MEMBRANE VIBRATIONS

by

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INTRODUCTION

The purpose of this report is to find the fundamental frequencies and the corresponding mode shapes of a membrane using a numerical method. The membrane is circular with an off-center circular clamped region. A finite difference approximation is used in conjunction with Rayleigh's Quotient. As a check the method is used first on a regular circular membrane and an experiment is performed to verify the computed results.

DERIVATION OF THE GOVERNING DIFFERENTIAL EQUATION

An element of the membrane in its deflected position is given below.

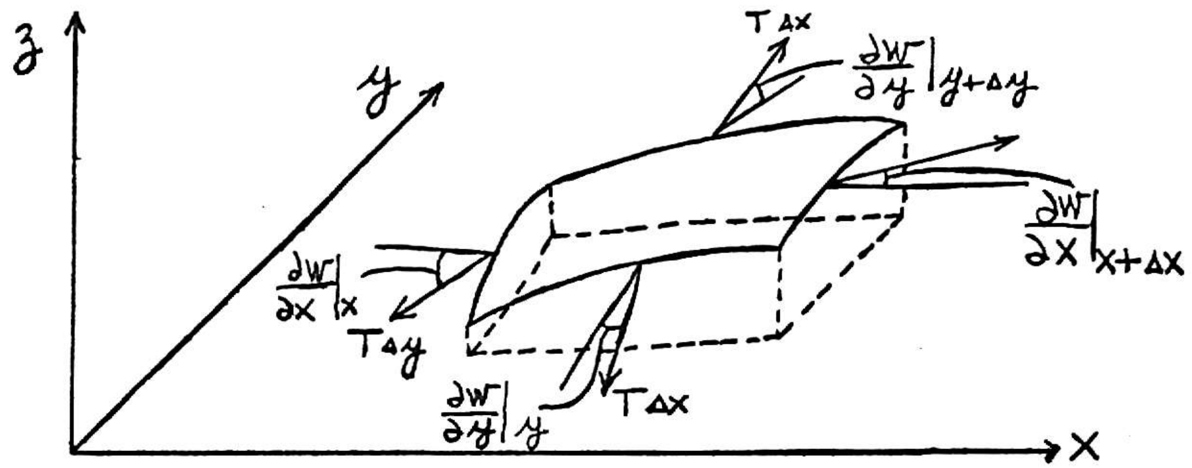


Fig. 1 Element of the membrane in its deflected position

In the derivation of the differential equation of motion of the membrane, the following assumptions are made:

1) the deflections of the membrane are small and

2) the tension is uniform and is unchanged by the small deflections,

3) the membrane is of uniform thickness,

4) the membrane has no bending stiffness.

An element of the membrane in its deflected position is shown in figure 1. Neglecting the gravitational force the sum of the forces in the z direction yields

$$T\Delta y \left.\frac{\partial w}{\partial x}\right|_{x\,+\,\Delta x} - T\Delta y \left.\frac{\partial w}{\partial x}\right|_{x} + T\Delta x \left.\frac{\partial w}{\partial y}\right|_{y\,+\,\Delta y} - T\Delta x \left.\frac{\partial w}{\partial y}\right|_{y} = \frac{\mu}{g}\,(\Delta x)(\Delta y)\,\frac{\partial^2 w}{\partial t^2}$$

where

T = force per unit length,

w = deflection of the membrane at right angles to the x,y plane, and

$$\frac{\mu}{g} = \text{mass per unit area.}$$

Dividing through by $(\Delta x)(\Delta y)T$ yields

$$\frac{\left(\left.\frac{\partial w}{\partial x}\right|_{x + \Delta x} - \left.\frac{\partial w}{\partial x}\right|_{x}\right)}{(\Delta x)} + \frac{\left(\left.\frac{\partial w}{\partial y}\right|_{y + \Delta y} - \left.\frac{\partial w}{\partial y}\right|_{y}\right)}{(\Delta y)} = \frac{\mu}{gT}\frac{\partial^2 w}{\partial t^2} \quad .$$

Let Δx and Δy both approach zero; then

$$\lim_{\Delta x \to 0} \left| \frac{\left(\left.\frac{\partial w}{\partial x}\right|_{x + \Delta x} - \left.\frac{\partial w}{\partial x}\right|_{x}\right)}{(\Delta x)} \right| = \frac{\partial^2 w}{\partial x^2}$$

and

$$\lim_{\Delta y \to 0} \left| \frac{\left(\left.\frac{\partial w}{\partial y}\right|_{y + \Delta y} - \left.\frac{\partial w}{\partial y}\right|_{y}\right)}{(\Delta y)} \right| = \frac{\partial^2 w}{\partial y^2} \quad .$$

The equation can then be written as

$$\left(\frac{\partial^2 w}{\partial x^2} + \frac{\partial^2 w}{\partial y^2}\right) = \frac{\mu}{gT}\frac{\partial^2 w}{\partial t^2}$$

or

$$\nabla^2 w = \frac{\mu}{gT}\frac{\partial^2 w}{\partial t^2}$$

where $\nabla^2 = \left(\frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2}\right)$, the two diemnsional form of the Laplacian.

SOLUTION

1) APPROXIMATION BY FINITE DIFFERENCES

A solution of the governing differential equation

$$\nabla^2 w = \frac{\mu}{gT} \frac{\partial^2 w}{\partial t^2}$$

is of the form

$$w = (A \cos \omega t + B \sin \omega t) X(x,y);$$

thus

$$\frac{\partial^2 w}{\partial t^2} = - \omega^2 (A \cos \omega t + B \sin \omega t) X(x,y) \quad ,$$

$$\frac{\partial^2 w}{\partial x^2} = (A \cos \omega t + B \sin \omega t) \frac{\partial^2 X(x,y)}{\partial x^2}$$

and

$$\frac{\partial^2 w}{\partial y^2} = (A \cos \omega t + B \sin \omega t) \frac{\partial^2 X(x,y)}{\partial y^2} \quad .$$

Substituting the above expressions into the governing differential equation gives

$$(A \cos \omega t + B \sin \omega t) \frac{\partial^2 X(x,y)}{\partial x^2} + (A \cos \omega t + B \sin \omega t) \frac{\partial^2 X(x,y)}{\partial y^2} = - \frac{\mu\omega^2}{gT} (A \cos \omega t + B \sin \omega t) X(x,y)$$

so that, after dividing by $(A \cos \omega t + B \sin \omega t)$,

$$\left(\frac{\partial^2 X}{\partial x^2} + \frac{\partial^2 X}{\partial y^2}\right) = - \frac{\mu\omega^2}{gT} X$$

or

$$\nabla^2 X = - \frac{\mu\omega^2}{gT} X \quad .$$

This equation can be put in a nondimensional form by replacing X by LX' and y by Ly' where L is some representative dimension of the membrane.

Also, let $\lambda = \frac{\mu\omega^2}{gT} L^2$ and drop the primes from X and y so that

$$\lambda X = - \nabla^2 X \quad .$$

An approximate solution can be obtained by using a finite difference method to reduce the continuous system to a system with a finite number of degrees of freedom. The membrane is divided into a network of squares with sides of length h. Each node of the network denotes a point for which an algebraic equation can be written. The finite difference approximation to ∇^2 (developed in Appendix I) for a node as shown in figure 3 is

$$(\nabla^2 X)_{j,k} = \frac{1}{h^2} \left\{ X_{j,k-1} + X_{j-1,k} - 4X_{j,k} + X_{j+1,k} + X_{j,k+1} \right\}$$

where j,k refer to a typical point on the membrane. An equation is written for each point except those on the boundary since X = 0 on the boundary. The equation

$$\lambda X = - \nabla^2 X$$

is thus approximated by

$$\lambda X_{j,k} = - \frac{1}{h^2} \left\{ X_{j,k-1} + X_{j-1,k} - 4X_{j,k} + X_{j+1,k} + X_{j,k+1} \right\} \quad .$$

This equation is good for one point. Rayleigh's Quotient (1) in finite difference form for the membrane is

$$\lambda h^2 = - \frac{\sum X_{j,k} \left\{ X_{j,k-1} + X_{j-1,k} - 4X_{j,k} + X_{j+1,k} + X_{j,k+1} \right\}}{\sum X^2_{j,k}} \quad .$$

The $\sum$ signs imply summation of the quantities in the numerator and denominator for all nodes of the net.

2) BOUNDARY APPROXIMATION

Figure 3 on page 7 shows the boundary approximations and the points used for finite difference approximation. The value of h is taken to be one eighth of the radius; all deflections at the boundary are zero and the deflections of points 1,...,14 are equal to the deflections of the corresponding points 29,...,42 due to symmetry. It should be noted that not all nodal points are at the center of a square net. Point 31, for example, is too close to the clamped region. In a case such as this the approximation for ∇^2 must be modified somewhat. This modification, as developed in Appendix I, is

$$\nabla^2 X_o = \frac{1}{h^2}\left[X_A + \frac{2X_B}{\phi(1+\phi)} + X_C + \frac{2X_D}{1+\phi} - 2X_o\left(\frac{1+\phi}{\phi}\right)\right] \quad .$$

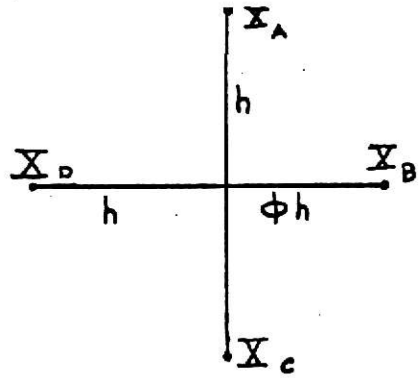


Fig. 2 Nodal point near a curved boundary

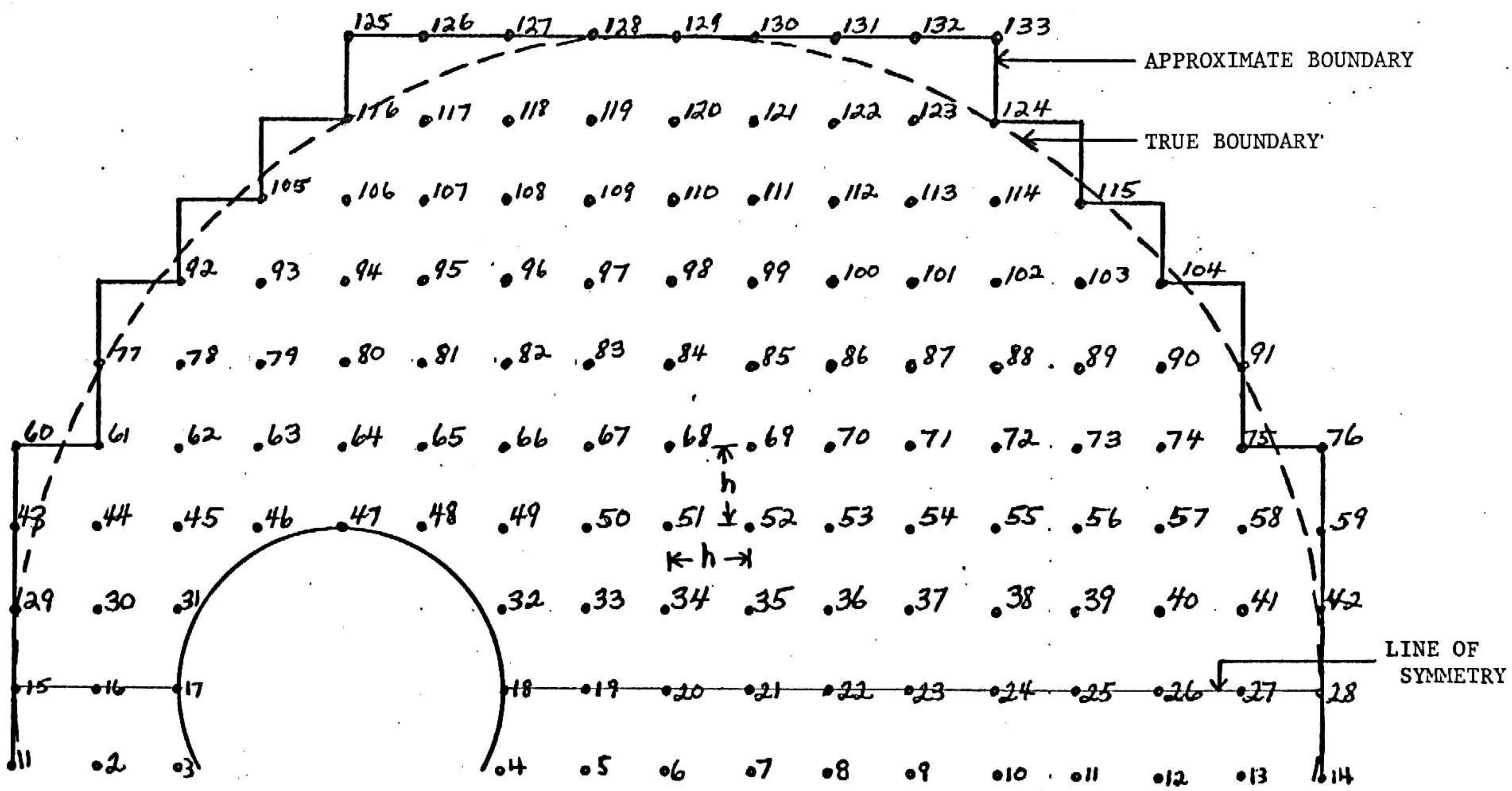


Fig. 3 Boundary approximation and points for finite difference approximation

SOLUTION PROCEDURE

The equation to be satisfied at each regular point is

$$\lambda X_{j,k} = -\frac{1}{h^2}\left(X_{j-1,k} + X_{j,k-1} - 4X_{j,k} + X_{j,k+1} + X_{j+1,k}\right) .$$

Solving for Xj,k yields

$$X_{j,k} = \frac{X_{j-1,k} + X_{j,k-1} + X_{j,k+1} + X_{j+1,k}}{(4 - \lambda h^2)} \qquad (A)$$

The equations for points such as 31 differ only in that the coefficients are different from 1 and 4.

The general procedure is to assume an eigenvector, X, and to use this eigenvector in Rayleigh's Quotient

$$\lambda h^2 = -\frac{\sum X_{j,k}\left(X_{j,k-1} + X_{j-1,k} - 4X_{j,k} + X_{j+1,k} + X_{j,k+1}\right)}{\sum X^2{}_{j,k}} \qquad (B)$$

to find the first approximation to λh^2 which is used in equation (A) to find a new eigenvector. This new eigenvector is normalized and compared with the previous eigenvector, and if the difference between any two corresponding elements is too big, the eigenvector is used in equation (A) to find a new vector which is then normalized, compared, etc.

After a few such iterations, the value of λh^2 is calculated again by using equation (B) and the process of finding a vector normalizing, comparing, etc. is continued until the difference between two successive eigenvectors differs by a predetermined small quantity at which time the final X is the eigenvector and λ is the corresponding eigenvalue. Since the assumed eigenvector and all others thereafter are positive, the final eigenvalue, λ, is the lowest and X is the corresponding eigenvector.

The above computation procedure is implemented by the computer program of Appendix II.

COMPUTER RESULTS

LAMDA
0.87534714E 01

EIGENVECTOR

0.0	0.19666932E-02	0.14546723E-02	0.10572410E 00
0.45439255E 00	0.70289087E 00	0.89807707E 00	0.95659399E 00
0.97001421E 00	0.85198611E 00	0.70741808E 00	0.47997665E 00
0.25028074E 00	0.0	0.0	0.99263084E-03
0.0	0.0	0.41033423E 00	0.71719748E 00
0.88663262E 00	0.99999994E 00	0.96366524E 00	0.89837950E 00
0.71289730E 00	0.51169133E 00	0.25545341E 00	0.0
0.0	0.19666932E-02	0.14546723E-02	0.10572410E 00
0.45439255E 00	0.70289087E 00	0.89807707E 00	0.95659399E 00
0.97001421E 00	0.85198611E 00	0.70741808E 00	0.47997665E 00
0.25028074E 00	0.0	0.0	0.49641356E-02
0.10320302E-01	0.96114166E-02	0.0	0.60604792E-01
0.26388621E 00	0.49342161E 00	0.71559334E 00	0.83827269E 00
0.92248648E 00	0.87824237E 00	0.80024600E 00	0.62103802E 00
0.43255383E 00	0.20775002E 00	0.0	0.0
0.73838122E-02	0.22861633E-01	0.50066084E-01	0.93061447E-01
0.18832922E 00	0.33470094E 00	0.52135098E 00	0.66211790E 00
0.78556448E 00	0.80326849E 00	0.78735572E 00	0.66445112E 00
0.52069294E 00	0.32130986E 00	0.14039046E 00	0.0
0.0	0.22820938E-01	0.63138604E-01	0.13036323E 00
0.22134840E 00	0.35271227E 00	0.47446251E 00	0.60118663E 00
0.65672958E 00	0.68760717E 00	0.62117720E 00	0.52464408E 00
0.35542691E 00	0.17957783E 00	0.0	0.0
0.46936922E-01	0.11374217E 00	0.20570081E 00	0.29867959E 00
0.40484560E 00	0.47225386E 00	0.52798378E 00	0.51006365E 00
0.46125589E 00	0.33605993E 00	0.18349189E 00	0.0
0.0	0.67706823E-01	0.14140975E 00	0.22028333E 00
0.28025913E 00	0.33732653E 00	0.35065460E 00	0.34371972E 00
0.27319795E 00	0.16167182E 00	0.0	0.0
0.66656828E-01	0.10978544E 00	0.14809352E 00	0.16804373E 00
0.18046635E 00	0.16138655E 00	0.11532068E 00	0.0
0.0	0.0	0.0	0.0
0.0	0.0	0.0	0.0
0.0			

Table 1. Fundamental eigenvalue and the corresponding eigenvector for the membrane with the region in place; read left to right, top to bottom.

EXPERIMENT

1) Apparatus

The membrane (4 mil plastic) is held in a pair of retaining rings (similar to the hoops used in crocheting). This assembly is placed over the top of a pipe flange of 5 inch radius. A retaining plate, with a 5 inch radius hole, placed on top of the membrane, is bolted to the pipe flange base. A speaker, positioned approximately 2 inches above the membrane, is used to excite the vibrations. An oscillator is connected to the speaker and an electronic counter is connected to the oscillator to measure the forcing frequency. An optical displacement detector is used in conjunction with an oscilloscope to facilitate finding the resonant frequencies of the membrane. A picture of the apparatus appears on page 18 and a block diagram in figure 4.

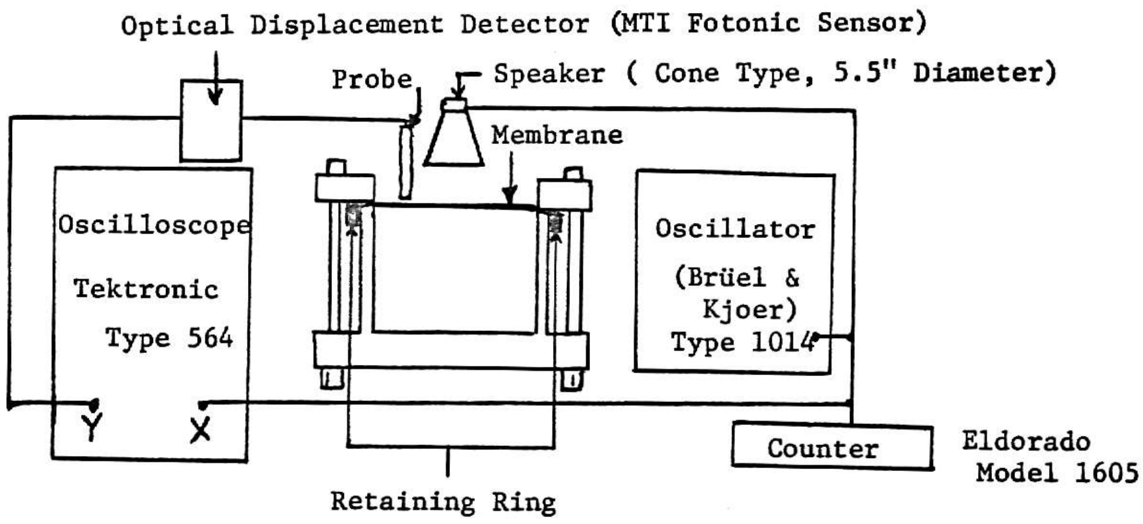


Fig. 4 Block diagram of experimental apparatus

2) Procedure

The probe of the optical displacement detector was placed close to a reflecting surface (a small white piece of paper) fastened to the membrane. The horizontal direction of the oscilloscope trace measures the oscillator while the vertical direction measures the output of the displacement detector. Sand, sieved through a number 100 sieve, is placed on the membrane to visually identify the modal patterns of the various modes.

The procedure is to vary the frequency of the driving force until a natural mode is found by observing the maximum vertical deflection on the oscilloscope. After the modes are found for the circular membrane, the off-center region is clamped in place. The region is clamped from above and below with the displacement detector being used to insure that no deflection is created while tightening the clamped region. The membrane is again vibrated and the modes identified by shape and frequency.

3) Experimental Results

The experimental results shown on the following pages are for both the circular membrane and the circular membrane with the off-center clamped region. The resonant frequencies in cycles per second appears below each figure for both, and the theoretical ratio of the frequency of the particular mode to the fundamental frequency for the circular case appears in parentheses.

4) Calculations

It is known that the frequency of the fundamental mode may be lowered by as much as 50% when the experiment is done in air instead of a vacuum[(3)]. Thus the fundamental frequency of the circular membrane of 54 cps cannot be used to calculate directly the tension in the membrane. The frequencies of the higher modes and the frequency ratios are used to estimate the undamped fundamental frequency.

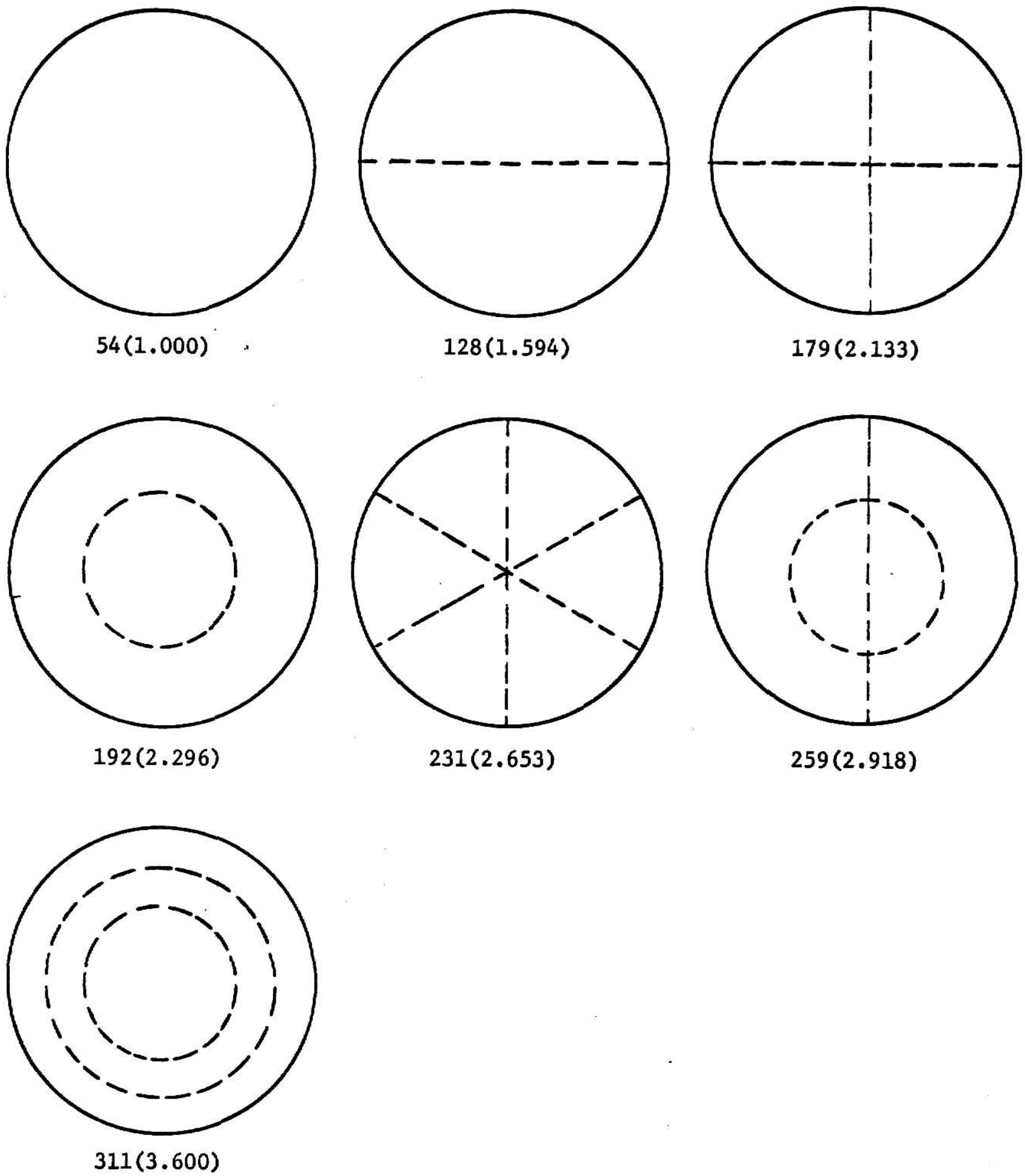


Fig. 5 Mode shapes, frequencies and theoretical ratios to the fundamental for a circular membrane

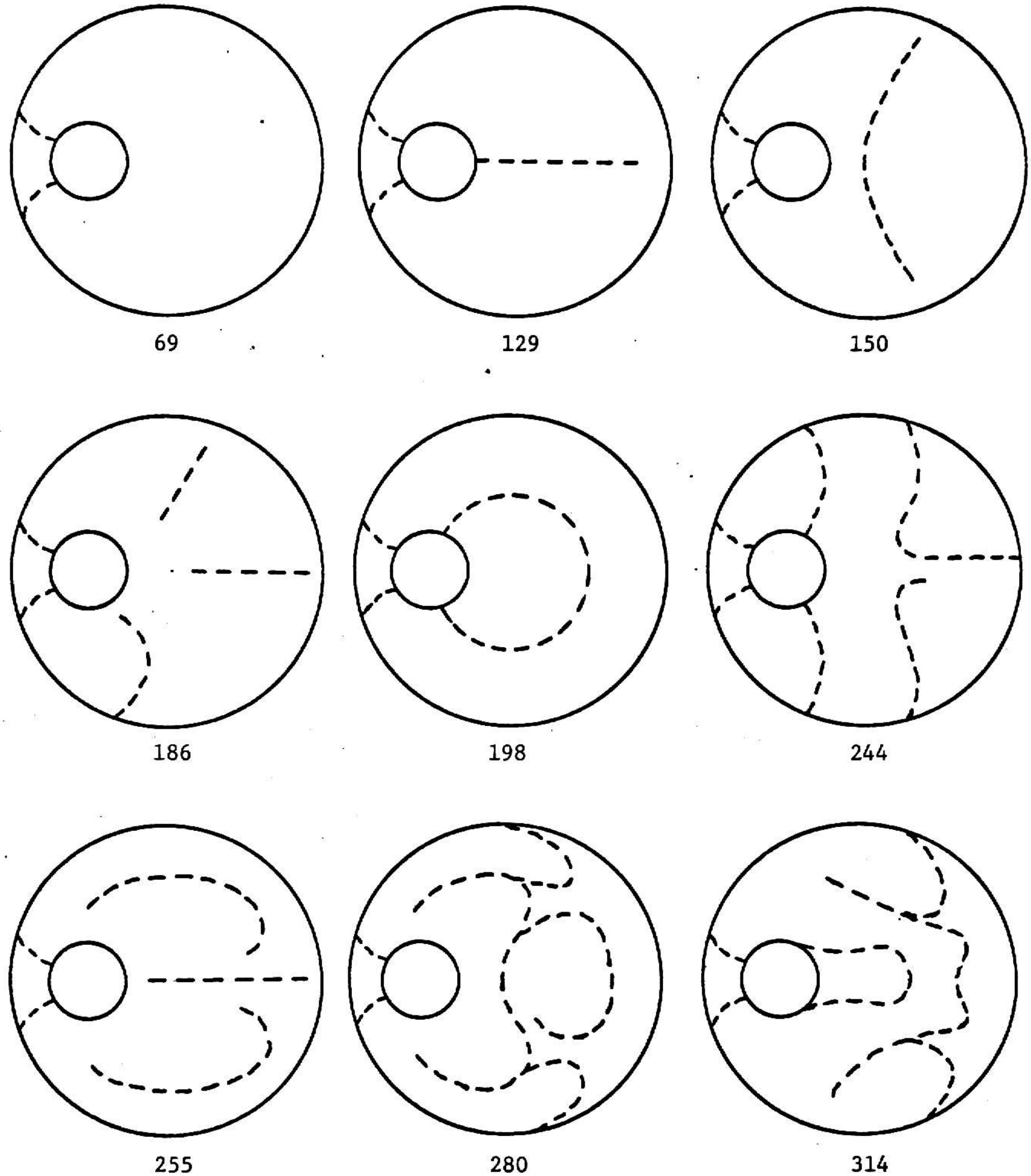


Fig. 6 Mode Shapes for the membrane with the clamped region in place

Using the six higher modes, and the particular ratio, it is possible to find six values of the fundamental frequency by using the results on page 12. An average estimated undamped fundamental frequency is calculated. The calculations appear below:

$$\frac{128}{1.594} \text{ cps} = 80.3 \text{ cps}$$

$$\frac{179}{2.133} \text{ cps} = 83.9 \text{ cps}$$

$$\frac{192}{2.296} \text{ cps} = 83.6 \text{ cps}$$

$$\frac{231}{2.653} \text{ cps} = 86.5 \text{ cps}$$

$$\frac{259}{2.918} \text{ cps} = 86.6 \text{ cps}$$

$$\frac{311}{3.600} \text{ cps} = 86.4 \text{ cps}$$

Average estimated undamped fundamental frequency $= \frac{\text{total}}{6}$

$$\text{Average} = \frac{507.3 \text{ cps}}{6} = 84.6 \text{ cps}$$

The average estimated undamped fundamental frequency is $84.6 \, \frac{\text{cycles}}{\text{sec.}}$.

$$\% \text{ Difference} = \frac{(84.6 - 54.0)}{54.0} (100) = 56.7\%$$

Now that the frequency for the circular case is known it is possible to calculate T in the membrane. Using the equation as written by Timoshenko

$$\omega = \alpha \sqrt{\frac{gT}{wA}}$$

where

w = weight / unit area

A = area

T = tension / unit length

g = acceleration due to gravity

ω = frequency in $\frac{\text{rad}}{\text{sec}}$

α = constant .

$$T = \frac{\omega^2}{\alpha^2} \frac{w}{g} A$$

$$T = \frac{(84.6 \text{ cps})^2 \left(2\pi \frac{\text{rad}}{\text{cyc}}\right)^2 \left(\frac{1.92(10^{-4})\text{lb}}{\text{in}^2}\right) \left(25\pi \text{ in}^2\right)}{\left(2.404 \sqrt{\pi}\right)^2 \left(32.174 \text{ ft/sec}^2\right)}$$

$$T = 7.3 \text{ lb/ft}$$

Now, the frequency of the membrane with the clamped region can be found by using

$$\lambda = \frac{w}{g} \frac{\omega^2}{T} L^2$$

where

λ = computed eigenvalue

L = radius of the membrane.

$$\omega = \sqrt{\frac{T}{L^2} \frac{\lambda}{w/g}}$$

$$\omega = \sqrt{\frac{(7.3 \text{ lb/ft}) \ (8.753) \ (32.174 \text{ ft/sec}^2)}{(25 \text{ in}^2)(1.92)(10^{-4})\text{lb/in}^2}}$$

$$\omega = 654 \text{ rad/sec}$$

$$\omega = 104.1 \text{ cyc/sec} \quad .$$

Experimentally

$$\omega = 69 \text{ cyc/sec } (1.567)$$

$$\omega = 108.1 \text{ cyc/sec} \quad .$$

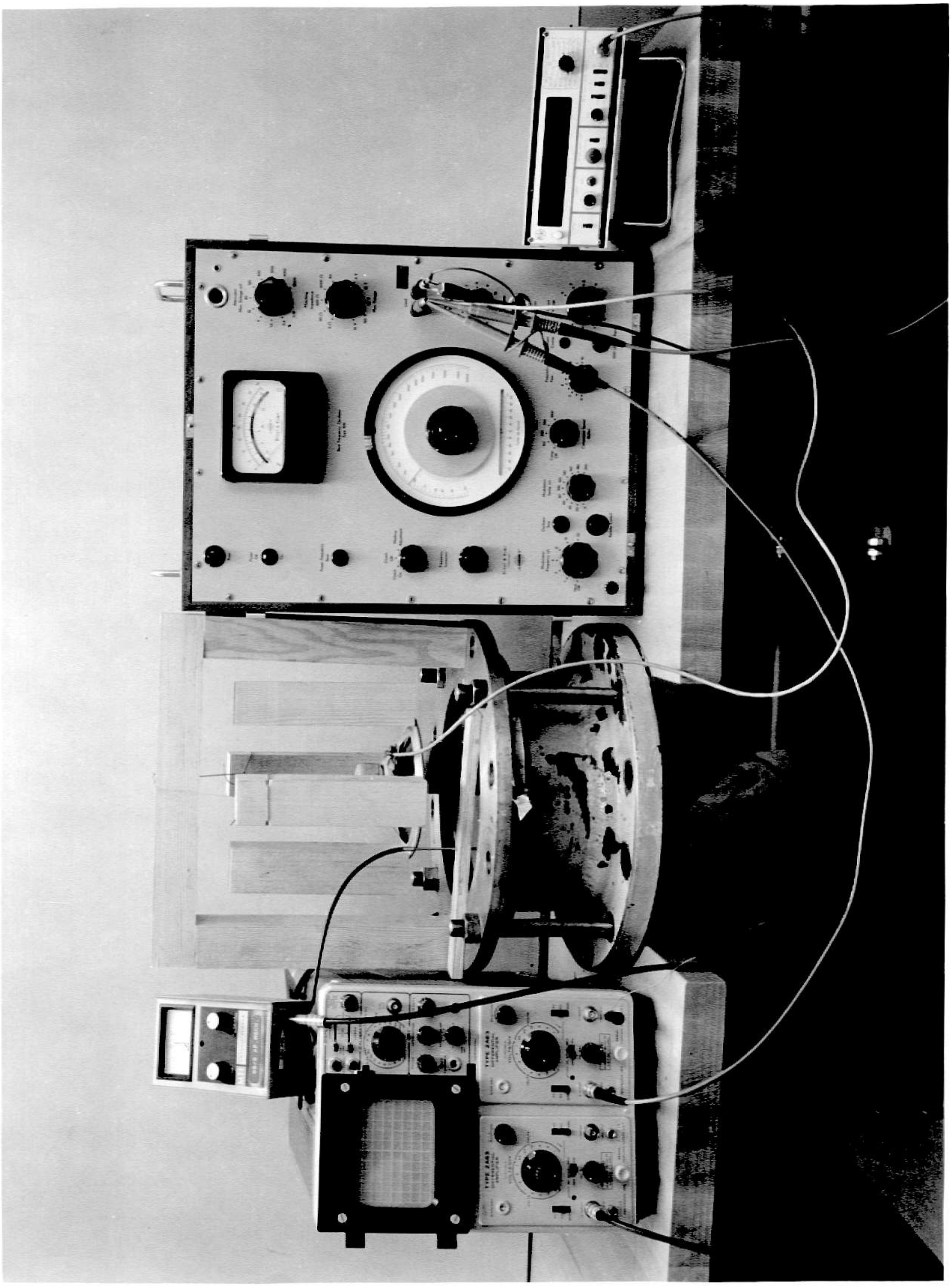
$$\% \text{ DIFFERENCE} = \frac{(108.1 - 104.1)}{(104.1)} (100)$$

$$\% \text{ DIFFERENCE} = 3.84\%$$

EXPLANATION OF PLATE I

PICTURE OF EXPERIMENTAL APPARATUS

PLATE I



CONCLUDING REMARKS

It is difficult to get uniform tension in the membrane. The retaining ring seemed to work rather well. An advantage might be gained if the membrane were larger; in this way the tension would be more uniform at the middle of the membrane.

The optical displacement detector was valuable in finding the modes and the corresponding frequency. For example, for the circular membrane, the second mode has a frequency of 128 cps, but the same mode shape appeared at 62 cps. However, at the lower frequency a horizontal figure 8 appeared on the oscilloscope indicating that the membrane frequency was twice the forcing frequency, thus the second mode occurs at 128 cps.

The mode shapes with the clamped region in place displayed good symmetry. It is interesting to note that some of the modes of both systems have nearly the same frequency. As expected, the fundamental frequency of the membrane with the clamped region was higher than the frequency of the circular membrane.

ACKNOWLEDGEMENT

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APPENDIX I

FINITE DIFFERENCE EQUATION FOR ∇^2

Consider point 0 in the figure below.

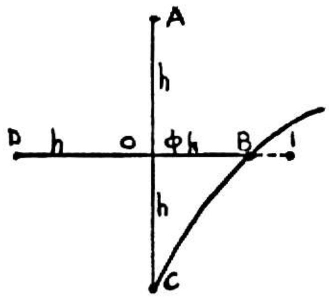

Fig. 7 Geometry of a point near a curved boundary

Point 1 is outside the domain but B can be used along with 0, A, D, C. In the neighborhood of any typical point, 0, the function X can be expanded in a Taylor's series:

$$X = X_o + \left(\frac{\partial X}{\partial x}\right)_o (x - x_o) + \frac{1}{\alpha!}\left(\frac{\partial^2 X}{\partial x^2}\right)_o (x-x_o)^2 + \frac{1}{3!}\left(\frac{\partial^3 X}{\partial x^3}\right)_o (x-x_o)^3 + \ldots,$$

Now let $x = x_o + \phi h$ where ϕh is the distance OB, and also let $x = x_o - h$; it is found that:

$$X_B = X_o + \left(\frac{\partial X}{\partial x}\right)_o (\phi h) + \frac{1}{\alpha!}\left(\frac{\partial^2 X}{\partial x^2}\right)_o (\phi h)^2 + 0(h^3) \ ,$$

and

$$X_D = X_o - \left(\frac{\partial X}{\partial x}\right)_o (h) + \frac{1}{\alpha!}\left(\frac{\partial^2 X}{\partial x^2}\right)_o (h)^2 + 0(h^3) \ .$$

The term $\left(\frac{\partial X}{\partial x}\right)_o$ may be eliminated from both equations so that:

$$\left(\frac{\partial^2 X}{\partial x^2}\right)_o = \frac{1}{h^2}\left[\frac{\alpha X_B}{\phi(1+\phi)} + \frac{\alpha X_D}{1+\phi} - \frac{\alpha X_o}{\phi}\right] \ .$$

In the vertical direction,

$$X = X_o + \left(\frac{\partial X}{\partial y}\right)_o \left(y-y_o\right) + \frac{1}{\alpha!}\left(\frac{\partial^2 X}{\partial y^2}\right)_o \left(y-y_o\right)^2 + \frac{1}{3!}\left(\frac{\partial^3 X}{\partial y^3}\right)_o \left(y-y_o\right)^3 + \dots \quad .$$

Now let $y = y_o + h$ and $y_o - h$ to get

$$X_A = X_o + \left(\frac{\partial X}{\partial y}\right)_o (h) + \frac{1}{\alpha!}\left(\frac{\partial^2 X}{\partial y^2}\right)_o (h)^2 + 0(h^3) \quad ,$$

and

$$X_C = X_o - \left(\frac{\partial X}{\partial y}\right)_o (h) + \frac{1}{\alpha}\left(\frac{\partial^2 X}{\partial y^2}\right)_o (h)^2 + 0(h^3) \quad .$$

Eliminating $\left(\frac{\partial X}{\partial y}\right)_o$ gives

$$\left(\frac{\partial^2 X}{\partial y^2}\right)_o = \frac{1}{h^2}\left|X_A + X_C - 2X_o\right| \quad .$$

Combining $\left(\frac{\partial^2 X}{\partial x^2}\right)_o$ and $\left(\frac{\partial^2 X}{\partial y^2}\right)_o$ gives

$$\left(\frac{\partial^2 X}{\partial x^2} + \frac{\partial^2 X}{\partial y^2}\right)_o = \frac{1}{h^2}\left|X_A + \frac{\alpha X_B}{\phi(1+\phi)} + X_C + \frac{\alpha X_D}{1+\phi} - \alpha X_o \left(\frac{1+\phi}{\phi}\right)\right| \qquad \text{(C)}$$

which is the approximation for ∇^2 for a point near a boundary. In this problem all points on the boundary are equal to zero. This equation is used for a point such as point 31. If $\phi = 1$, equation (C) above becomes

$$\left(\frac{\partial^2 X}{\partial x^2} + \frac{\partial^2 X}{\partial y^2}\right)_o = \frac{1}{h^2}\left|X_A + X_B + X_C + X_D - 4X_o\right| \quad ,$$

which is the equation used for a regular point such as point 51.

APPENDIX II

COMPUTER PROGRAM

```
C     HOPEFULLY, THIS PROGRAM WILL FIND THE LOWEST
C     EIGENVALUE AND THE CORRESPONDING EIGENVECTOR
C     FOR A CIRCULAR MEMBRANE WITH AN OFF-CENTER
C     CIRCULAR CLAMPED REGION. THE MEMBRANE IS
C     APPROXIMATED BY A SERIES OF SQUARES, HENCE THE
C     OUTER BOUNDARY IS A SERIES OF STRAIGHT LINES.
C     THE INNER CIRCULAR BOUNDARY IS USED AS IS.
C     THE INTERVAL IS EQUAL TO 0.125. RAYLEIGH'S
C     QUOTIENT IS USED WITH ITERATION TO FIND LAMDA,
C     THE PROBLEM CONSTANT(HENCE THE FREQUENCY), AND
C     AND THE EIGENVECTOR.
C     EP=TEST
C     X=FUNCTION
C     T=VALUE AT A POINT
C     LAMDA=TENSION/UNIT LENGTH/MASS/UNIT AREA
      REAL LAMDA,LHS
      DIMENSION X(133),T(133),H(133),C(133)
  100 FORMAT (5X,'LAMDA')
  102 FORMAT (5X,'EIGENVECTOR')
  103 FORMAT (1E16.8)
  104 FORMAT (4E16.8)
  105 FORMAT (1H-)
  106 FORMAT (1H1)
  107 FORMAT (10F8.5)
  110 FORMAT (5X,'C')
      HS=0.015625
      READ(1,107) (X(I),I=1,133)
      M=1
      EP=0.001
C     THE TRIAL FUNCTION HAS ALREADY BEEN READ INTO
C     THE PROGRAM; WE NEED ONLY CALCULATE LAMDA BY
C     USINGRAYLEIGH'S QUOTIENT. LAMDA WILL THEN BE
C     USED WITH ITERATION TO FIND THE EIGENVECTOR
C     AND LAMDA.
  268 Q=4
      K=9.111
  214 DO 698 I=1,133
  698 T(I)=0.0
      DO 699 I=16,30,14
  699 T(I)=X(I-14)+X(I-1)-Q*X(I)+X(I+1)+X(I+14)
      DO 700 I=19,27
  700 T(I)=X(I-14)+X(I-1)-Q*X(I)+X(I+1)+X(I+14)
      DO 701 I=33,41
  701 T(I)=X(I-14)+X(I-1)-Q*X(I)+X(I+1)+X(I+17)
      DO 708 I=44,45
  708 T(I)=X(I-14)+X(I-1)-Q*X(I)+X(I+1)+X(I+17)
      DO 702 I=49,58
  702 T(I)=X(I-17)+X(I-1)-Q*X(I)+X(I+1)+X(I+17)
      DO 703 I=61,75
  703 T(I)=X(I-17)+X(I-1)-Q*X(I)+X(I+1)+X(I+16)
      DO 704 I=78,90
  704 T(I)=X(I-16)+X(I-1)-Q*X(I)+X(I+1)+X(I+14)
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      DO 705 I=93,103
  705 T(I)=X(I-14)+X(I-1)-C*X(I)+X(I+1)+X(I+12)
      DO 706 I=106,114
  706 T(I)=X(I-12)+X(I-1)-C*X(I)+X(I+1)+X(I+10)
      DO 707 I=117,123
  707 T(I)=X(I-10)+X(I-1)-C*X(I)+X(I+1)+X(I+9)
      T(31)=X(17)+1.561*X(30)-K*X(31)+X(45)
      T(32)=X(18)-K*X(32)+1.561*X(33)+X(49)
      T(46)=X(45)-K*X(46)+X(47)+1.561*X(63)
      T(48)=X(47)-K*X(48)+X(49)+1.561*X(65)
      IF(Q.EQ.4) GO TO 300
      DO 709 I=30,41
  709 T(I-28)=T(I)
      DO 710 I=1,133
  710 C(I)=T(I)/R
      C(3)=T(3)/S
      C(4)=T(4)/S
      C(31)=T(31)/S
      C(32)=T(32)/S
      C(46)=T(46)/S
      C(48)=T(48)/S
      N=N+1
C     NORMALIZE C(I), AND COMPARE WITH THE
C     PREVIOUS EIGENVECTOR.
      DUM=ABS(C(1))
      DO 601 I=1,133
      RUM=ABS(C(I))
      IF(RUM.GT.DUM) GO TO 602
      GO TO 601
  602 DUM=ABS(C(I))
  601 CONTINUE
      DIV=1.0/DUM
      DO 605 I=1,133
  605 C(I)=C(I)*DIV
      DO 606 I=1,133
      H(I)=ABS(C(I)-X(I))
      IF(H(I).GT.EP) GO TO 800
  606 CONTINUE
      GO TO 900
  800 DO 801 I=1,133
  801 X(I)=C(I)
      IF(N.EQ.10) GO TO 268
      GO TO 214
  300 N=0
      SUMN=0.0
      SUMD=0.0
      DO 267 I=16,133
      SUMN=SUMN+(X(I)*T(I))
  267 SUMD=SUMD+(X(I)**2)
      LHS=-SUMN/SUMD
      R=4.0-LHS
      S=9.111-LHS
```

```
      LAMDA=LHS/RS
      IF(M.EQ.2) GO TO 750
      WRITE(3,100)
      WRITE (3,103) LAMDA
      WRITE (3,105)
      WRITE(3,102)
      WRITE(3,104) (X(I),I=1,133)
      WRITE (3,106)
      C=0
      K=0
      GO TO 214
  900 M=2
      GO TU 268
  750 WRITE (3,100)
      WRITE (3,103) LAMDA
      WRITE (3,105)
      WRITE (3,102)
      WRITE (3,110)
      WRITE(3,104) (C(I),I=1,133)
      STOP
      END
```

MEMBRANE VIBRATIONS

by

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AN ABSTRACT OF A MASTER'S REPORT

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ABSTRACT

This report considers the vibrations of a circular membrane with an off-center circular clamped region. A finite difference approximation to the governing differential equation and Rayleigh's quotient are used to find the fundamental eigenvalue and the corresponding eigenvector. The outer boundary is approximated by a series of straight lines while the inner boundary is approximated by irregular stars. Experimental mode shapes and frequencies are presented for both the circular membrane and the circular membrane with the clamped region.