THE EFFECTS OF DAMPING ON THE LATERAL VIBRATIONS OF A FREE-FREE BEAM

by 1264

Carol Ann Rubin

B. S., Columbia University, 1966

A MASTER'S REPORT

submitted in partial fulfillment of the

requirements for the degree

MASTER OF SCIENCE

Department of Mechanical Engineering

KANSAS STATE UNIVERSITY

Manhattan, Kansas

1969

Approved by:

Hugh S. Walker

Major Professor

TABLE OF CONTENTS

NOMENCLATURE

w	Lateral displacement of the beam
z	Horizontal coordinate of a differential beam element
ℓ	Length of beam
$M(z,t)$	Moment acting on beam
$V(z,t)$	Vertical component of thrust
$P(z,t)$	Horizontal component of thrust
α	Angle of beam axis with horizontal
q	Constant axial load
t	Time
u	Longitudinal displacement of beam
ρ	Mass per unit length of beam (constant)
$A(z)$	Cross sectional area of beam
$I(z)$	Moment of inertia of beam
A_o	Cross sectional area of beam at $z=\ell$
I_o	Moment of inertia of beam at $z=\ell$
n,m	Real numbers
T_c	Thrust exerted on beam at $z=\ell$
E	Modulus of elasticity (constant)
ξ	z/ℓ
τ	$t\sqrt{\frac{EI_o}{\rho A_o \ell^4}}$
$\bar{T}$	$\frac{\ell^2 T_c}{EI_o}$
$\bar{Q}$	$\frac{\ell^3 q}{EI_o}$

ψ	Function of ξ
F	Function of τ
λ^2	Eigenvalue of differential equation
k	Directional control parameter
T	Differential operator

INTRODUCTION

The problem of the influence of transverse vibrations on the dynamic stability of rockets has been dealt with recently in a paper by Huang and Walker [1]. This study idealized the rocket as a free-free elastic beam with a constant end thrust and a constantly varying cross-sectional area. It is the purpose of the present study to introduce an additional variable and, using the Huang-Walker paper as a basis for comparison, see what the effects are on the vibration and stability properties. This new variable provides an approximation of the effect that frictional drag will have on rockets. A constant axial load is assumed to act along the axis of the beam and in a direction opposite to the motion of the beam. The simplifying assumptions made here for the form of the drag force allow for the derivation and solution of the governing differential equations, and at the same time provide a basis for discussion of the effects of a motion-opposing force on a rocket.

The governing differential equation of motion is derived using simple beam theory; and the characteristic equation for the special case of a beam with a parabolic surface of revolution is obtained by means of Frobenius' method. In addition, a variable indicating the angle of applied thrust is introduced to further generalize the results.

DERIVATION OF THE EQUATION OF MOTION

Simple beam theory is used to obtain the governing differential equation. The effects of longitudinal vibrations are neglected and the motion is assumed to be planar. Fig. 1 shows a differential beam element with the relevant forces and couples acting. The plane of motion is the w,z-plane, as shown, where w is the distance from the z-axis to the beam axis and z is the distance along the beam (the beam begins at z=0 and ends at $z=\ell$ where the thrust is considered to be applied). M(z,t) is the moment acting on the beam at a distance z and a time t. V(z,t) and P(z,t) are the vertical and horizontal components, respectively, of the thrust at a given position and time. The angle α indicates the angle that the beam axis makes with the horizontal axis at any given point. A constant axial load representing the drag force is denoted by q and is acting in the direction α. D'Alembert forces are assumed to be acting as shown in Fig. 1, where u is the longitudinal displacement of a material element. It is assumed that u is independent of z, that is, that the rocket material is so stiff that vibrations due to bending occur long before longitudinal vibrations. Therefore, u will be considered dependent only upon t.

D'Alembert's principle yields the following equations of motion:

$$\rho A(z)\,\frac{\partial^2 u(t)}{\partial t^2} = \frac{\partial}{\partial z}\,[P(z,t)] - q \quad , \tag{1}$$

$$\rho A(z)\,\frac{\partial^2 w(z,t)}{\partial t^2} = -\frac{\partial}{\partial z}\,[V(z,t)] + q\,\frac{\partial w}{\partial z} \quad , \tag{2}$$

and

$$P\,\frac{\partial w}{\partial z}\,dz + \left(\frac{\partial P}{\partial z}\,dz\right)\left(\frac{\partial w}{\partial z}\,\frac{dz}{2}\right) - V\,dz - \left(\frac{\partial V}{\partial z}\,dz\right)\left(\frac{dz}{2}\right) + \frac{\partial M}{\partial z}\,dz = 0 \quad , \tag{3a}$$

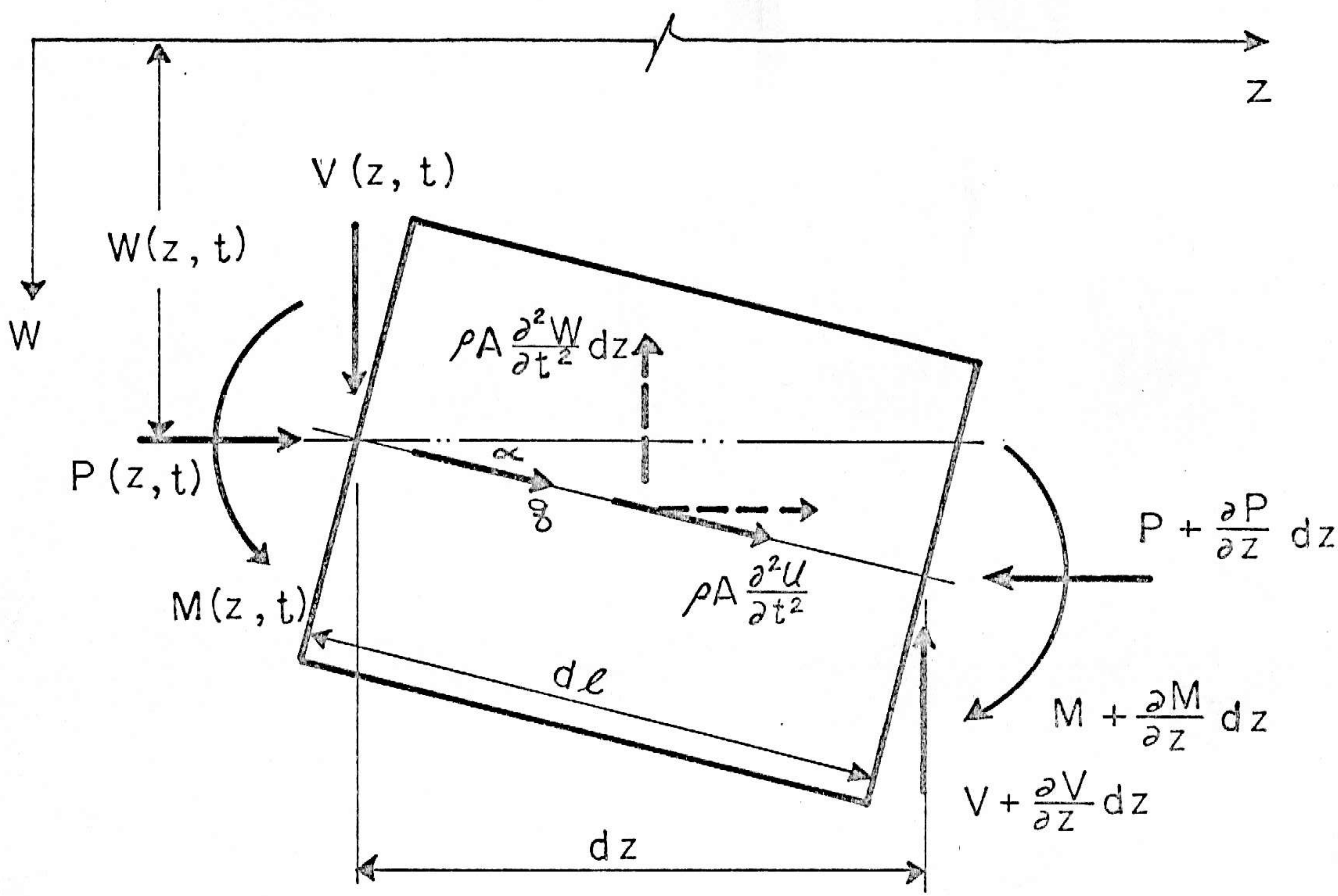

Fig. 1. DIFFERENTIAL ELEMENT OF THE TAPERED BEAM

where a small angle assumption has been made for α so that $\cos\alpha = \frac{dz}{d\ell} \cong 1$, and $\sin\alpha = \frac{dw}{dz} \cong \alpha$. Neglecting terms of higher order, Eq. (3a) becomes:

$$P(z,t)\,\frac{\partial}{\partial z}[w(z,t)] - V(z,t) + \frac{\partial}{\partial z}[M(z,t)] = 0 \quad . \tag{3}$$

The cross sectional area and the moment of inertia of the beam at a point z are denoted, respectively, as

$$A(z) = A_0\left(\frac{z}{\ell}\right)^n \quad ,$$

$$I(z) = I_0\left(\frac{z}{\ell}\right)^m \quad , \tag{4}$$

where A_0 and I_0 are the respective cross sectional area and moment of inertia of the beam at $z=\ell$. The constants m and n are real numbers and depend upon the choice of rocket shape; for a constant cross sectional area, $m=n=0$, and a conical cross section has $m=n=1$.

The following boundary conditions are compatible with the physical system and are assumed to be:

$$P(z,t)\Big|_{z=0} = 0 \quad \text{and} \quad P(z,t)\Big|_{z=\ell} = T_c \quad ,$$

where T_c is the thrust exerted on the rocket at $z=\ell$. Integrating Eq. (1) with respect to z yields

$$P(z,t) = \frac{\rho\ddot{u}A\ell}{n+1}\left(\frac{z}{\ell}\right)^{n+1} + qz + C_1 \quad . \tag{5a}$$

Evaluating this equation at the boundary points and substituting the boundary conditions gives

$$P(0,t) = C_1 = 0 \quad ,$$