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A REVIEW OF RESEARCH AND LITERATURE RELATED TO VERBAL PROBLEM SOLVING AND APPLICATIONS TO IMPROVEMENT OF STUDENT ABILITIES

By

MARION K. TOMMER

B.S.E., Kansas State Teachers College of Emporia, 1969

A MASTER'S REPORT

submitted in partial fulfillment of the

requirements for the degree

MASTER OF SCIENCE

College of Education

KANSAS STATE UNIVERSITY
Manhattan, Kansas

1972

Approved by:

Jackson A. Byars
Major Professor

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Chapter 1

INTRODUCTION

A major objective of any mathematics teacher today is developing the ability of students to solve verbal problems. There are numerous situations in the classroom in which the problem solving ability is necessary, and as the student leaves school he will be faced with opportunities to use this ability.

A verbal, or word, problem may be defined as a written question raised for inquiry, consideration, discussion, decision or solution. In each verbal problem a situation is described that involves a quantitative question for which the individual has no ready answer.

Developing good problem solving techniques should be a major part of a good mathematics curriculum for several reasons:

1. Learning to solve textbook word problems is important in the study of mathematics because they give the student an exposure to problem situations well beyond those possible through real life experience. At the same time the student is learning to deal with problems in a form he will encounter throughout his education.

2. Verbal problems provide practice in computational skill in a more interesting manner than a list of exercises.

3. Competence in living and working in an age with so great an emphasis on applied science is dependent upon a person's understanding and skill in using problem solving methods in personal, social, and

economic situations.

4. The verbal problem may demonstrate the practical part of mathematics and thus motivate a student to learn the skills necessary for the course he is taking and to retain these skills for further use.

The effectiveness of any education program must be judged by its success in developing the student's ability to function as a responsible citizen. This is also true of the mathematics program, it can be effective only if it develops the ability of the student to solve problems he will encounter in life activities.

The purpose of this report was to examine research studies and literature written concerning verbal problem solving and to relate them to improvement of student abilities.

Three areas of research and literature were examined: (1) student problem solving abilities--examining the relationship between a student's success in problem solving and other characteristics of his thinking and personality, (2) problem content--how the content of the problem is related to a student's ability to solve it, and (3) problem solving processes--studies of the processes used by students to solve verbal problems and how these processes can be improved.

After a review of the research and literature, student difficulties in verbal problem solving were examined to see how they might be corrected using the implications of the research and literature.

Chapter 2

A REVIEW OF RELATED RESEARCH AND LITERATURE

Many research studies have been conducted concerning the ability of students to solve verbal problems. In this report only those studies concerned with problem solving ability, problem content, and problem solving processes were examined. Much research found on the same topic did not agree as some articles completely refuted what another had concluded. Also, many experimenters found no statistical difference between their control and experimental groups. The sampling of research cited in this report is given for its direct application to the mathematics student having difficulty in verbal problem solving.

STUDENT PROBLEM SOLVING ABILITIES

Many factors make up a student's ability in solving verbal problems. The understanding of these factors and how they relate to problem solving can lead a teacher in finding the best method to help students improve their problem solving abilities.

Stevens

In a study conducted by B. A. Stevens[1] the relationships of ability in silent reading, power in the fundamental operations of arithmetic, power in solving reasoning problems in arithmetic, and

[1] B. A. Stevens, "Problem Solving in Arithmetic," Journal of Educational Research, 25:253-260, April/May, 1932.

"general intelligence" test scores were compared. A comparison of scores was made from a wide sampling of students in the fourth, fifth, sixth, and seventh grades in the different areas given above. The study used the correlation method in the comparison of the scores. Each score for a particular ability was obtained by a standardized test given to the pupils.

The experiment results showed that ability in fundamental operations in arithmetic was more closely correlated with ability in problem solving than was general reading ability. Test of problem analysis seemed to have higher correlation with the test of problem solving than did tests of general reading ability or of fundamental operations. Although in this test this correlation between problem analysis and problem solving did exist, other studies seem to indicate that special training in problem analysis does not result in higher achievement for normal pupils in problem solving. Stevens felt that this study and others suggest that a large proportion of our pupils may be solving reasoning problems in arithmetic by means of type solutions rather than by means of vital reasoning processes.

Tate and Stanier

A study by Merle W. Tate and Barbra Stanier was concerned with the kinds of errors made by good and poor problem solvers in judgements as measured by critical thinking and practical-judgement tests.[2] This study used data from a previous study in which 117 good problem solvers

[2]Merle W. Tate and Barbra Stanier, "Errors in Judgement of Good and Poor Problem Solvers," Journal of Experimental Education, 32:371-376, Summer, 1964.

and 117 poor problem solvers were selected. The median IQ of the good problem solvers was 114.8; that of the poor problem solvers was 113.6. The median chronological age of the good problem solvers was 156 months; that of the poor problem solvers was 154 months.

The test to examine the errors in critical thinking consisted of fourteen paragraphs dealing with science and social studies topics. Each paragraph was followed by a number of statements, 84 statements in all. The subjects were to mark each statement as true, probably true, not enough facts, probably false, and false on the basis of the information contained in the paragraph. They were to use the response not enough facts if there was not enough information in the paragraph to determine the truth or falsity of the statement.

As the authors expected, the test was relatively hard. The mean score of good problem solvers was 38.2 on the 84 items. The mean score of poor problem solvers was 29.9. The difference was significant, the t ratio being 9.01 with 209 degrees of freedom.

Test results showed that, in general, both groups checked as true or probably true statements frequently heard or commonly believed as true, even if there was no evidence to support the statement presented in the paragraph. Results of the test also showed that poor problem solvers tend to judge statements as true or false and avoid the other alternatives more frequently than the good problem solver. The good problem solvers seem to make better use of the information they are given.

The differences in responses of the good and poor problem solvers can perhaps be explained in terms of tempermental rather than cognitive differences. The good problem solver responded with not enough facts when he did not know the answer rather than the extreme true or false. On the other hand, the poor problem solver would reject not enough facts and respond with the true or false because of impulsiveness or intolerance of ambiguities.

The errors in practical judgement test consisted of 36 questions of the following type:

1. Why is a traffic officer superior to a traffic light?
   _He can help people cross the street.
   _He can think.
   _He can control traffic.
   _He can arrest a driver who does something wrong.

7. Why are skyscrapers built in large cities?
   _To save space.
   _To beautify the city.
   _To draw tourists.
   _Because land is expensive.

Subjects were to write 1 before the answer they considered the best and 2 before the next best answer. In scoring, the answers were ranked from 1 (best) to 4 (poorest) on a key. The differences between a subject's number and the key's number were summed and subtracted from 100. The mean score of the good problem solvers was 55.3 and the poor problem solvers was 45.0.

In addition to practical information and evaluation of past experience, success on the practical judgement test seemed to require the ability to delay a response until all of the answers had been read and impartially analyzed. It seemed probable that the differences between the errors of good and poor problem solvers were due to differences in susceptability to affective answers. The poor problem solvers preformed better on 11 of the 36 questions; actually only 18 questions having answers involving beliefs or feelings of the students elicited definitely better performance from the good problem solvers.

Alexander

Vincent E. Alexander analyzed characteristic differences between high and low achievers in problem solving.[3] For the study, pairs of high and low achievers in seventh grade problem solving were matched according to sex, IQ, and mental age in months. These pairs were compared with respect to certain factors and abilities. The measure of problem solving ability was determined by the pupil's score on a standardized test. Only the general conclusions for this study were given. All differences were statistically significant at or above the 5% level of confidence.

The results of the study are given in the following outline:

- A. Characteristics of high achievers
  - 1. Specific mental abilities
    - a. General reasoning ability
    - b. Ability to understand verbal concepts
  - 2. Quantitative skills
    - a. Understanding mathematical terms and concepts
    - b. Skill in computation
  - 3. General reading skills
    - a. Comprehension of reading material
    - b. Understanding words in context
  - 4. Problem solving reading skills
    - a. Comprehension of statements in problems
    - b. Selection of relevant details in problems
    - c. Selection of correct procedures to solve problems
  - 5. Interpretation of quantitative materials
    - a. Finding data from graphs, tables, charts, and maps
    - b. Perception of relationships involving comparison of data
    - c. Recognition of limitations of given data
- B. No significant difference
  - 1. Specific mental abilities
    - a. Ability to use words easily
    - b. Ability to visualize objects in two or three dimension
  - 2. Socio-economic status
  - 3. Quantitative skills
    - a. Timed addition of whole numbers
- C. Characteristics of low achievers

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[3]Vincent E. Alexander, "Seventh Graders Ability to Solve Problems," School Science and Mathematics, 60:603-606, November, 1960.

1. Interpretation of data errors
   a. Tendency to require more information than necessary to judge data
   b. Going beyond the data given (finding an answer even when there is not sufficient information)
   c. Inaccuracy due to carelessness, reading difficulties, or inability to see relationships

From this study, Alexander set up guides for the teacher in planning instruction in problem solving:

A differentiated program--Instruction must be adapted to the needs and abilities of students. Some of the assigned problems should be of low enough difficulty to be solved by all of the students. There should also be some problems difficult enough to challenge the high achiever. "Diagnosis of difficulties and activities to improve problem solving skills, instructional materials, learning experiences, and goals to be achieved should be selected and organized to meet individual variations in problem solving ability."

Selection of printed materials--Materials are to be selected so that the vocabulary and language structure are appropriate to the reading level of the pupils who will be using them.

Improvement of general reading ability--General reading can be improved through specific reading instruction.

Development of reading skills related to problem solving--The pupils are to have instruction in using the meaning of items and statements in verbal problems as clues to computation processes. The pupils need to have an understanding of basic and enriched meanings of mathematical terms and concepts.

Development of mathematical concepts--The pupils need to learn the relationships between quantities and processes.

Skill in fundamental operations--The pupils need the ability to employ processes with understanding.

Interpretation of quantitative materials--Pupils should be taught to interpret and visualize facts and relationships in charts, tables, graphs, and maps. They should be able to compare data, recognize limitations of given data, and discriminate between relevant and irrelevant data.

Kennedy, Eliot, and Krulee

A test was developed by George Kennedy, John Eliot, and Gilbert Krulee to find the error patterns in the problem solving processes of students. Their experiment had two purposes: (1) to determine if students differ with respect to their solution patterns for algebraic word problems; and (2) to determine how students use the information given to them in problem statements.[4]

Kennedy, Eliot, and Krulee gave the following five steps which students use in solving word problems:

1. The student reads the problem and forms a rough hypothesis.

2. The student looks for information requiring translating into mathematical symbols.

3. The student decides what type of relationships are needed to form an appropriate equation.

4. The student acertains whether he has identified the physical or logical inferences needed to solve the problem.

5. The student is ready to solve the equation and obtain a

[4]George Kennedy, John Eliot, and Gilbert Krulee, "Error Patterns in Problem Solving Formulations," Psychology in the School, 7:98-99, January, 1970.

solution.

The first purpose of the study, to determine if students differ with respect to their solution patterns for algebraic word problems, was divided into two parts: (a) that the difference between less and more able students is a function of their ability to recognize the relationships needed to form an appropriate equation, and (b) that the difference between less and more able students is a function of their ability to add or to identify the logical or physical inferences needed to solve the problem.

The subjects of the study consisted of 28 high school juniors from Evanston Township High School in Evanston, Illinois. All subjects were from the same teacher's average and advanced mathematics classes. The subjects were selected in order to have an equal number of average and advanced students and an equal number of boys and girls. All backgrounds of the students indicated that the majority were from families with professional backgrounds. All subjects took the College Entrance Examination and plan to go to college. Most of the students were planning to enter a professional field. The subjects reflected the upper to middle class families.

The test given consisted of the following six problems:

1. $\frac{3y - 4}{8} = \frac{4y + 8}{4}$, $y = ?$

2. A man is three times as old as his son. Eleven years from now he will be only twice as old as his son. How old is the son at present?

3. $B(X - B) = X - (2 - B)$, $X = ?$

4. Mary can wash the dishes in a half hour, and Tracy can wash them in 25 minutes. After Mary has worked for 10 minutes, Tracy begins to help her. How long will it take both girls to finish the dishes?

5. $A - X^2 = 4X - 21$
   $-X - 56 + 9X = 4X$
   $X = ?$

6. An automobile radiator contains 4 gallons of a mixture of water and antifreeze. If the mixture in now is 20% antifreeze, how much of the mixture must be drawn off and replaced by pure antifreeze to get a mixture containing 30% antifreeze?

Each student was taken alone into a soundproof recording room, and he was told that they were interested in the way in which people go about solving algebra problems. He was asked to solve the six problems, putting all his work on paper and saying anything which came to mind no matter how trivial. He was reminded that the interest was in how he worked the problem, not his answer.

After the tests were scored, it was found that the numerical problems offered little difficulty for any subjects. The word problems were more difficult for the less able student. Of problems 4 and 6, there were 15 of 28 correct solutions of honor students but only 5 of 28 for average students.

The first part of the first hypothesis (difference is a function of ability to recognize the relationships needed to form an appropriate equation) was not supported because both groups recognized the relationships needed for equations equally well. However, the second part (difference is a function of ability to add or to identify the logical or physical inferences needed to solve the problem) was supported. When comparing the cummulative frequency for each step of the problem solutions, the greatest discrepancy occured at the step requiring students to identify logical or physical inferences in the problem statement.

The hypothesis that less able students use facts in the order that they appeared in the problem was partially supported as only some of the less able students did formulate word problems sequentially as facts appeared.

The results of the study indicate that "teachers should be less concerned with teaching students to define the relationships between problem elements and more concerned with helping them to identify the logical and physical assumptions made in the problem statement." Although the sample size of this study was too small to be reliable, the results can be used to help some students that are having difficulty solving verbal problems.

The preceding research studies were all on topics concerned with some phase of the relation of a student's problem solving ability to the different factors which make up his ability in problem solving. These studies can be very valuable if they are used to correct the difficulties of the less able student in problem solving.

PROBLEM CONTENT

An examination of problem solving must include research studies concerning the content of the problem itself. This would include the vocabulary of the problem, the relation of the problem to the experiences of the student, and the preferences of the student as to the type of problem he likes to solve. The research given was to determine the relation between a problem's content and the student's ability to solve the problem.

Lyda and Church

A study was done by Wesley J. Lyda and Ruby Summers Church to determine the potency of direct, practical mathematical experiences, as distinguished from those activities encountered only in the classroom, as a factor in solving realistic verbal "reasoning" problems in arithmetic.[5]

The subjects for the experiment were Mrs. Church's fifth grade class of a Negro school in Fort Valley, Georgia. Students were divided into three groups: average, above average, and below average. The average students had an IQ of 90 to 109 on two Group Intelligence Tests, Arithmetical Computation Grade Equivalent of 5.0 to 5.5, and Reading Grade Equivalent of 5.0 to 5.5. The above average students had an IQ of 110 and above, Arithmetical Computation Grade Equivalent of 5.6 and above, and Reading Grade Equivalent of 5.6 and above. The below average students had an IQ of 70 to 89, Arithmetical Computation Grade Equivalent of 3.0 to 5.0, and Reading Grade Equivalent of 3.0 to 5.0. There were 16 average students, 5 above average students, and 9 below average students.

This study was designed in terms of three basic assumptions:

1. "Intelligence, arithmetic computation, reading, and direct, practical experiences in arithmetic, as distinguished from classroom activities, are presumably conditioning factors in success in solving realistic verbal 'reasoning' problems."

2. "Most of the pupils in schools are average in intelligence."

3. "Textbooks are to be written primarily for the average student."

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[5]Wesley J. Lyda and Ruby Summers Church, "Direct Practical Arithmetical Experiences and Success in Solving Realistic Verbal 'Reasoning' Problems in Arithmetic," Journal of Educational Research, 57:530-533, July-August, 1964.

To select the realistic verbal "reasoning" problems to be used in the experiment, the authors' used Brueckner and Grossnickle's Standards for Evaluating Problems and considered verbal "reasoning" problems in each of 11 units of the Row-Peterson text. The researchers selected a random sample of 150 of the most realistic verbal "reasoning" problems. The problems were then submitted to a panel consisting of a college instructor in mathematics who was very interested in education, an instructional supervisor, and a fifth grade teacher. The panel also used Brueckner and Grossnickle's Standards for Evaluating Problems and rated the problems as most realistic, realistic, and least realistic. For a problem to be included in the test it had to be rated most realistic by two of the three judges. The 25 most realistic verbal "reasoning" problems were selected in this way.

An arithmetical experience check list was designed to determine pupil experience based upon and parallel to the situations involved in the group of 25 problems. The pupils were to tell if they had had an experience often--three times or more, seldom--once or twice, or never--not at all. Some examples of the types of experiences are:

1. Selling tickets for a school play
2. Buying milk in the classroom or cafeteria
3. Planning a trip to the museum

The 25 problems were divided into series of five tests with five problems in each test. These tests were given daily for five days; a maximum time of thirty minutes was allowed for each test. Some examples of the problems that relate to the above experiences are:

1. The fifth grade girls sold 235 tickets for their play. The boys sold 309. How many more tickets did the boys sell than the girls? How many tickets did they sell together?

2. The pupils in our room paid $1.68 for milk today. If the pupils pay 8¢ for each bottle of milk, how many bottles of milk did they buy?

3. The 75 fifth grade pupils are planning a trip to the museum. It will cost $18.75 to hire a bus to take them. If each pupil pays an equal share, how much should each one pay?

In analyzing the percentage of average, above average, and below average pupils who had certain arithmetical experiences, the experimenters analyzed the data in terms of experiences in which 75% or more of the pupils indicated that they had a given experience often, seldom, or never. Table 1 shows the percent of arithmetical experiences for each group that were marked often, seldom, or never. For example, the table shows that 75% of the above average students marked 40% of the experiences often, 12% of the experiences seldom, and 12% of the experiences never.

Table 1

Arithmetical Experience

Group	Percent marked often	Percent marked seldom	Percent marked never
Above average	40	12	12
Average	28	16	12
Below average	20	12	16

Table 2 shows the data analyzed on the incidence of arithmetical experiences and success in problem solving. Again the data is for 75% or more of the pupils which marked a given experience as often, seldom, or never. The table shows the number of problems marked often, seldom, or never and the percent of students that worked those problems

Table 2

Arithmetical Experiences and Success
in Solving the Problems

Group	Problem experiences marked often		Problem experiences marked seldom		Problem experiences marked never	
	Number	Percent of students working problems correctly from those marked	Number	Percent of students working problems correctly from those marked	Number	Percent of students working problems correctly from those marked
Above average	10	90% - 7 80% - 1 75% - 3	3	75% - 3	3	100% - 1 75% - 1 60% - 1
Average	7	90% - 5 79% - 1 75% - 1	4	100% - 4	3	87% - 2 50% - 1
Below average	5	100% - 3 88% - 1 78% - 1	3	100% - 1 0% - 2	4	43% - 1 0% - 3

correctly.

In terms of the number of problems solved satisfactorily, it was found that 90% or more of the average group worked 10 or 40% of the problems correctly, 90% or more of the above average group worked 9 or 36% of the problems correctly, and 90% or more of the below average group worked 5 or 20% of the problems correctly.

In terms of the pupils of the study, the authors based the following conclusions on uniformities that occured throughout the study:

1. All of the students had certain arithmetical experiences in common.

2. Regardless of the group, there were pupils who had never had certain arithmetical experiences.

3. Direct, practical arithmetical experience seems to be a more potent conditioning factor in success in solving realistic verbal "reasoning" problems for those pupils in the average and below average groups than for those pupils in the above average group.

4. The probability of working satisfactorily realistic verbal "reasoning" problems in which the three groups had not had the corresponding direct, practical arithmetical experiences was greater for the average group than for the below average group; likewise, it was greater for the above average than for the average and below average groups respectively.

5. The below average's difficulties are greatly increased when they have not had the direct, practical arithmetical experiences corresponding to realistic verbal "reasoning" problems.

Bowman

A study was done by Herbert Lloyd Bowman to determine the relation of general intelligence to types of problems best liked and most successfully performed by pupils of the junior high school level.[6] The test used consisted of 50 problems arranged in two forms of 25 problems each. The test was created to measure the preferences of a student and his performance. The problems constructed consisted of five types: (1) problems based upon adult life activities, (2) problems based upon child life activities, (3) problems whose nature are to be found in the field of science, (4) problems so stated so as to take on the nature of a puzzle, and (5) problems of pure computation only, in which the directions for each procedure were given. There were five problems on each page with one of each type and each problem on a page was of the same difficulty. The student was to solve the five problems of a group and then indicate the one of a group that he liked best.

Three groups were used for the study. Group I of 100 students was made up of the upper quartile of IQ. Group II of 203 students consisted of the middle 50% of IQ. Group III of 110 students was of the lower quartile of IQ. A comparison of percentages of preference and performance of all three groups were made and the following conclusions were reached:

1. There is less variation in both preference and performance by pupils of higher intelligence with respect to types of problems than with lower intelligence.

2. Pupils of lower intelligence prefer problems involving few

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[6] Herbert Lloyd Bowman, "Reported Preference and Performance in Problem Solving According to Intelligence Groups," Journal of Educational Research, 25:295-299, April-May, 1932.

or no complex situations and they perform better on these problems.

3. Pupils of lower intelligence show little preference or ability on problems of a science or puzzle type. This would suggest that such problems should be constructed on a lower ability level when presented to pupils of this kind.

4. Pupils of high ability in problem solving and in intelligence show a comparatively high degree of preference and performance with respect to problems of scientific type. For this level of student, more problems of this type might be used to maintain interest.

5. Problems dealing with child life activities seem to be highly preferred and successfully performed by all pupils of junior high school grades. If arithmetic problems through their solutions offer to children means of meeting real needs they become important factors in the education of the child.

6. Building interest in problem solving is best accomplished by having children work problems which are not too difficult and which represent genuine childhood situations. This is primarily true for those students who are of average or below average ability.

Travers

Kenneth Travers studied the nature of preferences for problem solving situations--under what circumstances they exist and how they relate to problem solving success.[7] He did this study because he thought it might provide much needed information concerning the nature of pupils'

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[7]Kenneth Travers, "A Test of Pupil Preference for Problem Solving Situations in Junior High School Mathematics," The Journal of Experimental Education, 35:9-18, Summer, 1967.

thought processes as they attempt to solve mathematical problems.

The Test of Choice Behavior in Number Situations was devised in an attempt to identify pupil preferences for problem solving situations. The test involved three situations: (1) mechanical-scientific, (2) social-economic, and (3) abstract. The test was designed to yield a preference score from highest to lowest for each type of situation; a no preference score was possible. Fifteen pairs of word problems based on 5 topics in junior high school mathematics were used. Each pair of problems consisted of two of the three problem situations and all possible pairings of the three situations were made for each of the five topics. The pair of problems involved the same numerical combinations, used similar style of phrasing, employed the same verbal clues, and had comparative levels of difficulty of vocabulary. Each pair of problems had the same answer.

The student was to read each problem and then cross out the one he did not want to work. He then worked the remaining problem giving his solution. His test was scored to determine his preference; his choices were rated 1, 2, and 3 with 1 being his preference. If he had no preference, each situation was given a 2.

The test was also scored on the basis of number of correctly answered problems. A problem was correct if the correct numerical answer was given, or if it was evident from the pupils work that the appropriate mathematical operation had been applied to the right numbers. After the test was scored, two ratios were formed. One ratio was the number of correct answers of the first preference to the number attempted. The other ratio was the number of correct responses of the other two situations to the number attempted.