

INTRODUCTION TO THE SIMULATION OF CONTROL SYSTEMS
USING THE ANALOG COMPUTER

by

JOHN WAYNE HERMAN

B.S., Kansas State Teachers College, 1973

A MASTER'S REPORT

submitted in partial fulfillment of the

requirements for the degree

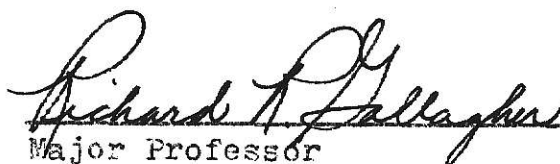
MASTER OF SCIENCE

Department of Electrical Engineering

KANSAS STATE UNIVERSITY
Manhattan, Kansas

1975

Approved by:


Major Professor

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ACKNOWLEDGMENTS

I wish to express my appreciation to Dr. Richard R. Gallagher of the Department of Electrical Engineering for his many hours of assistance in completing this paper. His scholarship and understanding contributed to my being able to successfully complete this degree. My thanks also go to Drs. W. W. Koepsel, Donald R. Hummels and Richard F. Sincovec for serving on my committee and for the many hours they spent in class and in private guiding me through the program.

I would also like to thank my wife for her encouragement, patience, and typing ability, all of which were of immeasurable value in my success.

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Chapter 1

1. INTRODUCTION

The purpose of this set of experiments is to introduce the student to the simulation of physical systems. Being able to approximate real physical systems by models is an important basis of engineering design since the exact mathematical model of the actual device either doesn't exist or is usually too complicated to yield usable results with a reasonable amount of work. For this reason, a major thrust of most design courses is to acquaint the student with some useful models. This also is the purpose of simulation, to create models of physical systems that yield meaningful results with a minimum of work and error. With this in mind, a definition of simulation for the purposes of these experiments is associated with the creation of a model of a physical system for the purpose of analyzing the operation of that system.

The tools that are used to approximate the real physical systems are the standard mathematical tools of the practicing engineer, those associated with algebraic and differential equations. The analog computer can be

used to solve these equations while most of the mathematical operations are done in the LaPlace transform domain. For these reasons the student should have a good practical grasp of differential equations and their relationship to LaPlace transform theory. The physical systems that are simulated relate to simple linear and non-linear feedback control systems. These are presented in most undergraduate engineering curricula and are not so complex that the student loses the intuitive feel for what is happening within the system. It must be remembered that these simple systems in no way approach the limitations of the analog computer or the LaPlace transform techniques. Nor are these the only techniques available to simulate physical systems; however, these tools are very useful in developing a first approximation to a physical system.

These experiments are divided into four major areas. The first of these is designed to familiarize the student with the analog computer. The second section is to use the analog computer to simulate simple position control systems and to demonstrate the effect of changing both the model and the parameters of the model on the open and closed loop response of the control system. The second part also includes a demonstration of the non-linear and delay effects encountered in most practical control systems. The third section treats the improvement of the system using the classical techniques of the system specification, gain, and lead and lag compensation along with the

more modern cost function techniques using the state variable formulation, the stability theories of Lyapunov, and basic matrix theory. The last section consists of a short description of some special projects the student may use to expand his knowledge of control theory or analog computer simulation techniques.

The first block of three experiments is designed to acquaint the student with the analogue computer. Experiment I deals with programming the solution of differential equations in the classical and transfer function form. In order to produce an accurate and usable solution to these differential equations this section covers amplitude and time scaling of these differential equations. Experiment II covers this same material with regard to non-linear equations. Experiment III deals with equations in state variable form.

The second block of experiments treats the simulation of simple models of position control systems. Experiment IV demonstrates the open and closed loop responses of types zero, one, and two control systems when excited by step, ramp, and sine wave inputs. This experiment also demonstrates the effect of changing the gain on the response of the system. Experiment V deals with comparison of the model with the actual system and with improving the model. Model improvement may consist of adding circuits that approximate non-linearities in the actual system or with slight changes in coefficients in a linear model creating a more realistic response to the input excitation. Experiment VI deals with

the effect of time delay on the operation of simple control systems. The system under consideration is a temperature control system with proportional control and cooling.

The next section emphasizes improvement of control system performance. Experiment VII deals with the subject of compensation from both the classical (frequency domain) point of view and the state variable point of view. The first experiment is a non-lab experiment involving the mathematical operations necessary to compensate the control system. Experiment VIII compares the performance of control systems compensated by classical and state variable techniques to demonstrate the relative advantages and disadvantages of each method and the performance of each with the specifications from both.

The last block consists of a number of descriptions of problems the student may attempt to increase his knowledge of analog computer techniques and control systems. Included in each experiment is a short description of the problem and a few references the student may use to increase his understanding of the problem. This is not intended to show the limitations of the analog computer but to demonstrate some techniques that may be used in conjunction with the analog computer to obtain solutions to problems in engineering.

Chapter 2

2. INTRODUCTION TO THE ANALOG COMPUTER

2.1 EXPERIMENT I: LINEAR EQUATIONS

2.1.1 Purpose

The purpose of this experiment is to acquaint the student with the programming of the analog computer. Upon performing this experiment the student should be able to scale a differential equation and program the equation on the analog computer.

2.1.2 Background Material

The operational amplifiers that are used in the analog computer are capable of performing four linear operations; namely, inversion, multiplication of a variable by a constant, addition, and integration. Additionally, the operational amplifiers that form the heart of the analog computer are not ideal so that they have a finite input impedance, gain, bandwidth, and output voltage. Also a non-zero output impedance, phase shift (except at DC), offset, and noise voltage are characteristic of the amplifiers. These limitations on the performance of the operational amplifiers partially justify this experiment since they contribute to the necessity for amplitude and time scaling.

2.1.3 Magnitude Scaling

Magnitude scaling consists of adjusting the coefficients of the differential equations in such a manner as to decrease the effect of the non-ideal characteristics of the operational amplifiers. This consists of increasing the relative amplitude of those variables that have a small maximum magnitude and to decrease the relative amplitude of those variables that have a large maximum magnitude. The maximum magnitudes are quite often known or may be accurately estimated. These may be measured in the case of an operating system, estimated from the specifications of a system to be built, or estimated from the differential equations.

If the maximum magnitudes of the variables are known or have been estimated then the following procedure may be used to scale the equations describing the system for programming on the analog computer.

(a) Solve the differential equations for the highest order derivative.

(b) Noting that each differential may be obtained by integrating the next highest order derivative and that each amplifier inverts its input to produce its output, write out each of these equations.

(c) Multiply and divide each variable by its maximum magnitude. Do not forget to multiply and divide the initial conditions.

(d) The divisor remains with the variable name and is the scale factor relating the output of the amplifier

to the real variable represented by the output of that amplifier.

(e) Solve each of the equations for the scaled variable on the left.

(f) Separate the multipliers of each variable into a pot setting whose value is between zero and one and a gain that is a power of the standard gains that are available on the amplifiers. Pot settings in the range between 0.2 and 0.8 are the most desirable because of accuracy considerations. Time scaling should be used where possible to obtain pot settings in this range. Where time scaling cannot be performed because of limitations of the devices used to record the output or fixed relationships between real time and machine time, multiply-divide circuits are available to bring the pot settings into the desirable range.

(g) Program the resulting equations on the analog computer using the pot settings and gains obtained during the scaling operation. (1:95-108)

2.1.4 Example of Magnitude Scaling

The following worked example may aid in understanding the scaling procedure. Given the equation

$$3\ddot{y} + 2\dot{y} + y = 4 \quad y(0) = y_0 = 1 \quad \dot{y}(0) = \dot{y}_0 = 0 \quad (2.1.1)$$

(a) Solve the Equation (2.1.1) for \ddot{y}

$$\ddot{y} = -\frac{2}{3}\dot{y} - \frac{1}{3}y + \frac{4}{3} \quad (2.1.2)$$

(b) Write the auxillary equations

$$y(t) - y(t_0) = \int_{t_0}^t \dot{y}(t) dt \quad (2.1.3)$$

$$\dot{y}(t) - \dot{y}(t_0) = \int_{t_0}^t \ddot{y}(t) dt \quad (2.1.4)$$

(c) Scale the variables by their respective maximums.

$$|y|_{\max} = 9 \quad (2.1.5)$$

$$|\dot{y}|_{\max} = 20 \quad (2.1.6)$$

$$|\ddot{y}|_{\max} = 45 \quad (2.1.7)$$

Equations 2.1.2, 2.1.3, and 2.1.4 then become

$$45 \left[\frac{\ddot{y}}{45} \right] = -\frac{2}{3} (20) \left[\frac{\dot{y}}{20} \right] - \frac{1}{3} (9) \left[\frac{y}{9} \right] + \left[\frac{4}{3} \right] \quad (2.1.8)$$

$$9 \left[\frac{y}{9} \right] - 9 \left[\frac{y_0}{9} \right] = \int_{t_0}^t 20 \left[\frac{\dot{y}}{20} \right] dt \quad (2.1.9)$$

$$20 \left[\frac{\dot{y}}{20} \right] - 20 \left[\frac{\dot{y}_0}{20} \right] = 45 \int_{t_0}^t \left[\frac{\ddot{y}}{45} \right] dt \quad (2.1.10)$$

(d) The expressions in square brackets are the outputs of the amplifiers with the possible exception of a plus or minus sign.

(e) Solving each of these equations for the dependent variables on the left yields

$$\left[\frac{\ddot{y}}{45} \right] = \frac{2}{3} \cdot \frac{20}{45} \left[\frac{\dot{y}}{20} \right] - \frac{1}{3} \cdot \frac{9}{45} \left[\frac{y}{9} \right] + \frac{4}{45} \quad (2.1.11)$$

$$\left[\frac{y}{9} \right] = \frac{20}{9} \int_{t_0}^t \left[\frac{\dot{y}}{20} \right] dt + \left[\frac{y_0}{9} \right] \quad (2.1.12)$$

$$\left[\frac{\dot{y}}{20} \right] = \frac{45}{20} \int_{t_0}^t \left[\frac{\ddot{y}}{45} \right] dt + \left[\frac{\dot{y}_0}{20} \right] \quad (2.1.13)$$

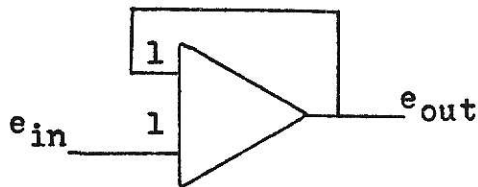
(f) Separating the multiplier of each variable in brackets into a decimal between zero and one and a power of a standard gain yields

$$\left[\frac{\ddot{y}}{45} \right] = -(0.2963) \cdot 1 \left[\frac{\dot{y}}{20} \right] - (0.06667) \cdot 1 \left[\frac{y}{9} \right] + (0.02962) \cdot \left[\frac{1}{1} \right] \quad (2.1.14)$$

$$\begin{bmatrix} \dot{y} \\ 9 \end{bmatrix} = (0.2222) \cdot 10 \int_{t_0}^t \begin{bmatrix} \dot{y} \\ 20 \end{bmatrix} dt + \begin{bmatrix} y_0 \\ 9 \end{bmatrix} \quad (2.1.15)$$

$$\begin{bmatrix} \dot{y} \\ 20 \end{bmatrix} = (0.2250) \cdot 10 \int_{t_0}^t \begin{bmatrix} \dot{y} \\ 45 \end{bmatrix} dt + \begin{bmatrix} y_0 \\ 20 \end{bmatrix} \quad (2.1.16)$$

Since the pot settings of the bracketed variable, y , and the forcing function are outside the desirable range, 0.2 to 0.8, either time scaling must be done or a multiply-divide circuit must be used. As stated above, the primary reason this is done is to increase accuracy. If the pot setting is below 0.2, the noise applied to the input of the following amplifier is increased relative to the desired signal and if the pot setting is greater than 0.8 the loading error is increased excessively due to the appearance of a virtual ground at the input of an operational amplifier. If the equation cannot be time scaled, one of the following circuits may be used to obtain a pot setting in the desired range.



$$e_{out} = -e_{in} - e_{out}$$

or

$$e_{out} = -e_{in}/2$$

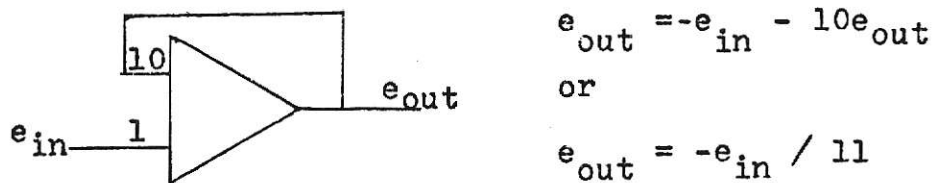


Figure 2.1.1 Divide by two and divide by eleven circuits.

Using the divide by eleven circuit for the bracketed variable containing y and the forcing function causes the following changes in Equation 2.1.14.

$$\left[\frac{\ddot{y}}{45} \right] = -(0.2963) \cdot 1 \left[\frac{\dot{y}}{20} \right] - (0.7334) \frac{1}{11} \left[\frac{y}{9} \right] + (0.3258) \frac{1}{11} \quad (2.1.17)$$

(g) Using the pot settings and gains found in step six, a scaled computer diagram may be constructed. A note of warning at this point, the signs of the outputs of the amplifiers and the inputs to the following amplifiers must be correct or compensation must be made in the circuit. Beginning the programming with Equation (2.1.15) gives the following scaled computer diagram.

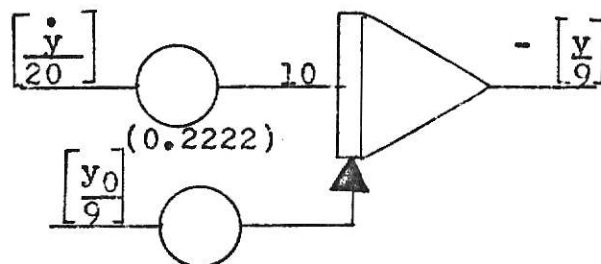


Figure 2.1.2. Scaled Computer Diagram Representing Equation (2.1.15).

The scaled computer diagram representing Equation (2.1.16) is illustrated in Figure 2.1.3.

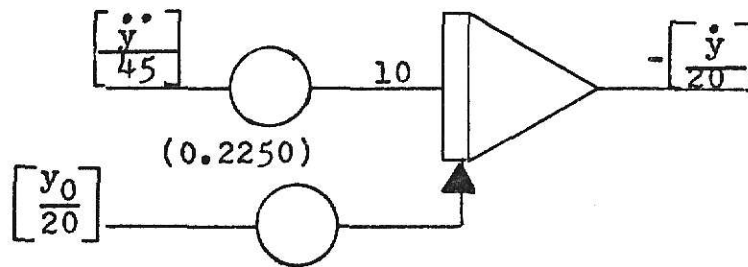


Figure 2.1.3. Scaled Computer Diagram Representing Equation (2.1.16).

The scaled computer diagram for Equation (2.1.17) is shown in Figure 2.1.4.

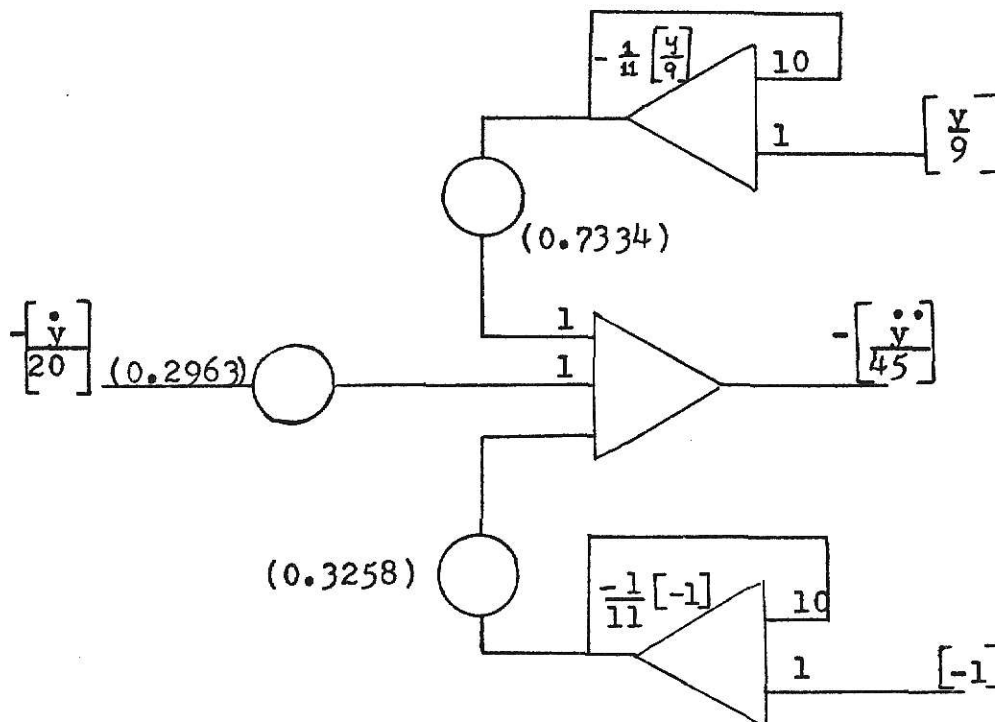


Figure 2.1.4. Scaled Computer Diagram Representing Equation (2.1.17)

2.1.5. Time Scaling

Time scaling adjusts the rate of solution of a system of differential equations. At times this adjustment is required in order to use a particular display device, to obtain reasonable solution rates, to reduce the dependence of the solution on high frequency components of the solution that may be distorted by the frequency response of the amplifiers, or to modify pot settings or required gains that lie outside the range of reasonable values. Examination of the operations performed by the analog computer reveals that integration is the only operation that depends directly upon time. Therefore, if we desire to time scale a problem with a time scaling factor, k , given by

$$k = \tau/t \quad (2.1.18)$$

where τ is the machine time and t is the problem time. Only the gains and pot settings associated with the inputs to the integrators need to be modified. Time scaling does not effect either the initial conditions or the maximum magnitudes associated with the problem variables. The modification, then, consists of multiplying the product of the gain and pot setting for each input to the integrators by a factor of $1/k$ and then developing a new pot setting and a new gain using the method described in Section 2.1.3, step f.

One note of caution, the same time scaling factor must be used for all parts of the problem so that if forcing functions are generated externally or coupled systems of equations are programmed, then these must all be time scaled using the time scaling factor, k .

2.1.6. Example of Time Scaling

Given the scaled computer diagram, Figure 2.1.5, time scale this problem with $k = 5$. Figure 2.1.5 is repeated here for convenience.

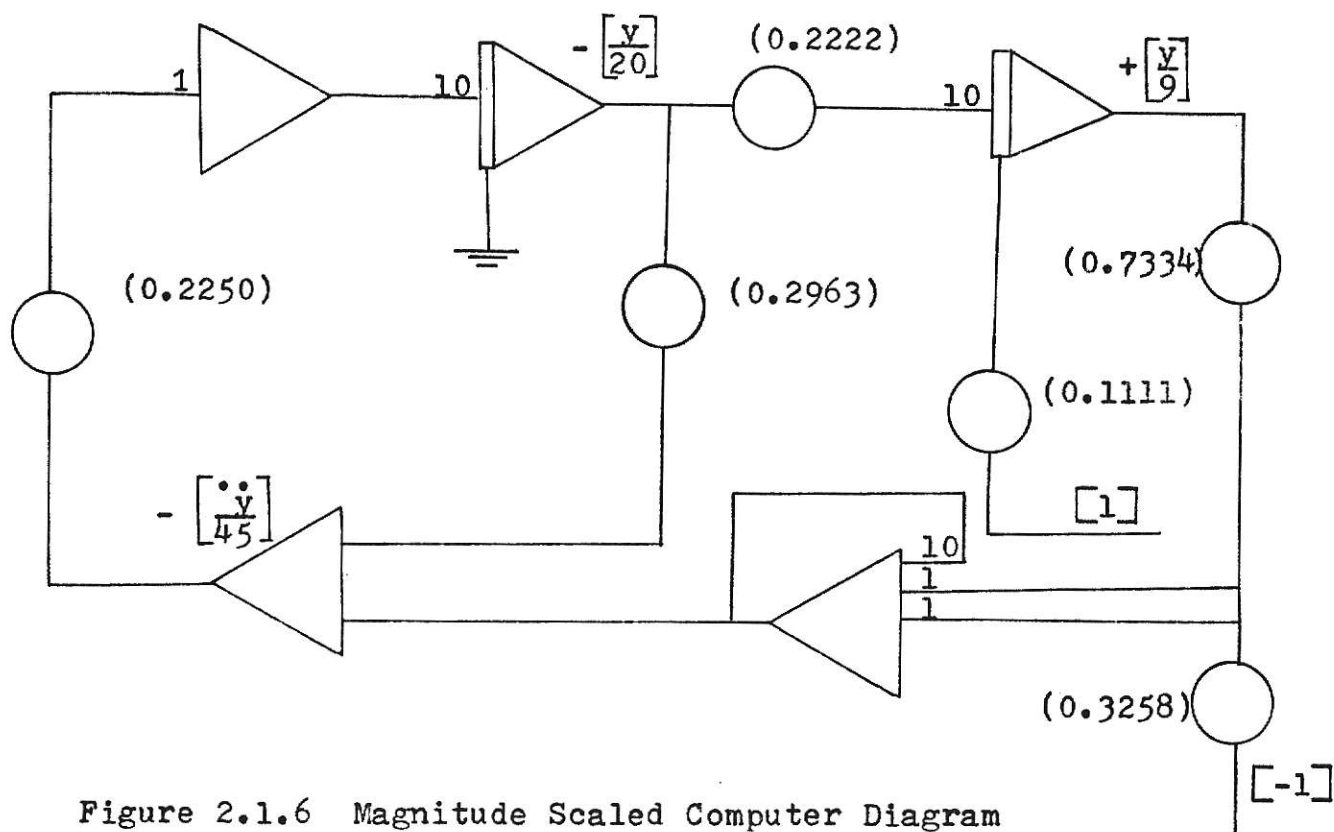


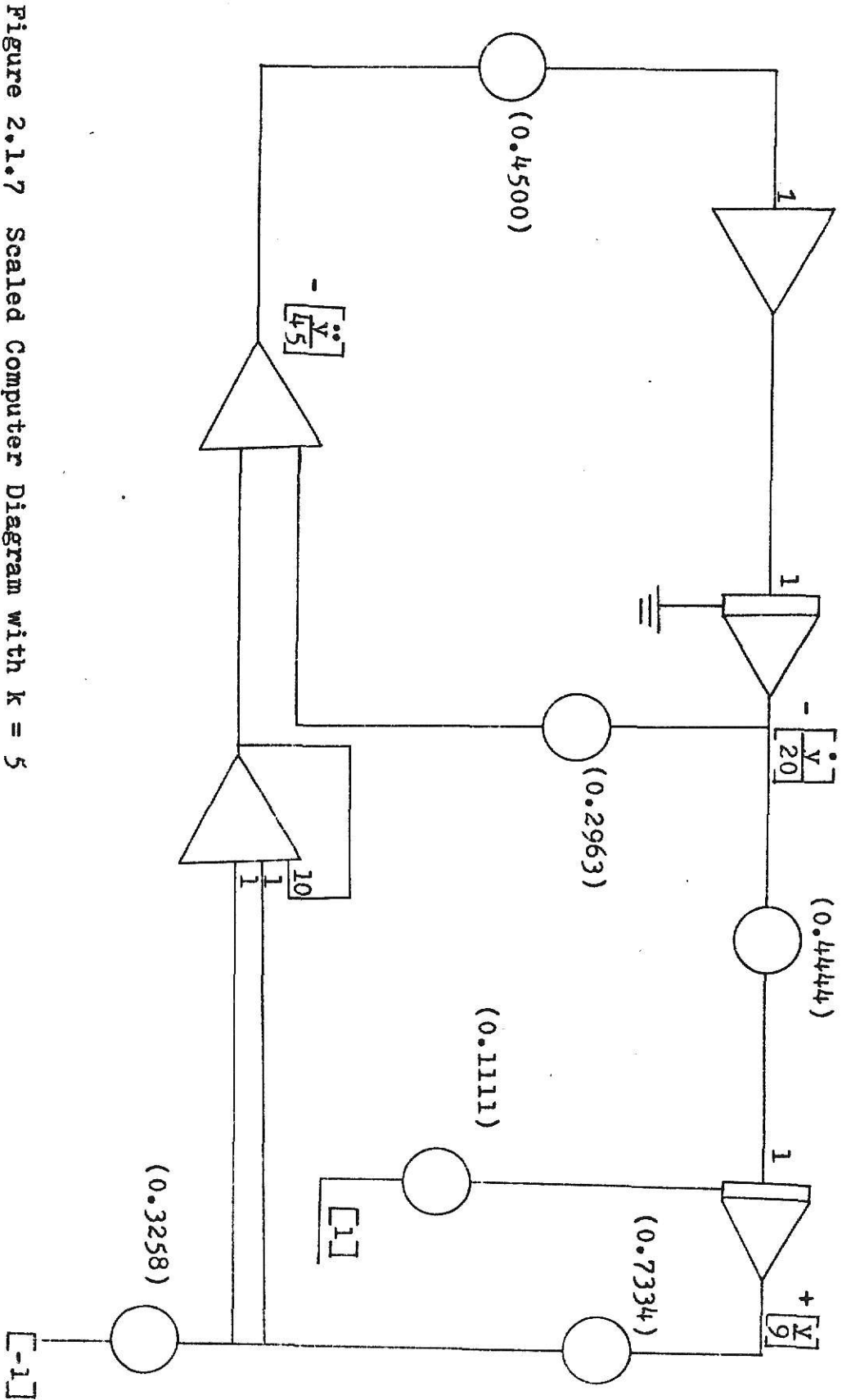
Figure 2.1.6 Magnitude Scaled Computer Diagram

As noted in Section 2.1.5, the time scaling operation consists of multiplying the product of the input gains and pot settings of each input to each integrator by a factor of $1/k$, where k is the ratio between machine time and problem (or real) time.

In Figure 2.1.6, there are only two integrators, one producing $[y/9]$ and one producing $[\dot{y}/20]$. The inverter at the input to the integrator producing $[\dot{y}/20]$ is ignored for time scaling purposes so that a pot setting of 0.2250 and a gain of 10 is associated with this input. A pot setting of 0.2222 and a gain of 10 is associated with the input to the integrator producing $[y/9]$. The products of these gains and pot settings are 2.250 and 2.222 respectively yielding 0.4500 and 0.4444 when multiplied by $1/5$. Therefore, a gain of 1 and a pot setting of 0.4500 is used for the integrator producing $[\frac{dy}{d\tau}/20]$ and a gain of 1 and a pot setting of 0.4444 is used for this integrator producing $[y(\tau)/9]$. Note that these variables are in terms of computer time, τ , instead of problem time, t , and that time scaling did not modify the unreasonable pot settings associated with the input to the summer. The scaled computer diagram is shown on the next page.

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2.1.7 Simulation of Systems Represented by Transfer Functions

A second way of representing the differential equations describing the system is the transfer function. Transfer functions describe the relationship between the input to the system and the output from the system by taking the LaPlace transformation of the system of equations that describe this relationship in the time domain. Generally, the transformed equations are easier to solve than the original time domain equations since they often consist of sums of powers of the transformed variables. For the purposes of this experiment all initial conditions associated with the original time domain equations are assumed to be zero and that the transformed equations may be expressed as

$$D(s) Y(s) = N(s) X(s) \quad (2.1.19)$$

where

$$D(s) = s^n + b_1 s^{n-1} + \dots + b_i s^{n-i} + \dots + b_{n-1} s + b_n \quad (2.1.20)$$