

ADVERTISING IN INDUSTRIAL SYSTEMS-
AN INDUSTRIAL DYNAMICS APPROACH

by

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CHAPTER 1

INTRODUCTION

This report is a study of Industrial Dynamics, and its application to some advertising practices. Industrial dynamics is the study of the time-varying behavior of systems within the framework of their feedback-loop structures. It was invented on the belief that the behavior of systems (physical, social, biological, and so on) can be investigated through the study of the cause-and-effect relationships between their components. The principles of feedback control, which are extensively used in engineering systems, might as well be applied to social, economic, and biological systems. Much, if not all, of the behavior of such complicated systems could be explained using the principles of feedback theory.

Chapter 2 of this report is a review of the principles of industrial dynamics. The four processes that make the foundation for industrial dynamics, i.e., theory of information-feedback systems, decision-making processes, simulation techniques, and availability of low-cost high-speed computers, are described. The last part of the chapter is devoted to defining the rules and guidelines that help to identify and represent the feedback characteristics of such systems, and finally build their dynamic models.

Since its invention during the period 1956-61 at Massachusetts Institute of Technology, industrial dynamics has found both pros and cons. Some (mostly the followers of J. W. Forrester, who has done the pioneering works on industrial dynamics) have praised it as a theory of structure of systems. On the contrary, some have considered industrial dynamics merely as a new

simulation technique, specifically because of the DYNAMO-compiler, which is designed to handle the industrial dynamics models. In Chapter 3, a review of the literature on industrial dynamics, both for and against, is presented.

Chapter 4 is an attempt to apply the industrial dynamics principles to a simple industrial system, and try to trace the behavior of the system through its feedback structure. The organization of the system, which deals with production and distribution is defined, the cause-and-effect interactions between system components are investigated, and the dynamic model of the system is presented. The model is later used to study the time-varying behavior of the system, and explore the possible ways of improvement. It should be pointed out that the content of Chapter 4 is essentially based on the analysis given in Chapters 2 and 15 of [13].

The second objective of this report is the application of industrial dynamics principles to some advertising models. The first part of Chapter 5, is the description of a simple advertising model proposed by Forrester [13]. The model assumes a direct relationship between advertising budget and production level at factory, at every moment of time. The effect of such an advertising practice on the production-distribution of Chapter 4 is investigated.

In the second part of Chapter 5, three other advertising models are introduced. In recent years, Operations Research has found some applications in the field of marketing. A study by Vidale and Wolfe [49,50], based on extensive experimentations, has proposed a mathematical model of sales response to advertising; however, it does not investigate the behavior of the system under the proposed model, which is of more interest as far as this report is concerned. It is basically a study of the

possible effects of advertising on retail sales. Here, the model is used, within the context of industrial dynamics, to investigate the behavior of the whole production-distribution system, and not only the retail sales, under three different types of advertising practices:

- 1 - Protracted advertising campaigns;
- 2 - Short-time, intensive advertising campaigns;
- 3 - Impulse-type advertising campaigns.

The last chapter offers a comparison between the results of the system simulation under three advertising models of Chapter 5. In doing so, certain criteria have been taken into consideration, such as total sales generated as a result of advertising, variations and fluctuations in the system parameters such as production rate, inventories, and so on, tendency of the system to fluctuate, etc.

Use of industrial dynamics models is closely associated with DYNAMO-compiler. This special-purpose language provides a fast and reliable means of simulation, although it is not the only one.

Throughout this report, DYNAMO is used to simulate the system behavior.

CHAPTER 2

GENERAL CONCEPTS OF INDUSTRIAL DYNAMICS

2.1. INTRODUCTION

"Industrial Dynamics" is the study of the time-varying behavior of the systems, with a view at their information feedback characteristics. As such, it proposes that the character of any organization (system) is determined by the dynamic interactions among its interconnected feedback networks (loops).

Forrester [13] defines "industrial dynamics" as:

"..... The study of the information-feedback characteristics of industrial activity to show how organizational structure, amplification in policies, and time delays (in decisions and actions) interact to influence the success of the enterprise. It treats the interactions between the flows of information, money, orders, materials, personnel, and capital equipment in a company, an industry, or a national economy.

Industrial dynamics provides a single framework for integrating the functional areas of management - marketing, production, accounting, research and development, and capital investment. It is a quantitative and experimental approach for relating organizational structure and corporate policy to industrial growth and stability."

As was pointed out, industrial dynamics focuses on the behavior of systems and their components within the framework of their information feedback characteristics. Therefore it is primarily based on the theory of information-feedback system; however, there are three other foundations

for industrial dynamics. They are:

- Decision-making Processes;
- Simulation Techniques;
- Availability of Low-cost Digital Computers.

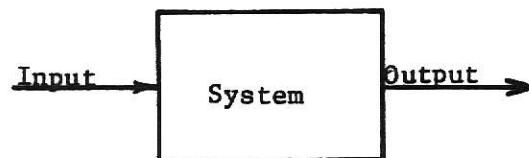
2.2. THEORY OF INFORMATION-FEEDBACK SYSTEMS

The concept of servomechanisms (information-feedback systems) was evolved during and after World War II.

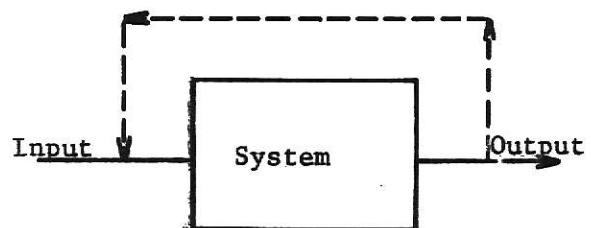
A system, as used here, is identified by a grouping of parts (physical as well as people) which function together for a common goal. This is a broad definition which encompasses almost any organization (physical, social, economic, etc.) with which we are familiar.

The first step toward understanding the concepts and principles of systems would require an orderly structure whereby the information about the system could be organized. This structure would serve as a basis for unifying the diversities of the physiological, industrial, and economic systems, just as the laws of physics provide a structure for today's technology [1,13].

Systems are classified as either "open" or "closed, or feedback" systems. In an open system, the output is affected by the input, but has no effect upon the input. Another words, the future action is not controlled by the past action (Fig. 2.1a). On the other hand, in a closed or feedback system, the information about the output of the system is brought back to control and maintain the future action. This implies a closed-loop structure as shown in Fig. 2.1b [4].



(a) Open System



(b) Closed System

Fig. 2.1. Open and Closed Systems

Definition of Feedback Control System. American Institute for Electrical Engineers (AIEE) defines the feedback control systems as follows [37]: "A feedback control system is a control system which tends to maintain a prescribed relationship of one system variable to another by comparing functions of these variables and using the difference as means of control."

Hammond [28] defines such systems in the following way: "A feedback system comprises one or more distinguishable elements which react on each other in a predetermined manner and are arranged so that a closed ring or loop of dependencies is formed."

Kuo's definition of feedback control systems is as follows [32]: "Systems comprising one or more feedback loops which compare the controlled signal c with the command signal r ; the difference ($e = r - c$) is used to drive c into correspondence with r ."

Forrester, within the context of industrial dynamics, states that [13]: "An information-feedback system exists whenever the environment leads to a decision that results in action which affects the environment and thereby influences future decision."

The structure of an information feedback loop is illustrated in Fig. 2.2.

There are two classes of feedback systems:

- 1 - Negative feedback systems, where a goal is sought, and the response of the system is in such a direction as to eliminate any discrepancy between the system state and the goal.
- 2 - Positive feedback systems, in which the result of the action generates still greater action i.e., the system generates a growth process.

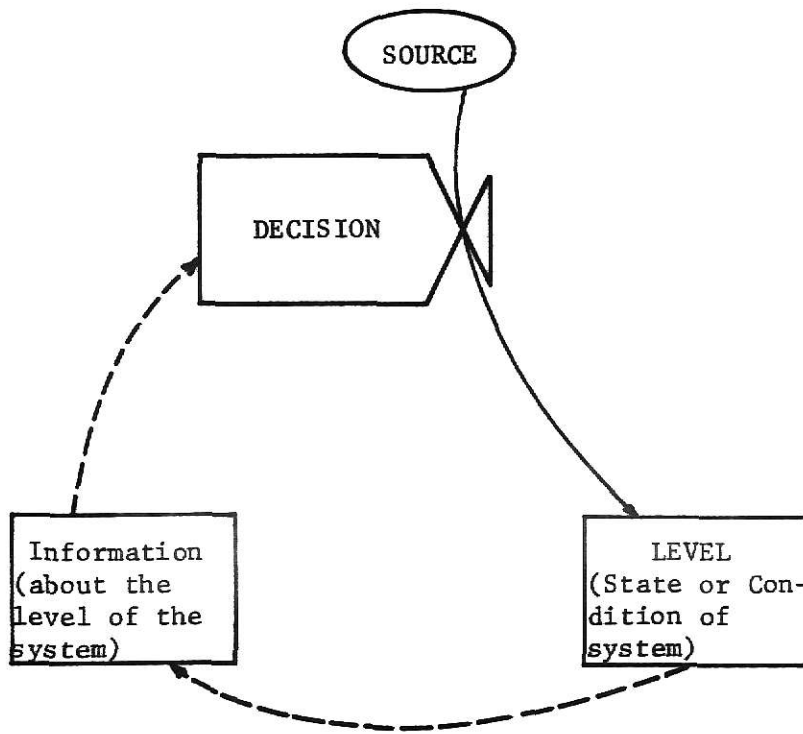


Fig. 2.2. Structure of a Feedback Loop

The concept of information-feedback systems, whether positive or negative, provides a basis for understanding and interpreting the behavior of social systems. The feedback theory has been successfully applied to mechanical and electrical systems over the past decades, but due to the complexity of the social organizations, it has been only during the last decade that the principles of feedback systems have been developed far enough to be applied to social systems as well.

In the study of information-feedback system, there are three important characteristics which determine the behavior of such systems - structure, delays, and amplifications. Structure of a system determines how the components are related to one another. Delays always exist in the transfer of information, in decision-making processes, and in the actions that follow the decisions. Amplifications manifest themselves whenever an action is more forceful than what is thought to be, considering the information that is the basis for decision-making. There are other concepts in the theory of feedback systems which are equally applied in social systems, such as noise spectrum, bandwidth, natural frequencies, filter distortion, gain, and transient response [11].

These general concepts provide the essential framework within which the behavior of social systems is analyzed. It is along this line that an organization can operate effectively if the control system structure of the organization is recognized, and the dynamic interactions between its components are understood [35].

2.3. DECISION-MAKING PROCESSES

The second foundation for industrial dynamics is the theory of decision-making, which evolved during the 1950's out of the process of automatizing

military tactical operations [13]. This was due to the necessity for formalizing the decision-making policy, which, in turn, was a result of the quicker pace of warfare, to which human organizations no longer could respond. Consequently it became more and more necessary to set the formal rules for decision-making, and divert from "tactical judgement and experience."

"A decade of time and thousands of people were involved in this interpretation of military decision processes and automatizing the general policies that are the basis for tactical military decision making. It has been amply demonstrated that carefully selected formal rules can lead to short-term tactical decisions, that excel those made by human judgement under the pressure of time, or with men having insufficient experience and practice, or in the rigidity of large organizations." [13]

The same rules apply to social systems as well. The complexity of these systems is now growing beyond the ability of human decision-making processes, and therefore attention is shifted toward making use of formal rules of decision-making in industrial and economic systems. The management of such systems is increasingly concerned with setting up the formal rules governing the decisions, and interpreting the effects of such policies on the systems.

2.4. SIMULATION TECHNIQUES VS. ANALYTICAL APPROACHES

Simulation techniques have long been known, and used. An example is the navigation tables which were used in the past.

Simulation is an experimental approach to representing the behavior of systems, whenever the mathematical analysis is not powerful enough to obtain an analytical solution, and wherever the cost is not a limiting factor.

In general, it is the process of substituting an experiment or a model for a system, because that experiment or model is easier to study and experiment with, than the system itself. Today, however, simulation techniques make use of mathematical models, used on a computer, to represent a physical or social system [11].

A mathematical model is a set of precise equations describing how the system varies, and how the conditions of system at one time could be used to determine the state of the system at a brief time later. But it cannot tell how to determine the condition of the system in some distant future, directly and without going through the step-by-step process of computing the intermediate states of the system. Whereas the analytical solution of the system determines its state at any time; however, it should be noted that the present social and industrial systems are of such degrees of complexity that the present-day mathematics is not adequate to find analytical solutions to these systems, nor is it powerful enough to develop, in some instances, the set of differential equations describing the behavior of the systems. Simulation techniques come to help whenever such difficulties arise.

In the beginning, the use of simulation was limited by the time and cost involved, and it was not until the invention of high-speed computers that these techniques were vastly applied to industrial as well as business systems. Now the time and cost are not limiting factors any more, and with

the invention of new special-purpose simulation languages, the use of this methods becomes more and more easy. However, identifying the operational aspects of a system, determining the assumptions, making the model, and finally analyzing the results of the simulation, need a great deal of skill and expertise.

This, of course, is part of the responsibilities of today's management.

2.5. LOW-COST COMPUTERS

Electronic digital computers provide the fourth foundation for industrial dynamics. Rapid development in this field was accomplished during 1950's, and technical performance of such computers constantly has increased, by a factor of 10 per year, over the same decade [13]. The progress has not stopped, however.

The low-cost, high-speed computers have opened new possibilities that are far beyond the current applications; computers are mostly used in the role of a system component, and very seldom as tools for the design of industrial and social systems. As active system components, they are widely used for data processing and information handling, but even in these cases, major areas of application for formal data processing are still open; probably 98% of the information flows that are important in determining the characteristics of today's social systems, lie outside of the formal data processing channels; some of these flows are among the most crucials in affecting the behavior of the system [11].

On the whole, the advent of computers:

".....was a technological change greater than that effected in going from chemical to atomic explosives. Society cannot absorb so big a change in a mere ten years. We have a tremendous untapped

backlog of potential devices and applications. It is now to be expected that machine progress will stay ahead of conceptual progress in industrial and economic dynamics. Computing machines are now so widely available, and the cost of computation and machine programming is so low relative to other costs, that the former difficulties in activating a simulation model need no longer determine our rate of progress in understanding system dynamics." [13]

2.6. STRUCTURE OF SYSTEMS

The importance of an orderly structure as a first step toward understanding the behavior of systems was already pointed out. Here the essentials of such structure are outlined.

A structure is necessary in order to interpret the observations. Without a structure any observation may, at first, seem meaningless, but if it can be determined which category it falls in, it can be identified and interpreted in a proper manner.

The structure of a system should be determined within a closed boundary; within a closed boundary, the feedback loops, i.e., the basic elements of system structure, are identified; levels and rates are the essential variables within a loop; and within the rate variable, goal, apparent condition, discrepancy, and action are the important constituents (Fig. 2.3) [21].

Closed Boundary. In formulating the structure of a system, the boundary of the system should be recognized; it would contain the smallest number of components within which the dynamic behavior of the system is to be observed.

The concept of closed boundary is important, because the interactions within the system, as viewed by industrial dynamics, are the causes of

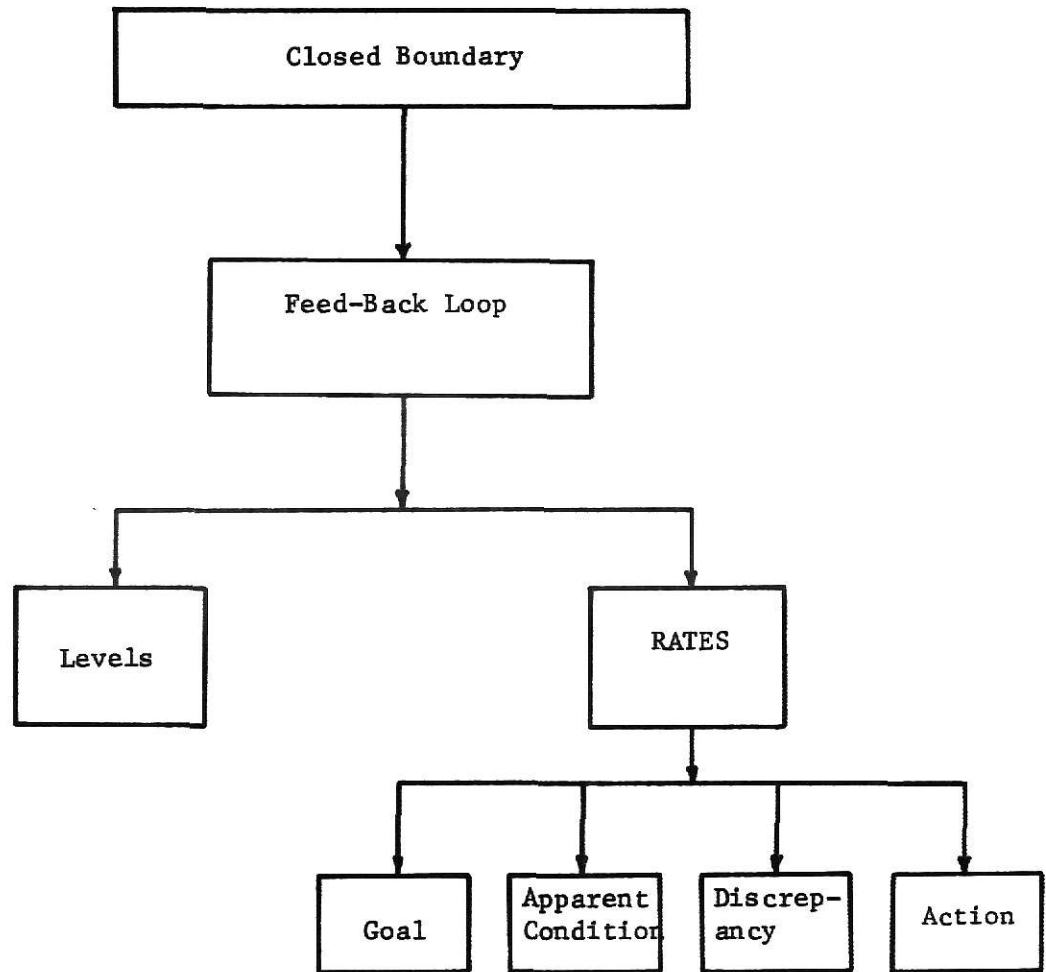


Fig. 2.3. Hierarchy of System Structure

dynamic behavior. The effect of exogenous variables on the system is of no interest at this stage. Therefore the closed boundary of the system should include in it all the interactions that are essential to the behavior of the system.

Feedback Loops. Feedback loop is the basic structural element within the closed boundary (Fig. 2.2). The feedback loop connects the decision to action, to state (or level) of the system, to information about the level, and back to decision process.

A system usually consists of a number of loops, and the interactions between the loops affects the dynamic behavior of the system.

The term "decision", as used in the loop, applies to a broader definition of the term than the simple human decision. It may be purely a human decision, or the one made by mechanical or electrical devices. Whatever the nature of such decision, it always takes place in a feedback loop.

Levels and Rates. A feedback loop, at a lower level of hierarchy, has its own substructure. There are two fundamental components within a feedback loop, i.e., the level variables and the rate variables.

Levels describe the state of the system at any particular time. They are accumulations of the results of actions. Inventories, number of orders on hand, amounts of information, bank balances, and goods in transit are just a few examples of level variables. Levels can represent "information" variables as well as "physical" ones; they may also represent such intangible and vague variables such as awareness of a product among people, feelings, satisfaction, reputation of a company. Mathematically speaking, level equations are equivalent to the process of integration.

Rate variables, on the other hand, determine the slope of the level variables, that is, they determine how the levels change with time. They are the activities, the decision functions of a system, and may represent such activities as movement of goods, sales, expenditure of money, hiring rate of people, and so on.

Because the rates act only by affecting the levels, they cannot interact directly. This also implies that the rates are dependent on levels, and constants, and not on any other rates.

Accordingly, the values of levels depend on their previous values, and the inflow and outflow rates. They do not depend on previous values of other levels.

Confusion may arise in distinguishing between levels and rates, especially in cases where they are both measured with the same units. An appropriate way of checking this is to imagine that all activity in the system is brought to rest; the levels will still exist, whereas the rates will stop flowing.

Goals, Apparent Condition, Discrepancy, and Action. Although the rate and level variables are the substructures of a feedback loop, it is possible to look for sub-substructures within these substructures. However, the structure of a level variable is straightforward, that is, the new value of a level variable is obtained only by adding the change in the level to its previous value; therefore, it is not useful to breakdown the structure of the level equations.

Rate equation, on the other hand, has a different structure and meaning; as was stated earlier, a rate equation is a decision function,

or a policy statement. As such, it tells how the available information is utilized to make the decision. The action follows the decision, but if there is any delay between decision and action, it would involve the presence of intermediate levels in the model. The rate equation is an algebraic equation, and free of delays and time-dependent changes. Such changes are only created by level equations. [21]

Within a policy statement (rate equation) these components are essential:

- a. Goal;
- b. Observed state of the system at any time;
- c. Discrepancy between the observed state of the system, and the goal;
- d. Action, based on the discrepancy.

Figure 2.4, shows the relation between these four components. More precisely, the above mentioned relation can be modified as in Fig. 2.5.

The rate equation sets a goal, makes a comparison between the apparent system condition (which is not necessarily the true system condition) and the goal, detects any possible discrepancy, and uses the information about the discrepancy to guide the next action.

In positive-feedback loop, "goal" is not the same as in a negative loop. In the latter case, goal is that state of the system toward which the policy is aimed, while in the former, goal is that state from which the system departs, and the discrepancy between apparent system condition and the goal will lead to further increase in discrepancy.

2.7. EQUATIONS, COMPUTATION, AND FLOW DIAGRAMS

In simulating the behavior of systems by using mathematical models, there must be a set of conventions, in the form of equations, in order to

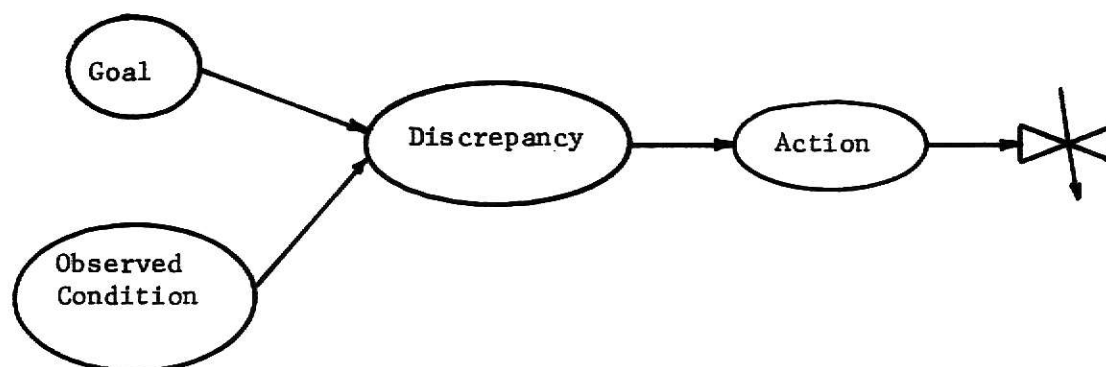


Fig. 2.4. Components of a Rate Equation

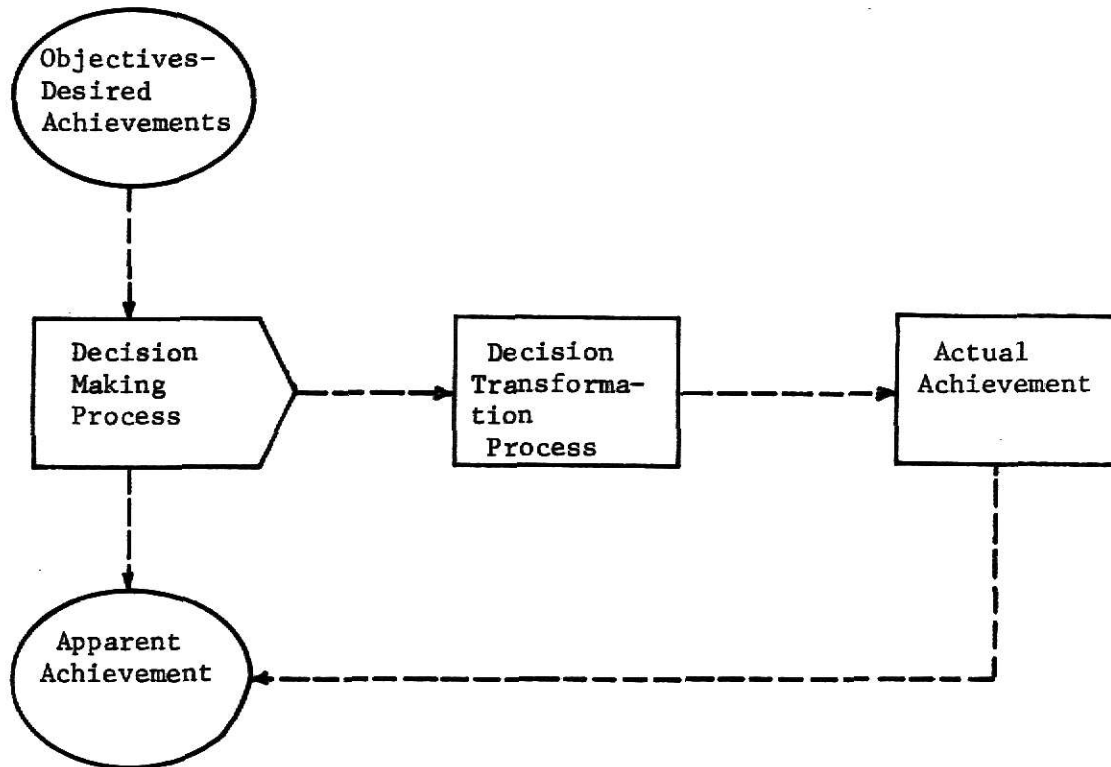


Fig. 2.5. Control System Structure

convey the structure of the model. These conventions should be capable of describing the situation, concepts, interactions, and decision processes in the system [13].

Simulation is essentially a step-by-step computation based on the system of equations describing the situation. This implies the need for a computing sequence, and the related symbols representing the model. In industrial dynamics simulation models we encounter two basic types of equations: level and rate equations; besides, there are other types of equations such as auxiliary and supplementary equations; constants and initial values are other parts of a model. Finally, the time step, i.e., the time interval between two consequent computations is of importance.

Equations, although necessary to describe the system, would be more meaningful if accompanied by some pictorial representation of the model, which shows the different kinds of movements among the system components. From here, arises the need for flow diagrams and related symbols.

Computing Sequence. Computing sequence determines how the computation of the system equations proceeds through time. The convention used in industrial dynamics is as shown in Fig. 2.6. Current time (present) is designated by 'K', past by 'J', and future by 'L'. 'J' is one DT behind 'K', and 'L' one DT ahead. 'DT' is the time step, as defined above, or the solution interval.

Starting at time 'K', the levels at time 'J', and the rates, existed over the time interval 'JK' are available from previous computations; now the levels at time 'K' could be determined because they are only dependent on their previous values, at time 'J', and the rates over the time interval 'JK'. After the levels are computed at time 'K', the rates that would

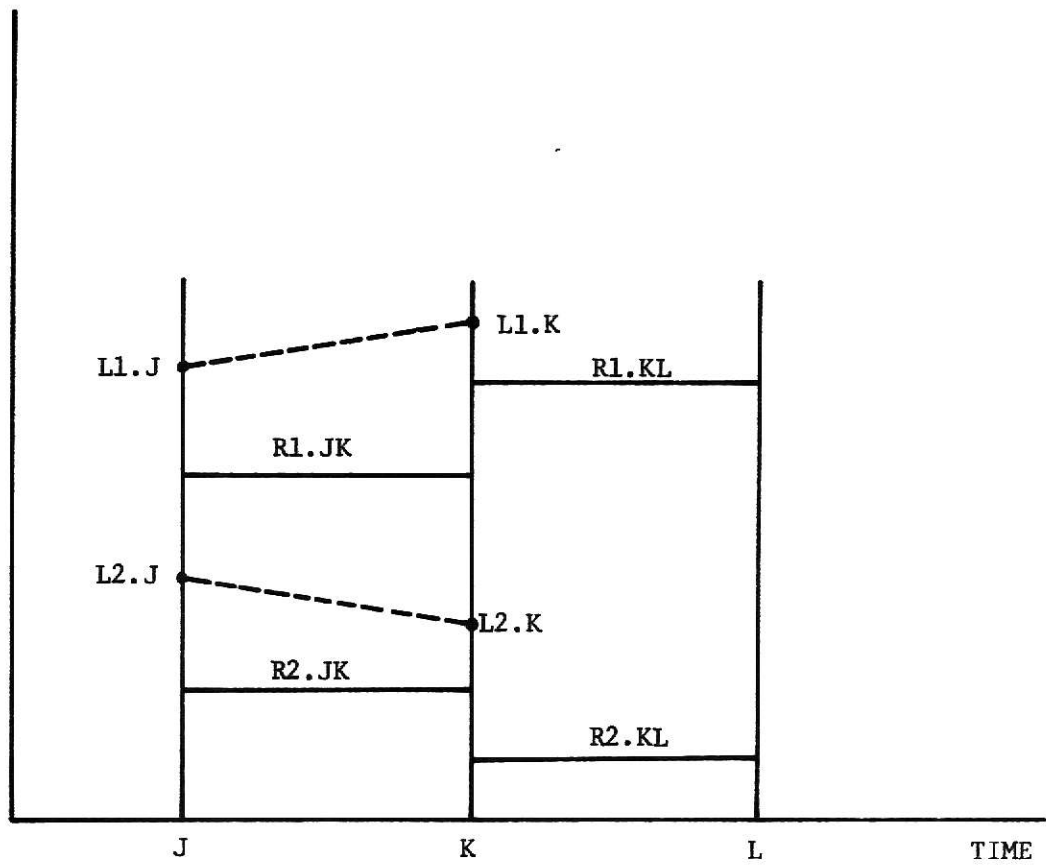


Fig. 2.6. Computation Sequence

exist over time interval 'KL' can be determined. The state of the system is now as shown in Fig. 2.6.

The sequence of computations now repeats for the next point in time; this will be done by advancing the time indicators, J, K, and L, one DT ahead, as in Fig. 2.7. The new 'J' is the old 'K', the new 'K' is the old 'L', and new 'L' would be one DT ahead of new 'K'.

The computations are exactly repeated in the same way. The previous rates over time interval 'KL' are now the rates over the time interval 'JK'.

Symbols and Time Notations in Equations. Variables and constants of the model should be represented by some symbols. Moreover, time notations, should specify the moment of time at which a variable applies. The time notations should also be specified such that they could be represented on computer printers. The following conventions^(*) have arbitrarily been adopted in industrial dynamics:

- A symbol representing a variable or constant will consist of at most six characters, the first of which is always alphabetic.
- Level variables always carry the single letter 'J' or 'K', separated from the variable name by a period, indicating their values at that time. Examples:

A.J	EMPLOY.J
LEVEL.K	INVEN.K
LE57.K	CASH.J

- Rates exist over a time interval, therefore the corresponding symbols carry two-letter time notations such as 'JK' or 'KL', separated from the variable name by a period; examples are:

*These are in accordance with the specifications of DYNAMO compiler, to be discussed in Appendix I.

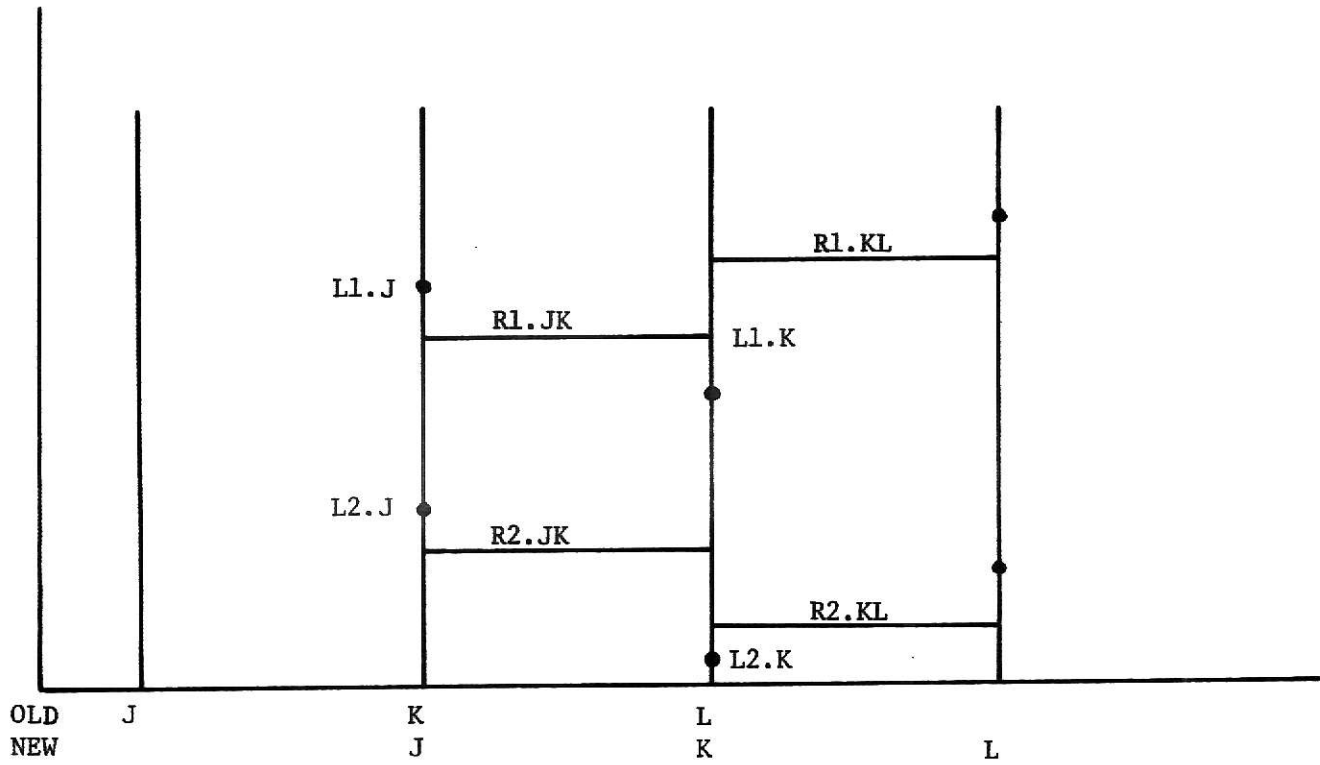


Fig. 2.7. Computation Sequence

RATE1.JK	B.KL
RATE2.KL	EMHR.JK
FLOW.JK	FEED.KL

- Constants carry no time notations, as their values are not changed with time. Examples:

CONST	MASH
ABCD	CASH
JEF	SALE

Level and Rate Equations. Levels represent reservoirs whose contents are varying by inflow and outflow rates. The value of a level depends on its previous value, and the change in its content during the time interval. The general form of a level equation would be:

$$\text{LEVEL.K} = \text{LEVEL.J} + (\text{DT})(\text{IR.JK} - \text{OR.JK}) \quad 2-1, \text{L}$$

LEVEL.K = Present value of the level, designated by 'LEVEL'; (units)

LEVEL.J = Value of level from previous time; (units)

DT = Solution interval between 'J' and 'K'; (time units)

IR = Incoming rate to level 'LEVEL'; (units/time)

IR.JK = Value of incoming rate over the time interval 'JK';

OR = Outgoing rate from level 'LEVEL'; (units/time)

OR.JK = Value of outgoing rate over the time interval 'JK';

(2-1,L) represents a number assigned to the equation for reference, (it can be any number), and 'L' represents the type of equation, i.e., a Level equation.

A level equation is equivalent to the process of integration. Another words, the above equation can be written in integral form, as:

$$\text{LEVEL} = \text{LEVEL}_0 + \int_0^t (\text{IR}-\text{OR})dt \quad 2-2$$

in which LEVEL_0 is the initial value of the level variable LEVEL. The time interval 'dt' corresponds to 'DT' in previous equation. 'DT', therefore, is only appeared in level equations and not in any other type.

Rate equations represent the flow rates between levels of system, therefore the input to such equations are only levels and constants. The general form of a rate equation is:

$$\text{RATE.KL} = f(\text{Levels \& Constants}) \quad 2-3,R$$

$$\text{RATE.KL} = \text{Value of rate 'RATE' over the time interval KL; (units/time)}$$

$$f = \text{any function, or relationship, of levels and constants}$$

Equation is assigned a number (2-3), and its type is designed by letter 'R'.

Auxiliary and Supplementary Equations. Often, the actual formulation of a rate equation may become complicated, and it would be more clear if the equation could be written in terms of its algebraic subdivisions, or parts. These subdivisions or parts are called "auxiliary equations." The following example will show the concept of auxiliary equations [13]:

$$\text{SSR.KL} = \frac{\text{UOR.K}}{\text{DFR.K}} \quad 2-4,R$$

$$\text{DFR.K} = \text{DHR} + \text{DUR} \frac{\text{IDR.K}}{\text{IAR.K}} \quad 2-5,A$$

$$\text{IDR.K} = (\text{AIR})(\text{RSR.K}) \quad 2-6,A$$

In this example, the value of the rate 'SSR' depends on the values of two levels, 'UOR' and 'DFR'. The value of 'DFR', however, depends on the constants 'DHR' and 'DUR', and levels 'IDR' and 'IAR'. The value of 'IDR' itself depends on the values of constant 'AIR' and level 'RSR'.

Writing equation (2-4,R) in this way, helps clarify its structure. It does not alter anything in the rate equation; it still depends on levels and constants, as can be seen by substituting (2-6,A) into (2-5,A) and the result into (2-4,R), to obtain:

$$\text{SSR.K} = \frac{\text{UOR.K}}{\text{DHR} + \text{DUR} \frac{(\text{AIR})(\text{RSR.K})}{\text{IAR.K}}} \quad 2-7,R$$

which shows the dependence of 'SSR' on constants and levels.

The appearance of letter 'A' indicates the type of the equation, i.e., "Auxiliary."

Supplementary equations define variables which are not actual variables in the system, but they contain information which is desired to be printed or plotted. As such, these variables are only used in printing or plotting instruction.

Supplementary equations will be denoted by letter 'S'.

Initial-Value Equations and Constants. All levels should be given initial values before the start of the simulation process. These initial values for levels are necessary to determine the flow rates over the first time interval (0 - DT). Initial-value equations would be used to serve this purpose.

The initial-value equations are also used to determine the values of some constants from other constants. These equations are designated by letter 'N'.