Design considerations for two-flight reinforced concrete stairs

by

Changmin Liao

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GE Johnson Department of Architectural Engineering and Construction Science
Carl R. Ice College of Engineering

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Approved by:

Co-Major Professor
Jongchul Song

Approved by:

Co-Major Professor
Kimberly Kramer

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Abstract

The reinforced concrete stairs are commonly used in the built environment for many years. In the early 20th century, the use of steel as the structural system for stairs has increased, and reinforced concrete has decreased as the most common structural material. However, the ability to make any shape, readily available materials, the compressive strength, and outstanding fire resistance of reinforced concrete ensure that it is a vital material in the built environment. Especially for stairs serving as egress for people to evacuate under severe conditions, reinforced concrete as a building material stands out. This report presents a design procedure for two-flight reinforced concrete stairs and provides a parametric study to analyze the relationship between flexural reinforcement and critical factors: reinforcement design methods, geographic locations, stair slab thicknesses, and boundary conditions. For this study, the materials are limited to normal weight concrete with ASTM 615 Grade 60 steel reinforcing with a specified yield strength of 60,000 psi. Twenty-four cases are designed with varying reinforcement design methods, geographic locations, stair slab thicknesses, and boundary conditions. The purpose of this report is to examine the differences in reinforcement design for reinforced concrete stairs.

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A_f = project area normal to the wind

A_g = gross area of concrete section

A_v = area of shear reinforcement within spacing

$A_{v,min}$ = minimum area of shear reinforcement within spacing

a_p = component amplification factor

b = width of compression face of member

b_w = web width or diameter of circular section

C_e = exposure factor

C_f = wind force coefficient

C_t = thermal factor

D = dead load

d = distance from extreme compression fiber to centroid of longitudinal tension reinforcement

d_b = nominal diameter of bar

E = earthquake load

E_h = effect of horizontal seismic force

E_v = effect of vertical seismic force

F_p = seismic design force

f_c' = specified compressive strength of concrete

f_r = modulus of rupture of concrete

f_y = specified yield strength for nonprestressed reinforcement

f_{yt} = specified yield strength for transverse reinforcement

G = gust-effect factor

h = average roof height of the structure with respect to the base

I_{cr} = moment of inertia of cracked section transformed to concrete

I_e = effective moment of inertia for calculation of deflection

I_g = area moment of inertia

I_p = component importance factor

I_s = importance factor

L = live load

L_r = roof live load

l = span length of beam or one-way slab

M_{cr} = cracking moment

M_n = nominal flexural strength at section

M_u = factored moment at section

N_u = factored axial force normal to cross section

p_f = snow load on flat roofs

p_g = ground snow load

q_z = the velocity pressure evaluated at height z, of the centroid of the area A_f

R = rain load

R_p = component response modification factor

S = snow load

S_{DS} = spectral acceleration, short period

s = center-to-center spacing of reinforcement

V_c = shear strength provided by concrete

V_n = equivalent concrete stress corresponding to nominal two-way shear strength

V_s = shear strength provided by reinforcement

V_u = maximum factored two-way shear stress calculated around the perimeter of a given critical section

W = wind load

W_p = component operating weight

y_t = distance from centroidal axis of gross section to tension face

z = height in structure

λ = modification factor to reflect the reduced mechanical properties of lightweight concrete relative to normal weight concrete of the same compressive strength

λ_s= factor used to modify shear strength based on the effects of member depth, commonly referred to as the size effect factor

Φ = strength reduction factor

$\int_{rqd}$ = required area ratio of reinforcement

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Chapter 1 - Introduction

Stairs are essential structural elements for vertical transportation within a building. It is necessary even if the building is equipped with elevator, escalator, mainly for the use of evacuation in case of a sudden emergency, or a natural disaster. The use of reinforced concrete in structures can be traced back to 1853 when Francois Coignet built the first iron reinforced concrete house in Paris. The building industry has been using and improving this composite material since Joseph Monier's patent, as it provides high tensile strength, compressive strength, and durability when exposed to the environment.

This report begins with the design considerations of the reinforced concrete stair structure: dimension, building loads, minimum reinforcement requirements, means of egress, fire protection, formwork, etc. *International Building Code 2018* (IBC 2018) and *Occupational Safety and Health Administration Standards* (OSHA 2014) provide the overall dimension requirement for the stairway. The building loads requirement is recommended by *Minimum Design Loads for Buildings and Other Structures* (ASCE/SEI 7-16). The minimum reinforcement requirement is recommended by *Building Code Requirements for Structural Concrete* (ACI 318-19).

Furthermore, the report covers the design procedure in accordance with ACI 318-19. A comparison of reinforcement designed by using a one-way slab classification and a concrete beam classification is included in this study to provide a better understanding of the considerations when designing the flexural reinforcement. Additionally, this report contains a parametric study, discussing the importance of four parameters: reinforcement design method referred to ACI 318-19 (one-way slab, or beam), geographic location (high seismic zone and low seismic zone), slab thickness, and boundary conditions. Results and discussions of the relationship of each parameter are presented bases on the calculation.

Finally, the conclusions of the study and recommendations for the better design are presented based on the data provided in the parametric study.

Chapter 2 - Background and Literature Review

Staircase Structural Analysis and Design, by M.Y.H. Bangash & T. Bangash defines a stair as "… constructed with steps rising without a break from floor to floor, or with steps rising to a landing between floors, with a series of steps rising further from the landing to the floor above."

The stairway plays a significant role for people and objects to pass from one level to another in the built environment. As regulations improved for the built environment, the promotion of the function of a stairway as a required means of egress in case of an emergency occurred. Even with an elevator equipped in a multi-story building, the need for a stairway is demanded since the fire could cause a short circuit to electricity so that the elevator would stop during a fire accident.

2.1 - Terminology

Various components and terminologies of the stairway exists; therefore, definitions are provided. For this research, the focus of the topic is the reinforced concrete stairway; thus, the elements and terminologies explained are limited to this topic.

As shown in Figure 2.1, the terminologies include:

- *Flight* - a series of steps or stairs between landings, levels, or stories.
- *Handrail* - a rail fixed parallel above the pitch line at the sides of a stair.
- *Landing* - an area of a floor or platform on top of or between the flight of stairs for changing direction.
- *Nosing* - an edge part of the tread that extends the riser beneath.

- *Riser* - the vertical part of the step between each tread. The rise height is measured as the vertical distance between tread nosings. Use a carpenter's level if the tread slopes to the front or the back.
- *Stringer* - the structural member that supports the treads and risers in standard staircases.
- *Tread* - the part of the stair that for people or objects to walk or stand on. The measurement of the tread refers to as tread depth, measured from the back of one tread to the end of another nearby.

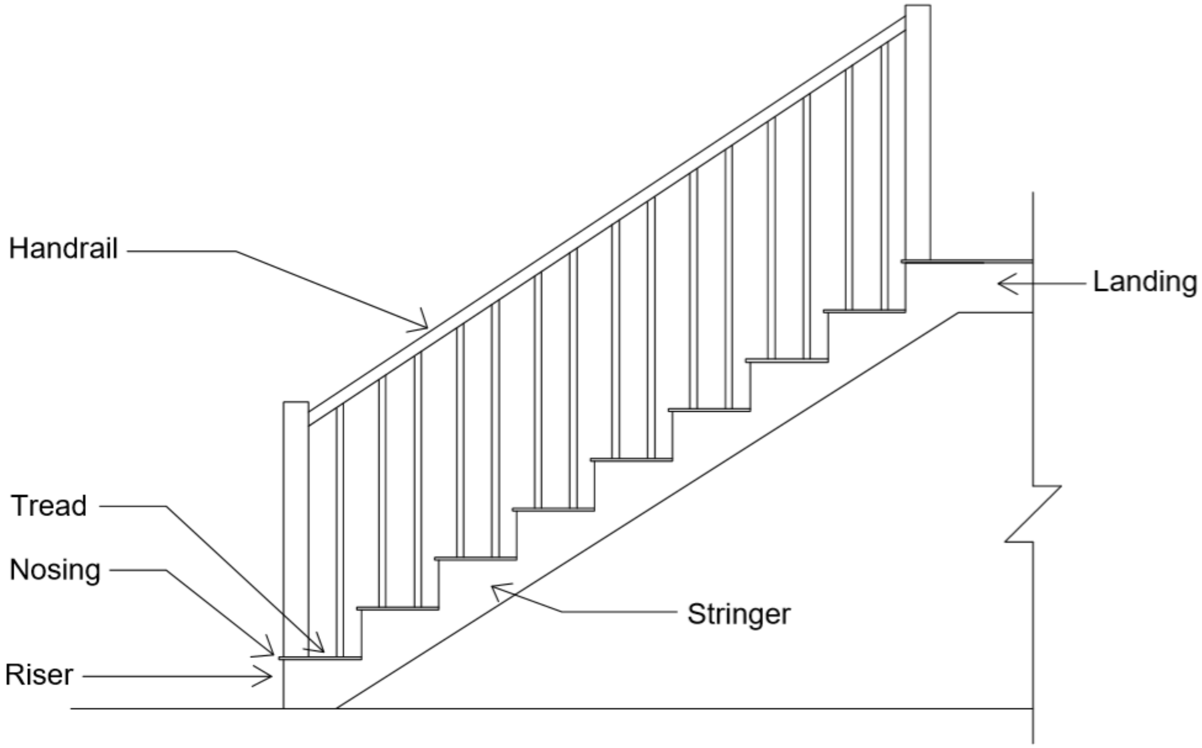


Figure 2.1 – Stairway components

2.2 - Design Considerations

This section introduces the design considerations for a reinforced concrete stair. Chapter 10 of *International Building Code 2018* (IBC 2018) and Section 1910 of *Occupational Safety and Health Administration Standards* (OSHA 2014) cover the overall dimension, arrangement, construction of the stairways, guards, and handrails. Generally, IBC 2018 is for the life and safety of the occupancy of the built structure, during its use, and the OSHA standards are for the life and safety of the construction workers while the structure is being built. Both are presented in this section. Additionally, Chapter 2 of *Minimum Design Loads for Buildings and Other Structures* (ASCE/SEI 7-16) which covers the combination of loads is given. Lastly, the minimum reinforcement requirement for the reinforced concrete stairs using Chapter 7 and Chapter 9 of *Building Code Requirements for Structural Concrete* (ACI 318-19) is presented.

2.2.1 - Dimension Requirement

As shown in Table 2.1, the dimensional requirements according to IBC 2018 and OSHA are given. IBC 2018 requires a minimum stair width of 44-inch while the OSHA Standards require 28-inch. During the life of the built structure, more occupants may be in the building and need to exit during a fire or other disaster compared to while the building is being constructed. The riser height maximum is 7-inch and 4-inch minimum for the IBC 2018. This range of riser height is required to encompass all sizes, small children to very tall adults, and ages of the people who will use the built structure during its life. Additionally, the IBC 2018 requires all risers between floors to be the same height. According to *Ergonomics for Children: Designing Products and Places for Toddler to Teens*, scaling-down stair geometries for children are not recommended, even for preschool-age facilities. The 7-inch United States of America “standard works reasonably well for children" (Lueder and Rice, 2007). During construction, only adults will be on the jobsite and using the stairs; therefore, the maximum riser height is 9.5-inch based on OSHA requirements. Due to the reasons that IBC stairs are typically applied in areas that are open to the public and with higher traffic than OSHA stairs, the minimum values of tread depth and landing length required by IBC 2018 are higher than OSHA Standards. However, the requirements for landing width by both standards are the same because of the structural uniformity of stair.

Table 2.1 - Overview of stair code requirements

Stairway Dimension Requirement		
Requirement	**IBC 2018, Sections 1011, 1014, and 1015**	**OSHA Standards 1910.25/.28/.29 and 1910.36 for Means of Egress**
Minimum width	44 in.	22 in. (between vertical barriers) or 28 in. (for exit access)
Riser height, vertically between nosings	7 in. maximum, 4 in. minimum	9.5 in. maximum
Tread depth, horizontally between nosings	11 in. minimum	9.5 in. minimum
Landing width	Matching stair width	Matching stair (platform)
Landing length	Straight run 48 in.	30 in. in the direction of travel

2.2.2 - Design Loads and Load Combinations

Four types of design loads should be taken into consideration for reinforced concrete stairway design: gravity loads, environmental loads, seismic loads, and thermal loads. The IBC 2018 refers to the American Society of Civil Engineers and Structural Engineering Institute *Minimum Design Loads and Associated Criteria for Buildings and Other Structures* 2016 Edition (ASCE/SEI 7-16) to provide design loads that are based on the geographic conditions, stair materials and function.

ASCE/SEI 7-16, Chapter 2 Combination of Loads, Section 2.3.1 Basic Combinations and Section 2.3.6, presents the load combinations for a stair structure design when using strength design methodology. ASCE/SEI 7-16 Chapter 2 Combination of Loads, Section 2.4.1 and 2.4.5 presents the load combinations for a stair structure for serviceability design items, such as deflection. Seven combinations for Strength Design and ten combinations for Allowable Stress Design (ASD) are given. The Strength Design, i.e. *Ultimate Strength Design*, method has been used in the American Concrete Institutes' Committee 318 document *Building Code Requirements for Structural Concrete* (ACI 318) since the 1971 (Pierce, 2015). Therefore, the Ultimate Strength Design method is adopted for this research.

The load combinations are:

1. $1.4D$
2. $1.2D + 1.6L + 0.5(L_r\ or\ S\ or\ R)$
3. $1.2D + 1.6(L_r\ or\ S\ or\ R) + (L\ or\ 0.5W)$
4. $1.2D + 1.0W + L + 0.5(L_r\ or\ S\ or\ R)$
5. $1.2D + 1.0E + L + 0.2S$
6. $0.9D + 1.0W$

7. $0.9D + 1.0E$

Additionally, for use in load combination 5 shown above, E shall be determined in accordance with Eq. 12.4-1 as follows:

$$E = E_h + E_v$$

(ASCE/SEI 7-16, Eq. 12.4-1)

For use in load combination 7 shown above, E shall be determined in accordance with Eq. 12.4-2 as follows:

$$E = E_h - E_v$$

(ASCE/SEI 7-16, Eq. 12.4-2)

2.2.2.1 - Gravity Loads

Gravity loads indicate dead (permanent) loads and live (temporary) loads of a building.

2.2.2.1.1 - Dead Loads

Dead loads indicate the weight of the structure, which includes its roofs, walls, beams, columns, finishes, insulations, sheathings, and MEP systems, etc. Commentary Chapter C3 and Table C3-1 of *Minimum Design Loads for Buildings and Other Structures* (ASCE/SEI 7-16) cover minimum design dead loads for various kinds of materials and structural elements. For this research, since the focus is reinforced concrete stairs, consideration of the dead loads would be the stair structure self-weight and the miscellaneous load (0 to 5 psf) produced by electronic devices in the buildings.

2.2.2.1.2 - Live Loads

Live Loads include any temporary or transient force that acts on a building or structural element. Typically, live loads include people, furniture, and almost everything else that can be moved throughout a building, indicate contents and occupancies of the structure. For this research, live loads of the stairway are induced by people. The minimum design live loads are given in ASCE/SEI 7-16, Table 4.3-1, and IBC 2018, Section 1607, Table 1607.1, and include platforms (landing), stair treads, handrails, guardrails, and grab bars. These live loads are based on the maximum load that will be imposed on the structure over its lifetime based on the average-sized person. The structural design of a stair system must consider the effects of uniform and concentrated loads on various components. Table 2.2 indicates a stair landing to be designed for a minimum uniform load of 100 psf. A 100 psf is similar to having a 150-pound person occupying 15-inch by 15-inch square (area). In other words, people shoulder to shoulder using the stairs to exit the building in case of a fire. As shown in Table 2.2, handrails, guardrails, and grab bars are designed for a uniform load of 50 psf which represents the force exerted by tightly grouped persons leaning on or pressing against the railing system. The concentrated load of 200 pounds represents the force exerted by a single individual leaning upon or over the rail or a person or object impacting upon the rail. This 200-pound concentrated load applied in any direction at any point along the top rail. The stair and exit way trends are designed for a 100 psf uniform load and a concentrated load of 300 pounds applied any point on the element.

Table 2.2 - Stairway live loads

Occupancy or Use	**Uniform load, psf**	**Concentrated load, lb**
Stair landing (platform)	100	-
Handrails, guardrails, and grab bars	50	200
Stairs and exit ways (tread)	100	300

2.2.2.2 - Environmental Loads

Environmental loads are structural loads that caused by natural forces, such as wind loads, snow loads, ice loads, rain loads, etc.

2.2.2.2.1 - Wind Loads

ASCE/SEI 7-16 covers the method to determine wind loads. For exterior stairways of buildings in different geographic locations, wind loads would be different. Since the role that stairs play in the building is not as load-resisting as other structural members such as columns or beams, unlikely the method applied for the stairway would be the methods demonstrated in Chapter 27, Chapter 28, or Chapter 31 of the ASCE/SEI 7-16. Therefore, the Direction Procedure for Other Structures (ASCE/SEI 7-16, Chapter 29, Section 29.5) is applied for the exterior stairway wind load calculation.

The wind load shall be by determined by the following equation:

$$F = q_z G C_f A_f$$

(ASCE/SEI 7-16, Eq. 29.4-1)

where

A_f = project area normal to the wind

C_f = wind force coefficient

G = gust-effect factor

q_z = the velocity pressure evaluated at height z, of the centroid of the area A_f

Furthermore, the minimum design wind force for exterior stairways shall not be less than 16 lb/ft^2 (0.77 kN/m^2) multiplied by the area A_f, as indicated by ASCE/SEI 7-16, Section 29.8. For the interior stairways, since the structures have no surface that exposed to the external environment, only the internal wind pressure shall be applied.

2.2.2.2.2 - Snow Loads

ASCE/SEI 7-16, Chapter 7, presents the method to determine snow loads. For exterior stairways, especially in the northern states that often snow, the ground snow load is used of 20 lb/ft^2 minimum. Certain regions may have different ground snow load determination due to the local authority having jurisdiction, such as Hawaii, that they have zero ground snow load. For exterior stairways on multi-story (mostly, the roof) other than just the ground, the snow load adjustment may be made by the following equation, with the accordance of ASCE/SEI 7-16, Section 7.3:

$$p_f = 0.7C_eC_tI_sp_g$$

(ASCE/SEI 7-16, Eq. 7.3-1)

where

C_e = exposure factor

C_t = thermal factor

I_s = importance factor

p_f = snow load on flat roofs

p_g = ground snow load

Moreover, for treads in different vertical distance, snow load adjustment may be made by snow drifts. According to ASCE/SEI 7-16, Section 7.7.1, the considerations of the load caused by snowdrifts are not required if h_c/h_b is less than 0.2 (where, h_c is the clear height from top of balanced snow load to closet point of adjacent upper tread; h_b is the height of balanced snow load determining by dividing the stairway snow load p_f by the snow intensity γ, in ft (m)). The snow intensity γ shall be determined by the following equation:

$$\gamma = 0.13p_g + 14 \text{ but not more than 30 pcf}$$

(ASCE/SEI 7-16, Eq. 7.7-1)

(in SI: $\gamma = 0.426p_g + 2.2$, but not greater than 4.7 kN/m^3)

Additionally, for locations where p_g is equal to or less than 20 lb/ft^2 (0.96 kN/m^2), but not zero, the exterior stairways with a sloping landing with slopes less than $W/50$ in ft (or $W/15.2$ in m) shall include a 5 lb/ft^2 (0.24 lb/ft^2) rain-on-snow surcharge load. This additional load needs not to be used in combination with snowdrift.

For interior stairways, no snow load shall be applied, unless the building is exposed to the external environment (such as a parking garage).

2.2.2.2.3 - Rain Loads

ASCE/SEI 7-16, Chapter 8, covers the rain loads. Generally, rain loads are not required unless the span of treads or landings is long enough to form a puddle of water.

2.2.2.2.4 - Ice Loads

ASCE/SEI 7-16, Chapter 10, covers the ice loads. An exterior stairway is defined as an ice-sensitive structure because of the long span of treads and landings, and the different vertical distances of each tread, it accumulates an excessive load of snow resulting in built-up ice.

2.2.2.3 - Seismic Loads

ASCE/SEI 7-16, Chapter 11, demonstrates the design criteria for seismic loads. It is known that most stairways do not serve as the main load-bearing structural elements in the building, and most of them are not even part of the seismic lateral force-resisting system. Therefore, the adequate method for stairway seismic design is demonstrated by Chapter 13, for nonstructural components. However, researches show that stairways effect on buildings' seismic behavior by their existence and locations inside the building, thereby changing the overall stiffness magnitude, stiffness distribution, and force distribution of the building. Furthermore, stairways serve as the main egress exit for emergencies such as fire, and natural disasters such as earthquakes; it is significant to pay attention in stairway seismic analysis and design so that this part of the structure remains functional and integrated with the seismic later force-resisting system.

The following equation determines the horizontal seismic design force:

$$F_p = \frac{0.4 a_p S_{DS} W_p}{\left(\frac{R_p}{I_p}\right)} \left(1 + 2\frac{z}{h}\right)$$

(ASCE/SEI 7-16, Eq. 13.3-1)

F_P is not required to be taken as larger than

$$F_p = 1.6 S_{DS} I_p W_p$$

(ASCE/SEI 7-16, Eq. 13.3-2)

and F_p shall not be taken as less than

$$F_p = 0.3 S_{DS} I_p W_p$$

(ASCE/SEI 7-16, Eq. 13.3-3)

The following equation determines the vertical seismic design force:

$$F_p = \pm 0.2 S_{DS} W_p$$

(ASCE/SEI 7-16, Section 13.3.1)

where

a_p = component amplification factor

h = average roof height of the structure with respect to the base

I_p = component importance factor

R_p = component response modification factor

S_{DS} = spectral acceleration, short period

W_p = component operating weight

z = height in structure

The component importance factor, I_p, shall be taken as 1.50 since egress stairways are required to function for life-safety purposes after an earthquake or a fire incident, with the accordance of ASCE/SEI 7-16, Section 13.1.3.

Steel framed stairs contain components that anchored to concrete foundation. For anchors in concrete, ACI 318-19 Chapter 17 provides the design requirements that govern concrete breakout and pullout strength in tension. Since the topic of this research is about reinforced concrete stairway design that often produced by formwork-making, reinforcement-placement, and concrete-pouring, the requirements for anchors in concrete are beyond the scope of this research.

2.2.3 - Minimum Reinforcement Requirement

Steel reinforcement provides strength against tensile stress that concrete lacks and assists concrete stairs to transfer loads to the ground. Moreover, steel reinforcement helps the concrete to improve its resistance to shear, and torsional force resulted from surrounding elements and boundary conditions.

ACI 318-19 does not provide an exclusive chapter to indicates the reinforcement design for the concrete stairways. However, structural behavior of a reinforced concrete stair may be taken as the same as either as a one-way slab, or a beam.

A one-way slab carries the load in one direction; thereby, it carries flexural stresses in one direction. Generally, the reinforcement designed for the one-way slab is for the spanning direction along with the main direction reinforcement parallel to the span, longitudinal, and the transverse direction reinforcement perpendicular to the span. As for a longitudinally spanning staircase, often two or more supports at both ends of the structure exist, including slab and landing. For each component, it bends in the same direction. As for a transversely spanning staircase, which often has the slab supported between two stringer beams or walls, it is comprehensive to determine whether it should be considered as a one-way or a two-way slab. However, as a rule of thumb for differentiating between one-way slabs and two-way slabs is if the length-to-breadth ratio (taken as long side divided by short side of span) is equal or greater than 2.0, the slab is considered as one-way, and vice versa.   IBC 2018 indicates a minimum clear width of 48 inches (IBC 2018, Section 1009.3) for a stairway between stories to satisfy the standard of an accessible means of egress. It also indicates a maximum vertical rise of 12 feet for the stairway between landings (IBC 2018, Section 1011.8). If performing a right triangle calculation based on the 4 inches minimum riser height and 11 inches minimum tread depth, the value of the hypotenuse would be around 12-inch, with 12-foot vertical rise height, the length of the slab would be around 35-foot, which is more than two times of the clear width. Therefore, a concrete stair may be considered as a one-way slab system.

A concrete beam is less structurally redundant and can span longer than a one-way slab. Generally, the concrete stair would require more reinforcement if applying the beam method instead of the one-way slab method because the increasing concrete cover of beam would result in a decreasing distance from extreme compression fiber to centroid of longitudinal tension reinforcement, thus requiring more strength provided by the reinforcement to resist the flexural strength acting on the stairs. Since a beam carries the load from the upper floor, it also experiences torsional strength, as one of the most significant design considerations. For example, a floor beam transfers load from the slab above to the spandrel beam in a concrete structural system. Depending on the stiffness and flexibility of the spandrel beam, torsion may be induced, which would be the moment transferred by the floor beam to its ends in contact with the spandrel beam. A stiff edge beam has more torsional resistance than a flexible edge beam.

In order to reduce crack size in concrete due to the initial dry shrinkage as concrete cures and future expansion caused by temperature changes, temperature and shrinkage reinforcement is often applied. ACI 318-19 Commentary states that the use of temperature and shrinkage reinforcement is intended for structural slabs only (ACI 318-19, R24.4.1). However, the need for temperature and shrinkage reinforcement might be significant for the concrete stairway designed by the concrete beam method as well. Because of the relatively large values of clear width and span length of the concrete stairway designed by IBC standard, even if treating a concrete stairway as a concrete beam, the large surface area of the concrete would probably result in the demand for distribution reinforcement to deal with the future expansion due to temperature changes. However, it might also cause an overdesign as it could take extra time and materials to affect the design and construction. The design and construction teams should communicate with each other whether the temperature reinforcement is necessary for a beam.

2.3 - Literature Review

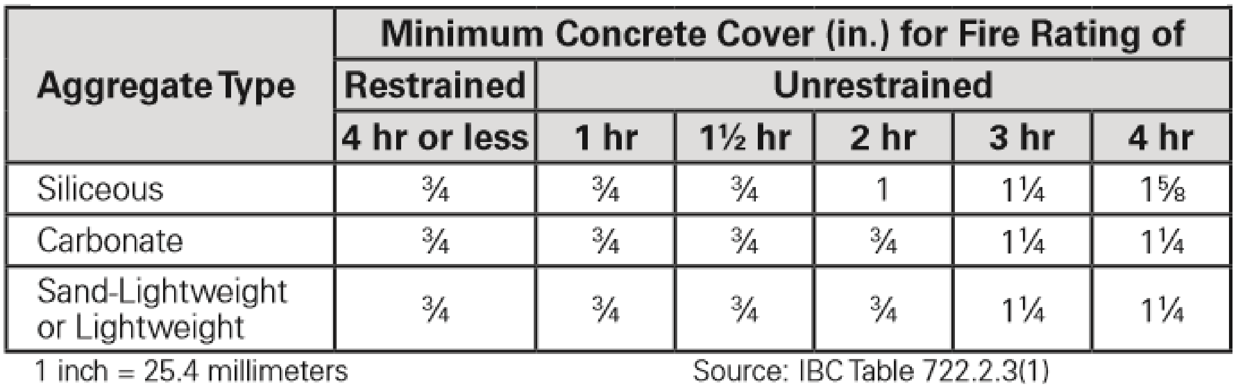
Engineers and designers should consider many factors when designing reinforced concrete stairs. Some factors have been defined, and the answers to those considerations have been provided by the building codes and standards, such as the minimum requirement of stairway sizing and load provided by IBC 2018. Some factors appear in the design process, depending on various design conditions or field conditions, they may not be covered thoroughly by the building codes and standards. Engineering judgment should be made by the engineers and designers, based on their professional experience. Meanwhile, relevant researches regarding the issues encountered in the design process should be reviewed as the problems might have been answered by the experts. The following review of literature presents the findings relevant to the design considerations of the reinforced concrete stairway.

Maintaining tolerance of the stairway is important to avoid a trip-and-fall accident. Section 7.2.2.3.6 of the Life Safety Code states, "There shall be no variation in excess of 3/16-in in the depth of adjacent risers, and the tolerance between the largest and smallest riser shall not exceed 3/8-in in any flight." However, the difficulty in meeting the requirement and the expensiveness of the correction action of the stairway remains tough. A study by Heather J. Brown and Bruce A. Suprenant (2007) managed to collect data from 10 sets of stairs chosen in each of the three general types of buildings: government agencies, schools, and offices/medical. The result shows that 30% of the adjoining stair measurements don't meet the requirement that adjacent risers can't differ more than 3/16-in, and 82% of the riser measurements don't meet the condition that the maximum-minus-minimum value can't exceed 3/8-in. The authors explain that it is difficult to form stair risers within tolerance under the code requirement because of the imperfect construction process. Stairway maintenance could also be a reason as the concrete erodes over time. When the out-of-tolerance condition happens, it is expensive to perform corrective actions since the modification of one tread elevation will influence the adjacent riser heights. The authors also point out that no studies are showing that the adjacent risers with a 3/16-in difference in height are less likely to cause a trip-and-fall accident than the adjacent risers with a 1/4-in difference or more. Therefore, the requirements of adjacent riser height and maximum-minus-minimum difference are not very practical to apply in the construction process.

The article *Effect of Staircase on The Seismic Behavior of RC Moment Frame Buildings* published by Azadeh Noorifard and Mohammad Reza Tabeshpour on July 19th, 2018, studied the effect of the staircase on seismic behavior of the structure. Although the professors who put effort into this study were not applying American building codes (instead they applied Australian Standard, Nepal National Building code, etc.), the idea behind this study is universal for different regions with different regulations. The authors described the effect of the stairway on seismic behavior of the structure in three aspects: increase in stiffness, changes in stiffness distribution, and changes in force distribution. Diving into more details about the effects. An increase in stiffness often represented as the reduction of the natural period and the reduction of lateral displacement. Changes in stiffness distribution lead to the geometry of the architectural and structural plan, the inclined slab of a staircase, infills around the stairs, number of structural bays, the dimension of the staircase frame, and the location of the staircase. Changes in force distribution mean the increase in internal forces of landing frame, reduction of internal effects of other structures, and short column. For stiffness, researchers selected a southern building with a width of 6 meters under the influences of bare frame, frame with staircase slabs, and frame with stringer beams, to study the impact on the reduction of the period and drift ratio of structure. The results showed an essential change in stiffness, drift ratio, as well as displace and period for the frame with staircase slabs since it works as a K-shaped bracing in longitude direction and as a delinked shear wall in the transverse direction of the staircase. As for stringer beams only performing as bracing in longitude direction thus its effect in the transverse direction is barely considerable. For distribution of stiffness, a fire stopping wall built around the staircase performs like an infilled wall against lateral forces. Therefore, it rearranges the stiffness focus in the structure. The researchers compared the results of period and mode shape, eccentricity, and the ratio of the maximum relative story drift to the average relative story drift in story three of the building under the two circumstances that either locating the staircase at the middle of the corner. Furthermore, more groups of modeling were studied by the researchers as they continued to compare the shear distribution and moment distribution under the seismic force. The reason why they could manage to do that was that internal force fundamentally changed under the influence of the increase of stiffness.

When comparing reinforced concrete with other structural materials, one topic is inevitable: fire resistance. According to Table 2.3 and Table 2.4, reinforced concrete tends to have 1 to 4 hours of fire-resistance rating depending on the different aggregate types, concrete covers, and beam widths. Since IBC requires at least a 1-hour fire-resistance rating for any connecting interior exit stairway, the reinforced concrete is an excellent material to satisfy that requirement. In order to study the size effect on fire performance of structural concrete, research by Dronnadula V. Reddy, Khaled Sobhan, Lixian Liu, and Jody D. Young Jr. (2015) concentrated on the size effect on the fire performance of axially loaded square RC columns and supported reinforced beams. The connection between their focuses and stairway is that the stairway could be treated as a beam since it is spanned toward horizontally, as mentioned in Section 2.2.3 for this report. The scope of their study includes the relationship between the fire endurance of RC beams and the cross-sectional size and the relationship between the fire endurance of RC beams and the concrete cover thickness. By comparing the curves developed by numerical modeling, the outcomes show that the fire endurance of RC beams increases slightly with the increase of cross-sectional width, and almost stays the same with the increase of cross-sectional depth. And, the fire resistance of RC beams improves significantly by properly increasing the concrete cover thickness. However, it is not practical to exorbitantly increase the concrete cover thickness to maximize the fire resistance of RC beams, as the authors clarified.

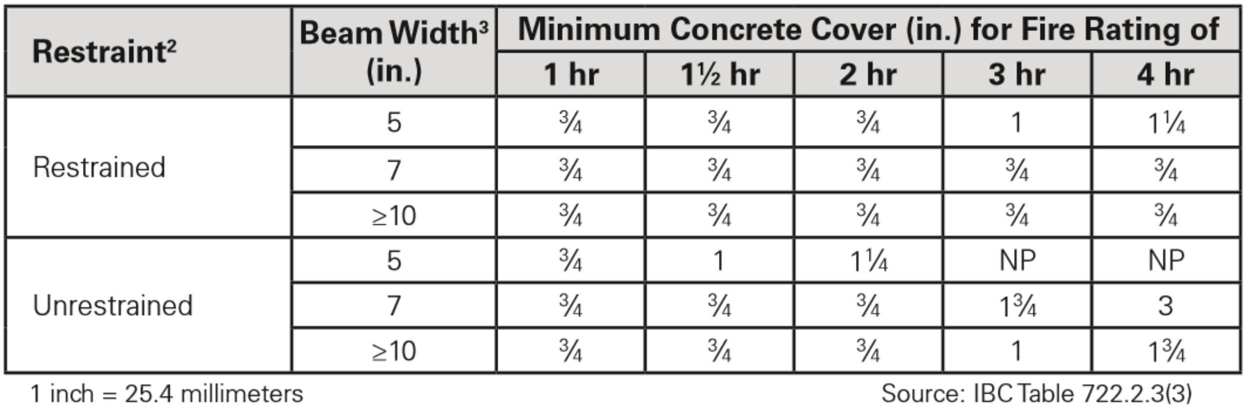
Table 2.3 - Minimum concrete cover for reinforced concrete floor and roof slabs



Aggregate Type	Minimum Concrete Cover (in.) for Fire Rating of					
	Restrained	Unrestrained				
	4 hr or less	1 hr	1½ hr	2 hr	3 hr	4 hr
Siliceous	¾	¾	¾	1	1¼	1⅝
Carbonate	¾	¾	¾	¾	1¼	1¼
Sand-Lightweight or Lightweight	¾	¾	¾	¾	1¼	1¼

1 inch = 25.4 millimeters Source: IBC Table 722.2.3(1)

Table 2.4 - Minimum concrete cover for reinforced concrete beams



Restraint²	Beam Width³ (in.)	Minimum Concrete Cover (in.) for Fire Rating of				
		1 hr	1½ hr	2 hr	3 hr	4 hr
Restrained	5	¾	¾	¾	1	1¼
	7	¾	¾	¾	¾	¾
	≥10	¾	¾	¾	¾	¾
Unrestrained	5	¾	1	1¼	NP	NP
	7	¾	¾	¾	1¾	3
	≥10	¾	¾	¾	1	1¾

1 inch = 25.4 millimeters Source: IBC Table 722.2.3(3)

Eurocode 2 and United Kingdom practices are used as a reference at the preparation stage of this research. Accordingly, the concrete Grade is 30/37 MPa (cylinder strength/cube strength) with a maximum aggregate of 20 mm (0.79-inch) for the general staircase uses. For the staircase in internal use, the nominal cover is 15 mm (0.59-inch) or bar size, plus Δ_{Cdev} (an allowance in design for deviation), which is either 5 mm (0.20-inch) or 10 mm (0.39-inch), determined by a contractor. Compared with the USA standard, the specified concrete cover for members not exposed to weather is either 0.75-inch or 1.5-inch, depending on member difference and reinforcement sizes and type of element (Table 20.6.1.3.1, ACI 318-19). The values of the concrete cover tend to be close. Furthermore, the formula of minimum area of tension reinforcement recommended by Eurocode 2 is:

$$A_{s,min} = 0.26 b_t d f_{ctm}/f_{yk} \geq 0.0013 b_t d$$

(Eq. 9.1N, Eurocode 2)

For concrete Grade 30/37 MPa and $f_{yk} = 500$ MPa, the formula should be adjusted as:

$$A_{s,min} = 0.0015 b_t d$$

Similarly, ACI standard recommends the $A_{s,min}$ to be $0.0014A_g$ (if it governs) with specified yield strength equal to or greater than 60,000 psi (414 MPa). However, the differences between the European standard and the USA standard are still excessive in terms of requirements of staircase dimensions, building load determinations, bar spacing, and so forth. Therefore, the parametric study does not use the Eurocode 2 as a reference to avoid excessive diversities.

Chapter 3 - Research Methodology

This chapter indicates the research methodology applied to this research. It describes the methodological approach, method of data collection, and method of data analysis.

3.1 - Methodological Approach

The core of this report is to provide a detailed guideline of reinforced concrete stairway design. Previously, countless research papers about the reinforced concrete staircase, in terms of properties, functioning, aesthetic aspect, etc. have occurred. The world has been designing concrete stairs for hundreds of years (The Ingalls Building, built in 1903). The design method of the reinforced concrete stairway is based on the existing standards and codes (IBC 2018, OSHA 2014, ASCE/SEI 7-16, and ACI 318-19). Fundamentally, the proposed research topic appears as a form of research but also based on an existing research subject. Therefore, the quantitative method was held for this study to explore the reinforcement design for the concrete stairway under different conditions.

3.2 - Data Collection Method

The existing data method was applied for this research since most of the information was collected from the current regulations or literature. Moreover, a parametric study was performed to analyze the difference between reinforcement designed by the one-way slab method and the beam method. The parameters include the reinforcement design method, geographic location, slab thickness, and boundary condition. More of this study is discussed in Chapter 4 Parametric Study.

3.3 - Data Analysis Method

The parametric study focuses on the cause-and-effect relationships between those parameters and the resulted reinforcement. Excel and RISA 3D were applied to perform the calculations of the study. As the results come out, an analysis targeting the number of layers of reinforcement, the relationship between required flexural reinforcement and actual flexural reinforced applied, governing minimum flexural reinforcement, and the changing tendency of reinforcement as slab thickness increases will be performed.

Chapter 4 - Parametric Study

This parametric study seeks to compare the difference of reinforcement for a concrete stairway designed under different conditions. The parameters include the reinforcement design method referred to ACI 318-19 (one-way slab, or beam), geographic location (high seismic zone and low seismic zone), slab thickness, and boundary condition. Consequently, 24 design cases are examined for this study. The design methods applied for this study are based on Chapter 7 and Chapter 9 of ACI 318-19. Appendix A and B of this report elucidate the design process for each case.

4.1 - Design Information

According to Figure 4.1, the parametric study emphasizes the design of an interior, 180-degree return stair made of reinforced concrete. The stairway is longitudinally spanning, with one or more supports at each end of the stair structure. The selected building locations include Manhattan, Kansas, and Los Angeles, California.

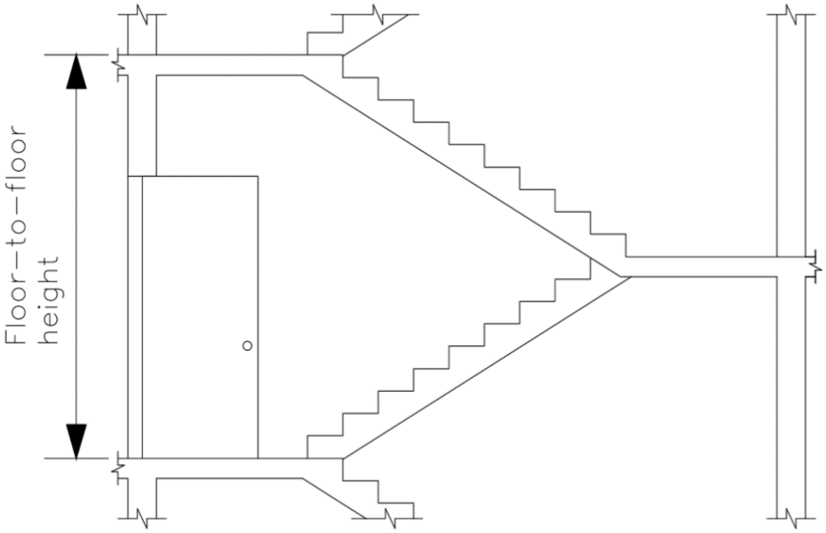


Figure 4.1 - 180-degree return stair

4.1.1 - Assumptions

(1) Longitudinally spanning stairway, where the torsional effect is neglected

(2) No slope for the step nosing, thereby treating the step as a right triangle

(3) Non-prestressed concrete

(4) Normal weight concrete, with specified compressive strength equals to 4,000 psi

(5) ASTM A615 Grade 60 reinforcement, with specified yield strength for non-prestressed reinforcement equals to 60,000 psi

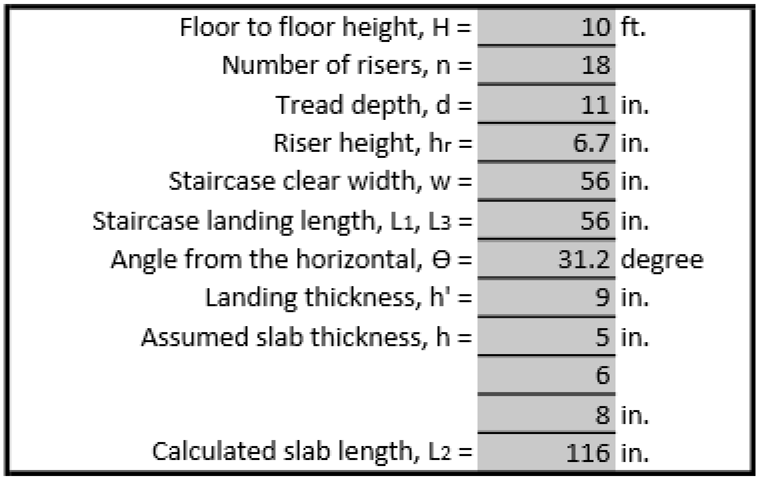
(6) Assume stair slab thickness to be 5-inch, 6-inch, and 8-inch

(7) No axial force acting on the structure

4.1.2 - Dimension

The dimensions of the stairway are shown in Table 4.1, Figure 4.2, and Figure 4.3.

Table 4.1 - Stairway dimensions



Floor to floor height, H =	10	ft.
Number of risers, n =	18	
Tread depth, d =	11	in.
Riser height, h_r =	6.7	in.
Staircase clear width, w =	56	in.
Staircase landing length, L_1, L_3 =	56	in.
Angle from the horizontal, ϴ =	31.2	degree
Landing thickness, h' =	9	in.
Assumed slab thickness, h =	5	in.
	6	
	8	in.
Calculated slab length, L_2 =	116	in.

As discussed in Chapter 2, Section 2.2.1, the assumed dimensions of the clear width, tread depth, and riser height satisfy with the IBC 2018. The height of one stair flight is 5 feet since the floor to floor height is 10 foot for the entire return stair (The Architect's Studio Companion, Six

Edition). In order to simplify the volume calculation for later building load calculations, the shape of the stair-step was set up to be a perfect triangle. Therefore, the angle from the horizontal turns out to be 31.2 degrees. In the end, the length of the stair slab was calculated to be 116 inches.

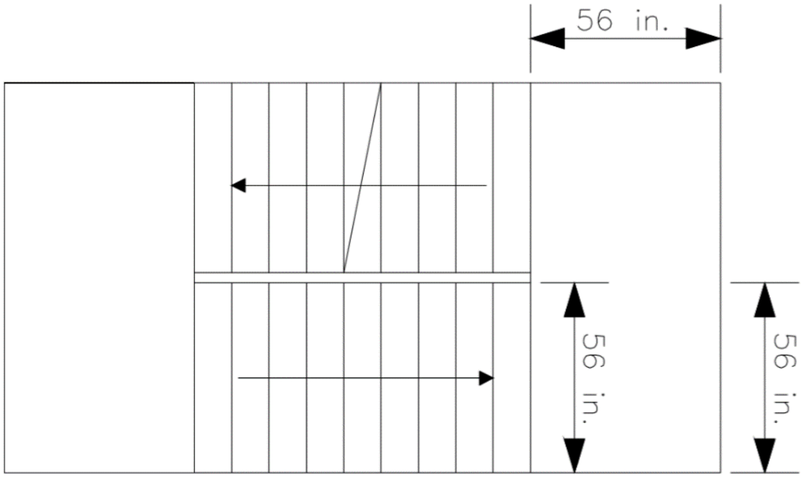


Figure 4.2 - Stair clear width, landing width/length

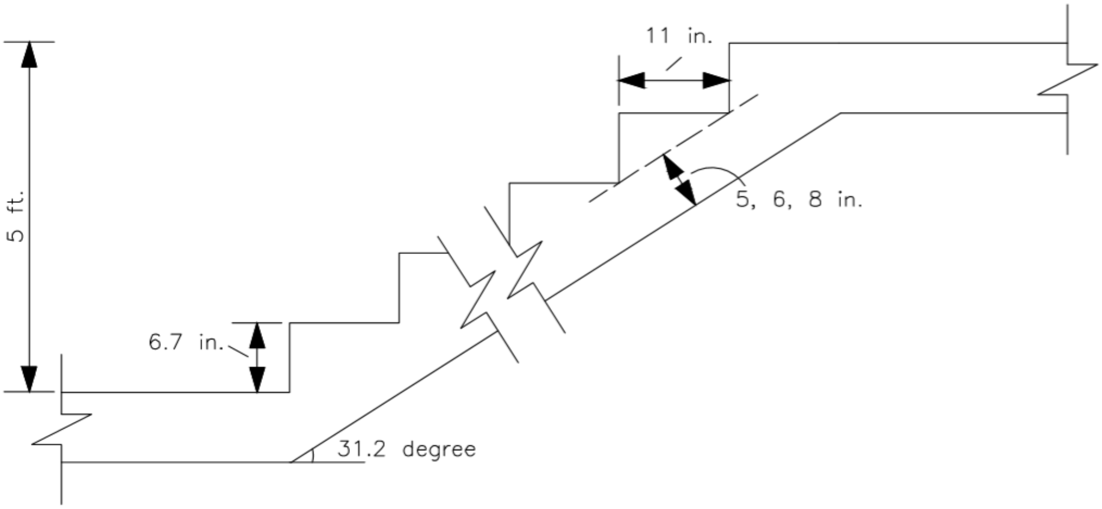


Figure 4.3 - Riser height, tread depth, slab thickness, angle

4.1.3 - Building Loads

As discussed in Chapter 2, Section 2.2.2, building loads of this project include vertical and horizontal loads. The majority of the vertical loads that the stairway experiences is its self-weight; at the same time, live load stands out for stairway since it serves as the egress exit for people to evacuate in case of a natural disaster or a life-critical emergency. Due to the precondition that this stairway is inside of a building, this study neglects the effect of rain load, snow load, and ice load. The wind pressure was not taken into consideration except the minimum internal wind force 16 $\mathrm{lb/ft^2}$ (ASCE/SEI 7-16, Section 29.8), for conservative design. The effect of seismic force shall be considered; however, it depends on the region where the structure is located, as it will influence the load combination. All building loads are determined in accordance with *Minimum Design Loads for Buildings and Other Structures* (ASCE/SEI 7-16).

4.1.4 - Load Combinations

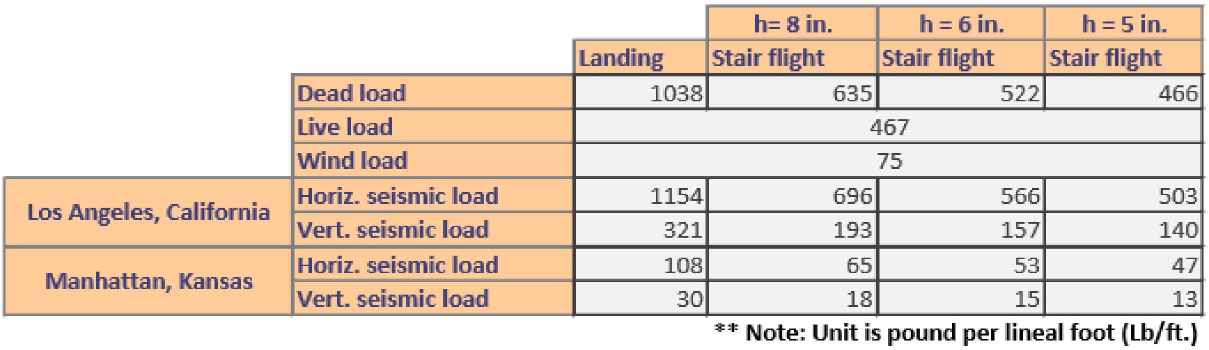
The load combinations applied to this parametric study were found in Section 2.3.2, Chapter 2 of ASCE/SEI 7-16, as well as in Table 5.3.1 of ACI 318-19. The list of load strength combinations shown as follows:

1. $1.4D$
2. $1.2D + 1.6L + 0.5(L_r \ or \ S \ or \ R)$
3. $1.2D + 1.6(L_r \ or \ S \ or \ R) + (L \ or \ 0.5W)$
4. $1.2D + 1.0W + L + 0.5(L_r \ or \ S \ or \ R)$
5. $1.2D + 1.0E + L + 0.2S$
6. $0.9D + 1.0W$

7. $0.9D + 1.0E$

Based on the significant impact that dead and live loads make for this study, load combination 2 will govern over the other combinations for the low seismic activity region. When taking seismic effect into considerations for the high seismic activity region, load combination 5 will govern as it highlights the influence of seismic force; meanwhile, the rest of the force remains the same. The effect of wind force is relatively low since only the minimum internal wind pressure was considered. Table 4.2 demonstrates the governing load combinations by showing the difference of seismic force impact between Los Angeles, California, and Manhattan, Kansas.

Table 4.2 - Building loads, from calculations in Appendix A



			h= 8 in.	h = 6 in.	h = 5 in.
		Landing	Stair flight	Stair flight	Stair flight
	Dead load	1038	635	522	466
	Live load	467			
	Wind load	75			
Los Angeles, California	Horiz. seismic load	1154	696	566	503
	Vert. seismic load	321	193	157	140
Manhattan, Kansas	Horiz. seismic load	108	65	53	47
	Vert. seismic load	30	18	15	13

** Note: Unit is pound per lineal foot (Lb/ft.)

4.2 - Parameters

The parameters of this study consist of the reinforcement design method, geographic location, slab thickness, and boundary condition. As shown in Figure 4.4, each of the parameters is labeled with a letter or number to distinguish from others. For the reinforcement design method, capital letter S means the one-way slab method, and capital letter B means the beam method. For the geographic location, a combined capital letter KS stands for Manhattan Kansas, and a combined capital letter CA stands for Los Angeles California. For slab thickness of stair, number 5 means 5-inch, figure 6 means 6-inch, and figure 8 means 8-inch. For boundary conditions, the lower-case letter i means the stairway is supported by four beams, and the lower-case letter ii means the stairway is supported by two beams. As a result, a total of 24 cases have been established for this study.

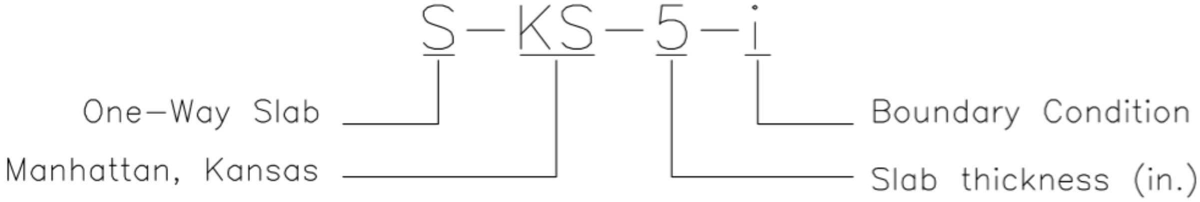


Figure 4.4 - Notations

4.2.1 - Reinforcement Design Method

As discussed in Chapter 2, two methods adopted for this study are: concrete one-way slab and concrete beam methods. For each method, there are six factors that differentiate from each other in terms of different equations applied under different conditions. These six factors include slab thickness, clear cover, flexural reinforcement, shear reinforcement, torsional reinforcement, and temperature reinforcement.

Slab thickness refers to the thickness of the stair stringer without counting treads. Table 4.3 and Table 4.4 (Table 7.3.1.1 and Table 9.3.1.1 ACI 318-19) present the minimum thickness of solid non-prestressed one-way slabs and beams, applicable for normal weight concrete and $f_y = 60{,}000$ psi. For this study, the values of slab thickness were selected to be 5-inch, 6-inch, and 8-inch.

Table 4.3 - Minimum thickness of solid non-prestressed one-way slabs
(Table 7.3.1.1 ACI 318-19)

Support condition	Minimum h
Simply supported	$l/20$
One end continuous	$l/24$
Both ends continuous	$l/28$
Cantilever	$l/10$

Table 4.4 - Minimum depth of non-prestressed beams
(Table 9.3.1.1 ACI 318-19)

Support condition	Minimum h
Simply supported	$l/16$
One end continuous	$l/18.5$
Both ends continuous	$l/21$
Cantilever	$l/8$

Concrete cover refers to the distance between the outermost surface of reinforcement and the outer surface of the concrete. The primary role that concrete cover plays is to protect the reinforcement from weather, such as moisture change and temperature change (ACI 318-19, R20.6.1.1). According to ACI 318-19, Table 20.6.1.3.1, the value of specified concrete cover shall be determined as follows:

Table 4.5 - Specified concrete cover for non-prestressed concrete members (Table 20.6.1.3.1 ACI 318-19)

Concrete exposure	Member	Reinforcement	Specified cover, in.
Cast against and permanently in contact with ground	All	All	3
Exposed to weather or in contact with ground	All	No. 6 through No. 18 bars	2
		No. 5 bar, W31 or D31 wire, and smaller	1-1/2
Not exposed to weather or in contact with ground	Slabs, joists, and walls	No. 14 and No. 18 bars	1-1/2
		No. 11 bars and smaller	3/4
	Beams, columns, pedestals, and tension ties	Primary reinforcement, stirrups, ties, spirals, and hoops	1-1/2

For this study, apply 3/4 in. for one-way slabs, and 1-1/2 in. for beams since the stairway is inside of the building, not exposed to outside weather or in contact with the ground. The difference of clear cover could also extend to the difference of the distance from extreme compression fiber to centroid of longitudinal tension reinforcement, d. As for one-way slabs, the value of d is determined by $d = h - clear\ cover - 1/2(dia)$, which means to subtract slab thickness by clear cover and half of the diameter of longitudinal tension reinforcement. For beams, the value of d is determined by $d = h - clear\ cover - 1/2\ (dia) - (dia')$, which means to subtract beam depth by clear cover, half of the diameter of longitudinal tension reinforcement, and diameter of the shear stirrup. With the same value of h, the value of d for the beam is less than the one-way slab. The lesser the distance from extreme compression fiber to centroid of longitudinal tension reinforcement, the more the required tension reinforcement is necessary by Eq. 4.2 and 4.3.

Flexural reinforcement refers to the main reinforcement focusing on resisting bending moments caused by factored loads. The essential requirement for flexural strength design is to make sure that $M_u \leq \emptyset M_n$. The value of $\emptyset$ is specified as 0.9 by Table 21.2.2 ACI 318-19 since the focus of this study is reinforcement in the tension-controlled region. The equations applied to determine the value of the required tension reinforcement ratio are listed as follows:

$$m = \frac{f_y}{0.85 f_c'}$$

(Eq. 4.1)

$$R_u = \frac{M_u}{\emptyset b d^2}$$

(Eq. 4.2)

$$\int_{rqd} = \frac{1}{m}(1 - \sqrt{1 - \frac{2R_u m}{f_y}})$$

(Eq. 4.3)

Appendix D provides the derivation process of these equations. Moreover, ACI 318-19 requires for minimum flexural reinforcement check shown as follows:

Table 4.6 - Minimum flexural reinforcement, $A_{s,min}$ for non-prestressed one-way slabs (Table 7.6.1.1 ACI 318-19)

Reinforcement Type	f_y, **psi**	$A_{s,min}$	
Deformed bars	$< 60{,}000$	$0.0020A_g$	
Deformed bars or welded wire reinforcement	$\geq 60{,}000$	Greater of:	$\frac{0.0018 \times 60.000}{f_y}A_g$
			$0.0014A_g$

Table 4.7 - Minimum flexural reinforcement, $A_{s,min}$ for non-prestressed beams (Section 9.6.1.2 ACI 318-19)

	$A_{s,min}$
Greater of:	$\frac{3\sqrt{f_c'}}{f_y}b_w d$
	$\frac{200}{f_y}b_w d$

Shear reinforcement is designed to resist shear force. Usually, shear reinforcement is provided in the form of stirrups to hold the longitudinal reinforcement together. For one-way slabs, minimum shear reinforcement shall be granted if $V_u > \emptyset V_c$ (ACI 318-19, Section 7.6.3.1). For beams, minimum shear reinforcement shall be provided if $V_u > 0.5\emptyset V_c$ (ACI 318-19, Section 9.6.3.1). The value of $\emptyset$ is specified as 0.75 by Table 21.2.1 ACI 318-19. Since this study limits the effect of torsional effect, the value of minimum shear reinforcement could be determined in accordance with Table 9.6.3.3 ACI 318-19, shown as follows:

Table 4.8 - Minimum shear reinforcement
(Table 9.6.3.3 ACI 318-19)

Beam type	$\boldsymbol{A_{v,min}/s}$	
Non-prestressed	Greater of:	$0.75\sqrt{f'_c}\,\dfrac{b_w}{f_{yt}}$
		$50\,\dfrac{b_w}{f_{yt}}$

Torsional effect is neglected for this study to simplify the calculations and focus on the comparison of tension reinforcement. However, torsional effect should be considered for future research targeted in comprehensive interaction of shear and torsional reinforcement for the reinforced concrete stairway.

As discussed in Chapter 2, Section 2.2.3, temperature reinforcement resists cracking caused by temperature and moisture changes in slabs. Table 24.4.3.2 ACI 318-19 provided the check for minimum shrinkage and temperature reinforcement shown as follows:

Table 4.9 - Minimum shrinkage and temperature reinforcement ratio
(Table 24.4.3.2 ACI 318-19)

Reinforcement type	$\boldsymbol{f_y}$**, psi**	**Minimum reinforcement ratio**	
Deformed bars	$< 60{,}000$	0.0020	
Deformed bars or welded wire reinforcement	$\geq 60{,}000$	Greater of:	$\dfrac{0.0018 \times 60{,}000}{f_y}$
			0.0014

For this study, temperature reinforcement for one-way slabs will be using #4 reinforcing rebar per 12 inches, as presented in calculations shown in Appendix A. Although temperature reinforcement is not required for beams by ACI, it is always feasible to add extra reinforcement if temperature and moisture changes are critical for the concrete member.

4.2.2 - Geographic Location

This study selects Manhattan, Kansas and Los Angeles, California as project locations based on their different seismic activity levels. As discussed in Chapter 2, Section 2.2.2.3, the determination of horizontal and vertical seismic force is provided by Chapter 13 ASCE/SEI 7-16 through Eq. 13.3-1 to Eq. 13.3-3 and Section 13.3.1. With the same equations applied, the main factor that differentiates the results calculated by the two locations is the spectral acceleration at a short period, S_{DS}. With the help of the OSHPD Seismic Design Map, developed by Structural Engineers Association of California's (SEAOC) and California's Office of Statewide Health Planning and Development (OSHPD), under the assumptions of Risk Category II and various soil conditions, the value of S_{DS} is 0.148 for Manhattan Kansas and is 1.579 for Los Angeles California. One thing that should be taken with caution is that despite the calculation of the values of S_{DS} provided by the OSHPD website are based on ASCE/SEI 7-16, no specific real-life examination is performed to verify their accuracy, according to the website's disclaimer. Predictably, load combination 5 will surpass load combination 2 as a better governing case when more seismic effect gets involved. Consequently, it will demand more reinforcement for the design.

4.2.3 - Slab Thickness

Slab thickness, h, is limited to 5-inch, 6-inch, and 8-inch for this study. There are two reasons behind this limitation: (1) deflection check; (2) number of reinforcement layers. According to Section 7.3.2.2 and 9.3.2.2 from ACI 318-19, deflection check shall be performed if the value of slab thickness is below the range provided by Table 7.3.1.1 and Table 9.3.1.1 ACI 318-19. It is necessary to have slab thicknesses available for situations whether a deflection check is required or not. Otherwise, the compassion of the results would be one-sided. Furthermore, the number of reinforcement layers could also be determined by the slab thickness, not just the bending moment. For this study, some of the cases show that a member is experiencing both positive and negative bending moments. Consequently, it will form two layers of tension reinforcement for the design. However, the spacing between reinforcement placed in two or more layers has been specified as at least 1-inch by Section 25.2.2 ACI 318-19. As discussed in Section 4.2.1, with the same h, the remaining space of d for stairways designed by beam approach is less than the ones designed by a one-way slab approach because of the more extensive clear cover and additional space taken by shear stirrups. With stairways of 5 inches thickness and designed by beam approach, it is unlikely to have more than one layer of reinforcement. As for those designed by a one-way slab approach, two layers of reinforcement are common.

4.2.4 - Boundary Condition

This study includes two boundary conditions, shown in Figure 4.5 and Figure 4.6. Boundary condition (i) produces less bending moment and shear strength than boundary condition (ii) because the main body (stair) of the former is both ends continuous, of the latter is simply supported. All bending moments and shear force are determined by RISA 3D. The results are shown in Table 4.10:

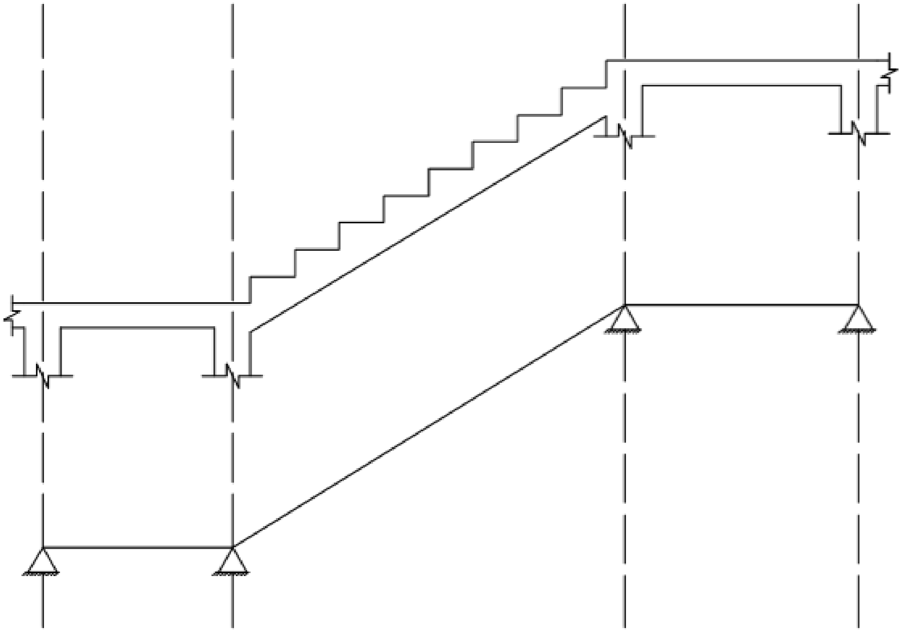


Figure 4.5 - Boundary condition i

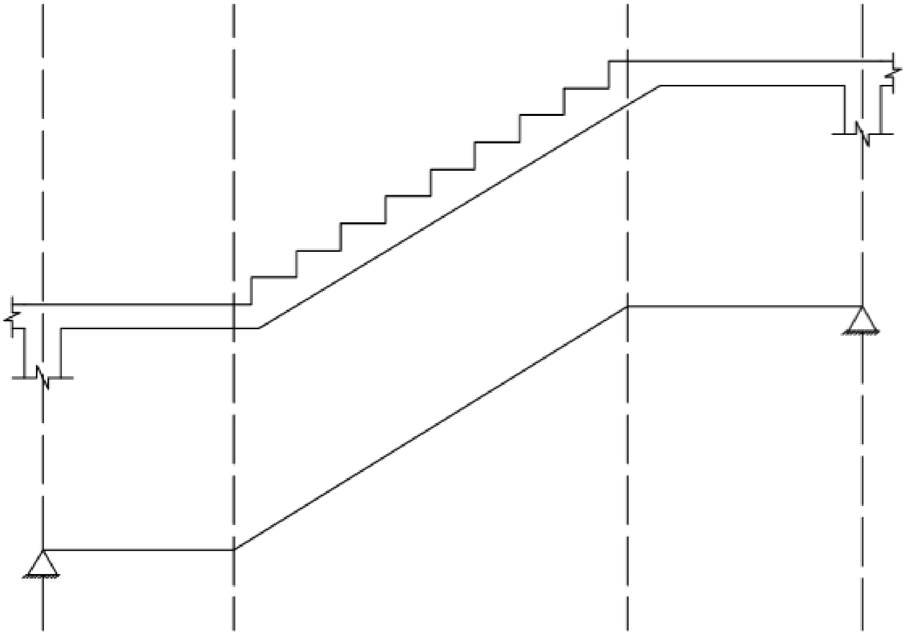
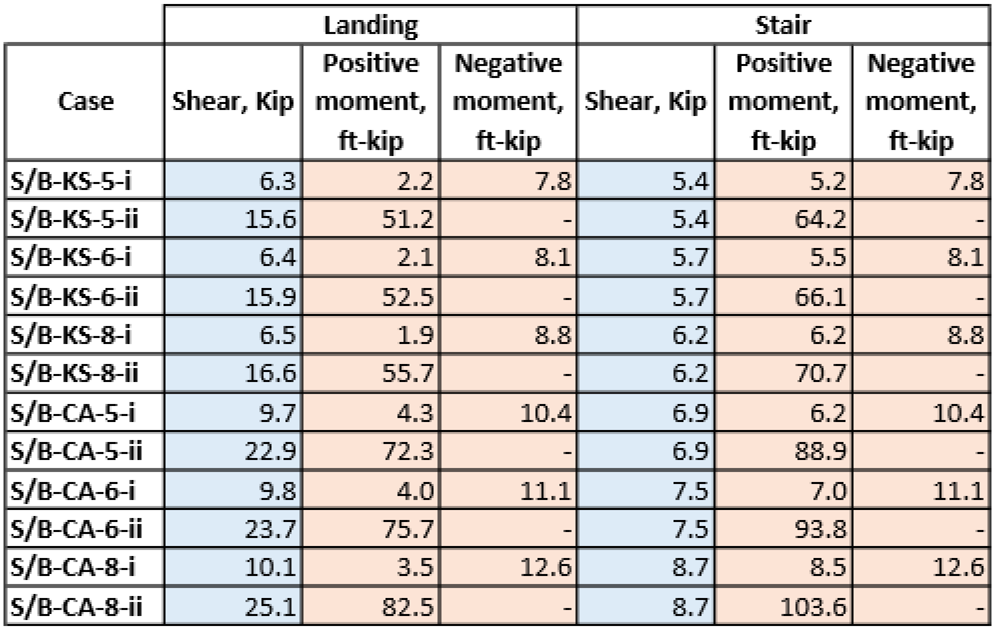


Figure 4.6 - Boundary condition ii

Table 4.10 - RISA results of shear and moment



	Landing			Stair		
Case	Shear, Kip	Positive moment, ft-kip	Negative moment, ft-kip	Shear, Kip	Positive moment, ft-kip	Negative moment, ft-kip
S/B-KS-5-i	6.3	2.2	7.8	5.4	5.2	7.8
S/B-KS-5-ii	15.6	51.2	-	5.4	64.2	-
S/B-KS-6-i	6.4	2.1	8.1	5.7	5.5	8.1
S/B-KS-6-ii	15.9	52.5	-	5.7	66.1	-
S/B-KS-8-i	6.5	1.9	8.8	6.2	6.2	8.8
S/B-KS-8-ii	16.6	55.7	-	6.2	70.7	-
S/B-CA-5-i	9.7	4.3	10.4	6.9	6.2	10.4
S/B-CA-5-ii	22.9	72.3	-	6.9	88.9	-
S/B-CA-6-i	9.8	4.0	11.1	7.5	7.0	11.1
S/B-CA-6-ii	23.7	75.7	-	7.5	93.8	-
S/B-CA-8-i	10.1	3.5	12.6	8.7	8.5	12.6
S/B-CA-8-ii	25.1	82.5	-	8.7	103.6	-

Table 4.10 illustrates that stairs and landings in boundary condition (i) experience bending moments with both signs; at the same time, the stairs and landings in boundary condition (ii) only experience a positive bending moment, but usually with a larger scale. For example, Figure 4.7 and Figure 4.8 indicate the different behaviors of bending moments for 5-inch slab thickness, located in Kansas, under different boundary conditions (i) and (ii).

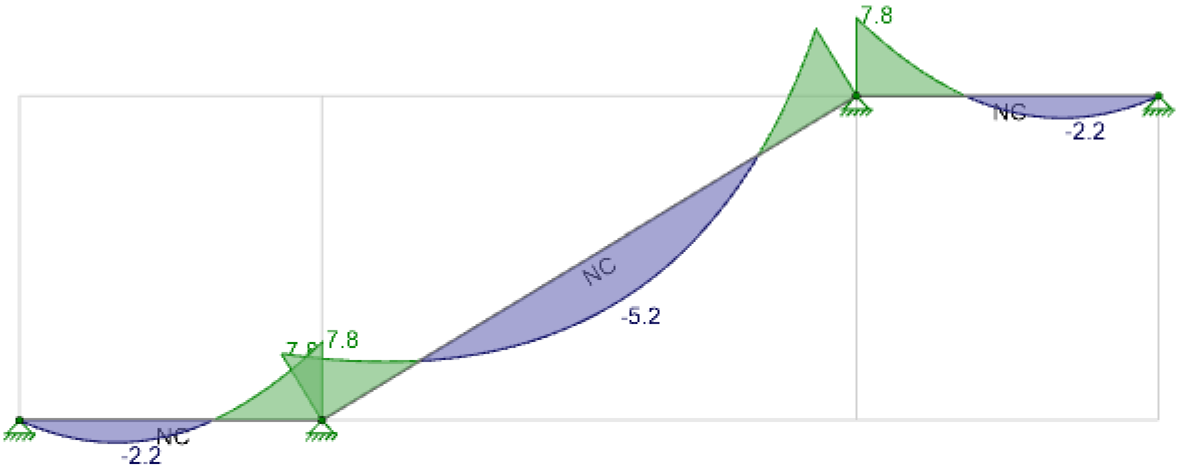


Figure 4.7 - RISA moment diagram of K-5-i (unit: ft-kips)

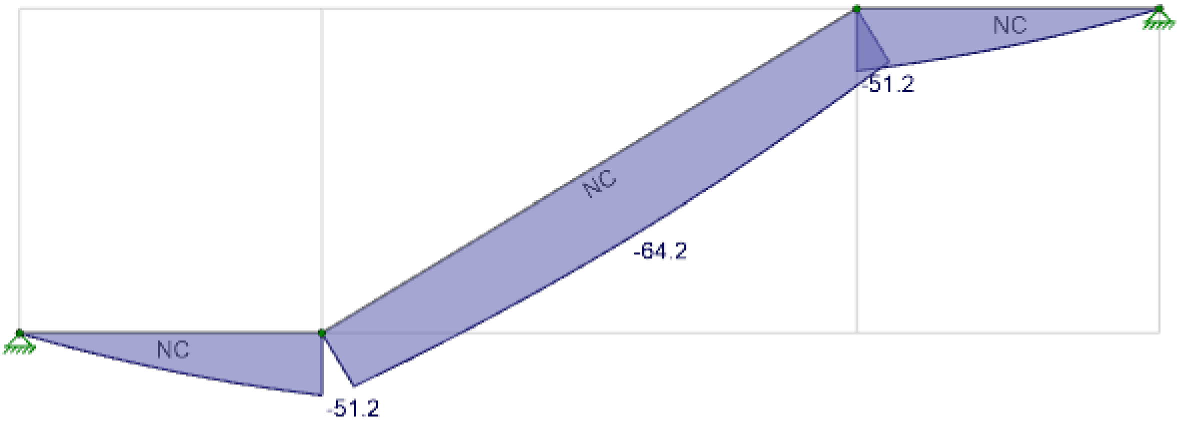
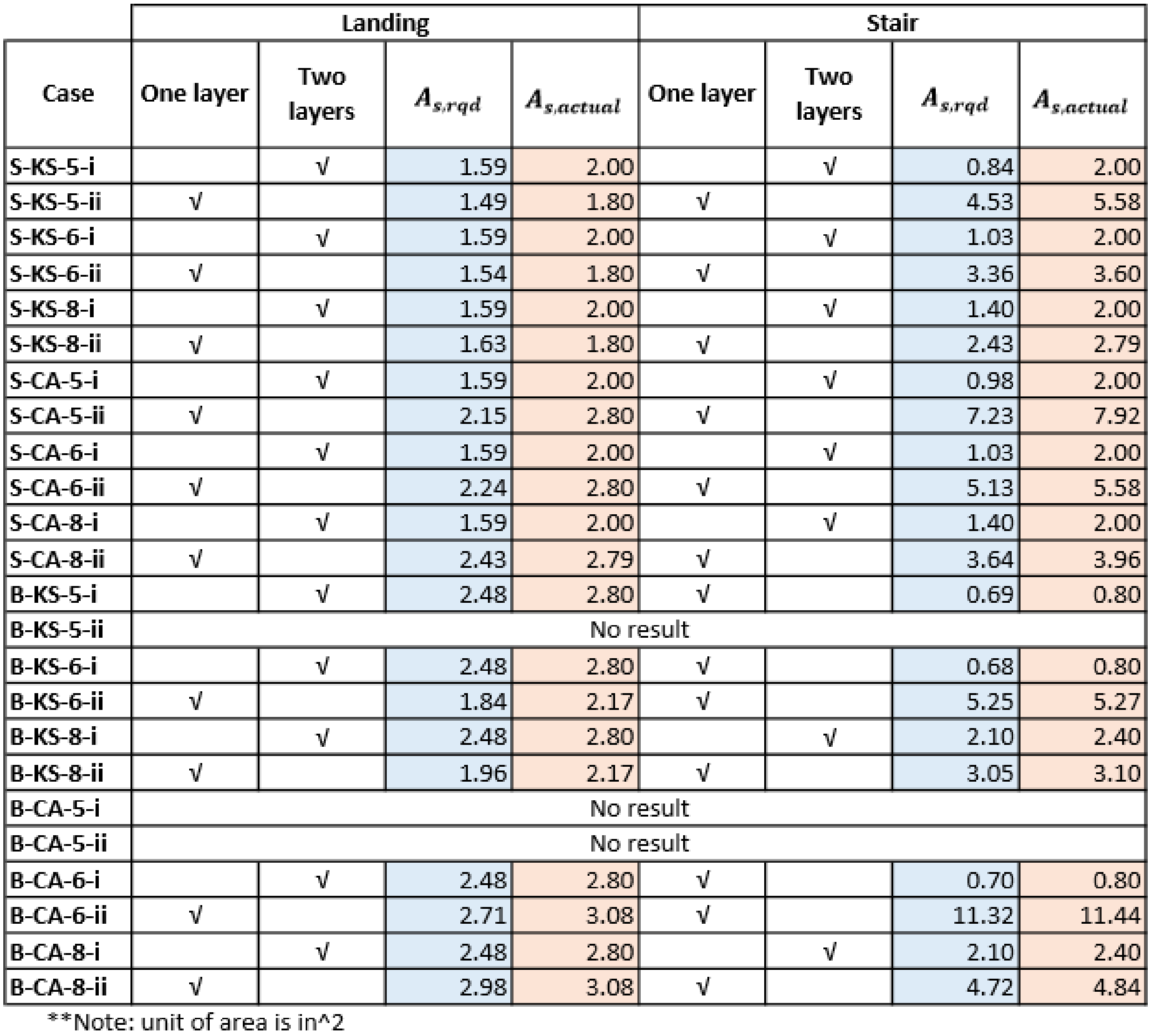


Figure 4.8 - RISA moment diagram of K-5-ii (unit: ft-kips)

4.3 - Results

Each case generates a required tension reinforcement area ($A_{s,rqd}$) determined by equations and standard requirement, and an actual tension reinforcement area ($A_{s,actual}$) determined by the actual size and number of reinforcing rebars inside the member. The rebar sizes are limited at a range of #4, #5, and #6 for both main and transverse reinforcement to minimize the impact of excessive rebar area. As mentioned previously, members experiencing both positive and negative bending moments might have two layers of tension reinforcement, depending on the thickness of its slab thickness/beam depth. For landing, the need for two layers of reinforcement is larger than stairs because of the depth of landing is limited to 9-inch, shown in Table 4.11.

Table 4.11 Summary of tension reinforcement for stairway



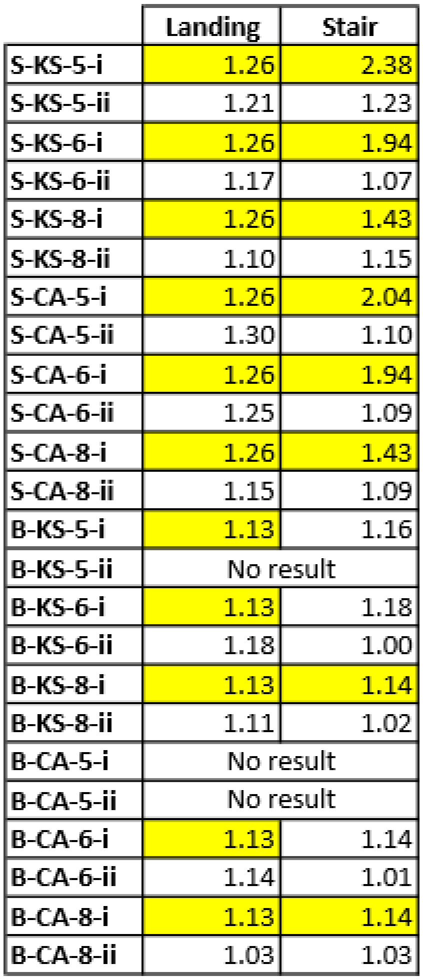
	Landing				Stair			
Case	One layer	Two layers	$A_{s,rqd}$	$A_{s,actual}$	One layer	Two layers	$A_{s,rqd}$	$A_{s,actual}$
S-KS-5-i		√	1.59	2.00		√	0.84	2.00
S-KS-5-ii	√		1.49	1.80	√		4.53	5.58
S-KS-6-i		√	1.59	2.00		√	1.03	2.00
S-KS-6-ii	√		1.54	1.80	√		3.36	3.60
S-KS-8-i		√	1.59	2.00		√	1.40	2.00
S-KS-8-ii	√		1.63	1.80	√		2.43	2.79
S-CA-5-i		√	1.59	2.00		√	0.98	2.00
S-CA-5-ii	√		2.15	2.80	√		7.23	7.92
S-CA-6-i		√	1.59	2.00		√	1.03	2.00
S-CA-6-ii	√		2.24	2.80	√		5.13	5.58
S-CA-8-i		√	1.59	2.00		√	1.40	2.00
S-CA-8-ii	√		2.43	2.79	√		3.64	3.96
B-KS-5-i		√	2.48	2.80	√		0.69	0.80
B-KS-5-ii	No result							
B-KS-6-i		√	2.48	2.80	√		0.68	0.80
B-KS-6-ii	√		1.84	2.17	√		5.25	5.27
B-KS-8-i		√	2.48	2.80		√	2.10	2.40
B-KS-8-ii	√		1.96	2.17	√		3.05	3.10
B-CA-5-i	No result							
B-CA-5-ii	No result							
B-CA-6-i		√	2.48	2.80	√		0.70	0.80
B-CA-6-ii	√		2.71	3.08	√		11.32	11.44
B-CA-8-i		√	2.48	2.80		√	2.10	2.40
B-CA-8-ii	√		2.98	3.08	√		4.72	4.84

**Note: unit of area is in^2

The Table 4.11 also illustrates the indeterminate result for case B-KS-5-ii, B-CA-5-i, and B-CA-5-ii. The calculations shown in Appendix A suggest the uncertain result is because of either the reinforcement not being in the tension-controlled region or the reinforcement is not adequate in deflection. This phenomenon shows more flexibility of slab thickness selection for a one-way slab approach since none of the indeterminate result happens in this method.

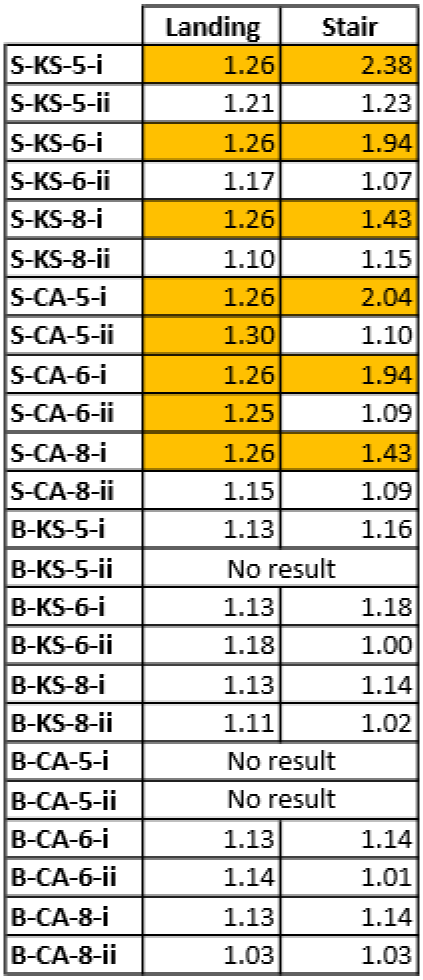
A gap between the required tension reinforcement area and the actual tension reinforcement area occurs because ASTM defines the nominal area for each size of the bar with certainty. In many situations, those two areas do not match perfectly. If the gap is too excessive, the member becomes over-reinforced, concrete crushes prior to steel yielding. The more layers of tension reinforcement a member has, the more likely the chance of over-reinforced a member can experience, despite the determination of tension reinforcement is satisfied with the minimum tension reinforcement are and cracking control spacing with the accordance of ACI 318-19. In Table 4.12, the ratio obtained by dividing the required area from the actual area of steel rebar is indicated. Numbers labeled with yellow color represents the ratios of two-layer reinforcement. Most of them are excessively high, except the two in the beam approach region. The ratio ranges from the lowest 1.13 to the highest 2.38, which means some of the members are more than two times over-reinforced than the required areas. To better show the difference of tension reinforcement area gap between the one-way slab method and beam method, the limit ratio is set up to be 1.25. When a member experiences a ratio of more than 1.25, it is categorized into the group of over-reinforced design. In Table 4.13, ratios beyond 1.25 are labeled with orange color. The statistics show the over-reinforced members are all from the stairways designed by the one-way slab method. Therefore, the beam method is a better way to minimize the impact of excessive tension reinforcement.

Table 4.12 - Ratio of $A_{s,actual}/A_{s,rqd}$, labeled with yellow



	Landing	Stair
S-KS-5-i	1.26	2.38
S-KS-5-ii	1.21	1.23
S-KS-6-i	1.26	1.94
S-KS-6-ii	1.17	1.07
S-KS-8-i	1.26	1.43
S-KS-8-ii	1.10	1.15
S-CA-5-i	1.26	2.04
S-CA-5-ii	1.30	1.10
S-CA-6-i	1.26	1.94
S-CA-6-ii	1.25	1.09
S-CA-8-i	1.26	1.43
S-CA-8-ii	1.15	1.09
B-KS-5-i	1.13	1.16
B-KS-5-ii	No result	
B-KS-6-i	1.13	1.18
B-KS-6-ii	1.18	1.00
B-KS-8-i	1.13	1.14
B-KS-8-ii	1.11	1.02
B-CA-5-i	No result	
B-CA-5-ii	No result	
B-CA-6-i	1.13	1.14
B-CA-6-ii	1.14	1.01
B-CA-8-i	1.13	1.14
B-CA-8-ii	1.03	1.03

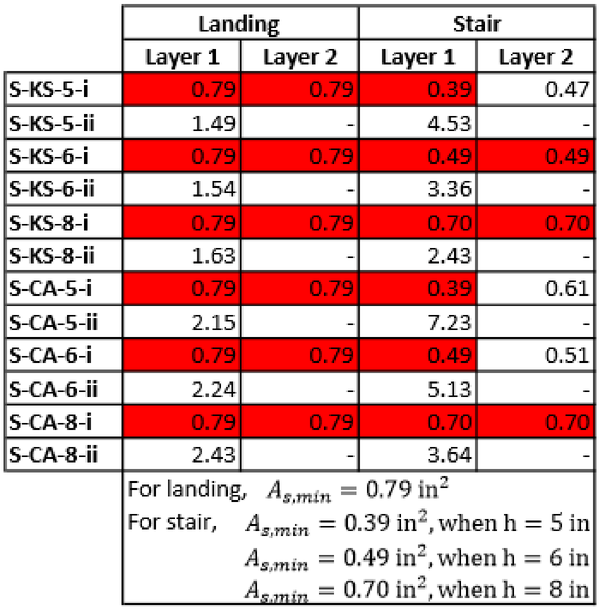
Table 4.13 - Ratio of $A_{s,actual}/A_{s,rqd}$, labeled with orange



	Landing	Stair
S-KS-5-i	1.26	2.38
S-KS-5-ii	1.21	1.23
S-KS-6-i	1.26	1.94
S-KS-6-ii	1.17	1.07
S-KS-8-i	1.26	1.43
S-KS-8-ii	1.10	1.15
S-CA-5-i	1.26	2.04
S-CA-5-ii	1.30	1.10
S-CA-6-i	1.26	1.94
S-CA-6-ii	1.25	1.09
S-CA-8-i	1.26	1.43
S-CA-8-ii	1.15	1.09
B-KS-5-i	1.13	1.16
B-KS-5-ii	No result	
B-KS-6-i	1.13	1.18
B-KS-6-ii	1.18	1.00
B-KS-8-i	1.13	1.14
B-KS-8-ii	1.11	1.02
B-CA-5-i	No result	
B-CA-5-ii	No result	
B-CA-6-i	1.13	1.14
B-CA-6-ii	1.14	1.01
B-CA-8-i	1.13	1.14
B-CA-8-ii	1.03	1.03

The required flexural reinforcement area also depends on the value of the minimum flexural reinforcement area. When comparing them, the greater one governs over another. In Table 4.12, the ratio of $A_{s,actual}/A_{s,rqd}$ presents the same value of 1.26 for the first four cases of landings with two layers of flexural reinforcement under boundary condition (i), designed by the one-way slab method. It indicates the required areas were governed by the minimum areas, thus resulting in the same actual areas for those four cases. The same tendency happens to the first three cases of landings under condition (i) designed by the beam method, and they share an equal value of 1.13 for the ratio. The purpose of minimum flexural reinforcement is to protect the member from failing in a brittle manner at the formation of first flexural cracks (NCHRP Research Report 906, 2019). For this study, more cases governed by minimum flexural reinforcement indicates less diversity of design as the minimum requirement only provides minimum protection, between nominal moment capacity and the cracking moment capacity. In Table 4.14, it shows the amount of the flexural reinforcement governed by minimum flexural reinforcement, marked by red color, in accordance with the one-way slab method. Twenty-one cases have been identified in a total amount of thirty-six cases, resulting in a governing percentage of 58%. In Table 4.15, it depicts the amount of the flexural reinforcement governed by minimum flexural reinforcement, marked by blue color, in accordance with the beam method. Fifteen cases have been governed among a total number of twenty-five cases, resulting in a governing percentage of 60%. However, most of the situations happen at the landing, which its clear depth is limited to 9-inch. That is, the landing experiences less amount of moment at the boundary condition (i), than the boundary condition (ii), but with both positive and negative values, as shown in Table 4.10. If ruling out the cases of landing for a better comparison, then the one-way slab method will have 50% (nine out of eighteen) of governing cases of minimum reinforcement, and the beam method will have 45% (five out of eleven) of governing cases of minimum reinforcement. Therefore, the beam method is more diversified than the one-way slab method in terms of decreasing governing cases of minimum flexural requirement.

Table 4.14 - Cases governed by minimum flexural reinforcement, for one-way slab



	Landing		Stair	
	Layer 1	Layer 2	Layer 1	Layer 2
S-KS-5-i	0.79	0.79	0.39	0.47
S-KS-5-ii	1.49	-	4.53	-
S-KS-6-i	0.79	0.79	0.49	0.49
S-KS-6-ii	1.54	-	3.36	-
S-KS-8-i	0.79	0.79	0.70	0.70
S-KS-8-ii	1.63	-	2.43	-
S-CA-5-i	0.79	0.79	0.39	0.61
S-CA-5-ii	2.15	-	7.23	-
S-CA-6-i	0.79	0.79	0.49	0.51
S-CA-6-ii	2.24	-	5.13	-
S-CA-8-i	0.79	0.79	0.70	0.70
S-CA-8-ii	2.43	-	3.64	-

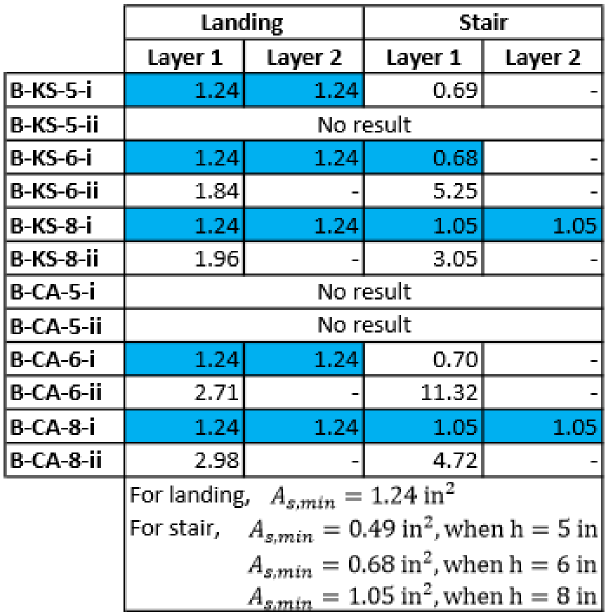
For landing, $A_{s,min} = 0.79 \text{ in}^2$

For stair, $A_{s,min} = 0.39 \text{ in}^2, \text{when h} = 5 \text{ in}$

$A_{s,min} = 0.49 \text{ in}^2, \text{when h} = 6 \text{ in}$

$A_{s,min} = 0.70 \text{ in}^2, \text{when h} = 8 \text{ in}$

Table 4.15 - Cases governed by minimum flexural reinforcement, for beam method



	Landing		Stair	
	Layer 1	Layer 2	Layer 1	Layer 2
B-KS-5-i	1.24	1.24	0.69	-
B-KS-5-ii	No result			
B-KS-6-i	1.24	1.24	0.68	-
B-KS-6-ii	1.84	-	5.25	-
B-KS-8-i	1.24	1.24	1.05	1.05
B-KS-8-ii	1.96	-	3.05	-
B-CA-5-i	No result			
B-CA-5-ii	No result			
B-CA-6-i	1.24	1.24	0.70	-
B-CA-6-ii	2.71	-	11.32	-
B-CA-8-i	1.24	1.24	1.05	1.05
B-CA-8-ii	2.98	-	4.72	-

For landing, $A_{s,min} = 1.24 \text{ in}^2$

For stair, $A_{s,min} = 0.49 \text{ in}^2, \text{when h} = 5 \text{ in}$

$A_{s,min} = 0.68 \text{ in}^2, \text{when h} = 6 \text{ in}$

$A_{s,min} = 1.05 \text{ in}^2, \text{when h} = 8 \text{ in}$

In examining Table 4.13 and Table 4.14, all cases where minimum flexural reinforcement governs over required flexural reinforcement happen in the stairway designed under boundary condition (i), regardless of the difference of design method, geographic location, and slab thickness. The reason behind this trend is that the actual bending moment that the stairway experiences is less than the minimum moment capacity the member required to have by ACI. In Figure 4.9, a gradual rise in required flexural reinforcement area for stair from 5-inch to 8-inch slab thickness is shown, designed by one-way slab method and under boundary condition (i), in both locations. However, for those designed under boundary condition (ii), the curve drops dramatically as the slab thickness goes up. The same tendency happens to stairs designed by beam method, shown in Figure 4.10 below. The drop of curve demonstrates a unique condition that the required flexural reinforcement decreases as the slab thickness rises, causing the increase of the distance from extreme compression fiber to centroid longitudinal tension reinforcement, overwhelming the influence brought by the rise of bending moment.

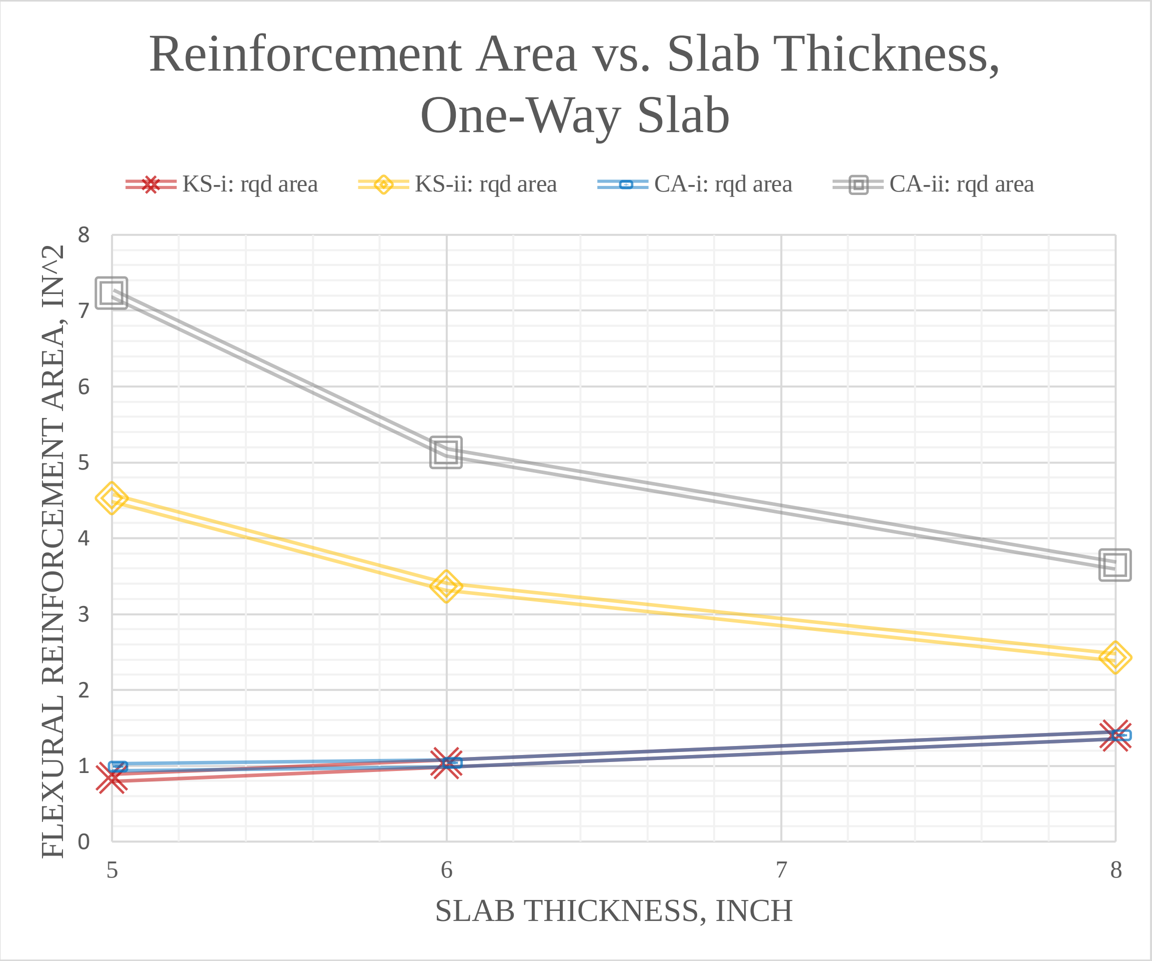


Figure 4.9 - Slab thickness vs. flexural reinforcement area for one-way slab method

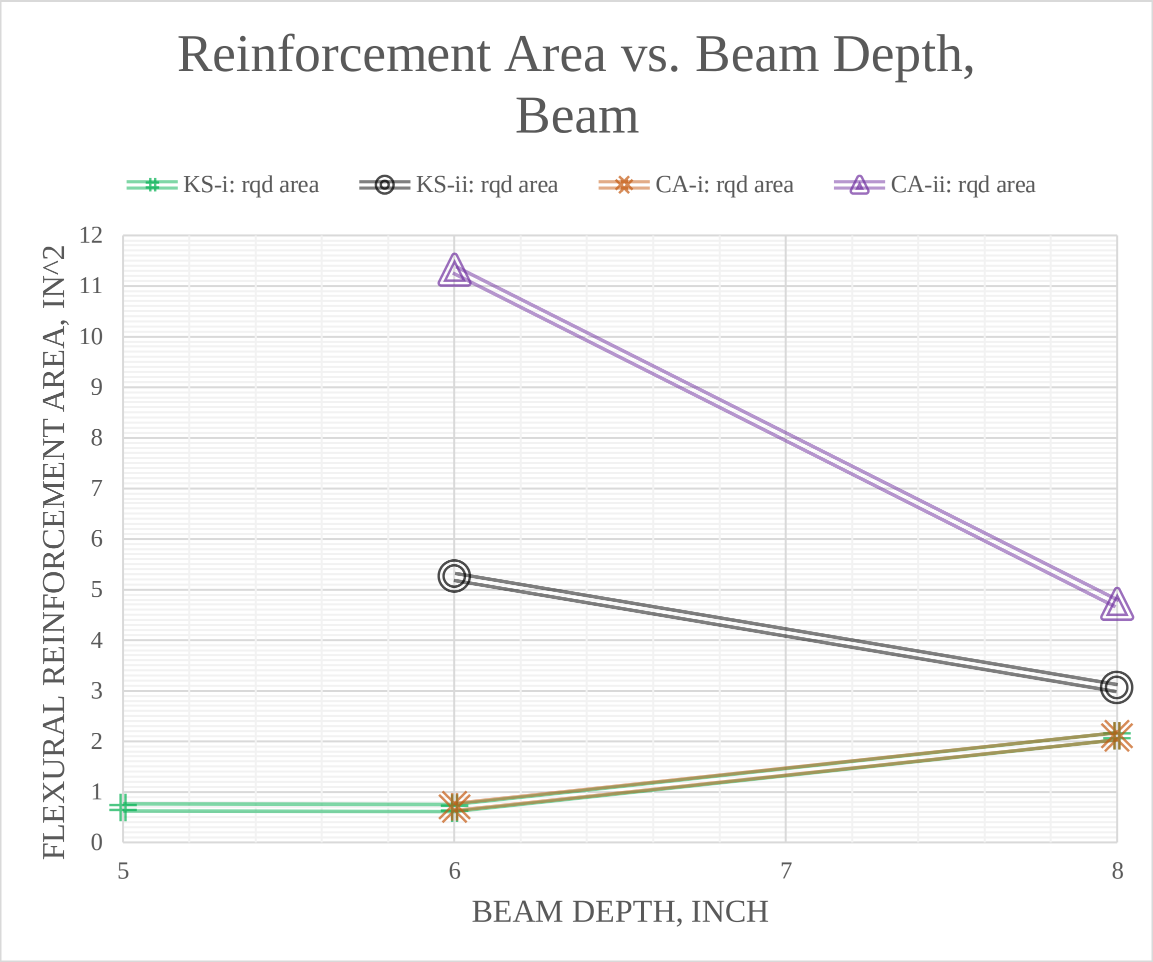


Figure 4.10 - Slab thickness vs. flexural reinforcement area for beam method

Table 4.16 presents the actual reinforcement information for each case. The use of temperature reinforcement is only applied for stairs and landings designed by one-way slab method. For beam method, the use of temperature reinforcement is not required. However, both methods require cracking control spacing checks for flexural reinforcement to prevent cracking due to high flexural stress by reducing flexural reinforcement spacing (Table 24.3.2 ACI 318-19). Calculations in Appendix A and B present the cracking check for flexural reinforcement. For shear reinforcement, the stairs and landings designed by one-way slab method seem not to have shear reinforcement since the shear strength provided by the concrete is sufficient enough to resist the actual shear force acting on the stairs and landings. The cases designed by beam method tend to have the same trending, except the landings of case B-CA-6-ii and case B-CA-8-ii. Although there are only two cases of shear reinforcement, the beam method tends to have a better performance in providing shear resistance for members. The reasons behind this conclusion is that, for one-way slab method, the shear reinforcement is required if the actual shear force acting on the member is greater than the factored shear strength provided by concrete (Section 7.6.3.1 ACI 318-19); for beam method, the shear reinforcement is required if the actual shear force acting on the member is greater than half of the factored shear strength provided by concrete (Section 9.6.3.1 ACI 318-19). Beam method turns out to be a more conservative method in terms of shear reinforcement design than one-way slab method. Since stairs are serving as exit access for people to evacuate from a high-rise building during an emergency, the need for shear force resistance is significant. Therefore, the beam method is a better approach for shear reinforcement design.

Table 4.16 - Tension/shear/temperature reinforcement for each case

	Landing			Stair flight		
Case	Tension reinforcement	Shear reinforcement	Temp. reinforcement	Tension reinforcement	Shear reinforcement	Temp. reinforcement
S-KS-5-i	(5)#4	-	#4 in every 12"	(5)#4	-	#4 in every 12"
S-KS-5-ii	(9)#4	-	#4 in every 12"	(18)#5	-	#4 in every 12"
S-KS-6-i	(5)#4	-	#4 in every 12"	(5)#4	-	#4 in every 12"
S-KS-6-ii	(9)#4	-	#4 in every 12"	(18)#4	-	#4 in every 12"
S-KS-8-i	(5)#4	-	#4 in every 12"	(5)#4	-	#4 in every 12"
S-KS-8-ii	(9)#4	-	#4 in every 12"	(9)#5	-	#4 in every 12"
S-CA-5-i	(5)#4	-	#4 in every 12"	(5)#4	-	#4 in every 12"
S-CA-5-ii	(14)#4	-	#4 in every 12"	(18)#6	-	#4 in every 12"
S-CA-6-i	(5)#4	-	#4 in every 12"	(5)#4	-	#4 in every 12"
S-CA-6-ii	(14)#4	-	#4 in every 12"	(18)#5	-	#4 in every 12"
S-CA-8-i	(5)#4	-	#4 in every 12"	(5)#4	-	#4 in every 12"
S-CA-8-ii	(9)#5	-	#4 in every 12"	(9)#6	-	#4 in every 12"
B-KS-5-i	(7)#4	-	-	(4)#4	-	-
B-KS-5-ii	-	-	-	-	-	-
B-KS-6-i	(7)#4	-	-	(4)#4	-	-
B-KS-6-ii	(7)#5	-	-	(17)#5	-	-
B-KS-8-i	(7)#4	-	-	(6)#4	-	-
B-KS-8-ii	(7)#5	-	-	(10)#5	-	-
B-CA-5-i	-	-	-	-	-	-
B-CA-5-ii	-	-	-	-	-	-
B-CA-6-i	(7)#4	-	-	(4)#4	-	-
B-CA-6-ii	(7)#6	#4 in every 3"	-	(26)#6	-	-
B-CA-8-i	(7)#4	-	-	(6)#4	-	-
B-CA-8-ii	(7)#6	#4 in every 3"	-	(11)#6	-	-

Chapter 5 - Conclusion and Future Research

This report provides a parametric study to differentiate the reinforcement design of the stairway based on one-way slab method and beam method following ACI 318-19. From a design standpoint, the beam method is better at minimizing excessive reinforcement area, avoiding design for minimum flexural reinforcement, and providing shear force resistance than the one-way slab method.

For future research, it is essential to take the torsional effect into considerations since it would be influenced by the stiffnesses of members and various boundary conditions, thus providing more comparisons for the parametric study. In addition, including more types of stairs (single flight stair, open-well staircase, and more) for diversity. The construction process is also significant since it brings other sides of perspective that differentiate from the design process.

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Appendix A - Stairway Parametric Calculations

Step	Computation	Reference
1. General Information	**1.1 Project statement:** Design the reinforcement of a concrete stairway in accordance with ACI 318-19. There are four parameters of this study: Geographic location (whether the seismic effect governs or not), reinforcement design method referred by ACI 318-19 (one-way slab, or beam), stair slab thickness (it depends on which method is used), and boundary conditions. To category the parameters and compare the results by each of them, there are sixteen cases been collected and shown as follows: S-KS-5-i S-KS-5-ii S-KS-8-i S-KS-8-ii S-CA-5-i S-CA-5-ii S-CA-8-i S-CA-8-ii B-KS-5-i B-KS-5-ii B-KS-8-i B-KS-8-ii B-CA-5-i B-CA-5-ii B-CA-8-i B-CA-8-ii where, B = Concrete beam method S = Concrete one-way slab method CA = Los Angeles, California KS = Manhattan, Kansas 5 = Stair slab thickness, 5 in. 8 = Stair slab thickness, 8 in. i = Boundary condition, shown in Figure ii = Boundary condition, shown in Figure **1.2 Project information:** (1) This is a 180 degree return stair, two flights with a half space landing between them (2) This is an interior stairway (3) This is a longitudinally spanning stairway, with one or more supports at each end of the stair **1.3 Project Location:** (1) Los Angeles, California State (high seismic activity region) (2) Manhattan, Kansas State (low seismic activity region)	

Step	Computation	Reference
1. General Information (cont'd)	**<u>1.4 Design parameters:</u>** Floor to floor height, H = 10 ft. Number of risers, n = 18 Tread depth, d = 11 in. Riser height, h_r = 6.7 in. Staircase clear width, w = 56 in. Staircase landing length, L_1, L_3 = 56 in. Angle from the horizontal, ϴ = 31.2 degree Landing thickness, h' = 9 in. Assumed slab thickness, h = 8 in. or 5 in. Calculated slab length, L_2 = 116 in. **<u>1.5 Assumptions:</u>** (1) Longitudinally spanning stairway, where torsional effect is neglected (2) No slope for the step nosing, thereby treating the step as a right triangle (3) Concrete member is nonprestressed (4) Normal weight concrete, with specified compressive strength equals to 4,000 psi (5) Specified yield strength for nonprestressed reinforcement equals to 60,000 psi (6) Assume stair slab thickness to be 5 in., and 8 in. (7) When checking deflection, only consider sections with stair steps and slabs. Since they are much longer spanning, if they are adequate in deflection, the landing will be, too. (8) No axial force (9) ASTM A615 Grade 60	*The Architect's Studio Companion, Six Edition, Exit Stairway Design Tables, Page 323*

Step	Computation	Reference
2. Building Loads	**2.1 Volume calculation:** For this part, take one flight and two half space landing as a whole (1) Vertical sectional area of treads: A_1 = 330 square in. (2) Vertical sectional area of landings: A_2 = 1008 square in. (3) Vertical sectional area of stair slab: when thickness is 8 in., A_3 = 926 square in. when thickness is 5 in., A_3 = 579 square in. (4) Overall vertical sectional area of stair flight (treads+slab): $A_{tot} = A_1 + A_3$ when thickness is 8 in., A_{tot} = 1256 square in. 9 square ft. when thickness is 5 in., A_{tot} = 909 square in. 6 square ft. (5) Volume: The value of volume equals to the area times clear width. Therefore, Volume of landing, V_L = 56448 cubic in. 33 cubic ft. when thickness is 8 in., volume of stair flight, V_S = 41 cubic ft. when thickness is 5 in., volume of stair flight, V_S = 29 cubic ft. **2.2 Dead load:** Assuming 145 pcf normal weight, load-bearing concrete, reinforced, and 5 psf of miscellaneous load. Therefore, Selfweight = 145 pcf Miscellaneous load = 5 psf (1) Convert to psf (pounds per square ft.) and add them together: Therefore, Dead load of landing, D = 223 psf when thickness is 8 in., dead load of stair flight, D = 136 psf when thickness is 5 in., dead load of stair flight, D = 100 psf (2) Convert to plf (pounds per linear ft.): Therefore, Dead load of landing, D = 1038 plf when thickness is 8 in., dead load of stair flight, D = 635 plf when thickness is 5 in., dead load of stair flight, D = 466 plf	

Step	Computation	Reference
2. Building Loads (cont'd)	**2.3 Live load:** According to IBC 2015, Table 1607.1, it indicates the live load for stair tread and landing is 100 psf. The live load for guards and handrails was not taken into consideration, because it should be provided by the manufacturers. Therefore, Live load, L = 100 psf 467 plf The landing and the stair flight have the same live load (plf) since they share the same clear width (56 in.)	*IBC 2015, Table 1607.1*
	2.4 Wind load: Since it is an interior stairway, only consider the minimum wind load, wind load, W = 16 psf 75 plf	*ASCE 7-16, Section 29.8*
	2.5 Snow load: Not required for an interior stairway.	
	2.6 Rain load: Not required for an interior stairway.	
	2.7 Seismic load: According to ASCE 7-16, Section 13.3.1, the horizontal seismic deisng force shall be determined by: $F_p = \frac{0.4a_p S_{DS} W_p}{\left(\frac{R_p}{I_p}\right)}\left(1 + 2\frac{z}{h}\right)$	*ASCE 7-16, Eq. 13.3-1*
	F_P is not required to be taken as larger than $F_p = 1.6 S_{DS} I_p W_p$	*ASCE 7-16, Eq. 13.3-2*
	and, F_p shall not be taken as less than $F_p = 0.3 S_{DS} I_p W_p$	*ASCE 7-16, Eq. 13.3-3*
	The vertical force shall be determined by: $\pm 0.2 S_{DS} W_p$	*ASCE 7-16, Section 13.3.1*
	where a_p = component amplification factor that in a range between 1.00 and 2.50, refer to ASCE 7-16, Table 13.5-1 or Table 13.6-1 F_p = seismic design force	

Step	Computation	Reference
2. Building Loads (cont'd)	h = average roof height of the structure in regard to the base I_p = component importance factor that in a range R_p = component response modification factor that in a range between 1.00 and 12, refer to ASCE 7-16, Table 13.5-1 or S_{DS} = spectral acceleration, short period, as determined from ASCE 7-16, Section 11.4.4 W_p = component operating weight z = height in structure or point of attachment of component with respect to the base. For items at or below the base, z should be taken as 0. The value of z/h need not exceed 1.0 For each high seismic activity region (LA) and low seismic activity region (manhattan), the main difference of them is the value of the spectral acceleration, S_{DS} For Log Angeles, S_{DS} = 1.579 For Manhattan, S_{DS} = 0.148 Other components, a_p = 1.0 R_p = 2.5 I_p = 1.5 z/h = 1.0 selfweight of landing, W_p = 4737 lbs when thickness is 8 in., selfweight of stair flight, W_p = 5903 lbs when thickness is 5 in., selfweight of stair flight, W_p = 4271 lbs Therefore, (1) For Los Angeles, CA: horizontal seismic force of landing, F_p = 5385 lbf ≤ 17950 lbf ≥ 3366 lbf vertical seismic force of landing, F_p = 1496 lbf When slab thickness = 8 in., horizontal seismic force of stair flight, F_p = 6710 lbf ≤ 22368 lbf ≥ 4194 lbf vertical seismic force of stair flight, F_p = 1864 lbf	*OSHPD Seismic Design Maps* *ASCE 7-16, Table 13.5-1 Egress Stair, and Section 13.1.3*

Step	Computation	Reference

2. Building Loads (cont'd)

When slab thickness = 5 in.,
horizontal seismic force of stair flight, F_p = 4855 lbf
≤ 16184 lbf
≥ 3034 lbf
vertical seismic force of stair flight, F_p = 1349 lbf

(2) For Manhattan, KS:
horizontal seismic force of landing, F_p = 505 lbf
≤ 1682 lbf
≥ 315 lbf
vertical seismic force of landing, F_p = 140 lbf

When slab thickness = 8 in.,
horizontal seismic force of stair flight, F_p = 629 lbf
≤ 2097 lbf
≥ 393 lbf
vertical seismic force of stair flight, F_p = 175 lbf

When slab thickness = 5 in.,
horizontal seismic force of stair flight, F_p = 455 lbf
≤ 1517 lbf
≥ 284 lbf
vertical seismic force of stair flight, F_p = 126 lbf

2.8 Building loads table:

From previous calculation, all building loads are shown as follows:
(Unit is plf.)

			h= 8 in.	h = 5 in.
		Landing	Stair flight	Stair flight
	D	1038	635	466
	L	467		
	W	75		
CA	Eh	1154	696	503
	Ev	321	193	140
KS	Eh	108	65	47
	Ev	30	18	13

*** Note: Convert seismic force from lbf to plf by dividing by the span length

Step | **Computation** | **Reference**

2. Building Loads (cont'd)

2.9 Load combinations:

According to ASCE 7-16, Chapter 2, Section 2.3.2, the load combinations are shown as following:

1. $1.4D$
2. $1.2D + 1.6L + 0.5(L_r \; or \; S \; or \; R)$
3. $1.2D + 1.6(L_r \; or \; S \; or \; R) + (L \; or \; 0.5W)$
4. $1.2D + 1.0W + L + 0.5(L_r \; or \; S \; or \; R)$
5. $1.2D + 1.0E + L + 0.2S$
6. $0.9D + 1.0W$
7. $0.9D + 1.0E$

where

D = dead load
E = earthquake load
L = live load
L_r = roof live load
R = rain load
S = snow load
W = wind load

ASCE 7-16, Chapter 2, Section 2.3.2

The primary loads for this project are dead load, live load, internal wind load, and seismic load.

For load combination 5 and 7, can be re-writen as:

For load combination 5 and 7, can be re-writen as:

5. $1.2D + E_v + E_h + L + 0.2S$
7. $0.9D - E_v + E_h$

where

E_h = horizontal seismic force
E_v = vertical seismic force

ASCE 7-16, Section 12.4.2

Therefore,

(1) For Los Angeles, the value of each load combination is shown as follows:

(unit is plf.)

		t = 8 in.	t = 5 in.	
	Landing	**Stair flight**	**Stair flight**	
LC-1	1454	889	652	
LC-2	1993	1509	1306	
LC-3	1713	1229	1026	
LC-4	1638	1154	951	
LC-5	3187	2118	1669	Governs
LC-6	1009	646	494	
LC-7	1768	1074	783	

As assumed previously, load combination 5 governs since Los Angeles is a high seismic activity region.

Step | **Computation** | **Reference**

2. Building Loads (cont'd)

(2) For Manhattan, the value of each load combination is shown as follows:

(unit is plf.)

		t = 8 in.	t = 5 in.	
	Landing	**Stair flight**	**Stair flight**	
LC-1	1454	889	652	
LC-2	1993	1509	1306	Governs
LC-3	1713	1229	1026	
LC-4	1638	1154	951	
LC-5	1851	1312	1086	
LC-6	1009	646	494	
LC-7	1013	619	453	

Load combination 2 governs since Manhattan is a low seismic activity region.

2.10 Governing load summary:

Convert plf (pounds per linear ft.) to klf (kips per lieanr ft.) for RISA 3D. The value of each governing load is shown as follows:

		t = 8 in.	t = 5 in.
	Landing	**Stair flight**	**Stair flight**
CA	3.19	2.12	1.67
KS	1.99	1.51	1.31

(Unit is klf.)

Step	Computation	Reference

3. Concrete One-way Slab Approach

3.1 General steps:

(1) Minimum slab thickness, h:

Acoording to ACI 318-19, Table 7.3.1.1 - Minimum thickness of solid nonprestressed one-way slabs:

Support condition	Minimum h
Simply supported	$l/20$
One end continuous	$l/24$
Both ends continuous	$l/28$
Cantilever	$l/10$

ACI 318-19 Table 7.3.1.1

(2) Check if L ≤ 3D, where L stands for live load, D stands for dead load:

ACI 318-19 Section 6.5.1

(3) Calculate shear and moment based on RISA 3D

(4) Calculate required area of reinforcement, $\int$, based on flexural moment:

$$m = \frac{f_y}{0.85 f_c'}$$

$$R_u = \frac{M_u}{\Phi b d^2}$$

$$\int_{rqd} = \frac{1}{m}[1 - \sqrt{1 - \left(\frac{2R_u m}{f_y}\right)}]$$

where

b = width of compression face of member, in.

d = distance from extreme compression fiber to centroid of longitudinal tension reinforcement, in.

f_y = specified yield strength for nonprestressed reinforcement, psi

f_c' = specified compressive strength of concrete, psi

M_u = actual moment, ft-kip

R_u, m = formula conversion parameter

$\int_{rqd}$ = required area ratio of reinforcement

Φ = strength reduction factor

(5) Check for flexural reinforcement limits:

According to ACI 318-19, Table 7.6.1.1 - $A_{s,min}$ for nonprestressed one-way slabs:

Reinforcement type	f_y ,psi	$A_{s,min}$	
Deformed bars	< 60,000	$0.0020A_g$	
Deofrmed bars or welded wire reinforcement	≥60,000	Greater of:	$\frac{0.0018 \times 60{,}000}{f_y} A_g$
			$0.0014A_g$

ACI 318-19, Table 7.6.1.1

<table>
<tr><th>Step</th><th>Computation</th><th>Reference</th></tr>
<tr>
<td>3. Concrete One-way Slab Approach (cont'd)</td>
<td>

(6) Determine center-to-center spacing, s of reinforcement:
Minimum spacing - shall be at least the greatest of 1 in., d_b and $(4/3)d_{agg}$
Maximum spacing - shall be the lesser of $3h$ and 18 in.
Minimum layer spacing - shall be at least 1 in. between layers for parallel nonprestressed reinforcement placed in two or more horizontal layers

(7) Check for shear limit:
According to ACI 318-19, Table 22.5.5.1, shear strength provided by concrete shall be determined by:

$A_v < A_{v,min}$

$$V_c = \left[8\lambda_s\lambda(\rho_w)^{1/3}\sqrt{f_c'} + \frac{N_u}{6A_g}\right]b_w d$$

$$\lambda_s = \sqrt{\frac{2}{1+\frac{d}{10}}} \leq 1$$

where
A_g = gross area of concrete section
A_v = area of shear reinforcement within spacing
$A_{v,min}$ = minimum area of shear reinforcement within spacing
b_w = web width or diameter of circular section
d = distance from exterme compression fiber to controid of longitudinal tension reinforcement
f_c' = specified compressive strength of concrete
N_u = factored axial force normal to cross section
ρ_w = ratio of A_s to $b_w d$
λ = modification factor to reflect the reduced mechanical properties of lightweight concrete relative to normal weight concrete of the same compressive strength
λ_s = factor used to modify shear strength based on the effect of member depth, commonly referred to as the size effect factor

(8) Check for minimum shrinkage and temperature reinforcement:
According to ACI 318-19, Section 7.6.4.1 and 24.4, reinforcement shall be provided to resist shrinkage and temperature based on:

<table>
<tr><th>Reinforcement type</th><th>f_y, psi</th><th colspan="2">Minimum rein. Ratio</th></tr>
<tr><td>Deformed bars</td><td>< 60,000</td><td colspan="2">0.0020</td></tr>
<tr><td rowspan="2">Deformed bars or welded wire reinforcement</td><td rowspan="2">≥60,000</td><td rowspan="2">Greater of:</td><td>$\frac{0.0018 \times 60,000}{f_y}$</td></tr>
<tr><td>0.0014</td></tr>
</table>

And, spacing of deformed shrinkage and temperature reinforcement shall not exceed the lesser of $5h$ and 18 in.

</td>
<td>

ACI 318-19 Section 7.7.2.3, 25.2.1, and 25.2.2

ACI 318-19 Table 22.5.5.1(c), Eq. 22.5.5.1.3

ACI 318-19 Table 24.4.3.2

Section 7.7.6.2.1

</td>
</tr>
</table>

Step | **Computation** | **Reference**

3. Concrete One-way Slab Approach (cont'd)

(9) Check for cracking control spacing requirement:

Reinforcement type	Maximum spacing s	
Deformed bars or wires	Lesser of:	$15\left(\frac{40{,}000}{f_s}\right) - 2.5c_c$
		$12\left(\frac{40{,}000}{f_s}\right)$

ACI 318-19 Table 24.3.2

(10) Check for deflection:
Deflection shall be calculated in accordance with ACI 318-19 Section 24.2 and shall not exceed the limits in 24.2.2.

ACI 318-19 Section 7.3.2.1, 24.2, 24.2.2

Assume the section is cracked, determine the cracking moment

$$f_r = 7.5\lambda\sqrt{f_c'}$$

ACI 318-19 Eq. 19.2.3.1

$$I_g = \frac{1}{12}bh^3$$

Area moment of inertia of rectangular section

$$M_{cr} = \frac{f_r I_g}{y_t}$$

Eq. 24.2.3.5b

where

f_c' = specified compressive strength of concrete, psi

f_r = modulus of rupture of concrete, psi

I_g = area moment of inertia

M_{cr} = cracking moment

y_t = distance from centroidal axis of gross section, neglecting reinforcement, to tension face, in.

λ = modification factor to reflect the reduced mechanical properties of lightweight concrete relative to normal weight concrete of the same compressive strength

Apply transformed area method to determine the distance from the exterme compression fiber to the neutral axis, C_{NA}

$$\bar{y} = \frac{\sum A_i y_i}{\sum A_i}$$

If take the very top line of the section as the reference line, then the equation becomes:

$$C_{NA} = \frac{(bC_{NA})\left(\frac{C_{NA}}{2}\right) + (nA_s)d}{(bC_{NA}) + (nA_s)}$$

Then, it becomes a quadratic equation:

$$\frac{b}{2}C_{NA}^2 + nA_sC_{NA} - nA_sd = 0$$

Step	Computation	Reference
3. Concrete One-way Slab Approach (cont'd)	The solution of C_{NA}: $C_{NA} = \frac{-nA_s \mp \sqrt{(nA_s)^2+2b(nA_sd)}}{b}$ Calculate moment of inertia of cracked section transformed to concrete, I_{cr} $I_{cr} = \sum I_i + A_i d_{yi}^2$ Then, it becomes: $I_{cr} = \frac{1}{12} bC_{NA}^3 + bC_{NA}(\frac{C_{NA}}{2})^2+nA_s(d - C_{NA})^2$ (The value of I_x is so small that can be neglected) or $I_{cr} = \frac{1}{3} bC_{NA}^3 + nA_s(d - C_{NA})^2$ Calculate effective moment of inertia, I_e $I_e = \frac{I_{cr}}{1-\left(\frac{(2/3)M_{cr}}{M_a}\right)^2 (1-\frac{I_{cr}}{I_g})}$ Then, check deflection According to ACI 318-19, Table 24.2.2, immediate deflection due to live load L is $l/360$ $l/360$ Only consider the middle span where the stairway slab is located, $\Delta_{max} = 5wl^4/384EI$ $E_c = 57{,}000\sqrt{f_c'}$ **3.2 Eight cases:** There are eight cases to be designed by one-way slab method for this project: S-KS-5-i S-KS-5-ii S-KS-8-i S-KS-8-ii S-CA-5-i S-CA-5-ii S-CA-8-i S-CA-8-ii	*ACI 318-19 Table 24.2.3.5(b)* *ACI 318-19 Eq. 19.2.2.1.b*

Step	Computation	Reference
3. Concrete One-way Slab Approach (cont'd)	**<u>3.3 Design for S-KS-5-i:</u>** (1) Minimum slab thickness, h: The boundary condition for slab is still both ends continuous. Therefore, when l = 116 in. h = 4.13 in. The assumption of the slab thickness was 5 in., which is greater than 4.13 in.. The assumption is OK. Therefore, h = 5 in. (2) Check if L ≤ 3D: L = 467 plf D = 466 plf 3D = 1398 plf O.K. (3) Shear and moment: According to RISA 3D model, see Figure 4.1 and Table 4.1 For stair slab: Shear strength, V_u = 5.4 kips Flexural positive strength, M_u = 5.2 ft-kips Flexural negative strength, M_u = 7.8 ft-kips If applying 1' strip method, then they become: Shear strength, V_u = 1.16 kips Flexural positive strength, M_u = 1.11 ft-kips Flexural negative strength, M_u = 1.67 ft-kips For landing: Shear strength, V_u = 6.3 kips Flexural positive strength, M_u = 2.2 ft-kips Flexural negative strength, M_u = 7.8 ft-kips If applying 1' strip method, then they become: Shear strength, V_u = 1.35 kips Flexural positive strength, M_u = 0.47 ft-kips Flexural negative strength, M_u = 1.67 ft-kips (4) Calculate required area of reinforcement, $\int_{rqd}$: The value of each parameter shown as follows: Cealr cover = 0.75 in. half of the assumed rebar dia. = 0.375 in. (#6) d'= 1.125 in.	

Step	Computation	Reference
3. Concrete One-way Slab Approach (cont'd)	h = 5 in. d= 3.875 in. b = 12.00 in. 1' strip f_c'= 4,000 psi f_y= 60,000 psi Tension controlled, Φ = 0.9	*Table 21.2.2*
	For stairway, postive moment: m= 17.65 R_u= 82.45 $\int_{rqd}$= 0.0014 $A_{s,rqd+}$= 0.06 square in.	
	For stairway, negative moment: m= 17.65 R_u= 123.68 $\int_{rqd}$= 0.0021 $A_{s,rqd-}$= 0.10 square in.	
	For landing, postive moment: m= 17.65 R_u= 8.45 $\int_{rqd}$= 0.0001 $A_{s,rqd+}$= 0.01 square in.	
	For landing, negative moment: m= 17.65 R_u= 29.95 $\int_{rqd}$= 0.0005 $A_{s,rqd-}$= 0.05 square in.	
	(5) Check for flexural reinforcement limit: $A_{s,min}$= greater of 0.084 square in. √ and 0.065 square in. For calculated values less than the area of minimum reinforcement, apply 0.084 in^2 instead. For landing, it's 0.17 in^2 (d = 7.875 in.).	
	Therefore, for stairway: $A_{s,rqd+}$= 0.08 square in. $A_{s,rqd-}$= 0.10 square in. for landing: $A_{s,rqd+}$= 0.17 square in. $A_{s,rqd-}$= 0.17 square in.	

Step	Computation	Reference
3. Concrete One-way Slab Approach (cont'd)	(6) Determine center-to-center spacing, s If pick bar size, no. = 4 diameter of bar = 0.500 in. area of bar = 0.20 square in. For stairway, Since, $A_{s,rqd+}$= 0.08 square in. for 1' strip slab No. of bar in 1 ft. strip = 1 Center-to-center spacing, s = 12 in. Since, $A_{s,rqd-}$= 0.10 square in. for 1' strip slab No. of bar in 1 ft. strip = 1 Center-to-center spacing, s = 12 in. For landing, Since, $A_{s,rqd+}$= 0.17 square in. for 1' strip slab No. of bar in 1 ft. strip = 1 Center-to-center spacing, s = 12 in. Since, $A_{s,rqd-}$= 0.17 square in. for 1' strip slab No. of bar in 1 ft. strip = 1 Center-to-center spacing, s = 12 in. Now, check for minimum spacing: The diameter of the aggregate is neglected for this project since it shall be determined by the manufacturer. So, s_{min} = greater of 1 in. √ and 0.500 in. Since $s = 12\ in. > s_{min} = 1\ in.$ O.K. Check for maximum spacing: s_{max} = lesser of 15 in. √ and 18 in. Since $s = 12\ in. < s_{max} = 15\ in.$ O.K. (7) Check for shear limit: Apply equation (b) shown in Section 3.1, Step (7), in accordance with ACI 318-19, Table 22.5.5.1 The value of each parameter shown as follows: b_w = 12 in. 1' strip	*ACI 318-19 Appendix A*

Step	Computation	Reference
3. Concrete One-way Slab Approach (cont'd)	d = 3.875 in. or 7.875 in. λ_s = 1.00 f'_c = 4,000 psi $\emptyset$ = 0.75 λ = 1.0 for stairway ρ_w = 0.0086 for landing ρ_w = 0.0042 Therefore, For stairway, $\emptyset V_c$ = 3.62 kips For landing, $\emptyset V_c$ = 5.80 kips Since both of them are greater than the value of V_u no need for shear reinforcement. (8) Minimum temperature and shrinkage reinforcement: $A_{ts,min}$= greater of 0.084 square in. √ and 0.065 square in. If pick bar size, no. = 4 diameter of bar = 0.500 in. area of bar = 0.20 square in. Since, $A_{ts,min}$= 0.084 square in. for 1' strip slab No. of bar in 1 ft. strip = 1 Center-to-center spacing, s = 12 in. Check for maximum spacing: s_{max}= lesser of 25 in. and 18 in. √ Since $s = 12\ in. < s_{max} = 18\ in.$ O.K. (9) Check for cracking control spacing requirement: Clear cover, c_c = 0.75 in. f_s = 40,000 psi s_{max}= lesser of 13.13 in. and 12 in. √ since $s = 12\ in. = s_{max} = 12\ in.$ for flexural reinforcement	*Eq. 22.5.5.1.3* *ACI 318-19* *Table 21.2.1* *Table 19.2.4.2* *ACI 318-19 7.6.3.1* *ACI 318-19 Appendix A*

Step	Computation	Reference
3. Concrete One-way Slab Approach (cont'd)	$s = 12\ in. = s_{max} = 12\ in.$ for temp. and shrinkage O.K (10) Check for deflection: According to ACI 318-19, Section 7.3.2.2, since the applied slab thickness satisfies Table 7.3.1.1, deflections occuring after the member becomes composite need not to be checked. (11) Conclusion: For stairway slab, use #4 reinforcing rebar in every 12 in. spacing for flexural reinforcement, and use #4 reinforcing rebar in every 12 in. spacing for temperature reinforcement. For landing, use #4 reinforcing rebar in every 12 in. spacing for flexural reinforcement, and use #4 reinforcing rebar in every 12 in. spacing for temperature reinforcement. Flexural reinforcement in both stair slab and landing has two layers, both acting as tention reinforcement.	*ACI 318-19 Section 7.3.2.2*

Step	Computation	Reference
3. Concrete One-way Slab Approach (cont'd)	**<u>3.4 Design for S-KS-5-ii:</u>** (1) Minimum slab thickness, h: The boundary condition for slab is simply supported. Therefore, when l = 116 in. h = 5.79 in. The assumption of the slab thickness was 5 in., which is less than 5.79 in.. Deflection check is required. Therefore, h = 5 in. (2) Check if L ≤ 3D: L = 467 plf D = 466 plf 3D = 1398 plf O.K. (3) Shear and moment: According to RISA 3D model, see Figure 4.2 and Table 4.2 For stair slab: Shear strength, V_u = 5.4 kips Flexural strength, M_u = 64.2 ft-kips If applying 1' strip method, Shear strength, V_u = 1.16 kips Flexural strength, M_u = 13.76 ft-kips For landing: Shear strength, V_u = 15.6 kips Flexural strength, M_u = 51.2 ft-kips If applying 1' strip method, Shear strength, V_u = 3.34 kips Flexural strength, M_u = 10.97 ft-kips (4) Calculate required area of reinforcement, $\int_{rqd}$: The values of each parameter are: Cealr cover = 0.75 in. half of the assumed rebar dia. = 0.375 in. (#6) d'= 1.125 in. h = 5 in. d= 3.875 in. b = 12.00 in. 1' strip f_c'= 4,000 psi	

Step	Computation	Reference
3. Concrete One-way Slab Approach (cont'd)	f_y= 60,000 psi	
	Tension controlled, Φ = 0.9	*Table 21.2.2*
	For stair:	
	m= 17.65	
	R_u= 1017.99	
	$\int_{rqd}$= 0.0208	
	$A_{s,rqd}$ = 0.97 square in.	
	For landing:	
	m= 17.65	
	R_u= 196.57	
	$\int_{rqd}$= 0.0034	
	$A_{s,rqd}$ = 0.32 square in.	
	(5) Check for flexural reinforcement limit:	
	$A_{s,min}$ = greater of	
	0.084 square in. √	
	and	
	0.0651 square in.	
	(For landing, it's 0.17 in^2 since the d = 7.875 in.)	
	Both stair slab and landing have greater required area of reinforcement than minimum area of reinforcement, so they are O.K.	
	(6) Determine center-to-center spacing, s	
	If pick bar size, no. = 5	
	diameter of bar = 0.625 in.	*ACI 318-19*
	area of bar = 0.31 square in.	*Appendix A*
	since, $A_{s,rqd}$= 0.97 square in. for stair slab	
	No. of bar in 1 ft. strip = 4	
	Center-to-center spacing, s = 3 in.	
	If pick bar size, no. = 4	
	diameter of bar = 0.500 in.	*ACI 318-19*
	area of bar = 0.20 square in.	*Appendix A*
	since, $A_{s,rqd}$= 0.32 square in. for landing	
	No. of bar in 1 ft. strip = 2	
	Center-to-center spacing, s = 6 in.	
	Now, check for minimum spacing:	
	The diameter of the aggregate is neglected for this project since it shall be determined by the manufacturer.	
	So, s_{min} = greater of	
	1 in. √	
	and	
	0.625 in. for #5	

Step	Computation	Reference
3. Concrete One-way Slab Approach (cont'd)	0.500 in. for #4	
	Since, $s = 3\ in.\ or\ 6\ in. > s_{min} = 1\ in.$ O.K.	
	(7) Check for shear limit:	
	Apply equation (b) shown in Section 3.1, Step (7), in accordance with ACI 318-19, Table 22.5.5.1	
	The value of each parameter shown as follows:	
	b_w = 12 in. 1' strip	
	d = 3.875 in.	
	λ_s = 1.00	*Eq. 22.5.5.1.3*
	f_c' = 4,000 psi	*ACI 318-19*
	$\emptyset$ = 0.75	*Table 21.2.1*
	λ = 1.0	*Table 19.2.4.2*
	for stair ρ_w = 0.0267	
	for landing ρ_w = 0.0042	
	Therefore,	
	For stairway,	
	$\emptyset V_c$ = 5.27 kips	
	For landing,	
	$\emptyset V_c$ = 5.80 kips	
	Since both of them are greater than the value of V_u no need for shear reinforcement.	*ACI 318-19 7.6.3.1*
	(8) Minimum temperature and shrinkage reinforcement:	
	$A_{ts,min}$= greater of	
	0.084 square in. √	
	and	
	0.065 square in.	
	If pick bar size, no. = 4	
	diameter of bar = 0.500 in.	*ACI 318-19 Appendix A*
	area of bar = 0.20 square in.	
	Since, $A_{ts,min}$= 0.084 square in. for 1' strip slab	
	No. of bar in 1 ft. strip = 1	
	Center-to-center spacing, s = 12 in.	
	Check for maximum spacing:	
	s_{max}= lesser of	
	25 in.	
	and	
	18 in. √	
	Since $s = 12\ in. < s_{max} = 18\ in.$ O.K.	

Step	Computation	Reference
3. Concrete One-way Slab Approach (cont'd)	(9) Check for cracking control spacing requirement: Clear cover, c_c = 0.75 in. f_s = 40,000 psi s_{max}= lesser of 13.13 in. and 12 in. √ since $s = 6\ in. < s_{max} = 12\ in.$ for flexural reinforcement $s = 12\ in. = s_{max} = 12\ in$ for temp. and shrinkage O.K (10) Check for deflection: (a) Idealize the surface of the concrete that shows the cross sectional area of the main bars Now it becomes a rectangularshape of concrete with: Clear width, b = 56 in. slab thickness, h = 5 in. No. of bar size = 5 Area of a single bar, A_s= 0.31 square in. No. of bars = 4 for 1' strip Total area of bars, A_s= 5.55 square in. (b) Determine the modification factor, n to convert steel to concrete For normal weight concrete, $E_c = 57{,}000\sqrt{f_c'}$ $n = \frac{E_s}{E_c}$ Since, f_c' = 4,000 psi E_c = 3,605 ksi E_s = 29,000 ksi So that, n = 8.04 (c) Assume the section is cracked, determine the cracking moment $f_r = 7.5\lambda\sqrt{f_c'}$ $I_g = \frac{1}{12}bh^3$ $M_{cr} = \frac{f_r I_g}{y_t}$ where f_c'= 4,000 psi λ = 1.0	 *ACI 318-19* *19.2.2.1.a* *ACI 318-19* *Eq. 19.2.3.1* *Area moment of inertia of rectangular section* *Eq. 24.2.3.5b*

Step	Computation	Reference

3. Concrete One-way Slab Approach (cont'd)

f_r= 474 psi
I_g= 583 in^4
y_t= 2.5 in.
M_{cr}= 9.22 ft-kips

Since, $M_{cr} = 9.22\, ft - kips < M_u = 64.2\, ft - kips$, it's cracked

(d) Apply transformed area method to determine the distance from the exterme compression fiber to the neutral axis, C_{NA}
From Section 3.1, Step (10),

$$\bar{y} = \frac{\sum A_i y_i}{\sum A_i}$$

$$C_{NA} = \frac{(bC_{NA})\left(\frac{C_{NA}}{2}\right) + (nA_s)d}{(bC_{NA}) + (nA_s)}$$

Then, it becomes a quadratic equation:

$$\frac{b}{2}C_{NA}^2 + nA_sC_{NA} - nA_sd = 0$$

The solution of C_{NA}:

$$C_{NA} = \frac{-nA_s \mp \sqrt{(nA_s)^2 + 2b(nA_sd)}}{b}$$

Therefore, C_{NA} = 1.81 in.

(e) Calculate moment of inertia of cracked section transformed to concrete, I_{cr}

$$I_{cr} = \sum I_i + A_i d_{yi}^2$$

Then, it becomes:

$$I_{cr} = \frac{1}{3}bC_{NA}^3 + nA_s(d - C_{NA})^2$$

(The value of I_x is so small that can be neglected)

Therefore, I_{cr} = 301.21 in^4

(f) Calculate effective moment of inertia, I_e

$$I_e = \frac{I_{cr}}{1 - \left(\frac{(2/3)M_{cr}}{M_a}\right)^2 \left(1 - \frac{I_{cr}}{I_g}\right)}$$

ACI 318-19 Eq. 24.2.3.5(b)

M_{cr}= 9.22 ft-kips
M_a = 64.20 ft-kips
I_g= 583 in^4
I_{cr}= 301 in^4
So that, I_e = 303 in^4

Step	Computation	Reference
3. Concrete One-way Slab Approach (cont'd)	(g) Check deflection: According to ACI 318-19, Table 24.2.2, immediate deflection due to live load L is $l/360$ l = 116 in. for slab span length $l/360$ = 0.32 Only consider the middle span where the stairway slab is located, see Figure $\Delta_{max} = 5wl^4/384EI$ $E_c = 57{,}000\sqrt{f_c'}$ f_c'= 4 ksi E_c= 3,605 ksi l = 116 in. I_e= 303 in^4 w= 1306 plf Δ_{max}= 0.23 Since, $\Delta_{max} = 0.23 < l/360 = 0.32$, this design satisfies with the deflection requirement (11) Conclusion: For stairway slab, use #5 reinforcing rebar in every 3 in. spacing for flexural reinforcement, and use #4 reinforcing rebar in every 12 in. spacing for temperature reinforcement. For landing, use #4 reinforcing rebar in every 6 in. spacing for flexural reinforcement, and use #4 reinforcing rebar in every 12 in. spacing for temperature reinforcement.	*ACI 318-19 Eq. 19.2.2.1.b*

Step	Computation	Reference
3. Concrete One-way Slab Approach (cont'd)	**<u>3.5 Design for S-KS-8-i:</u>** (1) Minimum slab thickness, h: The boundary condition for slab is still both ends continuous. Therefore, when l = 116 in. h = 4.13 in. The assumption of the slab thickness was 8 in., which is greater than 4.13 in.. The assumption is OK. Therefore, h = 8 in. (2) Check if L ≤ 3D: L = 467 plf D = 635 plf 3D = 1906 plf O.K. (3) Shear and moment: According to RISA 3D model, see Figure 4.3 and Table 4.3 For stair slab: Shear strength, V_u = 6.2 kips Flexural positive strength, M_u = 6.2 ft-kips Flexural negative strength, M_u = 8.8 ft-kips If applying 1' strip method, then they become: Shear strength, V_u = 1.33 kips Flexural positive strength, M_u = 1.33 ft-kips Flexural negative strength, M_u = 1.89 ft-kips For landing: Shear strength, V_u = 6.5 kips Flexural positive strength, M_u = 1.9 ft-kips Flexural negative strength, M_u = 8.8 ft-kips If applying 1' strip method, then they become: Shear strength, V_u = 1.39 kips Flexural positive strength, M_u = 0.41 ft-kips Flexural negative strength, M_u = 1.89 ft-kips (4) Calculate required area of reinforcement, $\int_{rqd}$: The value of each parameter shown as follows: Cealr cover = 0.75 in. half of the assumed rebar dia. = 0.375 in. (#6) d'= 1.125 in.	

Step	Computation	Reference
3. Concrete One-way Slab Approach (cont'd)	h = 8 in.	
	d= 6.875 in.	
	b = 12.00 in. 1' strip	
	f_c'= 4,000 psi	
	f_y= 60,000 psi	
	Tension controlled, Φ = 0.9	*Table 21.2.2*
	For stairway, postive moment:	
	m= 17.65	
	R_u= 31.23	
	$\int_{rqd}$= 0.0005	
	$A_{s,rqd+}$= 0.04 square in.	
	For stairway, negative moment:	
	m= 17.65	
	R_u= 44.33	
	$\int_{rqd}$= 0.0007	
	$A_{s,rqd-}$= 0.06 square in.	
	For landing, postive moment:	
	m= 17.65	
	R_u= 7.29	
	$\int_{rqd}$= 0.0001	
	$A_{s,rqd+}$= 0.01 square in.	
	For landing, negative moment:	
	m= 17.65	
	R_u= 33.79	
	$\int_{rqd}$= 0.0006	
	$A_{s,rqd-}$= 0.05 square in.	
	(5) Check for flexural reinforcement limit:	
	$A_{s,min}$= greater of	
	0.149 square in. √	
	and	
	0.116 square in.	
	For calculated values less than the area of minimum reinforcement, apply 0.149 in^2 instead. For landing, it's still 0.17 in^2.	
	Therefore,	
	for stairway:	
	$A_{s,rqd+}$= 0.15 square in.	
	$A_{s,rqd-}$= 0.15 square in.	
	for landing:	
	$A_{s,rqd+}$= 0.17 square in.	
	$A_{s,rqd-}$= 0.17 square in.	

Step	Computation	Reference
3. Concrete One-way Slab Approach (cont'd)	(6) Determine center-to-center spacing, s If pick bar size, no. = 4 diameter of bar = 0.500 in. area of bar = 0.20 square in. For stairway, Since, $A_{s,rqd+}$ = 0.15 square in. for 1' strip slab No. of bar in 1 ft. strip = 1 Center-to-center spacing, s = 12 in. Since, $A_{s,rqd-}$ = 0.15 square in. for 1' strip slab No. of bar in 1 ft. strip = 1 Center-to-center spacing, s = 12 in. For landing, Since, $A_{s,rqd+}$ = 0.17 square in. for 1' strip slab No. of bar in 1 ft. strip = 1 Center-to-center spacing, s = 12 in. Since, $A_{s,rqd-}$ = 0.17 square in. for 1' strip slab No. of bar in 1 ft. strip = 1 Center-to-center spacing, s = 12 in. Now, check for minimum spacing: The diameter of the aggregate is neglected for this project since it shall be determined by the manufacturer. So, s_{min} = greater of 1 in. √ and 0.500 in. Since $s = 12\ in. > s_{min} = 1\ in.$ O.K. Check for maximum spacing: s_{max} = lesser of 24 in. and 18 in. √ Since $s = 12\ in. < s_{max} = 18\ in.$ O.K. (7) Check for shear limit: Apply equation (b) shown in Section 3.1, Step (7), in accordance with ACI 318-19, Table 22.5.5.1 The value of each parameter shown as follows: b_w = 12 in. 1' strip	*ACI 318-19 Appendix A*

Step	Computation	Reference
3. Concrete One-way Slab Approach (cont'd)	d = 6.875 in.	
	λ_s = 1.00	
	f'_c = 4,000 psi	*ACI 318-19*
	Ø = 0.75	*Table 21.2.1*
	λ = 1.0	*Table 19.2.4.2*
	for stairway ρ_w = 0.0048 governs	
	for landing ρ_w = 0.0042 governs	
	Therefore,	
	For stairway,	
	$\text{Ø}V_c$ = 5.30 kips	
	For landing,	
	$\text{Ø}V_c$ = 5.80 kips	
	Since both of them are greater than the value of V_u no need for shear reinforcement.	*ACI 318-19* *7.6.3.1*
	(8) Minimum temperature and shrinkage reinforcement:	
	$A_{ts,min}$ = greater of	
	0.149 square in. √	
	and	
	0.116 square in.	
	If pick bar size, no. = 4	
	diameter of bar = 0.500 in.	*ACI 318-19*
	area of bar = 0.20 square in.	*Appendix A*
	Since, $A_{ts,min}$ = 0.149 square in.	
	for 1' strip slab	
	No. of bar in 1 ft. strip = 1	
	Center-to-center spacing, s = 12 in.	
	Check for maximum spacing:	
	s_{max} = lesser of	
	40 in.	
	and	
	18 in. √	
	Since $s = 12\ in. < s_{max} = 18\ in.$ O.K.	
	(9) Check for cracking control spacing requirement:	
	Clear cover, c_c = 0.75 in.	
	f_s = 40,000 psi	
	s_{max} = lesser of	
	13.13 in.	
	and	
	12 in. √	
	since $s = 12\ in. = s_{max} = 12\ in.$ for flexural reinforcement	

Step	Computation	Reference
3. Concrete One-way Slab Approach (cont'd)	$s = 12\ in. = s_{max} = 12\ in.$ for temp. and shrinkage O.K (10) Check for deflection: According to ACI 318-19, Section 7.3.2.2, since the applied slab thickness satisfies Table 7.3.1.1, deflections occuring after the member becomes composite need not to be checked. (11) Conclusion: For stairway slab, use #4 reinforcing rebar in every 12 in. spacing for flexural reinforcement, and use #4 reinforcing rebar in every 12 in. spacing for temperature reinforcement. For landing, use #4 reinforcing rebar in every 12 in. spacing for flexural reinforcement, and use #4 reinforcing rebar in every 12 in. spacing for temperature reinforcement. Flexural reinforcement in both stair slab and landing has two layers, both acting as tention reinforcement.	*ACI 318-19* *Section 7.3.2.2*

Step	Computation	Reference
3. Concrete One-way Slab Approach (cont'd)	**3.6 Design for S-KS-8-ii:** (1) Minimum slab thickness, h: The boundary condition for slab is simply supported. Therefore, when l= 116 in. h = 5.79 in. The assumption of the slab thickness was 8 in., which is greater than 5.79 in.. Deflection check is not required. Therefore, h = 8 in. (2) Check if L ≤ 3D: L = 467 plf D = 635 plf 3D = 1906 plf O.K. (3) Shear and moment: According to RISA 3D model, see Figure 4.4 and Table 4.4 For stair slab: Shear strength, V_u = 6.2 kips Flexural strength, M_u = 70.7 ft-kips If applying 1' strip method, Shear strength, V_u = 1.33 kips Flexural strength, M_u = 15.15 ft-kips For landing: Shear strength, V_u = 16.6 kips Flexural strength, M_u = 55.7 ft-kips If applying 1' strip method, Shear strength, V_u = 3.56 kips Flexural strength, M_u = 11.94 ft-kips (4) Calculate required area of reinforcement, $\int_{rqd}$: The values of each parameter are: Cealr cover = 0.75 in. half of the assumed rebar dia. = 0.375 in. (#6) d'= 1.125 in. h = 8 in. d= 6.875 in. b = 12.00 in. 1' strip f_c'= 4,000 psi	

Step	Computation	Reference
3. Concrete One-way Slab Approach (cont'd)	f_y= 60,000 psi	
	Tension controlled, Φ = 0.9	*Table 21.2.2*
	For stair:	
	m= 17.65	
	R_u= 356.14	
	$\int_{rqd}$= 0.0063	
	$A_{s,rqd}$ = 0.52 square in.	
	For landing:	
	m= 17.65	
	R_u= 213.85	
	$\int_{rqd}$= 0.0037	
	$A_{s,rqd}$ = 0.35 square in.	
	(5) Check for flexural reinforcement limit:	
	$A_{s,min}$ = greater of	
	0.149 square in. √	
	and	
	0.116 square in.	
	(Minimum area is 0.17 in^2 for landing)	
	Both stair slab and landing have greater required area of reinforcement than minimum area of reinforcement, so they are O.K.	
	(6) Determine center-to-center spacing, s	
	If pick bar size, no. = 5	
	diameter of bar = 0.625 in.	*ACI 318-19* *Appendix A*
	area of bar = 0.31 square in.	
	since, $A_{s,rqd}$= 0.52 square in. for stair slab	
	No. of bar in 1 ft. strip = 2	
	Center-to-center spacing, s = 6 in.	
	If pick bar size, no. = 4	
	diameter of bar = 0.500 in.	*ACI 318-19* *Appendix A*
	area of bar = 0.20 square in.	
	since, $A_{s,rqd}$= 0.35 square in. for landing	
	No. of bar in 1 ft. strip = 2	
	Center-to-center spacing, s = 6 in.	
	Now, check for minimum spacing:	
	The diameter of the aggregate is neglected for this project since it shall be determined by the manufacturer.	
	So, s_{min} = greater of	
	1 in. √	
	and	
	0.625 in. for #5	

Step	Computation	Reference
3. Concrete One-way Slab Approach (cont'd)	0.500 in. for #4 Since, $s = 6\ in. > s_{min} = 1\ in.$ O.K. (7) Check for shear limit: Apply equation (b) shown in Section 3.1, Step (7), in accordance with ACI 318-19, Table 22.5.5.1 The value of each parameter shown as follows: b_w = 12 in. 1' strip d = 6.875 in. λ_s = 1.00 f_c' = 4,000 psi $\emptyset$ = 0.75 λ = 1.0 for stair ρ_w = 0.0075 for landing ρ_w = 0.0042 Therefore, For stairway, $\emptyset V_c$ = 6.13 kips For landing, $\emptyset V_c$ = 5.80 kips Since both of them are greater than the value of V_u no need for shear reinforcement. (8) Minimum temperature and shrinkage reinforcement: $A_{ts,min}$= greater of 0.149 square in. √ and 0.116 square in. If pick bar size, no. = 4 diameter of bar = 0.500 in. area of bar = 0.20 square in. Since, $A_{ts,min}$= 0.149 square in. for 1' strip slab No. of bar in 1 ft. strip = 1 Center-to-center spacing, s = 12 in. Check for maximum spacing: s_{max}= lesser of 40 in. and 18 in. √ Since $s = 12\ in. < s_{max} = 18\ in.$ O.K.	*ACI 318-19* *Table 21.2.1* *Table 19.2.4.2* *ACI 318-19* *7.6.3.1* *ACI 318-19* *Appendix A*

Step	Computation	Reference
3. Concrete One-way Slab Approach (cont'd)	(9) Check for cracking control spacing requirement: Clear cover, c_c = 0.75 in. f_s = 40,000 psi s_{max}= lesser of 13.13 in. and 12 in. √ since $s = 12\ in. = s_{max} = 12\ in.$ for flexural reinforcement $s = 12\ in. = s_{max} = 12\ in.$ for temp. and shrinkage O.K (10) Check for deflection: According to ACI 318-19, Section 7.3.2.2, since the applied slab thickness satisfies Table 7.3.1.1, deflections occuring after the member becomes composite need not to be checked. (11) Conclusion: For stairway slab, use #5 reinforcing rebar in every 6 in. spacing for flexural reinforcement, and use #4 reinforcing rebar in every 12 in. spacing for temperature reinforcement. For landing, use #4 reinforcing rebar in every 6 in. spacing for flexural reinforcement, and use #4 reinforcing rebar in every 12 in. spacing for temperature reinforcement.	*ACI 318-19 Section 7.3.2.2*

Step	Computation	Reference
3. Concrete One-way Slab Approach (cont'd)	**<u>3.7 Design for S-CA-5-i:</u>** (1) Minimum slab thickness, h: The boundary condition for slab is still both ends continuous. Therefore, when l= 116 in. h = 4.13 in. The assumption of the slab thickness was 5 in., which is greater than 4.13 in.. The assumption is OK. Therefore, h = 5 in. (2) Check if L ≤ 3D: L = 467 plf D = 466 plf 3D = 1398 plf O.K. (3) Shear and moment: According to RISA 3D model, see Figure 4.5 and Table 4.5 For stair slab: Shear strength, V_u = 6.9 kips Flexural positive strength, M_u = 6.2 ft-kips Flexural negative strength, M_u = 10.4 ft-kips If applying 1' strip method, then they become: Shear strength, V_u = 1.48 kips Flexural positive strength, M_u = 1.33 ft-kips Flexural negative strength, M_u = 2.23 ft-kips For landing: Shear strength, V_u = 9.7 kips Flexural positive strength, M_u = 4.3 ft-kips Flexural negative strength, M_u = 10.4 ft-kips If applying 1' strip method, then they become: Shear strength, V_u = 2.08 kips Flexural positive strength, M_u = 0.92 ft-kips Flexural negative strength, M_u = 2.23 ft-kips (4) Calculate required area of reinforcement, $\int_{rqd}$: The value of each parameter shown as follows: Cealr cover = 0.75 in. half of the assumed rebar dia. = 0.375 in. (#6) d'= 1.125 in.	

Step	Computation	Reference
3. Concrete One-way Slab Approach (cont'd)	h = 5 in.	
	d= 3.875 in.	
	b = 12.00 in. 1' strip	
	f_c'= 4,000 psi	
	f_y= 60,000 psi	
	Tension controlled, Φ = 0.9	*Table 21.2.2*
	For stairway, postive moment:	
	m= 17.65	
	R_u= 98.31	
	$\int_{rqd}$= 0.0017	
	$A_{s,rqd+}$= 0.08 square in.	
	For stairway, negative moment:	
	m= 17.65	
	R_u= 164.91	
	$\int_{rqd}$= 0.0028	
	$A_{s,rqd-}$= 0.13 square in.	
	For landing, postive moment:	
	m= 17.65	
	R_u= 16.51	
	$\int_{rqd}$= 0.0003	
	$A_{s,rqd+}$= 0.03 square in.	
	For landing, negative moment:	
	m= 17.65	
	R_u= 39.93	
	$\int_{rqd}$= 0.0007	
	$A_{s,rqd-}$= 0.06 square in.	
	(5) Check for flexural reinforcement limit:	
	$A_{s,min}$= greater of	
	0.084 square in. √	
	and	
	0.065 square in.	
	For calculated values less than the area of minimum reinforcement, apply 0.084 in^2 instead. For landing, it's 0.17 in^2.	
	Therefore,	
	for stairway:	
	$A_{s,rqd+}$= 0.08 square in.	
	$A_{s,rqd-}$= 0.13 square in.	
	for landing:	
	$A_{s,rqd+}$= 0.17 square in.	
	$A_{s,rqd-}$= 0.17 square in.	

Step	Computation	Reference
3. Concrete One-way Slab Approach (cont'd)	(6) Determine center-to-center spacing, s If pick bar size, no. = 4 diameter of bar = 0.500 in. area of bar = 0.20 square in. For stairway, Since, $A_{s,rqd+}$= 0.08 square in. for 1' strip slab No. of bar in 1 ft. strip = 1 Center-to-center spacing, s = 12 in. Since, $A_{s,rqd-}$= 0.13 square in. for 1' strip slab No. of bar in 1 ft. strip = 1 Center-to-center spacing, s = 12 in. For landing, Since, $A_{s,rqd+}$= 0.17 square in. for 1' strip slab No. of bar in 1 ft. strip = 1 Center-to-center spacing, s = 12 in. Since, $A_{s,rqd-}$= 0.17 square in. for 1' strip slab No. of bar in 1 ft. strip = 1 Center-to-center spacing, s = 12 in. Now, check for minimum spacing: The diameter of the aggregate is neglected for this project since it shall be determined by the manufacturer. So, s_{min} = greater of 1 in. √ and 0.500 in. Since $s = 12\ in. > s_{min} = 1\ in.$ O.K. Check for maximum spacing: s_{max} = lesser of 15 in. √ and 18 in. Since $s = 12\ in. < s_{max} = 15\ in.$ O.K. (7) Check for shear limit: Apply equation (b) shown in Section 3.1, Step (7), in accordance with ACI 318-19, Table 22.5.5.1 The value of each parameter shown as follows: b_w = 12 in. 1' strip	*ACI 318-19 Appendix A*

Step	Computation	Reference
3. Concrete One-way Slab Approach (cont'd)	d = 3.875 in.	
	λ_s = 1.00	
	f'_c = 4,000 psi	*ACI 318-19*
	$\emptyset$ = 0.75	*Table 21.2.1*
	λ = 1.0	*Table 19.2.4.2*
	for stairway ρ_w = 0.0086 governs	
	for landing ρ_w = 0.0042 governs	
	Therefore,	
	For stairway,	
	$\emptyset V_c$ = 3.62 kips	
	For landing,	
	$\emptyset V_c$ = 5.80 kips	
	Since both of them are greater than the value of V_u no need for shear reinforcement.	*ACI 318-19* *7.6.3.1*
	(8) Minimum temperature and shrinkage reinforcement:	
	$A_{ts,min}$ = greater of	
	0.084 square in. √	
	and	
	0.065 square in.	
	If pick bar size, no. = 4	
	diameter of bar = 0.500 in.	*ACI 318-19*
	area of bar = 0.20 square in.	*Appendix A*
	Since, $A_{ts,min}$ = 0.084 square in.	
	for 1' strip slab	
	No. of bar in 1 ft. strip = 1	
	Center-to-center spacing, s = 12 in.	
	Check for maximum spacing:	
	s_{max} = lesser of	
	25 in.	
	and	
	18 in. √	
	Since $s = 12\ in. < s_{max} = 18\ in.$ O.K.	
	(9) Check for cracking control spacing requirement:	
	Clear cover, c_c = 0.75 in.	
	f_s = 40,000 psi	
	s_{max} = lesser of	
	13.13 in.	
	and	
	12 in. √	
	since $s = 12\ in. = s_{max} = 12\ in.$ for flexural reinforcement	

Step	Computation	Reference
3. Concrete One-way Slab Approach (cont'd)	$s = 12\ in. = s_{max} = 12\ in.$ for temp. and shrinkage O.K (10) Check for deflection: According to ACI 318-19, Section 7.3.2.2, since the applied slab thickness satisfies Table 7.3.1.1, deflections occuring after the member becomes composite need not to be checked. (11) Conclusion: For stairway slab, use #4 reinforcing rebar in every 12 in. spacing for flexural reinforcement, and use #4 reinforcing rebar in every 12 in. spacing for temperature reinforcement. For landing, use #4 reinforcing rebar in every 12 in. spacing for flexural reinforcement, and use #4 reinforcing rebar in every 12 in. spacing for temperature reinforcement. Flexural reinforcement in both stair slab and landing has two layers, both acting as tention reinforcement.	*ACI 318-19 Section 7.3.2.2*

Step	Computation	Reference
3. Concrete One-way Slab Approach (cont'd)	**<u>3.8 Design for S-CA-5-ii:</u>** (1) Minimum slab thickness, h: The boundary condition for slab is simply supported. Therefore, when l = 116 in. h = 5.79 in. The assumption of the slab thickness was 5 in., which is less than 5.79 in.. Deflection check is required. Therefore, h = 5 in. (2) Check if L ≤ 3D: L = 467 plf D = 466 plf 3D = 1398 plf O.K. (3) Shear and moment: According to RISA 3D model, see Figure 4.6 and Table 4.6 For stair slab: Shear strength, V_u = 6.9 kips Flexural strength, M_u = 88.9 ft-kips If applying 1' strip method, Shear strength, V_u = 1.48 kips Flexural strength, M_u = 19.05 ft-kips For landing: Shear strength, V_u = 22.9 kips Flexural strength, M_u = 72.3 ft-kips If applying 1' strip method, Shear strength, V_u = 4.91 kips Flexural strength, M_u = 15.49 ft-kips (4) Calculate required area of reinforcement, $\int_{rqd}$: The values of each parameter are: Cealr cover = 0.75 in. half of the assumed rebar dia. = 0.375 in. (#6) d'= 1.125 in. h = 5 in. d= 3.875 in. b = 12.00 in. 1' strip f_c'= 4,000 psi	

Step	Computation	Reference
3. Concrete One-way Slab Approach (cont'd)	f_y= 60,000 psi	
	Tension controlled, Φ = 0.9	*Table 21.2.2*
	For stair:	
	m= 17.65	
	R_u= 1409.64	
	$\int_{rqd}$= 0.0332	
	$A_{s,rqd}$ = 1.55 square in.	
	For landing:	
	m= 17.65	
	R_u= 277.58	
	$\int_{rqd}$= 0.0048	
	$A_{s,rqd}$ = 0.46 square in.	
	(5) Check for flexural reinforcement limit:	
	$A_{s,min}$ = greater of	
	0.084 square in. √	
	and	
	0.0651 square in.	
	As previously said, the minimum reinforcement area of landing is 0.10 in^2. Since the required area satisfy with the minimums, it is O.K.	
	(6) Determine center-to-center spacing, s	
	If pick bar size, no. = 6	
	diameter of bar = 0.750 in.	*ACI 318-19 Appendix A*
	area of bar = 0.44 square in.	
	since, $A_{s,rqd}$= 1.55 square in. for stair slab	
	No. of bar in 1 ft. strip = 4	
	Center-to-center spacing, s = 3 in.	
	If pick bar size, no. = 4	
	diameter of bar = 0.500 in.	*ACI 318-19 Appendix A*
	area of bar = 0.20 square in.	
	since, $A_{s,rqd}$= 0.46 square in. for landing	
	No. of bar in 1 ft. strip = 3	
	Center-to-center spacing, s = 4 in.	
	Now, check for minimum spacing:	
	The diameter of the aggregate is neglected for this project since it shall be determined by the manufacturer.	
	So, s_{min} = greater of	
	1 in. √	
	and	
	0.500 in. for #4	

Step	Computation	Reference
3. Concrete One-way Slab Approach (cont'd)	0.750 in. for #6	
	Since, $s = 3\ in.\ or\ 4\ in. > s_{min} = 1\ in.$ O.K.	
	(7) Check for shear limit:	
	Apply equation (b) shown in Section 3.1, Step (7), in accordance with ACI 318-19, Table 22.5.5.1	
	The value of each parameter shown as follows:	
	b_w = 12 in. 1' strip	
	d = 3.875 in.	
	λ_s = 1.00	
	f_c' = 4,000 psi	*ACI 318-19*
	$\emptyset$ = 0.75	*Table 21.2.1*
	λ = 1.0	*Table 19.2.4.2*
	for stair ρ_w = 0.0378	
	for landing ρ_w = 0.0063	
	Therefore,	
	For stairway,	
	$\emptyset V_c$ = 5.92 kips	
	For landing,	
	$\emptyset V_c$ = 6.64 kips	
	Since both of them are greater than the value of V_u no need for shear reinforcement.	*ACI 318-19 7.6.3.1*
	(8) Minimum temperature and shrinkage reinforcement:	
	$A_{ts,min}$= greater of	
	0.084 square in. √	
	and	
	0.065 square in.	
	If pick bar size, no. = 4	
	diameter of bar = 0.500 in.	*ACI 318-19*
	area of bar = 0.20 square in.	*Appendix A*
	Since, $A_{ts,min}$= 0.084 square in. for 1' strip slab	
	No. of bar in 1 ft. strip = 1	
	Center-to-center spacing, s = 12 in.	
	Check for maximum spacing:	
	s_{max}= lesser of	
	25 in.	
	and	
	18 in. √	
	Since $s = 12\ in. < s_{max} = 18\ in.$ O.K.	

Step | **Computation** | **Reference**

3. Concrete One-way Slab Approach (cont'd)

(9) Check for cracking control spacing requirement:

Clear cover, c_c = 0.75 in.

f_s = 40,000 psi

s_{max}= lesser of

13.13 in.

and

12 in. √

since $s = 4\ in. < s_{max} = 12\ in.$ for flexural reinforcement

$s = 12\ in. = s_{max} = 12\ in$ for temp. and shrinkage

O.K

(10) Check for deflection:

(a) Idealize the surface of the concrete that shows the cross sectional area of the main bars

Now it becomes a rectangularshape of concrete with:

Clear width, b = 56 in.

slab thickness, h = 5 in.

No. of bar size = 5

Area of a single bar, A_s= 0.44 square in.

No. of bars = 4 for 1' strip

Total area of bars, A_s= 7.92 square in.

(b) Determine the modification factor, n to convert steel to concrete

For normal weight concrete,

$$E_c = 57{,}000\sqrt{f_c'}$$

ACI 318-19 19.2.2.1.a

$$n = \frac{E_s}{E_c}$$

Since,

f_c' = 4,000 psi

E_c = 3,605 ksi

E_s = 29,000 ksi

So that, n = 8.04

(c) Assume the section is cracked, determine the cracking moment

$$f_r = 7.5\lambda\sqrt{f_c'}$$

ACI 318-19

Eq. 19.2.3.1

$$I_g = \frac{1}{12}bh^3$$

Area moment of inertia of rectangular section

$$M_{cr} = \frac{f_r I_g}{y_t}$$

Eq. 24.2.3.5b

where

f_c'= 4,000 psi

λ = 1.0

Step	Computation	Reference
3. Concrete One-way Slab Approach (cont'd)	f_r= 474 psi I_g= 583 in^4 y_t= 2.5 in. M_{cr}= 9.22 ft-kips Since, $M_{cr} = 9.22\, ft - kips < M_u = 88.9\, ft - kips$, it's cracked (d) Apply transformed area method to determine the distance from the exterme compression fiber to the neutral axis, C_{NA} From Section 3.1, Step (10), $\bar{y} = \frac{\sum A_i y_i}{\sum A_i}$ $C_{NA} = \frac{(bC_{NA})\left(\frac{C_{NA}}{2}\right) + (nA_s)d}{(bC_{NA}) + (nA_s)}$ Then, it becomes a quadratic equation: $\frac{b}{2}C_{NA}^2 + nA_sC_{NA} - nA_sd = 0$ The solution of C_{NA}: $C_{NA} = \frac{-nA_s \mp \sqrt{(nA_s)^2+2b(nA_sd)}}{b}$ Therefore, C_{NA} = 2.04 in. (e) Calculate moment of inertia of cracked section transformed to concrete, I_{cr} $I_{cr} = \sum I_i + A_i d_{yi}^2$ Then, it becomes: $I_{cr} = \frac{1}{3}bC_{NA}^3 + nA_s(d - C_{NA})^2$ (The value of I_x is so small that can be neglected) Therefore, I_{cr} = 373.00 in^4 (f) Calculate effective moment of inertia, I_e $I_e = \frac{I_{cr}}{1 - \left(\frac{(2/3)M_{cr}}{M_a}\right)^2 (1 - \frac{I_{cr}}{I_g})}$ M_{cr}= 9.22 ft-kips M_a = 88.90 ft-kips I_g= 583 in^4 I_{cr}= 373 in^4 So that, I_e = 374 in^4	*ACI 318-19 Eq. 24.2.3.5(b)*

Step	Computation	Reference
3. Concrete One-way Slab Approach (cont'd)	(g) Check deflection: According to ACI 318-19, Table 24.2.2, immediate deflection due to live load L is $l/360$ l = 116 in. for slab span length $l/360$ = 0.32 Only consider the middle span where the stairway slab is located, see Figure $\Delta_{max} = 5wl^4/384EI$ $E_c = 57{,}000\sqrt{f_c'}$ f_c'= 4 ksi E_c= 3,605 ksi l = 116 in. I_e= 374 in^4 w= 1669 plf Δ_{max}= 0.24 Since, $\Delta_{max} = 0.24 < l/360 = 0.32$, this design satisfies with the deflection requirement (11) Conclusion: For stairway slab, use #6 reinforcing rebar in every 3 in. spacing for flexural reinforcement, and use #4 reinforcing rebar in every 12 in. spacing for temperature reinforcement. For landing, use #4 reinforcing rebar in every 4 in. spacing for flexural reinforcement, and use #4 reinforcing rebar in every 12 in. spacing for temperature reinforcement.	*ACI 318-19 Eq. 19.2.2.1.b*

Step	Computation	Reference
3. Concrete One-way Slab Approach (cont'd)	**<u>3.9 Design for S-CA-8-i:</u>** (1) Minimum slab thickness, h: The boundary condition for slab is still both ends continuous. Therefore, when l = 116 in. h = 4.13 in. The assumption of the slab thickness was 8 in., which is greater than 4.13 in.. The assumption is OK. Therefore, h = 8 in. (2) Check if L ≤ 3D: L = 467 plf D = 635 plf 3D = 1906 plf O.K. (3) Shear and moment: According to RISA 3D model, see Figure 4.7 and Table 4.7 For stair slab: Shear strength, V_u = 8.7 kips Flexural positive strength, M_u = 8.5 ft-kips Flexural negative strength, M_u = 12.6 ft-kips If applying 1' strip method, then they become: Shear strength, V_u = 1.86 kips Flexural positive strength, M_u = 1.82 ft-kips Flexural negative strength, M_u = 2.70 ft-kips For landing: Shear strength, V_u = 10.1 kips Flexural positive strength, M_u = 3.5 ft-kips Flexural negative strength, M_u = 12.6 ft-kips If applying 1' strip method, then they become: Shear strength, V_u = 2.16 kips Flexural positive strength, M_u = 0.75 ft-kips Flexural negative strength, M_u = 2.70 ft-kips (4) Calculate required area of reinforcement, $\int_{rqd}$: The value of each parameter shown as follows: Cealr cover = 0.75 in. half of the assumed rebar dia. = 0.375 in. (#6) d'= 1.125 in.	

Step	Computation	Reference
3. Concrete One-way Slab Approach (cont'd)	h = 8 in.	
	d= 6.875 in.	
	b = 12.00 in. 1' strip	
	f_c'= 4,000 psi	
	f_y= 60,000 psi	
	Tension controlled, Φ = 0.9	*Table 21.2.2*
	For stairway, postive moment:	
	m= 17.65	
	R_u= 42.82	
	$\int_{rqd}$= 0.0007	
	$A_{s,rqd+}$= 0.06 square in.	
	For stairway, negative moment:	
	m= 17.65	
	R_u= 63.47	
	$\int_{rqd}$= 0.0011	
	$A_{s,rqd-}$= 0.09 square in.	
	For landing, postive moment:	
	m= 17.65	
	R_u= 13.44	
	$\int_{rqd}$= 0.0002	
	$A_{s,rqd+}$= 0.02 square in.	
	For landing, negative moment:	
	m= 17.65	
	R_u= 48.37	
	$\int_{rqd}$= 0.0008	
	$A_{s,rqd-}$= 0.08 square in.	
	(5) Check for flexural reinforcement limit:	
	$A_{s,min}$= greater of	
	0.149 square in. √	
	and	
	0.116 square in.	
	For calculated values less than the area of minimum reinforcement, apply 0.149 in^2 instead. For Landing, it is 0.17 in^2.	
	Therefore,	
	for stairway:	
	$A_{s,rqd+}$= 0.15 square in.	
	$A_{s,rqd-}$= 0.15 square in.	
	for landing:	
	$A_{s,rqd+}$= 0.17 square in.	
	$A_{s,rqd-}$= 0.17 square in.	

Step	Computation	Reference
3. Concrete One-way Slab Approach (cont'd)	(6) Determine center-to-center spacing, s If pick bar size, no. = 4 diameter of bar = 0.500 in. area of bar = 0.20 square in. For stairway, Since, $A_{s,rqd+}$ = 0.15 square in. for 1' strip slab No. of bar in 1 ft. strip = 1 Center-to-center spacing, s = 12 in. Since, $A_{s,rqd-}$ = 0.15 square in. for 1' strip slab No. of bar in 1 ft. strip = 1 Center-to-center spacing, s = 12 in. For landing, Since, $A_{s,rqd+}$ = 0.17 square in. for 1' strip slab No. of bar in 1 ft. strip = 1 Center-to-center spacing, s = 12 in. Since, $A_{s,rqd-}$ = 0.17 square in. for 1' strip slab No. of bar in 1 ft. strip = 1 Center-to-center spacing, s = 12 in. Now, check for minimum spacing: The diameter of the aggregate is neglected for this project since it shall be determined by the manufacturer. So, s_{min} = greater of 1 in. √ and 0.500 in. Since $s = 12\ in. > s_{min} = 1\ in.$ O.K. Check for maximum spacing: s_{max} = lesser of 24 in. and 18 in. √ Since $s = 12\ in. < s_{max} = 18\ in.$ O.K. (7) Check for shear limit: Apply equation (b) shown in Section 3.1, Step (7), in accordance with ACI 318-19, Table 22.5.5.1 The value of each parameter shown as follows: b_w = 12 in. 1' strip	*ACI 318-19 Appendix A*

Step	Computation	Reference
3. Concrete One-way Slab Approach (cont'd)	d = 6.875 in.	
	λ_s = 1.00	
	f'_c = 4,000 psi	*ACI 318-19*
	Ø = 0.75	*Table 21.2.1*
	λ = 1.0	*Table 19.2.4.2*
	for stairway ρ_w = 0.0048 governs	
	for landing ρ_w = 0.0042 governs	
	Therefore,	
	For stairway,	
	$\emptyset V_c$ = 5.30 kips	
	For landing,	
	$\emptyset V_c$ = 5.80 kips	
	Since both of them are greater than the value of V_u no need for shear reinforcement.	*ACI 318-19 7.6.3.1*
	(8) Minimum temperature and shrinkage reinforcement:	
	$A_{ts,min}$= greater of	
	0.149 square in. √	
	and	
	0.116 square in.	
	If pick bar size, no. = 4	
	diameter of bar = 0.500 in.	*ACI 318-19*
	area of bar = 0.20 square in.	*Appendix A*
	Since, $A_{ts,min}$= 0.149 square in.	
	for 1' strip slab	
	No. of bar in 1 ft. strip = 1	
	Center-to-center spacing, s = 12 in.	
	Check for maximum spacing:	
	s_{max}= lesser of	
	40 in.	
	and	
	18 in. √	
	Since $s = 12\ in. < s_{max} = 18\ in.$ O.K.	
	(9) Check for cracking control spacing requirement:	
	Clear cover, c_c = 0.75 in.	
	f_s = 40,000 psi	
	s_{max}= lesser of	
	13.13 in.	
	and	
	12 in. √	
	since $s = 12\ in. = s_{max} = 12\ in.$ for flexural reinforcement	

Step	Computation	Reference
3. Concrete One-way Slab Approach (cont'd)	$s = 12\ in. = s_{max} = 12\ in.$ for temp. and shrinkage O.K (10) Check for deflection: According to ACI 318-19, Section 7.3.2.2, since the applied slab thickness satisfies Table 7.3.1.1, deflections occuring after the member becomes composite need not to be checked. (11) Conclusion: For stairway slab, use #4 reinforcing rebar in every 12 in. spacing for flexural reinforcement, and use #4 reinforcing rebar in every 12 in. spacing for temperature reinforcement. For landing, use #4 reinforcing rebar in every 12 in. spacing for flexural reinforcement, and use #4 reinforcing rebar in every 12 in. spacing for temperature reinforcement. Flexural reinforcement in both stair slab and landing has two layers, both acting as tention reinforcement.	*ACI 318-19 Section 7.3.2.2*

Step	Computation	Reference
3. Concrete One-way Slab Approach (cont'd)	**<u>3.10 Design for S-CA-8-ii:</u>** (1) Minimum slab thickness, h: The boundary condition for slab is simply supported. Therefore, when l = 116 in. h = 5.79 in. The assumption of the slab thickness was 8 in., which is greater than 5.79 in.. Deflection check is not required. Therefore, h = 8 in. (2) Check if L ≤ 3D: L = 467 plf D = 635 plf 3D = 1906 plf O.K. (3) Shear and moment: According to RISA 3D model, see Figure 4.8 and Table 4.8 For stair slab: Shear strength, V_u = 8.7 kips Flexural strength, M_u = 103.6 ft-kips If applying 1' strip method, Shear strength, V_u = 1.86 kips Flexural strength, M_u = 22.20 ft-kips For landing: Shear strength, V_u = 25.1 kips Flexural strength, M_u = 82.5 ft-kips If applying 1' strip method, Shear strength, V_u = 5.38 kips Flexural strength, M_u = 17.68 ft-kips (4) Calculate required area of reinforcement, $\int_{rqd}$: The values of each parameter are: Cealr cover = 0.75 in. half of the assumed rebar dia. = 0.375 in. (#6) d'= 1.125 in. h = 8 in. d= 6.875 in. b = 12.00 in. 1' strip f_c'= 4,000 psi	

Step	Computation	Reference
3. Concrete One-way Slab Approach (cont'd)	f_y= 60,000 psi	
	Tension controlled, Φ = 0.9	*Table 21.2.2*
	For stair:	
	m= 17.65	
	R_u= 521.87	
	$\int_{rqd}$= 0.0095	
	$A_{s,rqd}$ = 0.78 square in.	
	For landing:	
	m= 17.65	
	R_u= 316.74	
	$\int_{rqd}$= 0.0056	
	$A_{s,rqd}$ = 0.52 square in.	
	(5) Check for flexural reinforcement limit:	
	$A_{s,min}$ = greater of	
	0.149 square in. √	
	and	
	0.116 square in.	
	Both stair slab and landing have greater required area of reinforcement than minimum area of reinforcement, so they are O.K.	
	(6) Determine center-to-center spacing, s	
	If pick bar size, no. = 6	
	diameter of bar = 0.750 in.	*ACI 318-19* *Appendix A*
	area of bar = 0.44 square in.	
	since, $A_{s,rqd}$= 0.78 square in. for stair slab	
	No. of bar in 1 ft. strip = 2	
	Center-to-center spacing, s = 6 in.	
	If pick bar size, no. = 5	
	diameter of bar = 0.625 in.	*ACI 318-19* *Appendix A*
	area of bar = 0.31 square in.	
	since, $A_{s,rqd}$= 0.52 square in. for landing	
	No. of bar in 1 ft. strip = 2	
	Center-to-center spacing, s = 6 in.	
	Now, check for minimum spacing:	
	The diameter of the aggregate is neglected for this project since it shall be determined by the manufacturer.	
	So, s_{min} = greater of	
	1 in. √	
	and	
	0.750 in. for #6	

Step	Computation	Reference
	0.625 in. for #5	
	Since, $s = 6\ in. > s_{min} = 1\ in.$ O.K.	
	(7) Check for shear limit:	
	Apply equation (b) shown in Section 3.1, Step (7), in accordance with ACI 318-19, Table 22.5.5.1	
	The value of each parameter shown as follows:	
	b_w = 12 in. 1' strip	
	d = 6.875 in.	
	λ_s = 1.00	
	f_c' = 4,000 psi	*ACI 318-19*
	$\emptyset$ = 0.75	*Table 21.2.1*
	λ = 1.0	*Table 19.2.4.2*
	for stair ρ_w = 0.0107	
	for landing ρ_w = 0.0066	
	Therefore,	
	For stairway,	
	$\emptyset V_c$ = 6.89 kips	
	For landing,	
3. Concrete One-way Slab Approach (cont'd)	$\emptyset V_c$ = 6.71 kips	
	Since both of them are greater than the value of V_u no need for	*ACI 318-19*
	shear reinforcement.	*7.6.3.1*
	(8) Minimum temperature and shrinkage reinforcement:	
	$A_{ts,min}$ = greater of	
	0.149 square in. √	
	and	
	0.116 square in.	
	If pick bar size, no. = 4	
	diameter of bar = 0.500 in.	*ACI 318-19*
	area of bar = 0.20 square in.	*Appendix A*
	Since, $A_{ts,min}$ = 0.149 square in.	
	for 1' strip slab	
	No. of bar in 1 ft. strip = 1	
	Center-to-center spacing, s = 12 in.	
	Check for maximum spacing:	
	s_{max} = lesser of	
	40 in.	
	and	
	18 in. √	
	Since $s = 12\ in. < s_{max} = 18\ in.$ O.K.	

Step	Computation	Reference
3. Concrete One-way Slab Approach (cont'd)	(9) Check for cracking control spacing requirement: Clear cover, c_c= 0.75 in. f_s= 40,000 psi s_{max}= lesser of 13.13 in. and 12 in. √ since $s = 6\ in. < s_{max} = 12\ in.$ for flexural reinforcement $s = 12\ in. = s_{max} = 12\ in.$ for temp. and shrinkage O.K (10) Check for deflection: According to ACI 318-19, Section 7.3.2.2, since the applied slab thickness satisfies Table 7.3.1.1, deflections occuring after the member becomes composite need not to be checked. (11) Conclusion: For stairway slab, use #6 reinforcing rebar in every 6 in. spacing for flexural reinforcement, and use #4 reinforcing rebar in every 12 in. spacing for temperature reinforcement. For landing, use #5 reinforcing rebar in every 6 in. spacing for flexural reinforcement, and use #4 reinforcing rebar in every 12 in. spacing for temperature reinforcement.	*ACI 318-19 Section 7.3.2.2*

Step | **Computation** | **Reference**

4. Concrete Beam Approach

4.1 General steps:

(1) Minimum slab thickness, h:

Acoording to ACI 318-19, Table 9.3.1.1 - Minimum thickness of solid nonprestressed one-way slabs:

Support condition	Minimum h
Simply supported	$l/16$
One end continuous	$l/18.5$
Both ends continuous	$l/21$
Cantilever	$l/8$

ACI 318-19 Table 9.3.1.1

(2) Check if L ≤ 3D, where L stands for live load, D stands for dead load:

ACI 318-19 Section 6.5.1

(3) Calculate shear and moment based on RISA 3D

(4) Calculate required area of reinforcement, $\int$, based on flexural moment:

$$m = \frac{f_y}{0.85 f_c'}$$

$$R_u = \frac{M_u}{\Phi b d^2}$$

$$\int_{rqd} = \frac{1}{m}[1 - \sqrt{1 - \left(\frac{2R_u m}{f_y}\right)}]$$

where

b = width of compression face of member, in.

d = distance from extreme compression fiber to centroid of longitudinal tension reinforcement, in.

f_y = specified yield strength for nonprestressed reinforcement, psi

f_c' = specified compressive strength of concrete, psi

M_u = actual moment, ft-kip

R_u, m = formula conversion parameter

$\int_{rqd}$ = required area of reinforcement

Φ = strength reduction factor

(5) Check for flexural reinforcement limits:

$A_{s,min}$ shall be the greater of (a) and (b), according to ACI 318-19, Section 9.6.1.2

(a) $$\frac{3\sqrt{f_c'}}{f_y} b_w d$$

(b) $$\frac{200}{f_y} b_w d$$

ACI 318-19 Section 9.6.1.2

Step | **Computation** | **Reference**

4. Concrete Beam Approach (cont'd)

(6) Shear reinforcement:
A minimum area of shear reinforcement, $A_{v,min}$ shall be provided in all regions where $V_u > 0.5\Phi V_c$

ACI 318-19 Section 9.6.3.1

If shear reinforcement is required, then:
Set

$$V_u = \emptyset V_n, \text{where } \emptyset = 0.75$$

then

$$V_n = V_c + V_s$$

Eq. 22.5.1.1

and

$$V_c = \left[2\lambda\sqrt{f_c'} + \frac{N_u}{6A_g}\right] b_w d$$

Table 22.5.5.1 (b)

$$V_s = \frac{A_v f_{yt} d}{s}$$

Eq. 22.5.10.5.3

which can be re-writen to:

$$\frac{A_v}{s} = \frac{V_s}{f_{yt} d}$$

Check with minimum shear reinforcement, in accordance with ACI 318-19, Section 9.6.3.3

Beam type	$A_{v,min}/s$	
Nonprestressed	Greater of:	$0.75\sqrt{f_c'}\frac{b_w}{f_{yt}}$
		$50\frac{b_w}{f_{yt}}$

ACI 318-19 Table 9.6.3.3

where
A_g = gross area of concrete section
A_v = area of shear reinforcement within spacing
b_w = web width or diameter of circular section
d = distance from exterme compression fiber to controid of longitudinal tension reinforcement
f_c' = specified compressive strength of concrete
f_{yt} = specified yield strength of transverse reinforcement, in.
N_u = factored axial force normal to cross section
s = center-to-center spacing of reinforcement, in.
V_c = shear srength provided by concrete
V_n = equivalent concrete stress corresponding to nominal two-way shear strength
V_s = shear srength provided by reinforcement
V_u = maximum factored two-way shear stress calculated around the perimeter of a given critical section
λ = modification factor to reflect the reduced mechanical properties of lightweight concrete relative to normal weight concrete of the same compressive strength
s = center-to-center spacing of reinforcement, in.

<table>
<tr><th>Step</th><th>Computation</th><th>Reference</th></tr>
<tr>
<td>4. Concrete Beam Approach (cont'd)</td>
<td>
(7) Determine center-to-center spacing, s of reinforcement:
Minimum spacing - shall be at least the greatest of 1 in., d_b and $(4/3)d_{agg}$
Minimum layer spacing - shall be at least 1 in. between layers for parallel nonprestressed reinforcement placed in two or more horizontal layers

(8) Check for cracking control spacing requirement:
<table>
<tr><th>Reinforcement type</th><th colspan="2">Maximum spacing s</th></tr>
<tr><td rowspan="2">Deformed bars or wires</td><td rowspan="2">Lesser of:</td><td>$15\left(\frac{40{,}000}{f_s}\right) - 2.5c_c$</td></tr>
<tr><td>$12\left(\frac{40{,}000}{f_s}\right)$</td></tr>
</table>

(9) Check for deflection:
Deflection shall be calculated in accordance with ACI 318-19 Section 24.2 and shall not exceed the limits in 24.2.2.

Assume the section is cracked, determine the cracking moment
$$f_r = 7.5\lambda\sqrt{f_c'}$$
$$I_g = \frac{1}{12}bh^3$$
$$M_{cr} = \frac{f_r I_g}{y_t}$$
where
f_c' = specified compressive strength of concrete, psi
f_r = modulus of rupture of concrete, psi
I_g = area moment of inertia
M_{cr} = cracking moment
y_t = distance from centroidal axis of gross section, neglecting reinforcement, to tension face, in.
λ = modification factor to reflect the reduced mechanical properties of lightweight concrete relative to normal weight concrete of the same compressive strength

Apply transformed area method to determine the distance from the exterme compression fiber to the neutral axis, C_{NA}
$$\bar{y} = \frac{\sum A_i y_i}{\sum A_i}$$
If take the very top line of the section as the reference line, then the equation becomes:
</td>
<td>
<i>ACI 318-19 Section 25.2.1, and 25.2.2</i>

<i>ACI 318-19 Table 24.3.2</i>

<i>ACI 318-19 Section 7.3.2.1, 24.2, 24.2.2</i>

<i>ACI 318-19</i>
<i>Eq. 19.2.3.1</i>
<i>Area moment of inertia of rectangular section</i>
<i>Eq. 24.2.3.5b</i>
</td>
</tr>
</table>

Step	Computation	Reference
4. Concrete Beam Approach (cont'd)	$C_{NA} = \frac{(bC_{NA})\left(\frac{C_{NA}}{2}\right) + (nA_s)d}{(bC_{NA}) + (nA_s)}$ Then, it becomes a quadratic equation: $\frac{b}{2}C_{NA}^2 + nA_sC_{NA} - nA_sd = 0$ The solution of C_{NA}: $C_{NA} = \frac{-nA_s \mp \sqrt{(nA_s)^2+2b(nA_sd)}}{b}$ Calculate moment of inertia of cracked section transformed to concrete, I_{cr} $I_{cr} = \sum I_i + A_i d_{yi}^2$ Then, it becomes: $I_{cr} = \frac{1}{12}bC_{NA}^3 + bC_{NA}(\frac{C_{NA}}{2})^2+nA_s(d - C_{NA})^2$ (The value of I_x is so small that can be neglected) or $I_{cr} = \frac{1}{3}bC_{NA}^3 + nA_s(d - C_{NA})^2$ Calculate effective moment of inertia, I_e $I_e = \frac{I_{cr}}{1 - \left(\frac{(2/3)M_{cr}}{M_a}\right)^2 (1 - \frac{I_{cr}}{I_g})}$ Then, check deflection According to ACI 318-19, Table 24.2.2, immediate deflection due to live load $L$is $l/360$ $l/360$ Only consider the middle span where the stairway slab is located, $\Delta_{max}= 5wl^4/384EI$ $E_c = 57{,}000\sqrt{f_c'}$	*ACI 318-19 Table 24.2.3.5(b)* *ACI 318-19 Eq. 19.2.2.1.b*

Step	Computation	Reference

4. Concrete Beam Approach (cont'd)

4.2 Eight cases:

There are eight cases to be designed by concrete beam method for this project:

B-KS-5-i
B-KS-5-ii
B-KS-8-i
B-KS-8-ii
B-CA-5-i
B-CA-5-ii
B-CA-8-i
B-CA-8-ii

4.3 Design for B-KS-5-i:

(1) Minimum slab thickness, h:

The boundary condition for slab is still both ends continuous. Therefore,

when l = 116 in.
h = 5.51 in.

The assumption of the slab thickness was 5 in., which is less than 5.51 in.. Need to check for deflection limit.

Therefore, h = 5 in.

(2) Check if L ≤ 3D:

L = 467 plf
D = 466 plf
3D = 1398 plf O.K.

(3) Shear and moment:

According to RISA 3D model, see Figure 4.1 and Table 4.1

For stair slab:

Flexural strength, M_u = 7.8 ft-kips
Shear strength, V_u = 5.4 kips

For landing:

Flexural strength, M_u = 7.8 ft-kips
Shear strength, V_u = 6.3 kips

(4) Calculate required area of reinforcement, $\int_{rqd}$

The values of each parameter are:

Cealr cover = 1.5 in.
half of the assumed rebar dia. = 0.375 in. (#6)
Dia. of stirrup = 0.5 in.
d'= 2.375 in.
h = 5 in.
d= 2.625 in.

Step	Computation	Reference
4. Concrete Beam Approach (cont'd)	b = 56.00 in. f_c'= 4,000 psi f_y= 60,000 psi Tension controlled, Φ = 0.9 For stair slab: m= 17.65 R_u= 269.52 $\int_{rqd}$= 0.0047 Therefore, $A_{s,rqd}$= 0.69 square in. For landing: m= 17.65 R_u= 42.31 $\int_{rqd}$= 0.0007 Therefore, $A_{s,rqd}$= 0.26 square in. **Note: For landing, the value of h is 9 inches. So, it has two layers of reinforcement. (5) Check for flexural reinforcement limit: $A_{s,min}$= greater of 0.46 square in. and 0.49 square in. √ Since, $A_{s,rqd} = 0.70\ in^2 > A_{s,,min} = 0.47\ in^2$ O.K. (For landing, the minimum area is 1.24 in^2. Therefore, change it to 1.24 in^2) (6) Minimum shear reinforcement: Since, b_w= 56.00 in. d = 2.63 in. f_c'= 4,000 psi λ = 1.0 ∅ = 0.75 So that, V_c= 18.59 kips $\emptyset V_c$ = 13.95 kips $0.5\emptyset V_c$ = 6.97 kips For stair slab $0.5\emptyset V_c$ = 17.60 kips For landing (when h = 9 in., and d = 6.625 in.) For stair slab: Since $V_u = 5.4\ kips < 0.5\emptyset V_c = 6.97 kips,$ shear reinforcement is not required. For landing: Since $V_u = 6.3\ kips < 0.5\emptyset V_c = 17.60 kips$, shear reinforcement is not required.	*Table 21.2.2* *ACI 318-19* *Table 19.2.4.2* *Table 21.2.1*

Step	Computation	Reference
4. Concrete Beam Approach (cont'd)	(7) Determine center-to-center spacing, s	
	For stair slab:	
	$A_{s,rqd}$= 0.69 square in.	
	If pick bar size, no. = 4	
	diameter of bar = 0.500 in.	*ACI 318-19* *Appendix A*
	area of bar = 0.20 square in.	
	No. of bar = 4	
	Center-to-center spacing, s = 12 in.	
	For landing:	
	$A_{s,rqd}$= 1.24 square in.	
	If pick bar size, no. = 4	
	diameter of bar = 0.500 in.	*ACI 318-19* *Appendix A*
	area of bar = 0.20 square in.	
	No. of bar = 7	
	Center-to-center spacing, s = 7 in.	
	Now, check for minimum spacing:	
	The diameter of the aggregate is neglected for this project since it shall be determined by the manufacturer.	
	So, s_{min} = greater of	
	1 in. √	
	and	
	0.500 in. #4 bars	
	Since $s = 12\ in. > s_{min} = 1\ in.$ O.K.	
	(8) Check for cracking control spacing requirement:	
	Clear cover, c_c = 1.5 in.	
	f_s = 40,000 psi	
	s_{max}= lesser of	
	11.25 in. √	
	and	
	12 in.	
	Since $s = 12\ in. > s_{max} = 11.25\ in.$ for flexural reinforcement reduce flexural reinforcement spacing to 11 in. to satisfy the standard.	
	(9) Check for deflection:	
	(a) Idealize the surface of the concrete that shows the cross sectional area of the main bars	
	Now it becomes a rectangularshape of concrete with:	
	Clear width, b = 56.00 in.	
	slab thickness, h = 5 in.	
	No. of bar size = 4	
	Area of a single bar, A_s= 0.20 square in.	
	No. of bars = 4	

Step	Computation	Reference

4. Concrete Beam Approach (cont'd)

Total area of bars, A_s = 0.80 square in.

(b) Determine the modification factor, n to convert steel to concrete
For normal weight concrete,

ACI 318-19 19.2.2.1.a

$$E_c = 57{,}000\sqrt{f_c'}$$

$$n = \frac{E_s}{E_c}$$

Since,

f_c' = 4,000 psi
E_c = 3,605 ksi
E_s = 29,000 ksi

So that, n = 8.04

(c) Assume the section is cracked, determine the cracking moment

ACI 318-19

Eq. 19.2.3.1

$$f_r = 7.5\lambda\sqrt{f_c'}$$

Area moment of inertia of rectangular section

$$I_g = \frac{1}{12}bh^3$$

Eq. 24.2.3.5b

$$M_{cr} = \frac{f_r I_g}{y_t}$$

where

f_c'= 4,000 psi
λ = 1.0
f_r= 474 psi
I_g= 583 in^4
y_t= 2.5 in.
M_{cr}= 9.22 ft-kips

Since, $M_{cr} = 9.22\ ft - kips > M_u = 7.8\ ft - kips$, it's not cracked.
No further calculation needed.

(10) Conclusion:
For stair slab, use #4 reinforcing rebar in every 11 in. for flexural reinforcement.
For landing, use #4 reinforcing rebar in every 7 in. for each layer (total two layers) for flexural reinforcement.
No shear reinforcement required for neither stair slab or landing.

Step	Computation	Reference
4. Concrete Beam Approach (cont'd)	**<u>4.4 Design for B-KS-5-ii:</u>** (1) Minimum slab thickness, h: The boundary condition for slab is simply supported. Therefore, when l= 116 in. h = 7.24 in. The assumption of the slab thickness was 5 in., which is less than 7.24 in.. Need to check for deflection limit. Therefore, h = 5 in. (2) Check if L ≤ 3D: L = 467 plf D = 466 plf 3D = 1398 plf O.K. (3) Shear and moment: According to RISA 3D model, see Figure 4.2 and Table 4.2 For stair slab: Flexural strength, M_u = 64.2 ft-kips Shear strength, V_u = 5.4 kips For landing: Flexural strength, M_u = 51.2 ft-kips Shear strength, V_u = 15.6 kips (4) Calculate required area of reinforcement, $\int_{rqd}$ The values of each parameter are: Cealr cover = 1.5 in. half of the assumed rebar dia. = 0.375 in. (#6) Dia. of stirrup = 0.5 in. d'= 2.375 in. h = 5 in. d= 2.625 in. b = 56.00 in. f_c'= 4,000 psi f_y= 60,000 psi Tension controlled, Φ = 0.9 For stair slab: m= 17.65 R_u= 2218.33 $\int_{rqd}$= #NUM! The value of $\int_{rqd}$ is indeterminate by excel, which means it is not in the tension-controlled region. Therefore, concrete beam method does not apply to this problem.	*Table 21.2.2*

Step	Computation	Reference
4. Concrete Beam Approach (cont'd)	**4.5 Design for B-KS-8-i:** (1) Minimum slab thickness, h: The boundary condition for slab is still both ends continuous. Therefore, when l= 116 in. h = 5.51 in. The assumption of the slab thickness was 8 in., which is greater than 5.51 in.. No need to check for deflection limit. Therefore, h = 8 in. (2) Check if L ≤ 3D: L = 467 plf D = 635 plf 3D = 1906 plf O.K. (3) Shear and moment: According to RISA 3D model, see Figure 4.3 and Table 4.3 For stair slab: Shear strength, V_u = 6.2 kips Flexural positive strength, M_u = 6.2 ft-kips Flexural negative strength, M_u = 8.8 ft-kips For landing: Shear strength, V_u = 6.5 kips Flexural positive strength, M_u = 1.9 ft-kips Flexural negative strength, M_u = 8.8 ft-kips (4) Calculate required area of reinforcement, $\int_{rqd}$ The values of each parameter are: Cealr cover = 1.5 in. half of the assumed rebar dia. = 0.375 in. (#6) Dia. of stirrup = 0.5 in. d'= 2.375 in. h = 8 in. d= 5.625 in. b = 56.00 in. f_c'= 4,000 psi f_y= 60,000 psi Tension controlled, Φ = 0.9 For stair slab, postive moment: m= 17.65 R_u= 46.65 $\int_{rqd}$= 0.0008 $A_{s,rqd+}$= 0.25 square in.	 *Table 21.2.2*

Step	Computation	Reference
4. Concrete Beam Approach (cont'd)	For stair slab, negative moment: m= 17.65 R_u= 66.22 $\int_{rqd}$= 0.0011 $A_{s,rqd-}$= 0.35 square in. For landing, postive moment: m= 17.65 R_u= 10.31 $\int_{rqd}$= 0.0002 $A_{s,rqd+}$= 0.06 square in. For landing, negative moment: m= 17.65 R_u= 47.74 $\int_{rqd}$= 0.0008 $A_{s,rqd-}$= 0.30 square in. (5) Check for flexural reinforcement limit: $A_{s,min}$= greater of 1.00 square in. and 1.05 square in. √ None of the areas satisfies with the minimum flexural reinforcement. Increase them to 1.05 square in. For landing, it is 1.24 square in.. Therefore, For stair slab: $A_{s,rqd+}$ = 1.05 square in. $A_{s,rqd-}$ = 1.05 square in. For landing: $A_{s,rqd+}$ = 1.24 square in. $A_{s,rqd-}$ = 1.24 square in. (6) Minimum shear reinforcement: Since, b_w= 56.00 in. d = 5.63 in. f_c'= 4,000 psi λ = 1.0 $\emptyset$ = 0.75 So that, V_c= 39.84 kips $\emptyset V_c$ = 29.88 kips $0.5\emptyset V_c$ = 14.94 kips For stair slab $0.5\emptyset V_c$ = 17.60 kips For landing (when h = 6 in., and d = 3.5 in.)	*ACI 318-19* *Table 19.2.4.2* *Table 21.2.1*

Step	Computation	Reference
4. Concrete Beam Approach (cont'd)	For stair slab: Since $V_u = 6.2\ kips < 0.5\emptyset V_c = 14.94 kips$, shear reinforcement is not required. For landing: Since $V_u = 6.5\ kips < 0.5\emptyset V_c = 17.60 kips$, shear reinforcement is not required. (7) Determine center-to-center spacing, s For stair slab: $A_{s,rqd}$ = 1.05 square in. If pick bar size, no. = 4 diameter of bar = 0.500 in. area of bar = 0.20 square in. No. of bar = 6 Center-to-center spacing, s = 8 in.	*ACI 318-19* *Appendix A*
	For landing: $A_{s,rqd}$ = 1.24 square in. If pick bar size, no. = 4 diameter of bar = 0.500 in. area of bar = 0.20 square in. No. of bar = 7 Center-to-center spacing, s = 7 in.	*ACI 318-19* *Appendix A*
	(8) Check for cracking control spacing requirement: Clear cover, c_c = 1.5 in. f_s = 40,000 psi s_{max} = lesser of 11.25 in. √ and 12 in. Since $s = 8\ in. < s_{max} = 11.25\ in.$ for flexural reinforcement O.K.	
	(9) Check for deflection: According to ACI 318-19, Section 9.3.2.2, since the applied slab thickness satisfies Table 9.3.1.1, deflections occuring after the member becomes composite need not to be checked.	*ACI 318-19* *Section 9.3.2.2*
	(10) Conclusion: For stair slab, it has two layers of #4 reinforcing rebar in every 8 in., both acting as tension reinforcement. For landing, it has two layers of #4 reinforcing rebar in every 7 in.. No shear reinforcement required for neither stair slab or landing.	

Step	Computation	Reference
4. Concrete Beam Approach (cont'd)	**<u>4.6 Design for B-KS-8-ii:</u>** (1) Minimum slab thickness, h: The boundary condition for slab is simply supported. Therefore, when l= 116 in. h = 7.24 in. The assumption of the slab thickness was 8 in., which is greater than 7.24 in.. No need to check for deflection limit. Therefore, h = 8 in. (2) Check if L ≤ 3D: L = 467 plf D = 635 plf 3D = 1906 plf O.K. (3) Shear and moment: According to RISA 3D model, see Figure 4.4 and Table 4.4 For stair slab: Flexural strength, M_u = 70.7 ft-kips Shear strength, V_u = 6.2 kips For landing: Flexural strength, M_u = 55.7 ft-kips Shear strength, V_u = 16.6 kips (4) Calculate required area of reinforcement, $\int_{rqd}$ The values of each parameter are: Cealr cover = 1.5 in. half of the assumed rebar dia. = 0.375 in. (#6) Dia. of stirrup = 0.5 in. d'= 2.375 in. h = 8 in. d= 5.625 in. b = 56.00 in. f_c'= 4,000 psi f_y= 60,000 psi Tension controlled, Φ = 0.9 For stair slab: m= 17.65 R_u= 532.02 $\int_{rqd}$= 0.0097 Therefore, $A_{s,rqd}$= 3.05 square in.	*Table 21.2.2*

Step	Computation	Reference
4. Concrete Beam Approach (cont'd)	For landing:	
	$m=$ 17.65	
	$R_u=$ 302.16	
	$\int_{rqd}=$ 0.0053	
	Therefore, $A_{s,rqd}=$ 1.96 square in.	
	(5) Check for flexural reinforcement limit:	
	$A_{s,min}=$ greater of	
	1.00 square in.	
	and	
	1.05 square in. √	
	Since both values of $A_{s,rqd}$ are greater than $A_{s,min}$, it is O.K..	
	(For landing, the minimum area is 1.24 in^2)	
	(6) Minimum shear reinforcement:	
	Since, $b_w=$ 56.00 in.	
	$d=$ 5.625 in.	
	$f_c'=$ 4,000 psi	*ACI 318-19*
	$\lambda=$ 1.0	*Table 19.2.4.2*
	$\emptyset=$ 0.75	*Table 21.2.1*
	So that, $V_c=$ 39.84 kips	
	$\emptyset V_c=$ 29.88 kips	
	$0.5\emptyset V_c=$ 14.94 kips	
	For stair slab	
	$0.5\emptyset V_c=$ 17.60 kips	
	For landing	
	(when h = 9 in., and d = 6.625 in.)	
	For stair slab:	
	Since $V_u = 6.2\ kips < 0.5\emptyset V_c = 14.94 kips$, shear reinforcement is not required.	
	For landing:	
	Since $V_u = 16.6\ kips < 0.5\emptyset V_c = 17.60 kips$, shear reinforcement is not required.	
	Therefore, for minimum shear reinforcement:	
	$A_{v,min}/s$ shall be the greater of $0.75\sqrt{f_c'}\frac{b_w}{f_{yt}}$	*ACI 318-19*
	or	*Table 9.6.3.3*
	$50\frac{b_w}{f_{yt}}$	
	Since,	
	$b_w=$ ~~56.00~~ in.	
	$f_c'=$ ~~4,000~~ psi	
	$f_{yt}=$ ~~60,000~~ psi	

Step	Computation	Reference
4. Concrete Beam Approach (cont'd)	$A_{v,min}/s$ = greater of 0.0443 and 0.0467 Assume ~~The maximum spacing of shear reinforcement should not be greater than d/2 or 24 in.. Since d/2 = 1.75 in. for landing, take 1.5 in.~~ s = ~~1.5~~ in. $A_{v,min}$ = ~~0.04~~ square in. for single rebar If pick bar size, no. = ~~4~~ diameter of bar = ~~0.500~~ in. area of bar = ~~0.20~~ square in. (7) Determine center-to-center spacing, s For stair slab: $A_{s,rqd}$= 3.05 square in. If pick bar size, no. = 5 diameter of bar = 0.625 in. area of bar = 0.31 square in. No. of bar = 10 Center-to-center spacing, s = 5 in. For landing: $A_{s,rqd}$= 1.96 square in. If pick bar size, no. = 5 diameter of bar = 0.625 in. area of bar = 0.31 square in. No. of bar = 7 Center-to-center spacing, s = 7 in. Now, check for minimum spacing: The diameter of the aggregate is neglected for this project since it shall be determined by the manufacturer. So, s_{min} = greater of 1 in. √ and 0.625 in. #5 bars Since $s = 5\ in. > s_{min} = 1\ in.$ O.K. $s = 7\ in. > s_{min} = 1\ in.$ O.K. (8) Check for cracking control spacing requirement: Clear cover, c_c = 1.5 in. f_s = 40,000 psi s_{max}= lesser of 11.25 in. √ and 12 in. The spacings calculated above satisfy with the minimum requirement.	*ACI 318-19 Appendix A* *ACI 318-19 Appendix A*

Step	Computation	Reference
4. Concrete Beam Approach (cont'd)	(9) Check for deflection: According to ACI 318-19, Section 9.3.2.2, since the applied slab thickness satisfies Table 9.3.1.1, deflections occuring after the member becomes composite need not to be checked. (10) Conclusion: For stair slab, use #5 reinforcing rebar in every 5 in. spacing for flexural reinforcement. No shear reinforcement is required. For landing, use #5 reinforcing rebar in every 7 in. spacing for flexural reinforcement. No shear reinforcement is required.	*ACI 318-19 Section 9.3.2.2*

Step | **Computation** | **Reference**

4.7 Design for B-CA-5-i:

(1) Minimum slab thickness, h:

The boundary condition for slab is still both ends continuous.

Therefore,

when l= 116 in.

h = 5.51 in.

The assumption of the slab thickness was 5 in., which is less than 5.51 in.. Need to check for deflection limit.

Therefore, h = 5 in.

(2) Check if L ≤ 3D:

L = 467 plf

D = 466 plf

3D = 1398 plf O.K.

(3) Shear and moment:

According to RISA 3D model, see Figure 4.5 and Table 4.5

For stair slab:

Flexural strength, M_u = 10.4 ft-kips

Shear strength, V_u = 6.9 kips

4. Concrete Beam Approach (cont'd)

For landing:

Flexural strength, M_u = 10.4 ft-kips

Shear strength, V_u = 9.7 kips

(4) Calculate required area of reinforcement, $\int_{rqd}$

The values of each parameter are:

Cealr cover = 1.5 in.

half of the assumed rebar dia. = 0.375 in. (#6)

Dia. of stirrup = 0.5 in.

d'= 2.375 in.

h = 5 in.

d= 2.625 in.

b = 56.00 in.

f_c'= 4,000 psi

f_y= 60,000 psi

Tension controlled, Φ = 0.9 *Table 21.2.2*

For stair slab:

m= 17.65

R_u= 359.36

$\int_{rqd}$= 0.0063

Therefore, $A_{s,rqd}$= 0.93 square in.

Step	Computation	Reference

4. Concrete Beam Approach (cont'd)

For landing:

m= 17.65

R_u= 56.42

$\int_{rqd}$= 0.0009

Therefore, $A_{s,rqd}$= $\boxed{0.35}$ square in.

(5) Check for flexural reinforcement limit:

$A_{s,min}$= greater of

0.46 square in.

and

0.49 square in. √

Since, $A_{s,rqd} = 0.93\ in^2 > A_{s,,min} = 0.49\ in^2$ O.K.

(However, adjust area of landing reinforcement to 1.24 in^2, since d = 6.625'')

(6) Minimum shear reinforcement:

Since, b_w= 56.00 in.

d = 2.63 in.

f_c'= 4,000 psi — *ACI 318-19*

λ = 1.0 — *Table 19.2.4.2*

$\emptyset$ = 0.75 — *Table 21.2.1*

So that, V_c= 18.59 kips

$\emptyset V_c$ = 13.95 kips

$0.5\emptyset V_c$ = $\boxed{6.97}$ kips

For stair slab

$0.5\emptyset V_c$ = $\boxed{17.60}$ kips

For landing

(when h = 9 in., and d = 6.625 in.)

For stair slab:

Since $V_u = 6.9\ kips < 0.5\emptyset V_c = 6.97 kips$, shear reinforcement is not required.

For landing:

Since $V_u = 9.7 kips < 0.5\emptyset V_c = 17.60 kips$, shear reinforcement is not required.

Therefore, for minimum shear reinforcement:

$A_{v,min}/s$ shall be the greater of $0.75\sqrt{f_c'}\frac{b_w}{f_{yt}}$ — *ACI 318-19 Table 9.6.3.3*

or

$50\frac{b_w}{f_{yt}}$

Since,

b_w= ~~56.00~~ in.

f_c'= ~~4,000~~ psi

f_{yt}= ~~60,000~~ psi

Step	Computation	Reference
4. Concrete Beam Approach (cont'd)	$A_{v,min}/s$ = greater of 0.0443 and 0.0467 Assume s = ~~12~~ in. $A_{v,min}$ = ~~0.28~~ square in. for single rebar If pick bar size, no. = ~~5~~ diameter of bar = ~~0.625~~ in. area of bar = ~~0.31~~ square in. (7) Determine center-to-center spacing, s For stair slab: $A_{s,rqd}$= 0.93 square in. If pick bar size, no. = 4 diameter of bar = 0.500 in. area of bar = 0.20 square in. No. of bar = 5 Center-to-center spacing, s = 10 in. For landing: $A_{s,rqd}$= 0.65 square in. If pick bar size, no. = 4 diameter of bar = 0.500 in. area of bar = 0.20 square in. No. of bar = 4 Center-to-center spacing, s = 12 in. Now, check for minimum spacing: The diameter of the aggregate is neglected for this project since it shall be determined by the manufacturer. So, s_{min} = greater of 1 in. √ and 0.500 in. #4 bars Since $s = 10\ in. > s_{min} = 1\ in.$ O.K. $s = 12\ in. > s_{min} = 1\ in.$ O.K. (8) Check for cracking control spacing requirement: Clear cover, c_c= 1.5 in. f_s= 40,000 psi s_{max}= lesser of 11.25 in. √ and 12 in. Adjust the center-to-center spacing to 11 in..	*ACI 318-19 Appendix A* *ACI 318-19 Appendix A*

Step	Computation	Reference
4. Concrete Beam Approach (cont'd)	(9) Check for deflection:	
	(a) Idealize the surface of the concrete that shows the cross sectional area of the main bars	
	Now it becomes a rectangularshape of concrete with:	
	Clear width, b = 56.00 in.	
	slab thickness, h = 5 in.	
	No. of bar size = 4	
	Area of a single bar, A_s= 0.20 square in.	
	No. of bars = 5	
	Total area of bars, A_s= 1.00 square in.	
	(b) Determine the modification factor, $n$to convert steel to concrete	
	For normal weight concrete,	
	$E_c = 57{,}000\sqrt{f_c'}$	*ACI 318-19* *19.2.2.1.a*
	$n = \frac{E_s}{E_c}$	
	Since,	
	f_c' = 4,000 psi	
	E_c = 3,605 ksi	
	E_s = 29,000 ksi	
	So that, n = 8.04	
	(c) Assume the section is cracked, determine the cracking moment	
	$f_r = 7.5\lambda\sqrt{f_c'}$	*ACI 318-19* *Eq. 19.2.3.1*
	$I_g = \frac{1}{12}bh^3$	*Area moment of inertia of rectangular section*
	$M_{cr} = \frac{f_r I_g}{y_t}$	*Eq. 24.2.3.5b*
	where	
	f_c'= 4,000 psi	
	λ = 1.0	
	f_r= 474 psi	
	I_g= 583 in^4	
	y_t= 2.5 in.	
	M_{cr}= 9.22 ft-kips	
	Since, $M_{cr} = 9.22\ ft - kips < M_u = 10.4\ ft - kips$, it's cracked.	
	(d) Apply transformed area method to determine the distance from the exterme compression fiber to the neutral axis, C_{NA}	
	From Section 3.1, Step (10),	

Step	Computation	Reference
4. Concrete Beam Approach (cont'd)	$\bar{y} = \frac{\sum A_i y_i}{\sum A_i}$ $C_{NA} = \frac{(bC_{NA})\left(\frac{C_{NA}}{2}\right) + (nA_s)d}{(bC_{NA}) + (nA_s)}$ Then, it becomes a quadratic equation: $\frac{b}{2}C_{NA}^2 + nA_sC_{NA} - nA_sd = 0$ The solution of C_{NA}: $C_{NA} = \frac{-nA_s \mp \sqrt{(nA_s)^2+2b(nA_sd)}}{b}$ Therefore, C_{NA} = 0.74 in. (e) Calculate moment of inertia of cracked section transformed to concrete, I_{cr} $I_{cr} = \sum I_i + A_i d_{yi}^2$ Then, it becomes: $I_{cr} = \frac{1}{3}bC_{NA}^3 + nA_s(d - C_{NA})^2$ (The value of I_x is so small that can be neglected) Therefore, I_{cr} = 36.15 in^4 (f) Calculate effective moment of inertia, I_e $I_e = \frac{I_{cr}}{1 - \left(\frac{(2/3)M_{cr}}{M_a}\right)^2 (1 - \frac{I_{cr}}{I_g})}$ M_{cr}= 9.22 ft-kips M_a = 10.40 ft-kips I_g= 583 in^4 I_{cr}= 36 in^4 So that, I_e = 54 in^4 (g) Check deflection: According to ACI 318-19, Table 24.2.2, immediate deflection due to live load L is $l/360$ l = 116 in. for slab span length $l/360$ = 0.32	*ACI 318-19 Eq. 24.2.3.5(b)*

Step	Computation	Reference
4. Concrete Beam Approach (cont'd)	Only consider the middle span where the stairway slab is located, see Figure $\Delta_{max} = 5wl^4/384EI$ $E_c = 57{,}000\sqrt{f_c'}$ f_c'= 4 ksi E_c= 3,605 ksi l = 116 in. I_e= 54 in^4 w= 1669 plf Δ_{max}= 1.68 Since, $\Delta_{max} = 1.68 > l/360 = 0.32$, this design does not satisfy with the deflection requirement (10) Conclusion: This design is not adequate in deflection.	*ACI 318-19 Eq. 19.2.2.1.b*

Step	Computation	Reference
4. Concrete Beam Approach (cont'd)	**4.8 Design for B-CA-5-ii:** (1) Minimum slab thickness, h: The boundary condition for slab is simply supported. Therefore, when l= 116 in. h = 7.24 in. The assumption of the slab thickness was 5 in., which is less than 7.24 in.. Need to check for deflection limit. Therefore, h = 5 in. (2) Check if L ≤ 3D: L = 467 plf D = 466 plf 3D = 1398 plf O.K. (3) Shear and moment: According to RISA 3D model, see Figure 4.6 and Table 4.6 For stair slab: Flexural strength, M_u = 88.9 ft-kips Shear strength, V_u = 6.9 kips For landing: Flexural strength, M_u = 72.3 ft-kips Shear strength, V_u = 22.9 kips (4) Calculate required area of reinforcement, $\int_{rqd}$ The values of each parameter are: Cealr cover = 1.5 in. half of the assumed rebar dia. = 0.375 in. (#6) Dia. of stirrup = 0.5 in. d'= 2.375 in. h = 5 in. d= 2.625 in. b = 56.00 in. f_c'= 4,000 psi f_y= 60,000 psi Tension controlled, Φ = 0.9 For stair slab: m= 17.65 R_u= 3071.81 $\int_{rqd}$= #NUM! The value of $\int_{rqd}$ is indeterminate by excel, which means it is not in the tension-controlled region. Therefore, concrete beam method does not apply to this problem.	*Table 21.2.2*

Step	Computation	Reference
4. Concrete Beam Approach (cont'd)	**4.9 Design for B-CA-8-i:** (1) Minimum slab thickness, h: The boundary condition for slab is still both ends continuous. Therefore, when l= 116 in. h = 5.51 in. The assumption of the slab thickness was 8 in., which is greater than 5.51 in.. No need to check for deflection limit. Therefore, h = 8 in. (2) Check if L ≤ 3D: L = 467 plf D = 635 plf 3D = 1906 plf O.K. (3) Shear and moment: According to RISA 3D model, see Figure 4.7 and Table 4.7 For stair slab: Shear strength, V_u = 8.7 kips Flexural positive strength, M_u = 8.5 ft-kips Flexural negative strength, M_u = 12.6 ft-kips For landing: Shear strength, V_u = 10.1 kips Flexural positive strength, M_u = 3.5 ft-kips Flexural negative strength, M_u = 12.6 ft-kips (4) Calculate required area of reinforcement, $\int_{rqd}$ The values of each parameter are: Cealr cover = 1.5 in. half of the assumed rebar dia. = 0.375 in. (#6) Dia. of stirrup = 0.5 in. d'= 2.375 in. h = 8 in. d= 5.625 in. b = 56.00 in. f_c'= 4,000 psi f_y= 60,000 psi Tension controlled, Φ = 0.9 For stair slab, postive moment: m= 17.65 R_u= 63.96 $\int_{rqd}$= 0.0011 $A_{s,rqd+}$= 0.34 square in.	*Table 21.2.2*

Step	Computation	Reference
4. Concrete Beam Approach (cont'd)	For stair slab, negative moment: m= 17.65 R_u= 94.81 $\int_{rqd}$= 0.0016 $A_{s,rqd-}$= 0.50 square in. For landing, postive moment: m= 17.65 R_u= 18.99 $\int_{rqd}$= 0.0003 $A_{s,rqd+}$= 0.12 square in. For landing, negative moment: m= 17.65 R_u= 68.35 $\int_{rqd}$= 0.0012 $A_{s,rqd-}$= 0.43 square in. (5) Check for flexural reinforcement limit: $A_{s,min}$= greater of 1.00 square in. and 1.05 square in. √ None of the areas satisfies with the minimum flexural reinforcement. Increase them to 1.05 square in. For laning, it is 1.24 in^2, O.K. Therefore, For stair slab: $A_{s,rqd+}$ = 1.05 square in. $A_{s,rqd-}$ = 1.05 square in. (6) Minimum shear reinforcement: Since, b_w= 56.00 in. d = 5.625 in. f'_c= 4,000 psi λ = 1.0 $\emptyset$ = 0.75 So that, V_c= 39.84 kips $\emptyset V_c$ = 29.88 kips $0.5\emptyset V_c$ = 14.94 kips For stair slab $0.5\emptyset V_c$ = 17.60 kips For landing (when h = 9 in., and d = 6.625 in.)	*ACI 318-19* *Table 19.2.4.2* *Table 21.2.1*

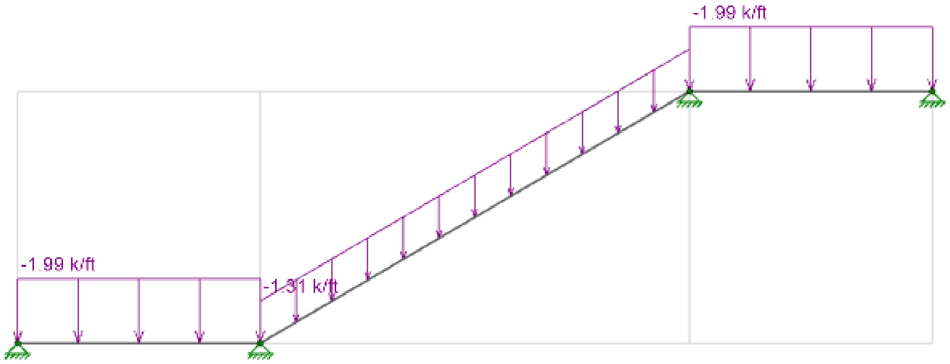
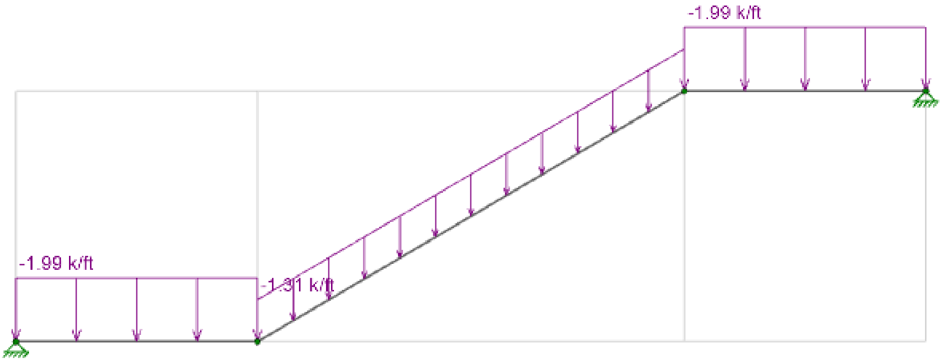
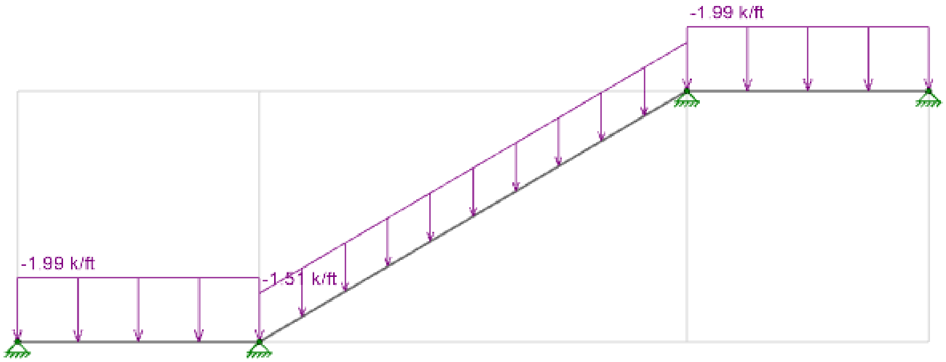
Step	Computation	Reference
4. Concrete Beam Approach (cont'd)	For stair slab: Since $V_u = 8.7\ kips < 0.5\emptyset V_c = 14.94 kips$, shear reinforcement is not required. For landing: Since $V_u = 10.1\ kips < 0.5\emptyset V_c = 17.60 kips$, shear reinforcement is not required. (7) Determine center-to-center spacing, s For stair slab: $A_{s,rqd}$ = 1.05 square in. If pick bar size, no. = 4 diameter of bar = 0.500 in. area of bar = 0.20 square in. No. of bar = 6 Center-to-center spacing, s = 8 in.	*ACI 318-19* *Appendix A*
	For landing: $A_{s,rqd}$ = 1.24 square in. If pick bar size, no. = 4 diameter of bar = 0.500 in. area of bar = 0.20 square in. No. of bar = 7 Center-to-center spacing, s = 7 in.	*ACI 318-19* *Appendix A*
	(8) Check for cracking control spacing requirement: Clear cover, c_c = 1.5 in. f_s = 40,000 psi s_{max} = lesser of 11.25 in. √ and 12 in. Since $s = 8\ in. < s_{max} = 11.25\ in.$ for flexural reinforcement O.K.	
	(9) Check for deflection: According to ACI 318-19, Section 9.3.2.2, since the applied slab thickness satisfies Table 9.3.1.1, deflections occuring after the member becomes composite need not to be checked.	*ACI 318-19* *Section 9.3.2.2*
	(10) Conclusion: For stair slab, it has two layers of #4 reinforcing rebar in every 8 in., both acting as tension reinforcement. For landing, it has two layers of #4 reinforcing rebar in every 7 in., both acting as tension reinforcement. No shear reinforcement required for neither stair slab or landing.	

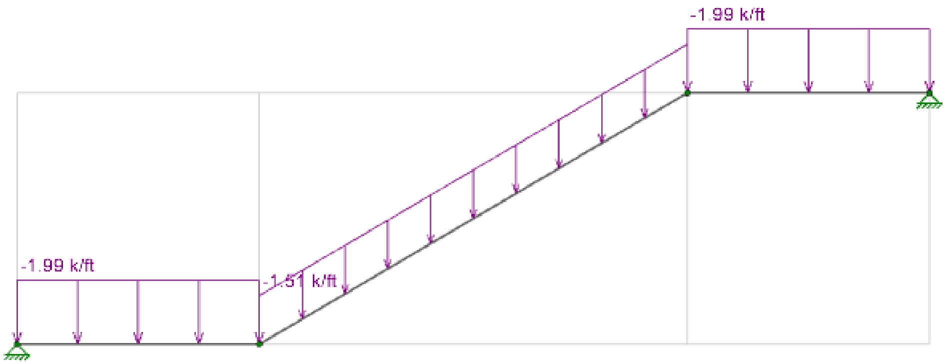
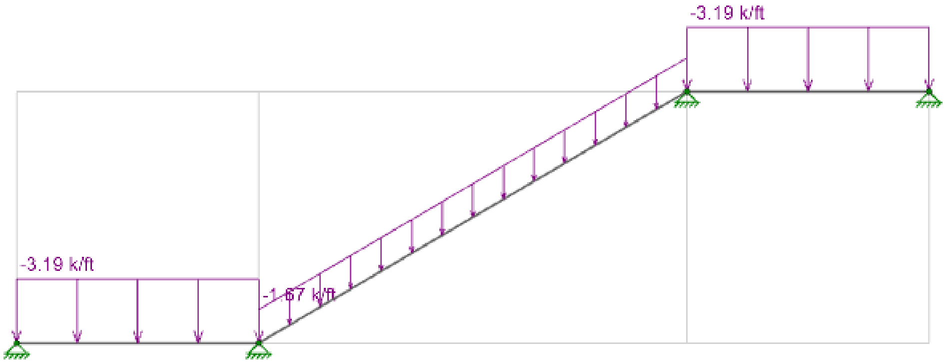
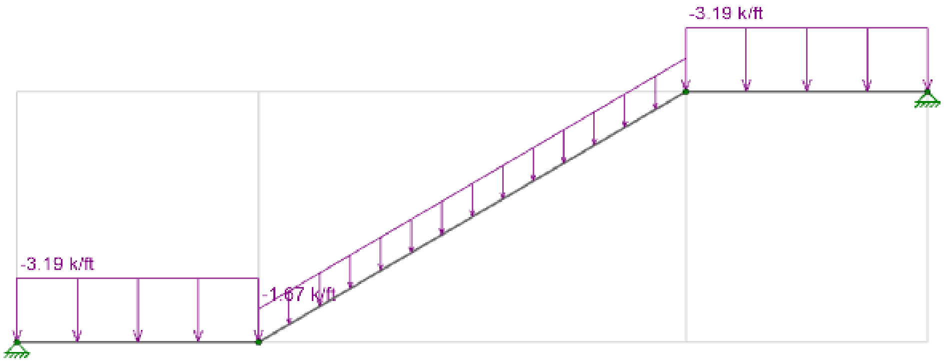
Step	Computation	Reference
4. Concrete Beam Approach (cont'd)	**4.10 Design for B-CA-8-ii:**	
	(1) Minimum slab thickness, h:	
	The boundary condition for slab is simply supported.	
	Therefore,	
	when l= 116 in.	
	h = 7.24 in.	
	The assumption of the slab thickness was 8 in., which is greater than 7.24 in.. No need to check for deflection limit.	
	Therefore, h = 8 in.	
	(2) Check if L ≤ 3D:	
	L = 467 plf	
	D = 635 plf	
	3D = 1906 plf O.K.	
	(3) Shear and moment:	
	According to RISA 3D model, see Figure 4.8 and Table 4.8	
	For stair slab:	
	Flexural strength, M_u = 103.6 ft-kips	
	Shear strength, V_u = 8.7 kips	
	For landing:	
	Flexural strength, M_u = 82.5 ft-kips	
	Shear strength, V_u = 25.1 kips	
	(4) Calculate required area of reinforcement, $\int_{rqd}$	
	The values of each parameter are:	
	Cealr cover = 1.5 in.	
	half of the assumed rebar dia. = 0.375 in. (#6)	
	Dia. of stirrup = 0.5 in.	
	d'= 2.375 in.	
	h = 8 in.	
	d= 5.625 in.	
	b = 56.00 in.	
	f'_c= 4,000 psi	
	f_y= 60,000 psi	
	Tension controlled, Φ = 0.9	*Table 21.2.2*
	For stair slab:	
	m= 17.65	
	R_u= 779.59	
	$\int_{rqd}$= 0.0150	
	Therefore, $A_{s,rqd}$= 4.72 square in.	

Step	Computation	Reference
4. Concrete Beam Approach (cont'd)	For landing: m= 17.65 R_u= 447.54 $\int_{rqd}$= 0.0080 Therefore, $A_{s,rqd}$= 2.98 square in. (5) Check for flexural reinforcement limit: $A_{s,min}$= greater of 1.00 square in. and 1.05 square in. √ Since both values of $A_{s,rqd}$ are greater than $A_{s,min}$, it is O.K.. **(For landing, the minimum reinforcement area is 1.24 in^2)** (6) Minimum shear reinforcement: Since, b_w= 56.00 in. d = 5.625 in. f'_c= 4,000 psi λ = 1.0 $\emptyset$ = 0.75 So that, V_c= 39.84 kips $\emptyset V_c$ = 29.88 kips $0.5\emptyset V_c$ = 14.94 kips For stair slab $0.5\emptyset V_c$ = 17.60 kips For landing (when h = 6 in., and d = 3.5 in.) For stair slab: Since $V_u = 8.7\ kips < 0.5\emptyset V_c = 14.94 kips$, shear reinforcement is not required. For landing: Since $V_u = 25.1\ kips > 0.5\emptyset V_c = 17.60 kips$, shear reinforcement is required. Therefore, for minimum shear reinforcement: $A_{v,min}/s$ shall be the greater of $0.75\sqrt{f'_c}\,\frac{b_w}{f_{yt}}$ or $50\frac{b_w}{f_{yt}}$ Since, b_w= 56.00 in. f'_c= 4,000 psi f_{yt}= 60,000 psi	*ACI 318-19* *Table 19.2.4.2* *Table 21.2.1* *ACI 318-19* *Table 9.6.3.3*

Step	Computation	Reference
4. Concrete Beam Approach (cont'd)	$A_{v,min}/s =$ greater of 0.0443 and 0.0467 √ Assume $s =$ 3 in. **The maximum spacing of shear reinforcement should not be greater than d/2 or 24 in.. Since d/2 = 3.3125 in. for landing, take 3 in.** $A_{v,min} =$ 0.07 square in. for single rebar If pick bar size, no. = 4 diameter of bar = 0.500 in. area of bar = 0.20 square in. √ (7) Determine center-to-center spacing, s For stair slab: $A_{s,rqd} =$ 4.72 square in. If pick bar size, no. = 6 diameter of bar = 0.750 in. area of bar = 0.44 square in. No. of bar = 11 Center-to-center spacing, $s =$ 4 in. For landing: $A_{s,rqd} =$ 2.98 square in. If pick bar size, no. = 6 diameter of bar = 0.750 in. area of bar = 0.44 square in. No. of bar = 7 Center-to-center spacing, $s =$ 7 in. Now, check for minimum spacing: The diameter of the aggregate is neglected for this project since it shall be determined by the manufacturer. So, $s_{min} =$ greater of 1 in. √ and 0.750 in. #6 bars Since $s = 4\ in. > s_{min} = 1\ in.$ O.K. $s = 7\ in. > s_{min} = 1\ in.$ O.K. (8) Check for cracking control spacing requirement: Clear cover, $c_c =$ 1.5 in. $f_s =$ 40,000 psi $s_{max} =$ lesser of 11.25 in. √ and 12 in. The spacings calculated above satisfy with the minimum requirement.	*ACI 318-19 Appendix A* *ACI 318-19 Appendix A*

Step	Computation	Reference
4. Concrete Beam Approach (cont'd)	(9) Check for deflection: According to ACI 318-19, Section 9.3.2.2, since the applied slab thickness satisfies Table 9.3.1.1, deflections occuring after the member becomes composite need not to be checked. (10) Conclusion: For stair slab, use #6 reinforcing rebar in every 4 in. spacing for flexural reinforcement. No shear reinforcement is required. For landing, use #6 reinforcing rebar in every 7 in. spacing for flexural reinforcement. Use #4 reinforcing rebar in every 3 in. for shear reinforcement.	*ACI 318-19 Section 9.3.2.2*

Step	Computation	Reference
4. Concrete Beam Approach (cont'd)	4.11 Graphs and tables:  Figure 4.1 Kansas-5-i  Figure 4.2 Kansas-5-ii  Figure 4.3 Kansas-8-i	

Step	Computation	Reference
4. Concrete Beam Approach (cont'd)	 Figure 4.4 Kansas-8-ii  Figure 4.5 California-5-i  Figure 4.6 California-5-ii	

Step	Computation	Reference
4. Concrete Beam Approach (cont'd)		

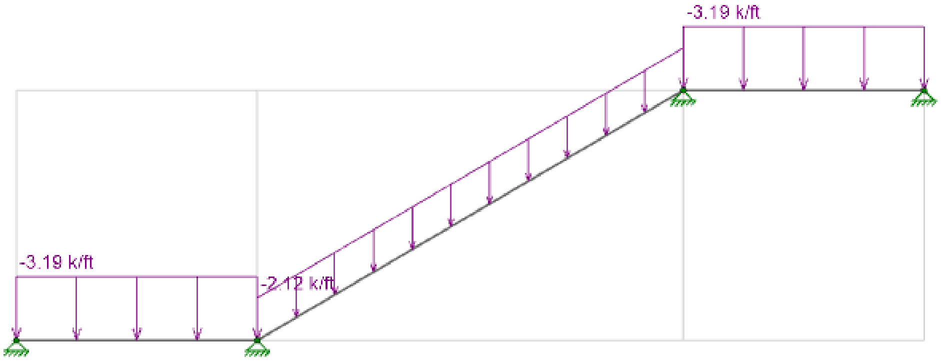


Figure 4.7 California-8-i

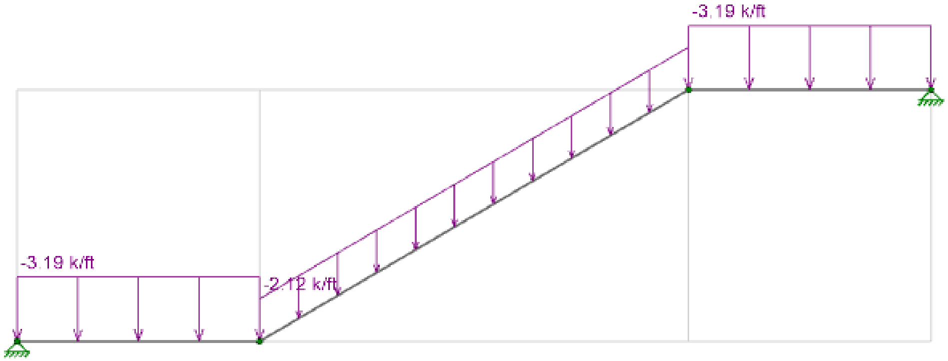
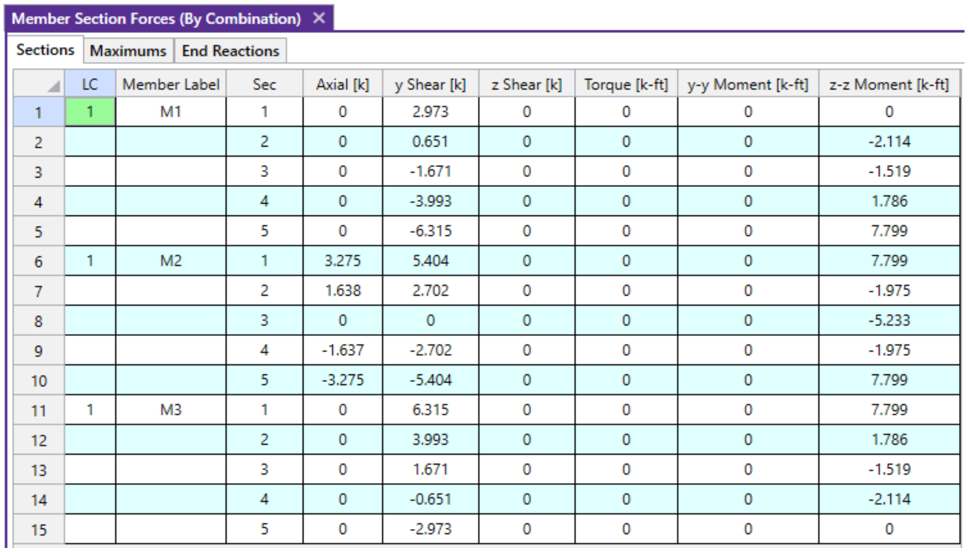


Figure 4.8 California-8-ii

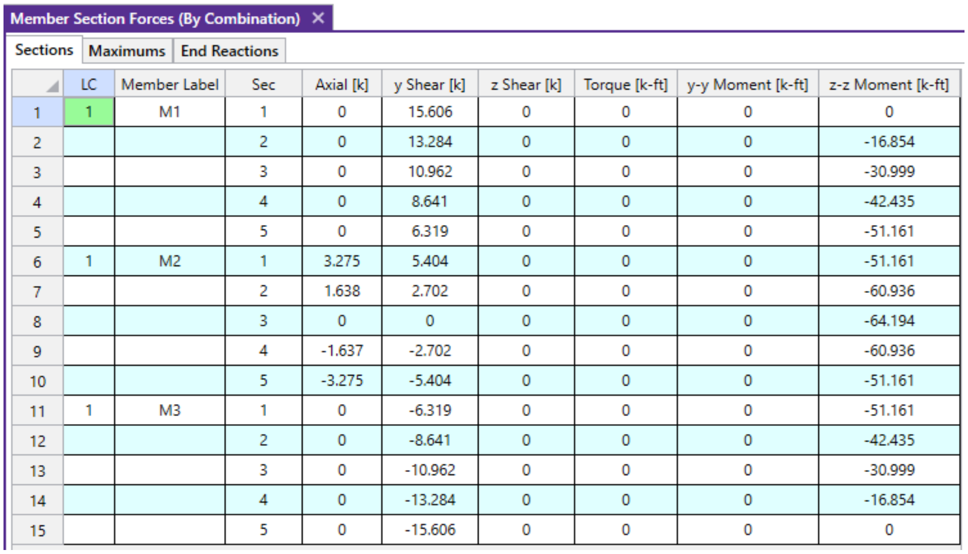
Table 4.1 Kansas-5-i member forces



	LC	Member Label	Sec	Axial [k]	y Shear [k]	z Shear [k]	Torque [k-ft]	y-y Moment [k-ft]	z-z Moment [k-ft]
1	1	M1	1	0	2.973	0	0	0	0
2			2	0	0.651	0	0	0	-2.114
3			3	0	-1.671	0	0	0	-1.519
4			4	0	-3.993	0	0	0	1.786
5			5	0	-6.315	0	0	0	7.799
6	1	M2	1	3.275	5.404	0	0	0	7.799
7			2	1.638	2.702	0	0	0	-1.975
8			3	0	0	0	0	0	-5.233
9			4	-1.637	-2.702	0	0	0	-1.975
10			5	-3.275	-5.404	0	0	0	7.799
11	1	M3	1	0	6.315	0	0	0	7.799
12			2	0	3.993	0	0	0	1.786
13			3	0	1.671	0	0	0	-1.519
14			4	0	-0.651	0	0	0	-2.114
15			5	0	-2.973	0	0	0	0

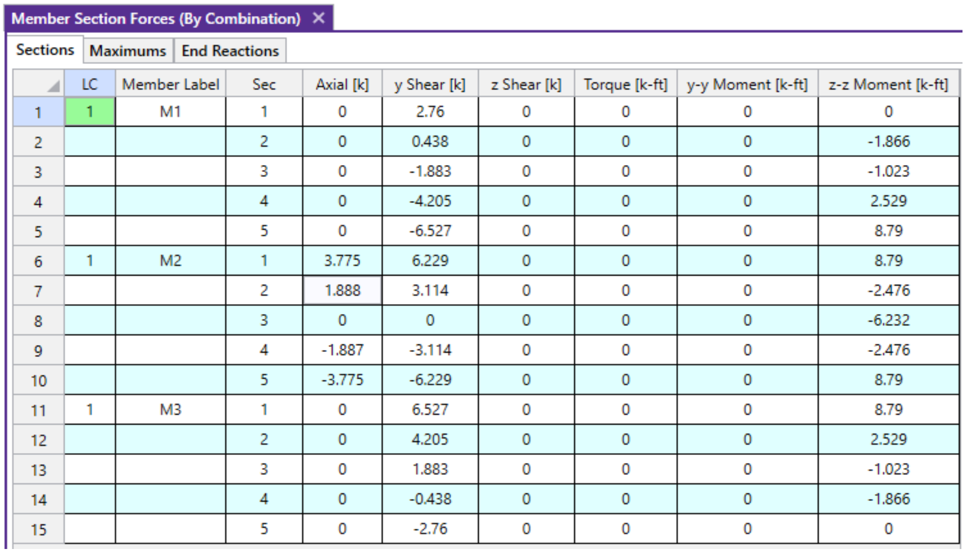
Step	Computation	Reference
4. Concrete Beam Approach (cont'd)	Table 4.2 Kansas-5-ii member forces Table 4.3 Kansas-8-i member forces	

Table 4.2 Kansas-5-ii member forces



	LC	Member Label	Sec	Axial [k]	y Shear [k]	z Shear [k]	Torque [k-ft]	y-y Moment [k-ft]	z-z Moment [k-ft]
1	1	M1	1	0	15.606	0	0	0	0
2			2	0	13.284	0	0	0	-16.854
3			3	0	10.962	0	0	0	-30.999
4			4	0	8.641	0	0	0	-42.435
5			5	0	6.319	0	0	0	-51.161
6	1	M2	1	3.275	5.404	0	0	0	-51.161
7			2	1.638	2.702	0	0	0	-60.936
8			3	0	0	0	0	0	-64.194
9			4	-1.637	-2.702	0	0	0	-60.936
10			5	-3.275	-5.404	0	0	0	-51.161
11	1	M3	1	0	-6.319	0	0	0	-51.161
12			2	0	-8.641	0	0	0	-42.435
13			3	0	-10.962	0	0	0	-30.999
14			4	0	-13.284	0	0	0	-16.854
15			5	0	-15.606	0	0	0	0

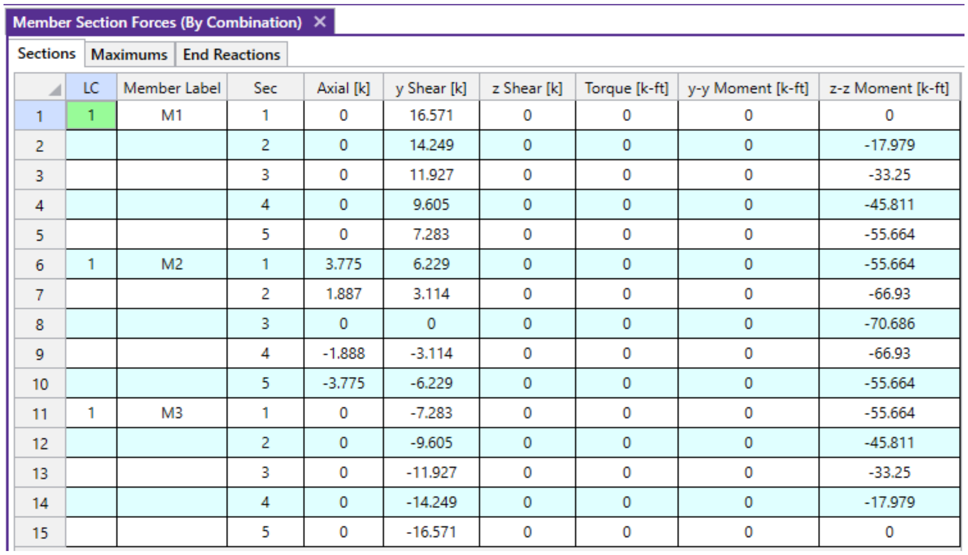
Table 4.3 Kansas-8-i member forces



	LC	Member Label	Sec	Axial [k]	y Shear [k]	z Shear [k]	Torque [k-ft]	y-y Moment [k-ft]	z-z Moment [k-ft]
1	1	M1	1	0	2.76	0	0	0	0
2			2	0	0.438	0	0	0	-1.866
3			3	0	-1.883	0	0	0	-1.023
4			4	0	-4.205	0	0	0	2.529
5			5	0	-6.527	0	0	0	8.79
6	1	M2	1	3.775	6.229	0	0	0	8.79
7			2	1.888	3.114	0	0	0	-2.476
8			3	0	0	0	0	0	-6.232
9			4	-1.887	-3.114	0	0	0	-2.476
10			5	-3.775	-6.229	0	0	0	8.79
11	1	M3	1	0	6.527	0	0	0	8.79
12			2	0	4.205	0	0	0	2.529
13			3	0	1.883	0	0	0	-1.023
14			4	0	-0.438	0	0	0	-1.866
15			5	0	-2.76	0	0	0	0

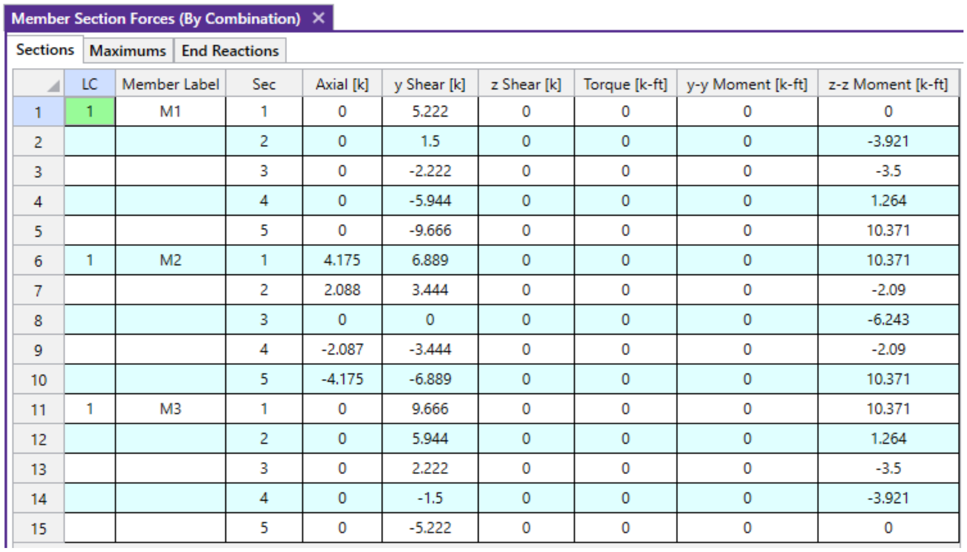
Step	Computation	Reference
4. Concrete Beam Approach (cont'd)	Table 4.4 Kansas-8-ii member forces; Table 4.5 California-5-i member forces	

Table 4.4 Kansas-8-ii member forces



	LC	Member Label	Sec	Axial [k]	y Shear [k]	z Shear [k]	Torque [k-ft]	y-y Moment [k-ft]	z-z Moment [k-ft]
1	1	M1	1	0	16.571	0	0	0	0
2			2	0	14.249	0	0	0	-17.979
3			3	0	11.927	0	0	0	-33.25
4			4	0	9.605	0	0	0	-45.811
5			5	0	7.283	0	0	0	-55.664
6	1	M2	1	3.775	6.229	0	0	0	-55.664
7			2	1.887	3.114	0	0	0	-66.93
8			3	0	0	0	0	0	-70.686
9			4	-1.888	-3.114	0	0	0	-66.93
10			5	-3.775	-6.229	0	0	0	-55.664
11	1	M3	1	0	-7.283	0	0	0	-55.664
12			2	0	-9.605	0	0	0	-45.811
13			3	0	-11.927	0	0	0	-33.25
14			4	0	-14.249	0	0	0	-17.979
15			5	0	-16.571	0	0	0	0

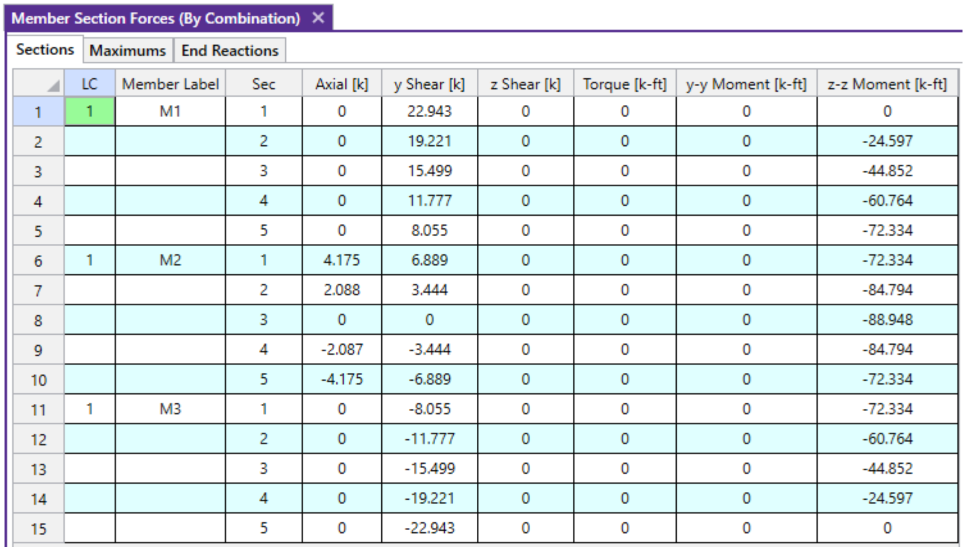
Table 4.5 California-5-i member forces



	LC	Member Label	Sec	Axial [k]	y Shear [k]	z Shear [k]	Torque [k-ft]	y-y Moment [k-ft]	z-z Moment [k-ft]
1	1	M1	1	0	5.222	0	0	0	0
2			2	0	1.5	0	0	0	-3.921
3			3	0	-2.222	0	0	0	-3.5
4			4	0	-5.944	0	0	0	1.264
5			5	0	-9.666	0	0	0	10.371
6	1	M2	1	4.175	6.889	0	0	0	10.371
7			2	2.088	3.444	0	0	0	-2.09
8			3	0	0	0	0	0	-6.243
9			4	-2.087	-3.444	0	0	0	-2.09
10			5	-4.175	-6.889	0	0	0	10.371
11	1	M3	1	0	9.666	0	0	0	10.371
12			2	0	5.944	0	0	0	1.264
13			3	0	2.222	0	0	0	-3.5
14			4	0	-1.5	0	0	0	-3.921
15			5	0	-5.222	0	0	0	0

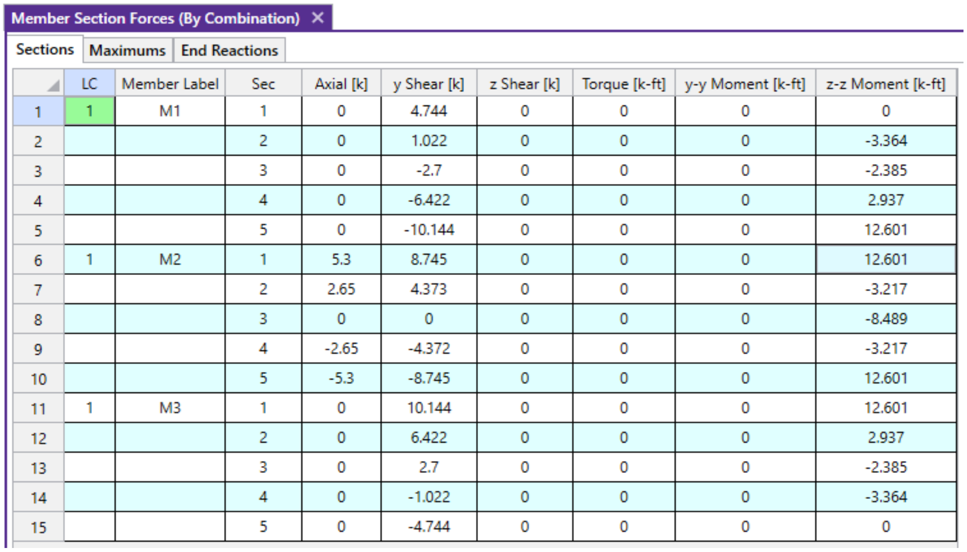
Step	Computation	Reference
4. Concrete Beam Approach (cont'd)	Table 4.6 California-5-ii member forces Table 4.7 California-8-i member forces	

Table 4.6 California-5-ii member forces



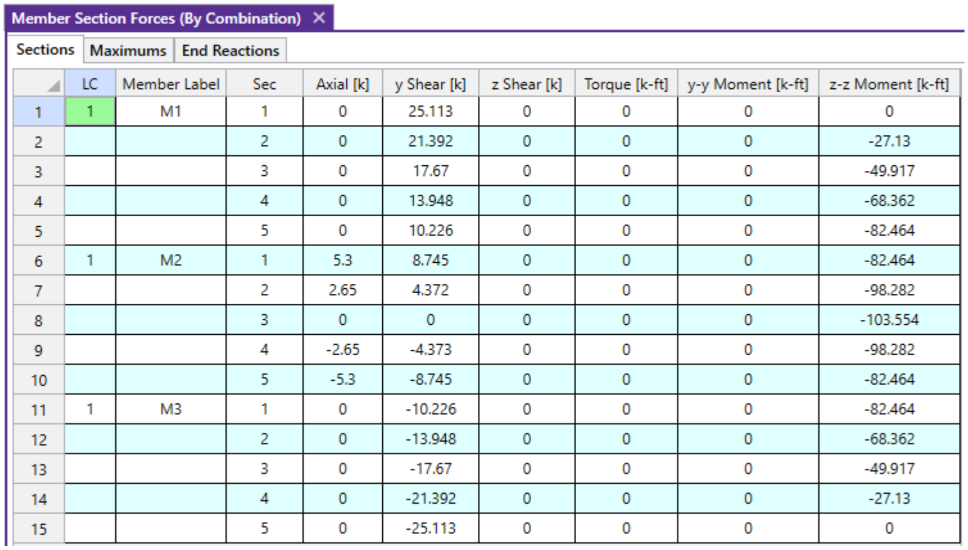
	LC	Member Label	Sec	Axial [k]	y Shear [k]	z Shear [k]	Torque [k-ft]	y-y Moment [k-ft]	z-z Moment [k-ft]
1	1	M1	1	0	22.943	0	0	0	0
2			2	0	19.221	0	0	0	-24.597
3			3	0	15.499	0	0	0	-44.852
4			4	0	11.777	0	0	0	-60.764
5			5	0	8.055	0	0	0	-72.334
6	1	M2	1	4.175	6.889	0	0	0	-72.334
7			2	2.088	3.444	0	0	0	-84.794
8			3	0	0	0	0	0	-88.948
9			4	-2.087	-3.444	0	0	0	-84.794
10			5	-4.175	-6.889	0	0	0	-72.334
11	1	M3	1	0	-8.055	0	0	0	-72.334
12			2	0	-11.777	0	0	0	-60.764
13			3	0	-15.499	0	0	0	-44.852
14			4	0	-19.221	0	0	0	-24.597
15			5	0	-22.943	0	0	0	0

Table 4.7 California-8-i member forces



	LC	Member Label	Sec	Axial [k]	y Shear [k]	z Shear [k]	Torque [k-ft]	y-y Moment [k-ft]	z-z Moment [k-ft]
1	1	M1	1	0	4.744	0	0	0	0
2			2	0	1.022	0	0	0	-3.364
3			3	0	-2.7	0	0	0	-2.385
4			4	0	-6.422	0	0	0	2.937
5			5	0	-10.144	0	0	0	12.601
6	1	M2	1	5.3	8.745	0	0	0	12.601
7			2	2.65	4.373	0	0	0	-3.217
8			3	0	0	0	0	0	-8.489
9			4	-2.65	-4.372	0	0	0	-3.217
10			5	-5.3	-8.745	0	0	0	12.601
11	1	M3	1	0	10.144	0	0	0	12.601
12			2	0	6.422	0	0	0	2.937
13			3	0	2.7	0	0	0	-2.385
14			4	0	-1.022	0	0	0	-3.364
15			5	0	-4.744	0	0	0	0

Step	Computation	Reference
4. Concrete Beam Approach (cont'd)	Table 4.8 California-8-ii member forces	



	LC	Member Label	Sec	Axial [k]	y Shear [k]	z Shear [k]	Torque [k-ft]	y-y Moment [k-ft]	z-z Moment [k-ft]
1	1	M1	1	0	25.113	0	0	0	0
2			2	0	21.392	0	0	0	-27.13
3			3	0	17.67	0	0	0	-49.917
4			4	0	13.948	0	0	0	-68.362
5			5	0	10.226	0	0	0	-82.464
6	1	M2	1	5.3	8.745	0	0	0	-82.464
7			2	2.65	4.372	0	0	0	-98.282
8			3	0	0	0	0	0	-103.554
9			4	-2.65	-4.373	0	0	0	-98.282
10			5	-5.3	-8.745	0	0	0	-82.464
11	1	M3	1	0	-10.226	0	0	0	-82.464
12			2	0	-13.948	0	0	0	-68.362
13			3	0	-17.67	0	0	0	-49.917
14			4	0	-21.392	0	0	0	-27.13
15			5	0	-25.113	0	0	0	0

Appendix B - Additional Calculations

Step | **Computation** | **Reference**

5. Additional Calculation

5.1 Problem statement:

Due to the indeterminate outcomes that case B-KS-5-ii and B-CA-5-ii have, perform additional calculations with 6 inches slab thickness to ensure the comparison is not one-sided.

5.2 Additional cases:

S-KS-6-i
S-KS-6-ii
S-CA-6-i
S-CA-6-ii
S-KS-6-i
S-KS-6-ii
S-CA-6-i
S-CA-6-ii

5.3 Design parameters:

All are the same except the slab thickness, 6 inches

Floor to floor height, H = 10 ft.
Number of risers, n = 18
Tread depth, d = 11 in.
Riser height, h_r = 6.7 in.
Staircase clear width, w = 56 in.
Staircase landing length, L_1, L_3 = 56 in.
Angle from the horizontal, Θ = 31.2 degree
Landing thickness, h' = 9 in.
Assumed slab thickness, h = 6 in.
Calculated slab length, L_2 = 116 in.

The Architect's Studio Companion, Six Edition, Exit Stairway Design Tables, Page 323

5.4 Assumptions:

All assumptions stay the same

5.5 Building loads:

Use the previous excel to calculate all loads.
With 6 in. slab thickness,

(1) Dead load:

Dead load of landing, D = 1038 plf
Dead load of stair, D = 522 plf

(2) Live load:

Live load, L = 467 plf

(3) Windload:

wind load, W = 75 plf

(4) Snow and rain loads:

Not required for an interior stairway

Step	Computation	Reference
5. Additional Calculation (cont'd)	(5) Seismic load:	
	For Los Angeles, CA:	
	horizontal seismic force of landing, Fp = 5385 lbf	
	≤ 17950 lbf	
	≥ 3366 lbf	
	vertical seismic force of landing, Fp = 1496 lbf	
	horizontal seismic force of stair flight, Fp = 5474 lbf	
	≤ 18245 lbf	
	≥ 3421 lbf	
	vertical seismic force of stair flight, Fp = 1520 lbf	
	For Manhattan, KS:	
	horizontal seismic force of landing, Fp = 505 lbf	
	≤ 1682 lbf	
	≥ 315 lbf	
	vertical seismic force of landing, Fp = 93 lbf	
	horizontal seismic force of stair flight, Fp = 513 lbf	
	≤ 1710 lbf	
	≥ 321 lbf	
	vertical seismic force of stair flight, Fp = 143 lbf	
	<u>5.6 Load combinations:</u>	
	From previous calculations, load combination 2 will govern for case of Manhattan Kansas, and load combination 5 will govern for case of Los Angeles California.	
	For Manhattan KS:	
	$1.2D + 1.6L + 0.5(L_r \; or \; S \; or \; R)$ = 1.99 klf for landing	*ASCE 7-16 Section 2.3.2*
	$1.2D + 1.6L + 0.5(L_r \; or \; S \; or \; R)$ = 1.37 klf for stair	
	For Los Angeles CA:	
	$1.2D + E_v + E_h + L + 0.2S$ = 3.19 klf for landing	*ASCE 7-16, Section 12.4.2*
	$1.2D + E_v + E_h + L + 0.2S$ = 1.82 klf for stair	

Step	Computation	Reference
5. Additional Calculation (cont'd)	**5.7 Design for S-KS-6-i:** (1) Minimum slab thickness, h: The boundary condition for slab is still both ends continuous. Therefore, when l = 116 in. h = 4.13 in. The assumption of the slab thickness was 6 in., which is greater than 4.13 in.. The assumption is OK. Therefore, h = 6 in. (2) Check if L ≤ 3D: L = 467 plf D = 522 plf 3D = 1566 plf O.K. (3) Shear and moment: According to RISA 3D model, see Figure 5.1 and Table 5.1 For stair slab: Shear strength, V_u = 5.7 kips Flexural positive strength, M_u = 5.5 ft-kips Flexural negative strength, M_u = 8.1 ft-kips If applying 1' strip method, then they become: Shear strength, V_u = 1.22 kips Flexural positive strength, M_u = 1.18 ft-kips Flexural negative strength, M_u = 1.74 ft-kips For landing: Shear strength, V_u = 6.4 kips Flexural positive strength, M_u = 2.1 ft-kips Flexural negative strength, M_u = 8.1 ft-kips If applying 1' strip method, then they become: Shear strength, V_u = 1.37 kips Flexural positive strength, M_u = 0.45 ft-kips Flexural negative strength, M_u = 1.74 ft-kips (4) Calculate required area of reinforcement, $\int_{rqd}$ The value of each parameter shown as follows: Cealr cover = 0.75 in. half of the assumed rebar dia. = 0.375 in. (#6) d'= 1.125 in. h = 6 in. d= 4.875 in. b = 12.00 in. 1' strip	

Step	Computation	Reference
5. Additional Calculation (cont'd)	f_c'= 4,000 psi	
	f_y= 60,000 psi	
	Tension controlled, Φ = 0.9	*Table 21.2.2*
	For stairway, postive moment:	
	m= 17.65	
	R_u= 55.10	
	$\int_{rqd}$= 0.0009	
	$A_{s,rqd+}$= 0.05 square in.	
	For stairway, negative moment:	
	m= 17.65	
	R_u= 81.15	
	$\int_{rqd}$= 0.0014	
	$A_{s,rqd-}$= 0.08 square in.	
	For landing, postive moment:	
	m= 17.65	
	R_u= 8.06	
	$\int_{rqd}$= 0.0001	
	$A_{s,rqd+}$= 0.01 square in.	
	For landing, negative moment:	
	m= 17.65	
	R_u= 31.10	
	$\int_{rqd}$= 0.0005	
	$A_{s,rqd-}$= 0.05 square in.	
	(5) Check for flexural reinforcement limit:	
	$A_{s,min}$= greater of	
	0.105 square in. √	
	and	
	0.082 square in.	
	For calculated values less than the area of minimum reinforcement, apply 0.105 in^2 instead. For landing, it is 0.17 in^2.	
	Therefore,	
	for stairway:	
	$A_{s,rqd+}$= 0.11 square in.	
	$A_{s,rqd-}$= 0.11 square in.	
	for landing:	
	$A_{s,rqd+}$= 0.17 square in.	
	$A_{s,rqd-}$= 0.17 square in.	

Step	Computation	Reference
5. Additional Calculation (cont'd)	(6) Determine center-to-center spacing, s Determine the spacing for stair and landing together since they have similar required reinforcement area If pick bar size, no. = 4 diameter of bar = 0.500 in. area of bar = 0.20 square in. since, $A_{s,rqd}$= 0.11 square in. No. of bar in 1 ft. strip = 1 Center-to-center spacing, s = 12 in. Now, check for minimum spacing: The diameter of the aggregate is neglected for this project since it shall be determined by the manufacturer. So, s_{min} = greater of 1 in. √ and 0.500 in. Since, $s = 12\ in. > s_{min} = 1\ in.$ O.K. (7) Check for shear limit: Apply equation (c) shown in Section 3.1, Step (7), in accordance with ACI 318-19, Table 22.5.5.1 The value of each parameter shown as follows: b_w = 12 in. 1' strip d = 4.875 in. f_c' = 4,000 psi ∅ = 0.75 λ = 1.0 Therefore, For stairway, $\emptyset V_c$ = 3.34 kips For landing, $\emptyset V_c$ = 4.60 kips Since both of them are greater than the value of V_u no need for shear reinforcement. (8) Minimum temperature and shrinkage reinforcement: $A_{ts,min}$= greater of 0.105 square in. √ and 0.082 square in. If pick bar size, no. = 4 diameter of bar = 0.500 in. area of bar = 0.20 square in.	*ACI 318-19 Appendix A* *ACI 318-19* *Table 21.2.1* *Table 19.2.4.2* *ACI 318-19 7.6.3.1* *ACI 318-19 Appendix A*

Step	Computation	Reference
5. Additional Calculation (cont'd)	Since, $A_{ts,min}$ = 0.105 square in. for 1' strip slab No. of bar in 1 ft. strip = 1 Center-to-center spacing, s = 12 in. Check for maximum spacing: s_{max} = lesser of 30 in. and 18 in. √ Since $s = 12\ in. < s_{max} = 18\ in.$ O.K. (9) Check for cracking control spacing requirement: Clear cover, c_c = 0.75 in. f_s = 40,000 psi s_{max}= lesser of 13.13 in. and 12 in. √ since $s = 12\ in. = s_{max} = 12\ in.$ for flexural reinforcement $s = 12\ in. = s_{max} = 12\ in.$ for temp. and shrinkage O.K (10) Check for deflection: According to ACI 318-19, Section 7.3.2.2, since the applied slab thickness satisfies Table 7.3.1.1, deflections occuring after the member becomes composite need not to be checked. (11) Conclusion: For stairway slab, use #4 reinforcing rebar in every 12 in. spacing for flexural reinforcement, and use #4 reinforcing rebar in every 12 in. spacing for temperature reinforcement. For landing, use #4 reinforcing rebar in every 12 in. spacing for flexural reinforcement, and use #4 reinforcing rebar in every 12 in. spacing for temperature reinforcement. Flexural reinforcement in both stair slab and landing has two layers, both acting as tention reinforcement.	*ACI 318-19 Section 7.3.2.2*

Step	Computation	Reference
5. Additional Calculation (cont'd)	**<u>5.8 Design for S-KS-6-ii:</u>**	
	(1) Minimum slab thickness, h:	
	The boundary condition for slab is simply supported.	
	Therefore,	
	when l = 116 in.	
	h = 5.79 in.	
	The assumption of the slab thickness was 6 in., which is greater than 5.79 in.. Deflection check is not required.	
	Therefore, h = 6 in.	
	(2) Check if L ≤ 3D:	
	L = 467 plf	
	D = 522 plf	
	3D = 1566 plf O.K.	
	(3) Shear and moment:	
	According to RISA 3D model, see Figure 5.2 and Table 5.2	
	For stair slab:	
	Shear strength, V_u = 5.7 kips	
	Flexural strength, M_u = 66.1 ft-kips	
	If applying 1' strip method,	
	Shear strength, V_u = 1.22 kips	
	Flexural strength, M_u = 14.16 ft-kips	
	For landing:	
	Shear strength, V_u = 15.9 kips	
	Flexural strength, M_u = 52.5 ft-kips	
	If applying 1' strip method,	
	Shear strength, V_u = 3.41 kips	
	Flexural strength, M_u = 11.25 ft-kips	
	(4) Calculate required area of reinforcement, $\int_{rqd}$	
	The values of each parameter are:	
	Cealr cover = 0.75 in.	
	half of the assumed rebar dia. = 0.375 in. (#6)	
	d'= 1.125 in.	
	h = 6 in.	
	d= 4.875 in.	
	b = 12.00 in. 1' strip	
	f'_c= 4,000 psi	
	f_y= 60,000 psi	
	Tension controlled, Φ = 0.9	*Table 21.2.2*

Step	Computation	Reference
5. Additional Calculation (cont'd)	For stair: m= 17.65 R_u= 662.22 $\int_{rqd}$= 0.0124 $A_{s,rqd}$ = 0.72 square in. For landing: m= 17.65 R_u= 201.56 $\int_{rqd}$= 0.0035 $A_{s,rqd}$ = 0.33 square in. (5) Check for flexural reinforcement limit: $A_{s,min}$= greater of 0.105 square in. √ and 0.0819 square in. For calculated values less than the area of minimum reinforcement, apply 0.105 in^2 instead. For landing, it is 0.17 in^2. (6) Determine center-to-center spacing, s If pick bar size, no. = 4 diameter of bar = 0.500 in. area of bar = 0.20 square in. since, $A_{s,rqd}$= 0.72 square in. for stair slab No. of bar in 1 ft. strip = 4 Center-to-center spacing, s = 3 in. If pick bar size, no. = 4 diameter of bar = 0.500 in. area of bar = 0.20 square in. since, $A_{s,rqd}$= 0.33 square in. for landing No. of bar in 1 ft. strip = 2 Center-to-center spacing, s = 6 in. Now, check for minimum spacing: The diameter of the aggregate is neglected for this project since it shall be determined by the manufacturer. So, s_{min} = greater of 1 in. √ and 0.500 in. Since, $s = 3\ in. or\ 6\ in. > s_{min} = 1\ in.$ O.K.	*ACI 318-19 Appendix A* *ACI 318-19 Appendix A*

Step	Computation	Reference
5. Additional Calculation (cont'd)	(7) Check for shear limit:	
	Apply equation (b) shown in Section 3.1, Step (7), in accordance with ACI 318-19, Table 22.5.5.1	
	The value of each parameter shown as follows:	
	b_w = 12 in. 1' strip	
	d = 4.875 in.	
	f_c' = 4,000 psi	*ACI 318-19*
	$\emptyset$ = 0.75	*Table 21.2.1*
	λ = 1.0	*Table 19.2.4.2*
	Therefore,	
	For stairway,	
	$\emptyset V_c$ = 5.31 kips	
	For landing,	
	$\emptyset V_c$ = 5.80 kips	
	Since both of them are greater than the value of V_u, no need for	*ACI 318-19*
	shear reinforcement.	*7.6.3.1*
	(8) Minimum temperature and shrinkage reinforcement:	
	$A_{ts,min}$= greater of	
	0.105 square in. √	
	and	
	0.082 square in.	
	If pick bar size, no. = 4	
	diameter of bar = 0.500 in.	*ACI 318-19*
	area of bar = 0.20 square in.	*Appendix A*
	Since, $A_{ts,min}$= 0.105 square in.	
	for 1' strip slab	
	No. of bar in 1 ft. strip = 1	
	Center-to-center spacing, s = 12 in.	
	Check for maximum spacing:	
	s_{max}= lesser of	
	30 in.	
	and	
	18 in. √	
	Since $s = 12\ in. < s_{max} = 18\ in.$ O.K.	
	(9) Check for cracking control spacing requirement:	
	Clear cover, c_c= 0.75 in.	
	f_s= 40,000 psi	
	s_{max}= lesser of	
	13.13 in.	
	and	
	12 in. √	

Step	Computation	Reference
5. Additional Calculation (cont'd)	since $s = 6\ in. < s_{max} = 12\ in.$ for flexural reinforcement $s = 12\ in. = s_{max} = 12\ in.$ for temp. and shrinkage O.K (10) Check for deflection: According to ACI 318-19, Section 7.3.2.2, since the applied slab thickness satisfies Table 7.3.1.1, deflections occuring after the member becomes composite need not to be checked. (11) Conclusion: For stairway slab, use #4 reinforcing rebar in every 3 in. spacing for flexural reinforcement, and use #4 reinforcing rebar in every 12 in. spacing for temperature reinforcement. For landing, use #4 reinforcing rebar in every 6 in. spacing for flexural reinforcement, and use #4 reinforcing rebar in every 12 in. spacing for temperature reinforcement.	*ACI 318-19 Section 7.3.2.2*

Step	Computation	Reference
5. Additional Calculation (cont'd)	**<u>5.9 Design for S-CA-6-i:</u>** (1) Minimum slab thickness, h: The boundary condition for slab is still both ends continuous. Therefore, when l = 116 in. h = 4.13 in. The assumption of the slab thickness was 6 in., which is greater than 4.13 in.. The assumption is OK. Therefore, h = 6 in. (2) Check if L ≤ 3D: L = 467 plf D = 522 plf 3D = 1566 plf O.K. (3) Shear and moment: According to RISA 3D model, see Figure 5.3 and Table 5.3 For stair slab: Shear strength, V_u = 7.5 kips Flexural positive strength, M_u = 7.0 ft-kips Flexural negative strength, M_u = 11.1 ft-kips If applying 1' strip method, then they become: Shear strength, V_u = 1.61 kips Flexural positive strength, M_u = 1.50 ft-kips Flexural negative strength, M_u = 2.38 ft-kips For landing: Shear strength, V_u = 9.8 kips Flexural positive strength, M_u = 4.0 ft-kips Flexural negative strength, M_u = 11.1 ft-kips If applying 1' strip method, then they become: Shear strength, V_u = 2.10 kips Flexural positive strength, M_u = 0.86 ft-kips Flexural negative strength, M_u = 2.38 ft-kips (4) Calculate required area of reinforcement, $\int_{rqd}$ The value of each parameter shown as follows: Cealr cover = 0.75 in. half of the assumed rebar dia. = 0.375 in. (#6) d'= 1.125 in. h = 6 in. d= 4.875 in. b = 12.00 in. 1' strip	

Step	Computation	Reference
5. Additional Calculation (cont'd)	f_c'= 4,000 psi f_y= 60,000 psi Tension controlled, Φ = 0.9 For stairway, postive moment: m= 17.65 R_u= 70.13 $\int_{rqd}$= 0.0012 $A_{s,rqd+}$= 0.07 square in. For stairway, negative moment: m= 17.65 R_u= 111.21 $\int_{rqd}$= 0.0019 $A_{s,rqd-}$= 0.11 square in. For landing, postive moment: m= 17.65 R_u= 15.36 $\int_{rqd}$= 0.0003 $A_{s,rqd+}$= 0.02 square in. For landing, negative moment: m= 17.65 R_u= 42.62 $\int_{rqd}$= 0.0007 $A_{s,rqd-}$= 0.07 square in. (5) Check for flexural reinforcement limit: $A_{s,min}$= greater of 0.105 square in. √ and 0.082 square in. For calculated values less than the area of minimum reinforcement, apply 0.105 in^2 instead. For landing, it is 0.17 in^2. Therefore, for stairway: $A_{s,rqd+}$= 0.11 square in. $A_{s,rqd-}$= 0.11 square in. for landing: $A_{s,rqd+}$= 0.17 square in. $A_{s,rqd-}$= 0.17 square in.	*Table 21.2.2*

Step	Computation	Reference
5. Additional Calculation (cont'd)	(6) Determine center-to-center spacing, s If pick bar size, no. = 4 diameter of bar = 0.500 in. area of bar = 0.20 square in. For stair and landing, Since, $A_{s,rqd}$ = 0.11 square in. for 1' strip slab No. of bar in 1 ft. strip = 1 Center-to-center spacing, s = 12 in. Since, $A_{s,rqd}$ = 0.17 square in. for 1' strip slab No. of bar in 1 ft. strip = 1 Center-to-center spacing, s = 12 in. (7) Check for shear limit: From previous calculations, For stairway, $\emptyset V_c$ = 3.34 kips For landing, $\emptyset V_c$ = 4.60 kips (8) Minimum temperature and shrinkage reinforcement: From previous calculations, apply #4 in every 12. spacing for temp. reinforcement (9) Check for cracking control spacing requirement: Clear cover, c_c = 0.75 in. f_s = 40,000 psi s_{max}= lesser of 13.13 in. and 12 in. √ since $s = 12\ in. = s_{max} = 12\ in.$ for flexural reinforcement $s = 12\ in. = s_{max} = 12\ in.$ for temp. and shrinkage O.K (10) Check for deflection: According to ACI 318-19, Section 7.3.2.2, since the applied slab thickness satisfies Table 7.3.1.1, deflections occuring after the member becomes composite need not to be checked.	*ACI 318-19 Appendix A* *ACI 318-19 Section 7.3.2.2*

Step	Computation	Reference
5. Additional Calculation (cont'd)	(11) Conclusion: For stairway slab, use #4 reinforcing rebar in every 12 in. spacing for flexural reinforcement, and use #4 reinforcing rebar in every 12 in. spacing for temperature reinforcement. For landing, use #4 reinforcing rebar in every 12 in. spacing for flexural reinforcement, and use #4 reinforcing rebar in every 12 in. spacing for temperature reinforcement. Flexural reinforcement in both stair slab and landing has two layers, both acting as tention reinforcement.	

Step	Computation	Reference
5. Additional Calculation (cont'd)	**<u>5.10 Design for S-CA-6-ii:</u>** (1) Minimum slab thickness, h: The boundary condition for slab is simply supported. Therefore, when l= 116 in. h = 5.79 in. The assumption of the slab thickness was 6 in., which is greater than 5.79 in.. Deflection check is not required. Therefore, h = 6 in. (2) Check if L ≤ 3D: L = 467 plf D = 522 plf 3D = 1566 plf O.K. (3) Shear and moment: According to RISA 3D model, see Figure 5.4 and Table 5.4 For stair slab: Shear strength, V_u = 7.5 kips Flexural strength, M_u = 93.8 ft-kips If applying 1' strip method, Shear strength, V_u = 1.61 kips Flexural strength, M_u = 20.10 ft-kips For landing: Shear strength, V_u = 23.7 kips Flexural strength, M_u = 75.7 ft-kips If applying 1' strip method, Shear strength, V_u = 5.08 kips Flexural strength, M_u = 16.22 ft-kips (4) Calculate required area of reinforcement, $\int_{rqd}$ The values of each parameter are: Cealr cover = 0.75 in. half of the assumed rebar dia. = 0.375 in. (#6) d'= 1.125 in. h = 6 in. d= 4.88 in. b = 12.00 in. 1' strip f_c'= 4,000 psi f_y= 60,000 psi Tension controlled, Φ = 0.9	*Table 21.2.2*

Step	Computation	Reference
5. Additional Calculation (cont'd)	For stair:	
	m= 17.65	
	R_u= 939.73	
	$\int_{rqd}$= 0.0188	
	$A_{s,rqd}$ = 1.10 square in.	
	For landing:	
	m= 17.65	
	R_u= 290.63	
	$\int_{rqd}$= 0.0051	
	$A_{s,rqd}$ = 0.48 square in.	
	(5) Check for flexural reinforcement limit:	
	$A_{s,min}$= greater of	
	0.105 square in. √	
	and	
	0.0819 square in.	
	For calculated values less than the area of minimum reinforcement, apply 0.105 in^2 instead. For landing, it is 0.17 in^2.	
	(6) Determine center-to-center spacing, s	
	If pick bar size, no. = 5	
	diameter of bar = 0.625 in.	*ACI 318-19* *Appendix A*
	area of bar = 0.31 square in.	
	since, $A_{s,rqd}$= 1.10 square in. for stair slab	
	No. of bar in 1 ft. strip = 4	
	Center-to-center spacing, s = 3 in.	
	If pick bar size, no. = 4	
	diameter of bar = 0.500 in.	*ACI 318-19* *Appendix A*
	area of bar = 0.20 square in.	
	since, $A_{s,rqd}$= 0.48 square in. for landing	
	No. of bar in 1 ft. strip = 3	
	Center-to-center spacing, s = 4 in.	
	Now, check for minimum spacing:	
	The diameter of the aggregate is neglected for this project since it shall be determined by the manufacturer.	
	So, s_{min} = greater of	
	1 in. √	
	and	
	0.625 in.	
	Since, $s = 3\ in. > s_{min} = 1\ in.$ O.K.	

Step	Computation	Reference
5. Additional Calculation (cont'd)	(7) Check for shear limit: From previous calculations, For stairway, $\varnothing V_c$ = 6.14 kips For landing, $\varnothing V_c$ = 6.64 kips (8) Minimum temperature and shrinkage reinforcement: From previous calculations, apply #4 in every 12. spacing for temp. reinforcement (9) Check for cracking control spacing requirement: Clear cover, c_c = 0.75 in. f_s = 40,000 psi s_{max}= lesser of 13.13 in. and 12 in. √ since $s = 4\ in. < s_{max} = 12\ in.$ for flexural reinforcement $s = 12\ in. = s_{max} = 12\ in.$for temp. and shrinkage O.K (10) Check for deflection: According to ACI 318-19, Section 7.3.2.2, since the applied slab thickness satisfies Table 7.3.1.1, deflections occuring after the member becomes composite need not to be checked. (11) Conclusion: For stairway slab, use #5 reinforcing rebar in every 3 in. spacing for flexural reinforcement, and use #4 reinforcing rebar in every 12 in. spacing for temperature reinforcement. For landing, use #4 reinforcing rebar in every 4 in. spacing for flexural reinforcement, and use #4 reinforcing rebar in every 12 in. spacing for temperature reinforcement.	*ACI 318-19 Section 7.3.2.2*

Step	Computation	Reference
5. Additional Calculation (cont'd)	**5.11 Design for B-KS-6-i:** (1) Minimum slab thickness, h: The boundary condition for slab is still both ends continuous. Therefore, when l= 116 in. h = 5.51 in. The assumption of the slab thickness was 6 in., which is greater than 5.51 in.. No need to check for deflection limit. Therefore, h = 6 in. (2) Check if L ≤ 3D: L = 467 plf D = 522 plf 3D = 1566 plf O.K. (3) Shear and moment: According to RISA 3D model, see Figure 5.1 and Table 5.1 For stair slab: Flexural strength, Mu = 8.1 ft-kips Shear strength, Vu = 5.7 kips For landing: Flexural strength, Mu = 8.1 ft-kips Shear strength, Vu = 6.4 kips (4) Calculate required area of reinforcement, $\int_{rqd}$ The values of each parameter are: Cealr cover = 1.5 in. half of the assumed rebar dia. = 0.375 in. (#6) Dia. of stirrup = 0.5 in. d'= 2.375 in. h = 6 in. d= 3.625 in. b = 56.00 in. f_c'= 4,000 psi f_y= 60,000 psi Tension controlled, Φ = 0.9 For stair slab: m= 17.65 R_u= 146.76 $\int_{rqd}$= 0.0025 Therefore, $A_{s,rqd}$= 0.51 square in.	*Table 21.2.2*

Step	Computation	Reference
5. Additional Calculation (cont'd)	For landing: m= 17.65 R_u= 43.94 $\int_{rqd}$= 0.0007 Therefore, $A_{s,rqd}$= 0.27 square in. (5) Check for flexural reinforcement limit: $A_{s,min}$ = greater of 0.64 square in. and 0.68 square in. √ For landing, the minimum reinforcement area is 1.24 in^2. (6) Minimum shear reinforcement: Since, b_w= 56.00 in. d = 3.625 in. f_c'= 4,000 psi λ = 1.0 $\emptyset$ = 0.75 So that, V_c= 25.68 kips $\emptyset V_c$ = 19.26 kips $0.5\emptyset V_c$ = 9.63 kips For stair slab $0.5\emptyset V_c$ = 17.60 kips For landing (when h = 9 in., and d = 6.625 in.) For stair slab: Since $V_u = 5.7\ kips < 0.5\emptyset V_c = 9.63 kips$, shear reinforcement is not required. For landing: Since $V_u = 6.4\ kips < 0.5\emptyset V_c = 17.60 kips$, shear reinforcement is not required. (7) Determine center-to-center spacing, s For stair: $A_{s,rqd}$= 0.68 square in. If pick bar size, no. = 4 diameter of bar = 0.500 in. area of bar = 0.20 square in. No. of bar = 4 Center-to-center spacing, s = 12 in. For landing: Apply (7)#4 in 7 in. spacing from previous calculations. Two layers.	*ACI 318-19* *Table 19.2.4.2* *Table 21.2.1* *ACI 318-19* *Appendix A*

Step	Computation	Reference
5. Additional Calculation (cont'd)	Now, check for minimum spacing: The diameter of the aggregate is neglected for this project since it shall be determined by the manufacturer. So, s_{min} = greater of 1 in. √ and 0.500 in. #4 bars Since $s = 7\ in. > s_{min} = 1\ in.$ O.K. (8) Check for cracking control spacing requirement: Clear cover, c_c = 1.5 in. f_s = 40,000 psi s_{max}= lesser of 11.25 in. √ and 12 in. Since $s = 12\ in. > s_{max} = 11.25\ in.$ for flexural reinforcement reduce flexural reinforcement spacing to 11 in. to satisfy the standard. (9) Check for deflection: According to ACI 318-19, Section 9.3.2.2, since the applied slab thickness satisfies Table 9.3.1.1, deflections occuring after the member becomes composite need not to be checked. (10) Conclusion: For stair, it has one layer of #4 reinforcing rebar in every 11 in.. For landing, it has two layers of #4 reinforcing rebar in every 7 in.. No shear reinforcement required for neither stair slab or landing.	*ACI 318-19 Section 9.3.2.2*

Step	Computation	Reference

5.12 Design for B-KS-6-ii:

(1) Minimum slab thickness, h:

The boundary condition for slab is simply supported.

Therefore,

when l = 116 in.

h = 7.24 in.

The assumption of the slab thickness was 6 in., which is less than 7.24 in.. Need to check for deflection limit.

Therefore, h = 6 in.

(2) Check if L ≤ 3D:

L = 467 plf

D = 522 plf

3D = 1566 plf O.K.

(3) Shear and moment:

According to RISA 3D model, see Figure 5.2 and Table 5.2

For stair slab:

Flexural strength, M_u = 66.1 ft-kips

Shear strength, V_u = 5.7 kips

5. Additional Calculation (cont'd)

For landing:

Flexural strength, M_u = 52.5 ft-kips

Shear strength, V_u = 15.9 kips

(4) Calculate required area of reinforcement, $\int_{rqd}$

The values of each parameter are:

Cealr cover = 1.5 in.

half of the assumed rebar dia. = 0.375 in. (#6)

Dia. of stirrup = 0.5 in.

d'= 2.375 in.

h = 6 in.

d= 3.625 in.

b = 56.00 in.

f_c'= 4,000 psi

f_y= 60,000 psi

Tension controlled, Φ = 0.9 *Table 21.2.2*

For stair slab:

m= 17.65

R_u= 1197.67

$\int_{rqd}$= 0.0259

Therefore, $A_{s,rqd}$= 5.25 square in.

Step	Computation	Reference
5. Additional Calculation (cont'd)	For landing: m= 17.65 R_u= 284.80 $\int_{rqd}$= 0.0050 Therefore, $A_{s,rqd}$= 1.84 square in. (5) Check for flexural reinforcement limit: $A_{s,min}$ = greater of 0.64 square in. and 0.68 square in. √ Both areas satisfy with the minimum reinforcement area. (6) Minimum shear reinforcement: Since, b_w= 56.00 in. d = 3.63 in. f_c'= 4,000 psi λ = 1.0 ∅ = 0.75 So that, V_c= 25.68 kips $\emptyset V_c$ = 19.26 kips $0.5\emptyset V_c$ = 9.63 kips For stair slab $0.5\emptyset V_c$ = 17.60 kips For landing For stair: Since $V_u = 5.7\ kips < 0.5\emptyset V_c = 9.63 kips$, shear reinforcement is not required. For landing: Since $V_u = 15.9\ kips < 0.5\emptyset V_c = 17.60 kips$, shear reinforcement is not required. Therefore, for minimum shear reinforcement: $A_{v,min}/s$ shall be the greater of $0.75\sqrt{f_c'}\frac{b_w}{f_{yt}}$ or $50\frac{b_w}{f_{yt}}$ Since, b_w= ~~56.00~~ in. f_c'= ~~4,000~~ psi f_{yt}= ~~60,000~~ psi	*ACI 318-19* *Table 19.2.4.2* *Table 21.2.1* *ACI 318-19* *Table 9.6.3.3*

Step	Computation	Reference
5. Additional Calculation (cont'd)	$A_{v,min}/s$ = greater of	
	0.0443	
	and	
	0.0467	
	Assume s = ~~1.5~~ in.	
	$A_{v,min}$ = ~~0.04~~ square in. for single rebar	
	If pick bar size, no. = ~~4~~	
	diameter of bar = ~~0.500~~ in.	
	area of bar = ~~0.20~~ square in.	
	(7) Determine center-to-center spacing, s	
	For stair slab:	
	$A_{s,rqd}$= 5.25 square in.	
	If pick bar size, no. = 5	
	diameter of bar = 0.625 in.	*ACI 318-19 Appendix A*
	area of bar = 0.31 square in.	
	No. of bar = 17	
	Center-to-center spacing, s = 3 in.	
	For landing:	
	$A_{s,rqd}$= 1.84 square in.	
	If pick bar size, no. = 5	
	diameter of bar = 0.625 in.	*ACI 318-19 Appendix A*
	area of bar = 0.31 square in.	
	No. of bar = 6	
	Center-to-center spacing, s = 8 in.	
	Now, check for minimum spacing:	
	The diameter of the aggregate is neglected for this project since it shall be determined by the manufacturer.	
	So, s_{min} = greater of	
	1 in. √	
	and	
	0.625 in. #5 bars	
	Since $s = 3\ in. > s_{min} = 1\ in.$ O.K.	
	$s = 8\ in. > s_{min} = 1\ in.$ O.K.	
	(8) Check for cracking control spacing requirement:	
	Clear cover, c_c = 1.5 in.	
	f_s = 40,000 psi	
	s_{max}= lesser of	
	11.25 in. √	
	and	
	12 in.	
	The spacings calculated above satisfy with the minimum requirement.	

Step	Computation	Reference
5. Additional Calculation (cont'd)	(9) Check for deflection: (a) Idealize the surface of the concrete that shows the cross sectional area of the main bars Now it becomes a rectangularshape of concrete with: Clear width, b = 56.00 in. slab thickness, h = 6 in. No. of bar size = 5 Area of a single bar, A_s= 0.31 square in. No. of bars = 17 Total area of bars, A_s= 5.27 square in. (b) Determine the modification factor, $n$to convert steel to concrete For normal weight concrete, $E_c = 57{,}000\sqrt{f_c'}$ $n = \frac{E_s}{E_c}$ Since, f_c' = 4,000 psi E_c = 3,605 ksi E_s = 29,000 ksi So that, n = 8.04 (c) Assume the section is cracked, determine the cracking moment $f_r = 7.5\lambda\sqrt{f_c'}$ $I_g = \frac{1}{12}bh^3$ $M_{cr} = \frac{f_r I_g}{y_t}$ where f_c'= 4,000 psi λ = 1.0 f_r= 474 psi I_g= 1008 in^4 y_t= 3 in. M_{cr}= 13.28 ft-kips Since, $M_{cr} = 13.28\, ft - kips < M_u = 66.1\, ft - kips$, it's cracked. (d) Apply transformed area method to determine the distance from the exterme compression fiber to the neutral axis, C_{NA} From Section 3.1, Step (10), $\bar{y} = \frac{\sum A_i y_i}{\sum A_i}$	*ACI 318-19 19.2.2.1.a* *ACI 318-19* *Eq. 19.2.3.1* *Area moment of inertia of rectangular section* *Eq. 24.2.3.5b*

Step	Computation	Reference
5. Additional Calculation (cont'd)	$C_{NA} = \frac{(bC_{NA})\left(\frac{C_{NA}}{2}\right) + (nA_s)d}{(bC_{NA}) + (nA_s)}$ Then, it becomes a quadratic equation: $\frac{b}{2}C_{NA}^2 + nA_sC_{NA} - nA_sd = 0$ The solution of C_{NA}: $C_{NA} = \frac{-nA_s \mp \sqrt{(nA_s)^2+2b(nA_sd)}}{b}$ Therefore, C_{NA} = 1.70 in. (e) Calculate moment of inertia of cracked section transformed to concrete, I_{cr} $I_{cr} = \sum I_i + A_i d_{yi}^2$ Then, it becomes: $I_{cr} = \frac{1}{3}bC_{NA}^3 + nA_s(d - C_{NA})^2$ (The value of I_x is so small that can be neglected) Therefore, I_{cr} = 248.80 in^4 (f) Calculate effective moment of inertia, I_e $I_e = \frac{I_{cr}}{1 - \left(\frac{(2/3)M_{cr}}{M_a}\right)^2 (1 - \frac{I_{cr}}{I_g})}$ M_{cr}= 13.28 ft-kips M_a = 66.10 ft-kips I_g= 1008 in^4 I_{cr}= 249 in^4 So that, I_e = 252 in^4 (g) Check deflection: According to ACI 318-19, Table 24.2.2, immediate deflection due to live load L is $l/360$ l = 116 in. for slab span length $l/360$ = 0.32 Only consider the middle span where the stairway slab is located, see Figure	*ACI 318-19 Eq. 24.2.3.5(b)*

Step	Computation	Reference
5. Additional Calculation (cont'd)	$\Delta_{max}= 5wl^4/384EI$ $E_c = 57{,}000\sqrt{f_c'}$ f_c'= 4 ksi E_c= 3,605 ksi l = 116 in. I_e= 252 in^4 w= 1370 plf Δ_{max}= 0.29 Since, $\Delta_{max}= 0.29 < l/360 = 0.32$, this design satisfies with the deflection requirement (10) Conclusion: For stair slab, use #5 reinforcing rebar in every 3 in. spacing for flexural reinforcement. No shear reinforcement is required. For landing, use #5 reinforcing rebar in every 8 in. spacing for flexural reinforcement. No shear reinforcement is required.	*ACI 318-19 Eq. 19.2.2.1.b*

Step	Computation	Reference
5. Additional Calculation (cont'd)	**5.13 Design for B-CA-6-i:**	
	(1) Minimum slab thickness, h:	
	The boundary condition for slab is still both ends continuous.	
	Therefore,	
	when l= 116 in.	
	h = 5.51 in.	
	The assumption of the slab thickness was 6 in., which is greater than 5.51 in.. No need to check for deflection limit.	
	Therefore, h = 6 in.	
	(2) Check if L ≤ 3D:	
	L = 467 plf	
	D = 522 plf	
	3D = 1566 plf O.K.	
	(3) Shear and moment:	
	According to RISA 3D model, see Figure 5.3 and Table 5.3	
	For stair slab:	
	Flexural strength, M_u = 11.1 ft-kips	
	Shear strength, V_u = 7.5 kips	
	For landing:	
	Flexural strength, M_u = 11.1 ft-kips	
	Shear strength, V_u = 9.8 kips	
	(4) Calculate required area of reinforcement, $\int_{rqd}$	
	The values of each parameter are:	
	Cealr cover = 1.5 in.	
	half of the assumed rebar dia. = 0.375 in. (#6)	
	Dia. of stirrup = 0.5 in.	
	d'= 2.375 in.	
	h = 6 in.	
	d= 3.625 in.	
	b = 56.00 in.	
	f_c'= 4,000 psi	
	f_y= 60,000 psi	
	Tension controlled, Φ = 0.9	*Table 21.2.2*
	For stair slab:	
	m= 17.65	
	R_u= 201.12	
	$\int_{rqd}$= 0.0035	
	Therefore, $A_{s,rqd}$= 0.70 square in.	

Step	Computation	Reference
5. Additional Calculation (cont'd)	For landing: m= 17.65 R_u= 60.21 $\int_{rqd}$= 0.0010 Therefore, $A_{s,rqd}$= 0.38 square in. (5) Check for flexural reinforcement limit: $A_{s,min}$= greater of 0.64 square in. and 0.68 square in. √ For landing, the minimum reinforcement area is 1.24 in^2. (6) Minimum shear reinforcement: Since, b_w= 56.00 in. d = 3.63 in. f_c'= 4,000 psi λ = 1.0 $\emptyset$ = 0.75 So that, V_c= 25.68 kips $\emptyset V_c$ = 19.26 kips $0.5\emptyset V_c$ = 9.63 kips For stair slab $0.5\emptyset V_c$ = 17.60 kips For landing For stair slab: Since $V_u = 7.5\ kips < 0.5\emptyset V_c = 9.63kips$, shear reinforcement is not required. For landing: Since $V_u = 9.8\ kips < 0.5\emptyset V_c = 17.60kips$, shear reinforcement is not required. (7) Determine center-to-center spacing, s For stair: $A_{s,rqd}$= 0.70 square in. If pick bar size, no. = 4 diameter of bar = 0.500 in. area of bar = 0.20 square in. No. of bar = 4 Center-to-center spacing, s = 12 in. For landing: From previous calculations, it has two layers of (7)#4 in 7 in. spacing.	*ACI 318-19* *Table 19.2.4.2* *Table 21.2.1* *ACI 318-19* *Appendix A*

Step	Computation	Reference
5. Additional Calculation (cont'd)	Now, check for minimum spacing: The diameter of the aggregate is neglected for this project since it shall be determined by the manufacturer. So, s_{min} = greater of 1 in. √ and 0.500 in. #4 bars Since $s = 7\ in. > s_{min} = 1\ in.$ O.K. (8) Check for cracking control spacing requirement: Clear cover, c_c = 1.5 in. f_s = 40,000 psi s_{max} = lesser of 11.25 in. √ and 12 in. Since $s = 12\ in. > s_{max} = 11.25\ in.$ for flexural reinforcement reduce flexural reinforcement spacing to 11 in. to satisfy the standard. (9) Check for deflection: According to ACI 318-19, Section 9.3.2.2, since the applied slab thickness satisfies Table 9.3.1.1, deflections occuring after the member becomes composite need not to be checked. (10) Conclusion: For stair, it has one layer of #4 reinforcing rebar in every 11 in.. For landing, it has two layers of #4 reinforcing rebar in every 7 in.. No shear reinforcement required for neither stair slab or landing.	*ACI 318-19 Section 9.3.2.2*

Step	Computation	Reference
5. Additional Calculation (cont'd)	**<u>5.14 Design for B-CA-6-ii:</u>** (1) Minimum slab thickness, h: The boundary condition for slab is simply supported. Therefore, when l= 116 in. h = 7.24 in. The assumption of the slab thickness was 6 in., which is less than 7.24 in.. Need to check for deflection limit. Therefore, h = 6 in. (2) Check if L ≤ 3D: L = 467 plf D = 522 plf 3D = 1566 plf O.K. (3) Shear and moment: According to RISA 3D model, see Figure 5.4 and Table 5.4 For stair slab: Flexural strength, M_u = 93.8 ft-kips Shear strength, V_u = 7.5 kips For landing: Flexural strength, M_u = 75.7 ft-kips Shear strength, V_u = 23.7 kips (4) Calculate required area of reinforcement, $\int_{rqd}$ The values of each parameter are: Cealr cover = 1.5 in. half of the assumed rebar dia. = 0.375 in. (#6) Dia. of stirrup = 0.5 in. d'= 2.375 in. h = 6 in. d= 3.625 in. b = 56.00 in. f_c'= 4,000 psi f_y= 60,000 psi Tension controlled, Φ = 0.9 For stair slab: m= 17.65 R_u= 1699.56 $\int_{rqd}$= 0.0558 Therefore, $A_{s,rqd}$= 11.32 square in.	*Table 21.2.2*

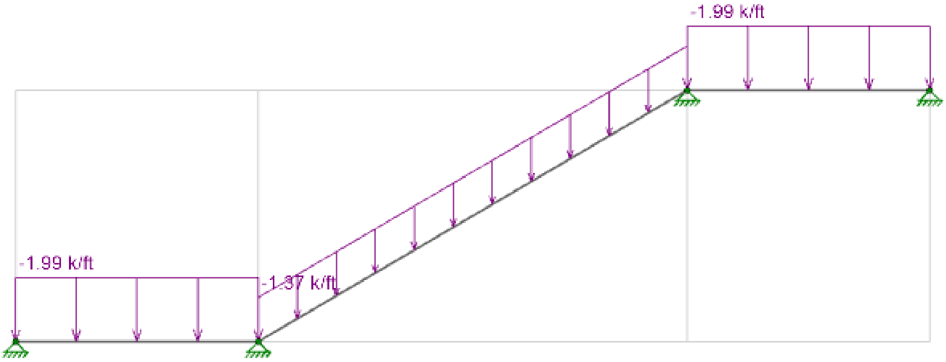
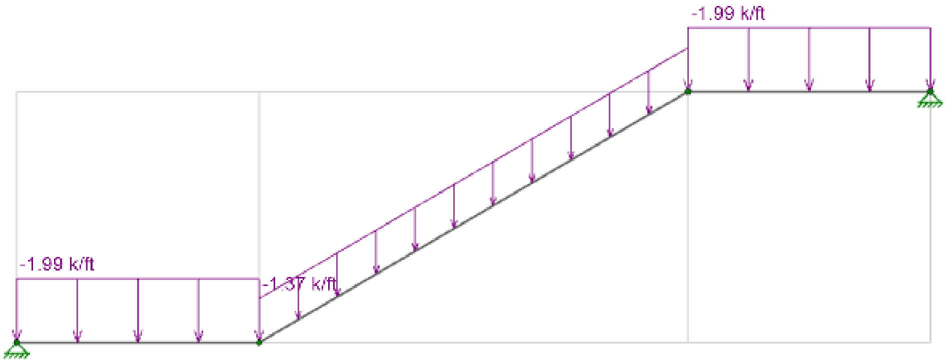
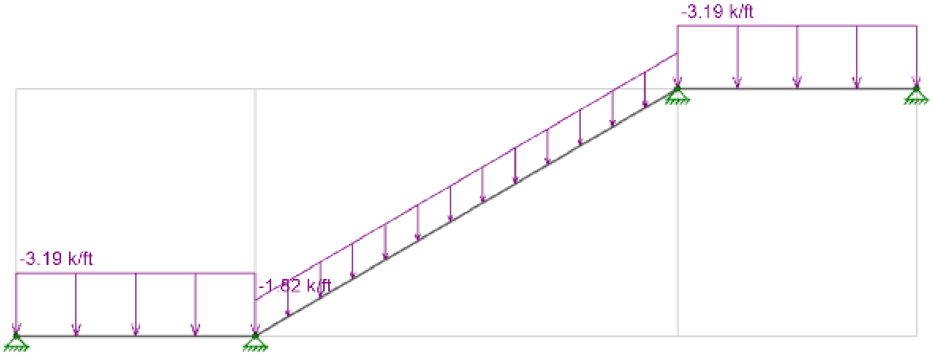
Step	Computation	Reference
5. Additional Calculation (cont'd)	For landing:	
	$m=$ 17.65	
	$R_u=$ 410.65	
	$\int_{rqd}=$ 0.0073	
	Therefore, $A_{s,rqd}=$ 2.71 square in.	
	(5) Check for flexural reinforcement limit:	
	$A_{s,min}=$ greater of	
	0.64 square in.	
	and	
	0.68 square in. √	
	For landing, the minimum reinforcement area is 1.24 in^2.	
	(6) Minimum shear reinforcement:	
	Since, $b_w=$ 56.00 in.	
	$d=$ 3.63 in.	
	$f_c'=$ 4,000 psi	*ACI 318-19*
	$\lambda=$ 1.0	*Table 19.2.4.2*
	$\emptyset=$ 0.75	*Table 21.2.1*
	So that, $V_c=$ 25.68 kips	
	$\emptyset V_c=$ 19.26 kips	
	$0.5\emptyset V_c=$ 9.63 kips	
	For stair slab	
	$0.5\emptyset V_c=$ 17.60 kips	
	For landing	
	For stair:	
	Since $V_u = 7.5\ kips < 0.5\emptyset V_c = 9.63 kips$, shear reinforcement is not required.	
	For landing:	
	Since $V_u = 23.7\ kips > 0.5\emptyset V_c = 17.60 kips$, shear reinforcement is required.	
	Therefore, for minimum shear reinforcement:	
	$A_{v,min}/s$ shall be the greater of $0.75\sqrt{f_c'}\frac{b_w}{f_{yt}}$	
	or	*ACI 318-19* *Table 9.6.3.3*
	$50\frac{b_w}{f_{yt}}$	
	Since,	
	$b_w=$ 56.00 in.	
	$f_c'=$ 4,000 psi	
	$f_{yt}=$ 60,000 psi	

Step	Computation	Reference
5. Additional Calculation (cont'd)	$A_{v,min}/s$ = greater of 0.0443 and 0.0467 √ Assume s = 3 in. The maximum spacing of shear reinforcement should not be greater than d/2 or 24 in.. Since d/2 = 3.3125 in. for landing, take 3 in. $A_{v,min}$ = 0.07 square in. for single rebar If pick bar size, no. = 4 diameter of bar = 0.500 in. area of bar = 0.20 square in. √ (7) Determine center-to-center spacing, s For stair slab: $A_{s,rqd}$= 11.32 square in. If pick bar size, no. = 6 diameter of bar = 0.750 in. area of bar = 0.44 square in. No. of bar = 26 Center-to-center spacing, s = 2 in. For landing: $A_{s,rqd}$= 2.71 square in. If pick bar size, no. = 6 diameter of bar = 0.750 in. area of bar = 0.44 square in. No. of bar = 7 Center-to-center spacing, s = 7 in. Now, check for minimum spacing: The diameter of the aggregate is neglected for this project since it shall be determined by the manufacturer. So, s_{min} = greater of 1 in. √ and 0.750 in. #6 bars Since $s = 2\ in. > s_{min} = 1\ in.$ O.K. $s = 7\ in. > s_{min} = 1\ in.$ O.K. (8) Check for cracking control spacing requirement: Clear cover, c_c= 1.5 in. f_s= 40,000 psi s_{max}= lesser of 11.25 in. √ and 12 in. The spacings calculated above satisfy with the minimum requirement.	*ACI 318-19 Appendix A* *ACI 318-19 Appendix A*

Step	Computation	Reference
5. Additional Calculation (cont'd)	(9) Check for deflection: (a) Idealize the surface of the concrete that shows the cross sectional area of the main bars Now it becomes a rectangularshape of concrete with: Clear width, b= 56.00 in. slab thickness, h= 6 in. No. of bar size = 6 Area of a single bar, A_s= 0.44 square in. No. of bars = 26 Total area of bars, A_s= 11.44 square in.	
	(b) Determine the modification factor, $n$to convert steel to concrete For normal weight concrete, $E_c = 57{,}000\sqrt{f_c'}$ $n = \frac{E_s}{E_c}$ Since, f_c' = 4,000 psi E_c = 3,605 ksi E_s = 29,000 ksi So that, n = 8.04	*ACI 318-19* *19.2.2.1.a*
	(c) Assume the section is cracked, determine the cracking moment $f_r = 7.5\lambda\sqrt{f_c'}$ $I_g = \frac{1}{12}bh^3$ $M_{cr} = \frac{f_r I_g}{y_t}$ where f_c'= 4,000 psi λ = 1.0 f_r= 474 psi I_g= 1008 in^4 y_t= 3 in. M_{cr}= 13.28 ft-kips Since, $M_{cr} = 13.28\,ft - kips < M_u = 93.8\,ft - kips$, it's cracked.	*ACI 318-19* *Eq. 19.2.3.1* *Area moment of inertia of rectangular section* *Eq. 24.2.3.5b*
	(d) Apply transformed area method to determine the distance from the exterme compression fiber to the neutral axis, C_{NA} From Section 3.1, Step (10), $\bar{y} = \frac{\sum A_i y_i}{\sum A_i}$	

Step	Computation	Reference
5. Additional Calculation (cont'd)	$C_{NA} = \dfrac{(bC_{NA})\left(\frac{C_{NA}}{2}\right) + (nA_s)d}{(bC_{NA}) + (nA_s)}$ Then, it becomes a quadratic equation: $\frac{b}{2}C_{NA}^2 + nA_sC_{NA} - nA_sd = 0$ The solution of C_{NA}: $C_{NA} = \dfrac{-nA_s \mp \sqrt{(nA_s)^2 + 2b(nA_sd)}}{b}$ Therefore, C_{NA} = 2.18 in. (e) Calculate moment of inertia of cracked section transformed to concrete, I_{cr} $I_{cr} = \sum I_i + A_i d_{yi}^2$ Then, it becomes: $I_{cr} = \frac{1}{3}bC_{NA}^3 + nA_s(d - C_{NA})^2$ (The value of I_x is so small that can be neglected) Therefore, I_{cr} = 385.55 in^4 (f) Calculate effective moment of inertia, I_e $I_e = \dfrac{I_{cr}}{1 - \left(\frac{(2/3)M_{cr}}{M_a}\right)^2 (1 - \frac{I_{cr}}{I_g})}$ M_{cr}= 13.28 ft-kips M_a = 93.80 ft-kips I_g= 1008 in^4 I_{cr}= 386 in^4 So that, I_e = 388 in^4 (g) Check deflection: According to ACI 318-19, Table 24.2.2, immediate deflection due to live load L is $l/360$ l = 116 in. for slab span length $l/360$ = 0.32 Only consider the middle span where the stairway slab is located, see Figure	*ACI 318-19 Eq. 24.2.3.5(b)*

Step	Computation	Reference
5. Additional Calculation (cont'd)	$\Delta_{max}= 5wl^4/384EI$ $E_c = 57{,}000\sqrt{f_c'}$ f_c'= 4 ksi E_c= 3,605 ksi l = 116 in. I_e= 388 in^4 w= 1820 plf Δ_{max}= 0.25 Since, $\Delta_{max}= 0.25 < l/360 = 0.32$, this design satisfies with the deflection requirement (10) Conclusion: For stair slab, use #6 reinforcing rebar in every 2 in. spacing for flexural reinforcement. No shear reinforcement is required. For landing, use #6 reinforcing rebar in every 7 in. spacing for flexural reinforcement. Use #4 reinforcing rebar in every 3 in. for shear reinforcement.	*ACI 318-19 Eq. 19.2.2.1.b*

Step	Computation	Reference
5. Additional Calculation (cont'd)	5.15 Graphs and tables:  Figure 5.1 Kansas-6-i  Figure 5.2 Kansas-6-ii  Figure 5.3 California-6-i	

Step	Computation	Reference
5. Additional Calculation (cont'd)		

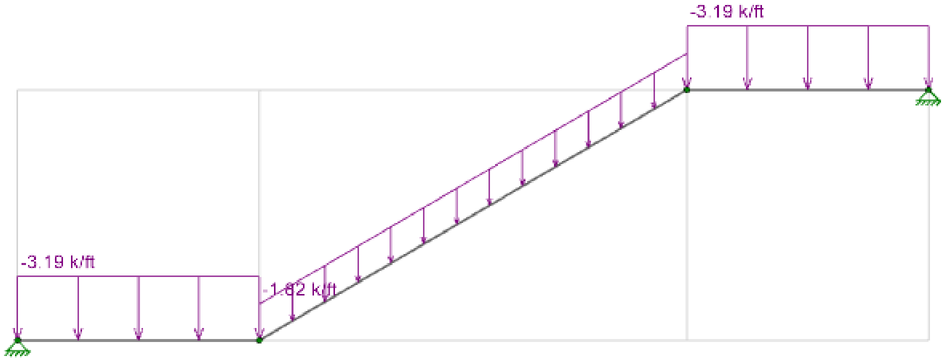
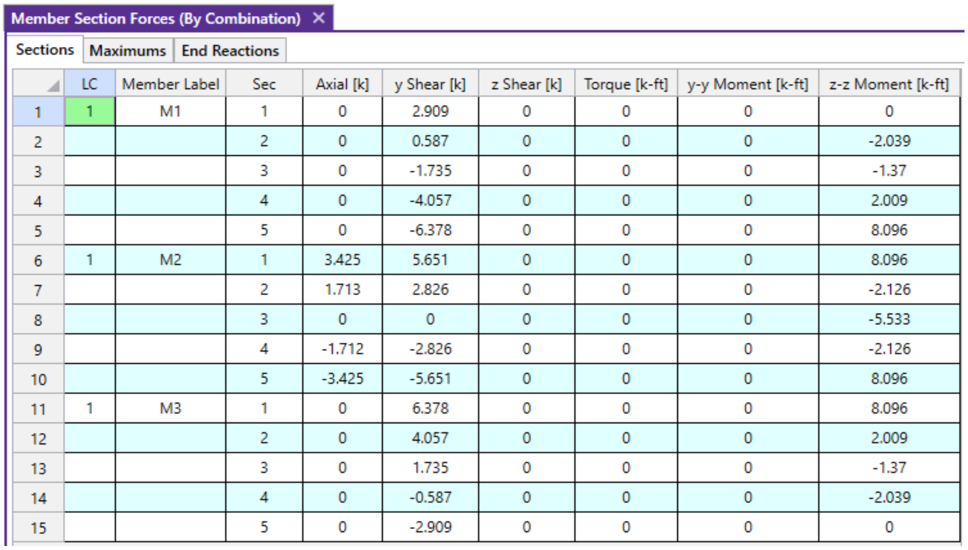


Figure 5.4 California-6-ii

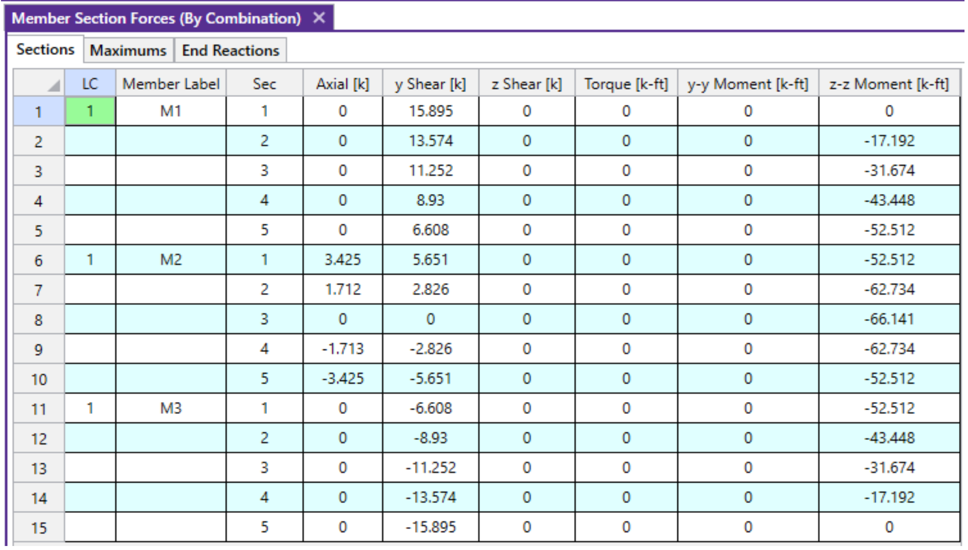
Table 5.1 Kansas-6-i member forces



	LC	Member Label	Sec	Axial [k]	y Shear [k]	z Shear [k]	Torque [k-ft]	y-y Moment [k-ft]	z-z Moment [k-ft]
1	1	M1	1	0	2.909	0	0	0	0
2			2	0	0.587	0	0	0	-2.039
3			3	0	-1.735	0	0	0	-1.37
4			4	0	-4.057	0	0	0	2.009
5			5	0	-6.378	0	0	0	8.096
6	1	M2	1	3.425	5.651	0	0	0	8.096
7			2	1.713	2.826	0	0	0	-2.126
8			3	0	0	0	0	0	-5.533
9			4	-1.712	-2.826	0	0	0	-2.126
10			5	-3.425	-5.651	0	0	0	8.096
11	1	M3	1	0	6.378	0	0	0	8.096
12			2	0	4.057	0	0	0	2.009
13			3	0	1.735	0	0	0	-1.37
14			4	0	-0.587	0	0	0	-2.039
15			5	0	-2.909	0	0	0	0

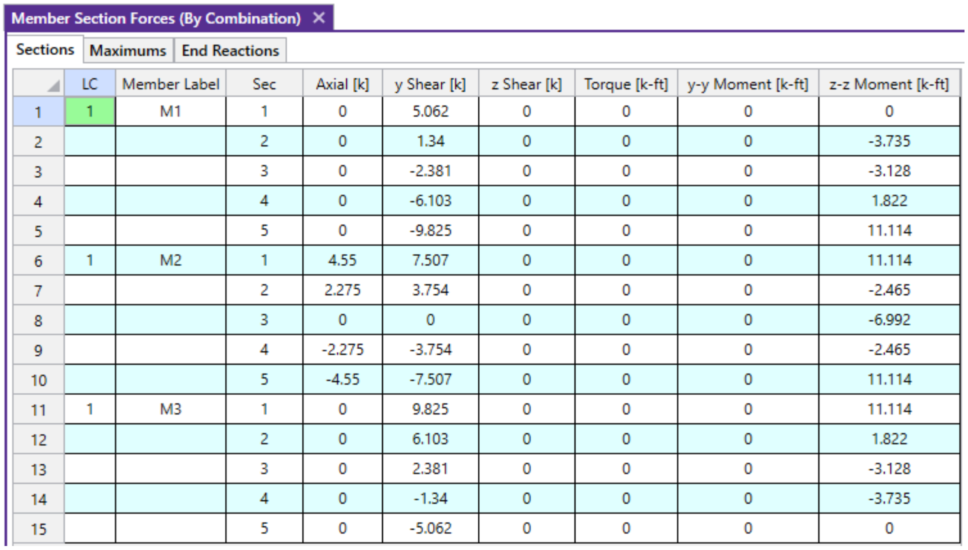
Step	Computation	Reference
5. Additional Calculation (cont'd)	Table 5.2 Kansas-6-ii member forces Table 5.3 California-6-i member forces	

Table 5.2 Kansas-6-ii member forces

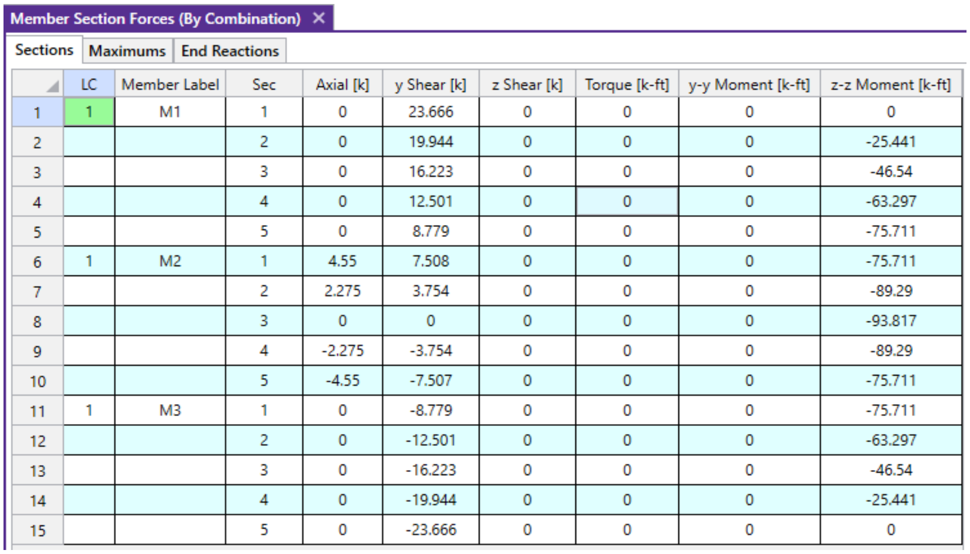


	LC	Member Label	Sec	Axial [k]	y Shear [k]	z Shear [k]	Torque [k-ft]	y-y Moment [k-ft]	z-z Moment [k-ft]
1	1	M1	1	0	15.895	0	0	0	0
2			2	0	13.574	0	0	0	-17.192
3			3	0	11.252	0	0	0	-31.674
4			4	0	8.93	0	0	0	-43.448
5			5	0	6.608	0	0	0	-52.512
6	1	M2	1	3.425	5.651	0	0	0	-52.512
7			2	1.712	2.826	0	0	0	-62.734
8			3	0	0	0	0	0	-66.141
9			4	-1.713	-2.826	0	0	0	-62.734
10			5	-3.425	-5.651	0	0	0	-52.512
11	1	M3	1	0	-6.608	0	0	0	-52.512
12			2	0	-8.93	0	0	0	-43.448
13			3	0	-11.252	0	0	0	-31.674
14			4	0	-13.574	0	0	0	-17.192
15			5	0	-15.895	0	0	0	0

Table 5.3 California-6-i member forces



	LC	Member Label	Sec	Axial [k]	y Shear [k]	z Shear [k]	Torque [k-ft]	y-y Moment [k-ft]	z-z Moment [k-ft]
1	1	M1	1	0	5.062	0	0	0	0
2			2	0	1.34	0	0	0	-3.735
3			3	0	-2.381	0	0	0	-3.128
4			4	0	-6.103	0	0	0	1.822
5			5	0	-9.825	0	0	0	11.114
6	1	M2	1	4.55	7.507	0	0	0	11.114
7			2	2.275	3.754	0	0	0	-2.465
8			3	0	0	0	0	0	-6.992
9			4	-2.275	-3.754	0	0	0	-2.465
10			5	-4.55	-7.507	0	0	0	11.114
11	1	M3	1	0	9.825	0	0	0	11.114
12			2	0	6.103	0	0	0	1.822
13			3	0	2.381	0	0	0	-3.128
14			4	0	-1.34	0	0	0	-3.735
15			5	0	-5.062	0	0	0	0

Step	Computation	Reference
5. Additional Calculation (cont'd)	Figure 5.4 California-6-ii member forces 	

	LC	Member Label	Sec	Axial [k]	y Shear [k]	z Shear [k]	Torque [k-ft]	y-y Moment [k-ft]	z-z Moment [k-ft]
1	1	M1	1	0	23.666	0	0	0	0
2			2	0	19.944	0	0	0	-25.441
3			3	0	16.223	0	0	0	-46.54
4			4	0	12.501	0	0	0	-63.297
5			5	0	8.779	0	0	0	-75.711
6	1	M2	1	4.55	7.508	0	0	0	-75.711
7			2	2.275	3.754	0	0	0	-89.29
8			3	0	0	0	0	0	-93.817
9			4	-2.275	-3.754	0	0	0	-89.29
10			5	-4.55	-7.507	0	0	0	-75.711
11	1	M3	1	0	-8.779	0	0	0	-75.711
12			2	0	-12.501	0	0	0	-63.297
13			3	0	-16.223	0	0	0	-46.54
14			4	0	-19.944	0	0	0	-25.441
15			5	0	-23.666	0	0	0	0

Appendix C - Beam Design Calculations

Step	Computation	Reference
6. Beam Design	**6.1 Problem statement:** Beams support stair slab and landings. And, these beams are part of the stairway design as well. This section provide a design example for one of the beams selected from case S-KS-8-i. **6.2 General information:** See Graph The selected beam supports the whole space of landing (not half) and two stairway going up and down. **6.3 Assumptions:** (1) Simply supported concrete beam (2) Torsional effect is neglected (3) When calculating load combination, flip the stairway to horizontal direction to simply the design (4) Assume the beam section is 9 inches in width, and 12 inches in depth (5) Concrete member is nonprestressed (6) Normal weight concrete, with specified compressive strength equals to 4,000 psi (7) Specified yield strength for nonprestressed reinforcement equals to 60,000 psi **6.4 Concrete beam dimensions:** Beam clear width, b_w= 9 in. Beam thickness, h = 12 in. Beam span, l = 112 in. **6.5 Building loads:** (1) Dead loads: From previous calculations, Dead load of landing, D = 223 psf when thickness is 8 in., dead load of stair flight, D = 136 psf As the beam carries loads from both stairway going up and down, and also the landing. Therefore, Area of stair slab carried = 6,483 square in. Area of landing carried = 3,136 square in. So that, Dead load of landing, D = 4,846 lbf when thickness is 8 in., dead load of stair flight, D = 6,128 lbf The beam also contains its selfweight, which is 145 pcf normal weight concrete. Selfweight = 145 pcf	

Step	Computation	Reference
6. Beam Design (cont'd)	Volume of beam = 7 cubic ft. Selfweight dead load = 1,015 lbf The total dead load carried by a beam is: D = 11,988 lbf 11.99 kips Convert kip to klf by dividing by the beam span length, 112 in. D = **1.28** klf	
	(2) Live load: From previous calculation, L = 100 psf Convert psf to plf (times beam clear width), L = 75 plf **0.075** klf	*IBC 2015, Table 1607.1*
	(3) Wind load: From previous calculation, W = 16 psf Convert psf to plf (times beam clear width), W = 12 plf **0.012** klf	*ASCE 7-16, Section 29.8*
	6.6 Load combinations: According to ASCE 7-16, Chapter 2, Section 2.3.2, the load combinations are shown as following: 1. $1.4D$ 2. $1.2D + 1.6L + 0.5(L_r \text{ or } S \text{ or } R)$ 3. $1.2D + 1.6(L_r \text{ or } S \text{ or } R) + (L \text{ or } 0.5W)$ 4. $1.2D + 1.0W + L + 0.5(L_r \text{ or } S \text{ or } R)$ 5. $1.2D + 1.0E + L + 0.2S$ 6. $0.9D + 1.0W$ 7. $0.9D + 1.0E$ where D = dead load E = earthquake load L = live load L_r = roof live load R = rain load S = snow load W = wind load	*ASCE 7-16, Chapter 2, Section 2.3.2*
	The result of each load combination shown as follows: LC-1 = **1.80** klf Governs LC-2 = 1.66 klf LC-3 = 1.62 klf	

Step	Computation	Reference
6. Beam Design (cont'd)		

LC-4 = 1.60 klf

LC-5 = 1.62 klf

LC-6 = 1.17 klf

LC-7 = 1.16 klf

6.7 Reinforcement design:

(1) Minimum slab thickness, h:

Acoording to ACI 318-19, Table 9.3.1.1 - Minimum thickness of solid nonprestressed one-way slabs:

ACI 318-19 Table 9.3.1.1

Support condition	Minimum h
Simply supported	$l/16$
One end continuous	$l/18.5$
Both ends continuous	$l/21$
Cantilever	$l/8$

$l/16$ = 7 in.

Since the assumed thickness of beam is 12 in., which is greater than the value of $l/16$, no need to check for deflection.

h = 12 in.

(2) Check if L ≤ 3D, where L stands for live load, D stands for dead load:

ACI 318-19 Section 6.5.1

L = 0.075 klf

$3D$= 3.85 klf

since, $L = 0.075klf < 3D = 3.35klf$ O.K.

(3) Calculate shear and moment based on RISA 3D:

See Figure 6.1, 6.2, and 6.3

Shear strength, V_u = 8.4 kips

Flexural strength, M_u = 19.6 ft-kips

(4) Calculate required area of reinforcement, $\int$, based on flexural moment:

$$m = \frac{f_y}{0.85f_c'}$$

$$R_u = \frac{M_u}{\Phi bd^2}$$

$$\int_{rqd} = \frac{1}{m}[1 - \sqrt{1 - \left(\frac{2R_u m}{f_y}\right)}]$$

The values of each parameter are:

Cealr cover = 1.5 in.

half of the assumed rebar dia. = 0.375 in. (#6)

Dia. of stirrup = 0.5 in.

d'= 2.375 in.

h = 12 in.

d= 9.625 in.

Step	Computation	Reference
6. Beam Design (cont'd)	b = 9.00 in.	
	f'_c= 4,000 psi	
	f_y= 60,000 psi	
	Tension controlled, Φ = 0.9	*Table 21.2.2*
	m= 17.65	
	R_u= 313.44	
	$\int_{rqd}$= 0.0055	
	Therefore, $A_{s,rqd}$= 0.48 square in.	
	(5) Check for flexural reinforcement limits:	
	$A_{s,min}$ shall be the greater of (a) and (b), according to ACI 318-19, Section 9.6.1.2	
	(a) $\frac{3\sqrt{f'_c}}{f_y} b_w d$	*ACI 318-19 Section 9.6.1.2*
	(b) $\frac{200}{f_y} b_w d$	
	$A_{s,min}$ = greater of	
	0.27 square in.	
	and	
	0.29 square in √	
	Since, $A_{s,rqd} = 0.48\ in^2 > A_{s,,min} = 0.29\ in^2$ O.K.	
	(6) Shear reinforcement:	
	A minimum area of shear reinforcement, $A_{v,min}$ shall be provided in all regions where $V_u > 0.5\Phi V_c$	*ACI 318-19 Section 9.6.3.1*
	If shear reinforcement is required, then:	
	Set	
	$V_u = \emptyset V_n, \text{where } \emptyset = 0.75$	
	then	
	$V_n = V_c + V_s$	*Eq. 22.5.1.1*
	and	
	$V_c = \left[2\lambda\sqrt{f'_c} + \frac{N_u}{6A_g}\right] b_w d$	*Table 22.5.5.1 (b)*
	$V_s = \frac{A_v f_{yt} d}{s}$	*Eq. 22.5.10.5.3*
	which can be re-writen to:	
	$\frac{A_v}{s} = \frac{V_s}{f_{yt} d}$	

Step	Computation	Reference

6. Beam Design (cont'd)

Check with minimum shear reinforcement, in accordance with ACI 318-19, Section 9.6.3.3

Beam type	$A_{v,min}/s$	
Nonprestressed	Greater of:	$0.75\sqrt{f'_c}\frac{b_w}{f_{yt}}$
		$50\frac{b_w}{f_{yt}}$

ACI 318-19 Table 9.6.3.3

Since,

b_w = 9.00 in.

d = 9.63 in.

f'_c = 4,000 psi

λ = 1.0

$\emptyset$ = 0.75

ACI 318-19 Table 19.2.4.2 Table 21.2.1

So that,

V_c = 10.96 kips

$\emptyset V_c$ = 8.22 kips

$0.5\emptyset V_c$ = 4.11 kips

Since $V_u = 8.4\,kips > 0.5\emptyset V_c = 4.11 kips$, shear reinforcement is required.

Therefore, for minimum shear reinforcement:

$A_{v,min}/s$ shall be the greater of

$$0.75\sqrt{f'_c}\frac{b_w}{f_{yt}}$$

or

$$50\frac{b_w}{f_{yt}}$$

ACI 318-19 Table 9.6.3.3

Since,

b_w = 9.00 in.

f'_c = 4,000 psi

f_{yt} = 60,000 psi

$A_{v,min}/s$ = greater of

0.0071

and

0.0075 √

Assume

s = 4 in.

$A_{v,min}$ = 0.02 square in for single rebar

If pick bar size, no. = 4

diameter of bar = 0.500 in.

area of bar = 0.20 square in √

Step	Computation	Reference

6. Beam Design (cont'd)

(7) Determine center-to-center spacing, s of reinforcement:
Minimum spacing - shall be at least the greatest of 1 in., d_b and $(4/3)d_{agg}$
Minimum layer spacing - shall be at least 1 in. between layers for parallel nonprestressed reinforcement placed in two or more horizontal layers

ACI 318-19 Section 25.2.1, and 25.2.2

$A_{s,rqd}$= 0.48 square in.
If pick bar size, no. = 5
diameter of bar = 0.625 in.
area of bar = 0.31 square in.
No. of bar = 2
Center-to-center spacing, s = 4 in.

ACI 318-19 Appendix A

Now, check for minimum spacing:
The diameter of the aggregate is neglected for this project since it shall be determined by the manufacturer.
So, s_{min} = greater of
1 in. √
and
0.625 in. #5 bars
Since $s = 4\ in. > s_{min} = 1\ in.$ O.K.

(8) Check for cracking control spacing requirement:

Reinforcement type	Maximum spacing s	
Deformed bars or wires	Lesser of:	$15\left(\frac{40{,}000}{f_s}\right) - 2.5c_c$
		$12\left(\frac{40{,}000}{f_s}\right)$

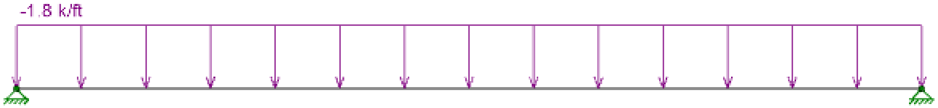
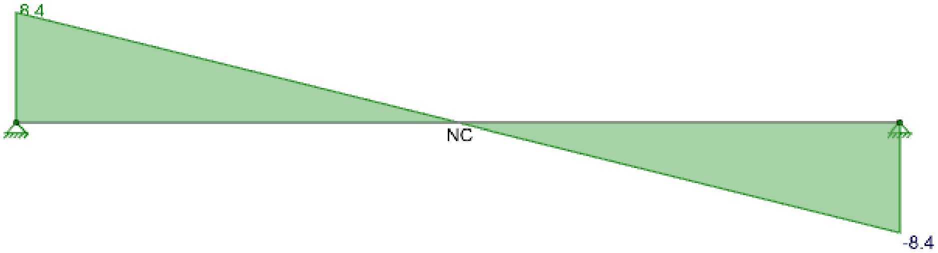
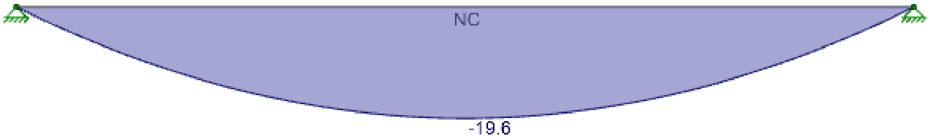
ACI 318-19 Table 24.3.2

Clear cover, c_c = 1.5 in.
f_s = 40,000 psi
s_{max}= lesser of
11.25 in. √
and
12 in.

Since $s = 4\ in. < s_{max} = 11.25\ in.$ for flexural reinforcement
It's O.K.

(9) Check for deflection:
According to ACI 318-19, Section 9.3.2.2, since the applied slab thickness satisfies Table 9.3.1.1, deflections occuring after the member becomes composite need not to be checked.

ACI 318-19 Section 9.3.2.2

Step	Computation	Reference
6. Beam Design (cont'd)	(10) Conclusion: For flexural reinforcement, use #5 reinforcing rebar in every 4 in. For shear reinforcement, use #4 reinforcing rebar in every 4 in. 6.8 Graphs and tables:  Figure 6.1 Beam uniform load  Figure 6.2 Beam shear diagram  Figure 6.3 Beam moment diagram	

Appendix D - Flexural Reinforcement Design Equation Derivation

Step	Computation	Reference
Derivation	**1. Problem:** Flexural reinforcement design equation derivation. **2. Assumptions:** (1) 60 Grade steel (2) Specified yield strength of nonprestressed reinforcement, $f_y = 60{,}000\ \text{psi}$ (3) modulus of elasticity of reinforcement and structural steel, $E_s = 29{,}000\ \text{ksi}$ (4) Assume tension equals compression, $T = C$ (5) Assume tension reinforcemen is yielding **3. Derivation:** Since tension reinforcement is yielding, the tension force in the reinforcement will be the area of reinforcement, $A_s$multiplied by the yielding stress of reinforcement, f_y Therefore, $T = A_s f_y$ (Eq. 1) And, $C = 0.85 f_c' ba$ If assume, $T = C$ the equation becomes, $A_s f_y = 0.85 f_c' ba$ To: $a = \frac{A_s f_y}{0.85 f_c' b}$ Multiply by $\frac{d}{d}$, it becomes: $a = \frac{A_s f_y}{0.85 f_c' b} \times \frac{d}{d}$ Since the equation of reinforcement ratio is $\int = \frac{A_s}{bd}$ (Eq. 2) Therefore, $a = \frac{\int f_y d}{0.85 f_c'}$ (Eq. 3) The expression of nominal moment is: $M_n = C(d - \frac{a}{2})$ or $M_n = T(d - \frac{a}{2})$ Because of the Eq. 1 and the assumption that tension equals to compression, the equation of nomimal moment becomes: $M_n = A_s f_y (d - \frac{a}{2})$ (Eq. 4)	

Step	Computation	Reference
Derivation	For design moment, multiply Eq. 4 by $\emptyset$ $\emptyset M_n = \emptyset A_s f_y (d - \frac{a}{2})$ (Eq. 5) Bring Eq. 3 into Eq. 5, it becomes: $\emptyset M_n = \emptyset A_s f_y (d - \frac{\int f_y d}{1.7 f_c'})$ (Eq. 6) Bring Eq. 2 into Eq. 6, it becomes: $\emptyset M_n = \emptyset \int b d f_y (d - \frac{\int f_y d}{1.7 f_c'})$ (Eq. 7) For design, $\emptyset M_n \geq M_u$ Assume $\emptyset M_n = M_u$ for this derivation Therefore, Eq. 7 becomes: $M_u = \emptyset \int b d f_y (d - \frac{\int f_y d}{1.7 f_c'})$ (Eq. 8) Divide both sides of Eq. 8 by $\emptyset b d^2$ $\frac{M_u}{\emptyset b d^2} = \frac{\emptyset \int b d f_y (d - \frac{\int f_y d}{1.7 f_c'})}{\emptyset b d^2}$ $\frac{M_u}{\emptyset b d^2} = \frac{\emptyset b d^2 \int f_y (1 - \frac{\int f_y}{1.7 f_c'})}{\emptyset b d^2}$ $\frac{M_u}{\emptyset b d^2} = \int f_y (1 - \frac{\int f_y}{1.7 f_c'})$ (Eq. 9) Set $\boxed{R_u = \frac{M_u}{\emptyset b d^2}}$ (Eq. 10) Now, Eq. 9 becomes: $R_u = \int f_y (1 - \frac{\int f_y}{1.7 f_c'})$ $(\int)^2 \frac{f_y^2}{1.7 f_c'} - (\int) f_y + R_u = 0$ (Eq. 11) Now it becomes a quadratic equation, the root is: $\int = \frac{f_y \pm \sqrt{f_y^2 - \frac{4 R_u f_y^2}{1.7 f_c'}}}{\frac{2 f_y^2}{1.7 f_c'}}$	

Step	Computation	Reference
Derivation	$\int = \frac{f_y \pm f_y\sqrt{1-\frac{4R_u}{1.7f'_c}}}{\frac{2f_y^2}{1.7f'_c}}$ $\int = \frac{1 \pm \sqrt{1-\frac{2R_u}{0.85f'_c}}}{\frac{f_y}{0.85f'_c}}$ $\int = \frac{1 \pm \sqrt{1-\frac{2R_u f_y}{0.85f'_c f_y}}}{\frac{f_y}{0.85f'_c}}$ (Eq. 12) Set $\boxed{m = \frac{f_y}{0.85f'_c}}$ (Eq. 13) Now, Eq. 12 becomes: $\int = \frac{1 \pm \sqrt{1-\frac{2R_u m}{f_y}}}{m}$ $\boxed{\int = \frac{1}{m}(1 \pm \sqrt{1-\frac{2R_u m}{f_y}})}$ (Eq. 14) **Take negative sign root for this equation** **4. Conclusion:** The required flexural reinforcement could be determined by: $\boxed{\begin{array}{c} m = \frac{f_y}{0.85f'_c} \\ R_u = \frac{M_u}{\emptyset b d^2} \\ \int_{rqd} = \frac{1}{m}(1 - \sqrt{1-\frac{2R_u m}{f_y}}) \end{array}}$	