THE EFFECT OF MODERN MATHEMATICS UPON ACHIEVEMENT IN ARITHMETIC COMPUTATION

by

THOMAS DALE HAWK

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Approved by:

Herbert E. Kaiser
Major Professor

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CHAPTER I

INTRODUCTION

Except, perhaps, for the reading controversy of several years ago that started from "Why Johnny Can't Read", nothing else in curriculum seems to have aroused so much concern as the new mathematics.[1] After passing through the stage of its awesome impact on curriculum planners, teachers, students, and parents, modern mathematics has emerged as a target of criticism concerning its value in the school curriculum.

Current magazine and newspaper articles have aroused public sentiment about the new programs and have even made sport of the puzzling new math. A humorous example of this attention is seen in the following playful, double parody of reading and new math written by Canadian mathematician R. A. Stall as it appeared in Newsweek: "Oh, see, Johnny has a set of marbles. See Johnny's set. Look, look, Billy has a set of marbles. See Billy's set. Here comes Mary. Mary gets all the marbles. Mary gets the union of Johnny's set and Billy's set."[2]

The effectiveness of the modern mathematics program compared to the traditional mathematics program had been a

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[1]Francis J. Mueller, "The Public Image of New Mathematics," The Mathematics Teacher, LIX (November, 1966), 618.

[2]"New Math--Does It Really Add Up?" Newsweek, LXV (May 10, 1965), 112.

recent concern of the teachers, counselors, and administrators at Manhattan Junior High School. The 1968-69 seventh-grade class was the first to have a complete modern mathematics program for everyone in the class with the exception of a few extremely slow students who took a remedial mathematics course which emphasized computational drill. Results on recent achievement tests indicated that seventh grade students were low in achievement skills involving arithmetic computation, arithmetic concepts, and arithmetic applications.[1]

Statement of the Hypothesis and Objectives

It was suspected that the modern mathematics program at Manhattan Junior High has been an influential factor in the low achievement in basic arithmetic skills as measured by the Stanford Achievement Test. It was the objective of this study to form an indication of the effect of modern mathematics on student achievement and to relate the other basic skills in other subject areas as measured by the Stanford Achievement Test to mathematical achievement. It was also the objective of this study to compare general intelligence scores to mathematical achievement.

Definition of Terms

Because there may be some ambiguity or confusion over

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[1]Mrs. Jo Dodge, Manhattan Junior High School Counselor, U.S.D. 383, Manhattan, Kansas.

a few terms, the following interpretations were used:

Modern mathematics or new mathematics refers to the recent approach to mathematics which develops the inherent structure through a study of the systems of numbers as developed from naive set theory. Special characteristics of modern mathematics include an emphasis on precision of definitions, terminology and notations along with a formation and foundation of mathematical concepts.[1]

Traditional mathematics refers to those older programs which concentrate on the method of solving a developed hierarchy of increasingly difficult problems. Generalizations are in the form of laws, axioms, and postulates which the student memorizes.[2]

Basic arithmetic skills are those skills of manipulation and computation with numbers requiring use of the operations of addition, subtraction, multiplication, division and the more complicated use of these skills as measured by the Stanford Achievement Test in computation, concepts, and

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[1]Kenneth E. Brown, "The Drive to Improve School Mathematics," The Revolution in Mathematics Education. A Report of Regional Orientation Conferences in Mathematics (Washington: National Council of Teachers of Mathematics, 1961), pp. 22-7, cited by William Harper Landis, "Secondary Students' Mathematical Competencies in Relation to Employment Tests," (Doctoral Dissertation, University of Southern California, 1967), p. 8, ERIC 016 784.

[2]Ibid.

applications.

CHAPTER II

REVIEW OF THE LITERATURE

Much of the literature pertinent to the topic of modern mathematics deals with its comparison to the traditional mathematics and its effect upon achievement. Another large portion of the current literature is critical of the modern programs and points out the shortcomings of the new mathematics. Before discussion of this pertinent literature, however, it is important to get a recent historical background of the modern mathematics.

Historical Background of Modern Mathematics

In the early fifties, many educators and mathematicians became aware of the deplorable state of the mathematical curriculum in the schools. Emphasis for several decades had been on pedagogy and the various psychological theories of learning with little importance given to content. In addition to this, society was rather indifferent and placed little importance upon mathematics.[1] Hancock studied the evolution of secondary mathematics curriculum and concluded that the aims of mathematics instruction seemed to change to meet the demands of society. During times when society has no pressing

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[1] Ryoichiro Sato, "Commentary on the International Study of Achievement in Mathematics," The Arithmetic Teacher, XV (February, 1968), 103-7.

need for mathematics, utilitarian aims were stressed; during periods of severe depression, the cultural aims of mathematics were emphasized; however, during the time when the need for mathematical instruction was readily apparent, attention to aims has diminished and there has been a tendency to assume that whatever mathematics could be taught was justifiable.[1]

The race for space supremacy between the United States and Russia and the increased awareness of technology indicated the need for mathematics instruction. Several special groups were organized to study the mathematics curriculum and make recommendations to improve the curriculum. Some of the more influential groups were the University of Illinois Committee on School Mathematics (UICSM), the Commission on Mathematics of the College Entrance Examination Board (CEEB), and the School Mathematics Study Group (SMSG) which was organized by the American Mathematical Society, the Mathematical Association, and the National Council of Teachers of Mathematics. These groups have aided in the planning and writing of new mathematics textbooks and programs and began the implementation of the present modern mathematics.[2]

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[1] John David Hancock, "The Evolution of the Secondary Mathematics Curriculum: A Critique," (Doctor's Thesis, Stanford University, 1961), Dissertation Abstracts 22:501-502, No. 2, 1961.

[2] Roger K. Meyer, "Modern Mathematics in the Secondary School," (unpublished Master's Report, Kansas State University, 1962), pp. 7-9.

Research of Achievement in Modern and Traditional Programs

Achievement in mathematics has been difficult to determine because of inappropriate or inadequate measuring devices to assess mathematical achievement. Most investigators used tests that were developed earlier even though the new mathematics produced changes in content and objectives. Problems in measurement will continue to exist until the behavioral changes which the new mathematics will produce are described.[1]

Much of the research at this stage of modern mathematics evaluation seems to be contradictory. Some researchers showed the modern program to be superior as Cassels and Jerman did with their experimental groups at the seventh-grade level in student achievement.[2] However, Shuff found just the opposite to be true with the traditional group scoring higher on achievement in his study.[3] Other studies by Flournoy[4]

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[1] Thomas A. Romberg, "Current Research in Mathematics Education," Review of Educational Research, 39 (August, 1969), 473-91.

[2] Russell Cassels and Mas. Jerman, "A Preliminary Evaluation of SMSG Instruction in Arithmetic and Algebra for 7th, 8th, and 9th Grade Pupils," California Journal of Educational Research, (November, 1963), 202-07.

[3] Robert V. Shuff, "A Comparative Study of Achievement in Mathematics at the Seventh and Eighth Grade Levels Under Two Approaches: SMSG and Traditional," (unpublished Doctoral Dissertation, Univ. of Minn., 1962), cited by Holland Payne, "What About Modern Programs in Mathematics?" The Mathematics Teacher, LVIII (May, 1965), 423.

[4] Francis Flournoy, "Understanding Relationships: An Essential for Solving Equations," Elementary School Journal, (January, 1964), 214-17.

and Peck[1] indicate elementary school children studying a modern program of mathematics achieve at least as well on traditionally oriented arithmetic tests as their traditional counterparts. McLaunchlin[2] found that elementary school children who had studied a modern program of arithmetic out-scored their traditional contemporaries on tests featuring traditional mathematics. Hungerman[3] and Graft and Ruddel[4] found similar results among a sample of sixth graders.

Many reasons for the gains in achievement can possibly be attributed to the Hawthorne effect rather than conclusively to the innovation of a modern program. Sparks found other factors which might significantly affect achievement. In comparing the achievement gains of schools taking the Iowa Test of Basic Skills, he found that the schools having the greatest gains required more mathematics study of their students, had students who spent more out-of-class time on

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[1]Hugh I. Peck, "An Evaluation of Topics in Modern Mathematics," The Arithmetic Teacher, X (May, 1963), 277-79.

[2]J. A. McLauchlin, "Can Johnny Still Add?" The Arithmetic Teacher, IX (December, 1962), 432.

[3]Ann D. Hungerman, "Achievement and Attitude of Sixth-Grade Pupils in Conventional and Contemporary Mathematics Programs," The Arithmetic Teacher, 14 (January, 1967), 30-39.

[4]William D. Graft and Arden K. Ruddel, "Cognitive Outcomes of the SMSG Mathematics Program in Grades 4, 5, and 6," The Arithmetic Teacher, 15 (March, 1968), 161-65.

math study, had teachers who were better prepared and had longer tenures, and had greater mutual respect and enthusiasm among students and teachers.[1]

Criticism of the Modern Mathematics Program in the Schools

As more people have become affected by modern mathematics and more people have had an opportunity to examine its content and goals, criticism has fallen heavily. Areas most frequently criticized are those involving the utility of much of the content and the student's ability and readiness to learn that content, the problems in teaching the modern mathematics, and finally the objectives and goals of the current modern mathematics programs.

A danger in the trend of modern mathematics, as viewed by Alfors and 64 other mathematicians, is that the mathematicians who are making the new curriculum are reacting to the previous dominance of mathematical education by professional educators who had stressed pedagogy at the expense of content. This reaction is now leading to the stressing of content at the expense of pedagogy and will be equally ineffective.[2] Kline echoed the same opinion as he denounced

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[1] Jack Norman Sparks, "A Comparison of Iowa High Schools Ranking High and Low in Mathematical Achievement," Dissertation Abstracts 21:1481-82, No. 6, 1960.

[2] Lars V. Ahlfors and Others, "On the Mathematics Curriculum of the High School," The Mathematics Teacher, LV (March, 1962), 191-95.

the role scientists have played in curriculum development:

> The curriculums have been taken over by professional scientists whose aim, judged by the curriculum they have produced, is to train professionals. These reformers assume that mathematics and science are ends in themselves, that students are automatically motivated, and that the goal is to rush the education so that 17-year-olds can start writing research papers . . . .
> The professors who have led the new curriculum movements have not even been wise men. Because they are the products of the narrow specialization which is characteristic of modern science education, their ignorance of the cultural significance of science may be excusable. But these men have shown a presumption and an egotism which is almost unbelievable. Most of them had never set foot in a high school or elementary classroom and had even disdained any interest in education. When they did decide to take an interest in curriculum they assumed that education is a simple, obvious matter. Of course the professional scientists have made a fiasco of reform.[1]

Before further criticism is leveled, an examination of some of the new content in modern mathematics might be valuable. Examples of some of the topics in modern mathematics is exemplified in the following: Modern Algebra, Linear Algebra, Point-Set Topology, Algebraic Topology, Finite Mathematical Systems, and Set Theory. These topics are an exceedingly abstract, logical, axiomatic, well-structured system of mathematics.[2] These topics are not

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[1]Morris Kline, "The Liberal Education Values of Mathematics, Science, and Technology for Youth," Addresses and Proceedings (Washington D.C.: National Education Association, 1965), pp. 65-66, cited by Herbert Smith, Curriculum Development and Instructional Materials," Review of Educational Research, 39 (October, 1969), 513.

[2]Howard F. Fehr, "Sense and Nonsense in a Modern School Mathematics Program," The Arithmetic Teacher, 13 (February, 1966), 84.

taught per se in the secondary or elementary schools, but some of their basic content has crept into the curriculum as new mathematics.

One potent topic of the newly discovered content of modern mathematics that is influencing school curriculum is set theory. In a recent article Geddes and Lipsey explained that, while the concept of set promises clarification, simplification, and unification in the teaching of mathematics, the use of sets did present some hazards. Some of these hazards were: (1) forcing the student to take the term of set as undefined because of the complexity of set theory, (2) confusing the student by the use of apparently contradictory terms such as "a set is undefined and is a well-defined collection of objects," (3) confusing students on the concept of addition with the similar idea of union, and (4) leading the student to illogical conclusions and confusion because some of set theory's assumptions are based upon paradoxical situations or lead to paradoxical conclusions.[1]

Fehr has examined parts of the new mathematics curriculum and concluded that many of its advanced notions are complete nonsense for elementary school mathematics. Some

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[1]Dorothy Geddes and Sally I. Lipsey, "The Hazards of Sets," The Mathematics Teacher, LXII (October, 1969), 454-59.

particular areas of nonsense mentioned by Fehr include the early introduction of sets through the use of letters and brace notation. While children should be learning to write numbers, they are struggling to make so-called curly-cue braces. Even though the recognition of collections of things is essential, the learning of the theory of sets is non-essential in the learning of school mathematics.[1]

A further area of nonsense as seen by Fehr was the introduction of formal logic into elementary school mathematics. Individuals need to experience and understand number, number relations, and geometrical figures a great deal before they see the need for reasoning about the acquired knowledge and sense how it may be arranged in formal structures. In education our emphasis is on understanding, use, and skill, not on abstract patterns. All past and present research on human learning indicates that until the human mind has acquired a vast reservoir of experimental knowledge and has matured to a mental age of ten or eleven years, the ability to do two-way reflective thinking is absent, and it is impossible to understand formal logic. With this in mind, Venn diagrams in set relations and Euler's circles in logical classifications is sheer nonsense before a mental

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[1]Fehr, op. cit., p. 84-85.

age of eleven years.[1]

Some of Bruner's thinking on the nature of human learning pointed in the same direction as Fehr has mentioned. When the child is in the concrete operational stage that Piaget described between the ages of six and ten years of mental age, he is able to give structure to the things he encounters, but he is not yet readily able to deal with possibilities not directly before him or not already experienced. It is futile to attempt to pass the child from his concrete thinking to more adequate modes of thought by presenting formal explanations based on a logic that is distant from the child's manner of thinking and sterile in its implications for him.[2]

Modern mathematics, in its use of symbolism, has attempted to pass the child into a more formal mode of thought. However, Bruner was aware that even though secondary school children could operate at this symbolic level alone, there was danger in simply instructing and learning at this level. This danger becomes readily apparent when, in the learning or problem solving, the symbolic method fails the person. At this time, he needs to be able to function

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[1] Ibid.

[2] Jerome S. Bruner, The Process of Education (Cambridge: Harvard University Press, 1961), pp. 37-38.

in a manipulative fashion.[1]

Further specific criticism of the modern mathematics content was also leveled by Fehr. He felt that the teaching of place systems of numeration to other bases than the decimal, and the computational algorisms in these other bases is nonsense. He argued that across the nation and the world in science and business, social circles and professions, the one number system that is used is the decimal system and this is the system that 95 per cent of the population will use for the rest of their lives, probably everyday. Granted, other systems are used in digital computers and special scientific studies, but to educate elementary school children as if everyone would become a computer programmer is not even logical. That learning other bases will help a student understand the base ten is a good hypothesis, but it has never been tested. Fehr concluded his argument on bases by pointing out that if generalizations on notations of number systems is deferred to high school, the use of algebra can make them a simple and easy matter to comprehend.[2]

As modern mathematics was introducing a new set of

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[1]Jerome S. Bruner, Towards a Theory of Instruction (Cambridge: Harvard University Press, 1966), cited by Thomas E. Kieren, "Activity Learning," Review of Educational Research, 39 (October, 1969), 513.

[2]Fehr, op. cit., p. 84.