ELASTIC BUCKLING OF RIGID FRAMES

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SYNOPSIS

The objective of this report is to get more accurate values in the stability analysis of rigid frames by a matrix formulation of the "exact" method using an electronic digital computer.

As the basic equations, slope deflection equations for an axially loaded member are derived expressing the end moments and shears as a function of its end slopes and lateral displacement considering the influence of axial force in the member. Since the stiffness of the axially loaded member depends on the axial force, the stiffness coefficient is expressed as a function of the axial force.

The well-known displacement method for the first-order frame analysis is modified to include the influence of axial forces. In the relationship for equilibrium between external and internal forces, a second-order spring matrix is added to account for the effect of axial forces on the end moments due to lateral displacement. The deformation matrix does not need to be modified and is still the transpose of the statics matrix as has been proved [1].* In the stiffness matrix, the stiffness coefficients for every axially loaded member are replaced with the modified values due to the axial forces. The buckling criterion is set up by equating the determinant of the load-displacement matrix to zero.

After discussing the computational procedure, a computer

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* Figures in brackets denote references in bibliography

program is described. The program is written in FORTRAN IV language for the IBM 1130 computer to get the buckling loads in the fundamental mode and the effective length ratio for every axially loaded member in the frame. A simple beam and two rigid frames are used as numerical examples, showing the input and output.

To compare the design by this "exact" method with the conventional design using the AISC Specification [2], a portal frame with an infinitely stiff horizontal member is analyzed for four different special conditions and a table for K values is made in each condition. Two typical numerical examples are designed by the two methods using the K factors offered by the existing AISC Specification [2] and the values in the tables provided by the "exact" method.