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THE PERIODOGRAM IN HARMONIC ANALYSIS

by 500

GERALD ROBERT HOBBS JR.

B. S., California State Polytechnic College

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Major Professor

TABLE OF CONTENTS

1. INTRODUCTION

Time series connected with physical occurances may often appear to repeat themselves at fixed time intervals. This type of trend is referred to as cyclic or oscillatory behavior. The very nature of surrounding events may dictate some type of cycle. Thus there is little question about the applicability of harmonic analysis.

It is reasonable that we describe cyclic events with functions that are themselves cyclic. We are then led to the use of trigonometric functions and Fourier series analysis. The first section of this paper is a review of classical Fourier methods in the approximation of analytic functions by infinite sums of trigonometric terms.

Early work in the area of harmonic analysis concerned itself with the idea that cyclic behavior was often obscured by random variation within the process. Schuester was the first to devise a scheme for ferreting out these "hidden periodicities". Hotelling, Wicksell and others attacked its use on the grounds that more time series were the result of erratic shocks than were the result of cyclic factors and thus variation in the form of irregular jerks would be more prevalent than variation in the form of smooth harmonics. Schuester's idea of the periodogram persisted, however. It was used more in theory than in practice because of the great amount of computational labor involved.

The popularity of the periodogram diminished in the early 1950's only to be revived in the last part of the decade when the topic of spectral analysis became of interest. The weighted periodogram was seen to be a consistant estimate of the spectral density function and Fourier relationships were seen to exist between the new spectral functions and the sample autocovariance function, the sample auto correlation function, etc.

Thus the periodogram and Fourier analysis have again become useful. Furthermore, modern computing techniques have made its use as a research tool much more tractable.

2. FOURIER ANALYSIS: A METHODS APPROACH

The method of approximating the value of a function at a point by using a power series expansion, such as a Taylor or MacLaurin series, is well known. Power series, however, are not always practically applicable. Many times in statistical work we find trends which are in some sense oscillatory. When this is the case we usually resort to an infinite series of trigonometric functions in order that we may approximate the values of the trend function. Leonhard Euler (1707-1783) was the first to recognize the fact that an analytic function can be represented by an infinite series of sines and cosines. However, the real pioneering work in this field was done by J. B. J. Fourier. In 1822, he published "Theorie analytique de la chaluer". This publication made popular what is now known as Fourier series analysis. We will define a Fourier series expansion to be a series of the form

$$\frac{a_o}{2} + \sum_{n=1}^{\infty} (a_n \cos nx + b_n \sin nx).$$

Not all functions possess valid Fourier expansions. Fortunately, however, expansions do exist for a wide class of functions arising in experimental work. Every function which has a valid Taylor series expansion also has a valid Fourier series expansion. Also some functions for which there exist no convergent Taylor expansions can be routinely expressed in a Fourier series. Without stating exactly under what conditions a valid Fourier expansion will exist we can state what is, for our purposes, the fundamental theorem of Fourier series analysis.