CHEAPER: A novel, mixed integer, linear program to minimize commercial building electricity costs under real-time conditions

by

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Abstract

Commercial buildings account for 35 percent of all U.S. retail electricity sales. The average commercial customer's electricity bill is $647 a month and utility costs, on average, represent 17 percent of a commercial building's annual operating budget. Technology that can save businesses money by automatically reducing or shifting building electricity use based on real-time pricing, weather and occupancy is highly desirable.

This thesis presents a novel, mixed integer, linear program called Cooling and Heating Efficiently through Automated Planning of Energy Resources (CHEAPER). CHEAPER creates optimal operating schedules for one or more building systems that minimize the total periodic electricity cost of system operation. The program uses building designs, system schedules, and local weather forecasts to model indoor temperature change based on outdoor conditions and building activities. Occupant comfort is addressed through use of one or more user-specified constraints pertaining to acceptable indoor thermal and visual conditions. Real-time pricing accessed through a utility web portal provides the 5-minute, electricity, spot prices necessary for cost planning over a 24-hour time horizon.

Due primarily to CHEAPER's size and RTP cost symmetries, the majority of problem instances do not solve fast enough to be practical for everyday use. To alleviate this issue, a relative optimality threshold, or gap, is used to relax the requirements for optimal CHEAPER schedules, which significantly decreases the program's runtime. With a 1.5 percent optimality gap, CHEAPER solutions are obtained, on average, within 45 seconds of program start. This gap size equates to an increase in daily electricity costs of $0.02 to $0.08.

Under these conditions, application of CHEAPER to a prototypical small office building located in northeast Kansas demonstrated daily cost savings of two to 55 percent as compared to the same building and systems operating with standard control strategies. Average savings of 22 percent were achieved. Cost savings are a result of three control strategies: occupancy control, light-level dimming and load shifting. For the average customer, use of CHEAPER schedules could result in an average, annual cost savings of $1,025. CHEAPER also produced consistent monthly energy savings, which ranged from 11 to 33 percent as compared to the baseline model.

The most important research need related to CHEAPER is the need for its demonstration in actual commercial buildings. The program must be tested in-situ to validate the approach and savings potential detailed in this thesis. In addition, CHEAPER currently includes a relatively small suite of control options and a single electricity-cost objective. Many other building features and optimization opportunities are possible such as expansion of the program to accommodate multiple building HVAC and lighting zones. Similarly, research to address the competing objectives of cost and carbon emissions reduction is needed to ensure CHEAPER can serve as a tool for meeting both energy and environmental goals.

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Chapter 1 Introduction

There are approximately 5.9 million commercial buildings in the U.S., spanning 97 billion square feet of floor space [1]. Last year, these buildings consumed 9.4 quads of energy, approximately 18 percent of all energy used in the United States [2]. The average commercial customer's electricity bill is $647 a month and utility costs, on average, represent 17 percent of a commercial building's annual operating budget. The median size of a commercial building is 5,400 square feet (sf).

In many areas of the country, building energy performance is regulated by local or state energy codes as part of policies to reduce energy use, improve environmental quality and similar objectives. However, energy codes do not regulate building operation post-occupancy, and very few U.S. municipalities have programs designed to improve or even maintain a building's energy performance over the building's life. While many will be retrofitted to update outdated and inefficient equipment like windows, lighting and insulation, utility costs still account for one of the largest recurring facility expenses for building owners over time [3]. Improvements in daily building operations that can intelligently reduce or shift energy use and peak demand away from costly parts of the day represent an enormous opportunity to save money while reducing strain on the electric grid. Energy reductions may also provide co-benefits like reduced carbon emissions, prolonged equipment life and improved indoor air quality.

From the supply side, utilities and regulators are addressing shifting and uncertain energy demand by deploying time variable pricing (TVP) programs. Across the U.S., utilities are transitioning customers to TVP in attempts to incentivize customer energy use during off-peak hours when energy is plentiful and costs are low. Real-time pricing (RTP), one type of TVP characterized by hourly or sub-hourly rate variations, is increasingly available to commercial and industrial customers. Utilities and regulators view RTP as a primary tool in combatting demand volatility, electric grid instability, and resource inequities prevalent in many communities. However, today there exist very few automation technologies or practical strategies available to commercial building owners for leveraging RTP to reduce operating costs.

Mathematical optimization is one strategy that is well suited for solving automation challenges, especially those with stochastic cost, schedule and resource variability. However, relatively little practical progress has been made within the commercial building sector to develop mathematical optimization models or deploy automation technologies based on optimization algorithms to better manage building operations. This is in stark contrast to industries such as transportation and financial management, which rely on optimization techniques to manage day-to-day business.

For commercial buildings, rule-based control strategies dominate. They are simple to understand and very practical to apply. Fixed schedules, for example, are widely used to reduce building energy use during non-business hours. Many commercial buildings set weekday, weekend and holiday schedules for lighting and HVAC systems to reduce system operation when the building is sparsely occupied or closed [3]. These strategies improve upon manual controls, but they cannot capitalize on periodic variations that affect actual building performance and costs.

Within a process control context, mathematical optimization is commonly referred to as model predictive control (MPC). MPC relies on detailed system models and methods such as linear programming, nonlinear programming and related search techniques to optimize processes over a finite time horizon. Just within the past two decades has MPC emerged as a research topic and tool for controlling individual building systems and devices. Limited MPC research exists with respect to integrated, multi-system or whole-building control [4].

Linear programming is a class of mathematical models that consist of a system of linear equations that describe a physical system's characteristics, operating constraints and their impact on objectives of interest. Linear programs can be solved quickly, relative to many other mathematical optimization techniques, and they can be solved to global optimality. This means that the solution is the best feasible solution meeting model objectives under stated constraints. A linear program's simplicity and computational speed is desirable when it comes to modeling a physical system, however, building processes rarely follow a linear trend. Therefore, MPC more often uses nonlinear modeling techniques such as genetic search heuristics. Nonlinear techniques can also be relatively effective at finding good feasible solutions, however unlike a linear program; they are not guaranteed to find the optimal solution. Additionally, nonlinear models are often highly customized and each may only be applicable to a small problem set or even a single problem instance whereas linear programs follow a well-defined structure [5].

1.1 Motivation

This research was motivated by a collection of converging issues facing the world of commercial building operations, some of which have been briefly introduced. First, policy makers are quickly advancing an agenda focused on clean, low-carbon energy coupled with a vision of the energy "prosumer" that is both energy efficient and energy flexible [6]. Yet, the building-level optimization technology necessary for balancing energy use, electricity costs and comfort is unavailable. Second, existing MPC methods for individual devices are complex and computationally intense, each often developed with only a single application or specific building system in mind. There exists no standardized model capable of being reasonably adapted for a variety of buildings using common, off-the-shelf tools [4]. Third, existing MPC methods rarely use real-time electricity and weather data to make control decisions, and nothing serving the building as a whole could be identified. Real-time conditions, in particular, are often the most important to building owners responsible for making difficult decisions that balance financial costs against occupant comfort and productivity.

1.2 Contributions

This thesis presents a new, practical method for optimizing commercial building operations using a novel, mixed integer linear program (MILP) model called Cooling and Heating Efficiently through Automated Planning of Energy Resources (CHEAPER). CHEAPER uses piecewise linear functions to model indoor temperature change based on outdoor conditions and building occupancy, which is new. While the set of piece-wise functions will be unique to every building, this thesis provides a replicable method for constructing and initializing the control functions using common design calculations and software tools. Over time, data on actual building performance can and should be used to replace initial function coefficients.

In addition, CHEAPER accesses publically available, real-time pricing and weather data to better predict building performance and costs, the combination of which has yet to be demonstrated in the literature. Because the program is predicting a relative change for a given time step based on real-time data and generalizations, the model is able to reduce compounding errors associated with many predictive control methods. This improves model accuracy within a given time horizon, allowing for use of predicted control over an extended period, which reduces the model update and high optimization frequency associated with MPC. This makes CHEAPER quicker to run and more practical for day-to-day use.

The CHEAPER model is a control building block. It should be populated with information on installed building equipment and replicated when multizone control is needed. Additional constraints can easily be added to address different control strategies and comfort conditions based on user preference.

To demonstrate the concept and savings potential for this thesis, CHEAPER was applied to a small office building with a single lighting and HVAC zone. Small buildings of 5,000 ft^2 or less make up nearly half of all buildings in the United States [1]. Results show a 11 to 36 percent improvement in average, monthly energy costs as compared to the same building operating with prototypical HVAC and lighting schedules. In addition, simulations show that CHEAPER reduces energy use by an average of 20 percent. At scale, this research demonstrates that CHEAPER has the potential to reduce U.S. annual energy consumption up to 1485 terawatt-hours (TWh) annually. Using the U.S. average commercial electricity rate of 17.35 cents per kilowatt-hour (kWh), CHEAPER could save businesses $286 billion per year.

1.3 Thesis Outline

This thesis is organized into five chapters. Chapter 2 provide the background necessary for understanding the motivation behind and research conducted for this thesis. The chapter describes the current state of commercial buildings in the U.S. including energy regulations and utility pricing programs. In addition, Chapter 2 includes a review of major building end-use systems included in this research. It concludes with an overview of linear programs, optimization algorithms and approximation heuristics that are commonly used to solve building science problems.

Chapter 3 describes the advances made by this research. This includes the linear MPC developed to optimally control building HVAC, lighting and ventilation. Work includes the building model definition, the model initialization procedure and methods for collecting and applying real-time electricity pricing and forecasted weather data for use within the model.

Chapter 4 focuses on application of the new model to a general use case that includes a small office building operating over the course of one year. Results are presented and compared to operation of the same building under a traditional, rule-based, control strategy. Electricity costs, energy use and associated savings are provided and discussed.

This thesis concludes with Chapter 5, which presents conclusions and areas for future research in whole-building optimization. This includes specific topics for improving and expanding the performance of the CHEAPER model.

Chapter 2 Background

2.1 Buildings and Energy

In 1974 in response to the first energy crisis of that decade, President Gerald Ford signed the Federal Energy Administration Act into law and set in motion the growth of new industry centered on building energy efficiency. The Act established the Federal Energy Administration (FEA) and authorized it to *"plan, direct and conduct programs related to conservation of all forms of energy"* [7]. The Act was amended in 1976 to require the Secretary of Housing and Urban Development to publish the first U.S. energy conservation standards for new commercial and residential buildings [8]. Two years later, the FEA merged with the newly developed U.S. Department of Energy (DOE), and to this day, with an annual budget exceeding $35 billion [9] , DOE continues to develop and manage commercial building energy programs and standards, among its many other duties.

One of the ways in which DOE supports the buildings sector is through its participation in development of model, building, energy codes [10]. Model building codes are developed and maintained by organizations independent from the jurisdictions in which the code is adopted [11]. These codes are updated periodically to include new, cost-effective, energy-efficient building equipment and design practices.

States or municipalities may also develop and implement their own unique energy codes. For example, California develops and maintains its own building energy code called the <u>Building Energy Efficiency Standards for Residential and Nonresidential Buildings</u> (Title 24). Title 24 has periodically exceeded the energy efficiency levels of model building codes [12]. This has been accomplished by incorporating requirements for more efficient equipment, advanced building control systems designed to reduce equipment energy use automatically under various conditions, or both [13].

Energy codes do not regulate building operation post-occupancy, and very few U.S. municipalities have energy programs designed to improve or even maintain building energy-efficiency over the building's life. However, because of related issues such as climate change, energy security and equitable distribution of energy resources, some jurisdictions are working toward regulations that will also require buildings and certain equipment to be automatically responsive to changing grid conditions, price signals or other critical events that impact the cost and availability of grid-provided electricity.

The state of California is one such jurisdiction. The California Energy Commission (Energy Commission) is in the process of updating its load management standards for buildings [14] and developing flexible demand appliance standards for equipment [15]. These standards are focused on regulating "when" energy is used, as opposed to regulating "how much". Regulations will address grid-connectivity, communication and operation of systems and devices in an effort to better control electricity demand.

Demand is an instantaneous power measurement that gives the rate at which electricity is consumed [16]. The combined demand of many devices operating simultaneously can create stress on the electric grid resulting in temporary loss of electrical service [17]. The maximum, instantaneous power demand of a system is called peak demand. California's Load Management Standards and Demand Flexible Appliance Standards are primarily intended to control peak demand in an effort to improve the reliability and stability of the State's electric grid.

Clearly, energy code development, code compliance and the new building technology behind it are big business. The commercial buildings, energy-efficiency industry is a multibillion dollar a year industry fueled, in part, by these mandatory regulations and rising energy costs [18]. Americans spent $51 billion on commercial building energy-efficiency investments in 2004 [19]. In 2012, The Rockefeller Foundation estimated that there was a $72 billion investment opportunity available in commercial building energy efficiency retrofits [20]. A 2019 market report developed by the Advanced Energy Economy cites $83 billion in revenue for the U.S. advanced building efficiency sector [18].

2.2 Utility Tariffs and Energy Prices

The Energy Efficiency Improvement Act, also known as the Better Buildings Act, authorized DOE to study the feasibility of significantly improving energy efficiency in commercial buildings by, among other things, installation and use of high-performance, energy-efficiency measures. The Act defines high-performance measures as products or practices resulting in substantial operational and utility cost savings. The Act also required DOE to establish energy conservation standards for grid-enabled water heaters [21].

While on its own, this last element may seem insignificant, however the inclusion of a grid-enabled, building-equipment, standard highlights the emergence of a new product class designed to capitalize on time-variable utility rate structures, as well as meet forthcoming load management standards previously discussed. Commonly referred to as grid-responsive technology or grid-interactive technology, this new product class includes highly automated, programmable devices or systems able to respond to changing grid conditions, and, as importantly, real-time utility price signals or similar time-variable pricing (TVP) programs whose rates are impacted by grid conditions. Buildings must utilize grid-interactive technology to automatically eliminate or reduce their energy use during the costliest times of the day;

automatically shift their energy use to inexpensive time periods; or make other automated, transient adjustments to fully realize the financial benefits of participation in TVP programs.

Time-variable pricing, also called dynamic pricing, represents a class of utility rate structures composed of energy rates that vary with time throughout the year. TVP programs are common throughout the U.S. and are most often used by commercial and industrial customers. There are varieties of TVP tariffs ranging from those that vary prices only with the season, to those with variability across every hour of the year. Examples include Time-of-Use Pricing (TOU), Variable Peak Pricing, Critical Peak Pricing and Real-Time Pricing (RTP) [22]. In contrast, fixed-rate utility pricing offers customers a constant or "fixed" rate per kWh over the term of a utility contract. The fixed rate is typically an average rate designed to reduce risk to the utility and provide price stability to the customer.

Real-time pricing is the most volatile TVP program and includes electricity prices that can change on an hourly or sub-hourly basis based on wholesale market prices [23]. Wholesale market prices, in turn, are affected by numerous factors including source fuel prices, power plant operating costs, transmission and distribution costs, and weather [24]. However, RTP is relatively predictable, where higher periodic prices generally accompany increased strain on the electric grid caused by high demand. Similarly, lower prices often coincide with periods of lower demand, even going so far as to become negative during periods with an extreme energy surplus.

Figure 2.1 provides an example of real-time prices for one 24-hour period in September 2020 in the ComEd Illinois service territory. Prices do not include capacity or demand changes. Beginning in early afternoon, prices rises, peaking at around 4 pm at the hottest part of the day. They begin to drop throughout the evening. Prices turn negative in the early morning hours of the following day between 2 a.m. and 4 a.m. Prices peak again at around 7 a.m., then decline, before starting another steady climb as the day progresses. Much of the variation can be correlated to demand, with peaks coming in early morning when customers wake and begin their workday, and again in late afternoon when temperatures are at their hottest and customers respond by increasing use of air conditioners and other cooling equipment. Data on forecasted electricity price, weather and temperatures for the same day are provided in Table 2.1.

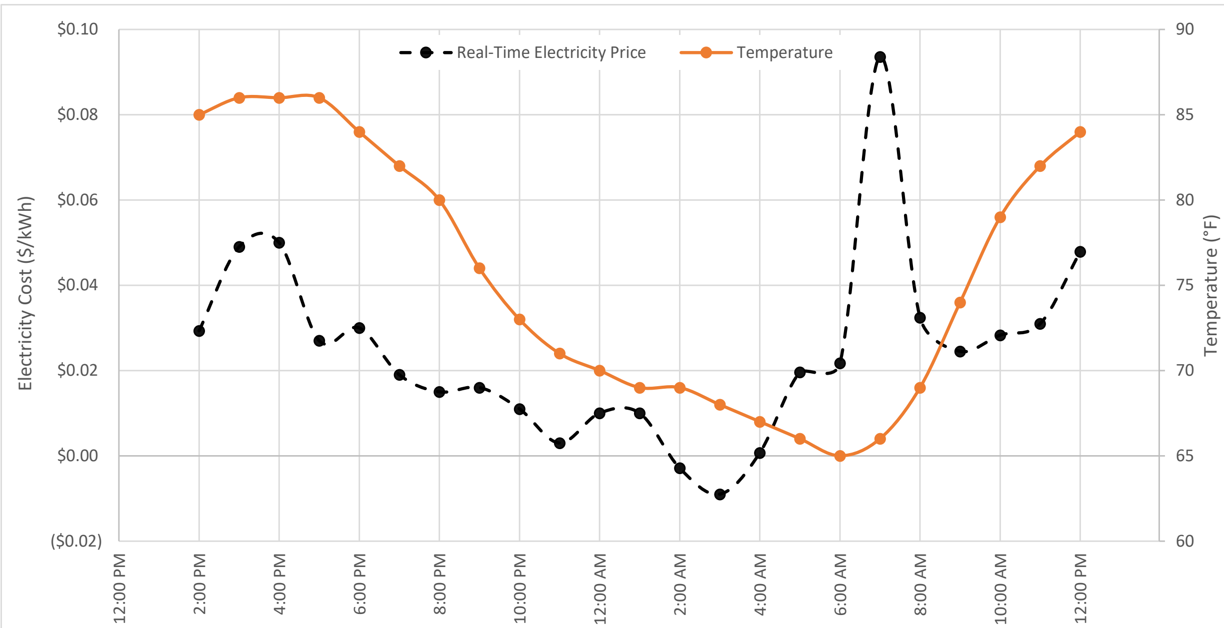

Figure 2.1. Example of Real Time Prices ($ / kWh) and weather (°F) for September 2, 2020. Source: ComEd Hourly Pricing Program and Weather.gov.

Today, RTP takes one of two forms. The first form, called "day-ahead" RTP, provides customers with a day-ahead price set, which contains up to 24 hours of prices for the forthcoming day. The prices are fixed for the day, regardless of the actual price to the utility. Day-ahead prices are released to customers, usually, around 4 p.m. on the day before the period to which they apply. Customers can then plan their energy use knowing how much they will pay for energy during each hour of the following day. This alleviates some customer risk, but also eliminates a customer's ability to capitalize on real-time prices that may be lower than those included in the day-ahead forecast. Day-ahead pricing is typically the default choice for customers enrolled in TVP programs [25].

Table 2.1. Real-time price and weather conditions graphed in Figure 2.1.

Time	Price Forecast ($/kWh)	Temperature (°F)	Sky Conditions
2:00 PM	0.0293	85	Mostly Sunny
3:00 PM	0.0490	86	Mostly Sunny
4:00 PM	0.0500	86	Mostly Sunny
5:00 PM	0.0270	86	Mostly Sunny
6:00 PM	0.0300	84	Partly Cloudy
7:00 PM	0.0190	82	Partly Cloudy
8:00 PM	0.0150	80	Mostly Clear
9:00 PM	0.0160	76	Mostly Clear
10:00 PM	0.0110	73	Mostly Clear
11:00 PM	0.0030	71	Mostly Clear
12:00 AM	0.0100	70	Mostly Clear
1:00 AM	0.0100	69	Mostly Clear
2:00 AM	-0.0029	69	Mostly Clear
3:00 AM	-0.0090	68	Mostly Clear
4:00 AM	0.0007	67	Mostly Clear
5:00 AM	0.0196	66	Mostly Clear
6:00 AM	0.0217	65	Sunny
7:00 AM	0.0936	66	Sunny
8:00 AM	0.0324	69	Sunny
9:00 AM	0.0245	74	Sunny
10:00 AM	0.0283	79	Sunny
11:00 AM	0.0310	82	Sunny
12:00 PM	0.0479	84	Sunny

The second form of RTP uses actual hourly prices called "spot" prices, which are applied to a customer's bill at the close of each hour. Customers are not provided a fixed hourly energy price in advance, and customers must estimate the energy cost for the forthcoming period in order to better control their energy costs. This is more risky for customers; however, RTP programs using spot prices also provide customers with more incentive to reduce energy use and more financial benefits when energy costs fall below the predicted levels.

With both programs, there are fixed and variable charges in addition to the hourly energy cost. The most significant of these charges, called demand or capacity charges, account for the utility's fixed cost to generate and transmit electricity to its customers. A customer's demand charge is typically based on the highest amount of electricity it drew during the billing cycle. Demand charges can account for 30 to 70 percent of a customer's bill [26].

Real-time pricing currently exists in many regions of the U.S.; however, it is not actively promoted in most markets. Utility programs in 31 states provide real-time pricing to commercial customers. Southern California Edison (SCE), a regional investor-owned utility (IOU) operating in Southern California, claimed the vast majority of commercial TVP customers reported to the U.S. Energy Information Administration in 2019. SCE accounts for approximately 568,000 of the 712,000 commercial customers enrolled in TVP programs in the U.S., however the data does not disaggregate RTP customers from the TVP total [22].

A 2004 study completed by Lawrence Berkeley National Laboratory (LBNL) provides some sense of RTP adoption in the U.S. The study included a survey of 43 of 70 U.S. utilities offering RTP programs in 2003. One third reported having zero RTP program participants, while a second 1/3 reported having less than 25 participants and less than one percent of the utility's system load enrolled. This suggests that RTP, historically, has been underutilized and that most customers today may be enrolled in TVP programs other than RTP [27].

One specific way to encourage use of RTP programs is for utilities to provide automated, easily accessible data on real-time prices. In doing so, technology developers can create products and services that automatically consider price during operation. Given the increasing available of high-speed, reliable internet access, utilities can provide real-time pricing to their customers and the public through websites and application programming interfaces (APIs). APIs standardize the way in which any two digital devices can connect, read and write data between themselves. Currently, very few utilities provide automated or publically available access to real-time prices for any time period. In addition, most utilities offering RTP only offer day-ahead programs.

Future RTP programs, including its variants like day-ahead pricing, are expected to increase significantly in the coming years due, in part, to the wide availability of advanced metering infrastructure (AMI), high –speed internet, and automated access to forecasted real-time prices. For example, in California, lawmakers are working towards statewide adoption of real-time pricing and the availability of standardized, digital RTP signals by 2022 [28]. This work is being conducted as part of their Load Management Standards update process. State regulators at both the Energy Commission and the California Public Utilities Commission (CPUC) expect all customers to have access to real-time price data via the internet or a secondary mechanism, such as FM-radio, so that they can better manage their energy use to reduce costs. In addition, studies show that even without making changes to when energy is used, customers can still save when enrolled in an RTP program [29], [30].

The emergence of RTP and methods for automated access to pricing data have created an opportunity for commercial buildings to decrease their annual electricity costs. By combining optimization techniques with automation hardware and software, building owners and tenants can better manage their building electricity use to reduce costs. This opportunity is available to even the most efficient buildings complying with modern energy codes and green building standards. However, to be successful, building owners and tenants must also consider occupant comfort.

2.3 Commercial Building Science

Building science constitutes a collection of knowledge, phenomena and disciplines focused on the building design, construction and operation including those of occupant comfort. Building science includes the materials, processes, and devices affecting building performance, as well as issues related to how a building is used, by who, and for how long. Because building science addresses the physical structure, the surrounding environment and its occupants, nearly every scientific discipline is involved in some aspect of building science. The following background information is limited to only a few topics affecting indoor occupant comfort in commercial buildings [31].

2.3.1 Building Occupants and Occupant Comfort

Within buildings, an occupant's physical comfort is influenced by indoor environmental conditions such as temperature, humidity and light level. These conditions are a product of many factors including those directly related to the outdoor environment, building construction, installed building equipment and building occupancy. In modern commercial buildings, for example, space conditioning and lighting system operation is often highly automated and tightly controlled in an attempt to maintain an appropriate indoor environment for building occupants.

For most people, indoor comfort conditions vary based on their age, health and activity level. For example, active people will generate more heat than those that are not, thus requiring cooler conditions to feel comfortable. Generally, an adult can generate 400 to 500 Btu per hour when performing basic to moderate activities such as general office, laboratory and retail work [32]. Most people performing these kinds of activities feel comfortable when the temperature is between 68 and 72 degrees and humidity is maintained between 30 and 60 percent. Comfortable lighting conditions are a bit more complicated, but for most general indoor activities, an illuminance of 20 to 50 foot-candles (fc) is appropriate, assuming minimal glare and uniformity below 10:1 [33].

2.3.2 Lighting Systems

Indoor lighting can generally be categorized into four primary types: ambient lighting, task lighting, architectural lighting and emergency lighting. Ambient lighting provides a general level of illumination for most tasks, while task lighting is used to supplement specific work or focus areas. Architectural lighting is designed to highlight individual building features including those for decorative purposes. Emergency lighting is used only during building evacuations and similar emergency events. It may be a set of stand-alone luminaires, or a subset of the other lighting types coupled to backup emergency power and controls.

Most new lighting systems are dimmable, and dimming, automatic scheduling, occupancy and daylighting controls are commonly used in many commercial lighting applications. Controllable, programmable and networked lighting systems also offer the ability to adjust light levels based on time of day, activity, or other transient events. A lighting system with dimming and network connectivity are necessary features for implementing automated, dynamic lighting control based on periodic occupant needs, TVP or other quickly changing conditions.

Lighting systems also generate heat, which can affect an occupant's thermal comfort. Waste heat from lighting that is released into the occupied space generates a temperature change that is a function of the maximum connected load of the lighting system; the split between convective and radiative heat generated and delivered to the conditioned space (vs to the plenum); the lighting system's dimming level, and the volume of the space in which the lighting is installed. Currently, most commercial new construction utilizes light-emitting diode (LED) sources, which produce much less radiative heat as compared to incandescent or fluorescent light sources. However, LED waste heat must still be considered when conditioning a space for occupant comfort.

2.3.3 Heating and Cooling Systems

In the absence of mechanical space conditioning, the building's indoor temperature changes over time due to heat gains from solar radiation, infiltration of outdoor air, people, lighting and other indoor equipment. Solar heat gains include heat transmitted to the interior of the building through windows and skylights and heat conducted through walls and ceilings. When in operation, building equipment such as lighting and computers also add waste heat to the indoor environment. As previously mentioned, people add varying amounts of heat to a building depending on the nature of their indoor activities. When space conditioning is provided by an HVAC system, indoor temperature is also impacted by heat transferred through powered introduction of ventilated air from the outdoors and the addition (or subtraction) of artificial heat provided by the HVAC unit.

Commercial heating and cooling systems, regardless of type, are designed to accommodate a building's worst-case conditioning needs. In the U.S., heating systems are designed to provide adequate heat for occupant comfort according to the weather and other characteristics associated with the "winter design day". Cooling systems are similarly designed and sized according to a "summer design day". Heating and cooling units may have one, two or more speeds. Heating and cooling may be combined into one device, such as packaged rooftop unit, or they may be provided by two or more different devices. These systems are controlled by thermostats and most often programmed to follow simple control rules based on the time and day of the week.

Building location relative to surrounding structures, as well as building orientation, can also have a significant impact on building heat gain, HVAC sizing, and subsequent energy use. In the Northern Hemisphere, south-facing windows, for example, are exposed to direct sunlight, which contributes a significant amount of heat gain to the building if suitable shading or other measures are not taken. During the winter, this can result in beneficial cost savings, because it reduces the amount of active space heating required to maintain a comfortable indoor temperature. In the summer, direct sunlight can increase cooling requirements and cooling costs. Shading from surrounding buildings or trees impacts building heating and cooling costs in a similar way.

Climate conditions also play an important role in selection and operation of space conditioning systems. The U.S. is commonly divided into eight climate zones defined by heating degree-days, average temperatures and precipitation [34]. A map of these zones is shown in Figure 2.2. California recognizes 16 climate zones for its building codes and standards purposes, which demonstrates the significant role that even minor changes in climate can play on building construction and energy use. Heating energy use is the largest building energy end-use in colder, northern U.S. climates, while cooling energy is more substantial in southern climate zones. Additionally, certain types of space conditioning equipment are only appropriate for use in specific climate zones. Other types may be used in multiple zones, but their efficiency may vary significantly depending on climate conditions.

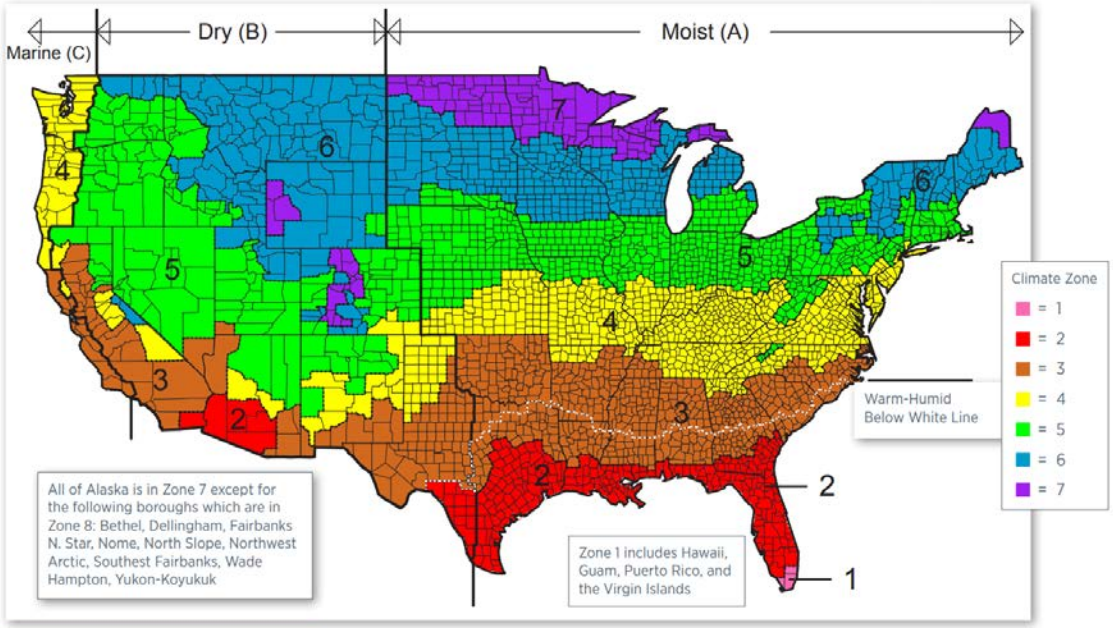

Figure 2.2. Eight climate zones in the United States as recognized by the International Code Council and ASHRAE.

Apart from HVAC and lighting, miscellaneous electric loads including plug loads, elevators, and security systems are the largest energy-consuming heat sources in general commercial buildings. Plug loads, often simply called "equipment" for building energy and modeling purposes, now consume more energy than lighting in most commercial buildings. Plug loads such as copiers, computers, and audio/visual equipment are always considered as heat sources when selecting and sizing space-conditioning equipment.

2.3.4 Mechanical Ventilation

Building ventilation, the introduction of fresh air to the building interior, may be provided as part of the heating and cooling system or it may be provided by dedicated outdoor air system (DOAS). In many systems, outdoor air is introduced using electric fans that force fresh air through a duct system and its output to various indoor building zones is controlled using dampers. Fans may have one, two or more speeds including variable air volume (VAV) units that can supply a continuous amount of fresh air between zero and maximum output.

Most often, ventilated air must be conditioned for temperature and humidity. However, some systems include economizers, which allow outdoor air to be introduced with minimal conditioning. Single and double speed fans are widely used. Other types of ventilation equipment such as VAV units are less common generally, but do have higher adoption rates in specific areas of the country among specific building types.

2.3.5 Building Energy Modeling

According to the U.S. DOE, building energy modeling (BEM) is physics-based software simulation of building energy use. A BEM program takes as input a building description that includes its geometry, construction materials, lighting, HVAC, refrigeration, water heating, and renewable generation system configurations and control strategies. It also takes descriptions of the building’s use and operation including schedules for occupancy, lighting, plug-loads, and thermostat settings. A BEM program combines these inputs with information about local weather and uses physics equations to calculate thermal loads, system response to those loads, and resulting energy use, along with related metrics like occupant comfort and energy costs. BEM programs complete a full year of performance calculations using an hourly or shorter time step. They also account for system interactions like those between lighting and cooling systems [35]. Common BEM software programs include EnergyPlus and eQuest [36]. BEM software is commonly coupled with a separate optimization engine, such as MATLAB's Optimization Toolbox, to estimate energy consumption values for use in buildings-related optimization [37], [38].

2.4 Optimization Techniques

There are many mathematical techniques available for solving building optimization problems. For this thesis, techniques that generate a proven, optimal solution use a mathematical procedure called an algorithm. Those that generate solutions that cannot be guaranteed as optimal utilize estimation procedures called heuristics. Often, the term algorithm is used for both within the literature; however, they are kept distinct for this thesis.

2.4.1 Linear Programming

Linear programming (LP) is one tool used to solve optimization problems. A linear program consists of a system of linear equations that describe a physical system's characteristics, operating constraints and their impact on objectives of interest. An LP contains an objective function composed of one or more decision variables and a set of one or more constraint functions that limit the feasible values that each decision variable may assume. There are multiple algorithms in existence to solve linear programs, one of the most commonly used being the Simplex Algorithm developed in 1947 by George Danzig [5], [39].

Each decision variable represents one decision about the physical system that must be made. An LP where all decisions variables must be integer is called an integer linear program. Decision variables representing yes or no decisions are often modeled as integer programs, where yes and no take on a value of one and zero, respectively. Integer programs can be difficult and time consuming to solve. An LP that includes both integer and non-integer decision variables is called a mixed integer linear program (MILP) [5].[1]

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[1] For more information on linear programming, see [5] and [38].

A linear program is defined as,

$$\min \boldsymbol{c}^T \boldsymbol{x}$$
$$\text{subject to } \boldsymbol{Ax} \leq \boldsymbol{b} \text{ and}$$
$$\boldsymbol{x} \in \mathbb{R}^n_+, where\ \boldsymbol{c} \in \mathbb{R}^n, \boldsymbol{A} \in \mathbb{R}^{mxn}\ and\ \boldsymbol{b} \in \mathbb{R}^m.$$

A linear program representing a complex system may contain millions of decision variables and constraints. An optimal solution to an LP will minimize or maximize the value of the objective function while ensuring that all constraints are satisfied. There may be an infinite number of feasible solutions to an LP, a finite set of feasible solutions, or none at all. Similarly, a subset of those solutions will form the optimal solution set. A given LP may have zero, one or more optimal solutions [39].

Optimal solutions to linear programs are guaranteed to be globally optimal. This means that the solution is the best solution to the stated problem under the stated constraints. Optimal solutions for many linear programs can be calculated quickly relative to other mathematical optimization tools [39].

2.4.2 Heuristics

A heuristic[2] is a problem solving method based on an intuitive "short-cut" process to arrive at a solution when a precise, algorithmic approach is impractical. Heuristics are often backed by a pragmatic justification for their form and use; however, they are not supported by a formal proof that guarantees the best solution found is also globally optimal [5]. Within

[2] For the purposes of this thesis, the term heuristic is meant to apply to only those techniques used in the fields of mathematical optimization and operations research.

optimization, heuristics are often used when available methods for arriving at the globally optimal solution are computationally expensive in terms of run-time, data storage or both.

Many heuristics are composed of general search techniques that iteratively sample the solution space for improving solutions to a problem instance, stopping the search only after an arbitrary number of consecutive searches yields a non-improving solution. Common heuristics include simulated annealing, genetic algorithms, and tabu search.

For building optimization problems, genetic algorithms (GA) are used extensively [38], [40]. A genetic algorithm is a randomized search heuristic based on the Darwinian natural selection process where good solutions are retained with each generation and poor solutions discarded. With each new generation, new solutions are based on the previous generation. Additionally, a GA may include mutation or crossover functions designed to introduce diversity into the solution space by broadening the search to areas outside the neighborhood of a solution obtained from the preceding generation. By doing so, the GA may discover an improving solution far from a locally optimal one dominating its current search. A GA is terminated based on some parameter defining a set of successive generations such as the size of the relative improvement between generations. Again, a good solution obtained from a GA is not guaranteed to be optimal [40], [41].

2.4.3 Solutions in the Time Domain

Both linear programs and many heuristics often employ a rolling time horizon to model systems that require solutions in the time domain. A time horizon contains the total number of time periods for which a particular solution must provide a decision variable value. For example, in a building operations model, the time horizon may be 12 hours. This could represent the duration of the building's normal weekday business hours. If the time period is one hour, a model solution would contain 12 values for each modeled building system, one for each hour in the time horizon. A rolling time horizon extends this concept to include optimization frequency. As time goes by, a particular solution will contain a decreasing number of future values. As the current time nears the horizon, the system is reoptimized for the forthcoming number of specified time periods and the time horizon rolls out into the future. A rolling time horizon is useful for replacing values near the time horizon boundary. These values may diverge significantly from those relevant to current or near-term conditions due to compounding error resulting from differences between modeled and actual conditions over time.

2.5 Buildings and Optimization

Optimization is just one piece of the much larger domain of building information modeling (BIM). The BIM domain includes three building lifecycle phases: design, construction and operation [42]. These phases can be used to categorize much of the existing research related to mathematical optimization and buildings. The literature contains three general types of optimization within this context: Architectural Design Optimization (ADO); optimization of a building's operational parameters like temperature set points, which is termed Commissioning-based Optimization (CBO) in this thesis; and Model Predictive Control (MPC) of individual building devices, systems or processes. A mapping of these optimization research areas to the building lifecycle is show in Figure 2.3.

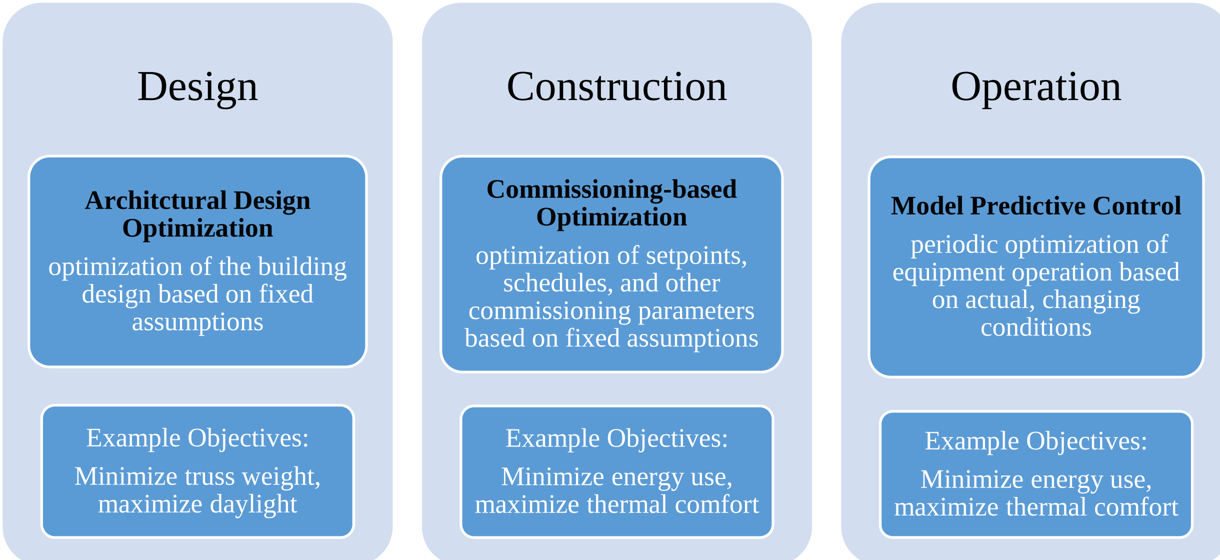

Figure 2.3. Mapping of building lifecycle phases to current optimization research applied to buildings.

Regardless of building phase, linear programming methods are rarely used to solve building optimization problems [38], [40]. Lack of use is due primarily to the fact that most building science is based on nonlinear processes. Heat transfer and water flow, for example, can be extensively modeled but the underlying physics for non-trivial problems are based on nonlinear functions [43]. In addition, due to dependencies among multiple building systems on building energy use and the indoor environment, the composite functions representing these complex systems may be difficult to articulate in mathematical terms [44]. Thus, researchers usually deploy heuristics to obtain good solutions to most problems. Generally, use of heuristics produces better solutions that use of no modeling technique at all, however, solutions cannot be guaranteed as optimal [38]. A survey of relevant examples, most of which employ heuristics, is provided below.

2.5.1 Architectural Design Optimization – Design Phase

ADO occurs during the building design phase. It uses parametric design, numerical simulations, and state and parameter estimation algorithms to identify the best design variants meeting certain performance criteria [45]. Common objectives include minimization of energy use, building weight, and construction material. ADO is most often used to identify feasible preliminary designs as opposed to optimal final designs [44]. A 2019 survey of people working with ADO, found that the most common uses of ADO pertain to building geometry, daylighting, and solar exposure. The most common optimization algorithms used to solve these problems include genetic algorithms and simulated annealing [46] and [47]. In practice, ADO use is rare[3]; however, future use is expected to increase as availability of lower-cost modeling tools and ADO education within architecture programs becomes more common [44]. The literature contains several recent examples of ADO applied to common problems associated with building geometry, envelop design and daylighting. This review excludes work consisting of parametric simulation only[4].

Dino and Ucoluk created a two-step, multi-objective optimization tool to generate good building layouts including window size that satisfied design constraints while minimizing energy use and maximizing daylight autonomy. In the first step, they generated multiple, good building layouts using a genetic algorithm. User-defined inputs for certain architectural design elements, including the maximum building height, length, width, facade direction and maximum number of building corner points, were used to generate corresponding constraints on building form, placement and topology. High performing building designs from step one were then used during step two to optimize each design's window-to-wall ratio (WWR) in order to minimize energy use and maximize daylight autonomy. Again, the researchers used a standard genetic algorithm to

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[3] 85 of 99 reviewed papers contained ADO applied to only simplified, fictitious building problems. For a more detailed review, see [39].

[4] For a good example of parametric simulation, see [54], which examined the impacts of glazing size, building height and atrium shape on interior daylighting using 75 models of one building, each with a different combination of the three key parameters. No mathematical optimization was performed.

explore the design space during step two, terminating their search after a fixed number of iterations. The work demonstrated that traditional compact forms performed worse than fragmented building designs when considering only the combined objectives of annual building energy use and building daylight autonomy [48].

Echenagucia et. al. examined the impact of building wall thickness and the quantity, type, shape and placement of windows on a commercial office building's total heating, cooling and lighting energy use. They used a multi-objective, genetic algorithm combined with building energy simulation software to evaluate various building configurations in four different climate zones, and then mapped two outputs ([heating, lighting], [cooling, lighting], [heating, cooling]) together to examine energy performance tradeoffs. Their results confirmed the significance of window arrangement and WWR on lowering building energy use across all climate zones [49].

Similar research has also been completed for residential buildings. In one example, researchers, Derazgisou, Bausys and Fayaz, used genetic algorithms to minimize residential building heating and cooling energy use by allowing changes in building orientation, placement, shape and arrangement with respect to the building site. Optimization was completed in a three-step process beginning with optimization of the form of individual one-story dwellings, moving to optimization of four units of the best-performing dwelling with respect to their placement on the building site, culminating with optimization of the form of six multistory apartment buildings, also located on the same site. Each optimization step used an initial population of 20 or 30 designs, and terminated the search for better results after between 15 and 30 non-improving generations. Results yielded savings of 21 percent, two percent, and 26 percent, respectively, as compared to the manually generated designs used to initialize each optimization step [50].

2.5.2 Building Commissioning Optimization

Selection of building equipment-operating parameters are first made during the commissioning or start-up process soon after the equipment is installed in the building. These or similar selections may be made again to correct operating issues, on a periodic maintenance schedule, or as part of a retro-commissioning project several years or more after initial installation. Commissioning decisions are rarely made based on the results of an optimization exercise. Typically, building owners or installation contractors set each device with values based on rules of thumb, minimum code requirements or experience gained from previous projects [51].

Some researchers have applied sensitivity analysis and optimization techniques to quantify the impact of equipment commissioning decisions. To the author's knowledge, no unifying term exists to describe this area of buildings optimization, and the term Building Commissioning Optimization (BCO) has been adopted for this purpose. BCO examples include optimization of heating and cooling temperature set points, daily and weekly operating schedules HVAC equipment, outdoor airflow ventilation rates, default maximum light levels and daylighting set points [45].

Papadopoulos and Azar examined the impact of temperature set points for heating and cooling systems on three objectives: annual energy use, occupant thermal comfort and productivity. The researchers utilized a genetic algorithm to search the solutions space for good set-point values for four decision variables representing heating and cooling set points for occupied and vacant building periods. Their model was designed to maintain indoor dry-bulb temperature within a specific range, however all variables were continuous within that range. They used energy modeling software coupled to their search algorithm to evaluate the annual energy use of each solution, and used a common comfort metric, the Predicted Percentage of Dissatisfied People (PPD), to determine thermal comfort along with a novel method developed from data collected as part of a separate study to determine productivity loss. As expected, they found that extreme temperature set points, while within the acceptable temperature range, minimized energy use but resulted in maximum productivity losses and PPD. However, they also found that very small changes in temperature set points around the middle of the acceptable range (less than 2 °C) resulted in significant improvements to energy savings with relatively small losses of productivity and thermal comfort[5] [41].

Gunay et al. completed an expanded, yet similar study to identify the best values for eight BCO parameters (from a small set of possible values) that minimized annual energy use, PPD, and unhealthy indoor CO_2 levels. BCO parameters examined included heating and cooling set points; weekday air-handling unit start and stop time; switchover day to cooling and switchover day to heating; minimum outdoor airflow rate and minimum outdoor airflow type. The subset of allowed values for each was determined by evaluating the BMS databases of 14 government buildings in Canada. In addition, researchers performed their optimization for 108 different building configurations composed of different combinations of building envelope, climate, and occupancy profile. An exhaustive search of all possible combinations would have resulted in 6.5 million building energy simulations; therefore, the researchers utilized a genetic search algorithm to identify good solutions to their problem more quickly. Results showed that good AHU start times were negatively correlated to climate zone, while good AHU stop times aligned well with occupancy schedules. Other significant results included the need for more than twice the normal

[5] Data tables for temperature set point values, PPD and productivity for each non-dominated solution were not provided by the study authors.

level of outdoor fresh air to maintain healthy indoor air conditions. Researchers state that additional work is needed to optimize these parameters with respect to peak demand and electricity costs, among other gaps [45].

2.5.3 Model Predictive Control – Operations Phase

Model Predictive Control (MPC) is a relatively recent advancement in building control methods that attempts to optimize building operations based predicted conditions. MPC uses a model of a specific building system or device paired with a rolling time horizon to estimate the near future state of the building environment and adjust device operation accordingly. After each adjustment, all model inputs are updated and the optimization is repeated to determine device settings for the next time period. With MPC, the time horizon is usually 12 hours or less, and only results for the first, modeled period are used. MPC approaches are most often applied to one specific building device or system such as a water heater [52]. They are often characterized by a large computational burden associated with the high frequency of optimization.

Ma et. al applied an economic model predictive control (EMPC) method to minimize HVAC energy and demand costs under a TOU pricing plan while maintaining appropriate indoor temperatures during occupied building hours. The TOU plan consisted of three periods each with a different demand charge.[6] The model adjusted HVAC set points during unoccupied hours to precool or preheat the building and minimize HVAC operation during the costliest hours of the day. Their work focused on use of a predictive linear model that estimated the average indoor temperature of future periods based on the thermostat set point from the previous period. Real-time, changing conditions that affect indoor building temperature such as weather and building

[6] The three times with different demand costs were low Off-Peak cost, a medium Mid-Peak cost, and a high On-Peak cost.

occupancy were ignored. The EMPC was tested over four summer days in an office building and compared to three summer baseline days for the same building operating with a fixed 72 °F thermostat set point. Results showed that the model was able to successfully reduce HVAC operation during the six-hour, On-Peak window by changing the thermostat set points for the hours prior to this window allowing the HVAC system to operate outside its normal range [53].

A similar EMPC was applied by Van Asselt to reduce the annual operating cost of a cool thermal energy storage system (CTES) under day-ahead and RTP electricity rate programs. The CTES was used to meet the cooling load of a large office building located in New York City, New York. She used a linear input model of the system's coefficient of performance (COP) versus the building's, part-load, cooling profile to generate optimal chiller part-load ratio values for each hour of a 24-hour operating cycle. Model constraints ensured that the building's cooling load was always met, and the system was fully charged at the end of each 24-hour period. Use of the EMPC control method combined with a day-ahead variable rate schedule resulted in 11 percent savings over a rule-based control method that allowed chiller charging at any time the electricity rate was below a user-specified, hourly, cost threshold. When used with the more volatile RTP rate structure, the EMPC produced 24 percent savings [54].

2.5.4 Linear and Mixed Integer Linear Programs for Buildings Problems

In contrast to the last two MPC examples, linear programming, including mixed integer linear programming, is rarely used to solve building optimization problems. A 2014 review of simulation-based optimization methods applied to building performance analysis found just six of 200 peer-reviewed building optimization studies used linear programming methods [38]. A similar review focused on sustainable building design and optimization identified only two building optimization studies that used linear programming methods [40]. Additionally, cost objectives within these studies were limited. One review noted that nine of 74 studies examined considered an operational cost objective [38].

Within the MPC context, when the model utilizes binary or integer decision variables to denote discrete device states or logical control decisions, it is called a hybrid model predictive control (HMPC). Only within the last decade, have researchers begun to apply LP, MILP and associated HMPC methods to solve building operations problems [37]. The work of Ma et. al. and Van Asselt provide good examples of this limited research.

Based on the existing literature, it is clear that no one has utilized HMPC or related predictive methods to minimize the electricity costs incurred to operate a commercial building under real-time conditions of electricity price, weather and occupancy. In addition, while heat-transfer equations are commonly used within BEM software to calculate annual building energy, they have not been used exclusively as an initialization tool for a whole-building operations model. No existing research could be identified that controlled light levels as part of an optimization of the building's thermal environment. Thus, the next chapter presents CHEAPER, the first predictive control model for commercial building operation based on a MILP that addresses heating, cooling, ventilation and lighting systems using real-time information on electricity costs, weather, and occupancy.

Chapter 3 Model

Every building has unique geometry, construction and location, which directly influences indoor environmental conditions. Although buildings may undergo retrofit or renovation, most building components remain fixed for at least several years, if not the entire life of the building. The net indoor temperature change per time period in the absence of people and equipment is, therefore, unique to each building and highly correlated to the weather and sun position. For this thesis, this relationship is called the building temperature function (BTF). The BTF is defined as the net indoor temperature change per outdoor temperature and hour of the year. It excludes buildings loads and operable components that affect the indoor environment such as people, electrical equipment, operable fenestration, and mechanical space conditioning systems. If the BTF is well defined, people can take proactive steps to effectively control select building systems to offset unwanted BTF impacts on visual and thermal comfort.

CHEAPER is a predictive control program designed to minimize a commercial building's monthly electricity cost. This is achieved by optimizing daily operation of lighting, heating, cooling and ventilation systems based on a building's BTF, real-time electricity prices, the local weather forecast, building occupancy and equipment schedules. In the U.S., these four building systems represent approximately 60 percent of the total electricity used by commercial buildings excluding refrigeration. Collectively, they may be considered a fair representation of the "whole-building" for many common building types [1]. As such, CHEAPER provides a robust example of a mixed integer linear program for optimizing commercial building energy use to minimize energy costs.

This chapter presents the CHEAPER model. Section 3.1 provides an overview of the model. Section 3.2 presents the necessary, external information required to estimate the BTF and populate the CHEAPER model. Section 3.3 and Section 3.4 describe each of the building systems included in the CHEAPER model. Section 3.5 describes the passive heat flows attributed to the building's location and construction, and Section 3.6 presents the complete MILP model developed to generate optimal daily operating schedules for controlled systems. For comparison, Section 3.7 includes a description of the baseline linear program developed to model the same commercial building operating with a traditional control system that includes thermostats and fixed schedules.

3.1 Model Summary

CHEAPER is a mixed integer linear program for commercial buildings that generates lighting, heating, cooling, and ventilation schedules to minimize electricity costs and maintain acceptable indoor temperature, ventilated air and lighting conditions. The model uses two types of linear functions to approximate the building's anticipated indoor temperature change as a function of equipment use, building occupancy and outdoor temperature. The first is a piecewise linear function, $f_{\varphi}(x)$ where $\varphi \in \Phi$, represents a heat-generating system that produces an indoor temperature change that varies with outdoor temperature. The set of all such systems, Φ, is given by the set of functions $F = \{f_{\varphi}(x)\}\ \forall\ \varphi \in \Phi$. Every building may be modeled by a unique set of such functions with each function containing an arbitrary number of segments R, where every single segment, $r \in R$, is linear. Each segment maps to a unique range of outdoor temperatures defined by a lower and upper bound (T_L, T_H), and all segments form a cover of the outdoor temperatures expected for the building location. For this thesis, these functions take the following form,

$$f_\varphi(x) = \begin{cases} \alpha_0^\varphi + \beta_0^\varphi x & for \quad t_0 \le x < t_1 \\ \alpha_1^\varphi + \beta_1^\varphi x & for \quad t_1 \le x < t_2 \\ & \vdots \\ \alpha_{r-1}^\varphi + \beta_{r-1}^\varphi x & for \quad t_{r-1} \le x \le t_r \end{cases} \qquad \forall\, \varphi \in \Phi$$

The second set of functions $G = \{g_\kappa(x)\}\ \forall\, \kappa \in K$, represents building heat flows, κ, which are a function of building utilization. Building utilization is expressed as a percentage of maximum occupancy or maximum connected load of equipment, both of which vary hourly based on a schedule that includes different values for weekdays and weekends. Generally, these functions take the following form,

$$g_\kappa(x) = V_\kappa x_{h,d} \qquad \forall\, \kappa \in K$$

where:

V_κ = maximum temperature change generated by the κ^{th} system at full load
$x_{h,d}$ = relative load (% of maximum) expected at hour *h*, on day *d*

For this thesis, CHEAPER uses six piecewise functions to model building heat flows that vary based on outdoor temperature: heating - low mode, heating – high mode, heating – AHU supplemental, cooling – low mode, cooling – high mode, and passive building heat flows. Each contains four segments. Additionally, CHEAPER includes four building systems that are a function of space utilization: lighting, ventilation, miscellaneous electric loads (MELs) and people.

Parameters for each function must be initialized. Several initialization methods exist including use of mass and heat transfer calculations based on data from specification sheets for each relevant piece of installed building equipment, building energy modeling, and in-situ data collection using sensors and data loggers. For this thesis, initialization is achieved using heat transfer equations of the form Q = C*ΔT, where Q is heat, C is a system-specific constant(s) defined by the building construction and installed equipment, and ΔT is the change temperature.[7]

Over time, initial BTF values can and should be replaced with building performance data derived from actual use. CHEAPER includes a function that allows it to search an external data file containing historical building performance and weather data to determine the expected indoor temperature change for a given set of conditions. If data on indoor building systems, weather and other required inputs and outputs is captured by sensors in the building and recorded, actual building response values can replace the function estimates, improving indoor occupant comfort and cost savings.

CHEAPER uses a 24-hour, rolling time horizon that reduces the required optimization frequency to as little as once per day. Building operations may be optimized, as frequently as once per time period, however, and each resulting solution will always contain operating schedules for the forthcoming 24-hour window. To achieve this, the day is partitioned into 276 5-minute time periods, *m*. The set of M periods requires specific building, environmental, and cost parameters as input to calculate the optimal ending indoor temperature, ventilated air and lighting levels for each hour in the horizon. CHEAPER minimizes electricity costs over the complete time horizon using a real-time spot price, P_m, applied to each controlled system in operation during period *m*. Each time building operation is optimized, external inputs are automatically updated and the prior period's ending indoor temperature, e_m, is used as the initial indoor temperature to begin the next optimization cycle.

[7] For this thesis, cooling load factors are not used.

3.2 Model Data

3.2.1 Electricity Prices

CHEAPER utilizes time-series, electricity price data to minimize building electricity costs. Electricity price data must be supplied from an external source. CHEAPER includes subroutines to read and import various data sources such as a comma-delimited text file or an Excel spreadsheet.

For this research, CHEAPER uses publicly accessible, real-time pricing provided by a large, Midwestern utility under their Hourly Pricing Program (HPP). CHEAPER automatically retrieves the hourly price data using a call to the utility's public API Web Service. These prices are then combined with additional demand and delivery price components taken from the same utility's Small Load Delivery Class (0-100kW) subprogram in order to provide a more complete estimate of the total hourly price [55]. CHEAPER can be easily modified to accept other utility pricing programs; however, the use of an API is essential for automatically accessing real-time data. For this reason, and to ensure consistency throughout the remainder of this thesis, the HPP described below is used for all calculations and discussion.

Per the utility's HPP tariff book, monthly billed costs are determined through application of multiple, unique delivery and supply charge rates, each of which is either a flat, fixed amount ($/month), or a rate applied to the total monthly electricity consumption ($/kWh) or the highest monthly demand ($/kW). Delivery charges include all three cost types, with the demand rate component based on the customer's highest, sustained, 30-minute demand from the previous month occurring between the weekday, non-holiday hours of 9 a.m. and 6 p.m.. Supply charges include the variable hourly electricity rate and capacity, transmission and miscellaneous procurement price components with rates that are fixed for 12 consecutive months. The monthly capacity charge rate (CCR) is based on the customer's highest demand occurring in one of ten select summer hours determined from the previous year. Electricity price details are provided in Table 3.1.

Table 3.1. RTP electricity price components

Type	No.	Value	Units	Description
Nonresidential Delivery Charges	F-1	24.30	($/month)	Customer Charge + Standard Metering Charge
	DR-1	8.11	($/kW)	Primary & Secondary Voltage Distribution Facilities Charges and Primary Voltage Transformer Charge
	ER_T - 1	0.00976	($/kWh)	State Electricity Distribution Tax Charge and miscellaneous fees
Supply Charges with Hourly Pricing	ER_T – 2	0.00935	($/kWh)	Transmission Services Charge + Miscellaneous Procurement Components Charge
	ER_T – 3	ER_m	($/kWh)	Electricity Rate, varies hourly
	DR-2	CCR	($/kW)	Monthly Capacity Charge, varies annually

CHEAPER uses 276 electricity prices for optimization, one for each 5-minute time period (spot price) in the forthcoming 24-hour time window. The total electricity cost *C* for time period *m* is given by

$$C_m = (P_m * X_m) + F_m$$

where,

X_m, total electric load of CHEAPER-controlled systems (kW) during time period *m*
P_m, periodic electricity price ($/kW) for time period *m*
F_m, fixed electricity cost ($) for time period *m*

The total 5-minute, electricity price per kWh, P_m, is the sum of the spot price, ER_m, a fixed delivery charge rate and a fixed supply cost rate, which are shown in rows three through five of Table 3.1.[8] In the CHEAPER model, the predicted spot price, ER_m, is an average of two values: one spot price taken from the previous week, same day and time, and one spot price taken from the previous year, same day and time. Combining two years of spot prices reduces the impact of an unusual spot price resulting from extreme weather or grid events that could skew the projected electricity price away from a more typical mean value representative of the forthcoming period.

CHEAPER also indirectly considers the monthly capacity charge, voltage distribution facilities charges and voltage transformer charge during the optimization. These price components listed as DR-1 and DR-2 in Table 3.1, respectively, are applied to a customer's bill based on the customer’s highest demand recorded during specific hours of the month. Because CHEAPER's time window is 24 hours, it cannot directly minimize demand charges, which are calculated from 30 days of electricity use. However, both demand charges are based on select hours that CHEAPER accounts for in one of two ways to minimize electricity use during these times. One method uses artificial price inflation; the other is addressed as part of the normal optimization process.

First, DR-1 is based on use during weekday, non-holiday hours of 9 a.m. through 6 p.m., so the ER during each of these hours is inflated by $1 prior to optimization in order to assure that electricity used during this time period is sufficiently penalized to account for monthly demand charges.[9] Inflating the electricity price ensures that electricity is only used during this time to obtain a feasible solution. Thus, demand charges are minimized over time. Similarly, DR-2 is based on 10 select summer hours coincident with the utility's overall highest hourly demand

---

[8] $ER_T\text{-}1 + ER_T\text{-}2 + ER_T\text{-}3 = ER_m{+}0.019326$

[9] $1 is roughly 10X higher than a typical "high" spot price.

during the previous year. Per the utility, these hours match closely with the most expensive hours of the year, so reducing electricity use each day based on real-time hourly costs also ensures that the monthly capacity charge and annual CCR is reduced over time [55]. All artificial penalty costs are subtracted from the calculated electricity costs reported by CHEAPER.

CHEAPER also includes a fixed cost per time period for standard customer and metering charges. This amount is represented by a constant, F_m, equal to the fraction of monthly fixed costs attributed to time period m. These costs do not influence the optimality of a reported solution and are included only for completeness. A graph of RTP spot prices over one 24-hour period is shown Figure 3.1.

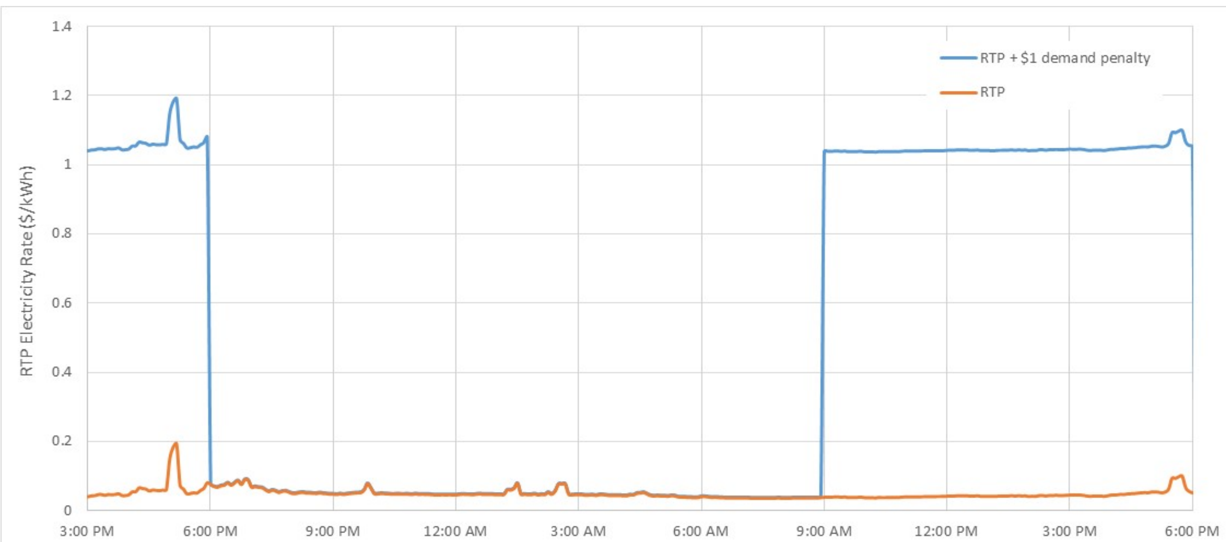

Figure 3.1. Twenty-four hours of RTP spot prices used as cost coefficients for CHEAPER's objective function. Rates are shown with and without a demand penalty. The demand penalty is used to reduce equipment-operating time during the hours that determine monthly demand charges.

3.2.2 Weather

CHEAPER uses local, real-time weather data to optimize building operations and maintain appropriate comfort levels. Weather data is retrieved from a public API web service managed by the National Weather Service (NWS). CHEAPER accesses one of approximately 50 NWS endpoints. Endpoints provide a variety of weather data including forecasts and alerts [56].

During each optimization cycle, CHEAPER accesses an endpoint that provides local, hourly weather information for 24 hours beginning with the hour the endpoint request was generated. Hourly forecasts for outdoor dry bulb temperature and sky condition are downloaded and stored. Dry bulb temperature is the ambient air temperature as measured with a thermometer. Sky condition includes a general description of the weather such as cloudy, partly cloudy, raining, snow and sunny. An example of processed weather data used in the CHEAPER model is given in Figure 3.2. Each hourly weather value is applied to twelve, consecutive, 5-minute, periods, which results visually in a stair-step pattern over time. A graph of outdoor and indoor temperatures for one 24-hour time window is shown in Figure 3.3.

```
Hourly Temperature Forecast:
 [70, 72, 74, 75, 75, 75, 74, 74, 71, 67, 64, 63, 61, 59, 59, 58, 57, 57, 57, 59, 64, 68, 72]

Hourly Forecasted Sky Conditions:
 ['Partly Sunny', 'Mostly Sunny', 'Sunny', 'Sunny', 'Sunny', 'Sunny', 'Clear', 'Mostly
Clear', 'Clear', 'Clear', 'Clear', 'Clear', 'Clear', 'Clear', 'Clear', 'Clear', 'Clear',
'Clear', 'Sunny', 'Sunny', 'Sunny', 'Sunny', 'Sunny']
```

Figure 3.2. An example of one day of filtered weather data used in the CHEAPER model.

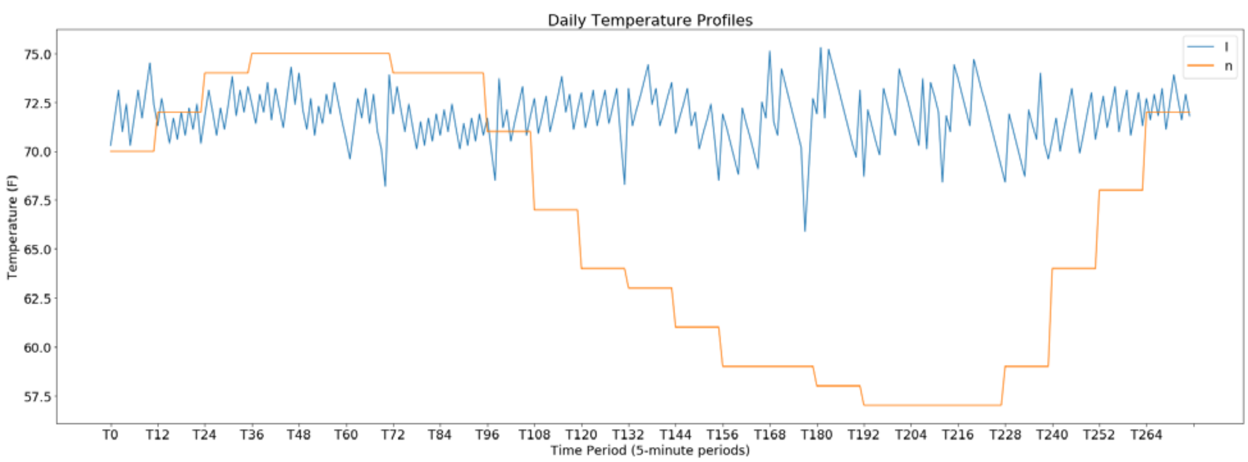

Figure 3.3. Indoor (blue) and outdoor (orange) temperatures for one day. Outdoor temperatures are for Topeka, Kansas in June 2021 as obtained from the National Weather Service. Indoor temperatures are the result of a standard thermostat control model.

CHEAPER uses the outdoor temperature along with certain building design and operations properties to predict the indoor temperature change per time period expected in the absence of mechanical space conditioning. For example, assume it is a normal workday and the weather forecast calls for a clear and sunny day with an outdoor temperature of 82 °F at 11:00 a.m., 84 °F by noon, and a high of 88 °F by 2 p.m. Also, assume that the indoor building temperature at 10 a.m. is 71 °F and the number of building occupants and activities is typical for a standard workday. According to CHEAPER's BTF, without any space conditioning, the building's ambient temperature will increase by approximately 2.4 °F over each of the next four hours resulting in an ending indoor temperature of 80.2 °F by 2 p.m.. CHEAPER uses these hourly estimates to determine the optimal combination of controlled building equipment, including space conditioning equipment, for each hour of the day up to and including 2 p.m.. The optimal solution is presented in the form of hourly equipment operating schedules, one for each CHEAPER-controlled device. These schedules minimize electricity costs while keeping the building within user-specified ranges for visual and thermal comfort. An example of system operation over time based on a CHEAPER schedule is show in Figure 3.4.

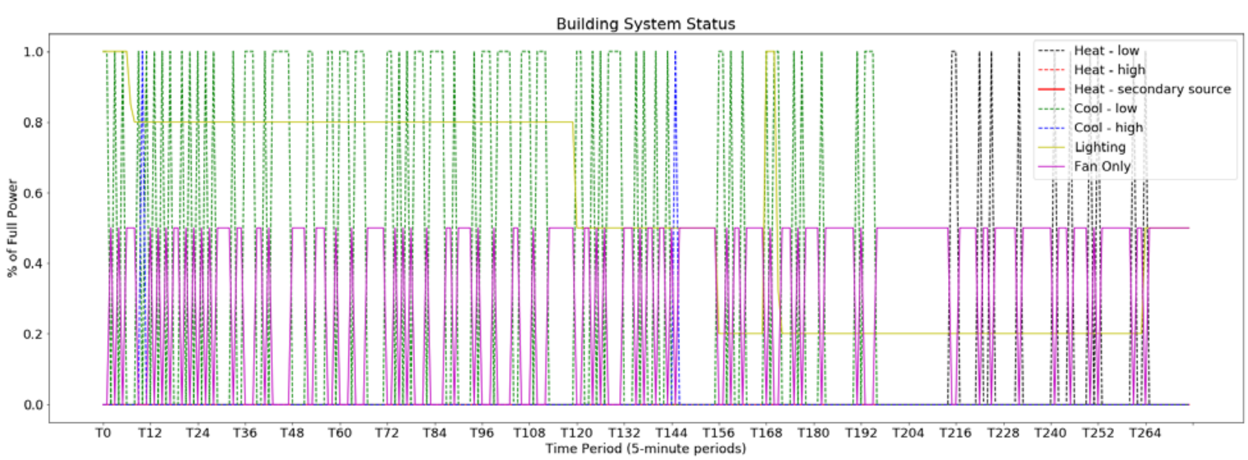

Figure 3.4. CHEAPER schedules for controlled building loads over 24 hours during a summer day. Schedules start at approximately 11 a.m. on a weekday morning.

3.3 Uncontrolled Building Loads

CHEAPER calculates and includes the indoor temperature change resulting from heat generated by building occupants and miscellaneous electric loads (MELs) as part of its optimization algorithm. Hourly occupancy schedules are user-specified as a percentage of maximum occupancy. CHEAPER accepts 24 unique values for each of three day types: weekday, weekend and holiday. This improves model accuracy, and can reduce electricity costs if a building has reduced occupancy on certain days and times of the week. For this implementation, only weekday and weekend schedules are utilized. Maximum occupancy and building size is also specified by the user. For this thesis, a small office building of 5000 square feet is assumed. The maximum occupancy is determined by assigning 150 sq. ft. per occupant. The hourly occupancy schedule used throughout this thesis is provided in Figure 3.5. The occupancy schedule matches that specified in the ASHRAE 90.1 User Manual for office occupancies [57].

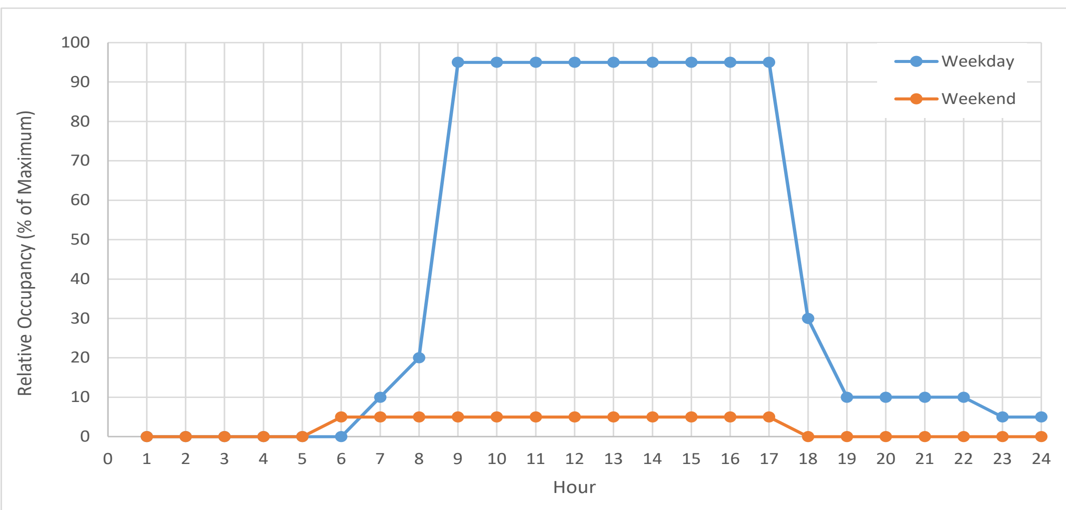

Figure 3.5. Building occupancy schedule used in the CHEAPER model. Schedule provided by ASHRAE 90.1 User Manual default schedule for office occupancy (Table G.1).

Currently, CHEAPER does not provide optimized schedules for miscellaneous electric loads; however, the heat generated by this equipment based on its size and operation is included to improve the accuracy of optimal HVAC and lighting schedules. For the current implementation, CHEAPER uses an hourly equipment load schedule (percent of full load) taken from a 2009 report by Pacific Northwest National Laboratory [58]. The total miscellaneous equipment load is required as user-specified model input and is currently modeled as 2.5 watts/ft^2, a typical value for small to medium office buildings. The hourly miscellaneous electric load schedule is shown in Figure 3.6.

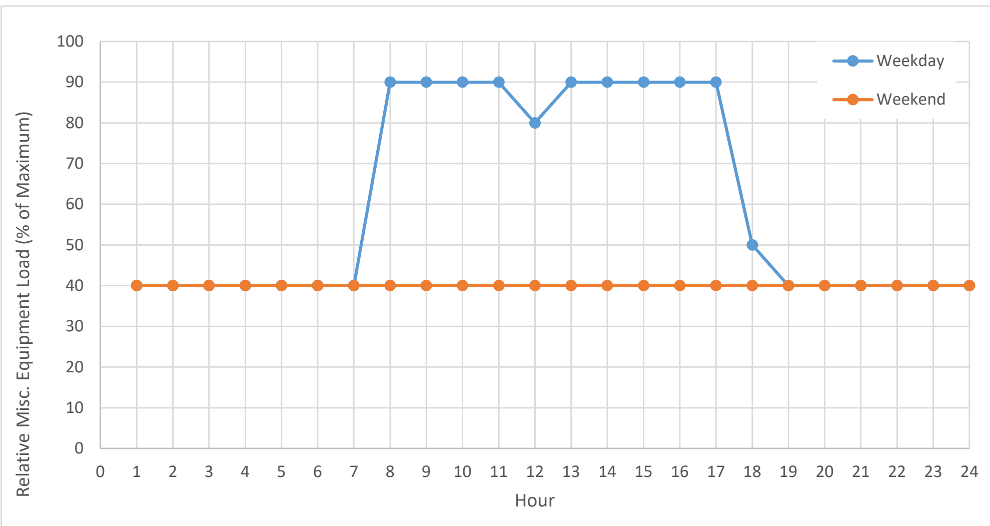

Figure 3.6. Miscellaneous electric load schedules used in the CHEAPER model. Schedule provided by Pacific Northwest National Laboratory as part of their report, "Technical Support Document: 50% Energy Savings Design Technology Packages for Medium Office Buildings".

3.4 Optimized Building Systems

To demonstrate the potential of the CHEAPER model, controlled building systems include heating and cooling, which is provided by a 5-ton, air-to-water, heat pump. The unit is combined with an air-handling unit (AHU) that provides air circulation and supplemental electric heat, when required. Lighting, designed with light-emitting diode (LED) sources and dimming controls, is also controlled as part of the CHEAPER model. In total, CHEAPER manages four building systems with a total of eight operating modes. Seven modes are managed as ON/OFF control and one as a continuously variable mode with output between 0 and 100 percent. A summary of these systems and operating modes is shown in Table 3.2.

Table 3.2. CHEAPER-controlled building systems and operating modes

System	Operating Mode(s)	Control Steps
Heating	High, Low	ON, OFF
Cooling	High, Low	ON, OFF
Mechanical Ventilation (AHU)	Heating/Cooling, Fan-Only, Supplemental Heat	ON, OFF
Lighting	Standard	Fully dimmable, 0-100% of full light output

3.4.1 Heating

The primary heating system is modeled as a high-efficiency, split-system, air-to-water heat pump with two heating modes and a maximum, rated heating capacity of 59,500 BTU per hour (BTU/h). System power (kW) and heat output (BTU/h) values are taken from the manufacturer's expanded heating data for system's operating under the standard conditions of sea level pressure and 70 °F entering, indoor, dry bulb temperature. The primary heating system's full load power is modeled as the sum of the heat pump power and AHU blower motor power. An air-to-water heat pump's coefficient of performance (COP) varies with outdoor ambient temperature. For simplicity, however, the heat pump power is modeled as a single value corresponding to the COP associated with the weighted average of monthly, mean, climate normals for Northeast Kansas occurring between 1991 and 2020 and the standard conditions previously stated. High-stage heating (High Mode) is specified at 100 percent capacity and low-stage heating (Low Mode) at 70 percent capacity. During heating operation, the AHU provides 1850 cubic feet per minute (CFM) of airflow and is assumed to deliver sufficient ventilated air necessary to maintain indoor air quality within healthy ranges.

The heat pump is supplemented by a secondary, electric, heat kit installed in the AHU. When needed, the AHU heat kit provides 19.2 kW of heating capacity at 240 Volts. A summary of heating performance parameters associated with the modeled heating equipment is provided in Table 3.3.

Table 3.3. Heating system performance data. Values taken or calculated from manufacturer's literature.

Heating Performance Parameter	High Mode (100% capacity)	Low Mode (70% capacity)	AHU Supplemental
System Power (kW)[10]	5.68 – 6.70	1.85 – 3.13	19.2
Average System Power used for modeling (kW)[10]	6.41	4.35	19.2
Total Heat Output (BTU/h)	16,340 – 77,420	6,810 – 57,010	65,510
Airflow (CFM)	1,850	1,850	1,850

For each heating mode, its contribution to the indoor temperature change (Δ°F) per time period is modeled as a multi-part piecewise function. Each part is characterized by a unique temperature change rate (°F/5-min) based on the average, hourly, outdoor temperature during the same time period. As previously discussed, the number of parts is arbitrary; however, functions

[10] Includes AHU blower fan power

with more pieces can often result in more accurate temperature change estimates. For this thesis, a four-part function is utilized to estimate the heating system components previously described. The parameters associated with this heating system piecewise approximation are provided in Table 3.4.

Table 3.4. Parameters of CHEAPER's piecewise functions used to model a building's heating systems.

<table>
<tr><th colspan="2">Heating – High Mode
(heat pump)</th><th colspan="2">Heating – Low Mode
(heat pump)</th><th colspan="2">Heating – Supplemental
(AHU)</th></tr>
<tr><th>Outdoor Temp. Range (°F)</th><th>Indoor Temp. Change (°F/5-min)</th><th>Outdoor Temp. Range (°F)</th><th>Indoor Temp. Change (°F/5-min)</th><th>Outdoor Temp. Range (°F)</th><th>Indoor Temp. Change (°F/5-min)</th></tr>
<tr><td>[-100,16]</td><td>[0.5, 2.0]</td><td>[-100,16]</td><td>[0.2, 1.3]</td><td rowspan="4">[-100, 50]</td><td rowspan="4">5</td></tr>
<tr><td>(16, 32]</td><td>(2.0, 3.0]</td><td>(16, 32]</td><td>(1.3, 2]</td></tr>
<tr><td>(32,49]</td><td>(3.0, 4.4]</td><td>(32,49]</td><td>(2, 3.1]</td></tr>
<tr><td>(49,100]</td><td>(4.4, 4.6]</td><td>(49,100]</td><td>(3.1, 4.2]</td></tr>
</table>

3.4.2 Cooling

Cooling is provided by the same high-efficiency, split-system heat pump with a maximum, rated cooling capacity 56,500 BTU/h.[11] The heat pump provides two cooling modes, High Mode and Low Mode. For each cooling mode, the indoor temperature change (Δ°F) per time period is modeled as a multi-part, piecewise function, identical to the methods described for heating modes. During operation, the system is assumed to deliver sufficient ventilated air necessary to maintain healthy indoor air quality. A summary of cooling performance parameters associated with the modeled cooling system is provided in Table 3.5. Parameters associated with the cooling system's piecewise temperature function are provided in Table 3.6.

[11] Rated at 95°F outdoor ambient temperature, 75°F indoor dry bulb temperature and 1,840 CFM.

Table 3.5. Cooling system performance data. Values taken or calculated from manufacturer's literature.

Performance Parameter	High Mode (100% capacity)	Low Mode (70% capacity)
System Power (kW)	3.23 – 5.75	2.61 – 4.55
Average System Power used for modeling (kW)[12]	5.44	3.09
Cooling Output (BTU/h)	30,070	21,230
Airflow (CFM)	2,000	1,350

Table 3.6. Parameters of CHEAPER's piecewise functions used to model a building's cooling system.

Cooling – High Mode		Cooling - Low Mode	
Outdoor Temp. Range (°F)	Indoor Temp. Change (°F/5-min)	Outdoor Temp. Range (°F)	Indoor Temp. Change (°F/5-min)
[-100, 60]	[3, 2.8]	[-100, 60]	[4, 3.8]
(60, 75]	(2.8, 2.7]	(60, 75]	(3.8, 3.6]
(75, 90]	(2.7, 2.5]	(75, 90]	(3.6, 3.4]
(90, 120]	(2.5, 2.0]	(90, 120]	(3.4, 3.2]

3.4.2 Mechanical Ventilation

The model includes an AHU that provides sufficient ventilation and air circulation using a multi-speed, circulating blower with a 1-HP blower motor rated at 6.9 FLA[13] at 240 V. The AHU selected was based on the assumptions of a well-mixed air distribution system with ventilation rates during occupied conditions matching or exceeding recommendations for office space types of 5 CFM per person plus 0.06 CFM per square foot of occupiable space [59]. The

[13] Full load amps (FLA)

AHU also meets minimum recommended airflow rates for the modeled heat pump. For a 5-ton heat pump running in High Mode - Cooling, approximately 2,000 CFM is recommended. Low Mode requires approximately 1,350 CFM. During heating mode, the blower provides 1,850 CFM, the AHU default for a 20 kW heating kit.

When heating or cooling is not required, the AHU continues to circulate ventilated air through the building based on the occupancy schedule. For example, at 4 p.m. on a mild spring weekday, no space conditioning may be required, however the building is 95 percent occupied. To ensure building occupants continue to receive adequate ventilation, the AHU operates in Fan-Only mode. The multi-speed blower must be configured at one of four speeds to ensure sufficient airflow during fan-only operation. For this model, the fan power is set to deliver 50 percent of full output during fan-only operation in order to meet requirements for the stated occupant density. Scaled AHU fan power and airflow assumes a standard fan performance curve with maximum values of 1.59 kW and 2000 CFM, respectively.

3.4.3 Lighting

The ambient, indoor lighting system is modeled at 0.5 Watts per square foot (W/sf), a reasonable lighting power density for commercial office spaces using LED luminaires. This value is required as input data to the model. The model assumes the system is fully dimmable from one to 100 percent of full light output. The model also requires a user-specified lighting schedule for weekdays and weekends in the form of a relative load profile. Minimum and maximum relative light levels can also be specified by the user. Both values are modeled relative to the specified lighting schedule. For example, if a user specifies 0.7 and 1.0 as the minimum and maximum light levels, respectively, then for each hour of the day, CHEAPER assumes that the light level can vary between 70 percent and 100 percent of the schedule value. A graph of the default lighting schedule is shown in Figure 3.7.

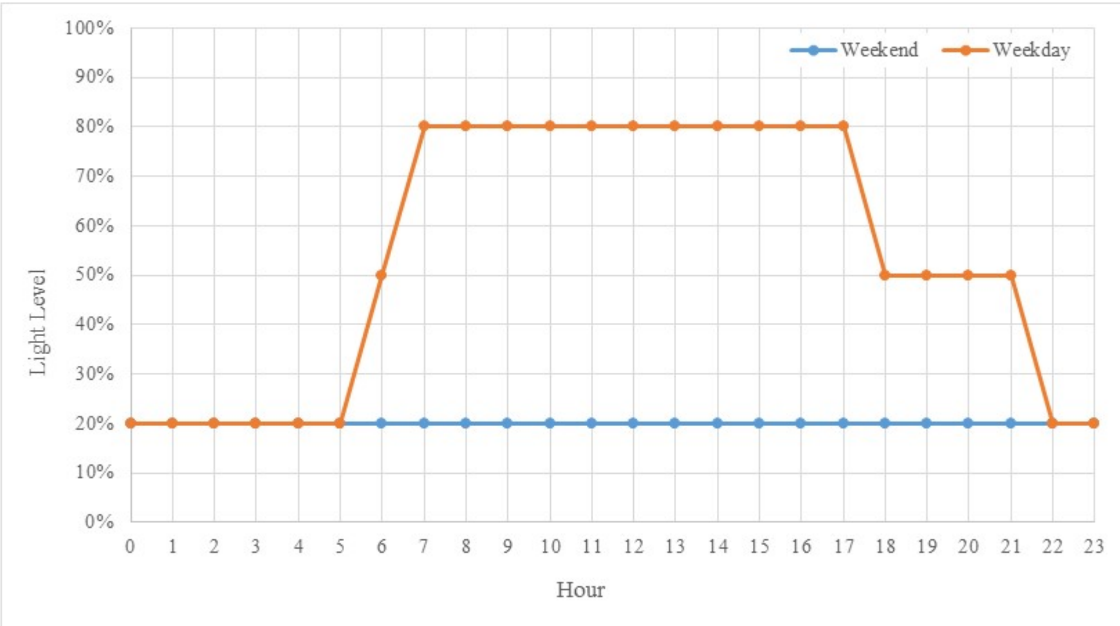

Figure 3.7. Default model lighting schedule for weekends and weekdays.

Lighting systems also generate waste heat in addition to visible light. CHEAPER controls the lighting system for two purposes. First, CHEAPER controls the lighting system based the building's occupancy schedule. Second, it controls the lighting system to manage the addition of lighting waste heat into the occupied space. Not all optical energy, however, contributes to heat in the conditioned space. A portion of the energy may be directed above the ceiling plenum, for example. Similarly, a portion is radiated as visible light that produces no heat. For this model, radiative/convective and conditioned space/plenum heat gain splits developed by Liu et. al. for common LED luminaires were used to calculate the amount of optical radiation contributing to heat gain in the conditioned lighting zone [60]. The combined, relative reduction to the maximum lighting load resulting from these splits ranges from 40 to 60 percent. This data is combined with building-specific parameters for the maximum lighting load, ceiling height and area of the conditioned space. The net heat contributed by the lighting system per hour, Q_{LH}, is determined through a straightforward calculation assuming all parameters, as previously described, are known. The equation used to determine Q_{LH} is

$$Q_{LH} = 3410\, L_{kW}/(c_p * A_L * D_L * L_{CP} * L_{CR})$$

where,

3410, kW to BTU/h conversion.
c_p, specific heat capacity of air at constant pressure (BTU/°F)
L_{kW}, maximum load of the lighting system (kW)
A_L, lighting zone area (ft^2)
D_L, lighting zone ceiling height (ft)
L_{CP}, percent of conditioned zone receiving optical radiation, and
L_{CR}, percent of optical radiation producing sensible heat gain

3.5 Solar Heat Gain and Other Passive Building Heat Flows

Two primary components of the BTF are solar heat gain and passive building heat flows such as infiltration. Solar heat gain is a function of the building's location, orientation, facade composition, and hour of the year. Passive building heat flow is the aggregated flow from three sources: ventilation (Q_V), infiltration (Q_I) and flow through the building envelop (Q_E). The heat flow is converted to indoor temperature change according to the following formula: $Q = C*\Delta T$, as previously described.

3.5.1 Ventilation

Ventilation is the intentional introduction of outside air into a building. The temperature impacts of ventilation can be mitigated in a number of ways. With many ducted HVAC systems, outdoor air is mixed with the return air prior to its introduction into the building reducing the temperature differential. Assuming that return air is approximately at room temperature, heat loss due to ventilation can be calculated assuming a temperature differential of 20 – 30 °F [61]. The hourly heat loss due to ventilation without heat recovery, Q_V, is given by

$$Q_V = 60\, \Delta T * c_p * \rho * f_a$$

where,

ΔT, temperature difference between indoor and outdoor ambient air (°F)
c_p, specific heat capacity of air at constant pressure (BTU/lb°F)
ρ, air density (lbs/ft^3)
f_a, air volume flow rate (ft^3/min)

3.5.2 Building Infiltration and Exfiltration

Infiltration is the unintentional introduction of outside air into a building through cracks in a building envelop or through doors used for passage. Exfiltration is the loss of indoor air to the outdoors, intentional and otherwise. Infiltration varies based on the building construction, rate of passage, and pressure difference between the building's interior and the outdoor environment.

The temperature change due to infiltration and exfiltration is a function of several variables including the temperature difference between the indoor, ambient temperature and the outdoor, ambient temperature; the volume of the indoor space; and the number of air change-outs resulting from the building's total air leakage. For this model, an average of 0.5 complete-volume air change-outs per hour is assumed [61]. The heat lost per hour to infiltration, Q_I, is given by

$$Q_I = \Delta T * V * n * \rho * c_p$$

where,

ΔT, temperature difference between indoor and outdoor ambient air

V, building volume (ft^3)
n, number of complete volume air change-outs per hour
ρ, air density ($lbs./ft^3$)
c_p, specific heat capacity of air at constant pressure (BTU/lb.°F)

3.5.3 Heat Flow through the Building Envelope

Heat flow due to infiltration and exfiltration is combined with heat flow occurring across the building walls and ceiling, which a function of the same temperature difference, plus the area of the exposed surface and the thermal resistivity of the composite surface. Heat flow across building surfaces can be expressed in terms of the R-value of insulation and building materials used to construct the walls, ceiling and floor [62], [63]. The heat lost across each of these surfaces, Q_E, is given by

$$Q_E = \frac{\Delta T * A}{R}$$

where,

ΔT = temperature difference between indoor and outdoor ambient air
A = area of the exterior walls or ceiling (ft^3)
R = combined R-value of the insulation and wall/ceiling material(s)

3.6 CHEAPER Linear Program

The CHEAPER linear program is described in this section including all necessary nomenclature, decision variables, the objective function and constraints. All model components, with one exception, are also used as part of the baseline model described in Section 3.7.

Table 3.7. CHEAPER Nomenclature

Symbol	Units	Type	Description
M		Set	$M = \{1,...,276\}$, the set of time periods in the CHEAPER time horizon
m	5-min	Parameter	one time period in M, $\forall\, m \in M$
R	-	Set	$R = \{1,...,n\}$, the set of segment indices for the piecewise linear function approximating the temperature change produced per time period by a CHEAPER-controlled building system or internal load
r	-	Parameter	The index of a segment in R, $\forall\, r \in R$
Y	-	Set	$Y = \{HL, HH, HS, CH, CL\}$ the set of CHEAPER-controlled building systems with performance/output correlated to outdoor, ambient temperature
y	-	Parameter	a CHEAPER-controlled building system/operating mode in Y, $\forall\, y \in Y$
G	-	Set	$G = \{hl, hh, hs, v, cl, ch\}$, the set of abbreviations representing CHEAPER-controlled building systems/operating modes with only ON/OFF functionality
g	-	Parameter	The abbreviation assigned to one CHEAPER-controlled building system/operating mode in G, $\forall\, g \in G$
HL	-	Parameter	Heating – low mode, a modeled system
HH	-	Parameter	Heating – high mode, a modeled system
HS	-	Parameter	Heating – supplemental (AHU), a modeled system
CL	-	Parameter	Cooling – low mode, a modeled system
CH	-	Parameter	Cooling – high mode, a modeled system
V_{AHU}	-	Parameter	AHU - fan-only mode, a modeled system
V_F	ft^3/min	Data	Minimum recommended ventilation rate per square foot of occupiable building space
V_P	ft^3/min	Data	Minimum recommended ventilation rate per person in the building
ER_T	$\$/_{kWh}$	Data	276 consecutive, 5-minute RTP rates (spot prices)
O	person	Data	Maximum building occupancy
o'_m	%	Data	Relative occupancy during time period $m\ \forall\, m \in M$
P_m	$\$/_{kWm}$	Data	Total electricity price during time period $m\ \forall\, m \in M$
W_m	°F	Data	Forecasted outdoor dry-bulb temperature for time period $m\ \forall\, m \in M$
W_{lr}^{y}	°F	Data	Lowest value of the outdoor temperature in segment r of OT_r^y for building system $y\ \forall\, y \in Y, \forall\, r \in R$
W_{hr}^{y}	°F	Data	Highest value of the outdoor temperature in segment r of OT_r^y for building system $y\ \forall\, y \in Y, \forall\, r \in R$
W_{lr}^{B}	°F	Data	Lowest value of the outdoor temperature in segment r of OT_r^B for building heat loss $\forall\, r \in R$

Symbol	Units	Type	Description
W_{hr}^{B}	°F	Data	Highest value of the outdoor temperature in segment r of OT_r^B for building heat loss $\forall\, r \in R$
$Q_{\%}$	-	Data	Fraction of lighting power contributing to indoor temperature change
Q_{LH}	BTU	Data	Heat gain in the conditioned zone produced by the lighting system
Q_{V}	BTU	Data	Heat loss in the conditioned zone produced by ventilated outdoor air
Q_{I}	BTU	Data	Heat loss in the conditioned zone produced by infiltration / exfiltration
Q_{E}	BTU	Data	Heat loss in the conditioned zone attributed to transfer through walls, ceilings and floors
A_{L}	ft^2	Data	The lighting zone area
D_{L}	ft.	Data	The lighting zone ceiling height
A_{B}	ft^2	Data	The building area
D_{B}	ft.	Data	The building ceiling height
HL_{kW}	kW	Data	Maximum load of the heating system in low mode
HH_{kW}	kW	Data	Maximum load of the heating system in high mode
HS_{kW}	kW	Data	Maximum load of the supplemental, electric heat kit provided by the AHU
CL_{kW}	kW	Data	Maximum load of the cooling system in low mode
CH_{kW}	kW	Data	Maximum load of the cooling system in high mode
L_{kW}	kW	Data	Maximum load of the lighting system
V_{kW}	kW	Data	Maximum load of the AHU in fan-only mode
S_{T}	°F	Data	Temperature change produced by miscellaneous electric equipment at full load
O_{T}	°F	Data	Temperature change produce by people at maximum building occupancy
L_{T}	°F	Data	Temperature change produced by lighting at full output (100% power)
T_L and T_H	°F	Data	Minimum and maximum allowed indoor temperature
L_L and L_H	%	Data	Relative minimum and relative maximum allowed indoor light level
l'_{m}	%	Data	Relative light level per the fixed schedule during time period m $\forall\, m \in M$
s'_{m}	%	Data	Relative miscellaneous equipment use per the fixed schedule during time period m $\forall\, m \in M$
IT_{r}^{y}	°F	Data	Indoor temperature change assigned to segment r of OT_r^y due to CHEAPER-controlled building system y $\forall\, y \in Y, \forall\, r \in R$
IT_{r}^{B}	°F	Data	Indoor temperature change assigned to segment r of OT_r^B due to building heat loss $\forall\, r \in R$

Symbol	Units	Type	Description
OT_r^y	°F	Data	Outdoor temperature range assigned to segment *r* of the function approximating indoor temperature change due to CHEAPER-controlled building system $y \; \forall \; y \in Y, \forall \; r \in R$
OT_r^B	°F	Data	Outdoor temperature range assigned to segment *r* of the function approximating indoor temperature change due to building heat loss $\forall \; r \in R$
hl_m	-	Decision Variable	State of the heating system – low mode during time period *m* $\forall \; m \in M$
hh_m	-	Decision Variable	State of the heating system – high mode during time period *m* $\forall \; m \in M$
hs_m	-	Decision Variable	State of the supplemental heating provided by the AHU during time period *m* $\forall \; m \in M$
v_m	-	Decision Variable	State of the AHU - fan-only mode during time period *m* $\forall \; m \in M$
cl_m	-	Decision Variable	State of the cooling system – low mode during time period *m* $\forall \; m \in M$
ch_m	-	Decision Variable	State of the cooling system – high mode during time period *m* $\forall \; m \in M$
l_m	-	Decision Variable	Relative light level during time period *m* $\forall \; m \in M$
b_m	-	Decision Variable	State of building heat loss occurring during time period *m* $\forall \; m \in M$
k_{rm}^y	-	Decision Variable	State of building system *y* during time period *m* with respect to outdoor temperature range $OT_r^y \; \forall \; m \in M, \forall \; r \in R, \forall \; y \in Y$
j_{rm}^y	°F	Decision Variable	The outdoor temperature during time period *m*, if it lies within the temperature range OT_r^y assigned to segment *r* and associated with CHEAPER-controlled building system $y \; \forall \; m \in M, \forall \; r \in R, \forall \; y \in Y$
k_{rm}^B	-	Decision Variable	State of building heat loss during time period *m* with respect to the outdoor temperature range assigned to segment *r* of $OT_r^B \; \forall \; m \in M, \forall \; r \in R$
j_{rm}^B	°F	Decision Variable	The outdoor temperature during time period *m* if it lies within the temperature range OT_r^B assigned to segment r and associated with building heat loss $\forall \; m \in M, \forall \; r \in R$
e_m	°F	Decision Variable	Indoor building temperature at the end of time period *m* $\forall \; m \in M$
C_m	$	Objective Function Value	Total electricity cost to operate CHEAPER-controlled building systems during time period *m* $\forall \; m \in M$

3.6.1 Decision Variables

$$k_{rm}^{y} = \begin{cases} 1, & \textit{if outdoor temperature is in range } OT_r^y \textit{ and building system } y \textit{ is ON during time period } m \\ 0, & \textit{else} \end{cases} \qquad \forall\, r \in R, \forall\, m \in M, \forall\, y \in Y$$

$$k_{rm}^{B} = \begin{cases} 1, & \textit{if outdoor temperature is in range } OT_r^B \textit{ and building heat loss is applied during time period } m \\ 0, & \textit{else} \end{cases} \qquad \forall\, r \in R, \forall\, m \in M$$

$$j_{rm}^{y} = \begin{cases} W_m, & \textit{if outdoor temperature is in range } OT_r^y \textit{ associated with building system } y \textit{ during time period } m \\ 0, & \textit{else} \end{cases} \qquad \forall\, r \in R, \forall\, m \in M, \forall\, y \in Y$$

$$j_{rm}^{B} = \begin{cases} W_m, & \textit{if outdoor temperature is in range } OT_r^B \textit{ associated with building heat loss during time period } m \\ 0, & \textit{else} \end{cases} \qquad \forall\, r \in R, \forall\, m \in M$$

$$hl_m = \begin{cases} 1, & \textit{if heating} - \textit{low mode is ON during time period } m \\ 0, & \textit{else} \end{cases} \qquad \forall\, m \in M$$

$$hh_m = \begin{cases} 1, & \textit{if heating} - \textit{high mode is ON during time period } m \\ 0, & \textit{else} \end{cases} \qquad \forall\, m \in M$$

$$hs_m = \begin{cases} 1, & \textit{if supplemental heating} - \textit{AHU is ON during time period } m \\ 0, & \textit{else} \end{cases} \qquad \forall\, m \in M$$

$$v_m = \begin{cases} 1, & \textit{if AHU fan} - \textit{only mode is ON during time period } m \\ 0, & \textit{else} \end{cases} \qquad \forall\, m \in M$$

$$cl_m = \begin{cases} 1, & \textit{if cooling system} - \textit{low mode is ON during time period } m \\ 0, & \textit{else} \end{cases} \qquad \forall\, m \in M$$

$$ch_m = \begin{cases} 1, & \textit{if cooling system} - \textit{high mode is ON during time period } m \\ 0, & \textit{else} \end{cases} \qquad \forall\, m \in M$$

$$b_m = \begin{cases} 1, & \textit{if building heat loss impacts the building during time period } m \\ 0, & \textit{else} \end{cases} \qquad \forall\, m \in M$$

$$l_m = \textit{relative light during time period } m \qquad \forall\, m \in M$$

$$e_m = \textit{indoor building temperature at end of time period } m \qquad \forall\, m \in M$$

3.6.2 Objective Function and Constraints

The detailed cost function for CHEAPER-controlled building systems during time period m is given by

$$C_m = P_m[(L_{kW} * l_m) + (HH_{kW} * hh_m) + (HL_{kW} * hl_m) + (HS_{kW} * hs_m) + (CH_{kW} * ch_m) + (CL_{kW} * cl_m) + (V_{kW} * v_m)]$$

The objective is to minimize the total electricity cost of these systems for all time periods,

$$min \sum_m C_m$$

subject to the following constraints:

$$L_L \leq l_m \leq L_H \qquad \forall\, m \in M \tag{1}$$

$$T_L \leq e_m \leq T_H \qquad \forall\, m \in M \tag{2}$$

$$e_m = e_{m-1} + (L_T * l_m) + (S_T * s'_m) + (O_T * o'_m) + \sum_r (IT_r^B * b_m) + \sum_r (k_{rm}^y * IT_r^y) \qquad \forall\, m \in M, \forall\, y \in Y \tag{3}$$

$$\sum_r k_{rm}^y = y_m \qquad \forall\, m \in M, \forall\, y \in Y \tag{4}$$

$$\sum_r j_{rm}^y = y_m * W_m \qquad \forall\, m \in M, \forall\, y \in Y \tag{5}$$

$$k_{rm}^y * W_{lr}^y \leq j_{rm}^y \leq k_{rm}^y * W_{hr}^y \qquad \forall\, r \in R, \forall\, m \in M, \forall\, y \in Y \tag{6}$$

$$\sum_r k_{rm}^B = b_m \qquad \forall\, m \in M \tag{7}$$

$$\sum_r j_{rm}^B = b_m * W_m \qquad \forall\, m \in M \tag{8}$$

$$k_{rm}^B * W_{lr}^B \leq j_{rm}^B \leq W_{hr}^B \qquad \forall\, r \in R, \forall\, m \in M \tag{9}$$

$$b_m = 1 \qquad \forall\, m \in M \tag{10}$$

$$\sum_{g \neq hs} g_m \leq 1 \qquad \forall\, m \in M \quad (11)$$

$$hs_m - hl_m - hh_m \leq 0 \qquad \forall\, m \in M \quad (12)$$

$$[(V_F A_B + (V_P v_m * Oo_m)) * (1 - \sum_{g \neq hs} g_m)] - V_{kW} v_m \leq 0 \qquad \forall m \in M, \forall g \in G \quad (13)$$

Optional:

$hh_m = 0$	*during hot season*	$\forall m \in M$	(14)
$hl_m = 0$	*during hot season*	$\forall m \in M$	(15)
$hs_m = 0$	*during hot season*	$\forall m \in M$	(16)
$ch_m = 0$	*during cold season*	$\forall m \in M$	(17)
$cl_m = 0$	*during cold season*	$\forall m \in M$	(18)

Light level and indoor temperature must be maintained within the allowed range, which is ensured by constraint sets (1) and (2), respectively. Constraint set (3) provides the calculation for the ending building temperature for each time period. This temperature is the sum of the previous period's ending temperature; temperature change from the lighting system; temperature change due to passive building heat flows; the temperature change from conditioned air provided by the space conditioning system; and temperature change due to heat gain from people and miscellaneous equipment. Passive building heat flows and each component of the space conditioning system is modeled as a piecewise linear approximation and described in more detail as part of constraint sets (4) through (10).

For the space conditioning system, it must be assured that the correct amount of heating or cooling is provided based on forecasted outdoor conditions. This is a three-step process. First, heating or cooling should be added to the indoor environment only when the HVAC unit is ON.

This is accomplished by requiring the sum of the decision variables k_{rm}^{y} for all segments *r* to equal the status of the heating or cooling system decision variable, y_m, for each time period *m* (4). Note, constraint set (4) does not ensure the correct k_{rm}^{y} is set to one (*r* options exist that ensure feasibility), only that exactly one k_{rm}^{y} is set to one if heating or cooling is ON. Second, the correct amount of temperature change is applied to the indoor environment by setting the temperature interval variable j_{rm}^{y} equal to the outdoor temperature if the HVAC unit is ON, and zero otherwise. Third, the issue noted previously with constraint set (4) is resolved by setting the correct k_{rm}^{y} to one through the set of inequalities shown in constraint set (6). For each time period, when the temperature interval variable j_{rm}^{y} is zero, the HVAC system is OFF and k_{rm}^{y} for all values of *r* must equal zero to maintain feasibility. When the HVAC system is ON, however, j_{rm}^{y} is equal to the forecasted outdoor temperature during time period *m*. Because the ranges described by OT_r^y do not overlap, yet cover all feasible values for the forecasted outdoor temperature, exactly one k_{rm}^{y} for each period must be set to one, and it must be the one with value of *r* corresponding to the range OT_r^y containing j_{rm}^{y}.

The process to determine a value for building heat loss, b_m, is identical to that described previously for the space conditioning system. Constraint sets (7) thru (9) follow this same logic. However, building heat loss must be applied for all time periods, which is ensured by constraint set (10).

The remaining constraint sets, (11) thru (18), are needed to ensure that the building systems operate in a manner consistent with standard practice for most systems. First, heating and cooling should not run at the same time. To ensure only one system and operating mode runs during each time period, the sum of decision variables for all heating and cooling modes must be less than or equal to one. This is ensured by constraint set (11). Note that this excludes the decision variable representing the supplementary heat kit provided by the AHU, because it can run at the same time as either of the heat pump's primary heating modes.

Additionally, the supplementary heat kit cannot run by itself. It can only be used in combination with the heat pump. This is ensured by constraint set (12). When the value of hh_m and hl_m is zero (systems are OFF), hs_m must also be zero. If hh_m or hl_m is set to one (a system is ON), hs_m may also be one (ON). This constraint set, when used in combination with constraint sets (2) and (11), sufficiently describes the operation of the supplemental heat kit and ensures it is used only after the primary heating is unable to maintain the indoor temperature within the acceptable range.

Second, the AHU must continue to provide airflow to the building when the heating and cooling systems are not in use and the building is occupied. This is ensured by constraint set (13). The constraint set includes a calculation necessary to ensure the minimum recommended ventilation rate is maintained. This rate is a function of the occupiable building area and the number of people in the building at any given time [59]. For each square foot of occupiable building area, V_F CFM must be provided, and for each person, V_P CFM. The total ventilation required therefore, is given by $V_F A_B + (V_P v_m * \, Oo_m)$. During heating and cooling operation, at least this much airflow is ensured by proper sizing of the heat pump and AHU. When these systems are not in operation, the AHU operates in Fan-Only mode, which for the AHU modeled as part of this thesis, may assume one of four values (25%, 50%, 75% or 100% of full output).

When any primary heating or cooling mode is on, the sum of the decision variables hh_m, hl_m, ch_m and cl_m, must equal one per constraint set (11). In this case, constraint set (13) reduces to

$-V_{kW} v_m \leq 0$. Because CHEAPER minimizes costs, v_m will always be set to zero so Fan-Only mode will be OFF. In contrast, when every heating and cooling mode is off, the constraint set reduces to $(V_F A_B + (V_P v_m * \; Oo_m)) - V_{AHU} v_m \leq 0$. Since V_{AHU} is at least equal to the minimum required ventilation rate for the building and occupancy, v_m must equal one for the inequality to be valid, which is the necessary value indicating that Fan-Only mode is operating and maintaining suitable airflow when all other space-conditioning systems are off.

Last, constraint sets (14 -18) may be used to turn off a particular space conditioning system for all time periods in the time horizon. For some commercial buildings, this is common practice, and heating and cooling systems only operate during specific times of the year. To do this, a constraint set is set equal to zero for all time periods. For example, during the summer season when temperatures are hot, the high (hh_m) and supplemental (hs_m) heating modes are not typically required. Setting constraint sets (14) and (16) equal to zero ensures these systems are not considered by the CHEAPER algorithm.

3.7 Baseline Model

The CHEAPER algorithm can be easily modified to represent a typical building control system that uses fixed schedules and thermostats. In this baseline model, lighting is controlled by a schedule composed of fixed hourly values for weekdays, weekends and holidays. Lighting is not allowed to vary from the schedule to control waste heat or reduce cost. Heating, cooling and ventilation is controlled by a thermostat programmed to maintain the building temperature at some user-specified value, T_s, independent of outdoor weather or electricity costs. These and similar modifications necessary to create a suitable baseline model for comparison to CHEAPER are described below. All other baseline model elements are identical to CHEAPER.

3.7.1 Additional Baseline Model Nomenclature

Symbol	Units	Type	Description
TH_L, TH_H	°F	Data	Minimum (TH_L) and maximum (TH_H) indoor temperatures associated with the thermostat span
T_S	°F	Data	Thermostat set-point temperature
t_m	°F	Decision Variable	Absolute temperature deviation from set-point during time period m $\forall\, m \in M$

3.7.2 Additional Baseline Decision Variables

$$t_m \quad = \quad absolute\ deviation\ of\ the\ indoor\ temperature\ from\ the\ set - point\ temperature\ during\ time\ period\ m \qquad \forall\, m\ \in M$$

3.7.3 Baseline Objective Function and Constraints

The cost function for the baseline model is identical to that utilized by CHEAPER. However, the baseline objective function seeks to minimize the sum of the absolute, indoor temperature deviation from the user-specified thermostat set-point temperature for all time periods *m* and ignores cost. Thermostat control is standard for U.S. commercial buildings. The baseline objective function is given by:

$$min \sum_{m} t_m$$

subject to CHEAPER constraint sets (2) thru (18) and the following additions,

$$l_m = l'_m \qquad \forall\, m \in M \qquad (1a)$$

$$TH_H - e_m \leq t_m \qquad \forall\, m \in M \qquad (19)$$

$$e_m - TH_L \leq t_m \qquad \forall m \in M \qquad (20)$$

For the baseline, light levels are set according to a fixed schedule that provides the relative light level, l'_m, as a percentage of maximum load for each hour of the day. Power is used as a proxy for light output, which is a typical control metric for dimmable lighting systems.

During each time period, relative lighting power, l_m, must match the schedule value, l'_m. This is ensured by constraint set (1a), which replaces the dynamic lighting control modeled using constraint set (1) of the CHEAPER model.

The final two constraint sets, (19) and (20), ensure that the objective function is minimized. Many digital thermostats include a span setting, which regulates how long a system will run based on indoor temperature. The span can vary from one to several degrees. For example, if a thermostat's span is set to 4°F with a set-point of 72°F in heating mode, the heating system would turn on at 70°F and turn off at 74°F. These lower and upper boundaries are given by [TH_L, TH_H], respectively. Constraint set (19) and (20) ensure that the absolute variance of the indoor temperature from the span boundary temperatures is minimized for all time periods. Constraint set (19) ensures that the indoor temperature does not stray far below the minimum span value, because as e_m gets small, t_m must grow to maintain feasibility. Similarly, constraint set (20) ensures that the temperature does not grow large, since as e_m grows, t_m must also. This combination results in a minimal value of t_m.

At present, the CHEAPER model includes approximately 16,000 decision variables and 10,000 constraints. The model is easily customizable with respect to building design and operational parameters. This makes CHEAPER a functional and flexible model that can accommodate different locations, energy-pricing programs, building types, building applications and installed building equipment. As presented, CHEAPER includes a relatively small suite of control options and a cost minimization objective. Chapter 4 presents the program results and discussion for a common office building. Many additional building system features and optimization opportunities are possible.

Chapter 4 Results

CHEAPER was tested with parameters and input data representative of a small, 5000 sf. office building located in northeast Kansas. Results were compared to those obtained for the same space and location using the standard building control model described in Section 3.7. Table 4.1 includes the building, application, and equipment parameters used to initialize and test the CHEAPER and baseline models. In addition, for some tests, certain building systems were turned off using the optional constraint sets previously described. These instances are noted in the applicable results tables and discussion.

Table 4.1. Building and related details used for CHEAPER and baseline model validation and testing.

Value	Units	Description	Acronym or Abbreviation	Model
5,000	$ft.^2$	Building area, lighting zone area	A_B, A_L	CHEAPER, Baseline
10	ft.	Ceiling height, lighting zone ceiling height	D_A, D_L	CHEAPER, Baseline
0.5	$W/ft.^2$	Lighting power density	LPD	CHEAPER, Baseline
2.3	$W/ft.^2$	MELs power density	MPD	CHEAPER, Baseline
0.8	-	Fraction of optical radiation reaching the lighting zone	L_{CP}	CHEAPER, Baseline
0.6	-	Fraction of optical radiation producing sensible heat gain	L_{CR}	CHEAPER, Baseline
70	°F	Starting indoor temperature at time m=0	e_0	CHEAPER, Baseline
68	°F	Minimum indoor temperature allowed during CHEAPER optimization	T_L	CHEAPER
76	°F	Maximum indoor temperature allowed during CHEAPER optimization	T_H	CHEAPER
0.7	-	Minimum light level allowed relative to the lighting schedule during CHEAPER optimization	L_L	CHEAPER
1.0	-	Maximum light level allowed relative to full output during CHEAPER optimization	L_H	CHEAPER
72	°F	Thermostat set-point	T_s	Baseline
+/- 2, (70,74)	°F	Thermostat span	TH_L, TH_H	Baseline

CHEAPER was written in the Python programming language, version 3.7.1 for the 64-bit Windows 10 operating system. The model was developed in Spyder version 3.2.2, a scientific Python programming environment that hosts all the necessary components for developing, executing and troubleshooting computer code written in Python [64]. Several Python programming packages were used to expedite model development. PuLP software, an open-source library of software development tools for building mathematical programs in Python, was critical to CHEAPER's development. PuLP translates Python code into the necessary format for processing by an external optimization solver such as CPLEX or Gurobi [65]. For this research, CHEAPER was coupled with the COIN OR optimization solver, which is an open-source solver developed and managed by COIN-OR Foundation [66].

CHEAPER development and testing was completed on a personal computer running with an Intel(R) Core(TM) i7-6600U processor and 16 GB of RAM. Tables, graphs and similar elements were created in Excel® using data and results from CHEAPER. CHEAPER accesses real-time weather and electricity price data through publically available APIs hosted by the National Weather Service, and Commonwealth Edison, Inc., respectively [56], [67].

4.1 Solutions and Optimality

Due primarily to CHEAPER's size and RTP cost symmetries, the majority of problem instances do not solve fast enough to be practical for everyday use. As a result, a relative threshold from optimality, also called an optimality gap, is used to reduce the program's runtime. This significantly improves the algorithm's runtime by reducing the precision required for an acceptable solution.

An appropriate gap size was determined by evaluating the solutions obtained for each of three, unique data sets. Each data set uses 16 maximum allowed optimality gap sizes varying from 0.25 percent to 10 percent of optimal resulting in up to 16 different solutions for the same day and 24-hour time horizon. The total runtime obtained using the 10 percent optimality gap was used as a basis for comparison.

Results demonstrate that solutions are obtained in ten seconds or less when a gap size of 1.0 percent or more is applied. In contrast, gap sizes less than 0.5 percent typically did not result in an optimal solution after five minutes of computation time; however, good feasible solutions were obtained in all cases. The relative improvement in accuracy as compared to the 10 percent optimality gap for all three data sets is shown in Figure 4.1. For all three, decreasing the gap size to 1.5 percent reduced the objective function value of an optimal solution by one to two percent. In terms of increased operating costs for CHEAPER-controlled systems, use of a 1.5 percent optimality gap equates to a daily cost increase of $0.02 to $0.08. Complete results for this analysis are provided in Appendix A. To be conservative in reporting results and ensure they are representative of those obtained from a reasonable algorithm runtime, a 1.5 percent optimality gap is applied to all problem instances reported in this thesis.

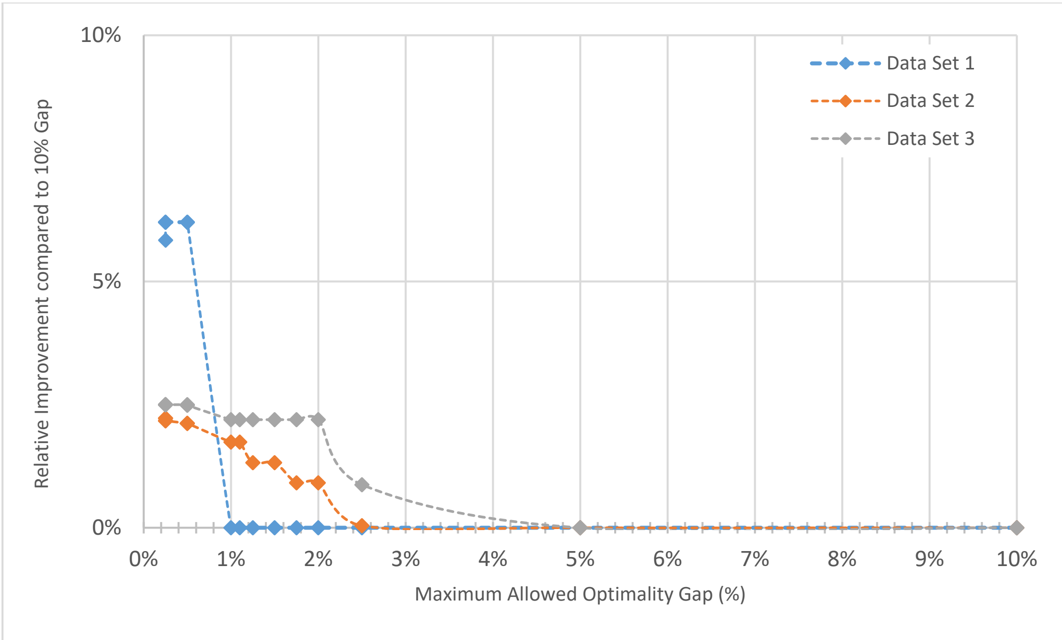

Figure 4.1. The relative improvement in a CHEAPER solution as compared to a solution obtained with a 10 percent gap size. Results include three unique data sets, each taken from a different day and 24-hour time span.

4.2 Cost Savings

To estimate the potential cost savings associated with optimized building operation using CHEAPER, six weeks of results were collected for the model building operating with CHEAPER and the baseline control strategies. The data collection period spanned from September 2020 through July 2021 and used historic weather data for Topeka, Kansas combined with RTP spot prices for the same periods.

On average, use of CHEAPER as compared to baseline control resulted in 22 percent cost savings. Savings ranged from two to 55 percent depending on the month and season. Average monthly savings increased throughout the year, peaking in late summer at approximately 36 percent. As temperatures cooled and electricity rates stabilized, savings declined to a low in mid-Winter. Daily and monthly relative cost savings for the modeled periods is shown in Table 4.2.

Table 4.2. Modeled daily cost savings by month including the monthly average achieved from use of the CHEAPER algorithm as compared to the baseline building control strategies.

Day / Month	Jan	March	May	July	September	November	Average
Monday	9%	21%	25%	32%	36%	19%	24%
Tuesday	12%	18%	17%	29%	39%	13%	21%
Wednesday	15%	22%	18%	26%	35%	19%	22%
Thursday	16%	19%	35%	26%	33%	7%	23%
Friday	9%	11%	23%	31%	30%	12%	19%
Saturday	15%	13%	23%	27%	25%	20%	20%
Sunday	2%	13%	12%	55%	52%	5%	23%
Average	**11%**	**17%**	**22%**	**32%**	**36%**	**14%**	22%

In absolute terms, CHEAPER-controlled systems cost \$15 to \$30 less to operate each month as compared to the baseline. Both models are based on use of the RTP electricity rate components previously described, which exclude demand charges that add 30 to 70 percent more costs to a customer's bill [26]. Considering reduced demand, monthly cost savings could reach \$85 for the building and systems modeled. For the average commercial electricity customer paying \$647 per month, adding CHEAPER control to 60 percent of their load equates to an average annual savings of \$1,024.

Cost savings are the result of three control strategies included in the CHEAPER model: occupancy control, light level dimming and load shifting. First, CHEAPER automatically reduces lighting and ventilation to minimum levels when the building is vacant. Similarly, CHEAPER reduces building ventilation costs by turning off the fan-only mode when the building is scheduled to be vacant. Occupancy-based control strategies save money regardless of when their used, because they reduce energy consumption.

Dimming is used to reduce light levels by up to 30 percent during daytime hours to reduce waste heat and costs. Dimming directly reduces lighting costs and, by reducing the amount of heat released into the building, it reduces cooling costs associated with the space conditioning system. CHEAPER also uses the dimmable lighting system as a micro space-conditioning system when small amounts of heat are required to maintain suitable indoor conditions and the cost of using the lighting system is less than the cost of using the heating system. Because of this feature, the lighting system cost savings may be less than that achieved from using a standard time clock or similar scheduling system to reduce lighting use during normally unoccupied hours of the days. At the building level, however, the net result is a decrease in electricity costs.

Figure 4.2 clearly shows the result of occupancy control and light level dimming during the early morning, weekday hours of 12 AM (T0) to 6 AM (T72). The top graph shows the baseline with constant ventilation provided by the space conditioning system operating in fan-only mode (magenta) and lighting at 20 percent of full output (yellow). The bottom graph shows that these systems are fully OFF in the CHEAPER model, except for a short period around 12 AM where the lighting is on at full power to supplement the heating system operating in low mode.

The third control strategy, load shifting, results when building equipment is operated during less costly periods or in different configurations to save money. For example, operating the cooling system in low mode for multiple time periods in advance of an expensive weekday afternoon peak as opposed to operating the system in high mode for a short time when needed is a typical result obtained with CHEAPER optimization. In contrast to occupancy-based control strategies, load shifting results in cost savings because of the temporal variability in electricity prices. It may not result in energy savings if the total operating time of each controlled device remains constant.

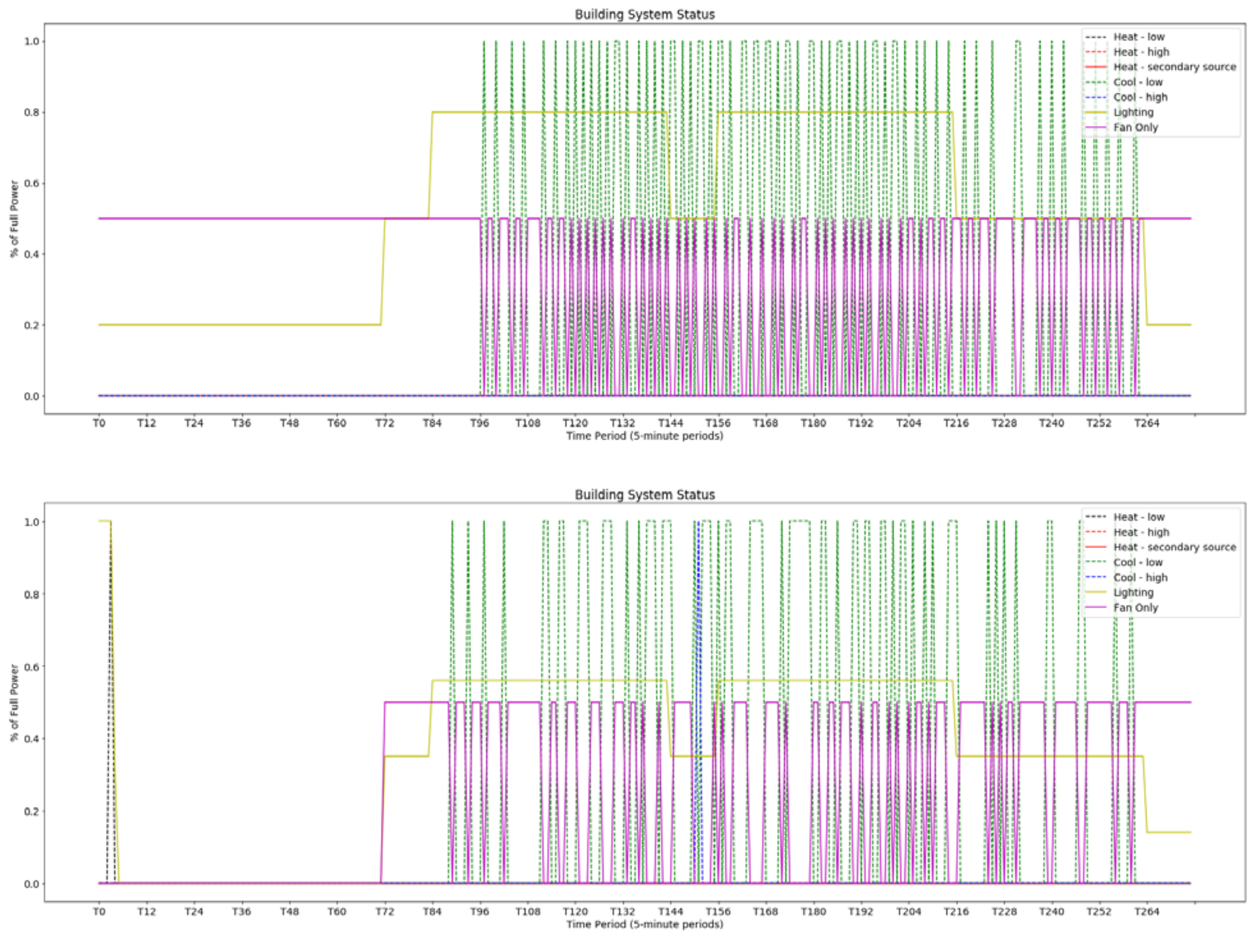

Figure 4.2. A baseline (top) and CHEAPER (bottom) weekday equipment schedule for the same day and time span that clearly show the use of occupancy control on lighting and ventilation systems during vacant nighttime hours of 12 AM to 6AM (T0-T72).

Consider the RTP prices for a Friday in July as shown in Figure 4.3. Prices begin to fluctuate significantly beginning around 7:00 AM and stabilize 4:00 PM. During this time, the spot price fluctuates between \$0.05 and \$0.20 per kWh. Outdoor temperatures rise steadily during this period, beginning around 70°F and peaking at 91°F.

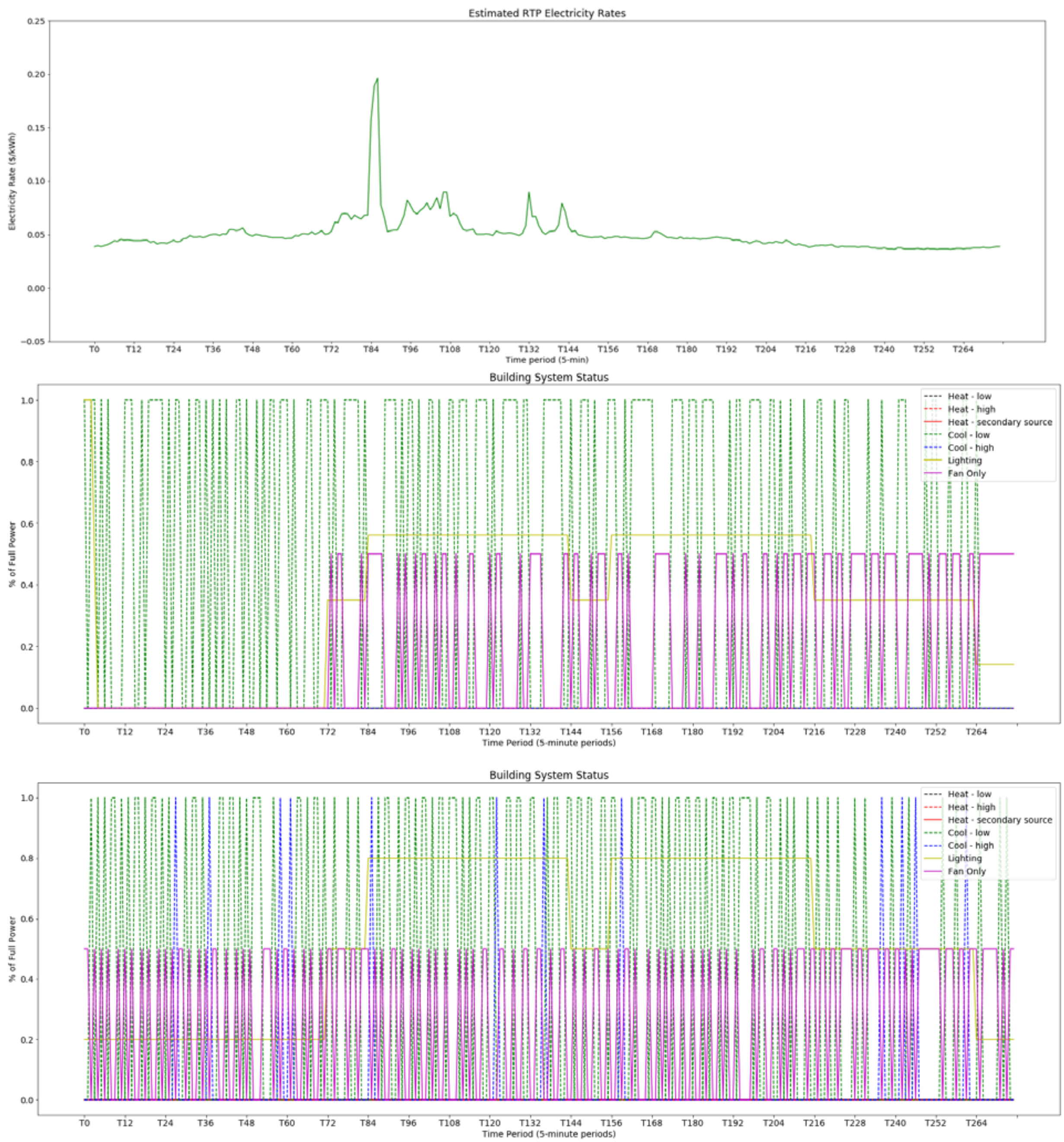

Figure 4.3. Graph of RTP prices (top), CHEAPER schedules (middle) for Friday, July 23, 2021, 12 AM to 11:59 PM.

To provide adequate cooling, the baseline model uses a combination of low and high cooling modes with a combined total of 660 minutes of operation. The CHEAPER model increases the total cooling system runtime to 680 minutes, but eliminates the use of the high cooling mode and shifts some use to avoid the costliest spot prices of the morning. Therefore, while the total system operating time was increased, the optimized schedule produced nine percent cost savings and maintained indoor building temperatures within the same comfort range as the baseline system. Figure 4.4 shows the baseline indoor temperatures in blue, which ranged from approximately 68 to 75°F. Figure 4.5 shows the CHEAPER indoor temperatures in blue, which ranged from 68 to 76°F.

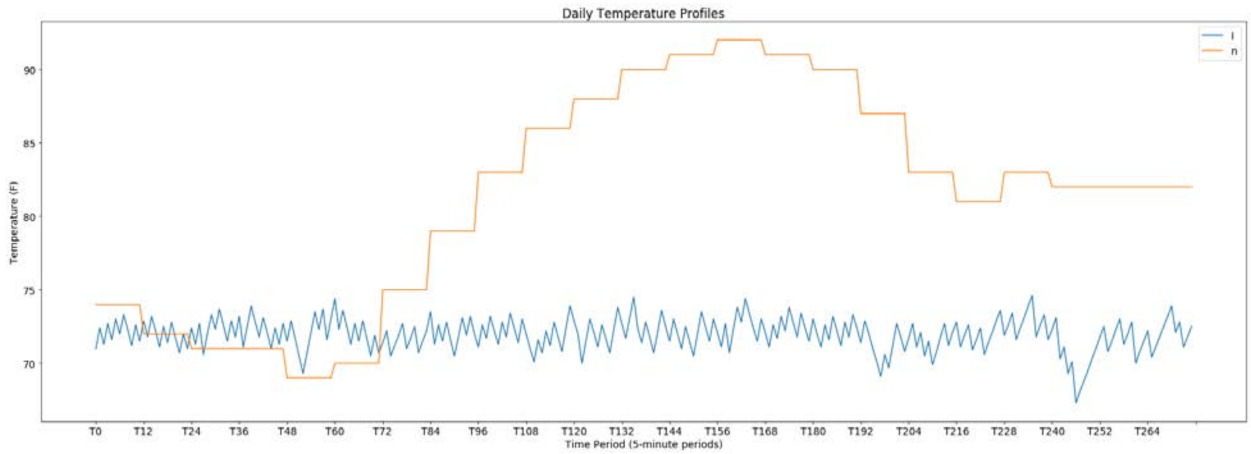

Figure 4.4. Outdoor and indoor temperatures for Friday, July 23, 2021. Indoor temperatures produced by the baseline model.

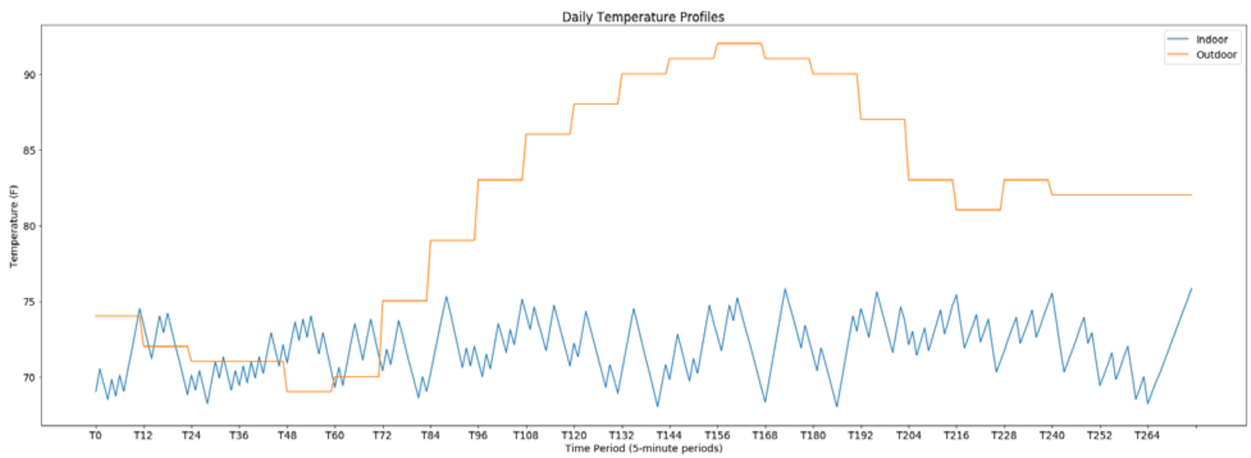

Figure 4.5. Outdoor and indoor temperatures for Friday, July 23, 2021. Indoor temperatures produced by the CHEAPER model.

The complete model results for this example, which include 20 percent cost savings for all CHEAPER-controlled systems, are summarized in Table 4.3. The daily cost savings excluding demand charges for all additional modeled periods is provided in Appendix A.

Table 4.3. Example of CHEAPER cost and energy results for one weekday in July.

	CHEAPER			Baseline			Savings	
	Operating Time (min)	Energy Use (kWh)	Cost	Operating Time (min)	Energy Use (kWh)	Cost	Energy Savings (kWh)	Cost Savings ($)
Heating - Low Mode	0	0.0	$ -	0	0.0	$ -	0.0	$ -
Heating - High Mode	0	0.0	$ -	0	0.0	$ -	0.0	$ -
Cooling - Low Mode	685	35.3	$ 1.73	600	30.9	$ 1.56	-4.4	$ (0.17)
Cooling - High Mode	0	0.0	$ -	60	5.4	$ 0.31	5.4	$ 0.31
Fan-Only Mode	488	2.3	$ 0.13	720	3.4	$ 0.17	1.1	$ 0.04
Lighting	530	22.1	$ 1.07	744	31.0	$ 1.64	8.9	$ 0.57
Total	1703	59.7	$ 2.93	2124	70.8	$ 3.68	11.1	$ 0.75
Savings		**11.1**	**$ 0.75**				**16%**	**20%**

4.3 Energy Savings

A clear, secondary benefit of CHEAPER optimization is energy savings. While seeking to reduce costly operating time, CHEAPER replaces energy-intensive operating modes with lower power alternatives and eliminates unnecessary equipment use during vacant periods. The same control strategies discussed in the previous section result in energy savings in many cases.

Due to occupancy-based lighting control, lighting system energy use was consistently reduced by an average of 33 percent as compared to the baseline. Similarly, ventilation energy use was consistently reduced each month. Savings from heating and cooling, because there was no occupancy or scheduled-based control used, may all be attributed to load shifting. Combined, load shifting of HVAC resulted in 25 percent average annual energy savings. Average annual energy savings for all CHEAPER-controlled building systems is shown in Figure 3.4.

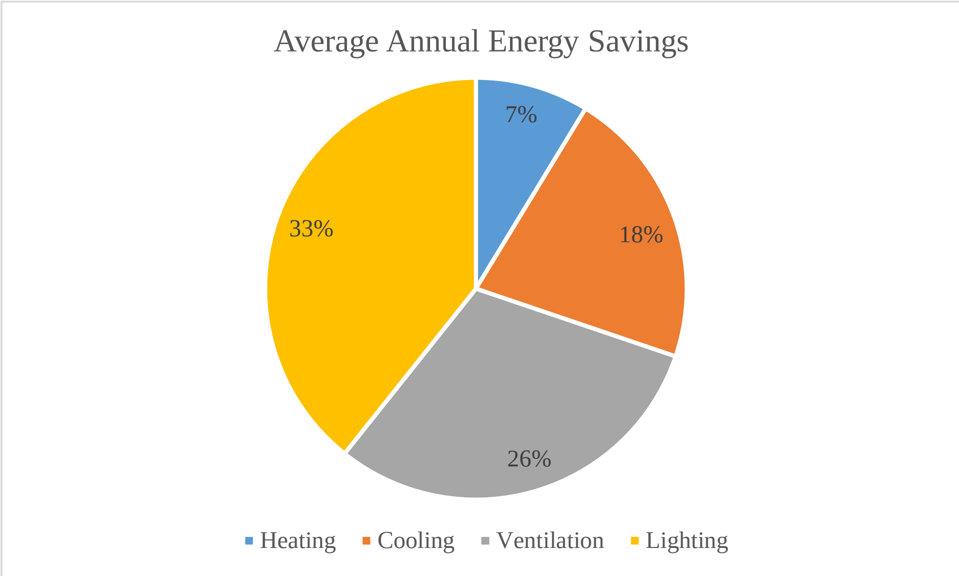

Figure 4.6. Average annual energy savings for CHEAPER-controlled building systems.

For the building and climate considered, energy savings are highest during the cooling season peaking in September. During this month, temperatures are generally mild with an average temperature of 66°F for the year considered. As compared to the baseline, CHEAPER did a better job of managing the building's cooling load using predictive methods than the baseline did using a standard thermostat with span setting. CHEAPER was able to forecast the building's heating and cooling needs over time, which was better suited for a month where both heating and cooling, are often required. With thermostat control, the building switched between heating and cooling modes when temperatures were in the 60s and low 70s, which increased energy costs considerably. Considerations on how to modify the baseline model to reduce frequent switching is an area of future research and discussed in Chapter 5.

No other significant energy savings trends are apparent in the results. Savings varied across all days of the week. Average monthly energy savings ranged from 11.5 to 32.7 percent.

A monthly results summary is provided in Table 4.4. The complete set of energy savings results are provided in Appendix A, Table A.2.

Table 4.4. Monthly energy savings for CHEAPER-controlled building systems.

Month	Monthly Savings (kWh)	Average Monthly Savings (%)	Relative Maximum Daily Savings (%)	Relative Maximum Savings Day
January	436.4	12.8%	32.2%	Friday
March	353.8	13.6%	18.7%	Wednesday
May	424.8	18.0%	31.5%	Thursday
July	447.6	30.1%	55.1%	Sunday
September	630.5	32.7%	49.0%	Sunday
November	258.44	11.5%	17.5%	Monday

In summary, CHEAPER produces both cost and energy savings in every month of the year. Through a combination of load shifting to avoid costly peak prices and occupancy control to reduce unnecessary energy use, average annual cost and energy savings of 22 and 20 percent, respectively, were achieved. Both weather and price volatility play an important role in creating cost and energy savings opportunities, which demonstrates the importance of using real-time conditions as part of any predictive control strategy. Without price and weather prediction capabilities, load shifting to save money would be a difficult objective to achieve successfully. Results demonstrate that whole-building, predictive control is possible and CHEAPER provides a good example to build from for future research and development.

Chapter 5 Conclusions and Future Research

The near-term reality of the smart electricity grid and grid-enabled buildings is driving demand for building automation solutions that will help owners and tenants take advantage of the technology. Some necessary technologies and programs are already in place. Real-time electricity pricing, in particular, provides a clear set of variable costs that can be leveraged to provide financial savings for utility customers and grid stability for grid operators. However, the publically available APIs needed to access and build upon this data are sparse. As a result, building automation solutions offering real-time optimization functions are limited. In the future, for buildings with existing BMS or plans to add them, the addition of an optimization engine may add relatively little additional capital cost, and pay immediate dividends. However, significant technology development is needed today to make optimization solutions available and affordable for most buildings.

This thesis presents the CHEAPER model, a mixed integer linear program for commercial buildings that generates lighting, heating, cooling, and ventilation schedules to minimize electricity costs and maintain acceptable indoor temperature, fresh air and lighting conditions. CHEAPER is an optimization tool with the potential to supplement or replace existing building control strategies to save money and energy consistently as part of daily commercial building operations. This thesis demonstrates that even small buildings equipped with efficient, modern equipment can potentially save 20 percent or more in electricity and energy costs by properly optimizing their operations over a finite time horizon using real-time weather and electricity price data. Savings are climate dependent and results show that under certain conditions, electricity cost savings could exceed 50 percent as compared to baseline conditions. In addition, while energy savings is not currently CHEAPER's primary objective, energy savings are achieved because the model includes control strategies that reduce unnecessary electricity use as well as time-dependent, load shifting techniques to minimize costs.

5.1 Future Research

Currently, CHEAPER demonstrates that linear optimization can be successfully applied as part of a predictive approach to integrated, whole-building control. However, significant research remains. Most importantly, CHEAPER, and optimization techniques generally, require real-world demonstration in actual buildings to validate the concepts and prove predictive control is a reliable alternative to existing building control solutions.

Technology demonstrations are key for several reasons related to technical feasibility and user acceptance. First, demonstrations are the best way to determine if CHEAPER can be implemented and maintained using existing building hardware and software tools. Second, demonstrations create an opportunity for building operators, installation contractors, building occupants and others to provide feedback on CHEAPER's performance in maintaining suitable indoor conditions. Third, sub-metering CHEAPER-controlled building equipment provides real energy use and costs that can be compared to the building's pre-retrofit performance or compared to similar buildings in the community to determine actual benefits.

Future research needs also include work to better analyze, improve and expand the CHEAPER model. Additionally, research to develop replicable data collection and processing methodologies for use in model initialization must be completed. The following sections detail several immediate and important research needs and gaps that remain to be filled.

5.1.1 Additional Data Analysis

In its current form, additional work is needed to analyze CHEAPER schedules and savings. For this thesis, cost and energy comparisons were completed using RTP as the baseline electricity tariff. However, most existing commercial ratepayers are not enrolled in RTP programs. Therefore, the results provided are very conservative and applicable to only a small subset of existing commercial customers. Additional data analysis is needed to compare the benefits of RTP combined with automated optimization against buildings with standard controls enrolled in common TOU and fixed-price utility rate programs. This analysis would provide a broad range of potential savings applicable to a larger portion of today's commercial building stock. Sensitivity analysis to also needed to determine the relative impact of rate variability on cost and energy savings. Similarly, sensitivity analysis is needed to better understand climate impacts, weather and occupancy on CHEAPER schedules and savings.

Research is also needed to compare CHEAPER results to other building modeling tools to determine the accuracy and efficacy of CHEAPER as compared to existing techniques. For example, Energy Plus software includes a parametric simulation function that could be used to examine the impacts of various thermostat set points. This is a reasonable alternative to mathematical set point optimization. Tools like Energy Plus are industry standards, and it would be good to understand how CHEAPER's optimization results compare to those obtained from existing tools.

5.1.2 Existing Model Improvements

Several important model improvements should be addressed in the near term. This includes improvements to address latent heating loads and the time delay associated with heat absorbed by the building's mass. Today, cooling factors are often used to address this time delay, and this, or a suitable alternative, should be incorporated into the existing model. In parallel, CHEAPER should also be updated to include HVAC schedules for weekdays, weekends and holidays with suitable temperature setback points.

With respect to the RTP prediction methodology, its accuracy has not been validated against historic RTP data. The methodology should be tested and refined, as needed, to ensure predicted values align well enough with actual. Alternatively, day-ahead RTP could easily replace RTP prediction in the CHEAPER program.

Last, the baseline model performs poorly when the outdoor temperature is at or near the indoor temperature set point. When the indoor building environment should be in equilibrium with the outdoor environment, the baseline model struggles to maintain a realistic balance in the use of heating and cooling systems. Currently, to alleviate frequent switching of these systems, the baseline model is run with certain heating and cooling systems turned OFF. Additional constraints on the HVAC system should be developed to mitigate these issues and present a more realistic model for baseline control. The BTF should also be modified to reflect better equilibrium conditions around the 65 to 75°F range.

5.1.3 Expansion of the CHEAPER Model

For wider applicability, the CHEAPER model should be expanded to include multi-zone control capabilities in terms of HVAC and lighting including the interactive effects among zones. Models are also needed to address other building systems such as water heating, refrigeration and plug loads. Models for appliances that use natural gas should also be addressed.

With respect to optimization objectives, CHEAPER could be expanded to consider multiple objective functions such as adding consideration of energy consumption directly. Minimization of carbon emissions is also an important topic for future research in building operations optimization. Due to the increasing variability of the U.S. source fuel supply and increase in renewable energy in particular, carbon emissions now vary significantly over the course of a day. Times with the smallest carbon footprint often differ from those offering a low electricity rate. Research to address the competing objectives of cost and carbon reduction is needed to ensure CHEAPER can serve as a tool for meeting both energy and environmental goals.

In closing, there are several tangential research needs associated with use of CHEAPER and optimization techniques in general. For example, model initialization is critical and current CHEAPER methods utilize basic heat transfer equations. Other, more accurate methods exist and methodologies should be developed that standardize the way in which this information is extracted for optimization purposes. Examples include the use of BIM software and use of actual building performance data collected with sensors installed in the building. Alternatively, research to develop and document a methodology for collecting building performance and other data using sensors and data loggers to create a history data file that can eliminate the need for piecewise linear functions of building temperature change could replace the BTF altogether.

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Appendix A - Results Data Tables

Table A.1 CHEAPER solutions obtained for various optimality gaps: A comparison of the impacts of gap size on a CHEAPER solution, its runtime and the resulting daily cost to operate CHEAPER-controlled building systems.

Data Set #	Maximum Optimality Gap Allowed (%)	Maximum Runtime Allowed (s)	Actual Runtime (s)	Solution Status	Final Objective Function Value (OFV)	Daily Operating Cost ($)	OFV Improvement compared to 10% Gap (%)	Cost Savings Improvement (%)	Best Possible Linear OFV	Estimated Daily Cost w/out Optimality Gap Allowance ($)
1	10.00%	300	5	Optimal	31.799	$ 2.74	-	-	-	-
1	5.00%	300	5.2	Optimal	31.799	$ 2.74	0.00%	0.00%	30.285	$ 2.61
1	2.50%	300	5.1	Optimal	31.799	$ 2.74	0.00%	0.00%	31.023	$ 2.67
1	2.00%	300	4.9	Optimal	31.799	$ 2.74	0.00%	0.00%	31.175	$ 2.69
1	1.75%	300	5.8	Optimal	31.454	$ 2.74	1.08%	0.00%	30.913	$ 2.69
1	1.50%	300	5.8	Optimal	31.454	$ 2.74	1.08%	0.00%	30.989	$ 2.70
1	1.25%	300	5.7	Optimal	31.454	$ 2.74	1.08%	0.00%	31.066	$ 2.71
1	1.10%	300	5.6	Optimal	31.454	$ 2.74	1.08%	0.00%	31.112	$ 2.71
1	1.00%	300	5.9	Optimal	31.454	$ 2.74	1.08%	0.00%	31.143	$ 2.71
1	0.50%	300	87.9	Optimal	31.282	$ 2.57	1.63%	6.20%	31.126	$ 2.56
1	0.50%	300	87.9	Optimal	31.283	$ 2.57	1.62%	6.20%	31.127	$ 2.56
1	0.50%	300	88.1	Optimal	31.283	$ 2.57	1.62%	6.20%	31.127	$ 2.56
1	0.25%	300	302.7	Feasible	31.253	$ 2.57	1.72%	6.20%	-	-
1	0.25%	300	303.0	Feasible	31.257	$ 2.57	1.70%	6.20%	-	-
1	0.25%	600	602.2	Feasible	31.257	$ 2.57	1.70%	6.20%	-	-
1	0.25%	600	602.1	Feasible	31.261	$ 2.58	1.69%	5.84%	-	-
2	10.00%	300	2.7	Optimal	28.860	$ 2.24	-	-	-	-
2	5.00%	300	2.7	Optimal	28.860	$ 2.24	0.00%	0.00%	27.486	$ 2.13
2	2.50%	300	3.6	Optimal	28.848	$ 2.27	0.04%	-1.32%	28.144	$ 2.21
2	2.00%	300	4.6	Optimal	28.597	$ 2.26	0.92%	-0.88%	28.036	$ 2.22
2	1.75%	300	4.4	Optimal	28.597	$ 2.26	0.92%	-0.88%	28.105	$ 2.22
2	1.50%	300	4.9	Optimal	28.479	$ 2.13	1.34%	5.16%	28.058	$ 2.10
2	1.25%	300	4.7	Optimal	28.479	$ 2.13	1.34%	5.16%	28.127	$ 2.10
2	1.10%	300	5.3	Optimal	28.358	$ 2.13	1.77%	5.16%	28.049	$ 2.11
2	1.00%	300	6.4	Optimal	28.358	$ 2.13	1.77%	5.16%	28.077	$ 2.11

Data Set #	Maximum Optimality Gap Allowed (%)	Maximum Runtime Allowed (s)	Actual Runtime (s)	Solution Status	Final Objective Function Value (OFV)	Daily Operating Cost ($)	OFV Improvement compared to 10% Gap (%)	Cost Savings Improvem ent (%)	Best Possible Linear OFV	Estimated Daily Cost w/out Optimality Gap Allowance ($)
2	0.50%	300	7.1	Optimal	28.248	$ 2.00	2.17%	12.00%	28.107	$ 1.99
2	0.50%	300	7.1	Optimal	28.248	$ 2.00	2.17%	12.00%	28.107	$ 1.99
2	0.50%	300	5.8	Optimal	28.248	$ 2.00	2.17%	12.00%	28.107	$ 1.99
2	0.25%	300	301.7	Feasible	28.233	$ 1.98	2.22%	13.13%	-	-
2	0.25%	300	302.0	Feasible	28.233	$ 1.98	2.22%	13.13%	-	-
2	0.25%	600	601.8	Feasible	28.218	$ 1.97	2.28%	13.71%	-	-
2	0.25%	600	601.8	Feasible	28.218	$ 1.97	2.28%	13.71%	-	-
3	10.00%	300	4.9	Optimal	27.476	$ 2.16	-	-	-	-
3	5.00%	300	4.8	Optimal	27.476	$ 2.16	0.00%	0.00%	26.168	$ 2.06
3	2.50%	300	5.1	Optimal	27.236	$ 2.12	0.87%	1.85%	26.572	$ 2.07
3	2.00%	300	5.3	Optimal	26.873	$ 2.15	2.19%	0.46%	26.346	$ 2.11
3	1.75%	300	5.6	Optimal	26.873	$ 2.15	2.19%	0.46%	26.411	$ 2.11
3	1.50%	300	5.8	Optimal	26.873	$ 2.15	2.19%	0.46%	26.476	$ 2.12
3	1.25%	300	5.5	Optimal	26.873	$ 2.15	2.19%	0.46%	26.541	$ 2.12
3	1.10%	300	5.5	Optimal	26.873	$ 2.15	2.19%	0.46%	26.581	$ 2.13
3	1.00%	300	5.5	Optimal	26.873	$ 2.15	2.19%	0.46%	26.607	$ 2.13
3	0.50%	300	301.5	Feasible	26.793	$ 2.07	2.49%	4.17%	-	-
3	0.50%	600	601.6	Feasible	26.791	$ 2.07	2.49%	4.17%	-	-
3	0.50%	600	601.6	Feasible	26.789	$ 2.06	2.50%	4.63%	-	-
3	0.25%	600	601.7	Feasible	26.790	$ 2.06	2.50%	4.63%	-	-
3	0.25%	600	601.7	Feasible	26.790	$ 2.07	2.50%	4.17%	-	-
3	0.25%	600	601.7	Feasible	26.788	$ 2.07	2.50%	4.17%	-	-
3	0.25%	600	601.8	Feasible	26.789	$ 2.06	2.50%	4.63%	-	-

Table A.2 Energy savings by building system and date for all CHEAPER solutions as compared to the baseline model results.

Date	Day / System	Energy Savings							Total Daily Savings	
		Heating - Low (kWh)	Heating - High (kWh)	Heating - AHU (kWh)	Cooling - Low (kWh)	Cooling - High (kWh)	Lighting (kWh)	Fan only (kWh)	Total Energy Savings (kWh)	Relative Energy Savings (%)
1/11/2021	Monday	27.91	-26.71	0.00	0.00	0.00	6.79	-0.02	7.97	7.0%
1/12/2021	Tuesday	17.40	-16.03	0.00	0.00	0.00	9.23	0.12	10.72	9.3%
1/13/2021	Wednesday	6.89	-8.01	3.20	0.00	0.00	10.75	0.43	13.26	12.9%
1/14/2021	Thursday	-16.68	13.35	6.40	0.00	0.00	9.94	0.81	13.83	13.4%
1/15/2021	Friday	40.24	-8.01	1.60	0.00	0.00	7.54	-0.48	40.89	32.2%
1/16/2021	Saturday	44.95	-68.37	36.80	0.00	0.00	7.21	0.67	21.25	14.1%
1/17/2021	Sunday	38.43	-35.79	0.00	0.00	0.00	-1.71	0.26	1.19	1.1%
3/9/2021	Monday	0.00	0.00	0.00	0.00	0.00	9.33	0.84	10.17	13.5%
3/10/2021	Tuesday	-4.71	4.81	0.00	0.00	0.00	9.69	0.84	10.62	13.2%
3/11/2021	Wednesday	-12.69	5.34	16.00	0.00	0.00	11.31	1.17	21.14	18.7%
3/12/2021	Thursday	-21.03	7.48	32.00	0.00	0.00	1.73	1.46	21.64	18.1%
3/13/2021	Friday	5.80	-5.34	0.00	0.00	0.00	8.10	0.19	8.75	8.5%
3/14/2021	Saturday	-3.99	4.27	0.00	0.00	0.00	6.65	1.58	8.51	11.6%
3/15/2021	Sunday	-9.43	11.22	0.00	0.00	0.00	3.81	2.01	7.61	11.9%
5/10/2021	Monday	5.44	0.00	0.00	2.06	2.72	9.81	-0.02	20.01	22.4%
5/11/2021	Tuesday	0.36	0.00	0.00	0.52	0.00	9.85	0.48	11.21	12.3%
5/12/2021	Wednesday	1.81	0.00	0.00	0.00	1.81	9.96	0.50	14.09	16.6%
5/13/2021	Thursday	9.43	0.00	0.00	2.06	7.71	10.88	-0.45	29.61	31.5%
5/14/2021	Friday	2.90	0.00	0.00	0.77	0.91	6.92	0.62	12.12	16.0%
5/15/2021	Saturday	2.54	0.00	0.00	1.03	1.36	3.75	1.65	10.33	16.4%
5/16/2021	Sunday	5.44	0.00	0.00	1.03	1.36	-0.06	1.08	8.84	10.9%
7/12/2021	Monday	1.09	0.00	0.00	3.09	-0.45	10.73	0.88	15.34	27.2%
7/13/2021	Tuesday	2.54	0.00	0.00	3.35	-0.45	10.50	1.24	17.18	29.0%
7/14/2021	Wednesday	1.09	0.00	0.00	2.32	0.00	8.13	1.41	12.94	22.3%
7/15/2021	Thursday	2.18	0.00	0.00	3.09	-0.91	8.46	0.98	13.80	21.3%
7/16/2021	Friday	2.54	0.00	0.00	4.64	-0.91	10.79	1.05	18.11	29.2%
7/17/2021	Saturday	0.36	0.00	0.00	1.80	-0.45	4.81	2.68	9.20	26.6%

Date	Day / System	Energy Savings							Total Daily Savings	
		Heating - Low (kWh)	Heating - High (kWh)	Heating - AHU (kWh)	Cooling - Low (kWh)	Cooling - High (kWh)	Lighting (kWh)	Fan only (kWh)	Total Energy Savings (kWh)	Relative Energy Savings (%)
7/18/2021	Sunday	7.98	0.00	0.00	9.01	-0.45	7.33	1.55	25.42	55.1%
9/16/2020	Monday	5.80	0.00	0.00	5.67	4.08	11.04	0.00	26.59	34.5%
9/17/2020	Tuesday	11.96	0.00	0.00	7.21	5.89	10.73	-0.57	35.22	42.8%
9/18/2020	Wednesday	3.63	0.00	0.00	1.03	3.17	9.56	0.88	18.28	29.5%
9/19/2020	Thursday	1.81	0.00	0.00	3.86	1.36	9.48	0.81	17.33	30.0%
9/20/2020	Friday	5.80	0.00	0.00	2.06	3.63	10.13	0.29	21.90	28.0%
9/21/2020	Saturday	1.45	0.00	0.00	0.77	0.45	4.42	1.87	8.96	15.3%
9/22/2020	Sunday	9.79	0.00	0.00	6.44	4.53	7.48	1.12	29.36	49.0%
11/9/2020	Monday	-1.81	2.14	0.00	0.00	0.00	8.69	1.20	10.21	17.5%
11/10/2020	Tuesday	29.36	-27.24	0.00	0.00	0.00	8.17	-0.24	10.05	9.1%
11/11/2020	Wednesday	2.90	-3.21	0.00	0.00	0.00	10.52	0.57	10.79	13.4%
11/12/2020	Thursday	-4.35	1.60	4.80	0.00	0.00	7.92	0.81	10.78	11.6%
11/13/2020	Friday	26.10	-42.73	1.60	0.00	0.00	25.06	1.22	11.25	9.6%
11/14/2020	Saturday	-5.08	5.34	0.00	0.00	0.00	6.85	2.18	9.30	16.2%
11/15/2020	Sunday	-3.99	4.81	0.00	0.00	0.00	-0.04	1.46	2.24	2.9%

Table A.3 CHEAPER and baseline model results for the period September 2020 through July 2021.

Model #	Date	Day	Start Time	CHEAPER Runtime* (s)	Optimized Daily Electricity Cost ($)	Baseline Daily Cost ($)	Daily Cost Savings ($)	Daily Cost Savings (%)	CHEAPER Solution**	Notes
1	1/11/2021	Monday	12:00 AM	4.2	4.59	5.04	0.45	9%	Optimal	All cooling systems OFF, All models
2	1/12/2021	Tuesday	12:00 AM	3.3	4.16	4.72	0.56	12%	Optimal	
3	1/13/2021	Wednesday	12:00 AM	6.2	3.37	3.96	0.59	15%	Optimal	
4	1/14/2021	Thursday	12:00 AM	3.8	3.67	4.38	0.71	16%	Optimal	
5	1/15/2021	Friday	12:00 AM	3.6	4.69	5.14	0.45	9%	Optimal	
6	1/16/2021	Saturday	12:00 AM	3.2	4.97	5.82	0.85	15%	Optimal	
7	1/17/2021	Sunday	12:00 AM	3.0	4.27	4.35	0.08	2%	Optimal	
Week 1	Average			3.9	4.25	4.77	0.53	11%		
8	3/9/2021	Monday	12:00 AM	11.4	2.65	3.34	0.69	21%	Optimal	All cooling systems OFF, All models
9	3/10/2021	Tuesday	12:00 AM	15	3.01	3.69	0.68	18%	Optimal	
10	3/11/2021	Wednesday	12:00 AM	3.4	3.70	4.73	1.03	22%	Optimal	
11	3/12/2021	Thursday	12:00 AM	2.8	3.70	4.57	0.87	19%	Optimal	
12	3/13/2021	Friday	12:00 AM	8	3.36	3.78	0.42	11%	Optimal	
13	3/14/2021	Saturday	12:00 AM	11.8	2.61	3.01	0.40	13%	Optimal	
14	3/15/2021	Sunday	12:00 AM	17.0	2.19	2.53	0.34	13%	Optimal	
Week 2	Average			9.9	3.03	3.66	0.63	17%		
15	5/10/2021	Monday	12:00 AM	13.8	2.95	3.94	0.99	25%	Optimal	Heating - High, Supp OFF, All models
16	5/11/2021	Tuesday	12:00 AM	4.8	3.08	3.73	0.65	17%	Optimal	
17	5/12/2021	Wednesday	12:00 AM	188.5	2.79	3.41	0.62	18%	Optimal	
18	5/13/2021	Thursday	12:00 AM	301.8	2.31	3.55	1.24	35%	Feasible	

Model #	Date	Day	Start Time	CHEAPER Runtime* (s)	Optimized Daily Electricity Cost ($)	Baseline Daily Cost ($)	Daily Cost Savings ($)	Daily Cost Savings (%)	CHEAPER Solution**	Notes
19	5/14/2021	Friday	12:00 AM	11.4	2.49	3.25	0.76	23%	Optimal	
20	5/15/2021	Saturday	12:00 AM	12.2	2.01	2.62	0.61	23%	Optimal	
21	5/16/2021	Sunday	12:00 AM	301.7	2.45	2.78	0.33	12%	Feasible	
Week 3	Average			119.2	2.58	3.33	0.74	22%		
22	7/12/2021	Monday	12:00 AM	6.7	2.01	2.94	0.93	32%	Optimal	Heating - High, Supp OFF on all models, Cooling - High OFF in Baseline to reduce frequent switching
23	7/13/2021	Tuesday	12:00 AM	3	2.11	3.09	0.98	32%	Optimal	
24	7/14/2021	Wednesday	12:00 AM	2.9	2.24	3.02	0.78	26%	Optimal	
25	7/15/2021	Thursday	12:00 AM	7.9	2.56	3.45	0.89	26%	Optimal	
26	7/16/2021	Friday	12:00 AM	8	2.21	3.21	1.00	31%	Optimal	
27	7/17/2021	Saturday	12:00 AM	302.3	1.06	1.45	0.39	27%	Feasible	
28	7/18/2021	Sunday	12:00 AM	302.3	0.89	1.98	1.09	55%	Feasible	
Week 4	Average			90.4	1.87	2.73	0.87	33%		
29	9/14/2020	Monday	12:00 AM	137.3	2.06	3.24	1.18	36%	Optimal	Heating - High, Supp OFF, All models
30	9/15/2020	Tuesday	12:00 AM	5.4	1.88	3.07	1.19	39%	Optimal	
31	9/16/2020	Wednesday	12:00 AM	3.3	1.71	2.63	0.92	35%	Optimal	
32	9/17/2020	Thursday	12:00 AM	3.7	1.66	2.46	0.80	33%	Optimal	
33	9/18/2020	Friday	12:00 AM	14	2.36	3.36	1.00	30%	Optimal	
34	9/19/2020	Saturday	12:00 AM	10.8	1.67	2.23	0.56	25%	Optimal	
35	9/20/2020	Sunday	12:00 AM	7.9	1.04	2.18	1.14	52%	Optimal	
Week 5	Average			26.1	1.77	2.74	0.97	36%		
36	11/9/2020	Monday	12:00 AM	18.7	1.79	2.22	0.43	19%	Optimal	

Model #	Date	Day	Start Time	CHEAPER Runtime* (s)	Optimized Daily Electricity Cost ($)	Baseline Daily Cost ($)	Daily Cost Savings ($)	Daily Cost Savings (%)	CHEAPER Solution**	Notes
37	11/10/2020	Tuesday	12:00 AM	5.7	3.91	4.52	0.61	13%	Optimal	All cooling systems OFF, All models
38	11/11/2020	Wednesday	12:00 AM	8.6	2.65	3.28	0.63	19%	Optimal	
39	11/12/2020	Thursday	12:00 AM	14.9	3.49	3.76	0.27	7%	Optimal	
40	11/13/2020	Friday	12:00 AM	6.1	4.30	4.89	0.59	12%	Optimal	
41	11/14/2020	Saturday	12:00 AM	7.9	1.85	2.30	0.45	20%	Optimal	
42	11/15/2020	Sunday	12:00 AM	44.5	2.75	2.91	0.56	5%	Optimal	
Week 6	Average			15.2	2.96	3.41	0.51	14%		

* Maximum 300 seconds allowed, **Optimality gap of up to 1.5 percent allowed