SPECTRAL ANALYSIS OF APERIODIC DIGITAL SIGNALS

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CHAPTER I

INTRODUCTION

This report can essentially be divided into two parts. In the first part, the notion of the time-varying Fourier transform which was introduced recently [1] for noise-free discrete signals is extended to the case when noise is present. This is achieved by introducing some damping in the filtering process associated with the time-varying Fourier transform. The second part presents a study of Arnold's on-line computational algorithm [2].

It is demonstrated that Arnold's algorithm can be used to good advantage prior to classification for signal representation of transient waveforms which are embedded in noise. It is especially useful for those cases in which the signal to noise ratio is low. Examples of such applications include the following [2]:

(1) The classification of moving traffic in the presence of a noisy background.

(2) The discrimination between classes of seismic events in a noisy background.

(3) The classification of both man-made and biologically generated transients in an ocean background.

If the transient waveforms are well above the noise to the extent that their epoch and duration can be easily determined, then the fast Fourier transform (FFT) technique [3] may be used as a means of signal representation prior to classification. However, in applications of the type cited above, the transients are usually well below the level of the noise. Consequently their epochs and durations are not known. To overcome this, one could choose an upper bound T_{max} as the duration of any transient of interest and then use the FFT technique. However, this is not satisfactory since the arbitrary segmentation could result in spectral distortion.

Next, it is demonstrated that the time-varying Fourier transform, on being modified as cited above, can be used as an effective tool to estimate the Fourier power and phase spectra of transients in the presence of noise. Again, it is shown that such estimation can be achieved with any desired frequency resolution. In conclusion, it is demonstrated that the spectral estimates so obtained are superior to those obtained by standard smoothing techniques.

The above applications are considered in Chapter IV of this report. In Chapters II and III some fundamental aspects of Arnold's algorithm and the time-varying Fourier transform are presented. Conclusions pertaining to the study undertaken and corresponding recommendations for future work are presented in Chapter V.

CHAPTER II

TIME-VARYING FOURIER TRANSFORM AND RELATED SPECTRA

2.1 Fourier Transform of a Discrete Signal

Let X(t) be a continuous real-valued signal. It can be approximated to a series of rectangular segments of width Δt as shown in Figure 2.1 (a). Now the signal can be represented by the sequence of impulses X*(t) shown in Figure 2.1(b). The impulse at the point $m\Delta t$ has a strength Δt X(m), which is area of a rectangle formed by the base Δt and the height X(m) at $m\Delta t$ in Figure 2.1(a).

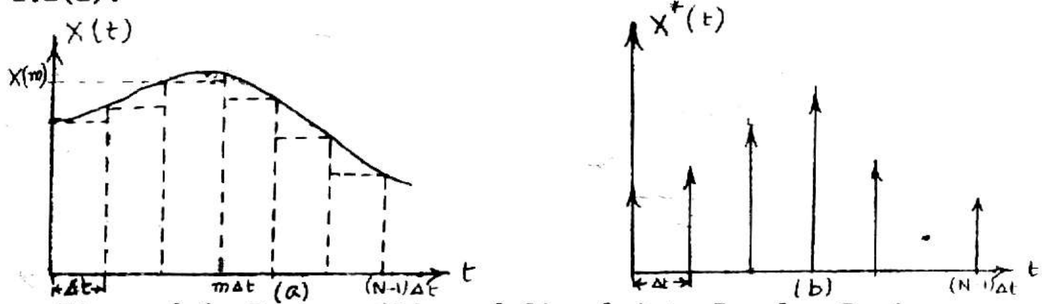

Figure 2.1 Decomposition of Signal into Impulse Train

The discrete signal X*(t) can be written as*

$$X^*(t) \simeq \sum_{m=0}^{N-1} \left[\Delta t\, X(m)\right] \delta(t-m\Delta t) \qquad (2\text{-}1a)$$

where $\delta(t)$ is the Dirac or impulse function. It X(t) has a bandwidth of B Hz, then from Shannon's Sampling theorem Δt must be less than or equal to 1/2B sec.

Fourier transform $F_x^*(\omega)$ of X*(t) is given by

$$F_x^*(\omega) = \Delta t \int_{-\infty}^{\infty} \sum_{m=0}^{N-1} X(m)\, \delta(t-m\Delta t)\, e^{-i\omega t}\, dt \qquad (2\text{-}1b)$$

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*See for example, "Lin. Network Analysis", by Seshu and Balabanian, John Wiley, 1959, p.205.

Interchanging the order of integration and summation in Eq. (2-1b) we obtain

$$F_{x*}(\omega) = \Delta t \sum_{m=0}^{N-1} X(m) \int_{-\infty}^{\infty} \delta(t-M\Delta t) e^{-i\omega t} dt$$

$$= \Delta t \sum_{m=0}^{N-1} X(m) e^{-im\omega\Delta t} \qquad (2\text{-}2)$$

$$= \Delta t \sum_{m=0}^{N-1} X(m) \mathrm{Cos}(\omega m\Delta t) - i \sum_{m=0}^{N-1} X(m) \mathrm{Sin}(\omega m\Delta t) \qquad (2\text{-}3)$$

that is,

$$F_{x*}(\omega) = \Delta t\{R(\omega) - iI(\omega)\} \qquad (2\text{-}4)$$

where

$$R(\omega) = \sum_{m=0}^{N-1} X(m) \mathrm{Cos}(\omega m\Delta t)$$

and

$$I(\omega) = \sum_{m=0}^{N-1} X(m) \mathrm{Sin}(\omega m\Delta t)$$

$F_{x*}(\omega)$ in Eq. (2-4) is the desired Fourier transform of a given discrete signal $X^*(t)$.

2.2 Time Varying Fourier Transform [1]

$F_{x*}(\omega)$ in Eq. (2-4) can be expressed in terms of a (2x1) vector to obtain,

$$\underline{F}_{x*}(\omega) = \Delta t \begin{bmatrix} R(\omega) \\ I(\omega) \end{bmatrix} \qquad (2\text{-}5)$$

Now, consider the (2x2) matrix

$$\left[L(\omega)\right] = \begin{bmatrix} \mathrm{Cos}(\omega\Delta t) & -\mathrm{Sin}(\omega\Delta t) \\ \mathrm{Sin}(\omega\Delta t) & \mathrm{Cos}(\omega\Delta t) \end{bmatrix} \qquad (2\text{-}6)$$

Since $\left[L(\omega)\right]$ is orthogonal, it has the property

$$\left[L(\omega)\right]^m = \begin{bmatrix} \text{Cos}(m\omega\Delta t) & -\text{Sin}(m\omega\Delta t) \\ \text{Sin}(m\omega\Delta t) & \text{Cos}(m\omega\Delta t) \end{bmatrix} \quad (2\text{-}7)$$

Thus, from Eqs. (2-5) and (2-7) it follows that,

$$\underline{F}_{x*}(\omega) = \Delta t \sum_{m=0}^{N-1} \left[L(\omega)\right]^m \underline{b} X(m) \quad (2\text{-}8)$$

where

$$\underline{b} = \begin{bmatrix} 1 \\ 0 \end{bmatrix}$$

We introduce the recurrence relation

$$\underline{Z}(\omega,k) = \left[L(\omega)\right] \underline{Z}(\omega,k-1) + \underline{b} X(N-1-k)$$
$$k = 0,1,2,\ldots,(N-1) \quad (2\text{-}9)$$

where k denotes the instant of time $t=k\Delta t$ and

$$\underline{Z}(\omega,-1) = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$$

With k=0, and k=1, Eq. (2-9) yields

$$\underline{Z}(\omega,0) = \left[L(\omega)\right] \underline{Z}(\omega,-1) + \underline{b} X(N-1) \quad (2\text{-}10)$$

and

$$\begin{aligned} \underline{Z}(\omega,1) &= \left[L(\omega)\right] \underline{Z}(\omega,0) + \underline{b} X(N-2) \\ &= \left[L(\omega)\right] \{ \left[L(\omega)\right] \underline{Z}(\omega,-1) + \underline{b} X(N-1) \} + \underline{b} X(N-2) \\ &= \left[L(\omega)\right]^2 \underline{Z}(\omega,-1) + \left[L(\omega)\right] \underline{b} X(N-1) + \underline{b} X(N-2) \end{aligned} \quad (2\text{-}11)$$

Proceeding on similar lines we arrive at the fundamental relationship

$$\underline{Z}(\omega,N-1) = \sum_{m=0}^{N-1} \left[L(\omega)\right]^m \underline{b} X(m) = \frac{1}{\Delta t} \underline{F}_{x*}(\omega) \quad (2\text{-}12)$$

where $\underline{F}_{x*}(\omega)$ is the (2x1) vector defined in Eq. (2-5). Eq. (2-12) states that, at the instant k=(N-1), $\underline{Z}(\omega,k)$ in Eq. (2-9) yields the conventional Fourier transform defined in Eq. (2-4). $\underline{Z}(\omega,k)$ can therefore be defined as