MERGER INCENTIVES OF COST ASYMMETRIC FIRMS UNDER PRODUCTION DIFFERENTIATION

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Abstract

This report examines merger incentives of cost asymmetric firms under product differentiation and their welfare implications. Considering a simple contract under which merger profit is distributed according to the proportions of differential marginal costs between duopolistic firms, we show in a stylized model that for almost all parameter ranges (in terms of market competition intensity and marginal cost differential), a low-cost firm may have no incentive to merge with a high-cost firm whereas the high-cost firm always finds merger to be profitable. Only when marginal cost differential is sufficiently low and the degree of product similarity is sufficiently high will both the low-cost firm and the high-cost firm share the common interest in merger. On the other hand, the merger equilibrium is not welfare-improving, regardless of whether the firms initially compete in quantities or prices. Viewed from the perspective of production efficiency, mergers with differentiated products thus create a fundamental conflict between the maximization of consumer and social welfare and the maximization of firm profits. We also examine the scenario that merger takes place when merger profit exceeds the sum of firm profits under duopoly, without considering how merger profit is distributed between the firms. We discuss the conditions under which mergers may or may not be welfare-improving.

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Chapter 1

Introduction

What are economic conditions under which two competing firms with asymmetry in production costs have an incentive to merge into a single entity when the products they sell are differentiated? What are effects of such a horizontal merger on consumer benefits under product differentiation? If the competing firms find it profitable to merge, would the society as a whole be better off? What are economic implications that horizontal mergers with differentiated products have for antitrust policies? In this report, we attempt to answer these questions by presenting an analysis of merger incentives when there involves cost asymmetry between firms under product differentiation.

Many contributions in the theoretical literature on horizontal mergers assume that products are homogeneous or that firms use the same technology and hence are cost symmetric. When firms engage in Cournot quantity competition, mergers are not profitable to the participating firms unless the number of the "insiders" constitutes the vast majority of the industry (Salant, Switzer, and Reynolds, 1983). Surprisingly, "outsiders" or firms not participating benefit from the merger. When firms engage in Bertrand price competition, mergers become profitable to the insiders, but outsiders make more profits than the insiders (Deneckere and Davidson, 1985). These results are considered as *merger paradox* in the industrial organization literature.[1] But if cost differential is substantial, a merger can enhance social welfare by improving efficiency in the Cournot equilibrium. Farrell and Shapiro (1990)

[1] Referring to the merger paradox, Pepall, Richards, and Norman (1999) state the following: "What may be surprising to you is that it is, in fact, quite difficult to construct a simple economic model in which there are sizable profitability gains for the firms participating in a horizontal merger that is not a merger to monopoly. This difficulty is what we call the merger paradox" (406).

and Perry and Porter (1985) give sufficient conditions for a merger to be profitable and welfare enhancing. Our analysis is complementary to the existing literature, but we focus on mergers of cost-asymmetric firms producing differentiated products.

Although most academic research focuses on mergers with homogeneous goods, a great deal of mergers was observed in industries with differentiated products. Kao and Menezes (2010) indicate that, according to the public register of merger cases for the Australian regulator, a significantly large proportion of about five hundred merger decisions since 2004 were markets with differentiated products. Moreover, it seems that merger activities continue to play an important role in the growth strategies of firms.

In the analysis, we consider two scenarios. One scenario is that before two firms decide whether or not to merge, they use cost-based shares to determine the proportion of merger profit that each firm receives after merger. Another scenario is that merger takes place when merger profit exceeds the sum of firm profits under duopoly, without considering how merger profit is distributed between the firms.

1.1 Relation to the literature

The contribution by Kao and Menezes (2010) is among the first to derive economic conditions for welfare-improving mergers with differentiated products when firms compete in quantities. The authors further show that if the firms compete in prices, mergers will always be welfare-diminishing. Farrell and Shapiro (1990) show that a merger may create two kinds of efficiency gains. First, a merger may lead to a cost-saving reorganization of product: for instance, a firm may shift its production from a high-cost facility to a low-cost facility or using cheap production factor (e.g., capital) instead of expensive factors. Second, a merger may create synergies through learning for instance, and make post-merger production greater than the sum of the pre-merger productions. Kao and Menezes point out that the positive welfare effect comes from first type of efficiency gain. Although a merger without creating synergies may increase the total welfare by improving the efficiency, this is not the case as we will address in the present report since the merger does not create a synergy. We will show that if merger is not economically feasible, two firms that provide differentiated goods will both choose to engage in quantity competition for almost entire parameter range in order to maximize profits.

Stressing the case of product differentiation, Kao and Menezes (2010) show conditions under which mergers are welfare-improving when firms compete in quantities. In this report, we further examine whether a profitable merger is welfare-improving when firms with cost asymmetry compete in quantity. If merger is allowable, low-cost and high-cost firms share the common interest in merger only under restrictive conditions when the intensity of product similarity is sufficiently high and when the marginal cost differential is sufficiently low. Kao and Menezes (2010) remark that the intensity of product market competition is an important factor in determining the welfare consequences of horizontal mergers. This suggests that the antitrust authority views more favorably horizontal mergers in industries where the product market competition is not intense. The analysis in the report shows a rigorous proof to the above statement. After analyzing the merger effect on consumer surplus and social welfare under the specific conditions, we find out that when two products become close substitutes, merger results in a decrease in consumer surplus. Moreover, the decrease in consumer surplus more than offsets the increase in profits, causing social welfare to decline. The findings of the present study have policy implications. In deciding whether a merger activity should be promoted, the government needs to set the goal of antitrust policy first. If the goal is to stimulate the industrial growth, the merger should be allowable. But if the goal is to maximize consumer and social welfare, the merger should be prohibited.

In addition to analyzing how strategic variables (quantities and prices) affect social welfare, as did in Kao and Menezes (2010), we further consider a contract under which merger profit is distributed according to the proportions of differential marginal costs between duopolistic firms in Chapter 2. These cost-based shares of the merged entity's profit allow us to examine the incentives of the firms in their merger decisions. Specifically, we compute each firm's profit before and after a merger for all possible ranges of parameters (in terms of market competition intensity and marginal cost differential). In Chapter 3, we identify the economic conditions that induce a profitable merger. We also analyze the merger effect on consumer surplus and overall welfare. In Chapter 4, we examine the profitability of merger when a merger decision depends on whether post-merger profit exceeds the sum of the firms' pre-merger profits under duopoly, without considering the distribution of the post-merger profit. Concluding remarks can be found in Chapter 5.

Chapter 2

The Analytical Framework

2.1 The Basic Assumptions

There are two domestic firms (denoted as 1 and 2) with market sizes of α_1 and α_2 selling differentiated products in their respective markets. The two firms may have different marginal costs of production c_1 and c_2, where $c_2 = (1+\beta)c_1$ and $\beta \geq 0$. For the ease of illustration, we assume that there are no fixed costs and that there involves only exist cost asymmetry. This assumption allows us to pay particular attention to the economic incentive for the two firms to merge. We further normalize the marginal cost of firm 1 to be $c_1 = 1$. This implies that firm 2's marginal cost of production is $c_2 = (1+\beta)$. Through the analysis, we use β to reflect the marginal cost differential or asymmetry between the two firms. For $\beta \geq 0$, firm 1 is referred to as a low-cost firm and firm 2 a high-cost firm.

We use a quadratic function of two differentiated products to describe a representative consumer's utility,

$$U = \alpha_1 q_1 + \alpha_2 q_2 - \frac{1}{2}\left(q_1^{\,2} + q_2^{\,2} + 2\gamma q_1 q_2\right) + m, \tag{1}$$

where $\alpha_i > 0$, q_1 and q_2 are the quantities of the products demanded, and m is a perfectly competitive numeraire good. The parameter γ measures the degree of product substitutability (or the intensity of market competition) between the differentiated products. For $\gamma = 0$, the two products are independent such that there is no market competition. For γ close to 1, the differentiated products become almost homogeneous or highly substitutable such that their market competition is severe.

Corresponding to the quadratic preferences in equation (1), we have the demand structure for the two products as follows:

$$p_1 = \alpha_1 - q_1 - \gamma q_2, \tag{2}$$

$$p_2 = \alpha_2 - q_2 - \gamma q_1. \tag{3}$$

To focus on cost asymmetry, we assume equality in market size such that $\alpha_1 = \alpha_2 = \alpha$. We further normalize the market size to be $\alpha = 2,$ which guarantees the quantity of output produced by a firm to be non-negative.

We define social welfare as the sum of consumer surplus and firm profits. It follows from the utility function (1) that social welfare is

$$SW = q_1 + (1-\beta) q_2 - \frac{1}{2}(q_1 + q_2)^2 + (1-\gamma) q_1 q_2. \tag{4}$$

Consumer surplus is given as

$$CS = \begin{cases} SW - \pi_1 - \pi_2 \text{ uner duopolitisitc competiton} \\ \\ SW - \Pi \text{ under merger} \end{cases} \tag{5}$$

where π_1 and π_2 are profits of the firms without having a merger and Π is profit when they merge.

We denote Q_i as the quantity choice of the merged entity and q_i as that of the differentiated duopolists in market $i \in \{1, 2\}$. For the merged entity, which is a monopolist over markets 1 and 2, it makes no difference between setting quantity and setting price. The optimization problem facing the monopolist is

$$\max_{\{Q_1,Q_2\}} \Pi = \left(p_1 - c_2\right)Q_1 + \left(p_2 - c_2\right)Q_2. \tag{6}$$

The first-order conditions for profit maximization yield Q_1^M and Q_2^M :

$$Q_1^M = \frac{1+(\beta-1)\gamma}{2\left(1-\gamma^2\right)} \quad \text{and} \quad Q_2^M = \frac{1-(\beta+\gamma)}{2\left(1-\gamma^2\right)}. \tag{7a}$$

Note that if $\beta > 1-\gamma$, merger of the asymmetric firms leads to an equilibrium where $Q_2{}^M = 0$. In this case, merger results in an elimination of the high-cost firm from the market.[2] But if $0 < \beta < 1-\gamma$, both firms produce positive quantities of their products.

The equilibrium merger profit is

$$\Pi = \frac{2(1-\beta)(1-\gamma)+\beta^2}{4(1+\gamma)(1-\gamma)}. \tag{7b}$$

Given that cost is the only element of asymmetry between the two firms, we assume that merger profit is shared on the basis of production efficiency. This means that the more efficient a firm is the higher share of the merger profit it will earn.[3] That is, we have the cost-based shares of the merger profits for firm 1 and firm 2 if they choose to merge. The merger contract can be denoted as $\{S_1, S_2\}$, where

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[2] We will show in the later part of the analysis that the two firms will not merge when $\beta > 1-\gamma$. Therefore, this case will be ruled out.

[3] An alternative approach to determine the shares of the merger profit is the use of a Nash bargaining. But this approach makes the analysis un-tractable. We therefore assume that the firms determine their shares according to their relative costs. We calculate shared profit for each firm in case they choose to merge. We then use shared profits to decide when merger is profitable for a firm, given the same set of parameters.

$$S_1 = \frac{1+\beta}{2+\beta} \text{ and } S_2 = \frac{1}{2+\beta}. \tag{8}$$

It follows from equations (7b) and (8) that the duopolists' profits when they choose to merge are given, respectively, as

$$\pi_{M1} = S_1\Pi = \frac{(1+\beta)\left[2(1-\beta)(1-\gamma)+\beta^2\right]}{4(\beta+2)(1+\gamma)(1-\gamma)} \tag{9}$$

and

$$\pi_{M2} = S_2\Pi = \frac{2(1-\beta)(1-\gamma)+\beta^2}{4(2+\beta)(1+\gamma)(1-\gamma)}. \tag{10}$$

The equilibrium value of consumer surplus is

$$CS_M = \frac{2(1-\beta)(1-\gamma)+\beta^2}{8(1+\gamma)(1-\gamma)}. \tag{11}$$

For $1-\gamma \le \beta \le 1-\frac{\gamma}{2}$, the first-order condition for profit maximization yields $Q_1^M = \frac{1}{2}$ and $Q_2^M = 0$, which implies that firm 2 will earn zero profit without signing a contract to share the merged entity's profit. Under this situation, firm 2 will choose to compete with firm 1 instead of merger. In response to this, firm 1 signs a contract with firm 2 promising to share the merger profit with firm 2 based on the relative cost shares S_1 and S_2 as given in equation (8). In the merger equilibrium, this gives firm 1's shared profit as

$$\pi_{M1} = \frac{\beta+1}{4(\beta+2)} \tag{12}$$

and firm 2's shared profit as

$$\pi_{M2} = \frac{1}{4(\beta+2)} \tag{13}$$

The equilibrium value for consumer surplus is

$$CS_M = \frac{1}{8}. \tag{14}$$

2.2 Quantity Competition

For a differentiated duopoly in which firms producing differentiated products compete by setting quantities, each firm i solves the following problem: $\max_{\{q_i\}} \pi_i = (p_i - c_i)q_i$. This yields the best response function: $q_i = \frac{2-\gamma q_j - c_i}{2}$, where $i=1,2$. The equilibrium levels of outputs produced by the two firms are given, respectively, as

$$q_1^{QC} = \frac{\gamma(\beta-1)+2}{4-\gamma^2} \text{ and } q_2^{QC} = \frac{2(1-\beta)-\gamma}{4-\gamma^2}.$$

Note that if $\beta \geq 1-\frac{\gamma}{2}$, the asymmetry between the two firms is so large that in equilibrium firm 2 is priced out of the market (i.e. $q_2^{QC}=0$). We rule out this possibility and examine the case when $\beta \leq 1-\frac{\gamma}{2}$ under which both firms produce positive quantities of their products.

The equilibrium profits of the two firms are:

$$\pi_1^{QC} = \frac{(-\gamma+\beta\gamma+2)^2}{(\gamma-2)^2(\gamma+2)^2}, \tag{15}$$

$$\pi_2^{QC} = \frac{\left(2\beta+\gamma-2\right)^2}{\left(\gamma-2\right)^2\left(\gamma+2\right)^2}. \tag{16}$$

This gives the equilibrium value of consumer surplus as

$$CS_{NM}^{QC} = \frac{\beta^2\left(4-3\gamma^2\right)+2\left(1-\beta\right)\left[\left(4-3\gamma^2\right)+\gamma^3\right]}{2\left(\gamma-2\right)^2\left(\gamma+2\right)^2}. \tag{17}$$

Combining this with the output decision of the merged entity analyzed in Section 2.1, we plot the different cases in Figure 2, depending on whether or not there is a corner solution. Our analysis focuses on the ranges of the parameters, (γ, β), for Case 1 and Case 2.

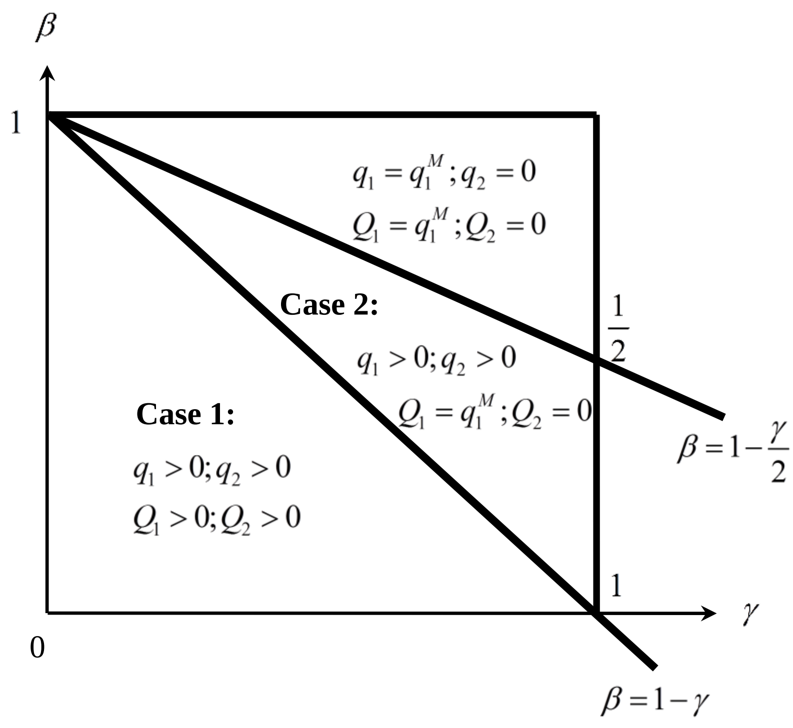


Figure 1. ***Output equilibrium in*** (γ, β) ***space under quantity competition***

2.2.1 Welfare Results

For the entire parameter range, industry profit increases and consumer surplus falls after merger. We show that under quantity competition, for a given parameter range two-to-one mergers can be welfare improving which implies the increase in profits dominates the fall in consumer surplus.

PROPOSITION 1: *For the range of the parameters that satisfy the following condition:* $\frac{2\left[12-\left(10\gamma+\gamma^2\right)\right]+3\gamma^2}{2\left(12-\gamma^2\right)} \le \beta \le 1-\frac{\gamma}{2}$, *the post-merger overall welfare increases under quantity competition.*

Proof: For $1-\gamma \le \beta \le 1-\frac{\gamma}{2}$, the profit-maximizing levels of outputs produced by the differentiated duopolists are $q_1 = q_1^{QC}$ and $q_2 = q_2^{QC}$. The equilibrium values of firm profits and consumer surplus are given in equations (15)-(17).

For the merged entity, the equilibrium levels of outputs under quantity competition are: $Q_1^M = \frac{1}{2}$ and $Q_2^M = 0$. The merged entity's profit equals $\Pi_M = \frac{1}{4}$ and the equilibrium value of consumer surplus equals $CS_M = \frac{1}{8}$. It follows that the post-merger overall welfare increases if

$$\Pi_M^{QC} + CS_M^{QC} > \pi_1^{QC} + \pi_2^{QC} + CS_{NM}^{QC}$$

which implies that

$$\frac{3}{8} \ge \frac{\left(-\gamma+\beta\gamma+2\right)^2}{\left(\gamma-2\right)^2\left(\gamma+2\right)^2} + \frac{\left(2\beta+\gamma-2\right)^2}{\left(\gamma-2\right)^2\left(\gamma+2\right)^2} + \frac{\beta^2\left(4-3\gamma^2\right)+2\left(1-\beta\right)\left[\left(4-3\gamma^2\right)+\gamma^3\right]}{2\left(\gamma-2\right)^2\left(\gamma+2\right)^2}.$$

This holds for

$$\beta \geq \frac{2\left[12-\left(10\gamma+\gamma^2\right)\right]+3\gamma^2}{2\left(12-\gamma^2\right)}$$

which further implies that

$$1-\gamma \leq \frac{2\left[12-\left(10\gamma+\gamma^2\right)\right]+3\gamma^2}{2\left(12-\gamma^2\right)} \leq 1-\frac{\gamma}{2}.$$

The possibility of a welfare improvement comes from the efficiency gain of shutting down the high-cost production. This suggests that only when the cost asymmetry between two firms is sufficiently large will there be welfare gains from mergers.

2.3 Price Competition

From the inverse demands given in equations (2) and (3), we obtain the following demand structure:

$$q_1 = \frac{1}{\left(1-\gamma^2\right)}\left[\left(2-2\gamma\right)-p_1+\gamma p_2\right] \text{ and } q_2 = \frac{1}{\left(1-\gamma^2\right)}\left[\left(2-2\gamma\right)-p_2+\gamma p_1\right].$$

Under price competition, firm i's optimization problem can be written as $\max_{pi}\left(p_i-c_i\right)q_i$. For $p_1 > c_1$ and $p_2 > c_2$, we have the equilibrium prices of the two products as follows:

$$p_1^{PC} = \frac{\beta\gamma-2\gamma^2-\gamma+6}{4-\gamma^2} \text{ and } p_2^{PC} = \frac{2\beta-2\gamma^2-\gamma+6}{4-\gamma^2}.$$

Note that the assumption $\beta \geq 0$ implies that

$$p_1 - c_1 \geq p_2 - c_2.$$

It should be noted that, under price competition, firm 1 may be able to charge a price low enough to drive firm 2 out of the market even if $\beta < 1 - \frac{\gamma}{2}$. This will occur when $q_2 \leq 0$ or

$$q_2 = \frac{(2-2\gamma) - \gamma p_1 - p_2}{\left(1-\gamma^2\right)} \leq 0.$$

This inequality holds for $p_2 \geq 2 - 2\gamma + \gamma p_1$. To ensure p_2 to be no greater than c_2, firm 1 has to set its price such that $p_1 \geq (2\gamma + \beta - 1)/\gamma$. In this case, firm 1 charges a price just low enough to drive firm 2 out of the market. According to Zanchettin (2006), the pricing behavior in this parameter range is considered as the limit-pricing equilibrium.

Substituting p_1^{PC} and p_2^{PC} back into the demand structure, we have the equilibrium levels of outputs produced by the two firms under price competition as follows:

$$q_1^{PC} = \frac{2-\gamma^2+\gamma(\beta-1)}{\left(1-\gamma^2\right)\left(4-\gamma^2\right)} \text{ and } q_2^{PC} = \frac{\left(2-\gamma^2\right)-\gamma-\beta\left(2-\gamma^2\right)}{\left(1-\gamma^2\right)\left(4-\gamma^2\right)}.$$

For $1 - \frac{\gamma}{2-\gamma^2} \leq \beta \leq 1 - \frac{\gamma}{2}$, we have the equilibrium results:

$$p_1 - c_1 = \frac{\gamma+\beta-1}{\gamma},\ q_1 = \frac{1-\beta}{\gamma},\ \text{and}\ p_2 - c_2 = q_2 = 0.$$

Note that in this parameter range, for quantity competition, both firms produce positive quantities of their products. The ability of firm 1 to exercise limit pricing is the key for Zanchettin's result that the efficient firm prefers price competition.

For $\beta < 1 - \frac{\gamma}{2-\gamma^2}$, we have the usual interior solution for differentiated Bertrand. The equilibrium prices are p_1^{PC} and p_2^{PC}, and the equilibrium outputs are $q_1^{PC} > 0$ and $q_2^{PC} > 0$.

We compare the price competition equilibrium to the merged entity's optimal choices in Figure 2. Focusing on the parameter range $\beta < 1 - \frac{\gamma}{2}$, there are three possible cases, according to the nature of equilibrium outcomes:

(I) the limit pricing behavior in which $q_1 > q_1^M$, $q_2 = 0$, $Q_1^M = q_1^M$, and $Q_2^M = 0$;

(II) $q_1 > 0$, $q_2 > 0$, $Q_1^M = q_1^M$, $Q_2^M = 0$; and

(III) an interior solution where $q_1 > 0$, $q_2 > 0$, $Q_1 > 0$, and $Q_2 > 0$.

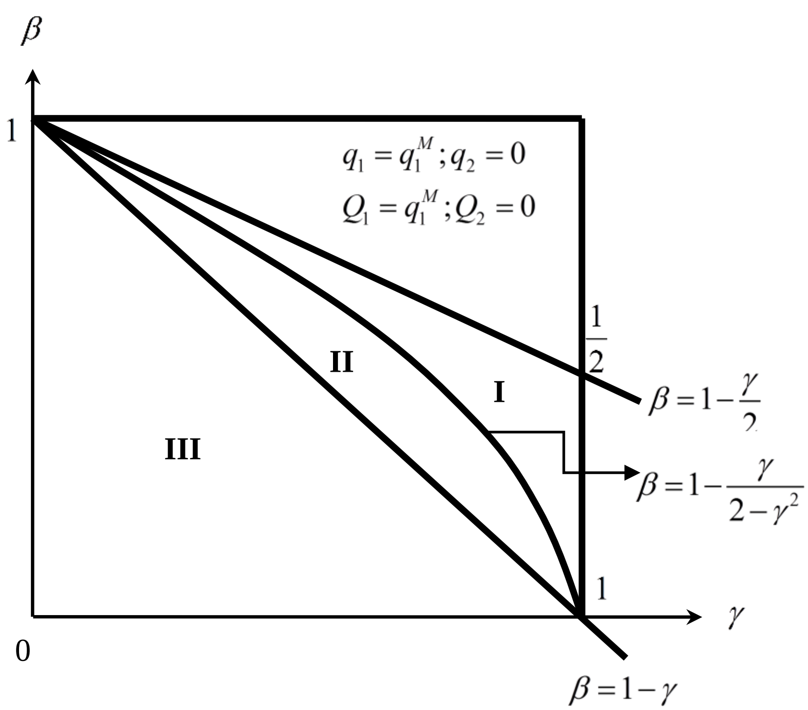


Figure 2. *Output equilibrium in* (γ, β) *space under price competition*

We use Figure 2 to illustrate these three cases.

Case 1: $1-\frac{\gamma}{2-\gamma^2} \le \beta \le 1-\frac{\gamma}{2}$

For the duopolists, $p_1 = \frac{2\gamma+\beta-1}{\gamma}$, Firm 1's profit is π_1^{PC} and $p_2 - c_2 = q_2 = 0.$ Consumer surplus is given by the one in equation (5). For the merged entity, the equilibrium is $Q_1^M = \frac{1}{2}$ and $Q_2^M = 0$ The equilibrium values of profits and consumer surplus are:

$$\pi_1^{PC} = \frac{(1-\beta)(\beta+\gamma-1)}{\gamma^2}, \tag{18}$$

$$\pi_2^{PC} = 0, \tag{19}$$

$$CS^{PC} = \frac{(\beta-1)^2}{2\gamma^2}. \tag{20}$$

The post-merger level of social welfare decreases if the following condition is satisfied:

$$\left(\pi_1^{PC} + CS^{PC}\right) - \frac{3}{8} = \frac{8\beta(1-\gamma)+\gamma(8-3\gamma)-4\left(\beta^2+1\right)}{8\gamma^2} \ge 0$$

This inequality holds since $8\beta(1-\gamma)+\gamma(8-3\gamma)-4\left(\beta^2+1\right) \ge 0.$

Case 2: $1-\gamma \le \beta \le 1-\frac{\gamma}{2-\gamma^2}$

For the case of duopolistic competition under price competition, we have $p_1 = p_1^{PC}$ and $p_2 = p_2^{PC}$ as shown in section 2.3 .The equilibrium values of profits and consumer surplus are:

$$\pi_1^{PC} = \frac{\left(-\gamma-\gamma^2+\beta\gamma+2\right)^2}{(1-\gamma)(1+\gamma)(\gamma-2)^2(\gamma+2)^2}, \tag{21}$$

$$\pi_2^{PC} = \frac{\left(-2\beta-\gamma-\gamma^2+\beta\gamma^2\right)}{(1-\gamma)(1+\gamma)(\gamma-2)^2(\gamma+2)^2}, \tag{22}$$

$$CS^{PC} = \frac{(\beta-1)\left(8-6\gamma^2-2\gamma^3\right)-\beta^2\left(4-3\gamma^2\right)}{2(\gamma-1)(\gamma+1)(\gamma-2)^2(\gamma+2)^2}. \tag{23}$$

The merged entity produces at $Q_1^M = \dfrac{1}{2}$ and $Q_2^M = 0$. The post-merger level of social welfare decreases if the following condition is satisfied:

$$\left(\pi_1^{PC}+\pi_2^{PC}+CS^{PC}\right)-\frac{3}{8}\geq 0.$$

That is,

$$\frac{4\beta(\beta-2)\left[12+\gamma^2\left(2\gamma^2-9\right)\right]-8\gamma(1-\beta)\left(8-3\gamma^2\right)-\gamma^4\left(11-3\gamma^2\right)+48}{8(1-\gamma)(1+\gamma)(\gamma-2)^2(\gamma+2)^2}\geq 0.$$

This inequality holds since

$$4\beta(\beta-2)\left[12+\gamma^2\left(2\gamma^2-9\right)\right]-8\gamma(1-\beta)\left(8-3\gamma^2\right)-\gamma^4\left(11-3\gamma^2\right)+48\geq 0.$$

<u>Case 3</u>: parameter range $\beta\leq 1-\gamma$

For the case of duopolistic competition under price competition, we have $p_1 = p_1^{PC}$ and $p_2 = p_2^{PC}$. The merged entity produces $Q_1 = Q_1^M$ and $Q_2 = Q_2^M$ as shown in equations (7a). The equilibrium values of profits and consumer surplus are:

$$\pi_1^{PC} = \frac{\left(-\gamma - \gamma^2 + \beta\gamma + 2\right)^2}{(1-\gamma)(1+\gamma)(\gamma-2)^2(\gamma+2)^2}, \tag{24}$$

$$\pi_2^{PC} = \frac{\left(-2\beta - \gamma - \gamma^2 + \beta\gamma^2\right)}{(1-\gamma)(1+\gamma)(\gamma-2)^2(\gamma+2)^2}, \tag{25}$$

$$CS^{PC} = \frac{(\beta-1)\left(8-6\gamma^2-2\gamma^3\right)-\beta^2\left(4-3\gamma^2\right)}{2(\gamma-1)(\gamma+1)(\gamma-2)^2(\gamma+2)^2}. \tag{26}$$

The post-merger level of social welfare decreases if the following condition is satisfied:

$$\Pi + CS_M \leq \pi_1^{PC} + \pi_2^{PC} + CS^{PC}.$$

That is,

$$\frac{2(1-\beta)(1-\gamma)+\beta^2}{4(1+\gamma)(1-\gamma)} + \frac{2(1-\beta)(1-\gamma)+\beta^2}{8(1+\gamma)(1-\gamma)}$$

$$\leq \frac{\left(-\gamma - \gamma^2 + \beta\gamma + 2\right)^2}{(1-\gamma)(1+\gamma)(\gamma-2)^2(\gamma+2)^2} + \frac{\left(-2\beta - \gamma - \gamma^2 + \beta\gamma^2\right)}{(1-\gamma)(1+\gamma)(\gamma-2)^2(\gamma+2)^2}$$

$$+\frac{(\beta-1)\left(8-6\gamma^2-2\gamma^3\right)-\beta^2\left(4-3\gamma^2\right)}{2(\gamma-1)(\gamma+1)(\gamma-2)^2(\gamma+2)^2}.$$

This holds for

$$\beta \leq \frac{-(1-\gamma)(4-3\gamma)(\gamma+2)^2 + \sqrt{(\gamma+2)^2(\gamma-2)^2(1+\gamma)(1-\gamma)(4+3\gamma)(4-3\gamma)}}{\gamma(12-5\gamma^2)}$$

Since

$$\frac{-(1-\gamma)(4-3\gamma)(\gamma+2)^2 + \sqrt{(\gamma+2)^2(\gamma-2)^2(1+\gamma)(1-\gamma)(4+3\gamma)(4-3\gamma)}}{\gamma(12-5\gamma^2)} \geq 1-\gamma.$$

This indicates that, for $\beta \leq 1-\gamma,$ the post-merger welfare decreases.

Based on the results of the analyses, we have the following proposition:

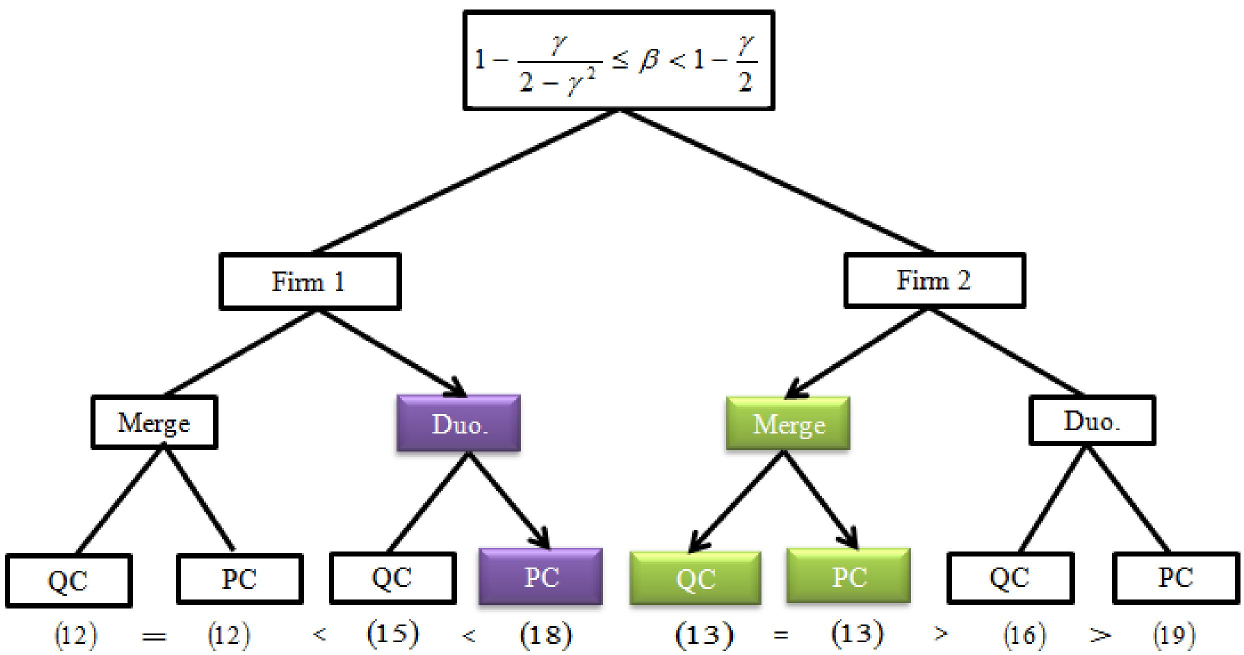
PROPOSITION 2: *when goods are substitutes, a merger from duopoly to monopoly is always welfare-diminishing if firms compete in price.*

2.4 Incentives of the Individual Firms to Merge

We analyze and compare profits of the duopolistic firms under imperfect competition with shared profit of the merged entity when the firms choose to merger. Our aim is identify conditions under which forming a merger is mutually profitable to the firms.

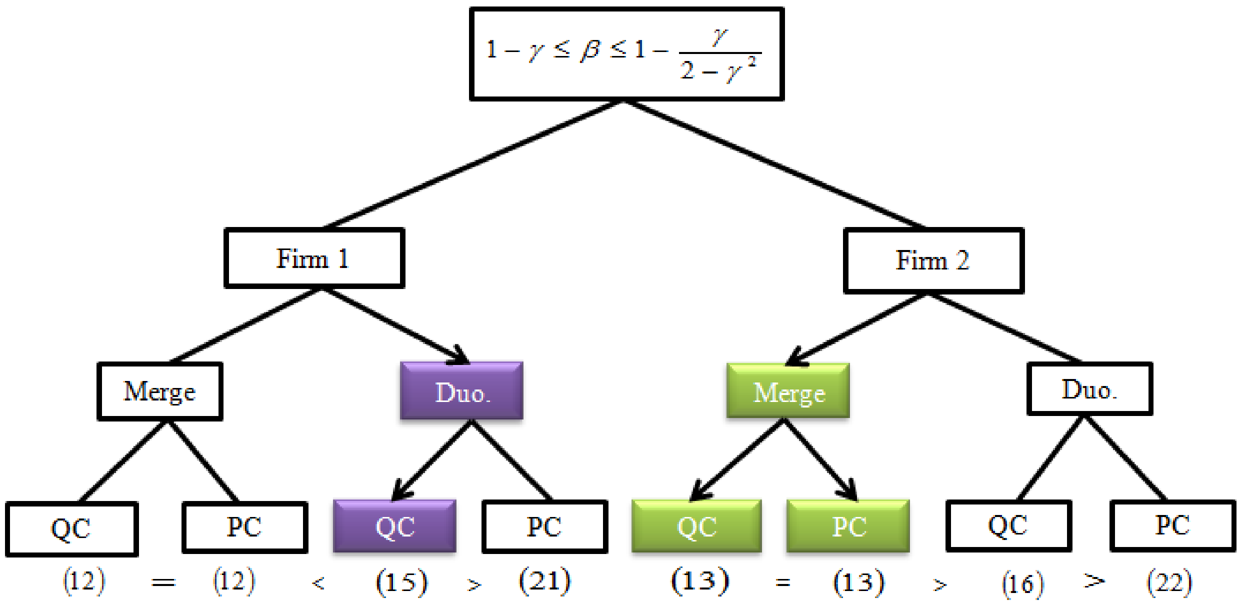
We use graphs to illustrate all possible cases in terms of the degree of cost asymmetry (β) and the intensity of market competition (γ). We have three different graphs. For each parameter range, the two firms decide whether they want to merger or compete. If they choose to compete, each firm has two strategies of engaging in competition: quantity or price. The numbers at the end of either quantity or price competition show equation numbers for the firms' profits as calculated in the previous sections.

Based on Figure 3, we find out that firm 2 (the high-cost firm) has an interest to merge regardless of competition strategies if merger is allowable; If merger is not feasible, firm 2 (the high-cost firm) prefers quantity competition to earn more profits. As for firm 1 (the low-cost firm), it has no interest to merge. Moreover, if two firms operate under duopoly, they make different competition strategies: firm 1 (the low-cost firm) prefers price competition whereas firm 2 prefers quantity competition.



Note: numbers in above Figure refer to equation numbers

Figure 3. W*hen the parameter range is:* $1-\dfrac{\gamma}{2-\gamma^2} \le \beta \le 1-\dfrac{\gamma}{2}$

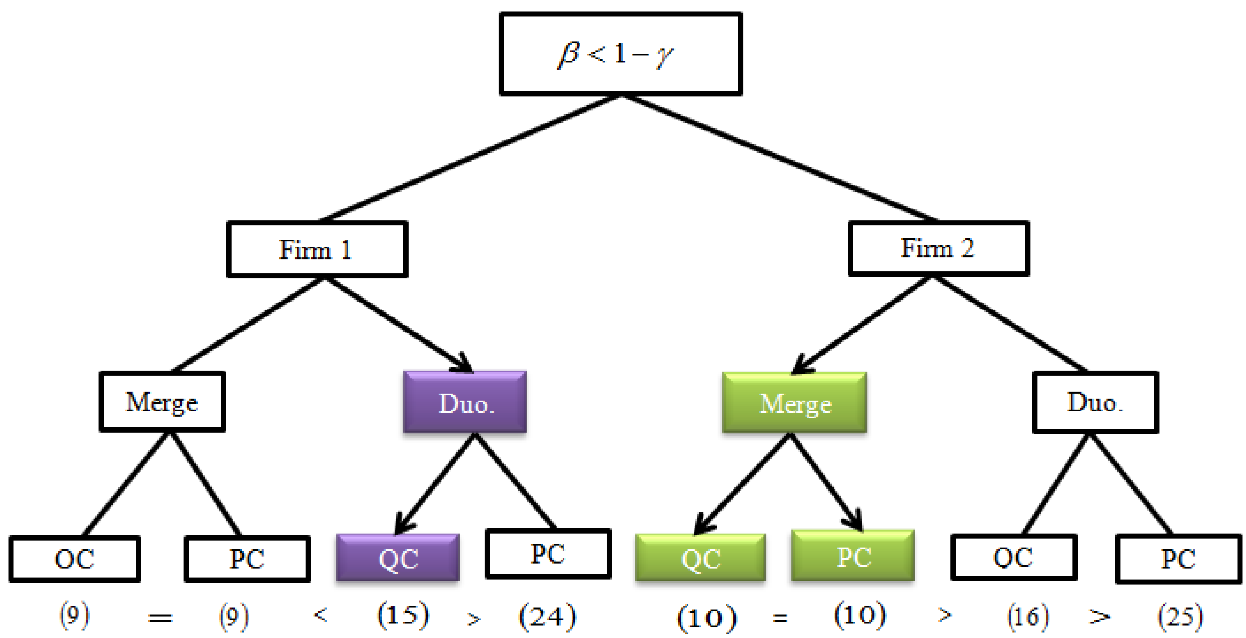


Note: Numbers in above Figure refer to equation numbers

Figure 4. ***The case when the parameter range is*** $1-\gamma \le \beta \le 1-\dfrac{\gamma}{2-\gamma^2}$

Under this parameter range, we got almost the same results as previous one: firm 2 (the high-cost firm) wants to merge while firm 1 (the low-cost firm) has no interest to merge. However, these two firms make the same competition strategies, quantity competition, when they operate under duopoly in order to maximize their profits.

As we can see from Figures 4 and 5, both firms find it's more profitable to engage in quantity competition under duopoly, regardless of whether the parameter range is $\beta < 1-\gamma$ or $1-\gamma \le \beta \le 1-\frac{\gamma}{2-\gamma^2}$. In these cases, the two firms have no common interest to merge.



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Note: Numbers in above Figure refer to equation numbers

Figure 5. ***The case when the parameter range is*** $\beta < 1-\gamma$

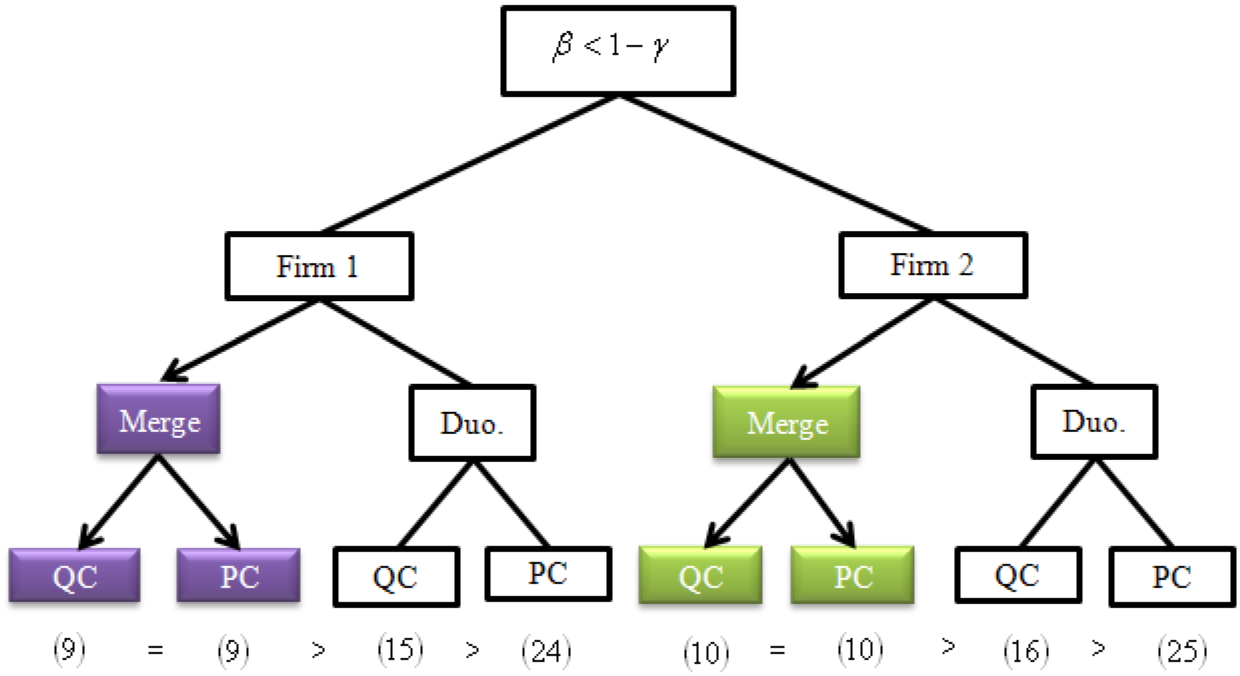
Although the low-cost firm and the high-cost firm may agree to share the merged entity's profit based on cost efficiency, the low-cost firm has no incentive to merge unless its post-merger profit is greater than its profit under duopoly. We find that for parameter range $\beta < 1-\gamma$, there is a special condition ($\gamma \ge 0.9$ and $\beta \le 0.05$) under which both low-cost and high-cost firm prefer to merge in order to maximize their joint profits. In the next section, we focus our analysis on this case.

Chapter 3

Will a Profitable Merger Increase Consumer and Social Welfare?

For the entire range of parameter values we consider, the high-cost firm always finds it profitable to merge with the low-cost firm regardless of whether the strategic variable is price or quantity. But the low-cost firm has no incentive to merge. The low-cost firm further finds it profitable to compete in quantity for almost all the parameter range. Under duopoly, only when the parameter range is $1-\frac{\gamma}{2-\gamma^2} \le \beta \le 1-\frac{\gamma}{2}$ will the low-cost firm be profitable to compete in price.

As have been shown in the analysis earlier, there are conditions under which both the low-cost and high-cost firms find it profitable to merge. That is, merger is profitable to both firms when (i) $\beta < 1-\gamma$, (ii) the marginal cost differential is sufficiently low ($\beta \le 0.05$), and (iii) the degree of product similarity is sufficiently high ($\gamma \ge 0.9$). Because this case emerges when both the low-cost and high-cost firms initially compete in quantity under duopoly (see Figure 3), we compare the post-merger profit of firm 1 (low-cost firm) with its pre-merger profit. In order to show it intuitively, we draw Figure 6. In this case, the low-cost firm shares the common interest with the high-cost firm in merger. Figure 6 illustrates that when product similarity is sufficiently high and the marginal cost differential is sufficient low, both firms have an incentive to merger as their post-merger profits are higher than their pre-merger profits.



Note: numbers in above Figure refer to equation numbers

Figure 6. ***The case that meets the merging conditions:*** $\beta < 1-\gamma$, $\beta \leq 0.05$, and $\gamma \geq 0.9$

In the subsequent analysis, we further examine the effect of such a merger on consumer benefit and social welfare. We find that the merger equilibrium is not welfare-improving because the decrease in consumer surplus more than offsets the increase in profits.

PROPOSITION 3: *For* $\beta < 1-\gamma$, $\gamma \geq 0.9$ *and* $\beta \leq 0.05$, *the post-merger total surplus decreases under quantity competition. In this case, the decrease in consumer surplus exceeds the increase in industry profits, causing social welfare to decline.*

Proof: For $\beta < 1-\gamma$, we compare consumer surplus between the case of the differentiated duopolists and that of a merged entity. Since we have

$$CS_M^{QC} = \frac{2(1-\beta)(1-\gamma)+\beta^2}{8(1+\gamma)(1-\gamma)}$$

and

$$CS_{NM}^{QC} = \frac{\beta^2\left(4-3\gamma^2\right)+2(1-\beta)\left[\left(4-3\gamma^2\right)+\gamma^3\right]}{2(\gamma-2)^2(\gamma+2)^2},$$

we calculate their difference to be

$$CS_M^{QC} - CS_{NM}^{QC} = \gamma\,\frac{2(1-\beta)\left\{4(4-5\gamma)-\gamma^2\left[4-\gamma(11-3\gamma)\right]\right\}+\beta^2\left(11\gamma^2-20\right)}{8(\gamma-1)(\gamma+1)(\gamma-2)^2(\gamma+2)^2} < 0$$

This indicates that the post-merger consumer surplus decreases.

As for social welfare, we have

$$SW_M = \Pi_M^{QC} + CS_M^{QC}$$

$$= \frac{(1-\beta)(1-\gamma)+\beta^2}{4(1+\gamma)(1-\gamma)} + \frac{2(1-\beta)(1-\gamma)+\beta^2}{8(1+\gamma)(1-\gamma)}$$

$$= \left(\frac{3}{8}\right)\frac{2(1-\beta)(1-\gamma)+\beta^2}{(1-\gamma)(1+\gamma)}$$

and

$$SW_{NM} = \left(\pi_1^{QC} + \pi_2^{QC}\right) + CS_{NM}^{QC}$$
$$= \frac{\left(-\gamma + \beta\gamma + 2\right)^2}{\left(\gamma - 2\right)^2 \left(\gamma + 2\right)^2} + \frac{\left(2\beta + \gamma - 2\right)^2}{\left(\gamma - 2\right)^2 \left(\gamma + 2\right)^2} + \frac{\beta^2 \left(4 - 3\gamma^2\right) + 2\left(1 - \beta\right)\left[\left(4 - 3\gamma^2\right) + \gamma^3\right]}{2\left(\gamma - 2\right)^2 \left(\gamma + 2\right)^2}$$

$$= \frac{2\left(1-\beta\right)\left[4\left(3-2\gamma\right) - \gamma^2\left(1-\gamma\right)\right] + \beta^2\left(12-\gamma^2\right)}{2\left(\gamma-2\right)^2\left(\gamma+2\right)^2}.$$

We calculate the difference in social welfare as

$$SW_M - SW_{NM}^{QC} = \gamma \frac{2\left(1-\beta\right)\left\{4\left(4-7\gamma\right) + \gamma^2\left[12 + \gamma\left(1-\gamma\right)\right]\right\} + \beta^2\gamma\left(\gamma^2 - 28\right)}{8\left(\gamma-1\right)\left(\gamma+1\right)\left(\gamma-2\right)^2\left(\gamma+2\right)^2} < 0.$$

This indicates that the post-merger welfare decreases.

Economically, there are two reasons for such an effect. First, the post-merger quantities of the products Q_1^{QC} and Q_2^{QC} decrease by $\frac{\gamma(2-\beta)}{2(\gamma+2)(\gamma+1)}$, compared to their pre-merger quantities. Second, the post-merger price of product 1 increases by $\frac{3}{2}$ and the post-merger price of product 2 increases by $\frac{\beta+3}{2}$.

PROPOSITION 4: *Mergers with differentiated products are welfare-diminishing due to the following reasons: decreases in quantities and increase in prices.*

Proof: For $\beta < 1-\gamma$, we compare the following pairs: $Q_1 + Q_2$ with $q_1 + q_2$, p_1^{POST} with p_1^{PRE}, and p_2^{POST} with p_2^{PRE}. Given that $Q_1 + Q_2 = \frac{2-\beta}{2(\gamma+1)}$ and $q_1 + q_2 = \frac{2-\beta}{\gamma+2}$, we calculate their difference to be

$$(Q_1 + Q_2) - (q_1 + q_2) = -\frac{\gamma(2-\beta)}{2(\gamma+2)(\gamma+1)} < 0.$$

This indicates that the post-merger quantities decrease.

For changes in the equilibrium price of product 1, we have

$$p_1^{POST} = \frac{3}{2} \text{ and } p_1^{PRE} = \frac{(6-\gamma^2) - \gamma(1-\beta)}{(2-\gamma)(2+\gamma)}.$$

We calculate price difference as

$$p_1^{POST} - p_1^{PRE} = \left(\beta + \frac{\gamma}{2} - 1\right)\frac{1}{(\gamma-2)(\gamma+2)} > 0.$$

Note that for $\beta < 1-\gamma$ we have $\beta + \gamma < 1$, which implies that $\beta + \frac{\gamma}{2} < 1$.

For changes in the equilibrium price of product 2, we note that

$$p_2^{POST} = \frac{\beta+3}{2} \text{ and } p_2^{PRE} = \frac{(6+2\beta) - \gamma(1+\gamma+\beta\gamma)}{(2-\gamma)(2+\gamma)}.$$

We calculate the difference in prices as

$$p_2^{POST} - p_2^{PRE} = \gamma\frac{2-\gamma(1-\beta)}{2(2-\gamma)(2+\gamma)} > 0.$$

These results indicate that post-merger prices are higher for both products.

Chapter 4

The Profitability of Merger without Considering Its Distribution

After analyzing the first scenario, in which firms use cost-based shares of merged entity's profit to decide on their merger decisions, we discuss merger incentives without considering how the entity's profit is split between the firms.

(i) For the parameter range $1-\frac{\gamma}{2-\gamma^2} \leq \beta \leq 1-\frac{\gamma}{2}$:

When firms compete in quantity, the merged entity's profit equals $\frac{1}{4}$ and the sum of firm profits under duopoly equals

$$\frac{(-\gamma+\beta\gamma+2)^2+(2\beta+\gamma-2)^2}{(\gamma-2)^2(\gamma+2)^2}.$$

A comparison between the profits reveals that

$$\frac{1}{4}-\frac{(-\gamma+\beta\gamma+2)^2+(2\beta+\gamma-2)^2}{(\gamma-2)^2(\gamma+2)^2}=\frac{(8(1-\beta)+2\gamma^2(1-\beta)+\gamma(\gamma^2-12))(2(\beta-1)+\gamma)}{4(\gamma-2)^2(\gamma+2)^2}>0.$$

This indicates that, when the firms compete in quantity, merger profit is greater than the sum of firm profits under duopoly.

When firms compete in price, the merged entity's profit equals $\frac{1}{4}$ and the sum of firm profits under duopoly equals

$$\left(\frac{(1-\beta)(\beta+\gamma-1)}{\gamma^2}+0\right).$$

A comparison between the profits reveals that

$$\frac{1}{4}-\frac{(1-\beta)(\beta+\gamma-1)}{\gamma^2}=\frac{(2\beta+\gamma-2)^2}{4\gamma^2}>0$$

This indicates that, when firms compete in price, merger profit is greater than the sum of firm profits under duopoly.

(ii) For the parameter range $1-\gamma\leq\beta\leq 1-\frac{\gamma}{2-\gamma^2}$:

When firms compete in quantity, the merged entity's profit equals $\frac{1}{4}$ and the sum of firm profits under duopoly equals

$$\frac{(-\gamma+\beta\gamma+2)^2+(2\beta+\gamma-2)^2}{(\gamma-2)^2(\gamma+2)^2}.$$

A comparison between the profits reveals that

$$\frac{1}{4}-\frac{(-\gamma+\beta\gamma+2)^2+(2\beta+\gamma-2)^2}{(\gamma-2)^2(\gamma+2)^2}=\frac{(8(1-\beta)+2\gamma^2(1-\beta)+\gamma(\gamma^2-12))(2(\beta-1)+\gamma)}{4(\gamma-2)^2(\gamma+2)^2}>0$$

This indicates that, when the firms compete in quantity, merger profit is greater than the sum of firm profits under duopoly.

When firms compete in price, the merged entity's profit equals $\frac{1}{4}$ and the sum of firm profits under duopoly equals

$$\left(\frac{8(1-\beta)(1-\gamma)+\beta^2(\gamma^2(\gamma^2-3)+4)-2\gamma^2(1-\beta)(3-\gamma(\gamma+2))}{(1-\gamma)(\gamma+1)(\gamma-2)^2(\gamma+2)^2}\right).$$

A comparison between the profits reveals that

$$\frac{1}{4}-\frac{8(1-\beta)(1-\gamma)+\beta^2(\gamma^2(\gamma^2-3)+4)-2\gamma^2(1-\beta)(3-\gamma(\gamma+2))}{(1-\gamma)(\gamma+1)(\gamma-2)^2(\gamma+2)^2}>0$$

This indicates that, when the firms compete in price, merger profit is greater than the sum of firm profits under duopoly.

(iii) For the parameter range $\beta \leq 1-\gamma$:

When firms compete in quantity, the Merged entity's profit equals $\frac{2(1-\beta)(1-\gamma)+\beta^2}{4(1+\gamma)(1-\gamma)}$ and the sum of two firms' profits under duopoly equal to $\frac{(-\gamma+\beta\gamma+2)^2+(2\beta+\gamma-2)^2}{(\gamma-2)^2(\gamma+2)^2}$. A comparison between the profits reveals that

$$\frac{2(1-\beta)(1-\gamma)+\beta^2}{4(1+\gamma)(1-\gamma)}-\frac{(-\gamma+\beta\gamma+2)^2+(2\beta+\gamma-2)^2}{(\gamma-2)^2(\gamma+2)^2}>0$$

This indicates that, when the firms compete in quantity, merger profit is greater than the sum of firm profits under duopoly.

When firms compete in price, the merged entity's profit equals $\frac{2(1-\beta)(1-\gamma)+\beta^2}{4(1+\gamma)(1-\gamma)}$ and the sum of firm profits under duopoly equals $\frac{(-\gamma-\gamma^2+\beta\gamma+2)^2+(-2\beta-\gamma-\gamma^2+\beta\gamma^2+2)^2}{(1-\gamma)(\gamma+1)(\gamma-2)^2(\gamma+2)^2}$. A comparison between the profits reveals that

$$\frac{2(1-\beta)(1-\gamma)+\beta^2}{4(1+\gamma)(1-\gamma)}-\frac{(-\gamma-\gamma^2+\beta\gamma+2)^2+(-2\beta-\gamma-\gamma^2+\beta\gamma^2+2)^2}{(1-\gamma)(\gamma+1)(\gamma-2)^2(\gamma+2)^2}>0$$

This indicates that, when the firms compete in price, merger profit is greater than the sum of firm profits under duopoly.

These results are summarized in the following:

PROPOSITION 5: *For the entire parameter range, firms have an incentive to merge without considering how to split merged entity's profit between the firms.*

Firms always have an incentive to merge into a single entity when we consider the case in which the overall post-merger profit is higher than the sum of their pre-merger profits. It is then to the firms to decide the distribution of the merger profit.

Chapter 5

Concluding Remarks

In this report, we analyze the incentives of duopolistic firms for merging into a single entity when the firms are asymmetric in production costs. We derive equilibrium profits for both high-cost and low-cost firms and compare these results under different parameter ranges and competition strategies. After we have identified conditions for merger profitability, we investigate the effects of the merger on consumer benefits and the society as a whole.

We find that for almost the entire parameter range, the high-cost firm has an economic incentive to merge with the low-cost firm. Nevertheless, the low-cost firm has no incentive to merge except under very restrictive conditions ($\beta < 1-\gamma$, where $\gamma \geq 0.9$ and $\beta \leq 0.05$). From the perspective of a low-cost firm, in a market providing differentiated products only consider merger when product similarity is sufficiently high and the cost differential is sufficiently low. From the perspective of a high-cost firm, it is always beneficial to merge with a low-cost firm. In the analysis, the share of merger profits to a firm is calculated on the basis of cost efficiency. The economic implications of the results are as follows. Only when the degree of product similarity is sufficiently high and the marginal cost differential is sufficiently low will both low-cost and high-cost firms share the common interest to merge. We find that consumers are hurt by a merger of two firms that provide closer substitutes ($\gamma \geq 0.9$) since the merged entity has an incentive to restrict the production and increase the prices as we have shown in Proposition 3. Our analysis indicates that upon a merger under strict conditions firms benefit from mergers at the expense of consumers and the society as a whole.

We also discuss the profitability of merger by looking at whether the merger entity's profit exceeds the sum of firm profits under duopolistic competition, without considering how the merger profit is distributed. In this case, we find that firms always find it profitable to merger because the overall post-merger profit exceeds the sum of pre-merger profits for the entire parameter range.

In terms of implications for antitrust policy, the government should decide its goal of regulating mergers with differentiated products first. For instance, if the goal is to protect consumer benefits, a merger under this condition should be prohibited; if the goal is to stimulate the industrial growth, the merger should be allowable. Viewed from the perspective of production efficiency, our analysis indicates that there exists the fundamental conflict between the maximization of consumer and social welfare and the maximization of merger profits. Some caveats should be mentioned, however. First, the analysis does not allow for the possibilities of cost synergies, which play an important role in affecting the merger decisions of competition firms. Second, the analysis does not consider the important element of product quality. Third, a potentially interesting extension is to allow for the endogeneity of market structure. The generality of the findings in this report thus requires future research.

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Appendix

This appendix presents detailed results for equilibrium profits and their differences under different parameter ranges and market strategies.

A - 1.

Table 1 – ***Summary of Profits when the firms engage in either quantity or price competition***

<table>
<tr><td colspan="5">Profit under different parameter range</td></tr>
<tr><td colspan="2">Parameter Range</td><td>$\beta < 1-\gamma$</td><td colspan="2">$1-\gamma \le \beta \le 1-\frac{\gamma}{2}$</td></tr>
<tr><td rowspan="4">Quantity Competition</td><td rowspan="2">Merged</td><td>$\pi_{M1} = \left(\frac{\beta+1}{4}\right)\frac{2(1-\beta)(1-\gamma)+\beta^2}{(\beta+2)(1+\gamma)(1-\gamma)}$ (9)</td><td colspan="2">$\pi_{M1} = \frac{\beta+1}{4(\beta+2)}$ (12)</td></tr>
<tr><td>$\pi_{M1} = \frac{2(1-\beta)(1-\gamma)+\beta^2}{4(\beta+2)(1+\gamma)(1-\gamma)}$ (10)</td><td colspan="2">$\pi_{M2} = \frac{1}{4(\beta+2)}$ (13)</td></tr>
<tr><td rowspan="2">Duopoly</td><td colspan="3">$\pi_1^{QC} = \frac{(-\gamma+\beta\gamma+2)^2}{(\gamma-2)^2(\gamma+2)^2}$ (15)</td></tr>
<tr><td colspan="3">$\pi_2^{QC} = \frac{(2\beta+\gamma-2)^2}{(\gamma-2)^2(\gamma+2)^2}$ (16)</td></tr>
<tr><td colspan="2">Parameter Range</td><td>$\beta \le 1-\gamma$</td><td>$1-\gamma \le \beta \le 1-\frac{\gamma}{2-\gamma^2}$</td><td>$1-\frac{\gamma}{2-\gamma^2} \le \beta \le 1-\frac{\gamma}{2}$</td></tr>
<tr><td rowspan="4">Price Competition</td><td rowspan="2">Merged</td><td>$\pi_{M1} = \left(\frac{\beta+1}{4}\right)\frac{2(1-\beta)(1-\gamma)+\beta^2}{(\beta+2)(1+\gamma)(1-\gamma)}$ (9)</td><td>$\pi_{M1} = \frac{\beta+1}{4(\beta+2)}$ (12)</td><td>$\pi_{M1} = \frac{\beta+1}{4(\beta+2)}$ (12)</td></tr>
<tr><td>$\pi_{M1} = \frac{2(1-\beta)(1-\gamma)+\beta^2}{4(\beta+2)(1+\gamma)(1-\gamma)}$ (10)</td><td>$\pi_{M2} = \frac{1}{4(\beta+2)}$ (13)</td><td>$\pi_{M2} = \frac{1}{4(\beta+2)}$ (13)</td></tr>
<tr><td rowspan="2">Duopoly</td><td>$\pi_1^{PC} = \frac{(-\gamma-\gamma^2+\beta\gamma+2)^2}{(1-\gamma)(1+\gamma)(\gamma-2)^2(\gamma+2)^2}$ (24)</td><td>$\pi_1^{PC} = \frac{(-\gamma-\gamma^2+\beta\gamma+2)^2}{(1-\gamma)(1+\gamma)(\gamma-2)^2(\gamma+2)^2}$ (21)</td><td>$\pi_1^{PC} = \frac{(1-\beta)(\beta+\gamma-1)}{\gamma^2}$ (18)</td></tr>
<tr><td>$\pi_2^{PC} = \frac{(-2\beta-\gamma-\gamma^2+\beta\gamma^2)}{(1-\gamma)(1+\gamma)(\gamma-2)^2(\gamma+2)^2}$ (25)</td><td>$\pi_2^{PC} = \frac{(-2\beta-\gamma-\gamma^2+\beta\gamma^2)}{(1-\gamma)(1+\gamma)(\gamma-2)^2(\gamma+2)^2}$ (22)</td><td>$\pi_2^{PC} = 0$ (19)</td></tr>
</table>

A - 2. *The case when* $1-\frac{\gamma}{2-\gamma^2} \le \beta \le 1-\frac{\gamma}{2}$

Firm 1:

$$(15) - (18) = \pi_1^{QC} - \pi_1^{PC} = \frac{(-\gamma+\beta\gamma+2)^2}{(\gamma-2)^2(\gamma+2)^2} - \frac{(1-\beta)(\beta+\gamma-1)}{\gamma^2}$$

$$= \frac{\left[2(\beta-1)+\gamma\right]\left[8(\beta-1)+4\gamma^2(1-\beta)-\gamma^4(1-\beta)+4\gamma\right]}{\gamma^2(\gamma-2)^2(\gamma+2)^2} < 0$$

$$(18) - (12) = \frac{(1-\beta)(\beta+\gamma-1)}{\gamma^2} - \frac{\beta+1}{4(\beta+2)} = -\frac{8(1-\gamma)+4\beta(\beta^2-3)+\gamma(1+\beta)(\gamma+4\beta)}{4\gamma^2(\beta+2)} > 0$$

Firm 2:

$$(13) - (16) = \frac{1}{4(\beta+2)} - \frac{(2\beta+\gamma-2)^2}{(\gamma-2)^2(\gamma+2)^2}$$

$$= \frac{\left[8(6\beta+4\gamma)+\gamma^4\right]-16(\beta^3+1)-4\gamma^2(4+\beta)-16\beta\gamma(1+\beta)}{4(\gamma-2)^2(\gamma+2)^2(\beta+2)} > 0$$

$$(16) - (19) = \frac{(2\beta+\gamma-2)^2}{(\gamma-2)^2(\gamma+2)^2} - 0 = \frac{(2\beta+\gamma-2)^2}{(\gamma-2)^2(\gamma+2)^2} > 0$$

A - 3. ***The case when*** $1-\gamma \le \beta \le 1-\frac{\gamma}{2-\gamma^2}$

Firm 1:

$$(15) - (21) = \frac{(-\gamma+\beta\gamma+2)^2}{(\gamma-2)^2(\gamma+2)^2} - \frac{(-\gamma-\gamma^2+\beta\gamma+2)^2}{(1-\gamma)(1+\gamma)(\gamma-2)^2(\gamma+2)^2}$$

$$= \gamma^3 \frac{\beta\gamma(2-\beta) - 2(\beta+\gamma-1)}{(1-\gamma)(1+\gamma)(\gamma-2)^2(\gamma+2)^2} > 0$$

$$(12) - (15) = \frac{\beta+1}{4(\beta+2)} - \frac{(-\gamma+\beta\gamma+2)^2}{(\gamma-2)^2(\gamma+2)^2}$$

$$= \frac{\gamma(1+\beta)(\gamma^3-16\beta)+4\beta\gamma^2(1-\beta^2)+16\left[\gamma(2-\gamma)-1\right]}{4(\gamma-2)^2(\gamma+2)^2(\beta+2)} < 0$$

Firm 2:

$$(16) - (22) = \frac{(2\beta+\gamma-2)^2}{(\gamma-2)^2(\gamma+2)^2} - \frac{\left(-2\beta-\gamma-\gamma^2+\beta\gamma^2\right)}{(1-\gamma)(1+\gamma)(\gamma-2)^2(\gamma+2)^2}$$

$$= \frac{\gamma^3\left[\beta\gamma(2\text{-}\beta)\text{-}2(\beta+\gamma\text{-}1)\right]}{(1-\gamma)(1+\gamma)(\gamma-2)^2(\gamma+2)^2} > 0$$

$$(13) - (16) = \frac{1}{4(\beta+2)} - \frac{(2\beta+\gamma-2)^2}{(\gamma-2)^2(\gamma+2)^2}$$

$$= \frac{16\left[(2\gamma-1)-\beta\left(\beta^2-3\right)-\beta\gamma(\beta+1)\right]-\gamma^2\left[4(\beta+4)-\gamma^2\right]}{4(\gamma-2)^2(\gamma+2)^2(\beta+2)} > 0$$

A - 4. ***The case when*** $\beta < 1-\gamma$

Firm 1:

$$(15) - (24) = \frac{(-\gamma+\beta\gamma+2)^2}{(\gamma-2)^2(\gamma+2)^2} - \frac{\left(-\gamma-\gamma^2+\beta\gamma+2\right)^2}{(1-\gamma)(1+\gamma)(\gamma-2)^2(\gamma+2)^2}$$

$$= \gamma^3 \frac{\beta\gamma(2-\beta)-2(\beta+\gamma-1)}{(1-\gamma)(1+\gamma)(\gamma-2)^2(\gamma+2)^2} > 0$$

$$(9) - (15) = \left(\frac{\beta+1}{4}\right)\frac{2(1-\beta)(1-\gamma)+\beta^2}{(\beta+2)(1+\gamma)(1-\gamma)} - \frac{(-\gamma+\beta\gamma+2)^2}{(\gamma-2)^2(\gamma+2)^2} < 0$$

Firm 2:

$$(16) - (25) = \frac{(2\beta+\gamma-2)^2}{(\gamma-2)^2(\gamma+2)^2} - \frac{\left(-2\beta-\gamma-\gamma^2+\beta\gamma^2\right)}{(1-\gamma)(1+\gamma)(\gamma-2)^2(\gamma+2)^2}$$

$$= \gamma^3 \frac{\beta\gamma(2-\beta)-2(\beta+\gamma-1)}{(1-\gamma)(1+\gamma)(\gamma-2)^2(\gamma+2)^2} > 0$$

$$(10) - (16) = \frac{2(1-\beta)(1-\gamma)+\beta^2}{4(\beta+2)(1+\gamma)(1-\gamma)} - \frac{(2\beta+\gamma-2)^2}{(\gamma-2)^2(\gamma+2)^2}$$

$$= \frac{16\beta\left[(1+\gamma)+\beta(1-\gamma)-\beta^2(1-\gamma^2)\right]-8\gamma^2(1-\beta^2)(2\gamma-1)-2\gamma^4\left[\gamma(1-\beta)-(5+\beta)+\beta\gamma^2(\beta\gamma^2-36)\right]}{4(\beta+2)(1+\gamma)(1-\gamma)(\gamma-2)^2(\gamma+2)^2} > 0$$

A - 5. *The case when* $\beta < 1-\gamma$ *where* $\gamma > 0.9$ *and* $\beta \leq 0.05$

Firm 1:

$$(9) - (15) = \left(\frac{\beta+1}{4}\right)\frac{2(1-\beta)(1-\gamma)+\beta^2}{(\beta+2)(1+\gamma)(1-\gamma)} - \frac{(-\gamma+\beta\gamma+2)^2}{(\gamma-2)^2(\gamma+2)^2} > 0$$