THE MONOTONIC PROPERTY OF A SYSTEM OF LINEAR EQUATIONS AND ITS APPLICATIONS

by 45

ROY LEE HARDER

B. S., Kansas State University, 1960

A MASTER'S REPORT

submitted in partial fulfillment of the

requirements for the degree

MASTER OF SCIENCE

Department of Mechanical Engineering

KANSAS STATE UNIVERSITY
Manhattan, Kansas

1969

Approved by:

F. C. Appl
Major Professor

TABLE OF CONTENTS

NOMENCLATURE

A	n x n matrix of real elements
B	inverse of the matrix A
n	integer, number of elements in a row or column of a matrix
x	n x 1 column matrix of real variables
c	n x 1 column matrix of real constants
z	n x 1 column matrix of real elements
q	n x 1 column matrix of real elements
y	n x 1 column matrix
w	n x 1 column matrix
u	n x 1 column matrix
d	n x 1 column matrix
i,j,k	index integers with range 1,2,...,n
r,s	index integers
v	n x 1 column matrix
g	n x 1 column matrix
$b^{(k)}, \bar{b}^{(k)}$	column vectors containing n elements
$y^{(k)}, \bar{y}^{(k)}$	column vectors containing n elements
$h^{(k)}$	column vector containing n elements
$\bar{q}^{(k)}, z^{(k)}$	column vectors containing n elements
σ	parameter for determining vectors for monotonicity
Q^U, Q^L	n x n matrices verifying upper and lower bounds of the matrix B

INTRODUCTION

It is often impossible to determine exact solutions of systems of linear equations due to round off errors and approximate procedures which are necessary to attempt a solution.

An approximate solution of such a system has more meaning if the error of the approximate solution with respect to the exact solution is known. Therefore, if an exact solution of a system of linear equations of the form

$$Ax = c$$

cannot be determined, then it is desirable to be able to determine upper and lower bounds for x. When upper and lower bounds are determined, the maximum possible error in the approximate solution can be determined. However, it may not always be possible to determine bounds for the solution of such a system, but if the matrix A is a "monotonic" matrix, or more accurately, if the problem is "written as a problem of a monotonic type", then upper and lower bounds for x can be determined [1].

A necessary and sufficient condition for a matrix to be monotonic is that all elements of the inverse of the matrix be non-negative [1]. Since in general it is only possible to determine the inverse of a matrix approximately, the sign if the elements of the inverse is not definitely known. Therefore, theorems

[ ] Numbers in brackets designate references at end of report.

stating simpler conditions sufficient to assure the monotonicity of a matrix have been developed [1]. However, the existing theorems are not general enough to include all monotonic matrices.

This paper presents a theorem stating sufficient conditions to assure monotonicity which are less restrictive than the conditions of existing theorems. This theorem includes monotonic matrices not included under the existing theorems. It is also more readily applicable to practical problems than the existing theorems.

The theorem presented states conditions which, if satisfied, assure that all elements of the inverse of the matrix are positive. It is only necessary to determine an approximation to the inverse of the matrix to apply this theorem.

The monotonicity of a matrix is a very important property, for if bounds can be determined for x then it follows that bounds can be determined for the inverse of the monotonic matrix. With the bounds of the inverse of the monotonic matrix known, bounds for any case involving this matrix may be determined directly.