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ON THE DSM VERSION OF NEWTON'S METHOD

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Abstract. The DSM (dynamical systems method) version of the Newton's method is for solving operator equation $F(u) = f$ in Banach spaces is discussed. If F is a global homeomorphism of a Banach space X onto X, that is continuously Fréchet differentiable, and the DSM version of the Newton's method is $\dot{u} = -[F'(u)]^{-1}(F(u) - f)$, $u(0) = u_0$, then it is proved that $u(t)$ exists for all $t \geq 0$ and is unique, that there exists $u(\infty) := \lim_{t\to\infty} u(t)$, and that $F(u(\infty)) = f$. These results are obtained for an arbitrary initial approximation u_0. This means that convergence of the DSM version of the Newton's method is global. The proof is simple, short, and is based on a new idea. If F is not a global homeomorphism, then a similar result is obtained for u_0 sufficiently close to y, where $F(y) = f$ and F is a local homeomorphism of a neighborhood of y onto a neighborhood of f. These neighborhoods are specified.

1 Introduction and results

The aim of this paper is to give new points of view on the Newton's method for solving nonlinear equations. There is a very large literature on the Newton's method. In calculus texts this method is described for solving equations $F(u) = 0$, where $u \in \mathbb{R}^n$, and $F : \mathbb{R}^n \to \mathbb{R}^n$ is a function. In the book [7] this method is discussed in detail. In [1], [6], [10], and in many other books, Newton's method is described for solving operator equations

$$F(u) = f, \qquad (1.1)$$

in Hilbert and Banach spaces.

The novel points in this paper include a new approach to justification of the global convergence of the Newton's method for the operators F, that are global homeomorphisms. For example, monotone coercive operators in Hilbert spaces have this property, see [1]. We also suggest a DSM version of the Newton's method with an accelerated convergence rate.

Our approach is outlined for the DSM (Dynamical Systems Method) version (1.4) of the Newton's method, which can be considered as a continuous analog of the classical Newton's method (1.2). The DSM version (1.4) of the Newton's method is very attractive both theoretically and from the numerical point of view. Theoretically it is attractive because it allows one to justify its convergence (see (1.3)) for wide classes of operators, the proofs are simpler than in the classical case, and the conditions, sufficient for convergence, are simpler and less restrictive. The novelty in our argument is also in the minimal smoothness assumptions on the nonlinear operator $F(u)$: usually, see, e.g., [1], [6], it is assumed that $F'(u)$ satisfies the Lipschitz condition. We assume only that $F'(u)$ is continuous with respect to u. Usually the convergence of Newton's method is proved locally, i.e., for the initial approximation sufficiently close to the solution. We prove global convergence of the DSM version of Newton's method for global homeomorphisms F.

The reader can appreciate this by reading a proof of Theorem 1.1 in this paper. Our proof is simpler and shorter than the proofs of the convergence of Newton's method in [1] and [6].

The numerical advantages of the DSM version of Newton's method were illustrated recently in [3], [4]. In these papers the reader can see that the DSM is applied successfully to the problems to which the usual Newton's method cannot be applied because $F'(u)$ is not a boundedly invertible operator.

The other novel point is the treatment of the Newton's method with a minimal assumption on the smoothness of F: we assume only that F is continuously Fréchet differentiable.

The theoretical and numerical developments of the DSM are presented systematically in the book [16].

The classical Newton's method for solving (1.1) is the iterative scheme:

$$u_{n+1} = u_n - [F'(u_n)]^{-1} (F(u_n) - f), \qquad u|_{n=0} = u_0, \tag{1.2}$$

where u_0 is an initial approximation. In [1] and [6] different proofs of convergence of the iterative process (1.2) to the solution y of (1.1) are given under the assumption that $F'(u)$ satisfies the Lipschitz condition, and several other assumptions, including the smallness of $\|u_0 - y\|$. One can give sufficient conditions for the existence of a solution to (1.1) (see [1] and [6]). We are mostly interested in a DSM version of process (1.2), which is (1.4) below. Since we do not assume that $F'(u)$ satisfies the Lipschitz condition, the convergence rate we obtain is exponential, slower than the superlinear (quadratic) convergence rate one gets for the process (1.2) under the assumption that $F'(u)$ satisfies the Lipschitz condition with respect to u. However, the convergence of the process (1.2) has been established under certain conditions, of which one is the sufficient closeness of the initial approximation u_0 to the solution y. If F is assumed a global homeomorphism, then convergence of the process (1.4) is global, i.e., the process converges to the (unique) solution y for *any* initial approximation u_0. Our arguments are also new for the case when F is a local homeomorphism of a neighborhood of y onto a neighborhood of $f = F(y)$. Our proofs are simple and short. The reader can appreciate the power of the DSM version (1.4) of the Newton's method because the proof of Theorem 1.1 is quite simple and gives global convergence.

The DSM was introduced systematically and applied to solving nonlinear operator equations in [8]-[13], where the emphasis was on convergence and stability of the DSM-based algorithms for solving operator equations, especially nonlinear and ill-posed equations. The DSM for solving an operator equation $F(u) = f$ in a Banach space X

consists of finding a nonlinear map $u \mapsto \Phi(t,u)$, depending on a parameter t in $[0,\infty)$, that has the following three properties:

(1) the Cauchy problem

$$\dot{u} = \Phi(t,u), \quad u(0) = u_0 \quad (\dot{u} := \frac{du(t)}{dt})$$

has a unique global solution $u(t)$ for a given initial approximation u_0;

(2) the limit $u(\infty) = \lim_{t\to\infty} u(t)$ exists; and

(3) this limit solves the original equation $F(u) = f$: $F(u(\infty)) = f$.

We will write these three properties as

$$\exists! u(t), \quad \forall t \geq 0; \quad \exists u(\infty); \quad F(u(\infty)) = f. \tag{1.3}$$

The operator $F : X \to X$ is a nonlinear map, The problem is to find a Φ such that the properties (1), (2), and (3) hold. Various choices of Φ for which these properties hold are proposed in [10], where the DSM is justified for wide classes of operator equations, in particular, for some classes of nonlinear ill-posed equations (i.e., equations $F(u) = f$ for which the linear operator $F'(u)$ is not boundedly invertible). By $A := F'(u)$ we denote the Fréchet derivative of the nonlinear map F at the element u.

The DSM version of the Newton's method (1.2) is the problem

$$\dot{u} = -[F'(u)]^{-1}[F(u) - f], \quad u(0) = u_0, \tag{1.4}$$

where f is an arbitrary given element of X.

This problem has been first discussed in [2], where convergence was established under the assumption that u_0 is sufficiently close to the solution y. The arguments in [2] differ from the ones in our paper.

Problem (1.4) is an example of the DSM with the following choice of Φ:

$$\Phi := -[F'(u)]^{-1}[F(u) - f].$$

Let us formulate

Assumptions A): Assume that F is a global homeomorphism of X onto X, the Fréchet derivative $F'(u)$ exists at every $u \in X$, and is boundedly invertible:

$$\|[F'(u)]^{-1}\| \leq m(\|u\|), \tag{1.5}$$

where $m(r) > 0$ is a continuous function which can grow arbitrarily fast as $r \to \infty$. We also assume that

$$\|F'(u) - F'(v)\| \leq \omega(\|u - v\|), \tag{1.6}$$

where $\omega(r) > 0$ is a continuous monotonically growing function,

$$\omega(0) = 0, \quad \lim_{r\to\infty} \omega(r) = \infty.$$

Without loss of generality one may assume that $\omega(r)$ is a smooth function.

The questions we wish to study are:

Does the process (1.4) *converge?*

What is its convergence rate?

Does the solution to (1.4) *satisfy* (1.3)*?*

Our first result is stated in the following theorem.

Theorem 1.1. *If Assumptions A) hold, then process* (1.4) *converges to* y *at an exponential rate for any initial approximation* u_0*, and conclusions* (1.3) *hold for any initial approximation* u_0.

It is of interest to note that even the proof of local existence of the solution to problem (1.4) seems out of reach if one does not assume the operator $[F'(u)]^{-1}[F(u)-f]$ to satisfy the Lipschitz condition, which requires in general $F'(u)$ to satisfy the Lipschitz condition. Additional difficulty is caused by a proof of global existence of the solution to problem (1.4). A new simple approach demonstrated in this paper removes these difficulties and requires only that $F'(u)$ is continuous with respect to u.

Our second result is of local nature. Let us formulate *Assumptions B).*

Assumptions B): *Suppose that* $F(y) = f$*, that* $\|[F'(y)]^{-1}\| \le m$*,* $m =$ *const* > 0*, inequality* (1.6) *holds for* $u, v \in B(y, R)$*, where* $B(y,R) = \{u : \|u-y\| \le R\}$*,* $\|h-f\| \le \rho := (1-q)R/m$*,* $R := \omega^{-1}(q/m)$*,* $q \in (0,1)$*,* $\omega^{-1}(r)$ *is the inverse function to* $\omega(r)$.

Theorem 1.2. *If Assumptions B) hold, then process* (1.4) *converges to* y *at an exponential rate for any initial approximation* $u_0 \in B(y,R)$*, and for process* (1.4) *conclusions* (1.3) *hold for any initial approximation* $u_0 \in B(y,R)$.

In Section 2 proofs of these theorems are given.

2 Proofs

Proof of Theorem 1. We start with the problem (1.4). One has to prove that the solution to (1.4) exists for all $t \ge 0$, and (1.3) holds. Existence of the solution to (1.4) is a non-trivial problem, because the usual methods to prove even local existence of the solution to (1.4) require that the operator in the right-hand side of (1.4) satisfies a Lipschitz condition, which means that $F'(u)$ has to satisfy this condition, while *Assumptions A)* guarantee only the continuity of $F'(u)$ as a function of u. Thus, a new approach is needed. Let us consider a function $v(t) = F(u(t))-f$, where $u(t) \in C^1(\mathbb{R}_+)$, $\mathbb{R}_+ = [0,\infty)$. Since *Assumptions A)* hold, F is a global homeomorphism, the function $v(t)$ defines uniquely the function $u(t)$ for any given $f \in X$, and $\dot{v} = F'(u(t)\dot{u}$. Let us choose $v(t) = v_0 e^{-t}$, where $v_0 := F(u_0) - f = v(0)$. Then $\dot{v} = -v,\ v(0) = v_0$. Let us define $u(t)$ as the unique solution of the equation

$$F(u(t)) = f + v_0 e^{-t}. \tag{2.1}$$

Then $F'(u(t)\dot{u} = -v$, so

$$F'(u(t)\dot{u} = -\left(F(u(t)) - f\right). \tag{2.2}$$

By *Assumptions A)*, the operator $F'(u(t)$ is invertible, so $u(t)$ solves equation (1.4) for all $t \ge 0$, $u(0) = u_0$, so $u(t)$ is the unique global solution to problem (1.4). Let us prove the rest of the conclusions (1.3). Existence of $u(\infty) := \lim_{t\to\infty} u(t)$ follows from the relation (2.1). Indeed, its right-hand side has a limit f as $t \to \infty$, and since F is a global homeomorphism, it follows that $u(\infty) := \lim_{t\to\infty} u(t)$ exists, and, moreover, $F(u(\infty)) = f$. Since F is a global homeomorphism, it follows that $u(\infty) = y$, where y is the unique solution to equation (1.1).

To finish our argument, let us prove that the rate of decay of

$$g(t) := \|v(t)\|, \qquad v(t) := u(t) - y.$$

is exponential.

Note that

$$[F'(u)]^{-1}(F(u) - F(y)) = v + [F'(u)]^{-1} \int_0^1 [F'(y + sv) - F'(u)]dsv, \tag{2.3}$$

and

$$\|F'(y + sv) - F'(u)\| \le \omega(g), \qquad u \in B(y, R). \tag{2.4}$$

Thus, (1.4) implies

$$\dot{v} + v = -[F'(u)]^{-1} \int_0^1 [F'(y + sv) - F'(u)]dsv. \tag{2.5}$$

Let $w := e^t v$. Then $\dot{w} = e^t(\dot{v} + v)$. Multiply (2.5) by e^t and get

$$\|\dot{w}\| \le m(R_1)\omega(g)g, \qquad g = \|v(t)\|, \tag{2.6}$$

where $R_1 = g + \|y\|$, because $\|u\| \le \|y\| + \|u - y\| \le \|y\| + g$.

One has

$$\frac{d\|w\|}{dt} \le \|\dot{w}\|,$$

where $\|w\| = e^t g(t)$. Thus, (2.5) implies

$$\dot{g} \le -g + m(R_1)\omega(g)g, \qquad t \ge 0, \qquad g(0) = \|u_0 - y\|. \tag{2.7}$$

Inequality (2.7) implies

$$g(t) \le cg(0)e^{-(1-\delta)t}, \tag{2.8}$$

where $\delta > 0$ is an arbitrary small number, and $c > 0$ is a constant depending on $g(0)$.

To derive esimate (2.8) we use a result from [12]. In formulation of this result we do not assume that $g = \|v\|$.

Assumptions A1). We assume that the function $g(t) \ge 0$ is defined on some interval $[0, T)$, has a bounded derivative $\dot{g}(t) := \lim_{s \to +0} \frac{g(t+s) - g(t)}{s}$ from the right at any point of this interval, and $g(t)$ satisfies inequality

$$\dot{g}(t) \le -\gamma(t)g(t) + \alpha(t, g(t)) + \beta(t),\ t \ge 0;\ g(0) = g_0;\ \dot{g} := \frac{dg}{dt}, \tag{2.9}$$

at all t at which $g(t)$ is defined. The functions $\gamma(t)$, and $\beta(t)$, are continuous, non-negative, defined on all of $\mathbb{R}_+$. The function $\alpha(t, g) \ge 0$ is continuous on $\mathbb{R}_+ \times \mathbb{R}_+$, nondecreasing with respect to g, and belongs to $L^1_{loc}([0, \infty))$ with respect to t for every $g \ge 0$.

Assumptions B1). There exists a $C^1(\mathbb{R}_+)$ function $\mu(t) > 0$, such that

$$\alpha\left(t, \frac{1}{\mu(t)}\right) + \beta(t) \le \frac{1}{\mu(t)}\left(\gamma(t) - \frac{\dot{\mu}(t)}{\mu(t)}\right), \quad \forall t \ge 0, \tag{2.10}$$

$$\mu(0)g(0) < 1. \tag{2.11}$$

Proposition 1. *If Assumptions A1) and B1) hold, then any solution $g(t) \geq 0$ to inequality (2.9) exists on all of $\mathbb{R}_+$, i.e., $T = \infty$, and satisfies the following estimate:*

$$0 \leq g(t) < \frac{1}{\mu(t)} \quad \forall t \in \mathbb{R}_+. \tag{2.12}$$

If $\mu(0)g(0) \leq 1$, then $0 \leq g(t) \leq \frac{1}{\mu(t)} \quad \forall t \in \mathbb{R}_+$.

If $\lim_{t\to\infty} \mu(t) = \infty$, then $\lim_{t\to\infty} g(t) = 0$.

Compare inequality (2.9) with (2.7) to conclude that

$$\gamma(t) = 1, \qquad \beta(t) = 0, \qquad \alpha(t, g) = m(R_1)\omega(g)g.$$

Applying Proposition 1 to inequality (2.7), one chooses

$$\mu(t) = \lambda e^{\delta t},$$

where $\delta \in (0, 1)$ is a number. Then conditions (2.10) and (2.11) read:

$$m(R_1)\omega(e^{-\delta t}/\lambda) \leq 1 - \delta,$$

and

$$\lambda\|u_0 - y\| = \lambda g_0 < 1.$$

The first condition holds if

$$m(R_1)\omega(1/\lambda) \leq 1 - \delta.$$

The second condition holds if, for example,

$$\lambda = 1/(2g_0).$$

Both conditions hold if

$$g_0 < 0.5\omega^{-1}\left(\frac{1-\delta}{m(R_1)}\right). \tag{2.13}$$

For any $\delta \in (0, 1)$, however small, one can choose $g_0 > 0$, so small that inequality (2.13) holds, because $m(R_1)$ is bounded uniformly with respect to R on compact sets, and $R_1 = g_0 + \|y\|$, so $m(R_1)$ is uniformly bounded, $m_0 \leq m(R_1) \leq m_1$, where m_0 and m_1 are positive constants, when $u(t)$ is in a bounded set. We have already proved that $u(t) \to y$ as $t \to \infty$. Therefore, for sufficiently large $t > t(R)$ the trajectory $u(t)$ remains in the ball $B(y, R)$, where $R > 0$ can be an arbitrary small number. If $R > 0$ is sufficiently small, then estimate (2.12) holds, i.e.,

$$g(t) < 2g_0 e^{-\delta t}. \tag{2.14}$$

Since $\delta \in (0, 1)$ can be chosen as small as one wants, the exponential rate of decay of $g(t)$ as $t \to \infty$ is established with the exponent $e^{-(1-\delta)t}$ with an arbitrary small $\delta > 0$.

Thus, we have proved Theorem 1.1. □

Remark 1. Under the assumptions of Theorem 1.1 one can construct a DSM version of the Newton's method with an accelerated convergence, see (2.15) below. Choose a continuous positive function $q(t) : \mathbb{R}_+ \to \mathbb{R}_+$, $\mathbb{R}_+ = [0, \infty)$, such that $Q(t) := \int_0^t q(s)ds$ will grow at a desirable rate. For example, let $q(t) = e^t$, then $Q(t) = e^t - 1$. Consider the problem

$$\dot{u} = -q(t)[F'(u)]^{-1}[F(u) - f], \quad u(0) = u_0. \tag{2.15}$$

Define $v(t) := F(u(t)) - f$. Then $\dot{v} = F'(u)\dot{u}$, and equation (2.15) takes the form $\dot{v} = -g(t)v(t)$. Its solution is $v(t) = v(0)e^{-Q(t)}$, where $v(0) := v_0 := F(u_0) - f$. Thus, $\|F(u(t)) - f\|$ tends to zero as $t \to \infty$ at a rate $e^{-Q(t)}$, which can be as fast as one wishes, if one chooses $q(t) > 0$ suitably. Consequently, integrating Cauchy problem (2.15) with an arbitrary u_0, one can get in a short time into a small neighborhood of the element $f = F(y)$. If $F(u(t))$ is in a small neighborhood of $F(y)$, then the element $u(t)$ is in a small neighborhood of the solution y to equation (1.1). Using Proposition 2 (see below) and *Assmptions B)*, one can claim that if $q = 1/2$ and $\|F(u(t)) - f\| \le \rho$, then $\|u(t) - y\| \le 2m\rho$, where $m = \|[F'(y)]^{-1}\| > 0$. Thus,

$$\|u(t) - y\| \le 2m\|F(u_0) - f\|e^{-Q(t)}$$

for all sufficiently large $t > 0$.

Remark 2. If the sequence u_n, defined by the process (1.2), has a limit $\lim_{n\to\infty} u_n = U$, then one can pass to the limit in equation (1.2) and get $U = U - [F'(U)]^{-1}(F(U) - f)$, so $[F'(U)]^{-1}(F(U) - f) = 0$. Apply the operator $F'(U)$ to this equation and get $F(U) = f$. Since F is a global homeomorphism, it follows that $U = y$.

Remark 3. Global convergence of process (1.2) is currently an open problem under the assumptions of Theorem 1.1.

Let us explain the role of *Assumptions B* in the proof of Theorem 1.2, given below. These assumptions allow one to prove the following version of inverse function theorem (see [15]).

Proposition 2. *If Assumptions B) hold, then equation* (1.1) *has a unique solution $u \in B(y, R)$ for any $h \in B(f, \rho)$, and*

$$\|[F'(u)]^{-1}\| \le \frac{m}{1-q}, \qquad \forall u \in B(y, R).$$

Proof of Theorem 1. This proof is similar to the proof of Theorem 1.1. Therefore, we only outline the basic new points. By Proposition 2, under the *Assumptions B)* the map F is a local homeomorphism of $B(y, R)$ onto $B(f, \rho)$. Thus, the proof of Theorem 1.1 yields the conclusions of Theorem 1.2 if one checks that the trajectory $u(t)$ stays in $B(y, R)$ for all $t \ge 0$. This follows from the inequality (2.7) and estimate (2.14), if $g(0)$ is sufficiently small.

Theorem 1.2 is proved. □

Remark 4. If *Assumptions A)* hold, then one can construct a sequence w_n, which converges to y, using the following method, similar to the one in the proof of Theorem 1.1. Let w_n be defined by the equation

$$F(w_n) - f = v_0 e^{-n} := v_n, \qquad w_0 := u_0, \quad v_0 := F(u_0) - f. \tag{2.16}$$

Then $\|F(w_n) - f\| \to 0$ as $n \to \infty$. Since F is a global homeomorphism, it follows that w_n tends to a limit w_∞, and $F(w_\infty) = f$. By the uniqueness of the solution to this equation, one concludes that $w_\infty = y$. Thus $\lim_{n\to\infty} w_n = y$.

One can calculate the sequence $z_n := F(w_n)$ analytically. Note that

$$v_{n+1} - v_n := -(1 - e^{-1})v_n.$$

Therefore,

$$F(w_{n+1}) - F(w_n) = -(1 - e^{-1})[F(w_n) - f]. \tag{2.17}$$

This difference equation can be written as

$$z_{n+1} = qz_n + (1 - q)f, \qquad z_n := F(w_n), \qquad q := e^{-1}. \tag{2.18}$$

The solution to this equation is

$$z_n = q^n z_0 + (1 - q^n)f. \tag{2.19}$$

Since $q \in (0, 1)$ it follows from (2.19) that $\|z_n - f\| \to 0$ as $n \to \infty$. This yield the conclusion $\|F(w_n) - f\| \to 0$ as $n \to \infty$ independently of the argument given above.

The relation between the sequence w_n and the sequence u_n, generated by the process (1.2) is not clear currently.

3 Conclusions

In this paper the DSM version of the Newton's method is discussed for global homeomorphisms, and its global convergence is proved under minimal assumptions on the smoothness of $F'(u)$: only the continuity of $F'(u)$ is assumed. The proofs are simple. The local convergence of the DSM version of the Newton's method is proved without assumption that F is a global homeomorphism and also under the minimal assumptions on the smoothness of $F'(u)$.

References

[1] K. Deimling, *Nonlinear functional analysis.* Springer-Verlag, Berlin, 1985.

[2] M. Gavurin, *Nonlinear functional equations and continuous analysis of iterative methods.* Izvestiya Vusov, Mathem., 5, (1958), 18 – 31 (in Russian).

[3] N.S. Hoang, A.G. Ramm, *The Dynamical Systems Method for solving nonlinear equations with monotone operators.* Asian Europ. Math. Journ., 3, no. 1, (2010), 57 – 105.

[4] N.S. Hoang, A.G. Ramm, *Dynamical systems method for solving nonlinear equations with monotone operators.* Math. of Comput., 79, 269, (2010), 239-258.

[5] N.S. Hoang, A.G. Ramm, *DSM of Newton-type for solving operator equations $F(u) = f$ with minimal smoothness assumptions on F.* International Journ. Comp.Sci. and Math. (IJCSM), 3, no. 1/2, (2010), 3 – 55.

[6] L. Kantorovich, G. Akilov, *Functional Analysis.* Pergamon Press, New York, 1982.

[7] J. Ortega, W. Rheinboldt, *Iterative Solution of Nonlinear Equations in Several Variables.* SIAM, Philadelphia, 2000.

[8] A.G. Ramm, *Dynamical systems method for solving operator equations.* Communic. Nonlinear Sci. and Numer. Simulation, 9 (2004), 383 – 402.

[9] A.G. Ramm, *Inverse Problems.* Springer, New York, 2005.

[10] A.G. Ramm, *Dynamical systems method for solving nonlinear operator equations.* Elsevier, New York, 2007.

[11] A.G. Ramm, *Dynamical systems method (DSM) and nonlinear problems.* Spectral Theory and Nonlinear Analysis (J. Lopez-Gomez, editor), World Scientific, Singapore, 2005, 201 – 228.

[12] A.G. Ramm, *A nonlinear inequality and evolution problems.* Journ. Ineq. and Special Funct., (JIASF), 1, no. 1 (2010), 1 – 9.

[13] A.G. Ramm, *Dynamical systems method and a homeomorphism theorem.* Amer. Math. Monthly, 113, no. 10 (2006), 928 – 933.

[14] A.G. Ramm, *Asymptotic stability of solutions to abstract differential equations.* Journ. of Abstract Diff. Equations and Applications (JADEA), 1, no. 1 (2010), 27 – 34.

[15] A.G. Ramm, *On the DSM Newton-type method.* J. Appl. Math. and Comp. (JAMC), DOI:10.1007/s12190-011-0494-z

[16] A.G. Ramm, N.S. Hoang, *Dynamical Systems Method and Applications.* Wiley-Interscience, New Jersey, 2012 (to appear).

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