ozimek's results were consistent with work by other researchers on transfer of learning.

2.6 Teaching Materials and Instructional Strategies

2.6.1. Calculus and Physics

Integrated curricula and textbooks have been developed to teach calculus and physics concurrently to maximize the possibility that students can apply their knowledge in calculus in the learning of physics (e.g. Rex and Jackson 1999; Dunn and Barbanel 2000). Dann and Barbanel (2000) argued that many of students' difficulties were because physics and calculus were taught as separate courses and the teachers in each of these courses probably knew little about the other course. They found that integrated physics and calculus could potentially be found useful. However, they also found that physicists and mathematicians usually speak different languages and use different notations. Thus, both students and faculty members felt that the integrated courses were very challenging

in terms of time and work load. After teaching three years of integrated calculus and physics courses, Yeatts and Hundhausen (1992) discussed their own insights in talking about the difficulties that students encounter when transferring their knowledge from calculus to physics. The difficulties cited by them can be summarized as follows:

- Notation and symbolism: Calculus and physics use different notation and symbols for the same concepts, which impedes transfer. For example, $V = \int E \cdot dl$ is related to the line integral $\int F(x)dx$, but students are often unable to recognize the relationship between them because of the different notations used.
- The distraction factor: Students tend to make unnecessary mistakes while paying attention to other unfamiliar aspect of the given problems. Some contextual features of the physics problems tend to grab students attention so that students' attention to mathematics sometimes is distracted by the physics problem.
- Compartmentalization of knowledge: Weaker students tend to compartmentalize their knowledge so they make distinction between calculus and physics. They frame knowledge in these two courses differently that prevents them from seeing connections between calculus and physics.

Although integrated curricula have proved useful, for practical reasons, calculus and physics are still taught as separate subjects in most of the colleges and universities. Therefore, it is important to take a close look at how students transfer the knowledge they learned in calculus class when they solving a problem in calculus-based physics courses, and find strategies to facilitate the transfer process given the constraints of most universities that require students to take calculus and physics asynchronously from different instructors, residing in different departments, and who may not necessarily communicate the goals and needs of their students with each other.

2.7 Chapter Summary

In this chapter, we reviewed the literature related to this study, including research on calculus education, problem solving in physics and transfer of learning. Calculus is the "language of change" and has been considered one of the fundamental courses for engineering and science majors. The calculus reform movement began because of mathematicians' dissatisfactions with undergraduate calculus education. The calculus reform movement suggested changes in calculus instruction that emphasized deep understanding of calculus concepts and their applications in problem situations. In spite of the advances of the calculus reform movement, multiple-choice and short-answer questions still appear to be the dominant tools to assess students' learning in calculus.

Similar to mathematics education, the teaching and learning of physics has also become a focus or research over the past few decades. There has been extensive research on problem solving in physics. Many studies found that students tended to use means-ends analysis to solve physics problems. Typically, they tended to grab an equation and plug in numbers when solving physics problems. Researchers agreed that traditional end-of-chapter problem did not help students to develop conceptual understanding of physics. Although these problems are still the most commonly used in physics courses, researchers have been exploring other types of non-traditional problems and instructional strategies to facilitate problem solving. Context-rich problems, Physics Jeopardy and other types of ill-structured problems have been developed to help students become successful problem solvers. In this study, we use both traditional end-of-chapter problems and non-traditional physics problems.

Closely tied with research on problems solving, is research in the area of transfer of learning. Transfer of learning is defined as the ability to apply what one has learned in one situation to a different situation. Traditionally transfer has been measured by examining whether students can successfully apply what they have learned to new isomorphic problems. Contemporary perspectives view transfer from the students' point of view as an active, dynamic process where students construct knowledge in the new situation, rather than merely applying prior knowledge. Bransford and Schwartz view transfer in terms of Preparation for Future Learning (PFL). They focus on whether

students can learn to problem-solve in a new context. Lobato's Actor-Oriented Transfer (AOT) model views transfer as the personal construction of similarities between the two contexts. Our own framework, views transfer as the dynamic processing by creation of associations between a learner's prior knowledge and information that is read-out by the learner from a new situation.

The assessment of transfer is influenced by the perspective one adopts to define transfer of learning. "One-shot" assessment and "graduated prompting" are the two commonly used techniques to assess transfer of learning. While the former is more consistent with the traditional perspective, we have used both of these methods in this study. Prior research has investigated students' transfer of knowledge from algebra to physics and from trigonometry to physics. However, there has been no significant research on transfer of learning from calculus to physics. Therefore, assessing transfer of learning from calculus to physics is the central focus of this study.

Tuminaro proposed a cognitive framework to analyze and describe introductory students' use and understanding of mathematics in physics. However, since he only used traditional physics problems in his research, it would be important to look beyond this framework in our research. In the next chapter we describe the theoretical framework we developed in this study and reframe our research questions through the lens of our framework.

CHAPTER 3 - THEORETICAL FRAMEWORK & REFRAMED RESEARCH QUESTIONS

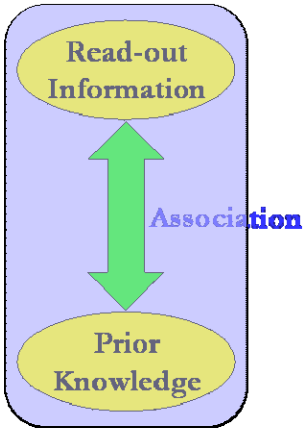
3.1 Chapter Overview

In this chapter, we present the theoretical framework that we developed and discuss its applications for characterizing transfer of learning during problem solving. The framework is grounded in the data that we collected in this study. This framework helps us reframe our original research questions in ways that are more meaningful to the project.

3.2 Theoretical Framework

Based on the contemporary views of transfer reviewed in Chapter 2, we developed a theoretical framework that distinguishes between different kinds of transfer processes relevant to problem solving (Rebello, Cui et al., in press). The framework is based on Redish's two-level framework of associations and control discussed in Chapter 2, as shown in Figure 3-1. Our framework focuses on the types of associations. Although we recognize that the factors that control the activation of these associations are important, they are beyond the scope of this study.

Figure 3-1: Association between Read-out Information and Prior Knowledge



3.2.1. Two Kinds of Associations

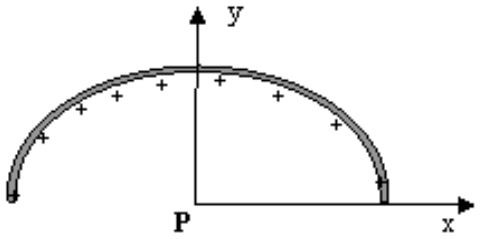
Our model of transfer is based on a framework presented by Redish (2003), who applied the research results from cognitive psychology to physics education. We view transfer as the dynamic creation of associations between prior knowledge and read-out information from a given problem by the learner. We found that there are two kinds of associations that a learner can create in a problem solving scenario.

3.2.1.1. First Kind of Association

One kind of association involves assigning information read out from a problem to an element of the learner's prior knowledge. An example is reading out a numerical value from the problem statement and assigning it to a particular physical quantity. For instance, in Figure 3-2, the learner needs to recognize that the integration limits are from 0 to π, and more specifically that these limits must be plugged into a particular equation. These kinds of associations are usually concrete, firmly established in the learner's mind and can be clearly articulated by the learner. These include, but are not limited to plug-and-chug type of associations.

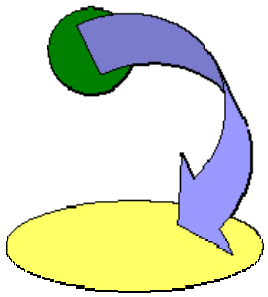
Figure 3-2: Sample problem that requires students to identify the limits of integration

A thin non-conducting rod is bent into a semicircle of radius R, charge Q spread uniformly along it. Find the magnitude and direction of electric field E at point P at the center of the semicircle.



This kind of association -- assigning the value of a problem variable to a known knowledge element -- is shown in Figure 3-3. We use the green circle to represent the read-out information, and the yellow oval to represent the knowledge element of the learner's prior knowledge. In the example described in Figure 3-2, the green circle would refer to π and the yellow oval refers to the upper limit of integration. So, the learner associates π with the upper limit of integration.

Figure 3-3: The First Kind of Association



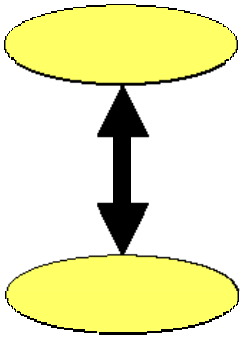
3.2.1.2. Second Kind of Association

The other kind of association occurs when the learner connects a knowledge element read-out from the problem statement with an element of the learner's prior knowledge. This association is more abstract and typically more tenuous than the first kind of association discussed above.

For instance, in the sample problem described above (Figure 3-2), the learner needs to think about what the relationship is between electric charge and electric field. To solve the problem, the learner has to know how these two physical constructs or knowledge elements are interrelated.

This kind of association between two different knowledge elements is shown in Figure 3-4. We use the two yellow ovals to represent the two knowledge elements. In the example described in Figure 3-2, one yellow oval refers to the electric charge and the other refers to the electric field.

Figure 3-4: The Second Kind of Association



3.2.2. Two Kinds of Transfer

These two kinds of associations that a learner might make in a problem solving scenario are related to two different kinds of transfer processes—horizontal and vertical transfer.

3.2.2.1. Horizontal Transfer

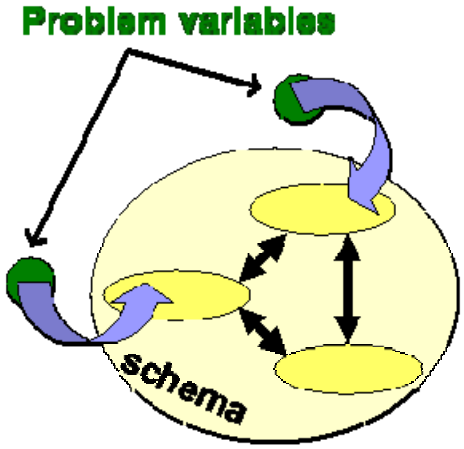
In horizontal transfer, the learner reads out information from a problem scenario that activates a pre-created schema[1] or internal representation that is aligned with the information provided in the problem and also what is asked for in the problem. This alignment between the provided information and the internal schema is the key to solving the problem. If such alignment or assignment does not naturally occur, i.e. if the external problem representation does not match the internal problem representation, the learner is left with no recourse to solve the problem, using their currently activated schema. A typical example of horizontal transfer occurs when learners solve plug-and-chug problems at the ends of chapters in some science and mathematics textbooks. The learner reads the problem statement, which explicitly provides information in terms of the required variables, though most likely without using the notation. For instance, in the sample

[1] We use the term 'schema' to refer to a pre-created set of tightly associated knowledge elements often activated simultaneously, a.k.a. 'mental model', 'internal representation', 'knowledge structure', 'coordination class' etc.

problem, the provided information includes the shape and the length of the electric charge distribution, the magnitude of the total electric charge, and clearly states the goal of the problem such as finding the electric field at certain point. After reading out this information from the problem, the learner activates a particular equation (which in this case is the mathematical representation of the learner's schema in this situation) from their memory and plugs the variables into this equation to solve for the required unknown variable. The learner does not need to consider the underlying assumptions of the situation where the equation may be applicable or even choose between several different equations.

Horizontal transfer is represented in Figure 3-5. We use the green circles to represent the read-out information i.e. the problem variables. The yellow oval represents the knowledge elements which form a certain schema and the black arrow represents the associations between knowledge elements. Schema is a set of knowledge elements, which was represented as the big buff circle. Figure 3-5 demonstrates that the horizontal transfer is nothing but repeated use of the first kind of association discussed above.

Figure 3-5: Horizontal Transfer: associations between problem variables and knowledge elements of a pre-existing schema

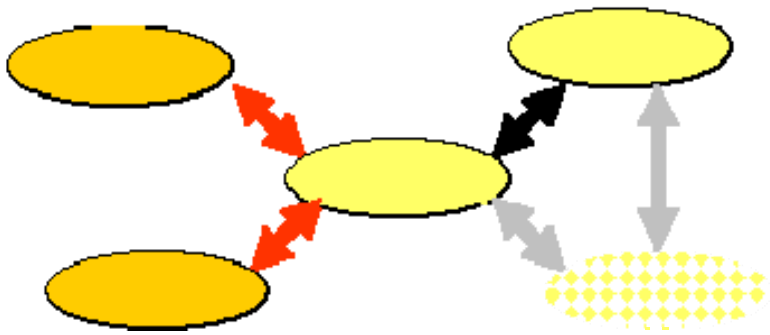


3.2.2.2. Vertical Transfer

In this kind of transfer -- vertical transfer -- the learner typically does not have a preconceived schema that aligns with the problem information. Rather, the learner recognizes features of the problem scenario and then constructs a new schema through successive activation and addition of associations between knowledge elements. Alternatively, the learner may activate more than one schema and go through an internal process to decide which schema is appropriate or blend them together to construct a new schema which has elements of both.

Vertical transfer is represented in Figure 3-6. The yellow ovals represent the knowledge elements which form a certain schema and the black arrow represents the associations between knowledge elements. In vertical transfer, new knowledge elements are incorporated into the schema and some old knowledge elements are discarded to form a new schema. In Figure 3-6, the faded yellow oval represents the abandoned knowledge element; the gray arrows represent the abandoned associations; the orange ovals represent the new knowledge elements; and the red arrows represent the new associations. The figure represents how the new schema was formed based on the old schema. At times a learner must choose between competing schemas for the problem situation. Choosing the most productive model or representation from several representations, depending upon the problem situation, is a key feature of vertical transfer.

Figure 3-6: Vertical Transfer: creation and suppression of associations between knowledge elements to change an existing schema into a new one.



Any problem solving process involves both horizontal and vertical transfer. However, when solving most end-of-chapter physics problems, students tend to use the means-ends analysis (Sweller, 1988). Students focus on finding a specific numerical answer, and this focus will direct their attention to the manipulation of equations instead of thinking in which situation those equations were applicable. They spend little effort on carrying out a qualitative analysis involving other representations. In this case, few problems in most science or mathematics textbooks require vertical transfer from students' perspective since students do not need to construct a new schema. On the other hand, most real world problems where there is no single easily identifiable equation or strategy known to the learner, involve vertical transfer. Often the learners must either create their own schema on the spot by associating individual knowledge elements, or decide between one or more schemas, or blend one or more schemas together. This process can often be long and difficult as the learner unsuccessfully tries a known schema or internal representation of the problem situation and then changes the internal representation to one that matches the external representation of the problem situation. After the required schema is constructed, the learner can engage in horizontal transfer to solve the problem. If the newly created schema is found to be useful and the associations are strong enough for the schema to be preserved, the learner may store the schema in the long-term memory and activate it as a whole for use in a later problem.

3.2.2.3. Alignment with Others' Views

Our ideas of horizontal and vertical transfer described above are not new. There is a vast body of literature on knowledge and conceptual change that expresses ideas along these lines.

Several decades ago Piaget (1952) proposed two mechanisms of conceptual change – assimilation and accommodation. Assimilation occurred when new information was incorporated into a learner's internal knowledge structure without modification of the knowledge structure. Accommodation meant new information resulted in the learner changing their internal knowledge structure to make sense of this new information. Although Piaget's ideas focused on conceptual change and not on transfer, the

mechanisms of assimilation and accommodation align closely with horizontal and vertical transfer respectively.

Gagne (1970) distinguished Lateral and Vertical transfer. Lateral transfer occurs when knowledge is transferred within a same difficulty level, which means there is no need to add a new knowledge element. Lateral transfer is similar to horizontal transfer. Vertical transfer is required when moving from a lower-level difficulty task to higher-level difficulty task. Vertical transfer is similar to our vertical transfer.

Broudy (1977) similarly identified at least two kinds of knowing – applicative (knowing what and how) and interpretive (knowing with) knowing. Applicative knowing includes clearly articulated schema that a learner uses in a given situation. Interpretive knowing, which is much more subtle and intangible, refers to a sense of intuition or gut instinct that a learner brings to bear as he/she makes sense of a new situation and frames the problem. Broudy's notions of applicative and interpretive knowing align closely with our ideas of horizontal and vertical transfer respectively.

The ideas of horizontal and vertical transfer are consistent with the ideas that have been used to design instruction for conceptual change. Karplus' (1974) Learning Cycle and more recently Hestenes' (1987) Modeling Cycle refer to the Model Development phase during which a learner constructs a model to explain their observations of phenomena. This phase is followed by the Model Deployment phase during which the learner applies the model in a new situation. Model Development involves vertical transfer since it relates to the learner building a new schema based on experiences. Conversely, model deployment involves horizontal transfer since the learner has to apply the schema to a new situation.

Salomon and Perkins (1989) distinguish between Low Road and High Road transfer. Low road or more typically near transfer occurs when the scenario in which original learning had occurred is similar to the new problem scenario so that the learner can successfully apply preconceived problem-solving processes. Low road transfer is similar to horizontal transfer. High road or more typically far transfer is much more challenging in that it requires the learner to abstract the new situation and engage in reflection and metacognition to help construct a way to solve the problem. High road

transfer is similar to vertical transfer. Thus, the distinction between low road and high road transfer align with the distinction between horizontal and vertical transfer respectively.

Bransford and Schwartz (1999) compared two measures of transfer – Sequestered Problem Solving (SPS) and Preparation for Future Learning (PFL). Sequestered problem solving (SPS) focuses on whether students can directly apply their learning to a new situation, without any scaffolding or support. Preparation for future learning (PFL) focuses on whether their learning has prepared them to learn in the future. To measure transfer from the PFL perspective we must observe whether a learner can bring to bear their earlier experiences to learn to construct new knowledge that would enable them to solve the problem in the new situation. Bransford and Schwartz point out that most traditional transfer measures focus on SPS rather than PFL and consequently fail to find evidence of transfer. SPS view of transfer focuses primarily on horizontal transfer in that it assesses whether a learner can apply their existing schema to new situations. SPS does not even consider the possibility that a learner may need to learn how to solve the problem in the new situation. Alternatively, PFL view of transfer focuses primarily on vertical transfer in that it assess whether a learner can create a new schema to solve the problem.

Jonassen (2003) has distinguished between well-structured and ill-structured problem solving, which also align with our ideas of horizontal and vertical transfer. Well-structured problems have clearly defined information and goals. Therefore, they are akin to problems that require mainly horizontal transfer. Ill-structured problems on the other hand have multiple solutions, may require the learner to choose between several competing internal representations and may require the learner to question several underlying assumptions about what model or representation is applicable in the given situation. Unstructured problems typically require significant vertical transfer.

DiSessa and Wagner (2005) distinguish between Class A and Class C transfer. Class A transfer, occurs when a learner applies "well prepared" knowledge such as a coordination class to a new situation. Class A transfer is similar to horizontal transfer. Alternatively, Class C transfer occurs when "relatively unprepared" learners use prior knowledge to construct new knowledge. Class C transfer is similar to vertical transfer.

Schwartz and Bransford (2005) suggested the notions of efficiency and innovation in transfer. Efficiency refers to a learner's ability to rapidly recall and apply their knowledge in a new situation, while innovation is their ability to restructure their thinking or reorganize the problem scenario so that it becomes more tractable than before. Developing efficiency in problem solving is analogous to engaging in horizontal transfer while innovation is analogous to vertical transfer.

Most recently, Jonassen (in press) also suggested that presenting examples or analogues of how similar problems were solved, which he called case reuse, was the most common strategy to develop students' problem solving abilities. He distinguished two kinds of case reuse—"Script Reuse of Cases" and "Schema Induction and Transfer from Worked Examples". "Script Reuse of Cases" means to retrieve cases (examples) from previous solved problems from memory and then directly reuse it in the new problem situation without any change. Script Reuse of Cases is analogous to horizontal transfer. "Schema Induction and Transfer from Worked Examples" means first to analyze the worked examples and then construct a new schema based on the given problem situation. "Schema Induction and Transfer from Worked Examples" is analogous to vertical transfer.

The next table (Table 3-1) summarizes how the horizontal and vertical transfers align with other researchers' views.

Table 3-1: Alignment of Horizontal and Vertical Transfer with Others' Views

Horizontal	Vertical	References
Assimilation of new experiences	Accommodation of new experiences	Piaget (1952)
Lateral Transfer	Vertical Transfer	Gagne (1970)
Uses Applicative knowledge	Uses Interpretive knowledge	Broudy (1977)
Involves Deductive reasoning: Model Deployment	Involves Inductive reasoning: Model Development	Hestenes (1987)
Low Road Transfer	High Road Transfer	Salomon & Perkins (1989)
Structured, traditional problems: Few internal representations activated repeatedly	Ill-structured, non-traditional problems: Choosing, using and constructing multiple internal representations	Jonassen (2003)
Sequestered Problem Solving	Preparation for Future Learning	Bransford & Schwartz (1999)
Class A Transfer	Class C Transfer	diSessa & Wagner (2005)
Efficiency	Innovation	Schwartz, Bransford & Sears (2005)
Script Reuse of Cases	Case Induction and Transfer from Worked Examples	Jonassen (in press)

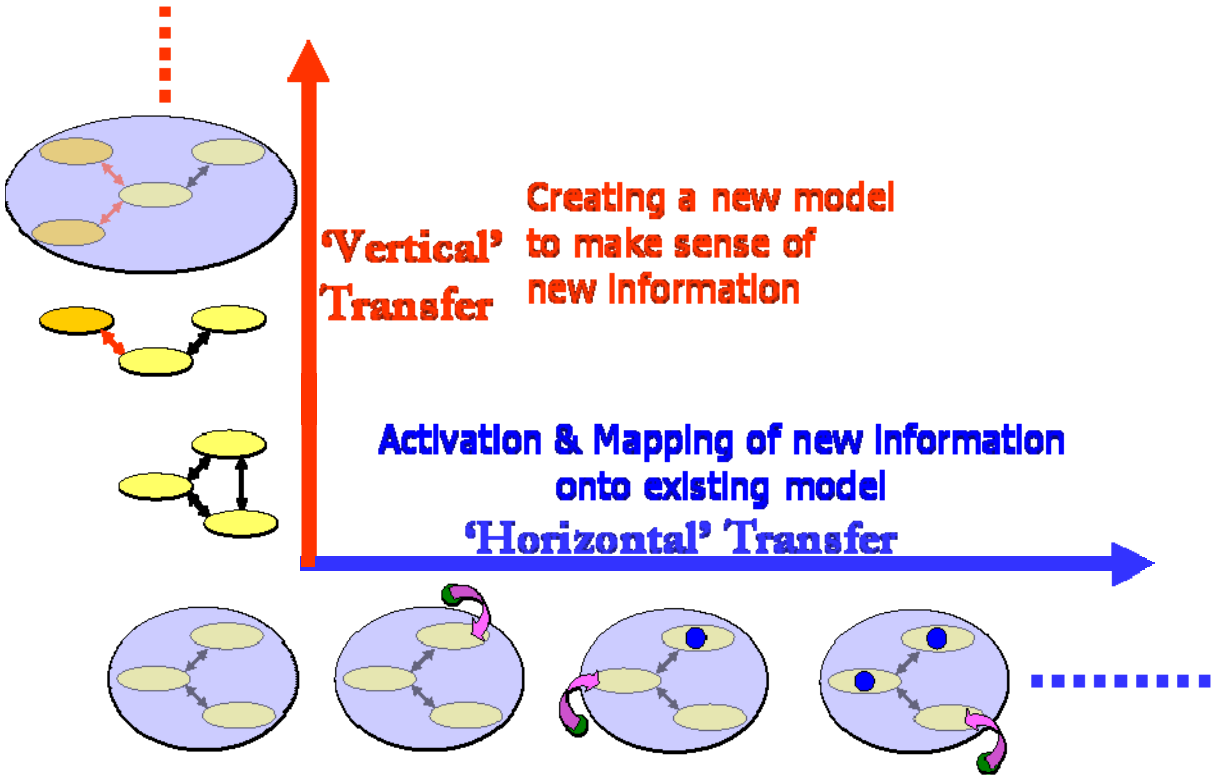
3.2.3. Theoretical Framework

Based on the notions of horizontal and vertical transfer, we represent the theoretical framework of this study as shown in Figure 3-7. Students need to employ both horizontal and vertical transfer when they solve any problem, and not just in the context of physics. This framework will help us differentiate horizontal and vertical transfer, and provide a lens that enables us to assess students' transfer of knowledge accordingly.

Figure 3-7 shows a metaphoric graph with horizontal and vertical axes each representing the corresponding type transfer. Near the origin of the graph is the learner's starting schema that is activated in a problem situation. If the learner engages in horizontal transfer, represented by progression of images along the horizontal axis, the schema itself remains unchanged, but different elements in the schema are associated with different input variables of the problem. On the other hand, if the learner is faced with a

problem situation in which the schema that was initially activated does not match the external problem representation then the learner must successively modify their schema by activating new and productive associations and suppressing old, unproductive associations until they arrive at a schema that is useful in solving the problem at hand. The conceptual trajectory of this learner, manifested in terms of his/her changing schema, is represented by a progression of images along the vertical axis. The value of the two-dimensional representation depicted in Figure 3-7 is that it allows one to visualize problem solving that may involve both horizontal and vertical transfer, as is often the case.

Figure 3-7: Theoretical Framework: showing the distinction between horizontal and vertical transfer



3.2.4. Some Other Points

It is worthwhile to mention this horizontal and vertical transfer framework evolved through our research. It is grounded in the data that we collected and helps us to describe transfer in the problem solving context. We first identified the distinction between horizontal and vertical transfer when we analyzed our interview data, then we found this idea was consistent with other researchers' views. The fact that our ideas align with those researchers lends credibility to our theoretical framework. We did not choose to use others' terminology because of the uniqueness of this framework. It is focused on model construction through the activation of associations between individual knowledge elements. There is no single model or framework that captures the essence of our framework. It can be applied not just in problem solving, but in any reasoning task. Moreover, it encapsulates many of the features of these other frameworks and shows how they are in fact closely related to one another.

There are a few issues we must keep in mind when we distinguish between horizontal and vertical transfer.

First, the two transfer processes, though distinct from each other, are not mutually exclusive in any way. A given problem scenario usually requires a learner to engage in both kinds of transfer processes. Sometimes one kind of transfer is more dominant than the other one in a given problem solving process. For example, students typically first need to activate certain calculus/physics schemas (which involves vertical transfer), and then assign the problem variables into their activated schemas (which involves horizontal transfer) when they solving any traditional physics problems. Students might not even recognize that they have already engaged in vertical transfer since the horizontal transfer seems more dominant during the problem solving process. However, we can not say a certain thinking process only involved one kind of transfer. Indeed, Schwartz, Bransford and Sears (2005) argue that we must prepare learners to engage in both kinds of transfer rather than one at the expense of the other. They point out that there is indeed value in developing efficiency, or horizontal transfer, because it frees up the mental resources that allow the mind to focus on other efforts, such as being more innovative in other ways.

Second, there are often no predefined universal normative criteria one can apply to identify whether a particular process involves horizontal or vertical transfer. If a learner already possesses a well-prepared schema, then from that learner's perspective, a particular task might require only horizontal transfer, i.e. applying this well prepared knowledge in the present scenario. However, a different learner who does not possess this schema or internal representation may need to construct a new one to solve the particular problem. Therefore, this learner has to engage in vertical transfer to solve the same problem. This criterion could be used to distinguish between experts and novices. A particular task that might be perceived as requiring horizontal transfer by an expert might in fact be perceived as requiring vertical transfer by a novice. In the same vein, what is perceived as vertical transfer by one expert may be perceived as horizontal transfer by another expert, depending upon their assumptions about the learner's level of expertise or intellectual development. Therefore, any distinction that we attempt to make between the two kinds of transfer must be tied to a particular perspective. In keeping with the contemporary transfer perspective, it is most useful to view transfer processes from the perspective of the learners who engage in it rather than from a researcher's perspective. In keeping with the contemporary transfer perspective, it is most useful to view transfer processes from the perspective of the learners who engage in it rather than from a researcher's perspective. Furthermore, as we mentioned before, any particular process involves both horizontal and vertical transfer, which means if one person uses a uniform normative criteria, she/he is likely to find both kinds of transfer within the same process. However, the fact that any particular process involves horizontal and vertical transfer does not imply that there is no need to define a normative criterion to identify the two types of transfer. This only means that the same process can be labeled differently (horizontal or vertical) by different people.

Finally, it is important to recognize that the distinction between horizontal and vertical transfer depends upon features of the overall learning context. These contextual features may include, but are not limited to, a learner's or teacher's expectations and culture of a given situation. For instance, in a mathematics course that focuses on learning how to solve quadratic equations, any problem that has a real world connection may be perceived as requiring vertical transfer. The same problem, however, in a physics course

that routinely expects students to solve word problems that invoke real-world situations might be seen as a regular plug-and-chug problem that requires only horizontal transfer.

The distinction between horizontal and vertical transfer has provided a theoretical framework for this study. In the following section we reframe our previously stated research questions from the point of view of horizontal and vertical transfer.

3.3 Reframed Research Questions

Based on the framework, we revisited the initial three research questions, and reframed them so as to assess students' knowledge transfer from calculus to physics in terms of both horizontal and vertical transfer. Ultimately, we looked for the instructional strategies that can facilitate both horizontal and vertical transfer. Therefore, this research had been divided into three phases.

3.3.1. Phase I: Horizontal Transfer

In phase I, we investigated student's horizontal transfer of calculus knowledge when they solving traditional physics problems. Horizontal transfer was explored by examining students' solutions to problems on tests and exams administered in class as well as problems that they were asked to solve during interviews. During the interview, students were asked to solve physics problems that were similar to their homework or exam problems. These problems required the use of simple integration or differentiation. At the outset of our research study these typical physics problems would involve horizontal transfer because from our (i.e. the researchers') perspective the problems did not require students to construct or to even choose between competing schemas or mental models to solve the problem.

Our original first research question was: *"To what extent do students retain and transfer their calculus knowledge while solving problems in introductory physics?"*

Now from the horizontal transfer perspective, it was reframed into two new research questions:

Q1: Have students retained their calculus schemas to solve calculus problems?

A student's initial knowledge is the precondition for transfer (Bransford, Brown et al., 1999). Therefore, we needed to investigate the extent to which the students had retained their calculus knowledge i.e. their schemas to solve calculus problems. We asked students to solve pure calculus problems when they were taking the physics course to ascertain to what extent they retained their calculus schema.

Q2: Can students associate their physics problem variables with their calculus schema i.e. can they engage in horizontal transfer?

In addition to investigating whether students had retained their calculus schemas, we also needed to examine the extent to which they could associate the physics problem variables with their calculus schema. In other words, we sought to investigate whether students could read out and assign the proper information from the physics problem to their calculus schema. We reviewed students' physics exam problems which involved calculus and conducted individual semi-structured interviews to explore this question.

3.3.2. Phase II: Vertical Transfer

In phase II, we investigated student's vertical transfer of calculus knowledge when they solved a physics problem. We used non-traditional physics problems which required students to engage in vertical transfer in several ways. Unlike end-of-chapter problems, the students could not apply a pre-constructed schema or mental model to solve these non-traditional problems. Because these problems were unfamiliar to students, they had to construct a schema or mental model on the spot to solve these problems. Thus, these problems provided a useful context in which to examine vertical transfer by the students.

Our original second research question was: *"What mental processes are involved as students transfer what they have learned in calculus to introductory physics?"* Now from the perspective of vertical transfer the question was reframed to two new research questions:

Q3: Can students appropriately activate their calculus schemas in the context of physics problems?

The ability to select and to activate appropriate schema from among competing schema pertains to vertical transfer. We presented students with "Compare and Contrast" problems to examine the process by which they made a decision of when to activate their calculus schema. The "Compare and Contrast" problem presented situations in which interviewees would either need to use integration or summation. We also asked students to articulate their underlying reasons for each.

Q4: Can students deconstruct and reconstruct their schemas to solve a physics problem?

The ability to construct new schema from old schema by activating and suppressing associations between knowledge elements pertains to vertical transfer. To address the aforementioned question, we asked students to solve physics Jeopardy questions to assess whether they could break down and reconstruct their schema to answer these questions. The Jeopardy problems presented interviewees with an intermediate step in the form of a mathematical integration and asked students to construct a physical scenario relevant to the integral provided. Therefore, they required students to deconstruct their existing schema (a mathematical expression) and construct a new using a different representation (physical scenario).

3.3.3. Phase III: Instructional Strategies

In phase III, we sought input from teachers regarding possible instructional strategies to facilitate both horizontal and vertical transfer. Our original third research question was: *"What strategies may facilitate students transfer from calculus to physics?"* Now from the perspective of both horizontal and vertical, it could be reframed to a new research question:

Q5: What instructional strategies can facilitate both horizontal and vertical transfer?

We interviewed both students and faculty for their feedback on instructional strategies and other relevant classroom practices. We interviewed experienced teachers from both mathematics and physics departments and asked them for their suggestions.

3.4 Chapter Summary

In this chapter, we introduced the notion of two kinds of associations -- assigning the problem variable to a knowledge element and associating two knowledge elements during problem solving. The first kind of association was intrinsic to what we called horizontal transfer. The second kind of association was intrinsic to what we called vertical transfer. Students need to employ both horizontal and vertical transfer when they solve any problem in the context of physics and accordingly our theoretical framework encompasses both horizontal and vertical transfer. During horizontal transfer, the schema itself remains unchanged, students need to associate different input variables of the given problem situation with the elements in the schema. During vertical transfer, students need to modify their schema by activating new and productive associations and suppressing old, unproductive associations until they arrive at a schema that is useful in solving the given problem. This framework provides a lens that enables us to assess students' transfer of knowledge accordingly. The notions of horizontal and vertical transfer are not new, and in fact are consistent with decades-old ideas of conceptual change. The fact that our ideas align with those of several researchers lends credibility to our theoretical framework.

In light of our theoretical framework of horizontal and vertical transfer, we also reframed our research questions. In the next chapter we describe the research design that we used to examine the reframed research questions.

CHAPTER 4 - RESEARCH DESIGN

4.1 Chapter Overview

In this chapter we describe the research setting (4.2) and participants (4.3) of this study, followed by a detailed description of research plan (4.4). We also discuss some common features (4.5) of all interviews and describe the interview analysis methods (4.6) used in this study.

4.2 Research Setting

This study focuses on assessing students' transfer of learning from calculus to physics at the college and university level. This study was conducted at Kansas State University (KSU), Manhattan, Kansas. KSU is a land grant research university, with an undergraduate and graduate student population exceeding 23,000. At Kansas State University, calculus courses and calculus-based physics courses are taught separately in the Mathematics and Physics Departments respectively.

4.2.1. Calculus Courses at KSU

At KSU, there are three sequential calculus courses: Analytic Geometry and Calculus I, II and III. These three courses are offered each semester, and are usually taken by engineering, mathematics and physical science majors. Each course is worth four credit hours. The enrollment is about 400 (often over 500) in Calculus I in the fall semester and is over 200 in Calculus I and II in each semester. Enrollment in Calculus III is sometimes under 200 and sometimes over 200 students for each course per semester. Each course is taught in a Lecture-Recitation format. Students attend two lectures and two recitation classes per week. The 50-minutes lectures are taught in a large lecture hall by the course instructor. Each recitation section is usually taught by a Teaching Assistant (graduate student in the Mathematics Department) and has up to 40 students enrolled. The format of the recitation depends on individual instructor and teaching assistant

4.2.1.1. Analytic Geometry and Calculus I (Calc I)

The prerequisite for Calc I is earning a B or better in College Algebra and C or better in Plane Trigonometry; or three years of college preparatory mathematics including trigonometry and calculus in high school. A score of 55 or higher on the ACT assessment; or a score of at least 26 on the mathematics placement test administered by KSU is required as per Website (http://courses.k-state.edu/catalog/undergraduate/as/math.html) of the Mathematics Department. According to the course description on the Mathematics Department website (http://www.math.ksu.edu/main/course_info/courses/supplcd1.htm), Calc I covers elementary concepts of analytic geometry and introduces the basic concepts of the differential and integral calculus of algebraic functions. The emphasis is on problem solving. The course description states:

> "...The idea of the derivative is introduced, motivated by considering rates of change and tangent lines, and the differentiation of algebraic functions is covered. Numerous problems involving applications of the derivative are assigned and explained in detail. These include a study of extrema of functions, graphing, related rates of change, and applications to physics, engineering and economics. The concept of the definite integral is introduced, and its basic properties are considered. The motivation for the integral and its relationship to the concept of the area under a curve are discussed, and the Fundamental Theorem of Calculus is proved. Finally, applications of the integral are considered and its relationship to the concepts of volume, work and other physical concepts is described."

4.2.1.2. Analytic Geometry and Calculus II (Calc II)

The prerequisite for Calc II is earning C or better in Calc I. According to the official course description, Calc II is a continuation of Calc I and introduces the differential and integral calculus in relationship to the transcendental functions and plane analytic geometry.

> "Logarithmic, exponential and trigonometric functions are defined, and their differential and integral properties are studied in detail. A considerable amount

of time is devoted to the development of techniques of integration, such as trigonometric substitution, integration by parts and partial fractions…"

4.2.1.3. Analytic Geometry and Calculus III (Calc III)

The prerequisite for Calc III is getting C or better in Calc II. According to the official course description, Calc III covers calculus for functions of many variables together with vector analysis in two and three dimensional space.

"These topics are basic for applied mathematics and geometry for we live in three spatial dimensions, not just one. Mechanics of particle motion is developed in detail including curvature and normal and tangential components of acceleration. A beautiful application is the derivation of Kepler's Laws of planetary motion from Newton's Law of gravitational attraction. The three dimensional geometry of surfaces, lines and tangent planes is included. The calculus of several variables, partial derivatives, chain rules and directional derivatives using the gradient are studied. Max-min problems and the method of Lagrange multipliers for extreme problems with constraint are considered. An extensive development and application of multiple integrations is presented. Finally, line integrals of a vector field along a curve, conservative force fields and Green's Theorem are studied."

4.2.2. Calculus-based Physics Courses at KSU

At KSU, there are two sequential calculus-based physics courses, which are Engineering Physics I (PHYS 213) -- EPI -- and Engineering Physics II (PHYS 214) -- EPII. These two courses are offered each semester, and are usually taken by engineering, and science majors. Each course is a combination of two hours lecture and four hours studio a week. Studio is a combination of recitation and laboratory. The enrollment is about 100-300 students for each course per semester (varies for each semester). Each class has a large-enrollment lecture which meets twice a week, followed by two two-hour sessions of Studio. Each Studio section has up to 40 students enrolled. The students work in groups of four students each at lab table. Each table is equipped with a computer and

Data Studio™ which allows for interfacing data collection probes with the computer. In addition to using the Data Studio™ students also use simple hands-on equipment and demonstrations at their table. The Studio lab manual contains brief instructions for each Studio lab exercise. Students are not provided detailed instructions or work sheets to fill out. They are expected to record their laboratory data in their notebook and turn it in to the teaching assistant after each Studio session. Each Studio section is taught by a lead instructor who is a senior graduate student, faculty or post-doc. A secondary lab TA primarily assists that lead instructor with the laboratory and grading (Churukian, 2002; Allbaugh, 2003).

These courses are worth five credit hours each. According to the course syllabus, "the goal of this course is to help you learn the fundamental knowledge of physics and how this knowledge can be applied in solving physics problems," "lecture will help you develop a conceptual understanding of physics while the studios will help you integrate conceptual understanding with problem solving skills and concepts of measurement." *Fundamentals of Physics* by Halliday, Resnick and Walker, 7th edition, was the assigned textbook (Halliday, Resnick et al., 2004).

4.2.2.1. Engineering Physics I (EPI)

The prerequisite for EPI is having taken Calc I or concurrent enrolling in Calc I. EPI covers mechanics, waves and oscillations, and thermodynamics.

4.2.2.2. Engineering Physics II (EPII)

The prerequisite for EPII are having taken EPI and Calc I. EPII covers Electricity, Magnetism, and Optics.

4.3 Participants

4.3.1. Students in EPII

Students who were enrolled in EPII during the interview period participated in this study. The reason we chose students from EPII was because the content covered in EPII requires a wider application of calculus knowledge compared with EPI, especially with topics such as Gauss's law, electric potential and magnetic flux. EPII students need to use knowledge of integration and differentiation to succeed in this course. Furthermore, according to the course requirement, students enrolling in EPII must take at least one semester of calculus, Calc I. In this study, we found that a majority (more than 80%) of students we interviewed have already taken Calc I and Calc II before they enrolled in EPII.

4.3.2. Instructors in Physics and Mathematics

It was not enough to assess transfer of learning only from the students' perspective. It was necessary to look at transfer from the instructors' point of view as well. Experienced teachers (including faculty members and teaching assistants) from mathematics and physics departments were interviewed in this study. We asked these individuals about their learning goals and expectations of their students in this class as well as the teaching strategies that they employed. Details of instructor interview are covered in next section.

4.4 Research Plan

A multi-methodological approach would be needed to adequately address the research questions proposed in Chapter 3, since the selection of a single research methodology may overlook other relevant factors. Because we consider transfer to be a dynamic process a qualitative approach -- individual interviews -- using graduated prompting were considered an appropriate way to assess transfer of learning. Interviews however, can only be used with a limited number of students, so we used a combination of quantitative and qualitative methods in this project. As discussed in Chapter 3 this

research was divided into three phases, we will address the research methods we used in each phase separately.

4.4.1. Phase I: Assessing Horizontal Transfer

In phase I, we investigated horizontal transfer of learning from calculus to physics. Particularly we sought to answer the first two research questions.

Q1: Have students retained their calculus schemas to solve calculus problems?

Q2: Can they associate their physics problem variables with their calculus schema?

Two studies were conducted in Phase I. Study I-1 uses a quantitative approach while Study I-2 uses a qualitative approach. Traditional physics problems, similar to homework and exam problems were used in this phase. As mentioned in Chapter 3, we deemed these problems to involve horizontal transfer because from our (i.e. the researchers') perspective the problems did not require students to construct or to choose between competing schemas or mental models to solve the problem.

4.4.1.1. Study I-1: Quantitative Study

We used a quantitative approach to cast a wide net as we examined data from a large population. The following sources of quantitative data were used:

- Exam performance in EPII: Performance on individual exam problems was assessed using rubrics that separately assessed their calculus performance and their physics performance. 416 students' exams were collected during Fall 2004 and Spring 2005.
- Online homework and exam data in Calc II: This included final scores on each online homework assignment, the number of attempts needed to achieve that final score on each assignment and scores of each problem in all exams. 45 participants in this study took Calc II using the online homework system.

We collected data from two semesters -- Fall 2004 and Spring 2005. Our analysis was based on the following premise:

- Statistically significant correlation between calculus and physics performance is a necessary condition for transfer of learning from calculus to physics. So the absence of statistically significant correlation is indicative of a lack of transfer.
- Statistically significant correlation between calculus and physics performance is not a sufficient condition for transfer of learning from calculus to physics. So a statistically significant correlation does not, by itself, imply transfer of learning from calculus to physics.

It is necessary to point out that the while correlation does not always indicate transfer it might be indicative of transfer from some other source to both calculus and physics i.e. it could imply that performance in both calculus and physics are dependent upon the same factor. Different types of academic success are expected to be correlated since they are both linked to a common external variable (diligence, intelligence, expectation, other epistemic factors and etc.). So a statistically significant correlation is not a sufficient condition for transfer. However, such correlation should be larger when considering assessments that are close in time or context. If two items from different times and/or contexts are more closely correlated to each other than they are to other assessments that are more closely matched in time or context, that would strongly suggest the occurring of transfer. Since not much research has been done to support this argument, we stand on the conservative side and suggest that statistically significant correlation is a necessary but not sufficient condition for transfer.

Students' EPII exam sheets of questions that explicitly needed calculus knowledge were photocopied and analyzed after each exam during Fall 2004 (see Appendix A) and Spring 2005 (see Appendix B). The physics exam problems represented traditional physics problems that are typically used in most undergraduate courses to assess learning in physics. Similar problems can also be found at the ends of chapters in most physics textbooks. In Fall 2004, we collected data from three exams and one quiz, for147 students. In Spring 2005, we collected data from three exams for 269 students.

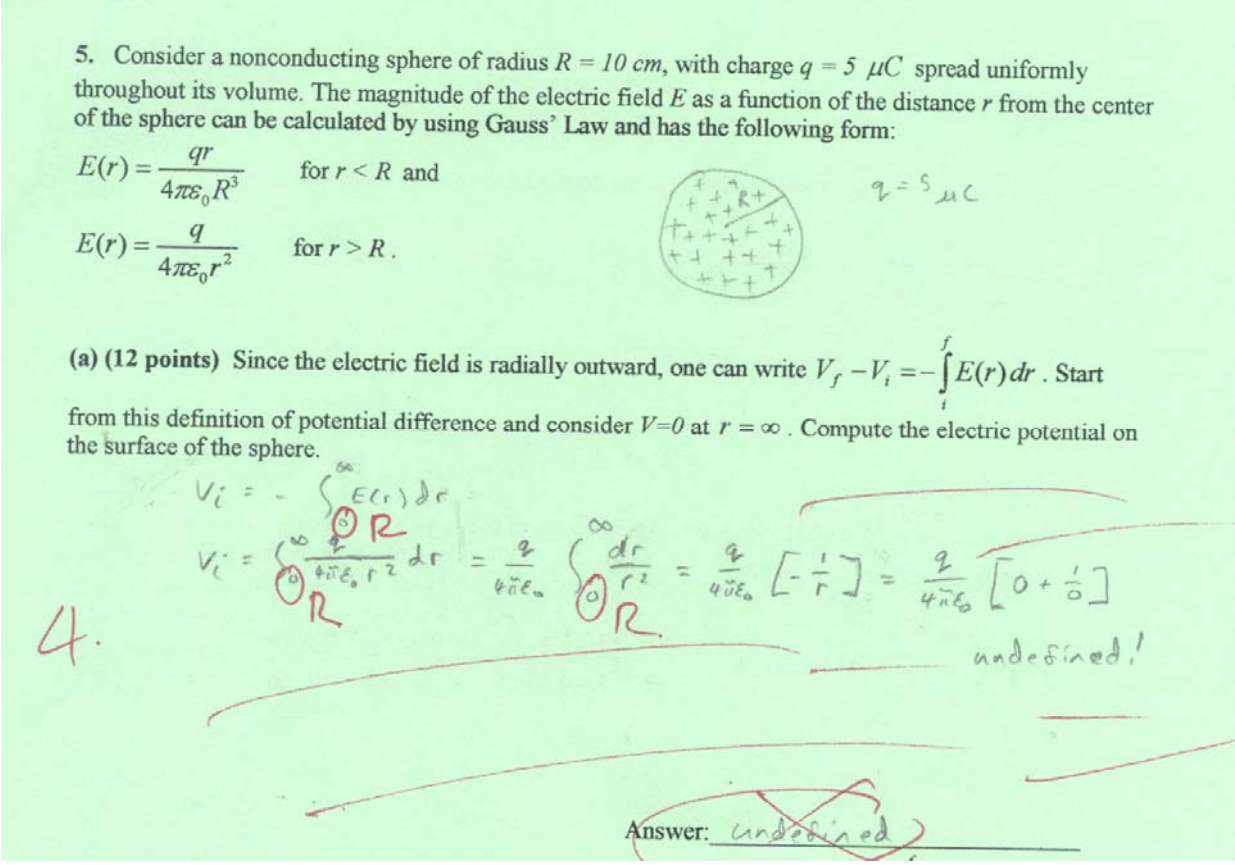
For each physics exam problem we examined students' physics and calculus performance separately. A statistically strong correlation between students' calculus and physics performance indicates the possibility of transfer according to the aforementioned hypotheses. To measure the correlation, we first developed a rubric to measure students' calculus and physics performance separately on each problem. Then we calculated Pearson correlation coefficient between calculus and physics performance for each problem. Additionally, we also used hierarchical cluster analysis to measure possible relationships between their calculus and physics performance on each problem and performance on online scores.

Step 1: Developing the rubric

To measure students' calculus and physics performance separately, we developed a four-point rubric to assess calculus performance and physics performance separately for each physics exam question, or each part of question. In other words, a calculus performance rubric and a physics performance rubric were developed separately to assess student calculus performance and physics performance within one physics problem. Three points were awarded for answering the problem completely in all respects, while zero points indicated that the student did the problem incorrectly in all respects. We established the face validity and inter-rater reliability of the rubric with other physics education researchers KSU. Based on rubric, we assigned the calculus and physics performance score for each question, or part of question.

Figure 4-1 shows an example of an exam question that is graded according the rubric in Table 4-1. The problem had two parts -- (a) and (b). Figure 4-1 shows part (a). The total grade for part (a) was 12 points, the assigned grade by the grader was four (4) points.

Figure 4-1: An example Problem on An EPII Exam



5. Consider a nonconducting sphere of radius $R = 10\ cm$, with charge $q = 5\ \mu C$ spread uniformly throughout its volume. The magnitude of the electric field E as a function of the distance r from the center of the sphere can be calculated by using Gauss' Law and has the following form:

$E(r) = \frac{qr}{4\pi\varepsilon_0 R^3}$ for $r < R$ and

$E(r) = \frac{q}{4\pi\varepsilon_0 r^2}$ for $r > R$.

$q = 5\ \mu C$

(a) (12 points) Since the electric field is radially outward, one can write $V_f - V_i = -\int_i^f E(r)\,dr$. Start from this definition of potential difference and consider $V=0$ at $r = \infty$. Compute the electric potential on the surface of the sphere.

$V_i = -\int_{\infty}^{R} E(r)\,dr$

$V_i = \int_{\infty}^{R} \frac{q}{4\pi\varepsilon_0 r^2}\,dr = \frac{q}{4\pi\varepsilon_0}\int_{\infty}^{R} \frac{dr}{r^2} = \frac{q}{4\pi\varepsilon_0}\left[-\frac{1}{r}\right] = \frac{q}{4\pi\varepsilon_0}\left[0 + \frac{1}{\infty}\right]$

4.

undefined!

Answer: undefined

Table 4-1: Example of the Physics and Calculus Rubrics for Assessing the Problem in Figure 4-1

Points	Physics Performance Criteria	Calculus Performance Criteria
3	Use the proper E(r), and choose the right limits from infinity to R	Finish the integration correctly, and correctly apply the limits.
2	Integrate the appropriate variable E(r), but use wrong limit; or negative sign	Do the indefinite integration correctly, but apply limits incorrectly.
1	No integral, use other formula like V=Ed; or use the wrong E	Perform the indefinite integration, but perform it incorrectly
0	Use a point charge formula to directly get V.	No use of integration

According to the physics rubric, the problem shown above was awarded two (2) points for the physics performance. The student chose the right E(r) (for r > R) but used the wrong limits from 0 to ∞, rather than from R to ∞. This error reflects the students' incorrect understanding of the physics, but not the mathematics.

According to the mathematics rubric, this problem was awarded three (3) points for the calculus performance. The student finished the integration correctly using the formula that he/she started with. The student did not make any mistake when calculating the integral.

Step 2: Calculating the Pearson Correlation

We calculated the Pearson correlation coefficient between students' calculus and physics performance for physics problem to see if they were statistically significantly correlated. If so, it indicated the possibility of transfer, particularly from Lobato's Actor-Oriented perspective. This perspective, which was discussed earlier in Chapter 3, is a value-neutral perspective in assessing transfer. As per this perspective, one examines whether a student has constructed "relations of similarity" between the physics and calculus aspects of the problem. Because a statistically significant correlation indicates similarities of performance between the physics and calculus aspects of a given exam problem, it might (though not necessarily) be evidence of constructions of similarity by the student between these two domains within the context of this question.

Step 3: Hierarchical Cluster Analysis

While correlation coefficients provide information about possible relationships between variables, they do not provide information on how all of the variables are inter-related. To get the big picture of how various variables are interrelated, we ran hierarchical cluster analysis (HCA) using Statistical Package for Social Sciences (SPSS) version 12.0 (Sciences)for all the recorded variables. We used HCA because it is appropriate for samples less than a few hundred. HCA is an exploratory tool designed to reveal natural groupings (or clusters) within a data set that would otherwise not be

apparent. Hierarchical cluster analysis begins by treating each of the individual data variables as a cluster by itself. At each stage of the analysis, the criterion by which variables are separated is relaxed in order to link the two most similar clusters until all of the variables are joined in a complete classification tree. The tree structure is called a dendrogram.

Data from the following variables was used in the HCA:

- Score on physics performance rubric for each EP II problem
- Score on the calculus performance rubric for each EP II problem
- Grade on relevant part for each EP II problem assigned by the grader
- Total grade on the for each EP II problem assigned by the grader
- Grade on online homework Calc II assignment
- Inverse time (i.e. reciprocal of number of attempts) on online homework Calc II assignment)
- Grade on each problem of all Calc II exams

If two variables are most closely joined in the HCA dendrogram, it indicates the highest correlation between these variables. We interpreted this correlation as evidence of the possibility of transfer from one variable to another. Again, this high correlation is only evidence of the possibility of transfer and not evidence of transfer per se. So for instance, if the grade on the online calculus assignment covering a given topic clustered closely with the EPII exam grade on a particular problem, we interpreted that as the possibility of transfer of learning from the relevant calculus concept to the context of the physics problem. The HCA dendrogram provides a clear picture on how different variables relate to each other. More discussion on how to read information from dendrogram will be described in Chapter 5.

We ran a bivariate correlation analysis using SPSS 12.0 to check the reliability of HCA analysis results.

4.4.1.2. Study I-2: Qualitative Study—Traditional Problems

To verify and complement the results in Study I-1, we conducted individual semi-structured interviews in Fall 2004. Semi-structured interviews provide a framework for using graduated prompting, which is the appropriate method to assess transfer consistent with a contemporary perspective. However, we were still using traditional, end-of-chapter physics problems to assess transfer, therefore we interpreted these as involving horizontal transfer as described in Chapter 3. We obtained informed consent from the interviewees consistent with the procedures established by the Institutional Review Board (IRB) on human subjects (see Appendix C). Students were paid $10 for each hour of their time for participating in the interviews. All interviews were videotaped and transcribed.

Eight paid volunteers were interviewed based on their availability. No attempt was made to select a representative sample from the class; rather interviewees were selected based on who was available at convenient times. Each interviewee was interviewed in two sessions; each session lasted about one hour long. In each session, interviewees were asked to solve two sets containing two problems each. Each set consisted of two problems that were isomorphic with respect to each other: a physics problem and a calculus problem. Both problems utilized the same calculus concept but the physics problem was contextualized in a physics context. In other words, the calculus problem was a "pure" calculus problem. The goal was to assess both the retention and transfer to physics. We assessed students' retention based on the extent to which students used their calculus schema through the solving of isomorphic calculus problem. Transfer was assessed based on whether the interviewee would apply the schema they had used in the calculus problem to solve the isomorphic physics problem.

All interviews followed a general structure. Each interviewee was left alone when solving the assigned problem. We left interviewees alone because we tried to mimic the situation when students were solving a real homework or exam problem. This would tell us how students typically approach the assigned problem. The interview problems were not easy and it would take interviewee a while to think through. The presence of interviewer might disturb students' thinking process since they might feel someone was monitoring them. After the interviewee solved each problem, the interviewer asked them

to explain what they had written down, and what difficulties they had when solving the problem. The problems also provided a context within which to discuss the overall connections between physics and calculus as seen from the students' perspective. General questions such as interviewee's calculus background, how they apply their calculus knowledge in physics classes were asked at the end of the interview, so as to avoid any form of stereotype threat (Steele, 1995) which might confound our results. The complete interview protocol is provided in Appendix D.

The interview physics problems we adopted from the textbook were all typical physics problem that need to use simple integration or differentiation. The four physics problem situations we used were:

1) Electric field caused by an arc of charge distribution
2) Electric potential caused by changing electric field
3) Magnetic field caused by a non-constant current distribution
4) Induced current caused by moving loop in a changing magnetic field

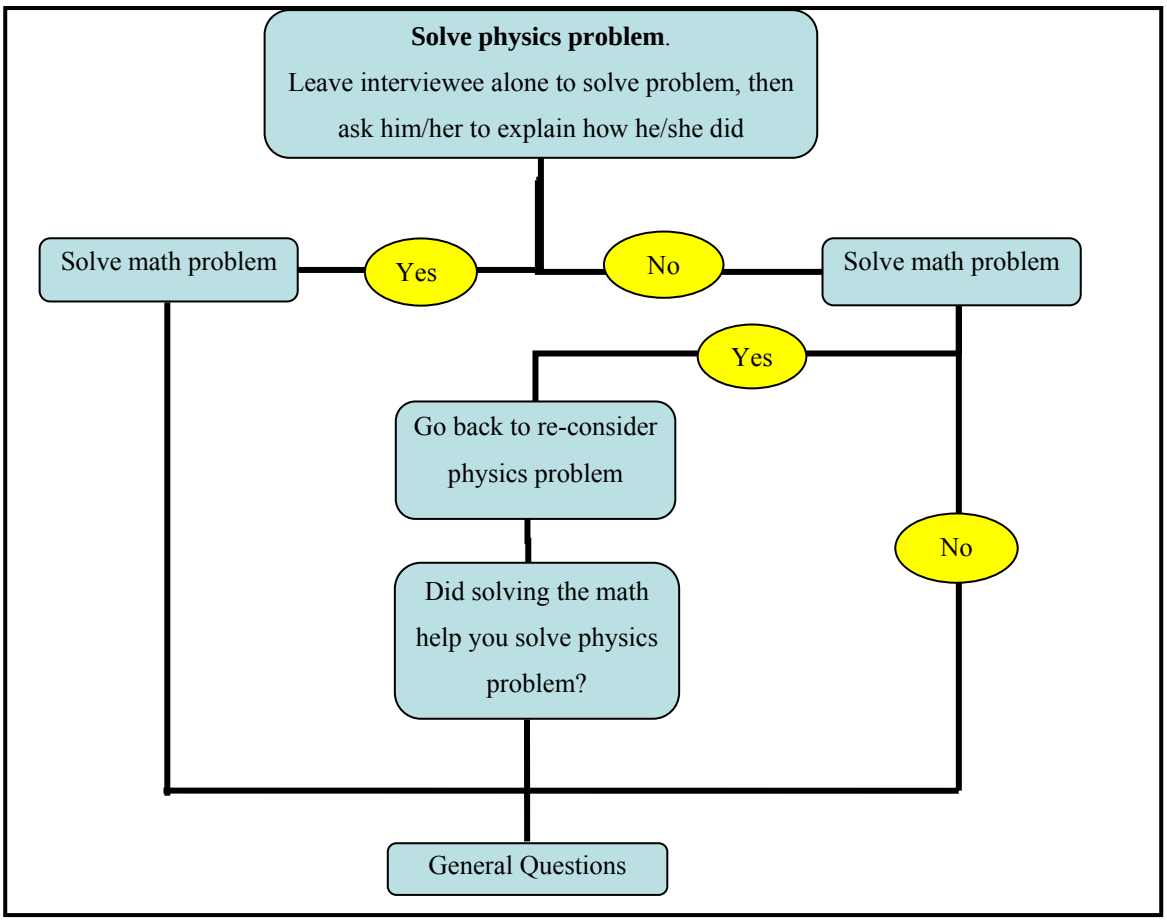
Each of the above situations is described in more detail in Chapter 5.

For each physics problem, we followed the following procedure as shown in Figure 4-2, when conducting interview. The interviewee first solved the given physics problem alone and then explained to the interviewer the process by which they solved the problem. Next, interviewee was asked to solve the isomorphic calculus problem regardless of whether she or he was able to solve the physics problem. If interviewee was able to solve the isomorphic calculus problem but was previously unable to solve the physics problem, we asked her or him to return to and reconsider the physics problem. We were interested in investigating whether solving calculus problems would help the interviewee to solve the physics problem. At last, we asked the interviewee if she or he saw any connections between the physics and calculus problems.

We did not start the flow chart with the calculus problem because if interviewee solved the physics problem after successfully solving the calculus problem, it would not be possible for us to ascertain the extent to which solving the calculus problem contributed to her successes in solving the physics problem. Rather we were interested in determining

whether the interviewer got any hints from the calculus problem and if so, whether these hints helped the interviewee in solving the physics problem. Starting with the physics problem would allow us to use graduated prompting using the calculus problem. This flow chart is also based on the transfer asymmetry found by Bassok & Holyok (1989), successfully solving the physics problems did not lead to doing better on the mathematics problems.

Figure 4-2: Flow Chart for Semi-Structured Interviews in Phase I



4.4.2. Phase II: Assessing Vertical Transfer

In phase II, we investigated vertical transfer of learning from calculus to physics. We sought to answer the third and fourth research questions.

Q3: Can students appropriately activate their calculus schemas in physics problems?

Q4: Can students deconstruct and reconstruct their schemas to solve a physics problem?

Two qualitative studies were conducted in Phase II: Study II-1 and Study I-2. Again, we used semi-structured interviews with graduated prompting, as an appropriate method to assess transfer consistent with contemporary perspectives.

Non-traditional physics problems – "Compare and Contrast" problems and Jeopardy problems were used in this phase. These two kinds of non-traditional problems required students to engage in vertical transfer. As mentioned in Chapter 3, unlike end-of-chapter problems, the students could not apply a pre-constructed schema or mental model to solve these non-traditional problems. Because these problems were unfamiliar to students, they had to construct a schema or choose between competing schemas to solve these problems.

4.4.2.1. Study II-1: Qualitative—"Compare and Contrast" Problems

We conducted individual think-out interviews in Spring 2005 using "Compare and Contrast" physics problems. We were looking at vertical transfer by assessing if students could choose between competing schemas for the problem situation.

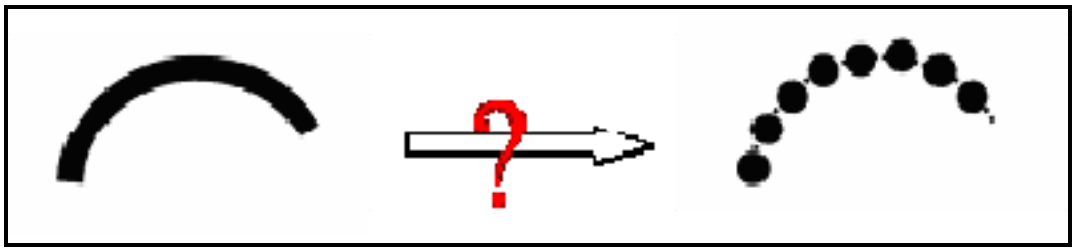
The interviews were organized and conducted in a way that was similar to the previous study. For this study we interviewed five male and three female paid volunteers based on their availability. Based on our results of Study I-2 we focused on exploring the origin of students' difficulties when they were solving physics problems. We used the same four physics problems as in Study I-2. Interviewees did not solve isomorphic calculus problems since, based on Study 1-2, we had found that students generally did not have any difficulties while solving them. Instead, after we asked students to describe how

they solved the problem, we presented them with variations of the problems that they had just solved.

These variations explored the criteria based on which interviewees used "integration" instead of "summation." The goal of this type of problem was to examine whether students could transition between two internal representations that are typically used to solve these kinds of problems. One internal representation involves point-wise summation or superposition. The other internal representation involves integration. Student who productively engage in vertical transfer are typically able to transition between different internal representations depending upon the external representation of the problem. We asked students to solve three variations of the physics problems below. Following are the three variations:

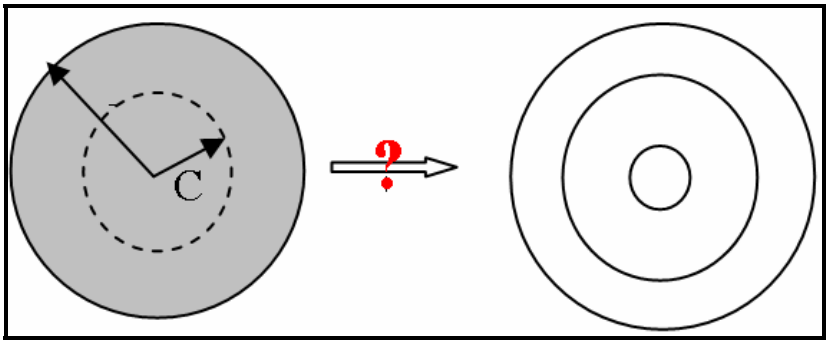
Variation I: As the variation of the "Electric field caused by an arc of charge distribution" question, we asked students whether they would use the same method if there were several point charges instead of an arc-shaped charge distribution. (See Figure 4-3)

Figure 4-3: Variation of the "Electric field caused by arc of charge distribution"



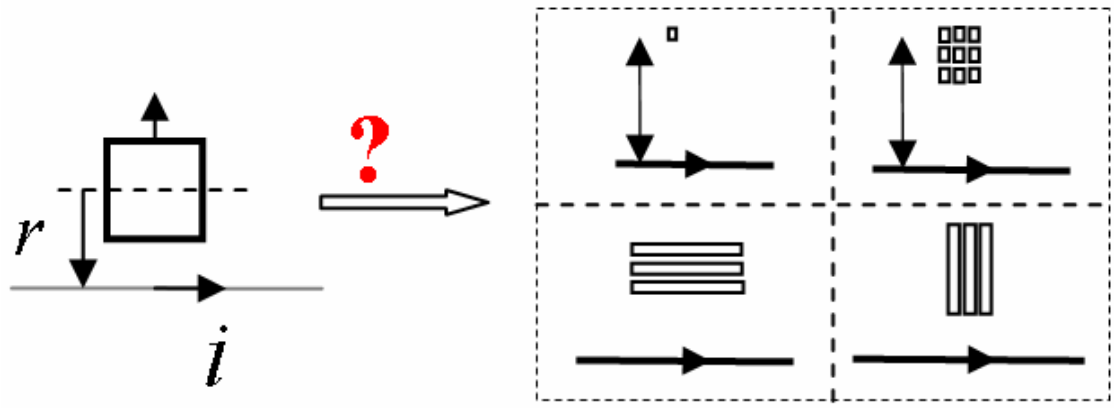
Variation II: As the variation of the "Magnetic field caused by a non-constant current distribution," we asked students what would be the difference if we changed the constant current distribution into a few very thin layers of current and why (See Figure 4-4).

Figure 4-4: Variation of the "Magnetic field by a non-constant current distribution"



Variation III: As a variation of the "Induced current caused by moving of the loop in a changing magnetic field" problem, we asked the students to consider what would be the difference for the four cases shown below, with the very small loops. (See Figure 4-5. In each of the cases on the right a small loop is being moved relative to the wire rather than large loop.

Figure 4-5: Variation of "Induced current by moving loop in a changing magnetic field"



At the end of the interview, we also asked the interviewees about their suggestions to help further EPII students better apply their knowledge learned in calculus class to problems in their physics class. The complete interview protocol for Study II-1 is in Appendix E.

4.4.2.2. Study II-2: Qualitative—Jeopardy and Graphical Representation Problems

We conducted individual semi-structured interviews in Fall 2005 using Jeopardy physics problems. Jeopardy physics problems require students to work backward. Instead of constructing and solving equations pertaining to a given physical situation, students are asked to construct a proper physical situation from a given equation or graph. According to Van Heuvelen & Maloney (1999), Jeopardy problems ensure that "students cannot use formula-centered, plug-and chug problem solving method, rather they must give meaning to symbols in the equation." Jeopardy problems "help students to learn to translate between representations in a more robust manner".

We designed Jeopardy problems that presented interviewees with an intermediate step in the form of a mathematical integration and asked students to come up with a physical scenario relevant to the integral provided. We were investigating vertical transfer by assessing if students could deconstruct their calculus/physics schemas and reconstruct a new schema to solve the Jeopardy problem provided.

The interviews were organized and conducted in a way that was similar to the previous study. For this study we interviewed eleven male and one female paid volunteers. We selected the interviewees in a way so they could present different performance groups in their EPII class. Interviewees were asked to solve a total of six (two in the first interview session and four in the second interview session) Jeopardy problems on electricity and magnetism instead of traditional physics problems. To be consistent with the previous interviews (Study I-2 and Study II-1) and to assess similar physics and calculus knowledge, we chose similar physics problem situations we used before:

1) Electric field caused by an arc of charge distribution
2) Electric potential caused by changing electric field
3) Magnetic field caused by line of current
4) Magnetic field caused by a non-constant current distribution
5) Magnetic flux caused by moving loop in a changing magnetic field
6) Induced current caused by moving loop in a changing magnetic field

In Jeopardy expression problems, students were provided with a mathematical expression and asked to construct an appropriate physical situation. We sought to examine how students understand calculus-based equations in physics. Two examples are shown below:

$$2\times\left[\int_0^{\frac{\pi}{6}}(8.99\times10^9\ N\cdot m^2\Big/C^2)\frac{(2\times10^{-10}\ C/m)(5\times10^{-2}\ m)\cos\theta d\theta}{(5\times10^{-2}\ m)^2}\right] \tag{4.1}$$

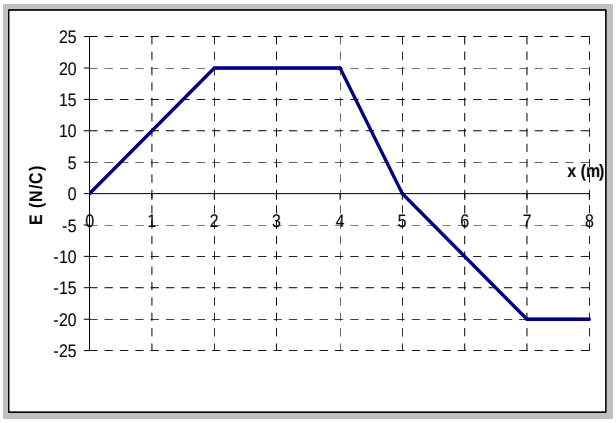
$$\frac{\mu_0\int_0^R J(r)\cdot(2\pi r dr)}{2\pi R} \tag{4.2}$$

The above two Jeopardy problems are described in further detail in Chapter 5.

To read out information from a given graph, like to read out information from a given equation, is another important aspect of students' knowledge transfer. We found it was very difficult to design a Jeopardy graph problem that required student to come up with a physical scenario relevant to the provided graph. As a solution, we designed Graphical Representation problems in which students were provided with a graph and asked to read out information from the graph so to find a certain variable. The Graphical Representation problems are more like traditional physics problems since students are given a problem situation and asked to find an answer. We were interested in students' ability to read out and analyze information from a given graph related to a physical situation. Two examples are shown below.

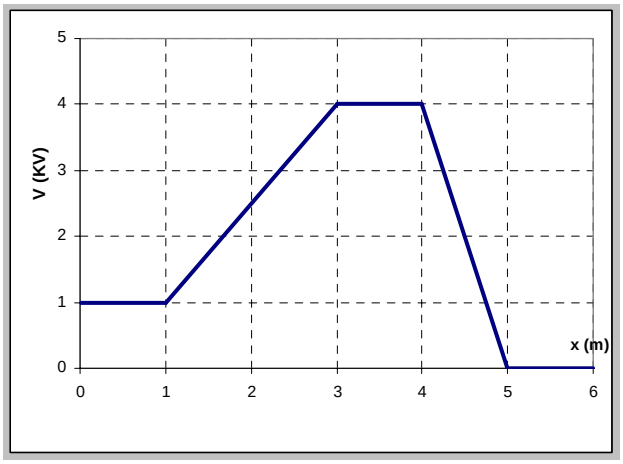
In the first example (Figure 4-6) the student is given the graph of electric field. The student is asked to describe the physical situation and also find the electric potential at different points in space.

Figure 4-6: Graphical Representation Problem 1



In the second example (Figure 4-7) the student is given the graph of electric potential. The student is asked to describe the physical situation and also find the electric field in different regions of space.

Figure 4-7: Graphical Representation Problem 2



It is important to mention that we understood that "Compare and Contrast" problems, Jeopardy problems and even Graphical Representation problems are very challenging. Our goal was not to find out whether our students could correctly solve these problems, rather we were more interested in the process they used to attempt the problems. The complete interview protocol for Study II-2 is in Appendix F.

4.4.3. Phase III: Faculty Interview

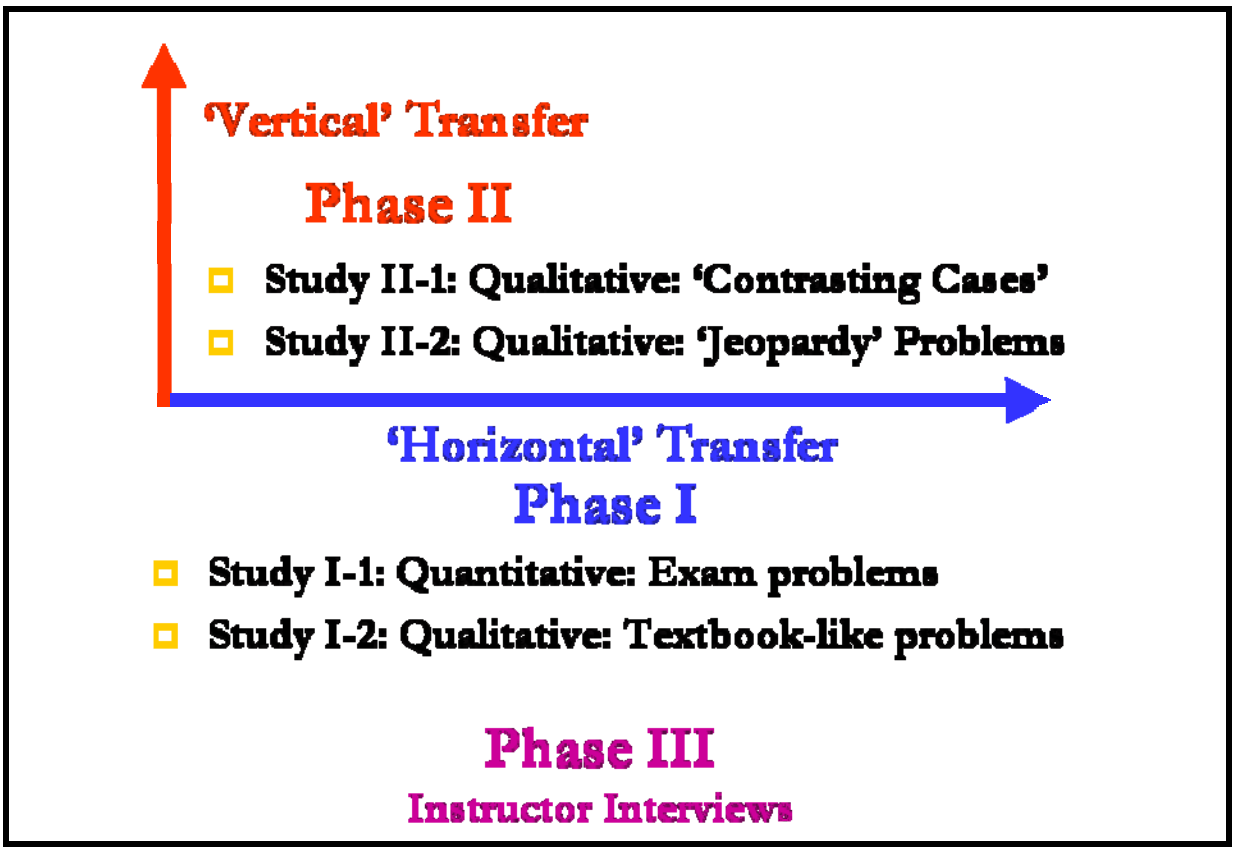
It was necessary to investigate mathematics and physics instructors' points of view with regard to transfer of learning from calculus to physics. In Spring 2006, we interviewed two physics faculty members and four studio physics instructors who had either previously taught or were currently teaching EPII. We also interviewed two mathematics faculty members and two teaching assistants who had either previously taught or were currently teaching calculus courses from the mathematics departments. All interviewees were volunteers for this study.

Each instructor interview was about 30 minutes long. All interviews were semi-structured and were audio-taped with IRB informed consent of the interviewee. Interviewees were asked a series of questions about their expectations and outcomes of the courses that they taught. We asked calculus instructors as to what knowledge they expected their students to have when finishing calculus, how they helped their students acquire that knowledge, and whether or not they were satisfied with the course outcomes. We were also interested in learning if calculus instructors were aware of the applications of calculus in other subjects. We asked physics instructors what knowledge they expected their students to have before coming to EPII, and whether or not the students they felt their students had acquired this knowledge. We also asked them for their suggestions for improving the calculus preparation of their students. The complete interview protocol for this phase is in Appendix G.

4.4.4. Summary of Research Plan

Figure 4-8 summarizes our overall research plan. We focused on horizontal transfer in Phase I, vertical transfer in Phase I, and instructional strategies in Phase III. Students from EPII and instructors from both mathematics and physics departments were participated in this study.

Figure 4-8: Research Design: Horizontal & Vertical transfer and Instructor views



4.5 Common Features for All the Interviews

Each interview protocol was developed to ensure a consistent and pleasant experience for both interviewee and interviewer. A safe, quiet, and convenient location was selected for all interviews. The interview room had suitable furniture, lighting, video and audio recording equipment. Interviewees were invited to participate in the interview at a time which was convenient to them. Before each interview each participant was given a consent form to sign (see Appendix C). In addition to providing them with the written formal consent form, the interviewer described the interview process to them. Student interviewees were informed that:

1) The interview would be completely confidential – no one other than the individuals whose names appeared on the consent form would view the

data. Also, their names as interview participants would not be divulged to anyone.

2) If we chose to share their data with other individuals, or in a publication, they would not be identified either by name or by student ID or by any other identifying feature.

3) Their performance in the interview would not affect their grade in any course in any way.

4) They would be paid $10 per hour in cash upon completion of the interview regardless of whether they chose to answer the questions or completed the interview.

5) The interview would be video and audio taped. They were shown the video on the screen and it was emphasized to them that their face would not be recorded in any way.

6) They could choose to leave the interview at any point and not face any penalty for doing so.

7) If they chose to withdraw their consent after the completion of the interview, they could also do so without any kind of penalty.

We began each interview with questions to relax the interviewee and form a rapport with her/him that would make them feel comfortable participating in the interview. The interview was brought to a close with a series of reflective questions. The interviewee was thanked for their participation and provided with follow-up information on their interview.

As mentioned before we used the individual semi-structured interview format for both students and faculty interview. The semi-structure interview allowed us to ask the same essential questions for each interviewee, but also allowed some flexibility depending on each individual's response. The interview questions were predominantly open-ended in nature. From the book titled *In-Depth Interviewing* (Minichiello, Aroni et al., 1995), we primarily used descriptive, structure, contrasting, opinion and probing questions, as described below:

- Descriptive Questions: Used primarily at the start of each interview or when moving to a new topic. This question type allows interviewees to discuss their experiences in their own words and from their perspective. An example of a descriptive question from this study is, "Can you describe to me how you solved this problem?"
- Background Demographic Questions: This is a form of descriptive questions, used to get the background information of the interviewee. An example of a background question from this study is, "What calculus courses have you taken before?" In keeping with the work on stereotype threat by Claude Steele (1995), background questions were typically reserved for the end of the interview.
- Knowledge Questions: Used to find out what factual information the interviewee has. An example of a knowledge question from this study is, "Can you explain what you mean by Ampere's Law?"
- Contrasting Questions: Used to enable the interviewee to make comparisons of situations. An example of a contrasting question from this study is, "What, if anything would you do differently to solve this question (compared with the previous question)?"
- Opinion or Value Questions: Used to determine what the subject thinks about a particular issue or person. This question type was used to elicit the subject's opinions and feelings, not just the correct answer. Examples opinion questions from this study are, "What kind of experiences did you have in your calculus class?"; "What might be helpful for future EPII students?"
- Probing Questions: Used to elicit information more fully, on a particular topic. This question type was used extensively in this study because we wanted to explore students' thinking process. An example of a probing question from this study is, "What were you thinking when you did this step in the problem?"

4.6 Analysis of Interview Data

Videotapes or audio-tapes of the interview were first transcribed and stored as a Word file. We tried to transcribe each interview as soon as it was completed. A cover page for the transcription was prepared to have details of the interview, such as date, time, duration, location, interviewer, interviewee (coded to conceal identity) and initial impressions of the interview. The cover page provided brief information that helped the interviewer recapture the events and feelings of the interview. Following the cover page we included the complete typed interview transcript, clearly indicating statements made by the interviewer and subject. The transcript was situated in the middle of the page with a column on either side for interviewer's notes and line numbers.

We used a phenomenographic approach to analyze all interview data. Phenomenographic analysis (Marton, 1986) yields a variation of students' ideas rather than researchers' conceptions about students' models. The categories for coding of the interactions emerge from the analysis of the responses. This strategy is consistent with contemporary views of transfer, such as Lobato's Actor-Oriented Transfer model since the researcher does not prejudge what ideas a student might transfer, but rather looks for what, if anything, the student has transferred.

We adopted Colaizzi's (1978) seven steps of phenomenological analysis to analyze interview transcriptions. The seven steps are as follows:

1) The researcher reviews the collected data and *becomes familiar* with it. Through this process she gains a feeling for the subject's inherent meanings.

2) The researcher returns to the data and focuses on those aspects that are seen as most important to the phenomena being studied. From the data she *extracts significant statements*.

3) The researcher takes each significant statement and *formulates meaning* in the context of the subject's own terms.

4) The meanings from a number of interviews are grouped or organized in a *cluster of themes*. This step reveals common patterns or trends in the data.

5) A detailed, analytic description is compiled of the subject's feelings and ideas on each theme. This is called an *exhaustive description*.

6) The researcher identifies the *fundamental structure* for each exhaustive description.

7) The findings are taken back to the subjects who check to see if the researcher has omitted anything. This is called a *member check*.

In our case we did not return to the student after the second round of each interview for a member check. Rather we performed a member check during the interview itself, we verified the meaning of students' statements especially the ambiguous ones. This variation of the member check procedure was used for logistical reasons, because it would be extremely difficult to request the interviewee to return for a third time.

We analyzed each individual transcript using the seven-step of phenomenological analysis after the interview was completed. The categories from the phenomenographic analyses were synthesized at the end of all interviews using thematic analysis until the dominant themes emerged.

4.7 Chapter Summary

We conducted this research at Kansas State University. Students who enrolled in Engineering Physics II (EPII) were chosen to participate in this study since EPII problems requires a certain amount of calculus knowledge. We collected both quantitative and qualitative data in this study because a multi-methodological approach would be needed to adequately address the research questions proposed in Chapter 3. A three-phase research plan was designed to collect data needed to address the research questions.

Phase I was designed to assess horizontal transfer of knowledge using traditional physics problems. Phase II was designed to assess vertical transfer of knowledge using non-traditional physics problems—"Compare and Contrast" problems, Jeopardy problems and Graphical Representation problems. So as not to limit our research to students' point

of view, we interviewed both physics and calculus instructors in Phase III. Pearson correlation and Hierarchical Cluster Analysis methods were used to analyze the quantitative data. A phenomenographic approach was adopted to analyze all the qualitative interview data. In the next chapter we present the results of our analysis of both the quantitative and qualitative studies conducted in this project.

CHAPTER 5 - RESULTS & DISCUSSION

5.1 Chapter Overview

First (Section 5.2), we discuss the results from the Phase I -- horizontal transfer, which includes Study I-1 (quantitative study) and Study I-2 (interviews using traditional physics problems). Then (5.3) we discuss the results from the Phase II -- vertical transfer, which includes Study II-1 (interviews using "Compare and Contrast" problems) and Study II-2 (interviews using Jeopardy problems). Finally (5.4), we describe the research findings from the instructor interviews.

5.2 Results of Phase I—Horizontal Transfer

Two studies were conducted in Phase I: Study I-1 (quantitative) and Study I-2 (qualitative) to assess horizontal transfer of knowledge from calculus to physics. Traditional physics problems which are similar to homework and exam problems were used in this phase. We treat the transfer measured using traditional homework or exam problems as horizontal transfer because from our perspective these problems typically require students to apply a pre-learned schema or strategy.

5.2.1. Results of Study I-1

Study I-1 uses a quantitative approach by using the idea of one-shot assessments. Students' EPII exams involving calculus knowledge were collected and reviewed. Rubrics were developed to separately assess students' calculus and physics performance when solving each physics problem. Pearson correlation analysis and hierarchical cluster analysis were both used to analyze the quantitative data.

5.2.1.1. Pearson Correlation Analysis Result

Traditional View of Transfer: We collected three EPII exam problems in Fall 2004 (n=147) and three EPII exam problems in Spring 2005 (n=269). We collected the data from 45 students who enrolled in EPII in Spring 2005 and had taken Calc II in Fall

2004. We obtained a complete record of these 45 students' Calc II performance, including their grade and time spent on each homework assignment and the grade on each problem on all of their exams.

We calculated the Pearson correlation between students' Calc II final course grade and their EPII grades for each calculus-based physics exam problems. This is the typical method to assess transfer from the traditional perspective as discussed in Chapter 3. The first exam (Exam 0, see Appendix H) in Calc II was designed to assess their Calc I knowledge retention. Therefore, we used their scores on this exam to represent their Calc I knowledge and calculated the correlation with the grades of their calculus-based physics exam problems.

The three collected exam problems in Table 5-1 are attached in Appendix B.

1) The collected problem in Exam 1 asked students to find the electric field caused by a spherical charge distribution. It used the calculus idea of surface and volume integral.
2) The collected problem in Exam 3 asked students to find the magnetic field caused non-constant current distribution. It used the calculus ideas of linear integral.
3) The collected problem in Exam 4 asked students to find the induced current in a loop in changing magnetic field. It used the calculus ideas of surface integral and simple differentiation.

Table 5-1: Pearson Correlation between Students' Calculus Grades and EPII Grades

Pearson Correlation between students' calculus course grades and EPII exam problem grades		
Calc II final course grade	EPII Exam 1 physics problem grade	**0.36**
	EPII Exam 3 physics problem grade	**0.29**
	EPII Exam 4 physics problem grade	0.18
Calc I knowledge score (tested at the beginning of Calc II class as exam 0 score)	EPII Exam 1 physics problem grade	0.18
	EPII Exam 3 physics problem grade	**0.35**
	EPII Exam 4 physics problem grade	0.05

*n=45, so r>0.29 indicates statistically significant correlation

- As shown in the top half of Table 5-1, statistically significant ($p < 0.05$) correlations (in bold) were found between students' Calc II final course grade and two out of three physics exam problems grade. These were the problems that explicitly required the knowledge of calculus.
- As shown in the lower half of Table 5-1, similarly statistically significant correlations (in bold) were also found between students' first exam grade at the beginning of Calc II (which was designed to assess their retention from Calc I) and one out of three physics exam grade. These problems explicitly required the knowledge of calculus.

In Table 5-1, we find that not all of the scores are statistically significantly correlated. No statistically significant correlation between the problem grade in EPII Exam 1 and either Calc I or Calc II course grades. Similarly no statistically significant correlation between the problem grades in EPII Exam 4 and Calc I course grade were found. There is no fundamental difference between the calculus knowledge used in EP II Exams 1, 3 or 4 since they all use basic integration and differentiation. Therefore, this result suggested that students' performance in their Calc I or Calc II was not a good predictor of how they would perform on EPII exam problems that required calculus. Based on correlation as a metric to assess transfer from the traditional perspective, we found weak evidence that students transferred their calculus knowledge to physics class since there was no consistently statistically significant correlation.

Actor-Oriented View of Transfer: As per the contemporary view, transfer is the learner's dynamic construction of similarities between the new situation and prior knowledge; this was consistent with the idea of actor oriented transfer by Lobato (2003). To assess the similarities constructed by the learner as they solved EPII problems, we calculated the Pearson correlation between measures of students' calculus performance and physics performance when solving an EPII problem. As explained in Chapter 4, a statically significant correlation between these two measures would indicate the possibility that the learner had internally connected these two (calculus and physics) pieces of a

problem. In other words, a statistically significant correlation is a necessary, though not sufficient condition for dynamic transfer.

Based on the rubric we developed to measure their calculus and physics performance on each problem, we found a statistically significant correlation between students' calculus and physics performance for relevant exam questions at the $p<0.05$ (two-tailed) significance level (See Table 5-2). For the final exam in Fall 2004, there were two problems (Q3 and Q4) that explicitly needed the use of calculus; on each of the other exams, only one problem explicitly needed to use of calculus as indicated in Table 5-2.

The statistically significant correlation between calculus and physics performance is a necessary condition for transfer of learning from calculus to physics. The particular EPII problems whose calculus and physics performance correlations are calculated in Table 5-2 are provided in Appendix A & B. So the strong correlation between the calculus and physics performance when solving a particular problem indicated the possibility of transfer from calculus to physics on these problems.

Table 5-2: Pearson Correlation between Calculus and Physics Performance

Pearson Correlation between Calculus and Physics Performance				
Fall 2004 (n=147)	Exam 2	Calculus in part (a)	Physics in part (a)	**0.42**
		Calculus in part (b)	Physics in part (b)	**0.36**
	Exam 4	Calculus in part (a)	Physics in part (a)	**0.38**
		Calculus in part (b)	Physics in part (b)	**0.53**
*r>0.18 indicates statistically significant correlation	Final Exam	Q3 Calculus	Q3 Physics	**0.26**
		Q4 Calculus	Q4 Physics	**0.64**
Spring 2005 (n=269)	Exam 1	Calculus in part (e)	Physics in part (e)	**0.66**
	Exam 2	Calculus in part (a)	Physics in part (a)	**0.88**
		Calculus in part (c)	Physics in part (c)	**0.94**
*r>0.23 indicates statistically significant correlation	Exam 3	Calculus in part (a)	Physics in part (a)	**0.82**
		Calculus in part (b)	Physics in part (b)	**0.84**

In examining the correlations between students grader assigned points on each problem and their calculus and physics rubric score assigned by us, we found that the students' grades on the problems, assigned by the graders, were more strongly correlated with their physics performance rubric score compared with their calculus performance rubric score. This observation is consistent with the fact that EPII graders would typically focus on correctness of the physics aspect of the problem, and not as much on the correctness of the calculus aspect of the problem. No significant correlation was found between students' calculus performance on these problems and how long ago they had taken calculus.

5.2.1.2. Hierarchical Cluster Analysis (HCA) Result

While correlation coefficients provide information on possible relationships between pairs of variables, they does not provide information on how these variables are interrelated. To examine the interrelationships between performance in calculus and physics we conducted Hierarchical Cluster Analysis (HCA). We performed HCA using the Statistical Package for Social Sciences (SPSS) version 12.0 for windows, chose Pearson correlation coefficient as a criteria to cluster the variables. Variables that were more closely correlated with each other were clustered. Once a cluster was formed it was treated as a composite variable and its correlation was calculated with all other variables. The dendrogram provides a graphical representation of the clustering and is discussed below.

HCA within each EPII exam problem (AOT view of transfer): We represented the HCA results of the quantitative data we obtained from Spring 2005 (N=269). Figure 5-1, Figure 5-2, and Figure 5-3 showed the dendrograms of the EPII Exams 1, 2 and 3 separately, and Table 5-3 presented the coding system we used in the dendrogram. For each exam problem, we had the following variables:

- Score on physics performance rubric for each EPII problem
- Score on the calculus performance rubric for each EPII problem
- Score on relevant part for each EPII problem assigned by the grader
- Total score on the for each EPII problem assigned by the grader.

Table 5-3: Sample Codes Used in Dendrogram

Code	Description
E1PHYSC	Physics performance in part C of EPII Exam 1 (by researcher)
E1MATHC	Calculus performance in part C of EPII Exam 1 (by researcher)
E1SCOREC	Assigned grade for part C of EPII Exam 1 (by grader)
E1TOTAL	Assigned grade for EPII Exam 1 (by grader)

Dendrograms are read from left to right. Vertical lines show joined clusters. The position of the vertical line on the scale indicates the distance at which clusters are joined, rescaled to a maximum of 25 units. The smaller the rescaled distance of a vertical line joining two variables, the more closely clustered these two variables are.

From Figure 5-1, Figure 5-2, and Figure 5-3 we see that students' mathematics performance and physics performance on the same problem are closely clustered, as shown in the rectangular boxes, compared to other variables,. In the other words, students' calculus performance is more correlated to students' physics performance on the same physics problem compared to other variables. From Lobato's "Actor-Oriented Transfer" (AOT) perspective that views transfer as the "personal construction of similarities" between the two contexts, the closely formed clusters between the calculus and physics performance when solving a particular problem indicated the possibility of transfer between calculus and physics on these problems. This result was consistent with our Pearson Correlation analysis result discussed earlier. From these figures, we can also see students' grades on the problems, assigned by the graders were more closely clustered with their physics performance compared with their calculus performance, which as discussed earlier is also expected.

Figure 5-1: Dendrogram for Spring 2005 Exam 1

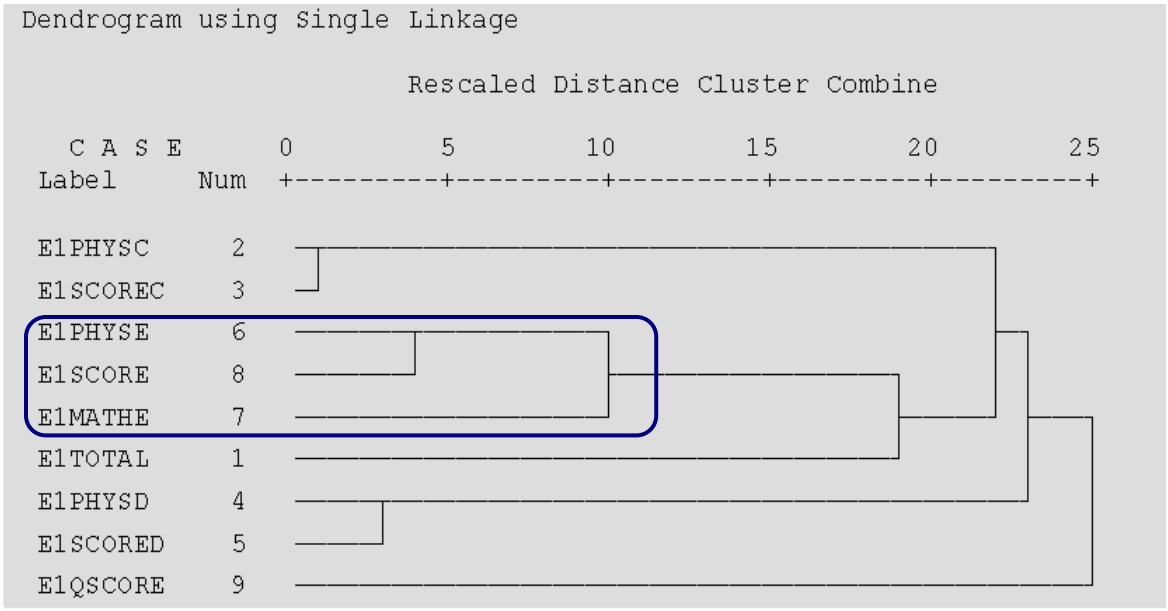


Figure 5-2: Dendrogram for Spring 2005 Exam 3

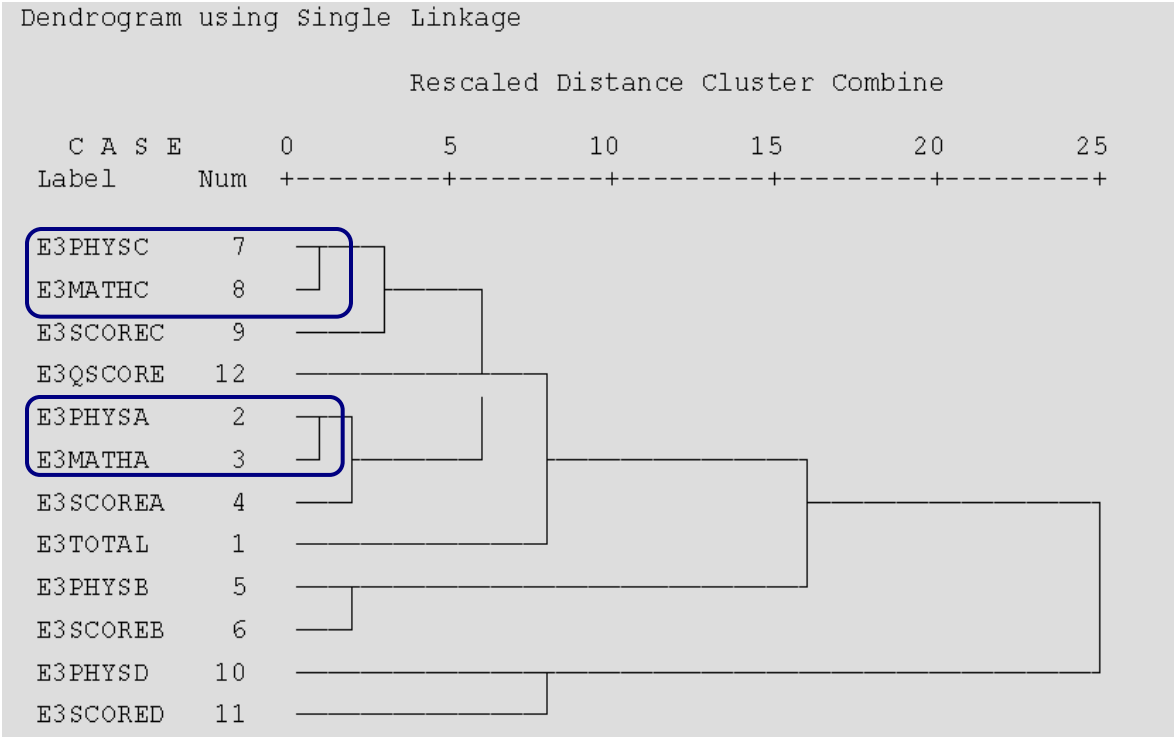
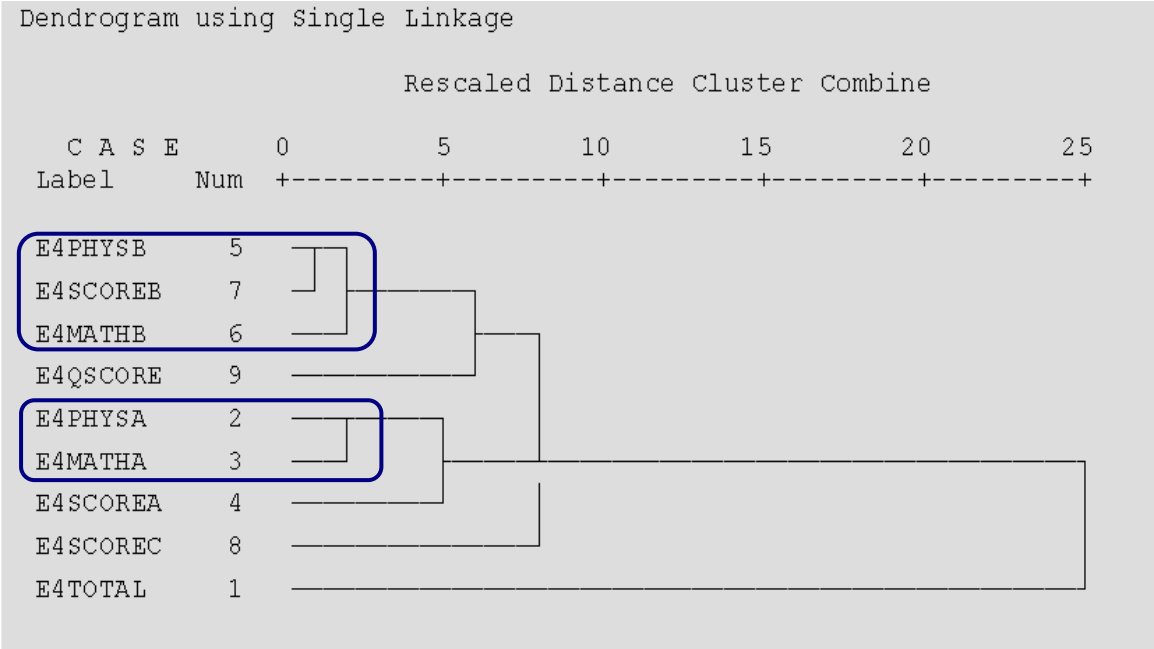


Figure 5-3: Dendrogram for Spring 2005 Exam 4



HCA using all possible variables from both calculus and physics course (PFL view of transfer): Using the data we got from the mathematics department (N=45), we conducted HCA for all possible variables. This HCA told us how the different variables relate to each other inside of one problem, in between problems and between two Calc II and EPII. The variables are:

- Score on physics performance rubric for each EPII problem
- Score on the calculus performance rubric for each EPII problem
- Score on relevant part for each EPII problem assigned by the grader
- Total score for each EPII problem assigned by the grader.
- Grade on each online homework Calc II assignment.
- Inverse time (i.e. reciprocal of number of attempts) on online homework Calc II assignment.
- Grade on each problem in all Calc II exams assigned by the grader.
- Total grade for all Calc II exams assigned by the grader.

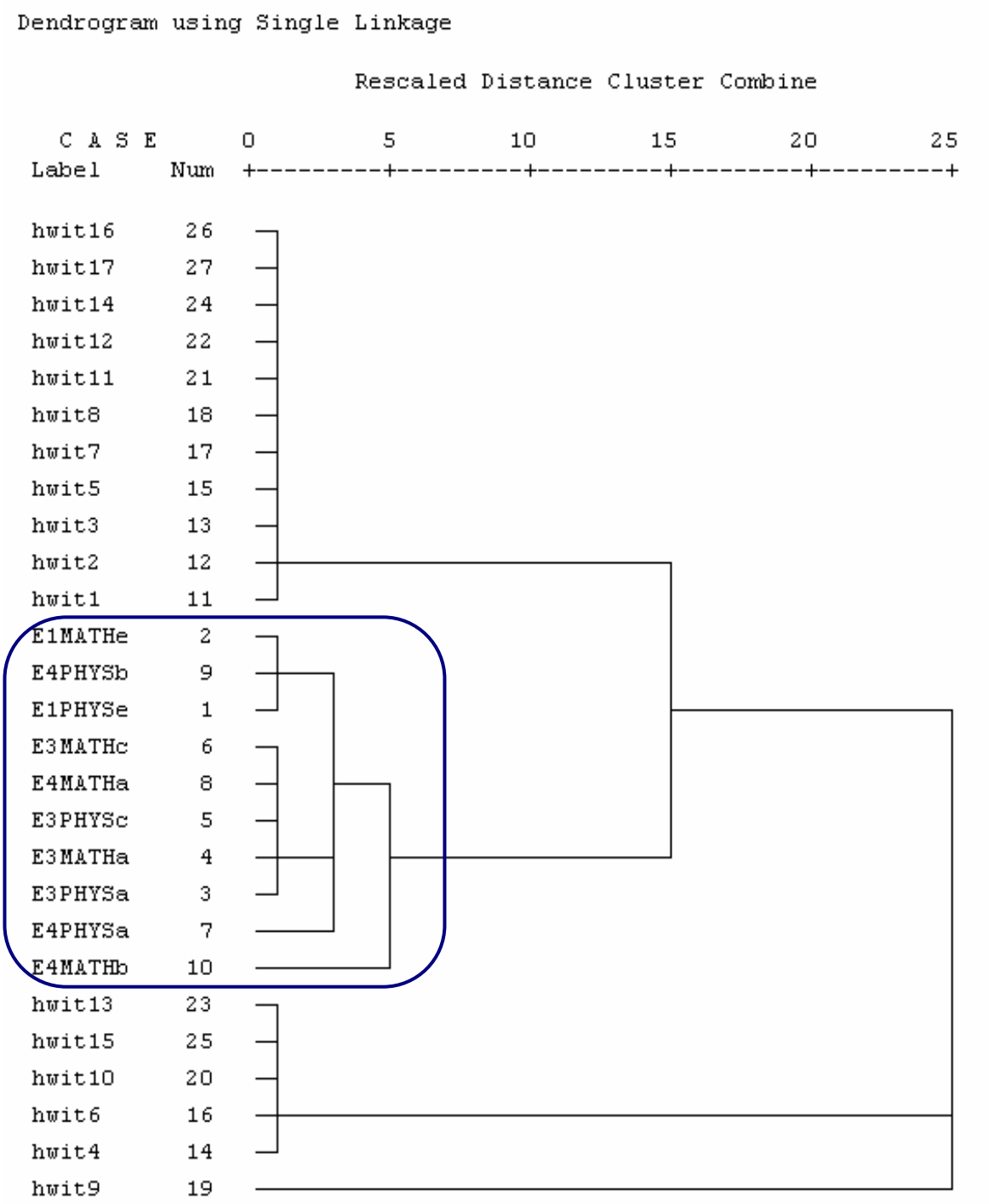
The result of HCA for all possible variables is attached as Appendix J since it is three pages long. From the dendrogram, it is clear that students' physics performance and calculus performance in each EPII problem were closely clustered. However, there was no explicit cluster formed between students' Calc II homework or exam performance, Calc I knowledge (as measured by their performance on Exam 0 in Calc II), and their EPII course performance. Relatively weaker evidence of PFL transfer was found. We used the bivariate correlation analysis from SPSS 12.0 to check the HCA results. The correlation matrix affirmed the results of HCA results, which are also consistent with the results from Pearson correlation analysis discussed earlier.

To simplify the three-page dendrogram presented in Appendix H, we chose the following two methods to run HCA. First we calculated the relationship between EPII and Calc II courses. We used the following variables:

- Score on physics performance rubric for each EPII problem (example code: E1PHYSe refers to the physics performance rubric score for part 'e' of a problem on Exam 1 in EPII.)
- Score on the calculus performance rubric for each EPII problem (example codes: E4MATHb, refers to the calculus performance rubric score for part 'b' of a problem on Exam 4 in EPII)
- Calc II online homework assignment (see Appendix I) scores averaged over number of attempts (example codes: hwit1, hwit2).

Figure 5-4 showed the dendrogram structure for EPII and Calc II. The variables that measure performance in the EPII are closely clustered, but there is no significant clustering between variables in EPII and Calc II. From Bransford and Schwartz's "Preparation for Future Learning" (PFL) view of transfer which focuses on whether students can learn to problem-solve in a new context, we must examine correlations between Calc II course variables and EPII variables. We found no evidence of transfer of learning from Calc II to EPII.

Figure 5-4: Dendrogram for EPII and Calc II



Secondly we calculated the relationship between EPII and Calc I courses. We had the following variables:

- Score on physics performance rubric for each EPII problem (example codes: E1PHYSe, E4PHYSb)
- Score on the calculus performance rubric for each EPII problem (example codes: E1MATHe, E4MATHb)
- Scores on each problem (p01 – p012) in the first exam of Calc II (see Appendix H), which was designed to test students' Calc I knowledge retention (sample code: p01, p02). We used these scores to represent students' Calc I knowledge.

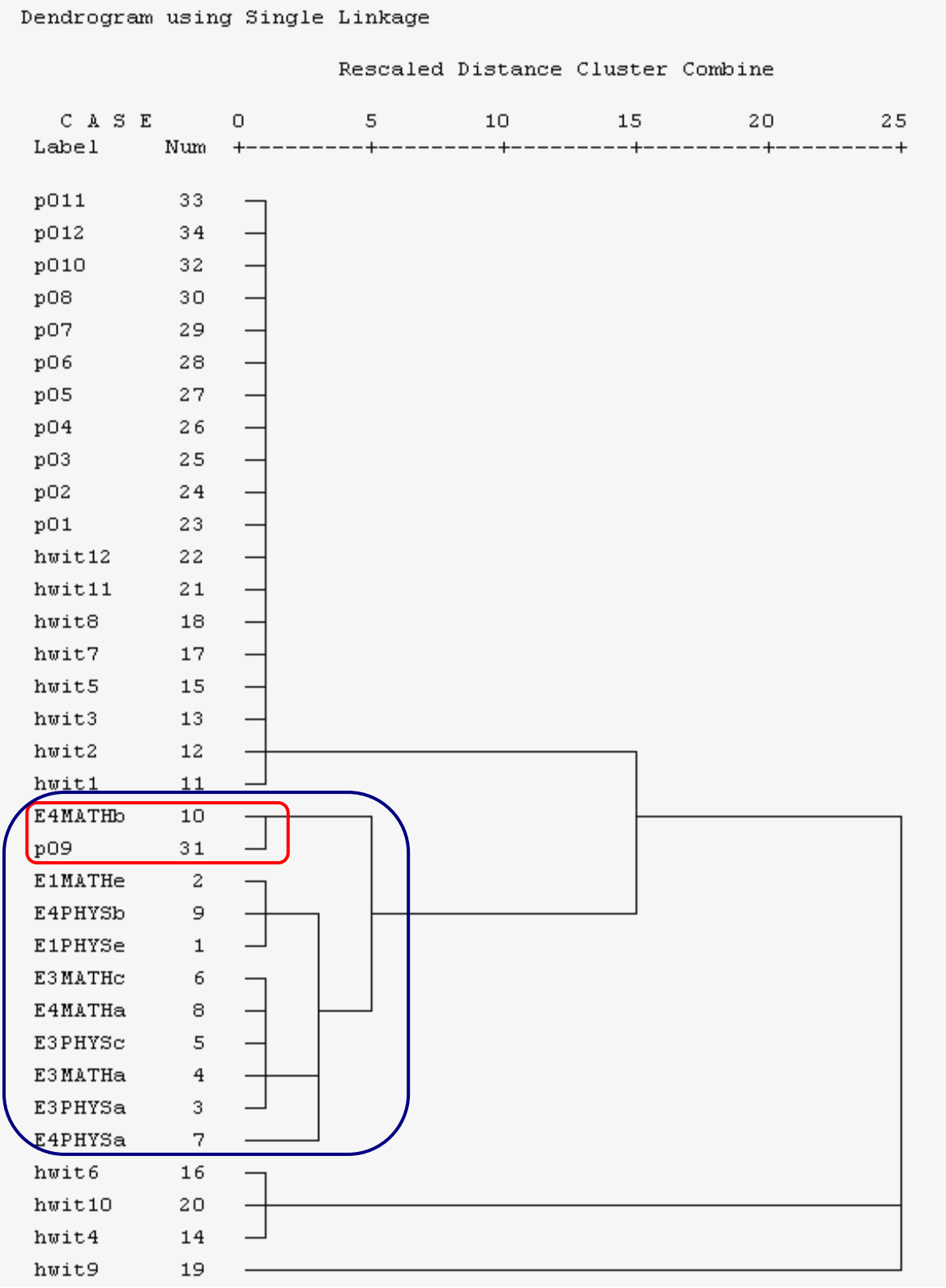
Figure 5-6 showed the dendrogram structure for EPII and Calc I. Similarly to Figure 5-4, the variables inside of the EPII closely clustered but there is no significant clustering between variables in EPII and Calc I. Again from the PFL view of transfer, we found relatively weaker evidence of transfer of learning from Calc I to EPII.

In Figure 5-6, as indicated by the red rectangular, E4MATHb (the calculus performance in the part b of EPII exam 4, see Appendix H) and p09 (the 9th problem in the first exam of Calc II as retention of Calc I knowledge) were very closely clustered. E4MATHb used simple differentiation to find the induced emf from the changing of magnetic flux; this was the only exam problem in EPII used differentiation. P09 (see Figure 5-5) asked to find the maximum value of a given function, needed to apply differentiation as well. Using simple differentiation was the common feature between E4MATHb and p09. It is interesting to note that there were other problems in that calculus exam that needed to do differentiation but scores on these problems were not closely clustered with E4MATHb.

Figure 5-5: 9th problem (p09) in the first exam of Calc II

9. The maximum value of $x^3 - x + 1$ in the interval [-1,2] is
   a. 0
   b. 1
   c. 7
   d. $\frac{9-2\sqrt{3}}{9}$
   e. $\frac{9+2\sqrt{3}}{9}$

Figure 5-6: Dendrogram for EPII and Calc I



Both from Pearson Correlation Analysis and Hierarchical Cluster Analysis, we found that assessing transfer of learning from calculus to physics must be examined from multiple perspectives of transfer. When viewed from a traditional perspective (correlating students' calculus course grade and physics exam problem grade), students appear to fail to transfer their learning from calculus courses to physics based the Pearson Correlation results. From results of HCA, we found the variables that assess performance in EPII and Calc I / Calc II did not closely cluster with each other, in the other words, what students did in their calculus course did not statistically significantly correlate with their performance in EPII. This also indicated relatively weaker evidence of transfer of learning was found when viewed transfer from Preparation for Future Learning (PFL) perspective. However, when viewed transfer from Actor-Oriented Transfer (AOT) perspective, which focuses on constructions of similarity between calculus and physics aspects on a given problem, we found evidence of transfer. We found statistically significant correlations between students' calculus and physics performance when solving a particular physics problem. Similarly, the dendrogram showed students' calculus and physics performances more closely clustered with each that with other variables.

The quantitative study could not give us a simple answer as to whether or not students were able to transfer their learning from calculus to physics. However, it did appear to indicate the possibility of transfer from calculus to physics. Thus at this point, the question – to what extend do students transfer their learning from calculus to physics -- was still an open one. To further investigate horizontal transfer using tradition physics problems, we conducted individual semi-structured interviews (Study I-2), the results of which are discussed in the next section.

5.2.2. Results of Study I-2

Eight paid volunteers who were enrolled in EPII participated in this study. Students were asked to solve four calculus-based physics problems during two interview sessions. After students solved the problem for a few minutes, we asked them to walk us through their solution. We used a phenomenographic approach to elicit common patterns in students' responses for the problems. We also asked the students to articulate any difficulties that they experienced as they solved the problem and possible reasons for these difficulties.

5.2.2.1. Results for Individual Questions

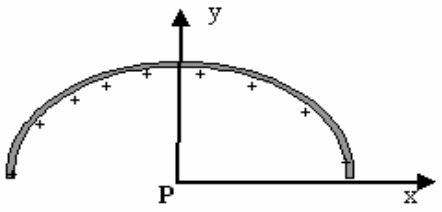
We first discuss below students' responses for each of the individual questions. Then, in the next section we discuss themes emerging from these responses.

5.2.2.1.1. Results for Q1: Electric field caused by an arc of charge distribution

Figure 5-7: Q1: Electric field callused by an arc of charge distribution

A thin non-conducting rod is bent into a semicircle of radius R, charge Q spread uniformly along it. Find the magnitude and direction of electric field E at point P at the center of the semicircle.

Note: As per Coulomb's Law the Electric Field E due to charge q at a distance r is: $E = \frac{kq}{r^2}$ where k is a constant.



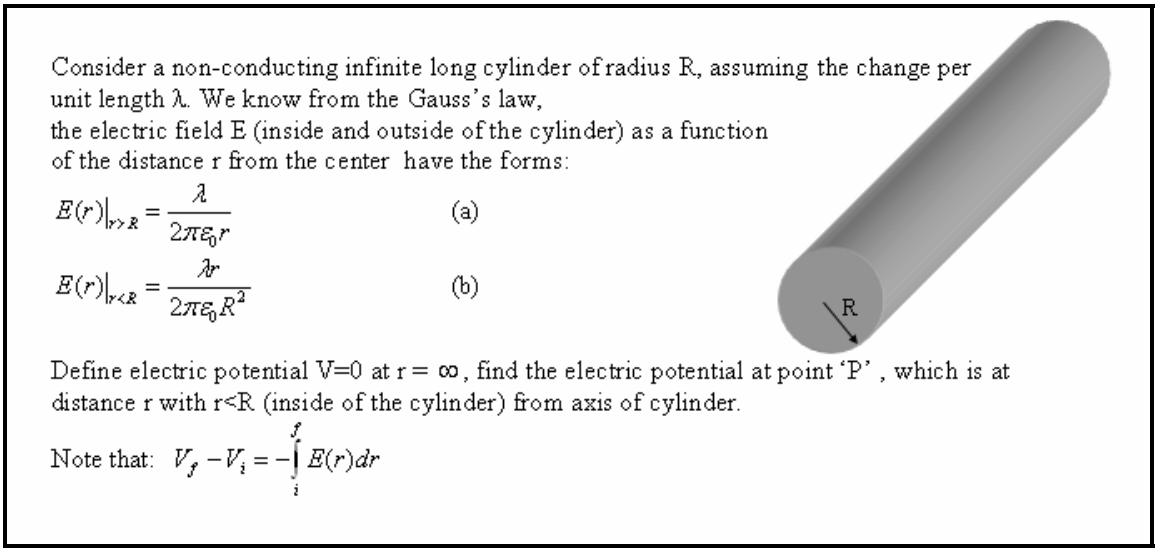
Two out of eight interviewees could solve this problem completely. Others could not solve it even after referring to the similar problem in the textbook. Interviewees spent an average of seven minutes on this problem. Almost all interviewees (seven out of eight) could write down $dE = \frac{kdq}{r^2}$ from the given equation $E = \frac{kq}{r^2}$ and $E = \int dE$.

Most interviewees (five out of eight) had difficulty defining dq in their equation. They could not get explain the physical and mathematical relationship between dq and ds, such that $dq = \lambda ds$. They also could not change variables of integration from ds to $d\theta$ such that $ds = \lambda r d\theta$.

One half of the interviewees did not use symmetry, so they ended up with an expression for dE instead of dE_y such that $dE_y = dE\cos\theta$. Overall, students experienced no difficulties while performing the integral. Only one of the eight interviewees made mistakes when calculating the integral.

5.2.2.1.2. Results for Q2: Electric potential within a uniformly charged cylinder

Figure 5-8: Q2: Electric potential in a uniformly charge cylinder



Only one out of eight interviewees could solve this problem completely. There was no similar sample problem in the textbook. Interviewees spent an average of seven minutes on this problem.

Most interviewees were confused between ds and dr. Interviewees were provided with the equation $V = -\int \vec{E} \cdot d\vec{s}$ from the textbook[1]. Three out of eight interviewees could not recognize that 'r' was the variable in this problem, thus they could not change the variables of integration from dr to ds in the integral.

Six out of eight interviewees used the wrong integration limits. The problem defined electric potential V=0 at r $=\infty$, which means the need to integrate equation (b) with the limits from ∞ to R, and integrate equation (a) from R to r. (Equations (a) and (b) were from the given problem.) The correct expression for the electric potential is:

$$V(r) = \left(-\int_{\infty}^{R} E(r)\big|_{r>R}\, dr \right) + \left(-\int_{R}^{r} E(r)\big|_{r<R}\, dr \right). \qquad (5.1)$$

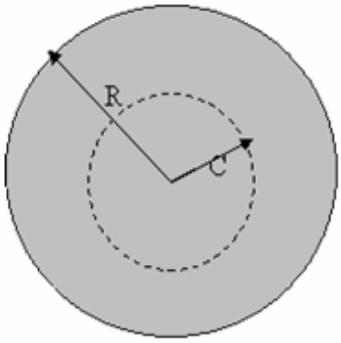
Most interviewees only used equation (a) in the question with the limits from 0 to r, which gave them $V(r) = -\int_{0}^{r} E(r)\big|_{r<R}\, dr$. They failed to recognize the physical meaning of potential as the work done in bringing a unit charge from infinity to a point. Two of the eight interviewees made mathematical errors when calculating the integral.

[1] We followed the notation from the textbook, Halliday, "Fundamental of Physics", 7th edition.

5.2.2.1.3. Results for Q3: Magnetic field caused by a non-constant current distribution

Figure 5-9: Q3: Magnetic field due to a non-constant current distribution

The figure right shows the cross section of a long conducting cylinder wire of radius R. The cylinder carries current insides of the page with a current density in the cross section is given by $J = ar^3 + br$, with a and b are constants. Find the magnetic field at the point c ($c < R$).



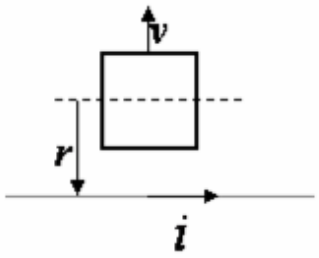
Five out of eight interviewees solved this problem correctly. Four interviewees referred to the similar problem in the textbook. Interviewees spent an average of five minutes on this problem. Half of interviewees wrote $B = \frac{\mu_0 i}{2\pi r}$ directly without referring to Ampere's law, $\oint B \cdot ds = \mu_0 i_{enclosed}$. In other words, they were referring to a closed form expression that is not valid under the problem situation of a non-uniform current.

All interviewees wrote down $i_{enclosed} = \int JdA$. However, two of the interviewees interpreted $dA = dr$ instead of $dA = 2\pi r dr$. In other words they had difficulty interpreting the two-dimensional geometry of the problem situation. All interviewees used the right limits --from 0 to C, in calculating $i_{enclosed} = \int JdA$, and none of the interviewees made mistakes when calculating the integration.

5.2.2.1.4. Results for Q4: Induced current caused by moving a loop in the vicinity of a conductor

Figure 5-10: Q4: Current induced a loop in the vicinity of a conductor

A square loop with length a, and resistance R is placed near an infinite long wire carrying current i. The distance from the long wire to the center of the loop is r. Find the induced current in the loop as it moves away from the long wire with the speed v.



Only one of the eight interviewees completed this problem. Three of eight interviewees only spent three minutes on this problem before they said they did not know how to solve it and were unwilling to try further. Others spent an average of ten minutes. Six out of eight interviewees wrote down $B = \frac{\mu_0 i}{2\pi r}$ directly. However, when calculating magnetic flux $\Phi = \int B \cdot dA$, only three out of these six could recognize that $dA = adr$ in this problem. Thus they could not relate r to v (velocity).

5.2.2.2. Emergent Themes of Students' Problem Solving Approaches (from observation)

In examining the ways in which students approached the problems and the difficulties that they experienced in solving these problems as described above, we arrived at the following themes.

5.2.2.2.1. Relying on equation sheet

We provided interviewees with an equation sheet (see Appendix C) because students were provided with equation sheet (prepared by the course instructor) during exams of the interview semester. We found all interviewees tended to refer to the

equation sheet right after reading the interview problems. They relied on the equation sheet to find the right equation instead of taking some time and thinking about the problem situation. Thus their immediate recourse in problem solving is to look for an existing schema or equation to solve the problem.

5.2.2.2.2. Pattern matching

Relying on the equation sheets led to the tendency of pattern matching. Interviewees read out certain information from the given problem and tried to find the matching value in the equation sheet. This pattern matching helped the interviewees to locate the formula they thought would be proper for the problem situation. For instance, if the problem was looking for electric field E, students would go to the equation sheet and search for the equation that had E in it. After students decided which formula to use, they tended to re-read out information from the given problem and tried to match it with the constants or variables in the formula. This problems solving behavior is similar to the often reported ends-means analysis (Sweller, 1988) that students typically tend to resort to.

5.2.2.2.3. Confusion about the meaning of different symbols

Interviewees did not have a clear understanding of what each symbol meant in the context of the problem. Symbols such as r, dr, s, ds, A or dA were often incorrectly used interchangeably by the students. This observation was consistent with the personal experience of Yeatts and Hundhausen (1992) that students had difficulties in transferring learning from calculus to physics because these two courses use different notation and symbolism.

5.2.2.2.4. Lack big picture of the problem

Majority interviewees could not solve the problems because they could not set up the problem. All of the interview problems needed multiple steps to complete but students did not seem to have a clear strategy to approach these problems. They were unable to step back from the problem and understand the big picture by thinking qualitatively first before attempting the problem quantitatively. This way of approaching the problem caused interviewees to have difficulty in setting up the problems.

5.2.2.2.5. Ability to perform calculus calculation

Almost all interviewees could correctly do the integration or differentiation and nobody asked to refer to an integration or differentiation table. This observation appears to indicate that students do know the mathematical processes for solving the problem, so the main hurdles they face are not related to their ability to integrate or differentiate.

5.2.2.3. Emergent Themes in Students' Responses (from discussion)

In addition to asking the students to solve the problems and explain and describe how they worked through them, we also asked students to articulate what they perceived as relevant issues to solving the aforementioned problems and the barriers they faced with regard to their preparation in calculus and physics. The following themes emerged from the phenomenographic analysis of students responses to these questions.

5.2.2.3.1. Self-confidence in calculus knowledge retention

All interviewees had taken Calc I and II before taking EPII. Three out of eight interviewees had positive experiences in their calculus classes, three had negative experiences and the other two were neutral. However, all of the interviewees stated that they were satisfied and confident of their calculus knowledge. The representative reasons why they were confident in their calculus knowledge retention are encapsulated in the following quotes:

"I have seen them a lot, they are just typical calculus problem";

"I have done it so many times, so I remember it well…";

"…they are just easy integrals…"

Interviewees' self-reflections described above were consistent with our observations. They typically spent several seconds to solve the assigned isomorphic calculus problems and answered them all correctly. This result indicated that students were able to retain their calculus schema pertaining to performing the differentiation and integration required for this course.

5.2.2.3.2. Realization that calculus is required in physics

All of the students realized that physics and mathematics were inextricably linked. As one student commented: *"Physics talks about why to solve it, math talks about how to solve it."* They also realized that they needed calculus knowledge to solve the physics problems. Representative comments were:

"*We use a lot of calculus in physics, more than use physics in calculus"*

"The math is kind of foundation of physics, do not understand math, you can not do physics".

5.2.2.3.3. Adequacy of knowledge learned in calculus class for the physics class

Seven out of eight interviewees thought their calculus knowledge was adequate for use in their physics class. Their comments were:

"The calculus I used in physics is not hard…"

"I have not come across many situations where I have no idea what the math means over there…"

Only one out of eight interviewees believed that their calculus knowledge alone was not sufficient to succeed in physics. She/he said: *"because it would teach you the basic mathematics, but at some point, I need them to teach me the different aspects as what's going on here (physics question)… although I am satisfied with my math, I think it is not enough to help me with physics…"*

5.2.2.3.4. Have seen similar physics problem before

All of the students had seen physics problems similar to the interview physics questions before. They commented that when faced with a new problem they would often try to find a problem that was similar to one they had seen before.

"Yeah, I have seen all similar problems in my EP2 homework and exams… this one (the interview problem) is very much like the one in my last exam"

This self-described strategy used by students appeared to be consistent with the pattern-matching problems solving process observed by the researchers.

5.2.2.3.5. Lack of confidence in setting-up physics problems

Although all of the students had seen physics problems similar to the interview physics questions before, none of them were confident about how to set up physics problems. Their typical comments were:

"I am not confident if I set up the problem right or wrong…";

"So many numbers and constants to taking account, I get confused, I lose objective of what I am actually looking for…";

"As soon as I set it up, there is no problem".

These comments suggested that students have difficulty associating their physics problem variables into their schema for solving calculus problems, although they retained their schema for solving calculus problems well.

Students' lack of confidence in applying their calculus knowledge to solving physics problems is consistent with our own observations. We also observed that interviewees were uncomfortable and had several difficulties when setting up the physics problem. However, they seldom had difficulty in carrying out the integral once it was set up.

We further probed students' views of the role of calculus in setting up a physics problem. So we added another question during the later interview.

5.2.2.3.6. Added question: Without calculus knowledge, it is possible to set up the physics problem?

Students were evenly split when asked whether it would be possible to set up the physics problems without calculus. Two out of five interviewees said it is possible to set up a physics problem without knowing calculus. Their reasons were:

"You can still set it up the relations although you do not understand calculus"

"You do not need to do real calculation. So setting it up is usually a physics thing. You can still understand (the physics) qualitatively"

Two out of five interviewees said it would not be possible to set up a physics problem without the knowledge of calculus. They remarked:

> *"Formula are all involved in calculus, if I do not know them, I will not understand the meaning of physics at all";*
>
> *"Although the set up part is basically physics, you still need certain math. Like 20% is math in the set up process…but you could not know what to do"*

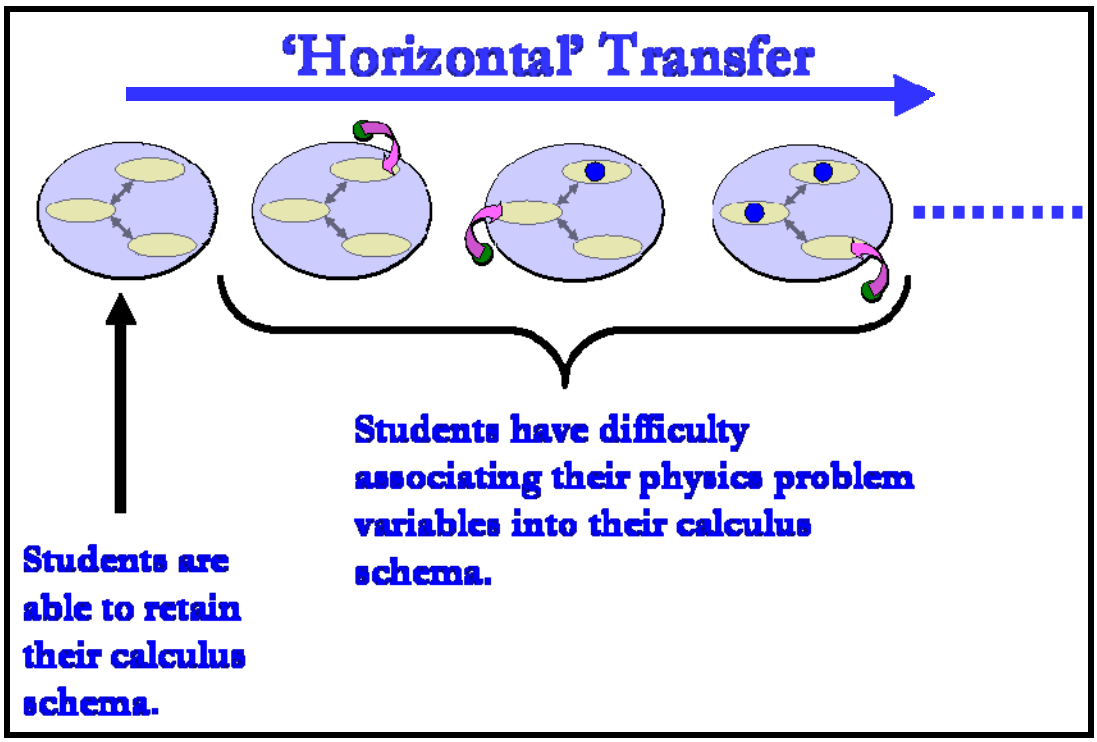
Furthermore, when asked to compare the physics and isomorphic calculus problems, only the students who successfully solved the physics problem could see the similarities in the problems. We also found that solving the isomorphic calculus problem did not help interviewees to solve the isomorphic physics problems.

5.2.3. Summary of Phase I Results

We investigated horizontal transfer of knowledge from calculus to physics in Phase I by using traditional physics problems. Horizontal transfer refers to the application of pre-constructed schema to solve problems in a new situation. Our quantitative analysis of student performance measures in calculus and physics (Study I-1) indicated a statistically significant correlation, which in turn indicated the possibility for transfer of learning from calculus to physics. To further investigate this possible transfer, we completed a qualitative research (Study I-2) using individual semi-structured interviews to further investigate horizontal transfer. We observed our interviewees were relying on the equations sheets and doing pattern matching. They were confused about the meaning of different symbols. Few interviewees had a big picture of the problem. Most interviewees were able to perform calculus calculation when solving traditional physics problems. Interviewees' answers to our probing questions suggested that students typically retained their problem solving calculus schemas for calculus problems. These students also appeared to realize calculus knowledge was needed to solve problems in physics and felt their calculus class had provided them with adequate calculus knowledge to do so.

However, these same students had difficulty setting up the physics problems that required calculus. It appeared that they had difficulty associating the variables provided in the physics problems to the calculus schema for solving integral, such as not being able to decide what variable to integrate or the limits of integration. Thus, these students were not confident in their ability to set-up calculus-based physics problems. This result is consistent with previous research on transfer of learning from mathematics to physics (Tuminaro, 2004). Tuminaro believed that student difficulties lay not in weak knowledge of mathematics, but rather in their ability to apply it in the new context. The results of Phase I (Horizontal transfer) are summarized in Figure 5-11.

Figure 5-11: Research Results of Phase I—Horizontal Transfer



Students seemed to fail to accomplish horizontal transfer successfully as per our definition of horizontal transfer. However, do the students see the problem solving as something different from horizontal transfer? The results in Phase I challenged our initial premise that solving end-of-chapter type of problems in EPII involved what we called horizontal transfer i.e. utilizing calculus knowledge to solve a physics problem involved

simply invoking an existing schema, and associating it with appropriate physical variables in the problem. Clearly, although we as researchers perceived typical end-of-chapter problems in EPII to involve what we called horizontal transfer, most students did not see it this way. For students, setting up the physics problem i.e. associating their physics problem variables with their calculus schema was the difficult part. In other words, what we researchers perceived as horizontal transfer was perhaps more accurately characterized as vertical transfer from these students' perspective.

The results in Phase I suggested that students had difficulties setting up the problem. We interpret setting up a problem to mean constructing an internal problem representation (or schema) that matches the external problem representation (i.e. the given problem situation). This difficulty in connecting the internal and external representations represented lack of vertical transfer. This realization led us to a further investigate vertical transfer in Phase II.

5.3 Results of Phase II—Vertical Transfer

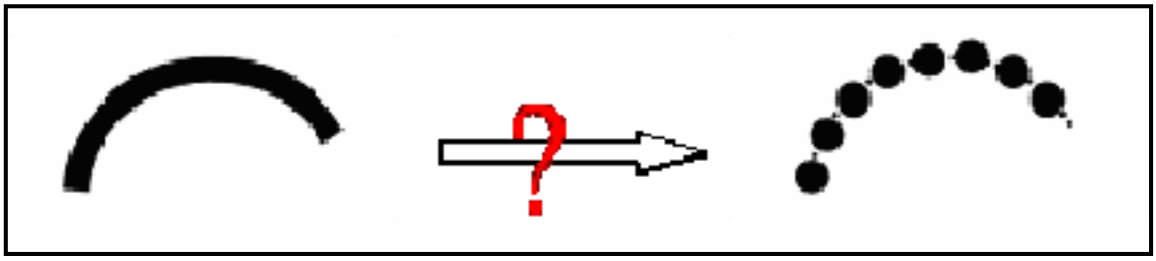
Two qualitative studies were conducted in phase II to assess vertical transfer of learning from calculus to physics. Because vertical transfer involves constructing new schema, or deciding between competing schemas, in previously unseen situations, we decided to examine students' problem solving approaches to non-traditional physics problems. These problems included variations of end-of-chapter problems or some completely different kinds of problems.

5.3.1. Results of Study II-1

In Study I-2, we had identified that students' difficulties in problem solving in EPII were mainly concerned with setting up the calculus-based physics problem rather than with calculus per se. In Study II-1, we further explored these difficulties using variations of the three problems described earlier. These problem variations helped us explore what we called vertical transfer from calculus to physics. One of the important aspects of engaging in vertical transfer is recognizing which schema is applicable in a given problem situation. Vertical transfer involves making judgments regarding the situations in which students believed integration was applicable to a physics problem. In Spring 2005, we conducted individual semi-structured interviews based on Bransford's (1989) idea of contrasting cases. We investigated vertical transfer by examining the thinking process of students as they decided whether or not use integration. Five male and three female paid volunteers were interviewed.

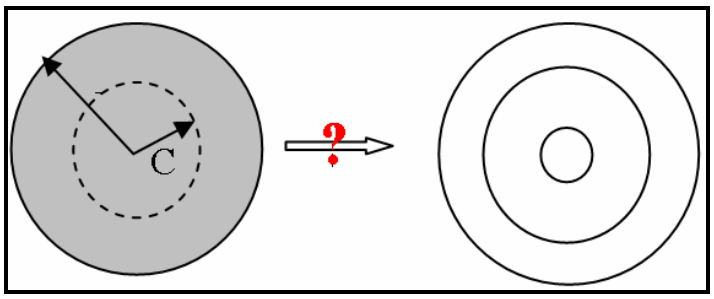
Variation I: As the variation of the "Electric field caused by an arc of charge distribution" question in Study I-2, we asked interviewees whether they would use the same method if there were several point charges instead of an arc-shaped charge distribution.

Figure 5-12: Variation I: Electric Field Caused by Arc of Charge Distribution



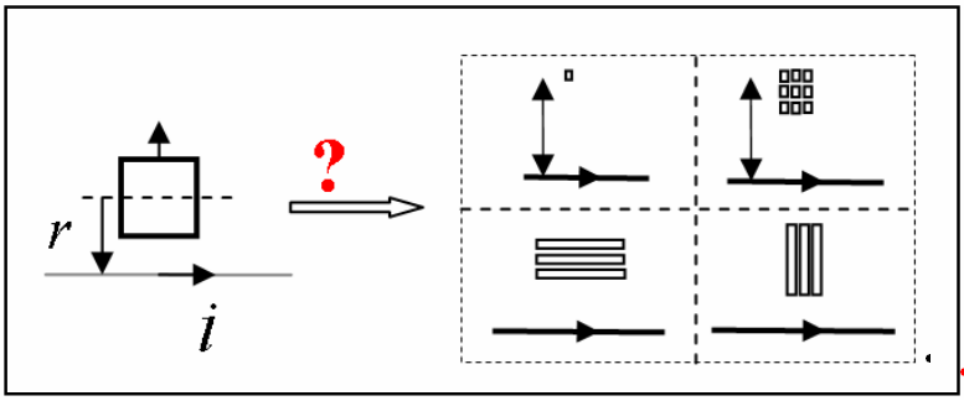
Variation II: As the variation of the "Magnetic field caused by a non-constant current distribution," problem in Study I-2, we asked students what would be the difference if we changed the constant current distribution into a few very thin layers of current and why.

Figure 5-13: Variation II: Magnetic Field by a Non-Constant Current Distribution



Variation III: As a variation of the "Induced current caused by moving of the loop in a changing magnetic field" problem in Study I-2, we asked the students to consider what would be the difference for the four cases shown below, with the very small loops moving close to a current carrying wire

Figure 5-14: Variation 3: Induced Current by Moving Loop in a Magnetic Field



5.3.1.1. Similar observations as study I-2

We used the same four physics interview problems as in Study I-2, and the interviewees' problem solving approaches were very similar. They tended to rely on the equation sheet, engaged in pattern matching, often having difficulties regarding the variables and limits of integration. While they were adept at performing the integral, they had difficulty setting up the problem and thinking qualitatively about the strategy that they would use before delving into the details of the problem. Students seemed to retain their calculus schema, but they had difficulties associating the read out information from a given problem to their calculus schema so to set up the physics problem. We had previously observed similar results for each individual problem as described in the results of Study I-2.

5.3.1.2. Emergent Themes in Students' Responses

The following themes emerged from the phenomenographic analysis of the students' responses to the contrasting cases presented to them in Study II-1.

5.3.1.2.1. Recognizing situations in which integration is appropriate

Seven out of eight interviewees appropriately used integration to solve the physics problems, while one student did not use calculus even after several hints. When the students that used calculus were asked about the criteria they used to decide why calculus

was applicable to the problem, four out of seven interviewees said the problems were similar to the examples they had seen in the textbook:

"Because it is the example in the book….I do not know the reason";

"I just know there is integral involved, I do not know why".

These four interviewees could not offer any other reason as to why they used integration in these problems. In other words, they were merely using integration because it reminded them of a similar problem they had seen previously that used integration.

Three out of seven interviewees had a rough idea as to why they needed to use integration in terms of adding up the infinitesimally small elements:

"You can not add up an infinite number…then I used integral…"

However, these interviewees were unable to further elaborate their criteria or explain in further detail what they meant by infinitesimally small elements. All of the interviewees replied in the negative when asked whether they had received any specific formal instruction on the topic of when integration rather than summation is an appropriate strategy.

All interviewees, even those who did not articulate the situations in which they would use integration, could solve the Variation I problem. They said they did not need to use integration if the problems involved point charges instead of a certain charge distribution,

Then we asked our interviewees, *"Now if you are smearing this point charge, to what extent would you choose to use integration instead of treating it as a point charge?"* Only one interviewee could clearly articulate the criteria.

> *"if they told us how far the smear, the distance, if that is way way smaller (than the distance between the smear and the target location)…I would consider it as point charge, if it is comparable, I would need to do integration"*

All other six interviewees could not articulate what would help one decide how large, or how close together the point charges (represented by the small dark circles)

should be before one needs to start using integration. They said: "*just very very small (charges), you know, point charges.*"

None of the interviewees could correctly solve the Variation II and III problems. The fact that the same students, who could solve the Variation I problem, were unable to solve Variation II and III problems appeared to indicate that they have difficulties adding discrete sources of magnetic or electric fields if these sources are not point charges. In other words, they are unable to generalize the process of discrete summation to geometries that are not point charges.

Overall, the interviewees' responses to the interview questions in this study appeared to indicate that they lacked a nuanced understanding of when integration would be applicable. Rather they typically tended to resort to pattern matching, and when pattern matching failed they had no overarching schema that they could invoke which would help them determine when and why integration was applicable.

5.3.1.2.2. Difficulties when applying integration in physics

The following themes emerged in students' responses when they were asked about their difficulties in applying integration in physics:

Determining the variable of integration. All interviewees complained that they had difficulty figuring out what was the "real" variable that needed to be integrated or differentiated. Representative comments were

> *"all constants (variables), I do not know what I should integrate although I know how to integrate"*
>
> *"I know how to integrate it, but it is just figuring out what to integrate, that is the hard part"*
>
> *"the physics use of calculus is not that bad, the thing is that you have to figure out what the variable goes where, what specifically you have to integrate with, that is what confused me, cause sometime you have, like* $E \cdot ds$*, which is really general, but what is* ds*, what should I substitute into it, and stuff like that, that really confuses me."*

A few interviewees who were able to figure out the variable of integration in the examples, stated that they *"got it from both calculus and physics, just look for whatever is changing"*.

Deciding the limits of integration. Most interviewees had difficulties in setting up the limits of integration. Furthermore, they usually did not realize that they had used the wrong limits. Calculus classes usually do not require students to set up the limits of integration; this did not mean students do not need to understand the meaning of limits. Students' difficulties in deciding the limits of integration in physics indicated their lack of understanding of limits.

Students' difficulties in determining the variable and limits of integration suggested students' lack of understanding the meaning of integration variables and limits and this impeded students' problem solving in physics. Each individual knowledge element (e.g. variables and limits) did not appear to be integrated into students' schema of integration.

Origin of difficulties. Six out of eight interviewees ascribed their difficulties to their physics class (EPII). One remarked that it has

> *"not really to do with my math class, just what variable you put there, cause when I got something to integrate, I know how to integrate it, but it is just figuring out what to integrate, that is the hard part, getting to the part.*"

Others felt that the calculus class was to blame. One of them remarked:

> *"Probably from math, because the concept of physics is pretty simple, because you can see the concept, I understand them well. …well, it is not physics is that hard, math is that hard, it is putting them together is hard, it is writing a equation for what I understanding is hard."*

5.3.1.2.3. Preference to use pre-derived algebraic relationship over calculus

Most interviewees tended to use pre-derived formulae rather than using calculus to derive the formulae from first principles. This tendency led to several difficulties. For instance, they would directly write:

$$B = \frac{\mu_0 i}{2\pi r} \qquad (5.2)$$

instead of using

$$\oint B.ds = \mu_0 i_{enclosed} \qquad (5.3)$$

and then applying it to derive the algebraic relationship. Also, when using the algebraic formula, they were not aware of the conditions in which the formula was applicable. When we asked interviewees why they preferred using the algebraic relationship rather than calculus they remarked that it was easier to go directly to the final answer rather than figure out the calculus. It is not surprising that students prefer to use a derived formula rather than starting from first principles, since it is the efficient thing to do and could decrease students' cognitive load. However, students appeared to be unaware of in which situation the pre-derived formula was applicable. This difficulty suggested students' lack of appropriate criteria to help decide when use of calculus was required and when the algebraic expression would suffice, indicated that students had difficulty deciding when to activate the appropriate problem solving schema – an important aspect of vertical transfer. Thus these students had difficulty engaging in vertical transfer.

5.3.1.2.4. Calculus in physics: Understanding or just plug-and-chug

Six out of eight interviewees felt that applying calculus in physics is more or less plug-and-chug. For instance, one of them said:

> *"I do not need to understand it, just how to do it. And I was doing good this way in calculus…"*

They were the same group of interviewees who believed that the origins of their difficulties came from physics class. Since these students believe applying calculus in physics is plug-and-chug, it made sense to them that the issue of understanding is only part of physics, not calculus. To this group of students, any problem solving that involved understanding is an issue in the physics class, not the calculus class.

Two out of eight interviewees believed they needed to understand calculus, or they would be *"confused"*. However they were unable to articulate why calculus was important and in what situations it would be necessary to use calculus.

5.3.1.2.5. Strategies to facilitate transfer from calculus to physics

At the end of the interview, we asked students about how their calculus or physics classes could be reformed to facilitate their learning. The following ideas emerged.

Learning how to set-up physics problems. Students would prefer more step-by-step scaffolding to help them solve problems in physics. This idea was mentioned by all of the interviewees.

Focus on understanding. Students would prefer a focus on understanding rather than on memorizing equations.

"Even in calculus, I had to understand why the differentiation of s^2 *equal to* $2s\ldots$*",*

"I need to know why integration and differentiation works (in physics)."

The aforementioned comments by students regarding the importance of conceptual understanding in calculus and physics seem to contradict the strategies that they appear to favor while solving physics problems. These strategies, such as resorting to algebraic relationships rather than deriving the calculus do not reflect a deeper conceptual understanding of either calculus or physics. The divergence between what students say that they value and what they appear to value based on their problem solving behaviors is similar to the dichotomy between students' epistemological beliefs and their personal epistemic resources reported by Hammer and Elby (2002). Hammer and Elby found that when asked students articulated epistemological beliefs that were similar to those of scientists, their personal epistemologies were more utilitarian i.e. they tended to do what works for them to succeed in the class rather than what they said they believed.

Course sequencing. Five out of eight interviewees stated that they would prefer to take calculus and physics concurrently because *"you will have more opportunities to use and understand it..."*

However, the other three students stated that they would prefer to take all calculus courses before taking any physics courses so that they would *"have some time to understand it."* This issue of appropriate course sequencing needs more investigation. We did not pursue the issue here because we were working within the constraints of an academic system at KSU that most likely would not be altered to accommodate changes suggested by these students, and therefore we focused on other changes and suggestions, discussed below that perhaps could be implemented within the individual courses.

More word problems in calculus. A majority of our interviewees would prefer more application-oriented problems in calculus to prepare them for future applications, because as one of them remarked:

> *"In word problems, you need to think about what integral you want to set up, so they can do that in calculus, that would be helpful, so when you go to physics, you are learning new material, like electricity, but you already know calculus."*

Thus, most of our interviewees appeared to recognize the value of word problems that required them to connect their knowledge in calculus to concrete applications.

5.3.1.3. Summary of Study II-1

In Study II-1 we investigated the extent to which students were able to examine a problem situation and decide the appropriate problem-solving schema e.g. integration vs. summation or calculus vs. algebra. Our results indicated that although students have retained their calculus knowledge, they do not understand, or are unable to articulate the criteria in which it is applicable for use in a physics problem and why. The strategies they use to decide whether a particular problem requires the use of calculus often rely on pattern matching i.e. comparing with similar examples they have seen before. Students appeared to suggest that their main difficulties in this area pertained to putting together their knowledge of mathematics and physics. While they were conversant with the techniques for integration, they had difficulties applying these techniques in the context of a physics problem. In other words, they had difficulty instantiating the appropriate

problem-solving schemas in the context of the physics problems in our interviews. Selecting the appropriate schema for a problem scenario is an important characteristic of vertical transfer. Thus, our students had difficulty engaging in vertical transfer in the context of these problems.

The other characteristic of vertical transfer is the ability to deconstruct one's existing schema and adapt it to a new problem scenario. In the next study (Study II-2) we investigate the extent to which students were able to solve problems that required them to deconstruct their schema to adapt it to a non-traditional problem scenario.

5.3.2. Results of Study II-2

We conducted individual semi-structured interviews in Fall 2005 by adapting the idea of Physics Jeopardy problems and Graphical Representation Problems. As mentioned in Chapter 4, Jeopardy problems present interviewees with an intermediate step in the form of a mathematical integration and ask students to come up with a physical scenario relevant to the integral provided. We were looking at vertical transfer by assessing if students could deconstruct their calculus/physics schemas and reconstruct a new schema to solve Jeopardy problems on the interview spot. In Graphical Representation problems, students were provided with a graph and asked to read out information from the graph so to find an answer of a certain variable. Eleven male and one female paid volunteers participated in this study. As mentioned before, we understood that Jeopardy problems and in some ways Graphical Representation problems were rather challenging problems to our interviewees. Our goal was not to find out whether they could correctly solve these problems, rather we were interested in process they used to attempt these problems.

5.3.2.1. Jeopardy Problems

We observed that most interviewees (ten out of twelve) had a difficult time solving the Jeopardy problems. They typically wrote down very little on the paper provided during the interview. This behavior was different from the traditional physics problems in Study I-2. It took interviewees an average of six minutes to try to solve the Jeopardy problems. Our observations of students' attempts to solve the problem did not provide

much information on how they would approach the problem. Thus our data were mainly obtained from the probing questions and discussions with interviewees.

All interviewees indicated they had never previously heard of Jeopardy problems and had never solved similar problems before. To help them become familiar with Jeopardy problems, we used a sample problem by presenting them with the expression (5.4) below:

$$60kg \times 9.8 m/s^2 \qquad (5.4)$$

Students were asked to describe a physical situation that when solved would result in equation (5.4) above being an intermediate step of the problem solution.

None of them had any difficulty solving this sample Jeopardy problem by describing physical situations as follows.

"Something falling, a block with 60 kg, accelerating,"

"So this is backward, so it is Jeopardy. You have a 60 kg object and you drop it"

Being satisfied that students were now aware of what a Jeopardy problem was, we presented them with several different Jeopardy problems in calculus-based physics. The following themes emerged from analyzing the ways in which students approached these Jeopardy problems.

5.3.2.1.1. Converting numerical representation into physical symbols

In the first set of interviews, we used the real numbers in the Physics Jeopardy problems. All interviewees tried to convert the numerical representation to the physical symbol.

In the following example (See expression 5-5), only one out of the sixteen students failed to recognize that $8.99 \times 10^9 N \cdot m^2/C^2$ was the constant k.

$$2\times\left[\int_{0}^{\frac{\pi}{6}}(8.99\times10^{9}\ N\cdot m^{2}\Big/C^{2})\frac{(2\times10^{-10}\ C/m)(5\times10^{-2}m)\cos\theta d\theta}{(5\times10^{-2}m)^{2}}\right] \qquad (5.5)$$

When asked about their strategy to approach this problem, interviewees reported that they needed to convert the numbers into symbols to make sense of the expression. As one student remarked:

"*They* (physical symbol) *are more straightforward …those numbers can be distracting"*.

To reduce the cognitive load of dealing with numbers and units while approaching this type of problem, in the second interview, we used typical symbols representing physical quantities in the Jeopardy problems, such as expression (5.6)

$$\int_{0}^{a}\frac{\mu_0\cdot I\cdot\dfrac{R}{\sqrt{s^2+R^2}}\cdot ds}{4\pi\cdot(s^2+R^2)} \qquad (5.6)$$

When asked to compare the Jeopardy problems with variables and symbols with those that used numbers, a vast majority of interviewees preferred the symbolic equation. One student commented, when asked about his preference for symbolic notation:

"I do not think it really makes difference to me, because if I see a number, I look the unit after it, and then I just translate that to what variable it is, so not much difference, and actually I like the variable method better, because I still need to write things down as variables".

We found this observation to be interesting because when solving traditional physics problems, most students prefer numbers instead of symbols and typically tend to substitute in numbers early in the problem solving process. Thus Jeopardy problem

appear to challenge students existing ways of approaching problems and the students seem to be changing their existing problem solving schemas in response to this challenge.

5.3.2.1.2. Using units to find the physical quantity

In the first set of interviews, when provided with numbers and units, most interviewees tried to 'play' with the units to find the answer. For example, one student described her strategy as follows:

"I take all the units and convert them to find what variable they are looking for."

This result was similar to the result above regarding the use of symbols in Jeopardy expression problems and converting them into physical quantity with a unit.

5.3.2.1.3. Using pattern matching but not explaining why

Pattern matching appeared to be the most commonly used technique by our interviewees used when solving Jeopardy problems. Students looked for the familiar terms that they could recognize and compared these with terms on the provided equation sheet[1]. One student described her/his strategy as follows:

"I look for pieces of terms that I recognize μ_0, J (current density)…they will tell what kind of problem they are, I just tend the recognize forms, like derivative…"

This pattern matching strategy sometimes helped interviewees find the right equation. For example, when interviewees noticed the numerical value of symbol for μ_0 in the problem, they could narrow down their search on the equation sheet by just looking for the equation which involving μ_0. Using this method, about one half of the interviewees was able find the right equation. However, they could not explain

[1] We gave interviewees an equation sheet because students were given an equation sheet during their exams. Students were not required to memorize equations in that particular semester. The equation sheet we used in the interview were very similar to the one given on the exam

corresponding to the physical situation. For instance, one remarked *"I do not know why those formulas work, I just use them."*

The other half of the interviewees was unable to use the pattern matching strategy to recognize the right formula. The reason was because when matching two equations, interviewees tended to focus on limited numbers of terms in the formula instead of considering all terms. They paid more attention to the equation constant instead of the variable of integration or differentiation. This tendency appeared to be a source of difficulties in deciding whether expression (5.7) referred to electrical field (E) or electrical potential (V) at a point, since all of the constants were same for both cases.

$$\left[\int_{\infty}^{(3\times10^{-2}m)}(8.99\times10^{9}\,N\cdot m^{2}\Big/C^{2})\frac{(5\times10^{-10}C)dr}{(3\times10^{-2}m)^{2}}\right]+\left[\int_{(3\times10^{-2}m)}^{(2\times10^{-2}m)}(8.99\times10^{9}\,N\cdot m^{2}\Big/C^{2})\frac{(5\times10^{-10}C)rdr}{(3\times10^{-2}m)^{3}}\right] \quad (5.7)$$

Expression (5.7) refers to the electric potential due to a charged sphere at a point inside the sphere. Most interviewees recognized that $8.99\times10^{9}N\cdot m^{2}/C^{2}$ was the dielectric constant k, $5\times10^{-10}C$ was the charge q, and $3\times10^{-2}m$ was the distance r, Then they matched the expression with the following formula (5.8) provided in the equation sheet:

$$E=\int dE=\int k\frac{dq}{r^{2}} \quad (5.8)$$

Thus they concluded the provided problem situation was to find electric field E, rather than the electric potential V.

> *"I saw the integral, and I look at these (formula), and only a few of them use integral and then I put this in, and I realize this is the integral for E field",*
>
> *"This constant k is usually used in electric field questions (so this problem is to find electric field)".*

In another question, students were provided expression (5.9), and asked to construct a physical situation corresponding to it.

$$\int_0^a \frac{\mu_0 \cdot I \cdot \frac{R}{\sqrt{s^2 + R^2}} \cdot ds}{4\pi \cdot (s^2 + R^2)} \qquad (5.9)$$

The physical situation that expression (5.9) corresponds to is the magnetic field due to a line of charge as evaluated using Biot-Savart's Law.

Half of the interviewees understood that *ds* is the small "chunk" that needs to be integrated. However, they were unable to explain the meaning of 'ds' any further in the context of this particular problem.

A similar situation occurred with the Jeopardy problem in expression (5.10).

$$\int_0^R \frac{J(r).(2\pi r dr)}{2\pi R} \qquad (5.10)$$

When asked to explain or draw $2\pi r dr$, few interviewees appeared to understand that $2\pi r dr$ represented an annulus of width 'dr' and radius '$2\pi r$' small ring shape. Two of the sixteen students appeared to realize that it had something to do with the circular geometry of the situation; however when asked to explain the situation more clearly, they appeared to be unclear, and stated:

"…just the circle …, that what the integration means",

"the circle dA (area) is always $2\pi r dr$, I do not know why".

All of the above results indicated that the students' had difficulties understanding the physical significance of variables of integration or differentiation such as *ds* or *dr*, and could not use these as clues to decipher the problem situation. This indicates that students faced difficulties in deconstructing and reconstructing their calculus-based problem solving schema in the context of previously unseen Jeopardy problems. Students' problem solving strategies appeared to rely heavily on pattern matching, which may have helped them in some situations but were seldom adequate in helping them construct the

physical situation represented by the expression. Thus, students had difficulty in engaging in what we call vertical transfer in the context of these problems.

5.3.2.1.4. Value of Jeopardy problems

Although interviewees generally agreed that Jeopardy problem were very hard to solve, most of them believed that solving jeopardy problems would help them better understand physics concepts. Some of the students' representative comments are given below.

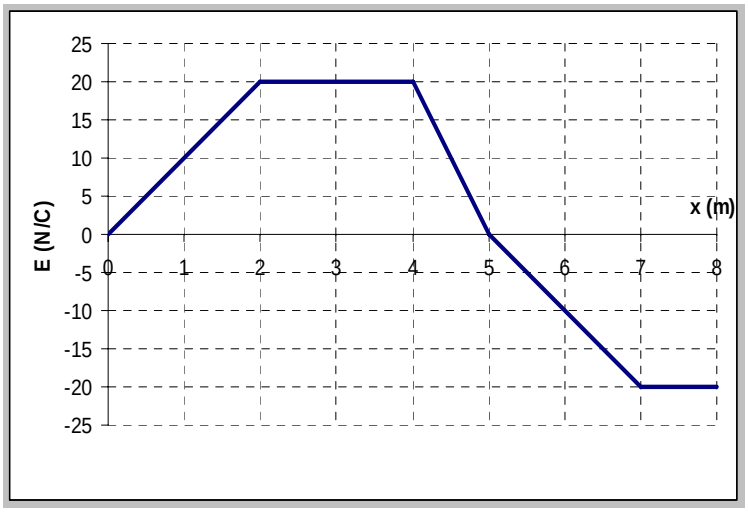
> *"Just working things backward, you have to understand it better, because if you just start with everything given and plug in the formula, you might get something better out of it, you might understand it better, but this way you will understand it really well, because you have to know where is everything come from"*
>
> *"because then we break down the problem and find out each part and then figure out why they multiple by some other parts, you can only truly understand something complicated only if you break it down to each part and why it uses in different cases. For the back of chapter problems (traditional physics problems), you manipulate formula…so two different procedure to get you to learn",*
>
> *"I have a test this afternoon, and now I feel much more prepared after just done these (Jeopardy) problems"*

5.3.2.2. Graphical Representation Problems

Two Graphical Representation problems were used in the interviews to assess students' ability to read out calculus-based information from a given graph and then relate the readout information to physics knowledge. We believed the ability to read out information from a given graph, like reading out information from a given equation, was another important aspect to assess students' knowledge transfer.

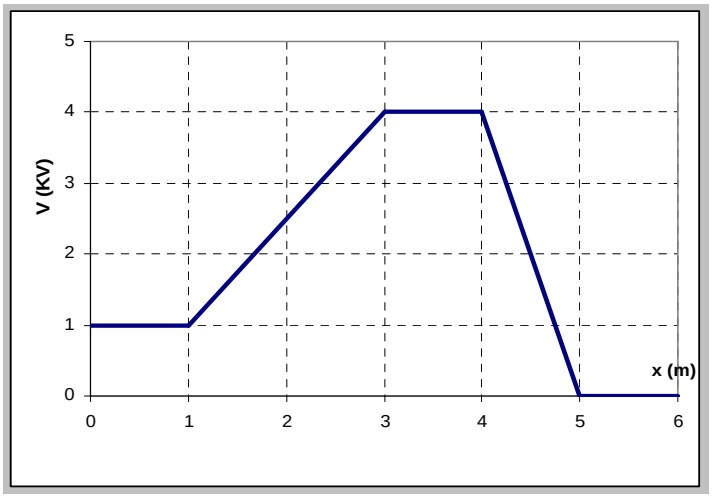
Graphical Representation Problem 1 (Figure 5-15): Given the graph of electric field, the student is asked to describe the physical situation and also find the electric potential at different points in space.

Figure 5-15: Graphical Representation Problem 1



Graphical Representation Problem 2: (Figure 5-16): Given the graph of electric potential, the student is asked to describe the physical situation and also find the electric field in different regions of space.

Figure 5-16: Graphical Representation Problem 2



It took interviewees an average of four minutes to try to solve the Graphical Representation problems. Interviewees tended to write the algebra calculation down to show how they got the answer. Similar to Jeopardy problems, our observations of students' attempts to solve the problem did not provide much information on how they would approach the problem because of the simplicity of the problems. The research data were mainly obtained from the probing questions and discussions with interviewees. We observed that only two of the twelve interviewees could successfully solve these two Graphical Representation problems correctly. The remaining students approached these problems in two kinds of ways.

One half of the students were unable to solve the problem because they appeared not to understand the relationship between electric field and electric potential, although they were provided with the equation sheet that clearly had equation (5.11) provided.

$$E_s = -\frac{\partial V}{\partial s} \qquad (5.11)$$

The other half of interviewees appeared to understand the relationship between E and V, given by equation (5.11) but they could not recognize that the slope of graph represented the differentiation and the area under the curve represented integration. Those

who were able to recognize this relationship graphically appeared to be unable to articulate the reason. As one student stated:

"This is kind of thing that I have known so long and I could not explain."

Most students said they learned this idea from their calculus courses. One student commented that she had not learned it until she was required to use the idea in one of her engineering courses.

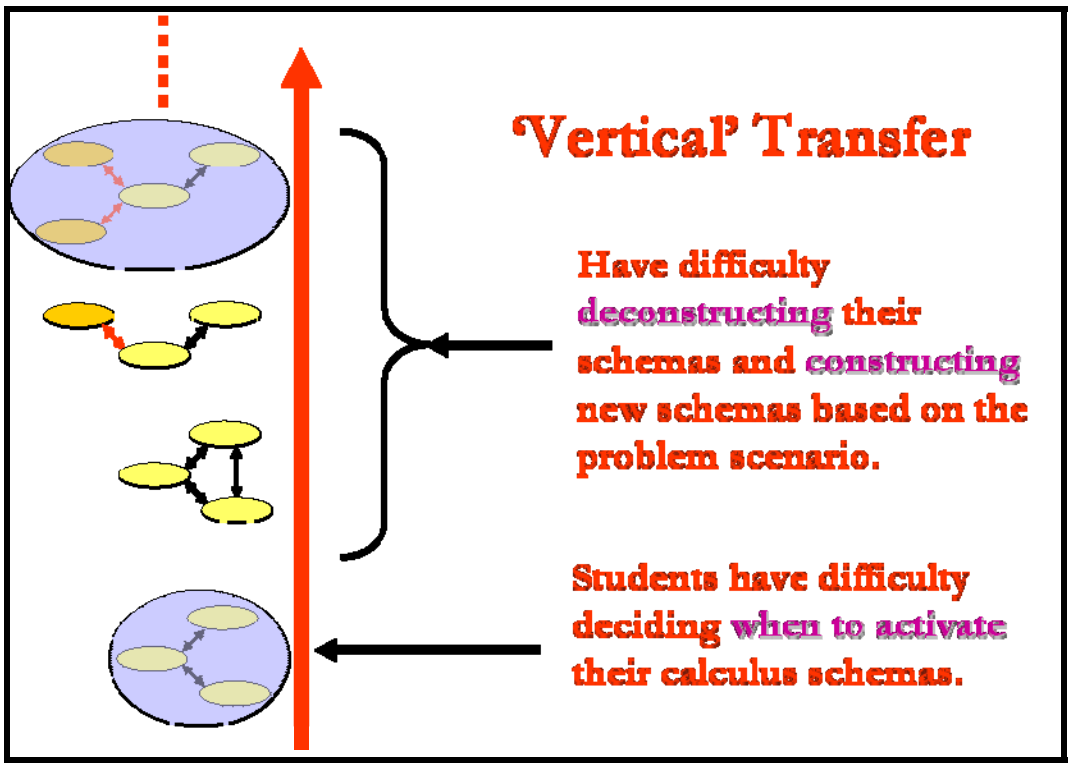
"I also had an exam last year, we were given a graph similar like this and ask to find E potential, and nobody could do this, so that stuck with me. That was not in physics though, that was in engineering class"

This comment appears to indicate that this student and perhaps others appeared to rely on a strategy where they tried to recall a similar question from before i.e. they resorted to pattern matching. However, when they were unable to recall a similar problem or perhaps recall the problem, but not where they had seen it before, they were unable to articulate their reasoning.

5.3.3. Summary of Phase II Results

We investigated vertical transfer of learning from calculus to physics in Phase II by using non-traditional physics problems. Non-traditional physics problems are useful for investigating vertical transfer because they require students to constructor deconstruct their existing schema to address the particular situation. Alternatively, in analyzing contrasting cases students must select one out of two or more schemas based on the situation. The results of Phase II are summarized in Figure 5-17.

Figure 5-17: Research Results of Phase II—Vertical Transfer



Results from our studies in Phase II indicate that students often recognize familiar features in a problem, and resort to pattern matching with earlier problems that they have seen before. Students often have difficulties in deciding when to activate appropriate problem solving schemas that use integration strategies. Other difficulties included determining the variable of integration or differentiation and the limits of integration. Students also appeared to have difficulties deconstructing and reconstructing their problem-solving schema based on the new problem scenario.

To facilitate transfer of learning from calculus to physics, our interviewees appear to believe that instruction should place greater emphasis on setting-up physics problems and on conceptual understanding in both calculus and physics courses, rather than merely on strategies. Interviewees also appeared to indicate that the inclusion of word problems in calculus courses might help prepare them to solve word problems that are commonly encountered in physics.

5.4 Results of Phase III—Instructors Interview

In Phase III, we investigated transfer of learning from calculus to physics from the instructors' point of view. We interviewed instructors and teaching assistants from both the mathematics department and physics department and asked them about their expectations and outcomes of their courses.

5.4.1. Interview results from the mathematics instructors

We interviewed two mathematics faculty members and two teaching assistants who had previously taught or were teaching calculus courses. The following themes emerged from the phenomenographic analysis of the interviews:

5.4.1.1. Experienced in teaching calculus

All interviewees indicated that they were rather experienced in teaching calculus. All of them had taught calculus courses (either Calc I or Calc II)[1] more than four times and they were positively disposed about their teaching experiences.

5.4.1.2. Challenges when teaching calculus

Students' weak backgrounds of algebra and trigonometry knowledge were mostly mentioned when interviewees were asked their challenges in teaching calculus. For instance, some of the interviewees said,

> *"Sometimes I had the impression that the students have very weak skills in (algebraic) computations, some students even do not know how to compute one divide one third, I would say they do not know much about algebra";*
>
> *"The things they do not like is trigonometry, anything to do with trig, they freaked out".*

[1] We were mainly interested in Cal I & Cal II because the concepts addressed in these two courses are relevant to physics.

All of the calculus instructors that we interviewed appeared to suggest that students' prior preparation in mathematics, or lack thereof was a major challenge in teaching this course.

5.4.1.3. Expectation after Calc I and Calc II

All interviewees indicated that they wanted their students to learn how to perform integration and differentiation after taking Calc I and Calc II. The topics mentioned by the interviewees were: *"the techniques of doing integration", "how to compute differentiation, and how to compute integrals", "a functioning ability to use calculus…to do integration and differentiation."*

Only one interviewee explicitly mentioned that students should *"have the basic concepts of derivative and integrals,"* and hoped that "*that understanding comes from Calculus I and Calculus II."* This interviewee added that he would *"want them* (the students) *to be familiar with the varieties of integral… so they can be comfortable with it"*.

Two out of four interviewees said they wanted their students be able to solve some simple applications.

> *"They should have a functioning ability to use calculus, so they should know how to do integration and differentiation; they should understand the application in a sense how to use calculus to find the maximum, minimum problems"*

> *"We do use examples to let them understand in what situation how we can use them, but I am not sure if they get from that, but they should at least understand looking at the graph, you find the peaks, and derivative represents the slope"*

5.4.1.4. Satisfied with calculus course outcomes

All interviewees appeared to be satisfied with the extent to which their calculus courses achieved their desired learning outcomes described above. They appeared to believe that a majority of their students had learned the ideas that they wanted to convey

in the courses. The general notion that while a *"majority students are doing quite well, there are always 20-30% that can not meet your expectation, in any course"* was expressed by all the calculus instructors that we interviewed.

5.4.1.5. Limited knowledge of calculus applications

When asked about the applications of calculus to other disciplines, all interviewees said that they were aware that calculus is used in *"a lot of things related to rates, usually used to describe physical situations."* However, when asked to provide specific examples, most interviewees could only offer a limited number of examples of such applications. Some of the responses provided by instructors were:

> *"How to compute energy, we even use this one in our Differential Equations (course), and another one is how to compute the volume, other than that, I can not think of anything else."*
>
> *"For further example, concrete examples, hum, because my lack of knowledge of physics, I really feel I do not know much real physics examples using calculus."*
>
> *"I do not have enough background to actually know where they are generally used"*

5.4.1.6. Limited use of word problems in calculus

Our student interviewees were virtually unanimous in suggesting that increased practice with word problems in calculus would better prepare them to solve word problems in physics. Therefore, we asked instructors about the use of word problems in their calculus courses. When talking about applications, a majority of interviewees agreed that word problems were important. However, they also conceded that few word problems were used in calculus homework and exams. They said that usually about 10% - 20% of the problems in calculus courses were word problems. We reviewed the past five years' calculus exams and found that this estimate was consistent with the actual number of word problems in these tests and exams. Compared with problems in physics courses, in which typically at least 70% were word problems, fewer than 20% of the problems were word problems in calculus. When asked why word problems were not asked more

frequently, almost all of the instructors we interviewed claimed that students often disliked word problems. One interviewee said:

> *"Students told me that they even do not want to try…something I never understood myself, cause that is the problem that you encounter in everyday life, but for some reason, translate a word problem into a mathematical problem is the big step…they do not do well on the word problems, so, as far as on exams, I mean I was trying to put some on them, but I do not make the exam too hard"*

We find the above representative comment particularly interesting because as mentioned earlier, students whom we interviewed in EPII had remarked that they would have preferred word problems in their calculus class. This view appeared to be completely contradictory to the interpretation of the students' views as expressed by the calculus instructors. We speculate that this discrepancy is because these students, while they were taking calculus found word problems to be difficult and therefore disliked having them on the exams. However, when students took EPII and were required to solve word problems, in retrospect they felt they would have preferred to have more word problems in their calculus courses.

5.4.1.7. Interested in hearing what physicists feel

Most interviewees indicated that they would like to know what other departments expected their students to have learned from a calculus class. They also appeared to believe that most departments were in someway not satisfied with the level of calculus preparation the students had acquired after completing their calculus course sequence. One interviewee explicitly said *"I would be very interested to hear what physicists feel that what we are not preparing them for it"*. This finding indicates that there might be potential for greater collaboration between departments in the future.

5.4.2. Interview results from the physics instructors

We interviewed two faculty members and three teaching assistants who had taught or were currently teaching EPII. The following themes emerged from a phenomenographic analysis of instructors' responses:

5.4.2.1. Experienced in teaching EPII

All interviewees indicated that they were experienced teachers in teaching EPII. Both faculty members had taught EPII more than three times. One teaching assistant had taught EPII studio three times and the other two teaching assistants had taught EPII studio two times before. All interviewed teaching assistants were lead instructors[1] in the EPII studio. Similar to the mathematics instructors interviewed earlier, the physics instructors appeared to be were satisfied about the success of their physics courses and the extent to which students were able to achieve the desired learning outcomes for these courses.

5.4.2.2. Expectations from calculus courses

When asked what knowledge and skills they expected their students have acquired from the calculus classes, all interviewees indicated that they expected their students to have "basic calculus knowledge" before they came to EPII. This "basic knowledge" included the simple techniques of performing integration and differentiation. They also expected their students to have a conceptual understanding of these operations. All of the interviewees were satisfied with their students' ability of doing calculus; however they were often dissatisfied with other aspects of their students' learning:

One interviewee pointed to students' conceptual understanding of calculus principles:

---

[1] A Lead instructor's main responsibility in Studio is to go over homework problems, design and grade quizzes. The teaching assistant is to assist with laboratory activities. Compared with the teaching assistant, the lead instructor needs to prepare for the recitation session and is typically more familiar with students' problem solving approaches and difficulties.

"I expect them to do simple mechanically differentiation and integrals, and most of them can do that, but I also expect them to conceptually understand what a derivative or integral means, and many students do not understand that."

Another interviewee alluded to students' difficulties in applying the calculus strategies to physics problems:

"The students did learn the calculus and they were able to do it, integrals or differentiations in the context of their math class, when you apply it to the physical problem, there is really a conceptual jump."

Yet another interviewee would have preferred students to possess superior problem solving skills after taking calculus courses.

"What I expect them to have seen some problem solving skills… from their lack of preparation, I would say they have not seen many word problems. These word problem does not seen to be in physics, could be in anything, but that they read a problem, from that, they need to set up the math, they lack that skill completely".

Typically, interviewees believed that only about one third of their students had the required calculus knowledge that they would have liked them to have when they began their EPII course.

5.4.2.3. Strategies to address students' difficulties in physics

When asked what they would do to help students overcome their difficulties in physics, the physics faculty members that we interviewed pointed to three strategies that they claimed to use:

1) Provide more concrete examples that would demonstrate how the concepts were applicable. One interviewee remarked:

   "Solve more problems, this is the only way that they can get though there, but also, especially EPII, they need some concrete examples"

2) Emphasize conceptual understanding rather than merely problem solving strategies, and

3) Utilize visualization strategies that help students connect their calculus knowledge to a physics situation.

5.4.2.4. Suggestions for the mathematics department

When asked to provide suggestions for mathematics faculty members who were teaching the calculus courses, most interviewees said that they would prefer to see more word problems in calculus to develop students' problem solving skills.

> *"I wish the first time when you teach integrals, we work hard on that, like a line of charge, but if they have seen the word problem in which the integral has been set up, as I said, it does not need to be a physics problem, can be anything, like a financial problem, then they are familiar with the process, I do not have that training in most of my students, that fact that you can break a charge into a very little point, little charges, seems a mystery to them but this should not be, since this is the basis of calculus,"*

Along the same lines, all interviewees suggested that the calculus courses should focus more on conceptual understanding of the principles underlying calculus rather than on strategies for merely doing calculus.

> *"I would be happier if the mathematicians put more emphasis on the theoretical basis of calculus, in terms of the exercises, more emphasis on simple problems"*

> *"You do need to have that mechanical ability, but actually more important is the conceptual thinking, if they have to go on to EPII, that is even more true, because there are programs, the computer programs are readily available, they will do all of the mechanics for them, what they need is to know how set up the problem, the mechanics can be automated"*

All of the physics instructors' suggestions, for the most part appeared to be consistent with those of student interviewees. However, as expected the instructors were much more articulate and cogent in their responses to the questions than the students.

5.5 Chapter Summary

In this chapter we discussed the results of this research project. The project was divided into two phases. In Phase I we investigated horizontal transfer of learning from calculus to physics, while in Phase II we investigated vertical transfer of learning.

Horizontal transfer, which we investigated in Phase I involves the application of a pre-constructed schema – in this case the schema for performing integrations or differentiation -- in a new context. For transfer to be characterized as horizontal we must choose a target context such that the problem representation in this context maps onto the learners' internal representation. Therefore, we used EPII exam problems to explore horizontal transfer.

In Phase I we developed a rubric to examine students' performance on the calculus and physics aspects of EPII exam problems that required calculus. Our results showed a statistically strong correlation between students' calculus and physics performance within an EPII exam problem. This appeared to indicate the possibility of transfer when viewed from the contemporary Actor-Oriented Transfer (AOT) perspective which focuses on students dynamic constructions of similarities between two aspects of their knowledge, which in this case was their knowledge of calculus and knowledge of physics. However, the correlation between student performance on EPII exams problems and their Calc I and Calc II course performance was not as significant. We found relatively weaker evidence for the possibility of transfer when viewed from traditional perspective and Preparation for Future Learning (PFL) perspective. This weaker evidence for the possibility of transfer when viewed from a more traditional perspective compared with stronger evidence from a contemporary perspective is consistent with previous research on transfer (Ozimek, 2004). This quantitative study also reaffirmed the results of previous research that "one-shot" assessment methods are insufficient to assess transfer. Rather transfer of learning from calculus to physics must be examined from multiple perspectives through the use of multiple research assessment strategies including individual semi-structured interviews.

In Phase I we also investigated students' problem solving processes of end-of-chapter EPII problems using individual semi-structured interviews. Our interview results suggested that students were able to retain their calculus schema for performing

integration. However, students had difficulty associating their physics problem variables into their calculus schema. This result is consistent with previous research on transfer of learning from mathematics to physics (Tuminaro, 2004). Students often had difficulties in setting up the physics problems, such as in deciding the variable or limits of integration. Student difficulties in Phase I appeared to indicate that from the students' point of view end-of-chapter EPII problems involved vertical and not horizontal transfer as we had previous assumed. This observation is consistent with our flexible framework of vertical and horizontal transfer which allows for divergent interpretations of the same task as involving either horizontal or vertical transfer when examined from the perspective of varying levels of expertise.

Vertical transfer, which we explored in Phase II, occurs when a learners' existing schema or internal representations do not match the external problem representations. In these situations the learners' may need to choose between multiple competing schemas or they may engage in cognitive processes that include reconstruction or deconstruction of their schemas.

In Phase II, we examined vertical transfer by asking interviewees to solve non-traditional calculus-based physics problems. Three kinds of non-traditional problems were used: contrasting cases, in which students had to decide in which situations integration would be appropriate and why; Jeopardy problems, in which students had to deconstruct the problem information provided in mathematical form and construct a physical situation corresponding to it; and graphical representation problems in which students had to use graphical representation and explain its connections with symbolic representations of the schemas of integration and differentiation.

We found that students had difficulty deciding when to activate their problem-solving schema utilizing integration and differentiation although they appeared to have retained these schemas. In examining the contrasting cases students appeared to have difficulties deciding when to use integration in a problem situation and performing an algebraic sum would suffice. In the jeopardy problems we found that students often had difficulty taking apart the problem and constructing the corresponding physics situation. We interpret these as difficulties to deconstruct and reconstruct schema based an

unfamiliar problem scenarios. In the same vein, students also appeared to have difficulties constructing connections between the meaning conveyed by the graphs and the corresponding symbolic representations.

In general, when faced with problems that required vertical transfer we found that students tended to rely on pattern matching i.e. searching for a similar problem that they had encountered below, without being able to articulate the underlying conceptual reason for their strategies and in what conditions these strategies would be applicable.

Students' suggestions for their instructors focused on providing more detailed instruction on how to set up physics problems, more focus on understanding in both calculus and physics, and more experiences with word problems in calculus courses.

In Phase III of the study we interviewed instructors who were teaching calculus and physics to get their perspectives. We found disconnects between what the calculus instructors do in their classes and what the physics instructors would prefer their students to have learned from their calculus classes. Calculus instructors appeared to focus more on the strategies doing calculus problems. They did not focus as much on understanding of the conceptual principles underlying these strategies. They also appeared to use a limited number of word problems on calculus tests and exams because they felt that students did not favor having these problems on test and exams. They also appeared to avoid real-world examples of where calculus could be used because majority of interviewees were not knowledgeable enough to understand these examples. Physics instructors appeared to be satisfied with students' ability to mechanically perform integration and differentiation. However, they would have preferred a deeper conceptual understanding in calculus and greater use of word problems in calculus.

In the next chapter we present these findings in the light of the research questions and discuss the implications for instruction and future research in the light of our framework on vertical and horizontal transfer.

CHAPTER 6 - CONCLUSIONS

This study investigated the retention and transfer of learning from a calculus course to a calculus-based physics course taken primarily by engineering and physics majors. We proposed a theoretical framework (horizontal and vertical transfer) that served as a lens with which to analyze our research results. We use a combination of qualitative and quantitative methods to examine transfer and problem solving. The participants in this study were students enrolled in a second-semester physics course (EPII), calculus instructors and physics instructors. A total of 416 EPII students' exam sheets were collected and reviewed. We also obtained the detailed records of 45 of these students' Calc II performance. Statistical methods (Pearson correlation and Hierarchical Cluster Analysis) were used to analyze the quantitative data. A total of 28 students and nine instructors were interviewed. Each student was interviewed over two sessions, each lasting about one hour. The interviewee was left alone to solve an assigned problem. Upon completion, we asked the interviewee to explain what they had written down and verbalize their thinking process. We also asked them to describe any difficulties they had when solving the problem. General questions about their calculus background and application of their calculus knowledge in physics were asked at the end of the interview. Each instructor interview lasted about half an hour. We asked instructors about the expectations and outcomes of their courses. A phenomenographic approach was used to analyze all of the interview data. We interpreted and analyzed our findings in light of a theoretical framework which is based on our model of transfer of learning.

Our model of transfer is based on a two-level structure of associations and control and is consistent with contemporary views of transfer of learning. Our model describes transfer as a dynamic cognitive process through which the learner constructs associations between new information that they read out from a current scenario and prior knowledge stored in their long-term memory. Our theoretical framework distinguishes between two kinds of associations that a learner might construct in a problem solving scenario. These two associations correspond to two kinds of transfer processes. In horizontal transfer, the

learner intuitively activates an existing internal representation or schema that aligns the external problem representation. The learner maps the variables of the problem to the knowledge elements of the schema. Solving plug-and-chug problems typically involves horizontal transfer. In vertical transfer the learner is unable to automatically activate a schema that matches the external problem representation. Rather the learner may have to decide between two or more schemas, modify an existing schema, combine elements of two or more schemas, or construct a completely new schema from its constituent knowledge elements. In the research presented in this dissertation we examine both horizontal and vertical transfer of learning from calculus to physics.

6.1 Addressing the Research Questions

A three-phase research plan was designed to address the research questions. Phase I is designed to assess horizontal transfer of knowledge using traditional physics problems, and to answer the first two research questions which evolved from the old research question #1 in light of our framework. Phase II is designed to assess vertical transfer of knowledge using non-traditional physics problems—"Compare and Contrast" problems, Jeopardy problems and Graphical Representation problems, and to answer research questions 3 and 4 which evolved from the old research question #2. In Phase III, we interviewed both physics and calculus instructors with regard to transfer of learning from calculus to physics, and to answer the last research question which evolved from the old research question #3.

6.1.1. Q1: Have students retained their calculus schemas to solve calculus problems?

We found students appeared to retain their calculus schemas well to solve calculus problems in Phase I. When interviewees were given pure calculus problems, they were able to solve the problems quickly and correctly. Furthermore, students self-reported that they were confident in their calculus knowledge retention because they remembered what they had learned in their calculus class and were able to do the calculus operations such as integrations and differentiations. On an average they ranked their calculus knowledge retention as 7 on a scale from 0 (most dissatisfied) to 10 (most satisfied). A majority of

students believed that the calculus knowledge they retained was enough for physics courses since they had not come across any situations in physics that required a level of calculus with which they were not comfortable. This result is consistent with previous research on transfer of learning from algebra to physics (Tuminaro, 2004).

6.1.2. Q2: Can students associate their physics problem variables with their calculus schemas?

We found that students had difficulty associating their physics problems variables with their calculus schemas. Students were not confident in setting up calculus-based physics problems; even though they may have seen similar problems previously. Students typically appeared to be misled by the various numbers or constants in the physics problems and they could not decide what variable they were looking for. They tended to resort to novice problem-solving strategies such as means-ends analysis. Students had difficulty reading out of information from the given physics problems and aligning it with their calculus schemas. More specifically, students could not decide the variable of integration and limits of the integral. These result is also consistent with Tuminaro's (2004) research for algebra-based physics courses, in which he found that students often failed to interpret their mathematics knowledge in a physical context.

6.1.3. Old Research Question #1: To what extent do students retain and transfer their calculus knowledge while problem solving in introductory physics?

Students did retain their calculus schema for performing integration and differentiation. But students had difficulties in transferring their calculus knowledge when solving a physics problem. We also found that assessing transfer of learning from calculus to physics must be examined from multiple perspectives of transfer and use multiple research methods. Our results showed a statistically strong correlation between students' calculus and physics performance within an EPII exam problem. This appeared to indicate the possibility of transfer when viewed from the contemporary Actor-Oriented Transfer (AOT) perspective which focuses on students dynamic constructions of similarities between two aspects of their knowledge. However, the correlation between

student performance on EPII exams problems and their Calc I and Calc II course performance was not as significant. We found relatively weaker evidence for the possibility of transfer when viewed from traditional perspective and Preparation for Future Learning (PFL) perspective. This weaker evidence for the possibility of transfer when viewed from a more traditional perspective compared with the evidence from a contemporary perspective is consistent with previous research on transfer (Ozimek, 2004).

Student difficulties in Phase I appeared to indicate that from students' point of view end-of-chapter EPII problems involved vertical and not horizontal transfer as we had previous assumed. This observation is consistent with our flexible framework of vertical and horizontal transfer which allows for divergent interpretations of the same task as involving either horizontal or vertical transfer when examined from the perspective of varying levels of expertise.

6.1.4. Q3: Can students appropriately activate their calculus schemas in physics problems?

We found that students had difficulty deciding when to activate appropriate calculus schemas. More than half interviewees admitted that they did not know the reason why they used integration in a given physics problems, other than they mimicked the strategy used in similar sample physics problem from lecture or textbook. Thus, students often resorted to pattern matching while approaching their problems.

Our interviewees generally had difficulties solving the non-traditional "Compare and Contrast" physics problems. They commented that they had not received any specific formal instruction on why to use integration instead of summation, and so they once again resorted to pattern matching by trying to recall to similar problems that they had seen and using them as a guide to decide whether integration was important. Most interviewees stated that they had not addressed these issues in their physics course.

We also found most of our interviewees tended to use pre-derived algebraic relationship rather than calculus to solve the problem. They were unable to explain the conditions under which the closed form expressions were applicable. So it appears that

students did retain their calculus schemas, but they did not have a clear understanding of when to activate their calculus schemas.

6.1.5. Q4: Can students deconstruct and reconstruct their schemas to solve a physics problem?

Students had difficulty in deconstructing and reconstructing their schemas from students' failure in physics Jeopardy problem. Again, students tended to rely on ends-means analysis without invoking deeper conceptual understanding. When trying to construct an appropriate physical situation corresponding to a given Jeopardy expression, we found students tended to focus on limited numbers of constants rather than of the variable of the integration or differentiation to help them construct the physical scenario. They often used dimensional analysis and unit matching to find out the physical quantity that was being calculated in the expression. Thus, students had difficulty in deconstructing their calculus schemas in Jeopardy problems of navigating multiple representations in the graphical representation problems.

6.1.6. Old Research Question #2: What mental processes are involved as students transfer what they have learned in calculus to introductory physics?

Based on the findings pertaining to Research Q1 through Q4 discussed above we can conclude that students had difficulty in engaging in both horizontal as well as vertical transfer of learning from calculus to physics. We observed that our interviewees were relying on the equations sheets and doing pattern matching. They were confused about the meaning of different symbols and lacking a big picture of the problem. Most of interviewees were able to perform calculus calculation when solving traditional physics problems. In case of horizontal transfer we found that students had difficulties in associating physics variables with their calculus schema, although they appeared to have no difficulty in recalling the required calculus schema for integration or differentiation. In case of vertical transfer we found that students were unable to articulate a set of criteria that would enable them to decide when to activate the appropriate calculus schema. They also faced difficulties in deconstructing and reconstructing their schemas. Finally,

students also appeared to have difficulties constructing connections between the meaning conveyed by the graphs and the corresponding symbolic representations.

Overall, students' problem solving behaviors appeared to suggest that they often resort to naïve strategies such as pattern-matching or ends-means analysis to solve problems. These problem solving behaviors appear to suggest that students searched for an appropriate schema to help them solve their problem. When they were unable to find the appropriate schema to solve a problem, they were often unable to construct or deconstruct an existing schema to address the problem at hand.

6.1.7. Q5: What strategies can facilitate both horizontal and vertical transfer?

In examining our student interviews from the perspective of our theoretical framework, we found that we need to assist students' understanding of why we use calculus in solving physics problems, the underlying assumptions when it is or is not used, what each knowledge element means in their calculus schema, and how to associate the physics problem variables with calculus schemas, to facilitate both horizontal and vertical transfer.

From the faculty interview, we found disconnects between what the calculus instructors do in their classes and what the physics instructors would prefer their students to have. Since calculus and physics are still taught in two departments, we do not suggest a radical approach that requires these departments to work together. Below we suggest approaches that each department can implement within their own courses.

6.1.7.1. Suggestions for the Mathematics Department

Although the calculus reform movement has been take place more than twenty years ago, we found, at least at Kansas State University, calculus reform has had minimal impact. The official calculus course descriptions at KSU state the importance of developing students' problem solving skills from Calc I though Calc III. However, from the physics students and instructors' perspectives, the calculus courses tend to focus on calculus strategies instead of conceptual understanding. This result would urge the mathematics educators to rethink the result of calculus reform, or the range of its impact.

Our research also provides some insights into strategies that students believe might be helpful to them as they transition from mathematics to physics classes where they apply their mathematics knowledge in relatively semi-structured problems. To adequately prepare them for these classes, mathematics classes that often focus on developing students' mathematical skills should also provide opportunities for helping students solve contextualized and semi-structured word problems. This is consistent with the suggestions from physics instructors. In studying transfer from one mathematics problem to another, Schoenfeld (1985) found that explicit instruction in recognizing similarities improved students' abilities to transfer ideas in solving novel problems. The students' requests for increased word problems in calculus may be related to their need for seeing such explicit instruction in recognizing similarities across contexts. More research should be carried out on to what extent that solving more word problems in calculus could prepare students to develop problem solving skills in physics. This is discussed on the recommendations for future research section.

6.1.7.2. Suggestions for the Physics Department

A Common belief among many physics instructors, not necessarily supported by research, is that their students do not enter their class with the adequate calculus preparation. Our results appear to indicate that the main difficulty that students have is not because of the lack of calculus knowledge or skills, rather it lies in their inability to understand how calculus is appropriately applied to physics problems. Students often do not understand the underlying assumptions and approximations that they might need to make in a physics problem before they apply a particular mathematical strategy.

We suggest that calculus-based physics courses should focus more on *why* calculus is used in physics, and the conditions and criteria for its use in physics. We also suggest that instructors provide more scaffolding on how to set up physics problems that use calculus. Finally, we recommend that physics instructors expand their repertoire of problem and use other problem types such as "Compare and Contrast" problems or Jeopardy problems, as suggested by Van Heuvelun (1999). These might help students develop a more nuanced understanding of not just how and why calculus is used, but also when it is used in a given problem situation. It will also enable students to learn how to

deconstruct or unpack their existing calculus schemas so that they are more aware of how each element in their schema can be associated or mapped on to the problem scenario. However, more research should be done to fully examine the effects of different problem types on the development of students' problem solving skills.

Physics courses, in general should facilitate students' development their problem-solving skills by helping them learn how to set-up ill-structured problems. Like mathematics courses, physics courses too should focus on helping students understand the concepts that underpin the strategies that they use rather than merely the strategies themselves. Finally, physics as well as mathematics courses should emphasize multiple representations in the homework and exam assignments by using non-traditional problems. These less-structured problems might help students break up their routine thinking and deconstructing their knowledge schemas. However, more research is needed to examine to what extent these less-structured problems could help students to be better problem solvers.

6.2 Broader Implications for Instruction

6.2.1. Broader Implications for Researchers

The framework – horizontal and vertical transfer--constructed from this study provides researchers a new lens and vocabulary to describe and assess transfer of learning. This framework is not limited from mathematics to physics. This dissertation also synthesizes previous research. It pulls together perspectives from various researchers such as conceptual change, modeling, transfer of learning and problem solving.

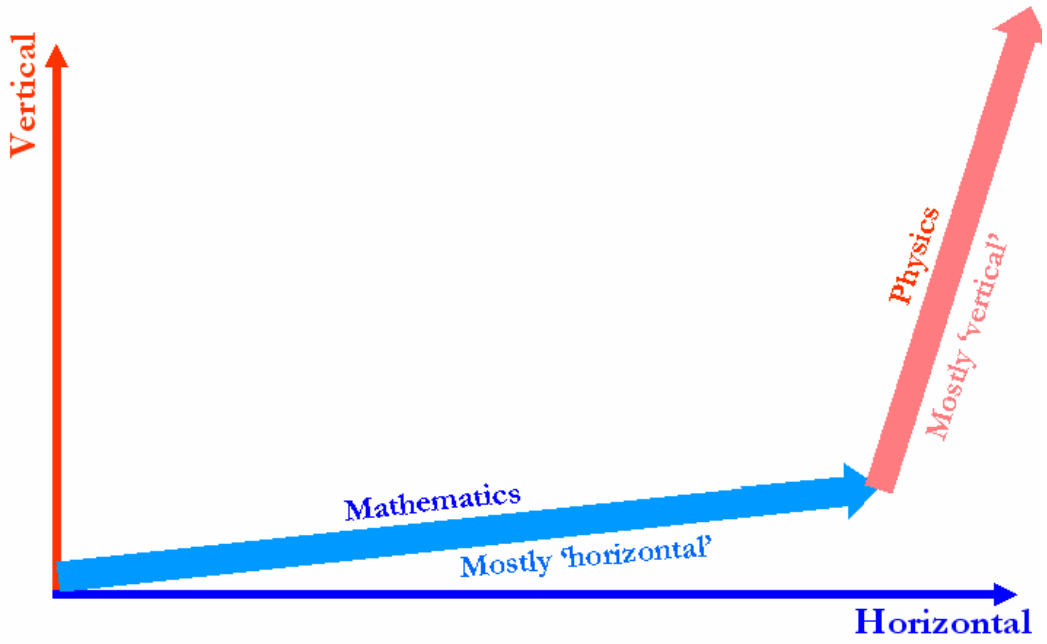
Our results demonstrate that transfer of learning from relatively abstract domains; such as mathematics to relatively concrete domains such as physics must be examined from multiple perspectives of transfer. When viewed from a traditional perspective, students often appear to fail to transfer what they have learned in one context to solve problems in another context. However, upon expanding our perspective to focus on

students abilities to learn how to solve problems in the new context we are more likely to find evidence of transfer.

6.2.2. Broader Implications for Educators

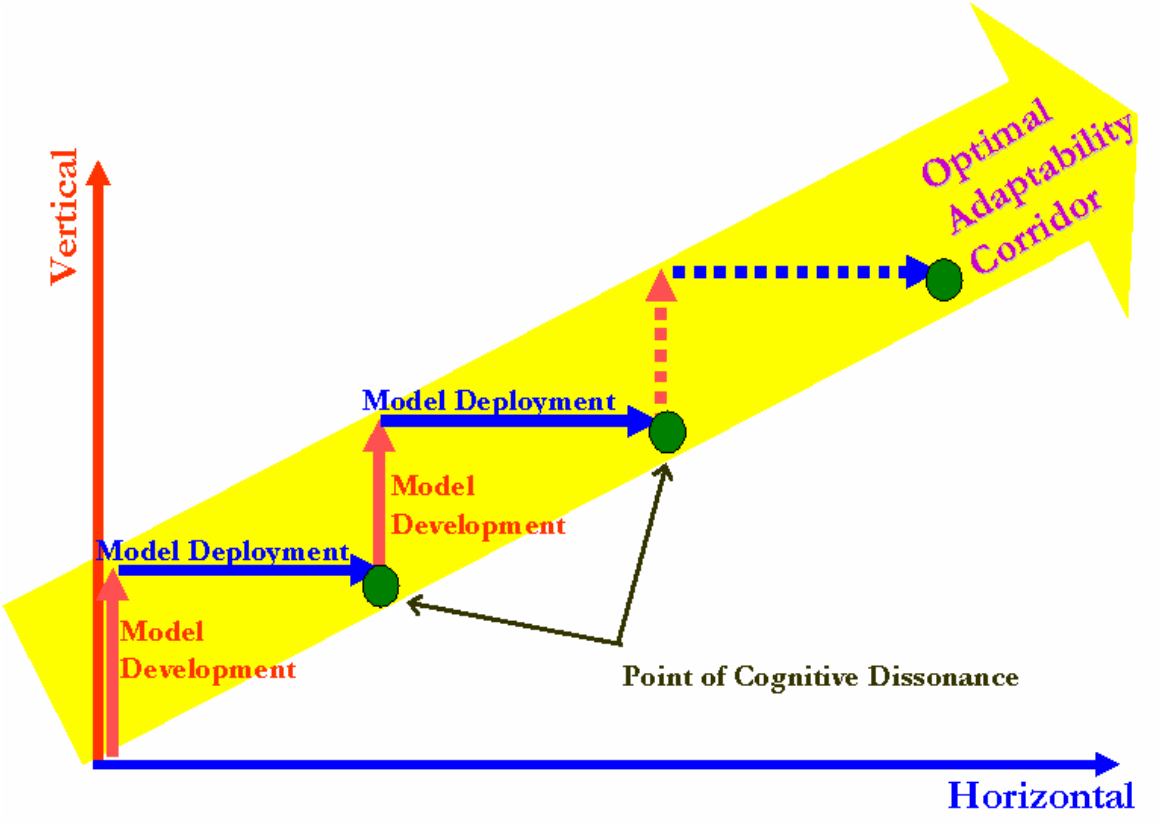
When viewed through the lens of our theoretical framework of horizontal and vertical transfer, this study seems to suggest that educators should balance both horizontal and vertical transfer when help students transfer their learning from calculus to physics, or more broadly from any structured domain to a relatively semi-structured or ill-structured domain. From a physics educator's perspective, our current mathematics education is mostly focused on horizontal transfer. However, when students come to our physics courses, we expect them to engage in vertical transfer. As shown in Figure 6-1, we speculate that the rather abrupt change in focus from horizontal to vertical transfer in going from one course to another cause students have difficulties because they have not gained enough training to engage in vertical transfer in their previous course.

Figure 6-1: Horizontal and Vertical Transfer in Mathematics and Physics courses



Schwartz, Bransford and Sears (2005) suggest that educators should follow what they call an Optimal Adaptability Corridor (OAC), shown as yellow arrow in Figure 6-2. The OAC provides a learning trajectory to develop from a novice into an adaptive expert though the balance of efficiency and innovation, or say, a balance of horizontal and vertical transfer as per our framework. Both mathematics and physics department should work on develop students' ability of horizontal and vertical transfer of learning.

Figure 6-2 Optimization for both Horizontal and Vertical Transfer



How does one facilitate students' navigation through Optimal Adaptability Corridor (OAC)? Educators can adapt proven successful pedagogical strategies such as the Hestenes' Modeling Cycle (Hestenes, 1987) to foster both horizontal and vertical transfer in the OAC through incremental steps of Model Development and Model Deployment in the OAC, which correspond to iterative modeling cycles. In the Model

Development step students develop a model on the new problem situation, while in the Model Deployment step they apply the developed model in different situation. Both mathematics and physics courses should use small steps of Model Deployment following Model Development to promote horizontal and vertical transfer. For example, after students learn how to find the electric field by a point charge using Coulomb's law, educators should give students several point changes to deploy their understanding of Coulomb's law. Similar exercise and homework problems do exist in most physics textbooks, usually with the limit of three charges. The model deployment stage seemed satisfying in this particular example.

The point of transition between the deployment of an old model and development of a new model is not arbitrary. Here too, proven theories and pedagogical strategies of conceptual development provide some clues. Piaget (1952) suggests that an internal conflict or cognitive dissonance due to a discrepant event -- a contradiction between observations and expectations -- provides the necessary motivation for students to abandon or modify their existing model or schema (which provided the basis for their expectations). Piaget's ideas of cognitive dissonance can be adapted to this model of instruction, in that by demonstrating the limitations of a particular model, we can provide students with the necessary impetus to modify their model. This realization of the inadequacy of a given model provides the necessary discrepant event that generates a point of inflection in students' learning trajectory and motivates them to develop a new model to address the new problem scenario. This would provide a reason to use integration over summation. These kinds of experiences would help students learn to both construct new models and recognize their underlying assumptions and limitations, thereby facilitating both horizontal and vertical transfer of learning. For example, after students' have applied Coulombs' law to find electric field at certain distance from several point charges, ask what they would do if they were given many closed spaced point charges. Students would realize it is unrealistic to add up a large amount of point charges and would be more amenable to develop an alternative model to calculate the electric field. We found lack use of this cognitive dissonance in most of the physics textbook. There were typical two kinds of homework problems in finding electric field: one was to find the electric field from less than three charges (which used summation), the other was to find

electric field from a certain continuous charge distribution (which used integration). We found a lack of model development using cognitive dissonance in this particular example.

6.3 Recommendations for Further Research

The purpose of this study was to examine students' retention and transfer from calculus to physics. As with most research, this study has not only answered the questions it posed, but also raised some interesting questions.

> Future research question #1: What would be the appropriate calculus and physics courses sequencing to facilitate the transfer from calculus to physics?

One of the issues that was raised in this study was course sequencing: Should the calculus and physics courses be taught concurrently or should student take all calculus courses (Calc I though Calc III) before entering physics classroom? During our study, both opinions have been expressed by our interviewees. Students who prefer to take calculus and physics courses concurrently believed that they would be more likely to use the ideas of calculus in physics if they are taking calculus concurrently with physics. However, students who prefer to take all calculus courses before any physics courses argued that they could get a better understanding or a bigger picture of calculus before they used it in physics. More research should be undertaken to explore this issue further and weigh the pros and cons of both possibilities.

One possible research strategy is to separate the participants (EPII students) into different groups depending on the calculus courses they are taking concurrently with EPII: students who are enrolling in Calc I (which should be very rare since typically students are required to take at least one calculus course before taking EPII), Calc II, Calc III, Differential Equation course (which is considered as a continuation of Calc III), and students who have finished all calculus courses. This research question could be answered by comparing different groups' EPII course performance and conducting individual interviews.

Future research question #2: Does assigning more word problems in calculus indeed help students develop problem solving skills in physics?

It seems all of our students and faculty interviewees agree that using more semi-structured word problems in calculus would help students develop problem solving skills in physics. No empirical research has been designed to investigate this hypothesis. A longitudinal study such as interviewing students during the calculus course, after finishing a calculus course, during a physics course and after finishing a physics course would be one possible way to study the effect of word problems. Another possible study that could be done is to compare students' problem solving abilities in physics courses using control and experimental groups. The control group is the students who were taking calculus course with a small percentage of word assignments (less than 20%), and the experimental group is the students who were taking calculus course with a large percentage of word assignments (more than 50%). The answer of the first future research question is critical to this research question since researcher needs to know which calculus course is more related to EPII to decide in which course to put more word problems.

Future research question #3: Do non-traditional physics problems (e,g. Jeopardy problem) indeed help students develop problem solving skills with understanding?

Van Heuvelen (1999) believed using Jeopardy problem in physics helped students become better problem solvers and our student interviewees agreed with his idea. However, to what extent can the non-traditional physics problems help students develop problem solving skills needs further investigation. Similar research methods could be adapted to address this research question from future research question #2.

Future research question #4: To facilitate horizontal and vertical transfer, is it indeed possible to help students navigate Schwartz's Optimal Adaptability Corridor (OAC) as proposed using successive stages of the Hestenes' Modeling Cycle as described?

One area of future research includes the validation of the instructional model discussed in the previous section. Is student learning and transfer enhanced using this instructional model, versus traditional instruction that is currently used in most courses? Several investigable questions could be pursued in this regard.

Another direction of future research is to expand on the content areas addressed by this project. Future possibilities include investigating the transfer of learning from upper-division mathematics courses (like Differential Equations) to upper-division physics courses (like Classical Mechanics), from physics courses to engineering courses to see if the issues found in this study would be similar or different.

Last but not least, the interview data collected in this study could be analyzed further. We were only look at the associations students made during problem solving process. But the controlling factors that control these associations, such as students' epistemic mode, motivation and others are also an interesting and important aspect to understand the transfer of leaning.

Appendix A - Reviewed Problems from EPII Exams of Fall 2004

Exam 2 Reviewed Problem

5. Consider a nonconducting sphere of radius $R = 10\ cm$, with charge $q = 5\ \mu C$ spread uniformly throughout its volume. The magnitude of the electric field E as a function of the distance r from the center of the sphere can be calculated by using Gauss' Law and has the following form:

$$E(r) = \frac{qr}{4\pi\varepsilon_0 R^3} \quad \text{for } r < R \text{ and}$$

$$E(r) = \frac{q}{4\pi\varepsilon_0 r^2} \quad \text{for } r > R .$$

(a) (12 points) Since the electric field is radially outward, one can write $V_f - V_i = -\int_i^f E(r)\,dr$. Start from this definition of potential difference and consider $V=0$ at $r = \infty$. Compute the electric potential on the surface of the sphere.

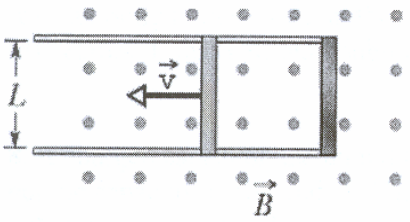
Answer:_____

(b) (12 points) Again start from the definition of potential difference $V_f - V_i = -\int_i^f E(r)\,dr$, but this time consider $V=0$ at $r = 0$, i.e. at the center of the sphere. Now compute the electric potential on the surface of the sphere.

Exam 4 Reviewed Problem

5. The figure shows a rod of length L = 10.0 cm that is forced to move at constant speed v = 5.00 m/s along horizontal rails. The rod, rails, and connecting strip at the right form a conducting loop. The rod has resistance 0.5 Ohm; the rest of the loop has negligible resistance. A magnetic field of magnitude B = 0.25 T points out of the page.

(a) (12 points) Find the magnitude and direction (i.e., clockwise or counterclockwise) of the induced current in the loop.



Answer:_____

(b) (6 points) What is the magnitude of the force that must be applied to the rod to make it move at constant speed?

Q3, Final Exam Reviewed Problem

3. (30 points) Consider a nonconducting sphere of radius $R = 8$ cm, with charge $q = 10\ \mu C$ spread uniformly throughout its volume. The magnitude of the electric field E as a function of the distance r from the center of the sphere can be calculated by using Gauss' Law and has the following form:

$$E(r) = \frac{qr}{4\pi\varepsilon_0 R^3} \quad \text{for } r < R \text{ and}$$

$$E(r) = \frac{q}{4\pi\varepsilon_0 r^2} \quad \text{for } r > R.$$

Since the electric field is radially outward, one can write $V_f - V_i = -\int_i^f E(r)\,dr$. Start from this definition of potential difference and consider $V=0$ at $r = 0$ i.e. at the center of the sphere. Compute the electric potential at a distance $r = 12$ cm from the center of the sphere.

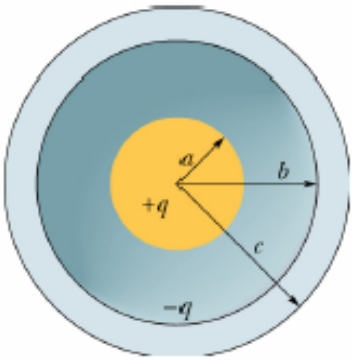
Q4, Final Exam Reviewed Problem

4. A long, straight wire has fixed positive charge with a linear charge density of magnitude + 10 .0 nC/m. The wire is enclosed by a thin, nonconducting cylinder of outer radius 2.0 cm, coaxial with the wire. The cylinder has negative charge *on its outside surface* with a surface charge density of - 20.0 nC/m^2.

(24 points) Draw an appropriate Gaussian surface and calculate the electric field at a distance of 5 cm from the axis of the cylinder.

Appendix B - Reviewed Problems from EPII Exams of Spring 2005

Exam 1 Reviewed Problem



13. (22 points total)

Consider the spherical distribution of charges shown in the figure. The region $r < a$ (region 1) is a solid piece of copper that carries a charge $+q$. The region $a < r < b$ (region 2) is empty space. The region $b < r < c$ (region 3) is an insulating spherical shell holding a charge $-q$ spread uniformly throughout its volume. The region $r > c$ (region 4) is again empty space. Your answers below should be in terms of q, a, b, and c (not all need appear in all answers. Use the symbol k for the constant $1/4\pi\varepsilon_0$.

(a) (4 points) Find the magnitude of the field in region 4 ($r > c$) as a function of r, the distance from the center of the sphere; i.e., find $E_4(r)$. You may give this answer with minimal or no calculation if you clearly state a one sentence reason for your answer. It is OK of course to do a calculation.

(b) (4 points) Find the magnitude of the field in region 1 ($r < a$) as a function of r, the distance from the center of the sphere; i.e., find $E_1(r)$. Again, you may give this answer with minimal or no calculation if you clearly state a one sentence reason for your answer.

(c) (3 points) Find the surface charge density on the outer radius of the conductor. Call this σ_1.

(d) (4 points) Find the magnitude of the field in region 2 ($a < r < b$) as a function of r, the distance from the center of the sphere; i.e., find $E_2(r)$. Again, you may give this answer with minimal or no calculation if you clearly state a one sentence reason for your answer.

(e) (3 points) Show that the volume charge density in region 3 ($b < r < c$) is $\rho_3 = -3q/\left[4\pi\left(c^3 - b^3\right)\right]$.

(f) (4 points) Show that the magnitude of the field (this you must calculate) in region 3 ($b < r < c$) is $E_3(r) = \frac{kq}{r^2}\left(1 - \frac{r^3 - b^3}{c^3 - b^3}\right)$.

Exam 3 Reviewed Problem

14. (16 points total) A wire of radius a carries a non-constant current density over its cross sectional area given by the function $J(r) = \frac{I_0}{\pi a r}$, where r is the distance from the center of the wire and I_0 is a constant.

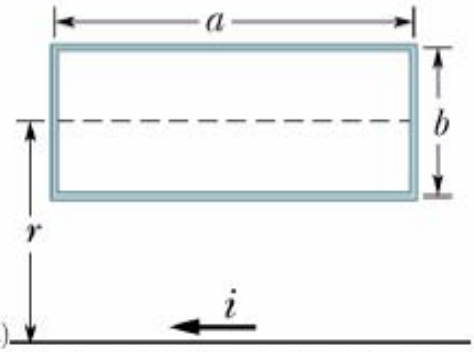
(a) (4 points) Show (using calculus) that the total current carried by the wire is $I = 2I_0$.

(b) (4 points) Find the magnitude of the magnetic field at a distance $2a$ from the center of the wire.

(c) (4 points) Show (more calculus) that the total current enclosed by a Amperian circle of radius $b < a$ sharing a common center with the wire is $I_{enc} = 2I_0\frac{b}{a}$.

(d) (4 points) Find the magnitude of the magnetic field at the center of the wire.

Exam 4 Reviewed Problem



13. (18 points total) A current $i(t) = Kt$ flows to the left in the long wire shown in the figure above, with K a positive constant and t the time. The center of a rectangular loop of area $a \times b$ is located a distance r away from the wire. The rectangular loop has a resistance R.

(a) (6 points) Show (using calculus) that the magnetic flux through the loop is given by $\Phi_B = \frac{\mu_0 Kta}{2\pi} \ln\left(\frac{r+b/2}{r-b/2}\right)$.

(b) (6 points) Find the magnitude of the induced current flowing in the rectangular loop.

(c) (6 points) The induced current in the loop will either flow clockwise or counter-clockwise. Choose the correct direction and explain your choice in one or two sentences.

Appendix C - Interview Consent Form

Kansas State University Informed Consent Form

KANSAS STATE UNIVERSITY

INFORMED CONSENT TEMPLATE

PROJECT TITLE: Assessing Student Transfer and Retention of Learning in Mathematics, Physics, and Engineering Courses

PRINCIPAL INVESTIGATOR: CO-INVESTIGATOR(S): N. Sanjay Rebello (PI)

CONTACT AND PHONE FOR ANY PROBLEMS/QUESTIONS: N. Sanjay Rebello
srebello@phys.ksu.edu
(785) 532 1612

IRB CHAIR CONTACT/PHONE INFORMATION: Clive Fullagar, Chair of Committee on Research involving Human Subjects
1 Fairchild Kansas State University, Manhattan KS, 66506, (785) 532-3224

Jerry Jaax, Associate Vice Provost for Research Compliance
1 Fairchild Kansas State University, Manhattan KS, 66506, (785) 532-3224

SPONSOR OF PROJECT: National Science Foundation

PURPOSE OF THE RESEARCH: The objective of this project is to develop effective online assessment tools to measure students. conceptual understanding and their likelihood of retaining their understanding and successfully applying learned material in new situations.

PROCEDURES OR METHODS TO BE USED: Interviews

ALTERNATIVE PROCEDURES OR TREATMENTS, IF ANY, THAT MIGHT BE ADVANTAGEOUS TO SUBJECT:

None

LENGTH OF STUDY: 60 - 120 min

RISKS ANTICIPATED: No known risks

BENEFITS ANTICIPATED: Deeper understanding of physical phenomena

CONFIDENTIALITY: The student's performance and/or statements during interview and in survey will not be disclosed with students' name or any identifying feature.

PARENTAL APPROVAL FOR MINORS: Not Applicable

PARTICIPATION: Voluntary

I understand this project is for research and that my participation is completely voluntary, and that if I decide to participate in this study, I may withdraw my consent at any time, and stop participating at any time without explanation, penalty, or loss of benefits, or academic standing to which I may otherwise be entitled.

I also understand that my signature below indicates that I have read this consent form and willingly agree to participate in this study under the terms described, and that my signature acknowledges that I have received a signed and dated copy of this consent form.

Participant Name: _____

Participant Signature: _____ **Date:** _____

Witness to Signature: (project staff) _____ **Date:** _____

Addendum to Informed Consent Form

I hereby state that:

- I have read, understood and signed the **Kansas State University, Informed Consent (Template)** Form.
- I have agreed to be *interviewed* for a total duration of *one/two hours in (interview semester)* in connection with the study described in the **Kansas State University, Informed Consent (Template)** Form.
- I understand that information collected from me during this interview/survey process, including any demographic information will be kept strictly confidential by the Project Staff. Audiotapes of the interview and their transcripts / originals and copies of the survey, will be stored in a secure place, and will be destroyed after the publication of the research resulting from this study.
- I understand that I will not be identified either by name or by any other identifying feature in any communication, written or oral, pertaining to this research.
- I understand that if I wish to withdraw from the study at any time, either before a scheduled interview/survey, during an interview/survey or after an interview/survey I can do so without explanation, penalty, or loss of benefits, or academic standing that I may otherwise be entitled.
- I understand that by signing this form, I have consented to have information learned from me during the process to be used by the Project Staff in their research and any resulting publications.
- I understand that if I give my consent to participate in this survey and for the use of my data, I will be earned $10 per hour.

Participant Name: _____

Participant Signature _____ Date: _____

Witness to Signature _____ Date: _____

(Project Staff)

Appendix D - Interview Protocol for Fall 2004

Interview Protocol for First Session

Thanks for coming

Request permission for Videotape of the interview

Go over and ask interviewee to sign the Consent Forms

Any questions before we start?

Leave the interviewee alone for about 7 minutes for solving each question.

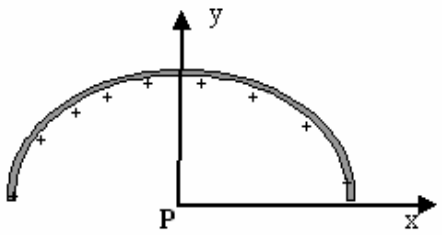
Ask him/her to verbalize their problem solving process ("why")

Guided Interview Questions:

1) Did you see any similarities between the math question and physics question?
2) What math classes have you taken? When did you take them? How do you think about your calculus class?
3) Are these math problems looks familiar to you?
4) Are you confident when you solve those math problems (as long as you have taken calculus before)? If so, why? If not, why?
5) Do you still remember the math knowledge that you need to solve this problem?
6) When you solve physics problem, do you feel the need to use your math knowledge? If so, please explain what math knowledge you need?
7) Are you satisfied with your math knowledge? If so, why? If not, why not?
8) Do you think what you learned in your math (calculus) class is enough to help you to solve this particular physics problem?
9) How much you have retain from your math class if give you a scale from 1-10?
10) Generally speaking, is there correlation between your calculus class and physics class? Why?

Interview Problems for First Session

A thin non-conducting rod is bent into a semicircle of radius R, charge Q spread uniformly along it. Find the magnitude and direction of electric field E at point P at the center of the semicircle.

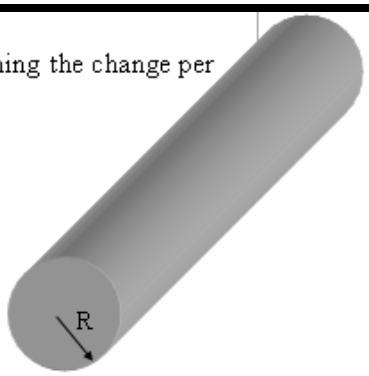


Note: As per Coulomb's Law the Electric Field E due to charge q at a distance r is: $E = \frac{kq}{r^2}$ where k is a constant.

Consider a non-conducting infinite long cylinder of radius R, assuming the change per unit length λ. We know from the Gauss's law,
the electric field E (inside and outside of the cylinder) as a function of the distance r from the center have the forms:

$$E(r)\big|_{r>R} = \frac{\lambda}{2\pi\varepsilon_0 r}$$

$$E(r)\big|_{r<R} = \frac{\lambda r}{2\pi\varepsilon_0 R^2}$$



Define electric potential V=0 at r = ∞, find the electric potential at point 'P', which is at distance r with r<R (inside of the cylinder) from axis of cylinder.

Note that: $V_f - V_i = -\int_i^f E(r)dr$

Evaluate the following integrals:

$$\int_a^b \frac{1}{x} dx = \_\_\_\_\_$$

$$\int_a^b x dx = \_\_\_\_\_$$

Evaluate the following integrals:

$$\int_0^{\pi/2} \sin\theta d\theta =$$

Interview Protocol for Second Session

Thanks for coming

Restate the content in the Consent Forms

Any questions before we start?

Leave the interviewee alone for about 7 minutes for solving each question.

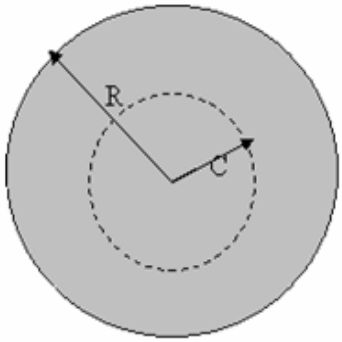
Ask him/her to verbalize their problem solving process ("why")

Guided Interview Questions:

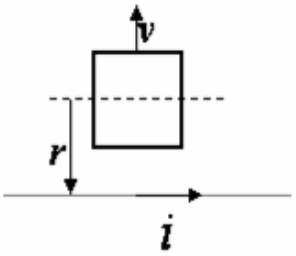
1) Are those problems are math problems or physics problem to you?
2) Did you see any similarities between the sets math question and physics question?
3) Are these math problems looks familiar to you?
4) Are you confident when you solve those math problems (as long as you have taken calculus before)? If so, why? If not, why?
5) Are you confident when you solve those physics problem? If so, why? If not, why?
6) Do you still remember the math knowledge that you need to solve this problem?
7) Do you still remember the physics knowledge that you need to solve this problem?
8) When you solve physics problem, do you feel the need to use your math knowledge? If so, explain what math knowledge you need?
9) Do you think what you learned in your math (calculus) class is enough to help you to solve this particular physics problem?
10) Generally speaking, is there correlation between your calculus class and physics class? Why?

Interview Problems for Second Session

The figure right shows the cross section of a long conducting cylinder wire of radius R. The cylinder carries current insides of the page with a current density in the cross section is given by $J = ar^3 + br$, with a and b are constants. Find the magnetic field at the point c $(c < R)$.



A square loop with length a, and resistance R is placed near an infinite long wire carrying current i. The distance from the long wire to the center of the loop is r. Find the induced current in the loop as it moves away from the long wire with the speed v.



Evaluate the following integrals:

$$\int_a^b (ax^3 + bx)dx =$$

Evaluate the following integrals:

(a) $\int_a^b \frac{1}{x} dx =$

(b) $\frac{d}{dx}\ln\left(\frac{x+a}{x+b}\right) =$

Appendix E - Interview Protocol for Spring 2005

Interview Protocol for First Session

Thanks for coming

Request permission for Videotape of the interview

Go over and ask interviewee to sign the Consent Forms

Any questions before we start?

Leave the interviewee alone for about 7 minutes for solving each question.

Ask him/her to verbalize their problem solving process ("why")

If solve correctly….	**If could not solve……..**
Ask to explain step by step Why you use integration? Integrate over what, why? Generally, when you need to use integration? Where you learn that?	Why you could not solve? Where is the trouble? Show the textbook if they feel they need Give some cues, (if just pick an element, can you find the E? How to do that...) Find out where they stuck, and how can help them out

Guided Interview Questions:

1) What do you think about this problem?
2) Is this problems looks familiar to you? Why?
3) Are you confident when you solve them? If so, why? If not, why?
4) What math/calculus classes have you taken? When did you take them? How do you think about your calculus class?
5) Do you still remember the math/calculus knowledge that you need to solve this problem?
6) Do you still remember the physics knowledge that you need to solve this problem?
7) When you solve physics problem, do you feel the need to use your math knowledge? If so, explain what math knowledge you need?
8) Do you think what you learned in your math (calculus) class is enough to help you to solve this particular physics problem?
9) Generally speaking, is there correlation between your calculus class and physics class? Why?

Interview Protocol for Second Session

Thanks for coming

Restate the content in the Consent Forms

Any questions before we start?

Leave the interviewee alone for about 7 minutes for solving each question.

Ask him/her to verbalize their problem solving process ("why")

If solve correctly…	**If could not solve…**
Ask to explain step by step Why you use integration? Integrate over what, why? Generally, when you need to use integration? Where you learn that?	Why you could not solve? Where is the trouble? Show the textbook if they feel they need Give some cues, (if just pick an element, can you find the E? How to do that...) Find out where they stuck, and how can help them out

Guided Interview Questions:

1) What do you think about this problem?
2) Did you see any similarity/difference between these two questions?
3) Is this problems looks familiar to you? Why?
4) Are you confident when you solve them? If so, why? If not, why?
5) Do you still remember the calculus knowledge that you need to solve this problem?
6) Do you still remember the physics knowledge that you need to solve this problem?
7) When you solve physics problem, do you feel the need to use your math knowledge? If so, explain what math knowledge you need?
8) Do you think what you learned in your math (calculus) class is enough to help you to solve this particular physics problem?
9) Generally speaking, what is the relationship between your calculus class and physics class? Why?

Appendix F - Interview Protocol for Fall 2005

Interview Protocol for First Session

Thanks for coming

Request permission for Videotape of the interview

Go over and ask interviewee to sign the Consent Forms

Any questions before we start?

Some explanation/example of Jeopardy Question

Have you solved this kind of questions (Jeopardy) before?

Leave the interviewee alone for about 7 minutes for solving each question.

Ask him/her to verbalize their problem solving process ("why")

Guided Interview Questions:

1) Do these questions look familiar to you? Before and after you solved it?
2) Did you see any similarities and differences between the questions?
3) Are you confident when you solving them? Why or why not?
4) What calculus classes you have taken before? When and where? How do you think about your calculus class?
5) Are you satisfied your calculus knowledge? Are they enough for your EP2 class?
6) What is the relationship between your calculus and physics classes?

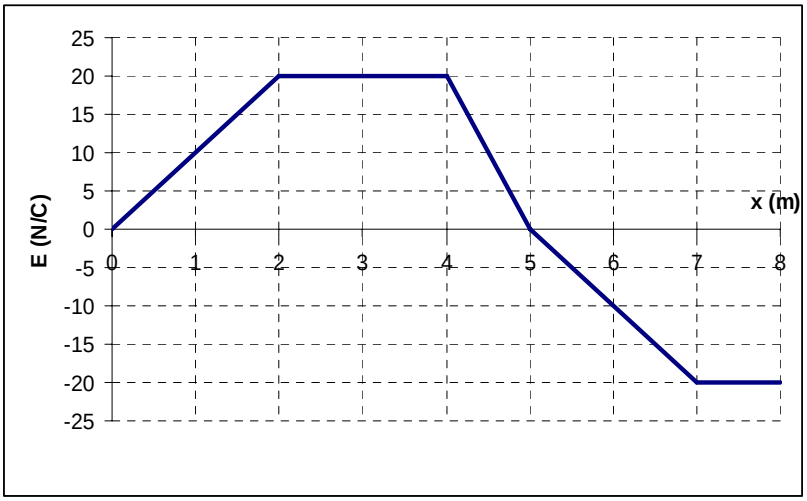
Interview Problem for First Session

Please construct an appropriate physical situation that is consistent with the following expression.

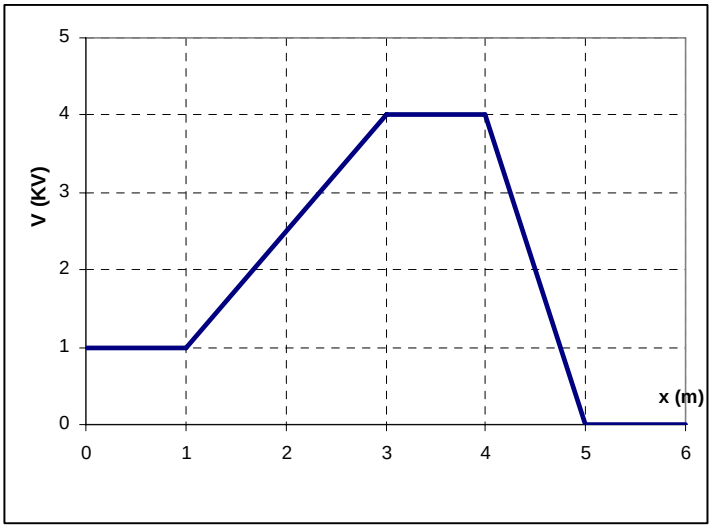
$$2\times\left[\int_{0}^{\frac{\pi}{6}}(8.99\times10^{9}\ N\cdot m^{2}\Big/C^{2})\frac{(2\times10^{-10}\ C/m)(5\times10^{-2}\ m)\cos\theta d\theta}{(5\times10^{-2}\ m)^{2}}\right]$$

$$\left[\int_{\infty}^{(3\times10^{-2}\ m)}(8.99\times10^{9}\ N\cdot m^{2}\Big/C^{2})\frac{(5\times10^{-10}C)dr}{(3\times10^{-2}\ m)^{2}}\right]+\left[\int_{(3\times10^{-2}\ m)}^{(2\times10^{-2}\ m)}(8.99\times10^{9}\ N\cdot m^{2}\Big/C^{2})\frac{(5\times10^{-10}C)rdr}{(3\times10^{-2}\ m)^{3}}\right]$$

The following graph shows the <u>electric field</u> as a function of x. If the electric potential at the origin (x=0) is 0 V, what is the electric potential at x=2m? x=4m? x=6m? x=8m?



The following graph shows the electric potential as a function of x, what is the magnitude of electric field at region 1, 2, 3, 4, and 5 individually?



Equation Sheet for First Session

Useful Formulas and Constants:

$$\mu_0 = 4\pi \times 10^{-7}\, T \cdot m/A$$

$$dB = \frac{\mu_0}{4\pi}\frac{id\vec{s} \times \vec{r}}{r^3}$$ Biot-Savart law

$$B = \frac{\mu_0 i}{2\pi R}$$ Long straight wire

$$\oint \vec{B} \cdot d\vec{s} = \mu_0 i_{enc}$$ Ampere's Law

$$E_s = -\frac{\partial V}{\partial s}$$

Interview Protocol for Second Session

Thanks for coming

Restate the content in the Consent Forms

Any questions before we start?

Leave the interviewee alone for about 7 minutes for solving each question.

Ask him/her to verbalize their problem solving process ("why")

Guided Interview Questions:

1) Do these questions look familiar to you? Before and after you solved it?
2) Did you see any similarities and differences between the questions?
3) Are you confident when you solving them? Why or why not?
4) Recall your (thinking) procedure to solve a typical physics problem which involves calculus.
5) Recall your (thinking) procedure to solve a Physics Jeopardy problem.
6) What is the major difference (on your thinking process) between Jeopardy question and your homework question?
7) Do you think Physics Jeopardy Question promotes your thinking, or say understand the concept better?
8) What are your major (conceptual) difficulties when you solving a physics problem this semester (in electricity and magnetism)?
9) What do you do to overcome these difficulties?
10) What would be your suggestions to better prepare you (further EP2 students) before they come to EP class?

Interview Problem for Second Session

Please construct an appropriate physical situation that is consistent with the following expression.

$$\int_0^a \frac{\mu_0 \cdot I \cdot \dfrac{R}{\sqrt{s^2 + R^2}} \cdot ds}{4\pi \cdot (s^2 + R^2)}$$

$$\frac{\mu_0 \int_0^R J(r) \cdot (2\pi r dr)}{2\pi R}$$

$$\int_1^2 \frac{\mu_0 \cdot I \cdot a \cdot dr}{2\pi r}$$

$$-\frac{d\left(\int_a^b \dfrac{\mu_0 \cdot I \cdot x(t) \cdot dr}{2\pi r}\right)}{dt}$$

Equation Sheet for Second Session

Useful Formulas and Constants:

$$\mu_0 = 4\pi \times 10^{-7}\, T \cdot m/A$$

$$dB = \frac{\mu_0}{4\pi}\frac{i d\vec{s} \times \vec{r}}{r^3}$$ Biot-Savart law

$$B = \frac{\mu_0 i}{2\pi R}$$ Long straight wire

$$\oint \vec{B} \cdot d\vec{s} = \mu_0 i_{enc}$$ Ampere's law

$$\Phi_B = \int \vec{B} \cdot d\vec{A}$$ Magnetic flux

$$\varepsilon = -\frac{d\Phi_B}{dt}$$ Faraday's law

Appendix G - Faculty Interview Protocol

Calculus Instructor Interview Protocol

Thanks for sharing your time with me

Request permission for audiotape of the interview

Go over and ask interviewee to sign the Consent Forms

Any questions before we start?

Guided Interview Questions:

1) When did you last teach calculus courses? For how long have you been teaching?
2) What background *(math)* do students have when they enter your class?
3) What is the format of your class? *(Lecture, help room…)*
4) What topics do you cover in you class?
5) What difficulties/challenges did you have while you were teaching EP2?
6) What knowledge/skills do you expect your students to have after they finish their course work? Why?
7) How do you help your students to acquire these knowledge/skills?
8) Are you satisfied with the knowledge/skills that demonstrated by your students when they finished?
9) Are you aware how the knowledge/skills learned in your class *(calculus)* are used in other subjects by your students?
10) Do you think your students are adequately prepared in calculus, so they are able to apply their knowledge to other subjects if needed? Especially in physics?

Ask for their sample calculus exam sheets.

Physics Instructor Interview Protocol

Thanks for sharing your time with me

Request permission for audiotape of the interview

Go over and ask interviewee to sign the Consent Forms

Any questions before we start?

Guided Interview Questions:

1) When did you last teach EP2 class? For how long have you been teaching EP2?
2) What is the format of your class? (Lecture, studio, help room…)
3) What topics do you cover in you class?
4) What difficulties/challenges did you have while you were teaching EP2?
5) What background *(math)* do students have when they enter your class?
6) What *(calculus)* knowledge/skills do you expect your students to have before they come to your engineering physics class?
7) Do you think your students have the required *(calculus)* knowledge/skills? Why or why not?
8) What problems/difficulties *(related to calculus)* do your students *have (when they solving physics problems)* in your class?
9) What strategies do you use to help your students overcome their difficulties?
10) What would you prefer to see things been done differently *(in their calculus classes)* to help your students better prepared for physics class?

Appendix H - Exam 0 in Calc II of Fall 2004 (Calc I Knowledge)

Math 221 – Exam 0
August 24, 2004

1. The tangent line to the graph $y = 3x^2 - 4\sqrt{x}$ at (4,40) is
   a. $y = 23x - 52$
   b. $y = 63x - 212$
   c. $y = 23x + 40$
   d. $y = 63x + 40$
   e. $y = -63x + 292$

2. The derivative of $f(x) = \sin(x)\cos(x)$ is
   a. $f'(x) = -\sin(x)\cos(x)$
   b. $f'(x) = 1$
   c. $f'(x) = \cos^2(x) - \sin^2(x)$
   d. $f'(x) = \frac{1}{2}\sin(2x)$
   e. $f'(x) = -\cos(\cos(x))\sin(x)$

3. If $f(x) = \dfrac{x^2 + 2}{3x - 1}$ then $f'(2)$ is
   a. 1.2
   b. $\dfrac{2}{25}$
   c. 1.52
   d. -0.08
   e. $\dfrac{4}{3}$

4. The derivative of $f(x) = \cos^3(2x)$ is
   a. $f'(x) = -6\cos^2(2x)\sin(2x)$
   b. $f'(x) = 3\cos^2(2x)$
   c. $f'(x) = -3\cos^2(2x)\sin(2x)$
   d. $f'(x) = 6\cos^2(x)$
   e. $f'(x) = -3\sin^2(2x)$

5. Evaluate $\int 2x\sqrt{3x^2+1}\,dx$.

   a. $\frac{2}{3}(3x^2+1)^{3/2}+C$

   b. $\frac{1}{6}(3x^2+1)^{-1/2}+C$

   c. $\frac{2}{3}x^2(x^3+x)^{3/2}+C$

   d. $x^2\sqrt{x^3+x}+C$

   e. $\frac{2}{9}(3x^2+1)^{3/2}+C$

6. Evaluate $\int_0^4 x^2-\sqrt{x}\,dx$.

   a. 7.75
   b. 14
   c. $\frac{58}{3}$
   d. -14
   e. 16

7. If $x^2-2x\cos(y)+y^2=1$, then $y'(x)$ is

   a. $-\frac{x}{y+\sin(y)}$

   b. $\frac{\cos(y)-x}{x\sin(y)+y}$

   c. $2x+2x\sin(y)-2\cos(y)+2y$

   d. $2x+2x(\sin(\cos(x))-2\cos(x)$

   e. $4x-2\cos(x)+2x\sin(x)$

8. The minimum value of x^3-x+1 in the interval [-1,2] is

   a. 0
   b. 1
   c. 7
   d. $\frac{9-2\sqrt{3}}{9}$
   e. $\frac{9+2\sqrt{3}}{9}$

9. The maximum value of $x^3 - x + 1$ in the interval [-1,2] is
   a. 0
   b. 1
   c. 7
   d. $\frac{9-2\sqrt{3}}{9}$
   e. $\frac{9+2\sqrt{3}}{9}$

10. Grain is pouring into a conical pile with radius equal to twice the height. If the pile is 4 feet high and rising at a rate of 1 foot per minute, how fast is the grain pouring? The formula for volume of a cone is $V = \frac{1}{3}\pi r^2 h$.
    a. $\frac{32}{3}\pi$ ft^3 / min
    b. $\frac{64}{3}\pi$ ft^3 / min
    c. 64π ft^3 / min
    d. $\frac{256}{3}\pi$ ft^3 / min
    e. $\frac{320}{3}\pi$ ft^3 / min

11. A 13 foot ladder is leaning against a wall. If the ladder is pulled away from a wall at the rate of 6 feet per second, how fast will the top of the ladder be sliding along the wall when the base is 5 feet from the wall?
    a. -12 ft / sec
    b. -9.6 ft / sec
    c. -2.5 ft / sec
    d. $-\frac{30}{13}$ ft / sec
    e. $-\frac{5}{12}$ ft / sec

12. The volume of the solid of revolution formed by rotating the region between the $x-$axis , the line $x = 2$, and the curve $y = x^2$ about the $x-$ axis is

a. $\frac{8}{3}$ units^3

b. $\frac{16}{3}\pi$ units^3

c. $\frac{32}{5}\pi$ units^3

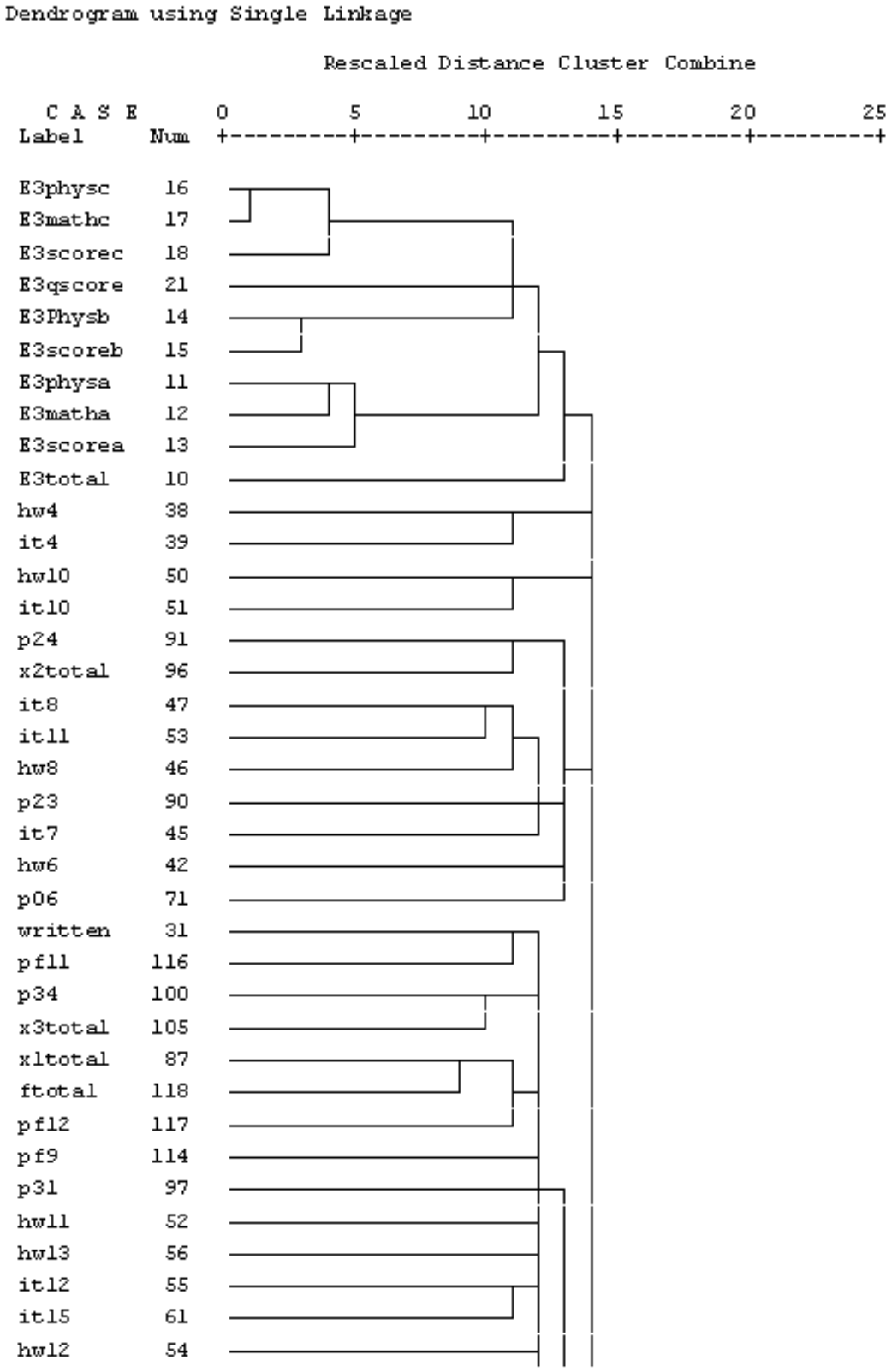
d. 8π units^3

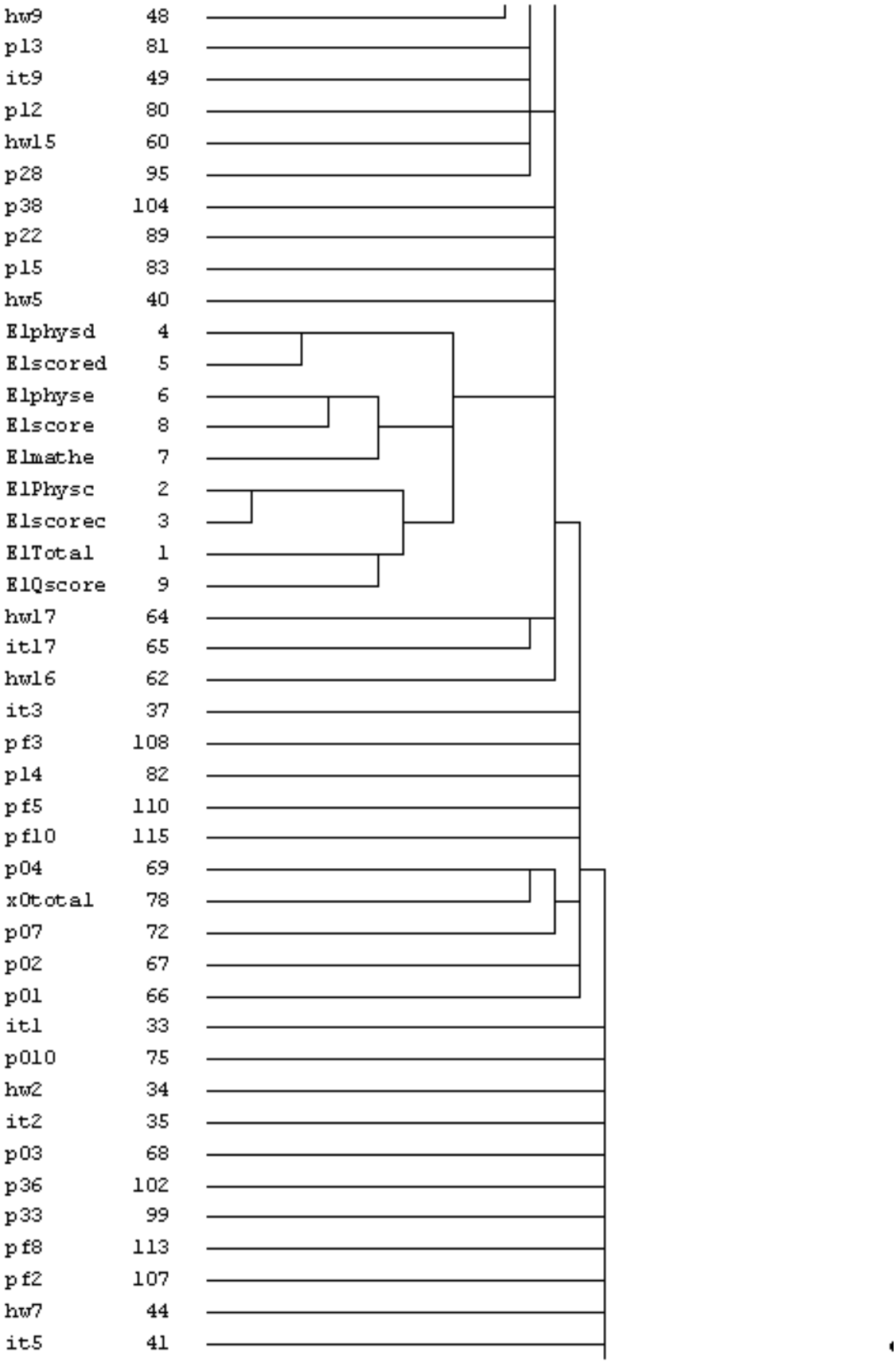
e. $\frac{128}{5}\pi$ units^3

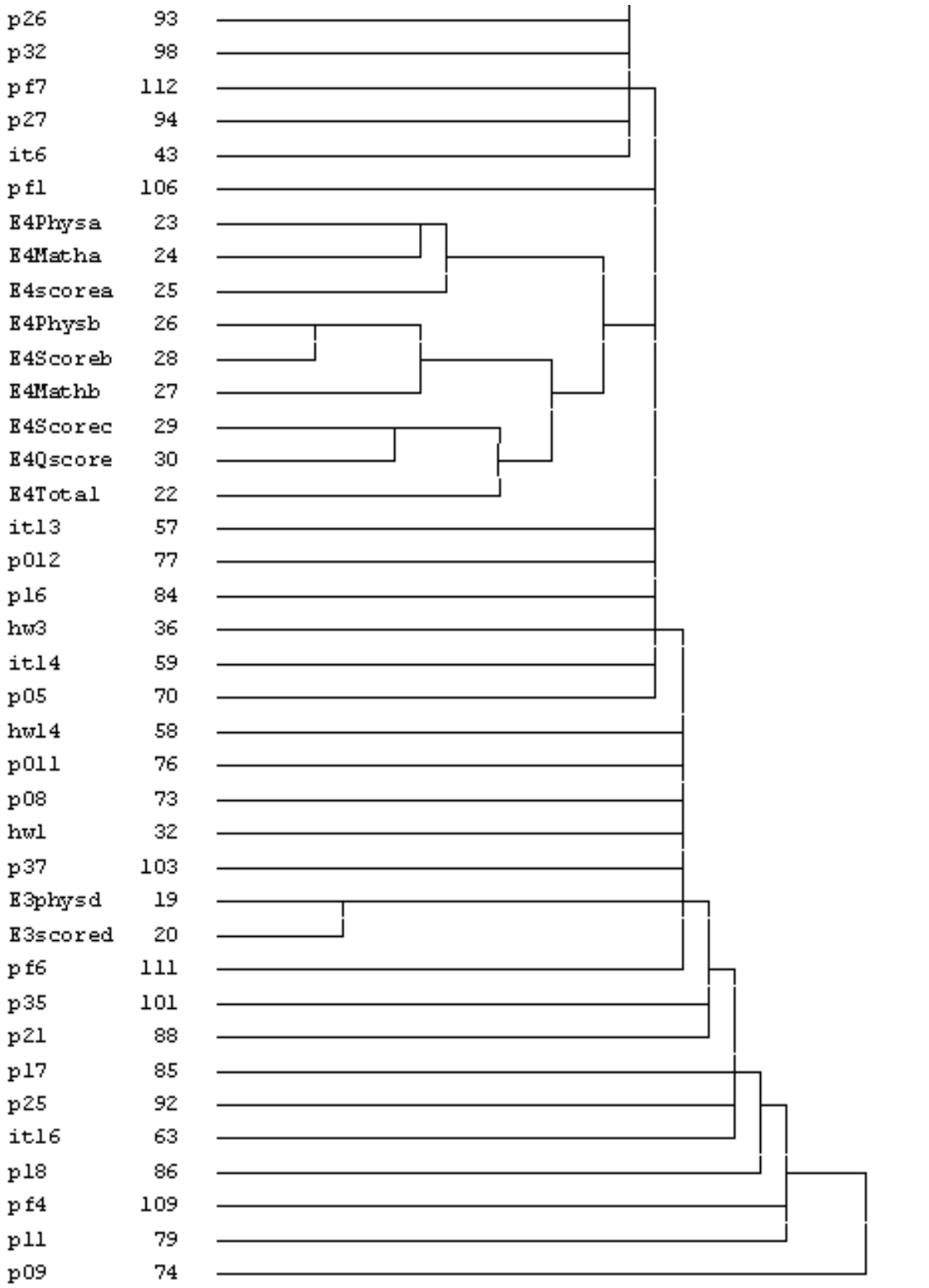
Appendix I - Calc II Knowledge in Each Online Homework

Homework Set	Knowledge Covered
1	Natural Logarithms
2	Inverse Functions
3	Exponential Functions
4	Separation of Variables
5	Inverse Trig Functions
6	Centroids
7	Integration By Parts
8	Trig Identities
9	Trig Substitution
10	Partial Fraction
11	L'Hopital's Rule
12	Improper Integrals
13	Sequences and Series
14	Ratio Test
15	Taylor Polynomials
16	Radius of Convergence
17	Parametric Curves

Appendix J - Dendrogram for all possible variables




hw9 48
p13 81
it9 49
p12 80
hw15 60
p28 95
p38 104
p22 89
p15 83
hw5 40
Elphysd 4
Elscored 5
Elphyse 6
Elscore 8
Elmathe 7
ElPhysc 2
Elscorec 3
ElTotal 1
ElQscore 9
hw17 64
it17 65
hw16 62
it3 37
pf3 108
p14 82
pf5 110
pf10 115
p04 69
x0total 78
p07 72
p02 67
p01 66
it1 33
p010 75
hw2 34
it2 35
p03 68
p36 102
p33 99
pf8 113
pf2 107
hw7 44
it5 41


p26 93
p32 98
pf7 112
p27 94
it6 43
pf1 106
E4Physa 23
E4Matha 24
E4scorea 25
E4Physb 26
E4Scoreb 28
E4Mathb 27
E4Scorec 29
E4Qscore 30
E4Total 22
it13 57
p012 77
p16 84
hw3 36
it14 59
p05 70
hw14 58
p011 76
p08 73
hw1 32
p37 103
E3physd 19
E3scored 20
pf6 111
p35 101
p21 88
p17 85
p25 92
it16 63
p18 86
pf4 109
p11 79
p09 74

References

Adams, L., J. Kasserman, et al. (1988). "The effects of facts versus problem-oriented acquisition." Memory and Cognition **16**: 167-175.

Allbaugh, A. (2003). The Problem Context Dependence of Students' Application of Newton's Second Law. Physics. Manhattan, KS, Kansas State University.

Ashcraft, M. A. and E. P. Kirk (2001). "The relationships among working memory, math anxiety, and performance." J. Exp. Psychol. Gen. **130**(224–237).

Asiala, M., Cottril, J, Dubinsky, E., Schwingendorf, K. (1997). "The development of students' graphical understanding of the derivative." Journal of Mathematical Behavior **16**(4): 399-431.

Askey, R. (1997). "What Do We Do About Calculus? First, Do No Harm." The American Mathematical Monthly **104**(8): 738-743.

Bao, L., K. Hogg, et al. (2002). "Model Analysis of Fine Structures of Student Models: An Example with Newton's Third Law." American Journal of Physics **20**(7): 766-778.

Bao, L. and E. F. Redish (1999). "Model Analysis: A Quantitative Approach to Study Student Understanding of Physics." AAPT Announcer **29**(2): 95.

Bascones, J. and J. D. Novak (1985). "Alternative instructional systems and the development of problemsolving

skills in physics." European Journal of Science Education **7**: 253–261.

Bassok, M. (1990). "Transfer of domain-specific problem-solving procedures." Journal of Experimental Psychology: Learning: 522-533.

Bassok, M. and K. Holyoak (1989). "Interdomain transfer between isomorphic topics in algebra and physics." Journal of Experimental Psychology: Learning: 153-166.

Beichner, R. (1994). "Testing student interpretation of kinematics graphs." American Journal of Physics **62**: 750-762.

Bookman, J., Friedman, C. (1994). A comparison of the problem solving performance of students in lab based and traditional calculus. Research in collegiate mathematics education. A. S. E. Dubinsky, J. Kaput. Providence, RI, American Mathematical Society. **I:** 101-116.

Bransford, J. D., A. L. Brown, et al., Eds. (1999). How People Learn: Brain, Mind, Experience, and School. Washington, DC, National Academy Press.

Bransford, J. D., J. J. Franks, et al. (1989). New approaches to instruction: Because wisdom can't be told. Similarity and Analogical Reasoning. S. Vosniadou and A. Ortony. Cambridge, U.K., Cambridge University Press.

Bransford, J. D. and D. Schwartz (1999). "Rethinking transfer: A simple proposal with multiple implications." Review of Research in Education **24**: 61-100.

Breidenbach, D., Dubinsky, E., Hawks, J., Nichols, D (1992). "Development of the process conception of function." Educational Studies in Mathematics **23**: 247-285.

Broudy, H. S. (1977). Types of knowledge and purposes of education. Schooling and the acquisition of knowledge. C. Anderson, R. J. Spiro and W. E. Montague. Hillsdale, NJ, Erlbaum.

Brown, A. L., Bransford, J. D., Ferrera, R. A., & Campione, J. C. (1983). Learning, remembering, and understanding. Handbook of Child Psychology. M. J. H. Flavell, E. M. New York, Wiley. **Vol. 3 Cognitive Development**.

Brown, A. L. and M. J. Kane (1988). "Preschool children can learn to transfer: Learning to learn and learning from example." Cognitive Psychology **20**: 493-523.

Bruer, J. T. (1993). Schools for thought. Cambridge, MA, MIT Press.

Byrnes, J. P. (1996). Cognitive Development and Learning in Instructional Contexts. Boston, MA, Allyn and Bacon.

Campione, J., Brown, A. L. (1987). Linking dynamic assessment with school achievement. Dynamic Assessment. C. S. Lidz. New York, NY, Guilford.

Carlson, M. (1998). A cross-sectional investigation of the development of the function concept. Research in collegiate mathematics education. J. K. A. H. Schoenfeld, & E. Dubinsky. Washington, DC, Conference Board of the Mathematical Sciences. **III:** 114-162.

Chen, Z. and M. W. Daehler (1989). "Positive and negative transfer in analogical problem solving." Cognitive Development **4**: 327-344.

Chi, M. T. H., Feltovich, P. J., Glaser, R. (1981). "Categorizations and representation of physics problems by experts and novices." Cognitive Science **5**: 121-152.

Churukian, A. (2002). Interactive Engagement in an Introductory University Physics Course: Learning Gains and Perceptions. Physics. Manhattan, KS, Kansas State University.

Clark, J., Cordero, F., Cottrill, J., Czarnocha, B., DeVries, D, St. John, D, Tolias, G., Vidakovic, D. (1997). "Constructing a schema: The case of the chain rule?" Journal of Mathematical Behavior **16**(4): 345-364.

Colaizzi, P. F. (1978). Psychological Research as a Phenomenologist Views It. Existential Phenomenological Alternatives for Psychology. R. S. Valle and M. King. New York, Oxford University Press**:** 48-71.

Davis, R. B. (1985). Calculus at University High School. Toward a lean and lively calculus. R. G. Douglas. Washington, DC, The Mathematical Association of America**:** 27-40.

diSessa, A. and J. Wagner (2005). What Coordination Has to Say about Transfer. Transfer of Learning from a Modern Multidisciplinary Perspective. J. P. Mestre. Greenwich, CT, nformation Age Publishing Inc.

diSessa, A. A. (1988). Knowledge in pieces. Constructivism in the computer age. G. Forman and P. B. Pufall. Hillsdale, NJ, Lawrence Erlbaum Associates**:** 49-70.

Douglas, R. G., Ed. (1986). Toward a lean and lively calculus. Washington, D. C., The Mathematical Association of America.

Dubinsky, E., Ralston, Anthony (1992). Undergraduate Mathematics - Now and Then.

Dunn, J. W. and J. Barbanel (2000). "One model for an integrated math/physics course focusing on electricity and magnetism and related calculus topics." American Journal of Physics **68**: 749.

Engelhardt, P. V. and R. Beichner (1996). Determing and Interpreting Resistve Electric Circuits Concepts Test.

Even, R. (1998). "Factors involved in linking representations of functions." Journal of Mathematical Behavior **17**(1): 105-121.

Frid, S. (1994). Three approaches to calculus instruction. Research in collegiate mathematics education. A. S. E. Dubinsky, J. Kaput. Washington, DC, Conference Board of the Mathematical Sciences. **I:** 69-100.

Gagn´e, R. M. (1970). The conditions of learning. New York, Holt, Rinehart & Winston.

Gibson, D. (1998). Students' use of diagrams to develop proofs in an introductory analysis course. Research in collegiate mathematics education. J. K. A. H. Schoenfeld, E. Dubinsky. Washington, DC, Conference Board of the Mathematical Sciences. **III:** 284-307.

Greeno, J. G., J. L. Moore, et al. (1993). Transfer of situated learning. Transfer on trial: Intelligence, cognition and instruction. D. K. Detterman and R. J. Sternberg. Norwood, NJ, Ablex**:** 99-167.

Halliday, D., R. Resnick, et al. (2004). Fundamentals of Physics, Wiley.

Hammer, D. (1995). "Epistemological considerations in teaching introductory physics." Science Education **79**(4): 393-413.

Hammer, D. and A. Elby (2002). On the form of a personal epistemology. Personal Epistemolgy: The Psychology of Beliefs about Knowledge and Knowing. P. R. Pintrich and B. K. Hofer. Mahwah, N.J., Lawrence Erlbaum**:** 169-190.

Harel, G., & Sowder, L. (1998). Students' proof schemes: Results from exploratory studies. Research in collegiate mathematics education. J. K. A. H. Schoenfeld, E. Dubinsky. Washington, DC, Conference Board of the Mathematical Sciences. **III:** 234-283.

Heller, P. and M. Hollabaugh (1992). "Teaching Problem Solving Through Cooperative Grouping - 2: Designing Problems and Structuring Groups." American Journal of Physics **60**(7): 637-644.

Heller, P., R. Keith, et al. (1992). "Teaching Problem Solving Through Cooperative Grouping - 1: Group versus Individual Problem Solving." American Journal of Physics **60**(7): 627-636.

Hestenes, D. (1987). "Toward a modeling theory of physics instruction." American Journal of Physics **55**: 440?54.

Hestenes, D., M. Wells, et al. (1992). "Force concept inventory." The Physics Teacher **30**(3): 141-166.

Hughes-Hallett, D., et al. (1998). Calculus single and multivariate. New York, John Wiley & Sons, Inc.

Jonassen, D. (in press). "Facilitating Case Reuse During Problem Solving." Technology, Instruction, Cognition & Learning.

Jonassen, D. H. (2000). "Toward a design theory of problem solving." Educational Technology: Research & Development, **48**(4): 63.

Jonassen, D. H. (2003). "Using cognitive tools to represent problems." Journal of Research in Technology in Education **35**(3): 362-381.

Karplus, R. J. (1974). "Science teaching and development of reasoning." Journal for Research in Science Teaching **12**: 213-218.

Kilpatrick, J. (1992). A history of research in mathematics education. Handbook of research on mathematicsteaching and learning. D. Grouws. New York, Macmillan**:** 3-38.

Klein, D. and J. Rosen (1997). "Calculus Reform For the $Millions." Notices of the American Mathematical Society **44**(10): 1323-1325.

Larkin, J., McDermott, J., Simon, D.P., Simon, H.A. (1980). "Expert and novice performance in solving physics problems." Science: 1335-1342.

LaTorre, D., Kenelly, J., Fetta, I., Carpenter, L., Harris, C. (1998). Calculus concepts: An informal approach to the mathematics of change. New York, Houghton Mifflin Company.

Laws, P. (1991). "Workshop Physics: Learning introductory physics by doing it." Change Magazine: 20-27.

Lobato, J. E. (1996). Transfer Reconceived: How "sameness" is produced in mathematical activity, Ph.D. Dissertation. Education. Berkeley, CA, University of California, Berkeley**:** 254.

Lobato, J. E. (2003). "How Design Experiments Can Inform a Rethinking of Transfer and Vice Versa." Educational Researcher **32**(1): 17-20.

Maloney, D. P. (1987). "Ranking tasks: A new type of test item." Journal for College Science Teaching **16**: 510-514.

Maloney, D. P. (1993). Research on problem solving: Physics. Handbook of Research on Science Teaching and Learning. D. L. Gabel. New York, NY, MacMillan Publishing Company.

Maloney, D. P., T. L. O'Kuma, et al. (2001). "Surveying students' conceptual knowledge of electricity and magnetism." American Journal of Physics **69**(7): S12-S23.

Marton, F. (1986). "Phenomenography- a research approach to investigating different understanding of reality." Journal of Thought **21**: 29-39.

McDermott, L. C. (1984). "Research in conceptual understanding in mechanics." Physics Today **37**(7): 24-32.

McDermott, L. C. (1996). Physics by Inquiry. New York, Wiley.

McDermott, L. C., M. L. Rosenquist, et al. (1987). "Student difficulties in connecting graphs and physics: Examples from kinematics." American Journal of Physics **55**: 503-513.

McKeough, R. E., J. Lupart, et al. (1995). Teaching for transfer: Fostering generalization in learning. Mahawah, NJ, Erlbaum.

Meel, D. E. (1998). Honors students?calculus understandings: Comparing Calculus & Mathematica and traditional calculus students. Research in collegiate mathematics education. J. K. A. H. Schoenfeld, E. Dubinsky, A. Schoenfeld, J. Kaput. Washington, DC, Conference Board of the Mathematical Sciences. **III:** 163-215.

Mestre, J. (2002). "Probing adults' conceptual understanding and transfer of learning via problem posing." J. Appl. Dev. Psych. **23**: 9-50.

Mestre, J. P. (1994). Cognitive aspects of learning and teaching science. Teacher Enhancement for Elementary and Secondary Science and Mathematics: Status, Issues, and Problems. S. J. Fitzsimmons and L. C. Kerpelman. Washington, DC, National Science Foundation (NSF 94-80).

Minichiello, V., R. Aroni, et al., Eds. (1995). In-depth Interviewing. Melbourne, Addison Wesley Longman.

Newell, A., Simon, H.A. (1972). Human Problem Solving, Prentice Hall.

Newmann, D., Griffin. P., Cole, M. (1989). The Construction Zone: Working for Cognitive Change in School. New York, Cambridge University Press.

Nisbett, R. E., G. T. Fong, et al. (1987). "Teaching reasoning." Science **238**: 625-630.

Novick, L. (1988). "Analogical transfer, problem similarity, and expertise." Journal of Experimental Psychology: Learning, Memory & Cognition **14**: 510-520.

Ozimek, D. J. (2004). Student Learning, Retention and Transfer from Trigonometry to Physics. Physics. Manhattan, KS, Kansas State University.

Park, K., Travers, K. (1996). A comparative study of a computer based and a standard college first-year calculus course. Research in collegiate mathematics education. A. S. E. D. J. Kaput. Washington, DC, Conference Board of the Mathematical Sciences. **II:** 155 - 176.

Perfetto, G. A., Bransford, J. D., Franks, J. J. (1983). "Constraints on access in a problem solving context." Memory and Cognition **11**: 12-31.

Piaget, J. (1952). The Origins of Intelligence in Children. New York, International Universities Press, Inc.

Rebello, N. S., L. Cui, et al. (2007). Transfer of learning in problem solving in the context of mathematics and physics. Learning to solve complex, scientific problems. D. H. Jonassen. Mahwah, NJ, Lawrence Earlbaum Associates.

Rebello, N. S., S. F. Itza-Ortiz, et al. (2003). Students' mental models of Newton's Second Law: Mechanics to Electromagnetism. NARST, Philadelphia, PA, In Print.

Rebello, N. S., D. A. Zollman, et al. (2005). Dynamic Transfer: A Perspective from Physics Education Research. Transfer of Learning from a Modern Multidisciplinary Perspective. J. P. Mestre. Greenwich, CT, Information Age Publishing Inc.

Rebello, N. S., D. A. Zollman, et al. (2005). A Model for Dynamic Transfer of Learning. Annual Meeting of the National Association for Research in Science Teaching, Dallas, TX, NARST Publications.

Redish, E. F. (1994). "The Implications of Cognitive Studies for Teaching Physics." American Journal of Physics **62**(6): 796-803.

Redish, E. F. (2003). A Theoretical Framework for Physics Education Research: Modeling Student Thinking. International School of Physics, "Enrico Fermi", Course CLVI, Varenna, Italy, Italian Physical Society.

Redish, E. F., J. M. Saul, et al. (1998). "Students expectations in introductory physics." American Journal of Physics **66**(212-224).

Reed, S. K. (1993). A schema-based theory of transfer. Transfer on trial: Intelligence, Cognition and Instruction. D. K. Detterman and R. J. Sternberg. Norwood, NJ, Ablex**:** 39-67.

Reed, S. K., G. W. Ernst, et al. (1974). "The role of analogy in transfer between similar problem states." Cognitive Psychology **6**: 436-450.

Reif, R. and J. I. Heller (1982). "Knowledge Structure and Problem Solving in Physics." Educational Psychologist **17**(2): 102-127.

Rex, A. and M. Jackson (1999). Integrated Physics and Calculus, Addison-Wesley

Salomon, G. and D. N. Perkins (1989). "Rocky roads to transfer: Rethinking mechanisms of a neglected phenomenon." Educational Psychologist **24**(2): 113-142.

Schoenfeld, A. (1985). Mathematical Problem Solving,. Orlando, Academic Press.

Schoenfeld, A. (1997). Student Assessment in Calculus, A Report of the NSF Working Group on Assessment in Calculus. Washington, DC, Mathematical Association of America.

Schultz, K., Lockhead, J. (1991). A view from physics. Toward a unified theory of problem solving: Views from the content domains. M. U. Smith. Hillsdale, NJ, Lawrence Erlbaum Associates**:** 99-114.

Schwartz, D., J. D. Bransford, et al. (2005). Efficiency and Innovation in Transfer. Transfer of Learning from a Modern Multidisciplinary Perspective. J. P. Mestre. Greenwich, CT, Information Age Publishing.

Schwingendorf, K. E., McCabe, G., Kuhn, J. (2000). A longitudinal study of the C^4L calculus reform program: Comparisons of C^4L and traditional students. Research in collegiate mathematics education. A. H. S. E. Dubinsky, J. Kaput. Washington, DC, Conference Board of the Mathematical Sciences. **IV:** 63-76.

Sciences, S. P. f. S. from http://www.spss.com/.

Selden, J., Selden, A., Mason, A. (1994). Even good calculus students can 抉 solve nonroutine problems. Research issues in undergraduate mathematics learning. E. D. J. Kaput. Washington,

DC, Mathematical Association of America**:** 17-26.

Singley, K. and J. R. Anderson (1989). The Transfer of Cognitive Skill. Cambridge, MA, Harvard University Press.

Steele, C., Aronson J. (1995). "Stereotype threat and the intellectual test performance of African-Americans." Journal of Personality and Social Psychology **69**: 797-811.

Steen, L. (1987). Calculus for a new century: A pump not a filter. Washington, DC, The Mathematical Association of America.

Sweller, J. (1988). "Cognitive load during problem solving: Effects on learning." Cognitive Science **12**: 257-285.

Throndike, E. L. and R. S. Woodworth (1901). "The influence of improvement in one mental function upon the efficacy of other functions." Psychological Review **8**: 247-261.

Tucker, A., Leitzel, J. (1995). Assessing Calculus Reform Efforts: A Report to the Community. Washington, DC, The Mathematical Association of America.

Tuminaro, J. (2004). A cognitive framework for analyzing and describing introductory students' use and understanding of mathematics in physics. Physics. College Park, University of Maryland.

Van Heuvelen, A. (1991). "Learning to think like a physicist: A review of research-based instructional strategies." American Journal of Physics **59**: 891–897.

Van Heuvelen, A. (1991). "Overview, case study physics." American Journal Physics **59**: 898-907.

Van Heuvelen, A., Maloney D. (1999). "Playing Physics Jeopardy." American Journal of Physics **67**: 252-256.

Wertheimer, M. (1959). Productive Thinking. New York, NY, Harper & Row.

Williams, S. R. (1991). "Models of limit held by college calculus students." Journal for Research in Mathematics Education **22**(3): 219-236.

Wilson, J. M. (1994). "The CUPLE Physics Studio." The Physics Teacher **32**: 518-523.

Wilson, R. (1997). "Reform Calculus" Has Been a Disaster, Critics Charge. Chronicle of Higher Education.

Wu, H. (1996). "The mathematician and the mathematics education reform." Notices of the American Mathematical Society **43**: 1531-1537.

Yeatts, F. R. and J. R. Hundhausen (1992). "Calculus and physics: Challenges at the interface." American Journal of Physics **60**: 716.

Zazkis, R., Dubinsky, E. (1996). Dihedral groups: A tale of two interpretations. Research in collegiate mathematics education. A. S. E. D. J. Kaput. Washington, DC, Conference Board of the Mathematical Sciences. **II:** 161-182.

Zollman, D. A., N. S. Rebello, et al. (2002). "Quantum Mechanics for Everyone: Hands-On Activities Integrated with Technology." American Journal of Physics **70**(3): 252-529.

Zollman, R. F. a. D. (1995). Physics InfoMall. Armonk, NY, The Learning Team.