Flat Plate Voided Slabs: A Lightweight Concrete Floor System Alternative

by

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B.S., Kansas State University, 2018

A REPORT

submitted in partial fulfillment of the requirements for the degree

MASTER OF SCIENCE

Department of Architectural Engineering
College of Engineering

KANSAS STATE UNIVERSITY
Manhattan, Kansas

2018

Approved by:

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Abstract

In structural engineering, it can be challenging to incorporate a sustainable design without sacrificing structural integrity. However, flat plate voided slabs are an interesting alternative to standard flat plate concrete slab systems due to the reduction in concrete and the recycled plastic void formers that are located inside the slab. This research is necessary because an increased use of voided slabs in concrete structures would help fight climate change by reducing the CO_2 emissions caused from cement production.

This report will discuss the advantages and disadvantages of implementing plastic void formers into solid flat plate slabs and examine a parametric study comparing voided flat plate slabs to solid flat plate slabs. The design of the voided slabs follows the *CRSI Design Guide for Voided Concrete Slabs* while also referencing the ACI 318-14 *Building Code Requirements for Structural Concrete*. Three different slabs for typical square bay sizes of 25 feet, 30 feet, and 35 feet are designed to compare the effectiveness of voided slabs to traditional solid slabs.

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Acknowledgements

I am very thankful for the encouragement and support I have received throughout my college career from my family, friends, classmates, and professors. I would especially like to thank my major professor, Dr. Bill Zhang, and committee members, Professor Kimberly Kramer and Associate Professor Fred Hasler, for all of the knowledge they have shared with me throughout this process. A lot of the information I found about voided slabs came from Michael Rusillo, President of Cobiax USA, which was very helpful in writing this report.

Chapter 1 - Introduction

Reinforced concrete is the most commonly used building material in the world. Over 2 billion tons of concrete is produced annually and that quantity will continue to increase in years to come. Its use can range from a common sidewalk to the Burj Khalifa, the tallest manmade structure in the world (Crow, 2008). Reinforced concrete has a wide variety of uses and is a common material for structural floor systems. Common reinforced concrete floor systems include post-tensioned slabs, one-way pan-joists, hollow core planks, one-way flat plate slabs, and two-way flat plate slabs. Each of these floor systems have advantages and disadvantages and are efficient systems for certain scenarios. This report will primarily focus on two-way flat plate slabs and the implementation of putting plastic hollow voids inside the slab to reduce the self-weight of the structure by reducing the volume of concrete and these advantages and disadvantages are further discussed in Chapter 3.

The concept of putting voids into concrete slabs is not new, but recent innovations have increased the efficiency of the voided slab concept. Chapter 2 examines the history and the evolution of voided slabs from ancient times through present day. There are multiple producers of plastic void formers in the United States and they each have a variety of products, which will be discussed in Chapter 4. Chapter 5 discusses current projects that have implemented flat plate voided slabs in North America.

From a statics perspective, voided slabs eliminate concrete in the areas where it is not necessary, while optimizing the slab's thickness and volume of concrete (Mota, 2013). By reducing the self-weight of the slab, the gravity loads transferred into the columns and foundation are also minimized, which will result in smaller columns and foundations. Ultimately, the overall volume of building material will be reduced, which will save the owner money.

Chapter 6 discusses the design process for properly designing flat plate voided slabs with the *CRSI Design Guide for Voided Slabs*. The results for the reduction in dead load by implementing voids into the traditional flat plate slab is presented in Chapter 7. The reduction in the volume of concrete reduces the amount cement production needed. Chapter 8 discusses the sustainability factor of voided slabs since cement production is a major cause of carbon dioxide emissions into the atmosphere.

The reduction in the dead load of the structure will also decrease the seismic loads in the building and reduce the size of the members in the lateral force resisting system. For structures in high seismic regions, the lateral force resisting system is often the controlling factor in cost over the gravity system. Therefore, any reduction in the dead load will decrease the seismic base shear, which will minimize the forces onto the lateral force resisting system and reduce the cost of the lateral members. Chapter 9 further discusses the economics and constructability of flat plate voided slabs.

Chapter 2 - The History and Evolution of Voided Slab Systems

The concept of putting voids in slabs dates back to ancient times. Builders had difficulty spanning long distances due to the relatively high dead weights of the available building materials. Over the years, building materials have increased in quality when it comes to strength and lighter materials can be implemented into structures. However, there has still been a lot of innovation and evolution in the voided slab concept to decrease the dead load and volume of material used.

The Pantheon

The first known example of a voided slab system dates back to the construction of the coffered ceiling dome of the Pantheon in Rome, which was completed in 125 AD. The dome spans over 140 feet and was the largest unreinforced concrete dome in the world until the construction of the Florence Cathedral in 1436. The engineers and constructors of the Pantheon eliminated the concrete at locations in the slab where the entire cross section is not needed to resist the loads. The voids in the dome were formed with external voided formwork adding a decorative element to the roof due to its unique shape. Unreinforced concrete domes would never be designed today due to code restrictions, but the dome of the Pantheon is still standing strong nearly 2000 years later.

Contemporary Voided Slabs

The 1950s brought a lot of change in the structural engineering world, especially when it came to structural floor systems. The one-way pan-joist increased in popularity due to the modular removable forms. However, the constructability and labor costs were an issue that stemmed from the close spacing of the joists that needed modified. The wide-module “skip” joist system was introduced in the 1960s, which increased the spacing of the joists allowing for quicker construction and thicker slabs that met the stringent fire codes. The “joists” in these systems would no longer be considered joists in modern concrete codes due to the spacing and would now be considered beams since shear reinforcement would be required.

Pan-joist systems often require deep girders for greater spans to minimize deflection. These deep members increase the floor-to-floor heights of buildings, which ultimately increases the cost due to the increase in material needed. To create a more efficient structural system, post-tensioning can be implemented into the girders to decrease the depth of the members to the same height as the pan-joists to create simpler formwork and lower floor-to-floor heights (CRSI, 2014). Pan-joist systems are still common in the Midwest, but are not as popular as they once were due to a change in architectural aesthetics and innovations in concrete floor systems.

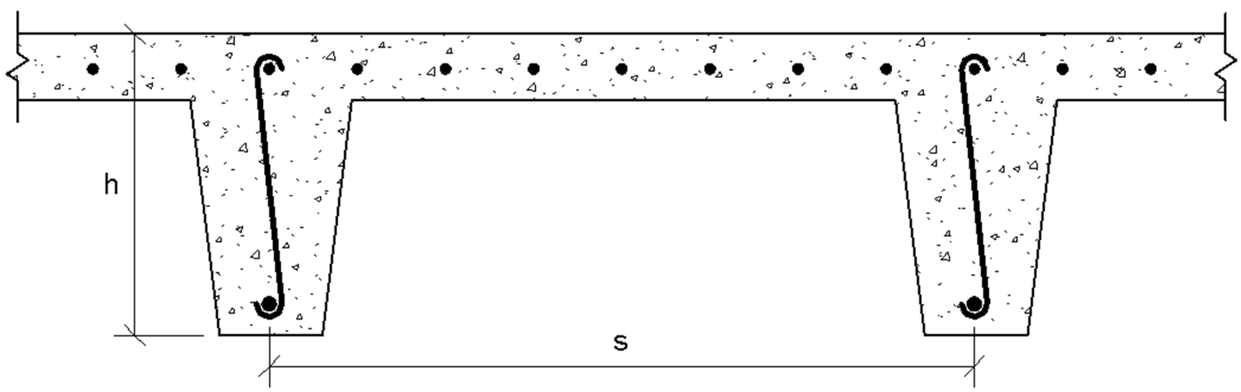


Figure 2-1: Pan-Joist System

Waffle slabs are a two-way joist system that rose to popularity around the same time as the pan-joist system. Waffle slabs are an economical system for long spans with high loads. Waffle slabs are a similar concept to the pan-joist system in that they have removable formwork. These systems are typically left exposed for a particular architectural look. The most well-known exposed waffle slab is in the roof of the Washington DC Metro Station, which is shown in Figure 2-2 below.

Figure 2-2: Waffle Slab in Washington DC Metro Station

To speed up construction, precast floor systems increased in popularity in the 1950s. The most common precast voided slab system is the hollow-cored plank, which can be used in steel-framed or concrete-framed structures. These one-way precast planks have voids running linearly along the length of the slab. There are many advantages to hollow-core planks, which include prefabrication and the elimination of on-site formwork and a flat soffit, but the spans of the planks are limited. Hollow-core planks are still a viable and popular floor-system choice in the United States.

Modern Voided Slabs

A new type of concrete voided slabs was created in the early 1990s in Europe that puts the voids inside the concrete with leave-in formwork. These flat plate voided slabs were pioneered primarily by Cobiax and Bubbledeck, who led design and production of the hollow void formers made of high-density polyethlyene (HDPE) plastic that are locked inside the concrete slabs. This variation of voided slab found success in a wide variety of buildings and can span up to 50 feet with traditional reinforcement and 60 feet with post-tensioning.

Throughout the 1990s and 2000s, flat plate voided slabs were utilized all over Europe and South America. This system gradually found its way into North America in the late 2000s. The first voided slab project constructed in North America was the Perez Art Museum in Miami, which was completed in 2011. Miami has strict environmental limits in its building codes and long clear spans were needed for this project, so the structural engineers decided to implement the innovative European voided slab system into an American project for the first time, which resulted in a reduction of material and cost. This project will be discussed in more depth in Chapter 5.

Chapter 3 - Advantages and Disadvantages

Every structural system has its advantages that set itself apart from other systems, but all systems also have disadvantages. However, some have more disadvantages than others do. Some systems are only effective for certain regions, which is often dependent on available materials and skilled labor. System selection is also determined based on the type of the building and the available budget. Flat plate voided slabs offer many advantages that make it a viable system for many types of buildings, but there are also some disadvantages that need to be weighed against the advantages by the architects, engineers, and owners involved.

Advantages

The most obvious advantage to flat plate voided slabs is the reduction in concrete. If designed efficiently, voided slabs can reduce the volume of concrete up to 35% compared to solid flat plate concrete slabs with similar spans and strength. The reduction in concrete results in less dead load applied to the rest of the structure, which will decrease the size of the columns and foundations. Lower self-weight of the slab will also reduce the effective seismic weight of the structure leading to a lower base shear for the seismic lateral loads. This would also reduce the size of the lateral force resisting system members, which would result in even more concrete savings and ultimately, reduce the cost of the structure.

Reducing the amount of concrete needed for a project, would also have a positive environmental impact. One of the main components of concrete is Portland cement, which creates high levels of carbon-dioxide emissions into the environment during production. The sustainability benefits of voided slabs will be discussed more in Chapter 8, but this is another important advantage to implement voids into the slabs to reduce the volume of concrete.

Another advantage to flat plat voided concrete slabs compared to other systems is the reduction in floor-to-floor heights, especially for two-way slabs. Since it is a flat plate slab system, there are no beams, which allows for the architects to lower the floor-to-floor heights or create more plenum space for the mechanical, electrical, and plumbing systems. A typical composite steel structure would have a concrete filled steel deck at least 4” deep that is supported by steel members ranging from 12 inches to 30 inches, depending on the loads and span. A concrete pan-joist system would have joists ranging from around 18 inches to 28 inches deep, which is another version of a one-way voided slab.

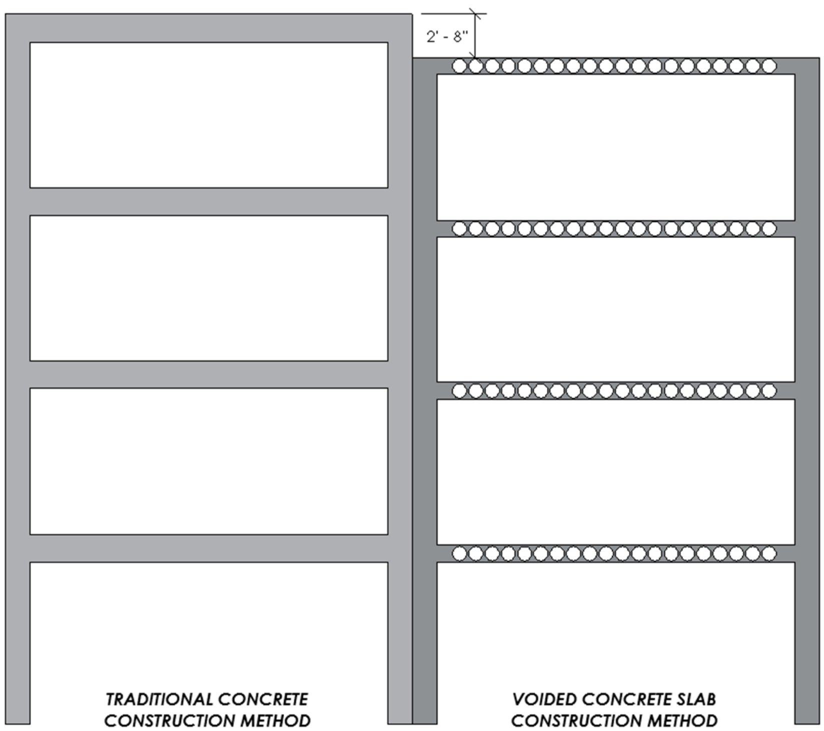


Figure 3-1: Floor-to-Floor Height Reduction of Flat Plate Voided Slabs

From the parametric study comparing flat plate voided slabs to flat plate solid slabs for typical bays of 25 feet, 30 feet, and 35 feet, the slab depth for the voided slabs ranged from 10 inches to 16.5 inches. The depth of the flat plate voided slabs could reduce the height of the structural system up to one foot per floor compared to a traditional concrete floor system, which could drastically reduce the overall height of the building for a mid-rise or high-rise structure. Figure 3-1 provides a visual representation of the difference in floor-to-floor height for a flat plate voided concrete slab next to a traditional concrete system. Reducing the height of the building would save a lot of money for the owner by reducing the area of the exterior façade and other building materials needed for construction. Flat plate voided concrete slabs also simplify the design for the other systems by creating a flat surface on the bottom side of the slab, unlike other structural systems that have beams and joists that create road blocks for the ductwork, pipes, and conduit.

Concrete is inherently the best building material for fire resistance and vibration control. Concrete is a prime material for fire resistance due to it being non-combustible and having a slow rate of heat transfer and the high mass required compared to steel and timber. Flat plate concrete slabs have a greater thickness than that of slabs in traditional beam systems and pan-joist systems, which makes flat plate slabs a top choice for fire resistance in buildings.

There have been many fire resistance tests conducted by BubbleDeck and Cobiax on flat plate voided slabs to determine a fire rating and to see how the voids effect the fire resistance. These tests have been in accordance with provisions in *Fire Behavior of Building Components; Definitions, Requirements, and Tests*, and have demonstrated a fire-resistance rating of at least 2 hours for slabs with a clear cover of at least ¾ inches.

More testing on flat plate voided slabs was conducted at the Fire Testing Laboratory of NGC Testing Services in Buffalo, NY in June 2017. The test was performed on an 8 inch slab with 4 inch thick ellipsoidal voids and a bay size of 14 feet by 18 feet. This slab thickness was chosen because it is the minimum slab thickness than can be specified for this type of voided slab. During the test, an 80 psf load was applied to the entire slab. The results from the test were quite positive for the voided slab and proved that a 2-hour fire rating is conservative for this structural system. Testing was terminated after 2 hours and 52 minutes and a maximum deflection of 3.5 inches was recorded and there was no sign of collapse throughout the test. One of the plastic void formers did melt during the test from the high temperatures, but none of the reinforcement at this location showed any signs of damage (Fanella, 2018).

Disadvantages

While there are many pros to using a flat plate voided concrete slab system, there are some cons that need to be considered by the parties involved. The most obvious disadvantages come down to the cost of the system and coordination with other systems when placing the voids.

Voided slabs are a light weight concrete system that allow for flexibility in future renovations in that the voids can be drilled through and punctured, unlike post-tensioned slabs that could be severely damaged if the tendons are punctured. However, there is still a lot of coordination required in placing the voids. If heavy equipment is being hung from the ceiling and anchored into the bottom of the slab, there needs to be enough concrete in the tension face at that specific location to allow for adequate anchorage. This is achieved by not putting voids in that area of the slab or by increasing the depth of the slab in that location, but leaving an area unvoided is typically the best option. The structural engineer of record is in charge of the design of the voided slab and coordinating with the other building systems to locate where the unvoided areas are needed, but the structural engineer would send this information to the manufacture of the void formers to design the layout of the voids. The layout of the voids would be reviewed again by the structural engineer of record for approval. This process is similar to the shop drawing process for steel structures. Once the voids are shipped to the job site, the contractor then has to properly arrange the void forms, which can be a daunting process if there are a high amount of irregularities. This process can be labor intensive and a cost analysis would have to be conducted to determine if the cost of the voids and the extra cost of labor is offset by the material savings of the reduced volume of concrete.

Another factor that is limiting the implementation of flat plate voided slab systems in the United States is the unfamiliarity and newness of this system to many American engineers. Structural engineers' number one goal is safety, so designing a system that is a new idea that the engineer is not fully comfortable with can be a difficult sell.

Additionally, the flat plate voided slab system is not programmed into finite element analysis software yet. Structural engineers rely on analysis software to check the adequacy of the structure, so not having this available can be intimidating to most engineers. However, Cobiax has provided tips on how to model voided slabs into finite element analysis software to accurately portray the system. This can be achieved by modeling the slab as a solid slab with a dead load reduction in the areas where there are voids to account for the decreased self-weight of the slab. Also, the modulus of elasticity will be decreased by 10% and a shear resistance reduction of 50% will be included into the analysis. The decreased modulus of elasticity will result in a stiffness reduction of the diaphragm. The shear resistance reduction accounts for the reduced cross sectional area of the slab, which results in less shear strength.

As the project portfolio of flat plate voided slabs continues to grow in America, more knowledge will be available for structural engineers to learn more about this system and more engineers will implement it into structures. Voided slabs are not the best system for every project, but a wide variety of buildings could benefit from the advantages this system has to offer.

Chapter 4 - Variations in Flat Plate Voided Slabs

Although flat plate voided systems are a very specific type of structural system, there can be numerous variations in the slabs. There are multiple manufactures of the void formers to choose from, two different ways to construct the slabs, various shapes and sizes of void formers, and two different ways to distribute the gravity loads.

Manufacturers

There are two primary manufactures of the plastic void formers for flat plate voided slabs: BubbleDeck and Cobiax. Both of these companies originated in Europe and now have offices and production all across the globe. The BubbleDeck technology was created by a Danish structural engineer, Jorgen Breuning, which was the first true practical, biaxial hollow slab. Breuning is credited with the invention of the modern voided slab system and his design was first implemented into the construction of Millennium Tower in Holland, which completed construction in 1999. The flat plate voided slab system in Millennium Tower won many awards for structural engineering innovation, which gained traction for this weight-reducing system across Europe. BubbleDeck completed their first project in the United States in 2011, which is the Labahn Hockey Arena at the University of Wisconsin. BubbleDeck has an American office in Kirkland, Washington.

Cobiax is the other pioneer of the flat plate voided slab system. Cobiax technology became market ready in 2005 and its main office is located in Germany. The Cobiax technology is based on the physics of a bird's wing in that its bone structure forms a "solid frame made out of hollows and struts." The wings do not "carry unnecessary weight, yet is completely stable" (Mota, 2013). The ideas of the bird wing structure are similar to a voided slab in maximizing weight optimization without any static performance decline. The Perez Art Museum in Miami, Florida was the first project in the United States to implement Cobiax technology, which also completed construction in 2011. Cobiax has an American office in Dedham, Massachusetts. Standard void former sizes are also produced in this location for North American projects. For the parametric study in Chapter 7, Cobiax void formers are used for the design of the flat plate voided slabs.

Construction Methods

Both BubbleDeck and Cobiax provide products for two different construction methods. Method one is a slab that is completely cast-in-place with removable formwork. The second method is a partially cast-in-place slab with a bottom layer of pre-cast concrete. The semi pre-cast method allows for quicker construction due to the pre-fabrication process. In fabrication, the bottom layer of pre-cast concrete is cast in modular plank sizes and the void formers and reinforcement are secured in place in top of the bottom layer of the slab. The slab planks are then delivered to the site and lifted into place. This process eliminates the need for on-site removable formwork, which is a time and cost saver. However, there are limitations to when this construction method can be implemented. The semi pre-cast planks have dimensional limitations and can typically only be used for spans up to 28 feet since the planks have to be transported to the site on trucks. Figure 4-1 below shows a section of the semi pre-cast voided slab.

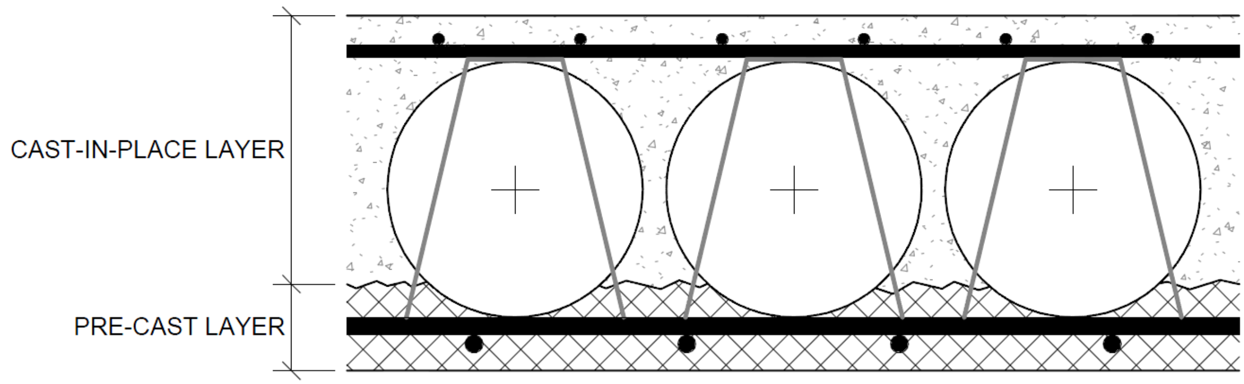


Figure 4-1: Cobiax Semi Pre-Cast Voided Slab Section

The completely cast-in-place slabs are typically poured in two layers to combat honeycombing issues. These slabs require removable on-site formwork that is costly. This construction method is less efficient in terms of cost and construction time, but it allows for more irregularities in the layout of the voids, and flexibility in the spans. Figure 4-2 below shows a section of the completely cast-in-place voided slab and how these slabs are poured in two layers.

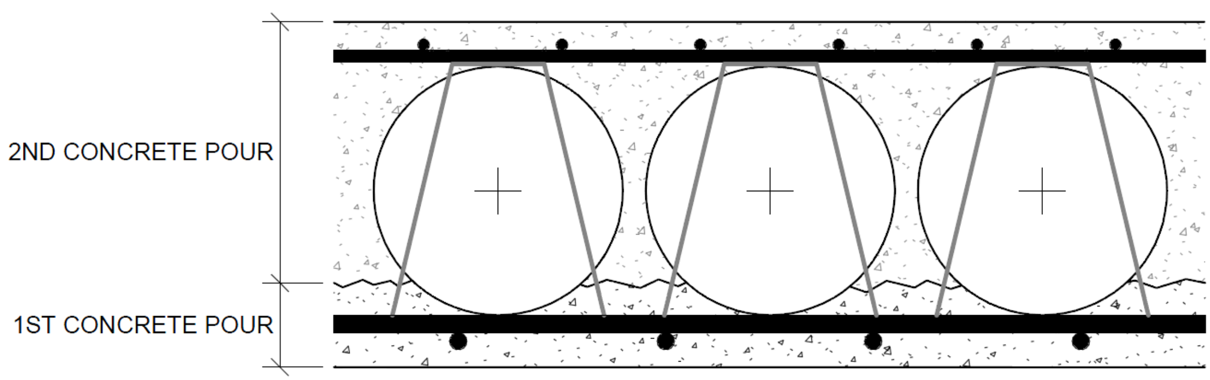


Figure 4-2: Cobiax Cast-in-Place Voided Slab Section

The first step of the construction process of the flat plate voided slabs involves placing the formwork and shoring. For the cast-in-place version of the voided slab system, the formwork is quite simple since there is a flat soffit at the bottom of the slab. For the semi pre-cast version of the voided slab system, formwork is minimal since the pre-cast bottom layer of the slab acts as leave-in-place formwork. Shoring is required for two-way flat plate concrete slabs since there are no beams and the formwork needs supported until the concrete cures.

For cast-in-place voided slabs, the void former cages must be properly arranged, which can be a complex, time-consuming process. The void formers are modularly preassembled in positioning steel wire cages that are typically eight feet long. Once all of the cages are in place on top of the bottom layers of steel reinforcement, the top layers of reinforcement must be arranged. Assembling the top layers of rebar for voided slabs is actually a much simpler process than assembling the top layers of rebar for solid flat plate slabs since the void former cages act as chairs for the reinforcement and the reinforcing bars can be laid directly on top of the cages.

Once all of the void formers and reinforcement are in place, the slab is ready to be poured. In order to ensure that there are no air bubbles in the bottom of the concrete slab under the void formers, the cast-in-place voided flat plate slabs must be poured in two layers. The first layer is thinner and usually just goes slightly above the bottom of the void former. This layer serves the purpose of anchoring the void former cages into place so the concrete vibrators can adequately remove all air bubbles out of the concrete in order to prevent honeycombing in the bottom of the soffit. The second concrete lift can occur after the first concrete lift is complete. The joint between the two layers is intentionally left rough to increase shear friction through aggregate interlock. The joint for the semi pre-cast voided slabs is also left rough for the same purpose. Pouring the slab in two different layers requires a lot more time and can increase the construction schedule.

Void Formers

In addition to construction methods, Cobiax also provides various options in the shapes and sizes of the void formers. The original product line of void formers was the Eco-Line, which is a spherical void that are most economical for slabs that are at least 14 inches in depth. Eco-Line void formers are best for long spans and heavy loads that require relatively deep slabs.

Cobiax created another product line to make thinner slabs possible for voided slabs, which is referred to as the Slim-Line. The Slim-Line void formers are ellipsoidal in shape and can be used in slabs that are as slim as 7 inches. From the Slim-Line product line, three different variations of this product have been created for different slab thicknesses. The thinnest is the Shell-line void former, which is a semi-ellipse shape void that is economical for slabs 7 inches to 8.5 inches deep. The Click-line void former is the same as the original Slim-Line void former and is most efficient for slabs 8 inches to 16 inches thick. The Stack-line void former is two ellipsoidal Click-line void formers stacked on top of each other and is best for long span slabs that are at least 16 inches thick. An analysis should be conducted to determine which product should be used in order to save the most volume of concrete and reduce the most dead load of the structure.

Chapter 5 - Flat Plate Voided Slab Projects in the United States

Flat plate voided concrete slab systems have increased in popularity among the construction industry in recent years, especially in the United States. The first building to implement this system completed construction in 1999 in Europe and the first flat plate voided slab project in the United States completed construction in 2011. Since 2011, there have been numerous projects in all different regions across America that have implemented this design into the structural system.

Perez Art Museum

The Perez Art Museum in Miami was the first project in the United States to utilize Cobiax void formers. Construction for this project was completed in 2013 by general contractor John Moriarty & Associates. This project implemented void formers into the slab in 80,000 of the 120,000 total square feet and reduced the volume of concrete by 935 cubic yards. Slag was also used to replace a portion of the Portland cement in order to lighten the slab. The reduction of concrete and the implementation of slag reduced the axial load on the foundation piles by 3,900 kips (Mota, 2013).

The Perez Art Museum was designed by Swiss architectural firm, Herzog & de Meuron, which wanted to created large open spans in the museum of over 50 feet. In order to achieve these long clear spans without increasing floor-to-floor heights, the structural engineering team from ARUP decided to implement a flat plate voided slab. Over 40 different structural systems were analyzed to determine which system would be most economical with the architectural constraints before the design team selected the voided flat plate slab. To create the most efficient design, ARUP implemented six different void former sizes ranging from 8 inch SlimLine voids to 18 inch EcoLine voids.

The implementation of the Cobiax voided slab technology was an innovative design decision that helped kick-start the flat plate voided slab system in North America. The reduction of greenhouse gas emissions from reduced concrete usage and innovation in environmental performance helped this project reach LEED Gold accreditation. The Perez Art Museum is a great example of how to implement sustainability into structural engineering.

Figure 5-1: Perez Art Museum Voided Slab

Columbia University Medical Center Graduate Education Building

The Columbia University Medical Center Graduate Education Building (CUMGEB) in New York City is a 100,000 square foot, 15-story tower with many geometric complexities. The design team included Diller Scofidio + Renfro (DS+R) in collaboration with executive architect Gensler. Leslie E. Robertson Associates (LERA) provided structural engineering services. F.J. Sciame Construction is the project's construction manager.

A major structural challenge was to provide long, open floor spans with minimal structural depth that could simultaneously accommodate the tight deflection performance requirements of the all-glass façade. The façade engineer concluded that the long-term deflection limit would be 1 ¼” for the slab, which cantilevers up to 26 feet in some locations. The cascading part of the structure has no perimeter columns, which results in a unique sequence of cantilevered voided concrete flat plate slabs. To meet the slab performance requirements, the cantilevered slabs are reinforced with a bonded post-tensioning system. Cobiax void formers are placed between bands of post-tensioning to create long span, beam-like framing with flat formwork and to reduce the structure’s self-weight. The cantilevered slabs utilize high strength concrete and taper in thicknesses from 24 inches at supports to 8 inches at the free end of the cantilevers. (Sesil, 2014).

Figure 5-2: CUMCGEB Cantilevered Flat Plate Voided Slab

Labahn Hockey Arena

The Labahn Hockey Arena serves as the facility for the University of Wisconsin hockey team in Madison, Wisconsin. Construction of this project was completed in 2012 and was the first structure in the United States to implement Bubbledeck voided slab technology. The new Labahn Arena needed to be connected to the existing Kohl Center Arena via an underground walkway. Above the underground walkway, a road capable of supporting the extensive weight of emergency vehicles was needed. This meant the Graef structural team needed to determine how to produce a structure that could support loads from above and provide space for pedestrian and small vehicle traffic below in the underground tunnel.

After analyzing many structural systems, a 21 inch flat plate voided slab proved to be the most efficient system. The structural team selected the semi pre-cast version of the Bubbledeck system to speed up construction and reduce the amount of formwork. The slab spans 35 feet and supports a 250 PSF live load and 3 ½ feet of soil above. The 11,000 square foot voided slab for this walkway resulted in a reduction of 174 cubic yards of concrete and shaved off three days from the construction schedule. These savings in materials and labor resulted in a cost savings of approximately $25,000, or $2 per square foot (Mota, 2013).

Figure 5-3: Bubbledeck Voided Slab at Labahn Hockey Arena

Harvey Mudd College Teaching and Learning Building

The Harvey Mudd College Teaching and Learning Building in Claremont, California was the first academic building in the United States to implement Bubbledeck voided slab technology. This four story, 80,000 square foot structure was completed in 2013 by Matt Construction and was designed by Boora Architects.

The structural engineering team from KPFF decided to utilize a voided slab in the structural system to reduce the seismic weight of the structure since Claremont is located in a high seismic region. The design called for five different void former sizes in the slabs ranging from 9 inches to 20 inches, which reduced the total volume of concrete by 750 cubic yards. The self-weight reduction of the slab resulted in thinner shear walls and lower floor-to-floor heights for the 35 foot clear spans. The owner and the architecture team were pleased with the structural system selection because the flat soffit of the bottom of the semi pre-cast slab was left exposed and the voids in the slab will allow for easier future renovations.

Figure 5-4: Semi Pre-Cast Voided Slab Construction at Harvey Mudd College

Chapter 6 – Design Process

The design process of two-way flat plate voided concrete is similar to the design of a solid two-way flat plate voided slab. However, there are some additional steps required to design the voided slab due to the irregularities in the slab. The design process followed the *CRSI Design Guide for Voided Concrete Slabs* and the ACI 318-14 *Building Code Requirements for Structural Concrete*.

Voided Slabs

Slab Thickness

The first step in the flat plate voided slab design process is determining the slab thickness. There is a minimum slab thickness based on the clear span of the slab, shown in Equation 6-1 (ACI 318-14 Table 8.3.1.1) below. There are no edge beams in the parametric study in Chapter 7 and there are no drop panels around the columns, so the clear span is divided by 30 to determine the minimum thickness.

$$h_{min} = \frac{l_n}{30} \qquad \text{(Equation 6-1)}$$

After the minimum slab thickness is determined, a preliminary slab thickness is selected that is greater than the minimum thickness. However, this thickness might be increased later in the design to create an efficient system that meets the punching shear and deflection criteria.

Void Former Selection

Once a slab thickness is selected, a void former size and shape can be selected. The void former shapes and sizes can be viewed in the Cobiax and Bubbledeck product catalogs. The maximum void former depth is determined by subtracting the required clear cover for the top and bottom of the slab, as well as the diameter of both layers of steel reinforcement from the slab thickness. For slabs not exposed to the exterior, the clear cover is typically 3/4 inches according to ACI 318-14 Table 20.6.1.3.1.

Self-Weight

The self-weight of the slab must be determined based on the reduction of concrete from the void formers. The self-weight of the concrete slab is determined from the equivalent slab thickness and the density of the concrete, which is 150 pcf for normal weight concrete. To determine the equivalent slab thickness, the percentage of concrete reduction must be calculated from the volume of the void compared to the gross volume of the system. For spherical voids with no irregularities, the volume of the void can be determined from Equation 6-2 below.

$$V_{void} = \frac{4}{3}\pi r^3 \qquad \text{(Equation 6-2)}$$

The gross volume for a void section is determined from the slab thickness and the spacing of the voids. The percentage of concrete reduction is the ratio of the volume of the void over the gross volume. The equivalent slab thickness is determined from one minus the percentage of concrete reduction multiplied by the slab thickness. Once the equivalent slab thickness is converted from inches to feet, it can be multiplied by the density of the concrete to determine the self-weight of the voided slab.

Moments at Critical Sections

Now that the self-weight has been determined, the factored distributed load can be calculated from the applied loads. The factored distributed load is calculated from the load and resistance factor design (LRFD) load factors in ACI 318-14 Table 5.3.1 that are applied to the dead and live load in Equation 6-3 below.

$$q_u = 1.2D + 1.6L \qquad \text{(Equation 6-3)}$$

From the factored distributed load, q_u, center-to-center span length in direction perpendicular to the direction of analysis, l_2, and clear span of the slab, l_n, the total factored static moment can be determined from Equation 6-4 (ACI 318-14 Eq. 8.10.3.2) below.

$$M_0 = \frac{q_u l_2 l_n^2}{8} \qquad \text{(Equation 6-4)}$$

To determine the applied moments at each location of the slab, the total factored static moment is multiplied by the distribution factors shown in Table 6-1 below, which is from ACI 318-14 Section 8.10.5.

Table 6-1: Summary of Distribution Factors at Critical Moment Sections

Location			Coefficient
End Span	Column Strip	Exterior Negative	0.26
		Positive	0.31
		Interior Negative	0.53
	Middle Strip	Exterior Negative	0.00
		Positive	0.21
		Interior Negative	0.17
Interior Span	Column Strip	Positive	0.21
		Negative	0.49
	Middle Strip	Positive	0.14
		Negative	0.16

Flexural Design

The flexural reinforcement in the slab is designed to be adequate for each critical section. However, it is more efficient to only design for the largest negative and largest positive moment because it is easier to construct a more uniform system. The reinforcement for the largest positive moment will be used in the bottom of the slab in both directions and the reinforcement for the largest negative moment will be used in the top of the slab in both directions. Once the flexural reinforcement is selected, the neutral axis needs to be determined to verify that it is only in the rectangular portion of the slab. If the neutral axis is above the bottom of the voids, the calculation would become much more complicated due to the irregular shape of the resultant shape. To determine the distance from the fiber of maximum compressive strain to the neutral axis, c, the depth of the equivalent rectangular stress block, a, needs to be calculated from Equation 6-5 (ACI 318-14 Section 22.2.2.4.1) below. The area of reinforcing steel, A_s, the steel yield strength, f_y, the concrete compressive strength, f'_c, and the width of the slab, b, are all variables that effect the depth of the rectangular stress block. A width of 12 inches is used for the slab width.

$$a = \frac{A_s f_y}{0.85 f'_c b} \qquad \text{(Equation 6-5)}$$

The neutral axis depth can now be determined from the depth of the equivalent stress block and the factor for the equivalent rectangular concrete stress distribution, β_1, which is dependent of the compressive strength of the concrete. The depth of the neutral axis is determined from Equation 6-6 (ACI 318-14 Eq. 22.2.2.4.1) below.

$$c = \frac{a}{\beta_1} \qquad \text{(Equation 6-6)}$$

The CRSI Design Guide for Voided Concrete Slabs recommends limiting the distance from the fiber of maximum compressive strain to the neutral axis to less than 1.5” to keep the shape of the compression block rectangular since that is the typical clear cover for interior slabs. The tensile strength needs to also be determined to verify that the section is tension-controlled. The tensile strength is calculated from Equation 6-7 (ACI 318-14 Section 21.2.2) below, where d is the distance from the extreme compression fiber to the centroid of the tension reinforcement.

$$\varepsilon_t = 0.003\left(\frac{d}{c} - 1\right) \qquad \text{(Equation 6-7)}$$

If the tensile strain is greater than or equal to 0.005, then the section is tension-controlled and all assumptions are correct.

Two-way Shear

Two–way shear, or punching shear, is often the limiting factor in biaxial flat plate concrete slabs. Punching shear is a type of failure for reinforced concrete slabs subjected to high localized forces, which is often prevalent around the perimeter of the columns. In order to combat this in flat plate voided concrete slabs, the area around the column is left solid without voids in order to increase the shear strength of the slab in these locations. To calculate how much area around the columns needs to be solid, the two-way shear capacity of the reinforced concrete slab needs to be determined. The length of the perimeter of the critical section, b_o, is determined from the depth of the slab, d, and the dimension of the square column, c, which is shown in Equation 6-8 below.

$$b_O = 4(c + d) \qquad \text{(Equation 6-8)}$$

The allowable concrete two-way shear direct strength is determined from Equation 6-9 (ACI 318-14 Table 22.6.5.2) below.

$$\phi V_C = \phi 4\lambda\sqrt{f'_C} b_O d \qquad \text{(Equation 6-9)}$$

The solid area around the column is determined from the tributary area of the column, A_T, the shear reduction factor, α_{bw}, the allowable direct shear force, ϕV_c, and the total factored uniformly distributed load, q_u. Equation 6-10 below shows how to properly calculate this solid area around the column. The shear reduction factor is the ratio of the remaining concrete section area and the total cross section area. The minimum value which is geometrically possible, and thus the relevant one, can be found at an inclination of 45° in cross-section.

$$Solid\ Area\ Around\ the\ Column = A_T - \frac{\alpha_{bw}(\phi V_c)}{q_u} \qquad \text{(Equation 6-10)}$$

The two-way shear strength of the slab must now be checked. This check is determined by calculating the total factored shear stress and verifying this applied stress is less than the allowable shear stress. The total factored shear stress is a combination of the direct shear stress and the shear stress due to the fraction of the unbalanced moment transferred by eccentricity of shear. In order to calculate the total factored shear stress, the factored shear force due to gravity loads at an edge column must be determined. The factored shear force, V_u, is determined from Equation 6-11 below. The factored shear force is dependent of the total factored uniformly distributed load, q_u, tributary area of the edge column, A_T, and the dimensions of the critical section, b_1 and b_2.

$$V_u = q_u(A_t - b_1 b_2) \qquad \text{(Equation 6-11)}$$

The direct shear stress is determined by dividing the factored shear force by the area of the critical section of the slab, A_c, which is determined from Equation 6-12 below.

$$A_c = (2b_1 + b_2)d \qquad \text{(Equation 6-12)}$$

To determine the shear stress due to the fraction of the unbalanced moment transferred by eccentricity of shear, the property of assumed critical section analogous to polar moment of inertia, J_c, needs to be calculated. Equation 6-13 (ACI 318-14 R8.4.4.2.3) below shows the formulation of this section property.

$$J_c = \frac{2{b_1}^2 d(b_1 + 2b_2) + d^3(2b_1 + b_2)}{6b_1} \qquad \text{(Equation 6-13)}$$

The factor for the eccentricity of shear at slab-column connections, γ_v, and the unbalanced moment at the edge column, M_u, need to be calculated to determine the shear stress from eccentricity. The unbalanced moment is determined from the Direct Design method in Equation 6-14 below, which is based upon the total factored static moment.

$$M_u = 0.3M_0 \qquad \text{(Equation 6-14)}$$

The factor for the eccentricity of shear at the slab-column connection is determined from Equation 6-15 (ACI 318-14 Eq. 8.4.4.2.2) below.

$$\gamma_v = 1 - \frac{1}{1 + \frac{2}{3}\sqrt{b_1/b_2}} \qquad \text{(Equation 6-15)}$$

All of the variables for the total factored shear stress have now been defined, so this stress can now be calculated, which is shown below in Equation 6-16 (ACI 318-14 R8.4.4.2.3).

$$v_u = \frac{V_u}{A_c} + \frac{\gamma_v M_u}{J_c c_{AB}} \qquad \text{(Equation 6-16)}$$

The total factored shear stress needs to be less than the allowable shear stress, ϕv_c, in order to be an adequate design for two-way shear forces. The allowable shear stress is determined from Equation 6-17 (ACI 318-14 Table 22.6.5.2) below.

$$\phi v_c = \phi 4\lambda\sqrt{f'_c} \qquad \text{(Equation 6-17)}$$

The two-way shear stresses need to be verified for each column location in order to prevent punching shear failures. The corner columns typically do not have issues with punching shear, which results in a small amount of solid area around these columns.

Deflection

The deflection of a two-way flat plate concrete slab is a key serviceability check. The maximum total deflection of a two-way flat plate slab is L/240 according to Table 1604.3 in the 2015 International Building Code. This deflection limit is for floor members. To determine the actual deflection in a two-way flat plate concrete slab, a complex design process is required, so it is best to utilize analysis software for this calculation. The deflection check for voided slabs is the same as solid slabs, except the moment of inertia will be adjusted due to the presence of voids. Conservatively, the moment of inertia is evaluated at a section where the void is the largest, i.e. at the hemisphere of the spherical void.

Solid Slabs

The design process for a solid flat plate concrete slab is very similar to the process for the voided concrete slab, but less complex due to the lack of geometrical irregularities in the slab. Like the voided slabs, the thickness of the slab is typically controlled by the two-way punching shear requirements, and the design check follows the same procedure as introduced above.

Chapter 7 – Parametric Study

A parametric study was conducted to compare the effectiveness of a flat plate voided concrete slab to a solid flat plate concrete slab for various spans. This parametric study tested spans of 25 feet, 30 feet, and 35 feet. These are common spans for concrete structural systems, where 30 feet is universally known as an optimum span for steel and concrete systems. The structure chosen for a consistent design between the three different spans consists of five bays by five bays and is four stories tall. The plan view of this structure can be seen in Figure 7-1 below.

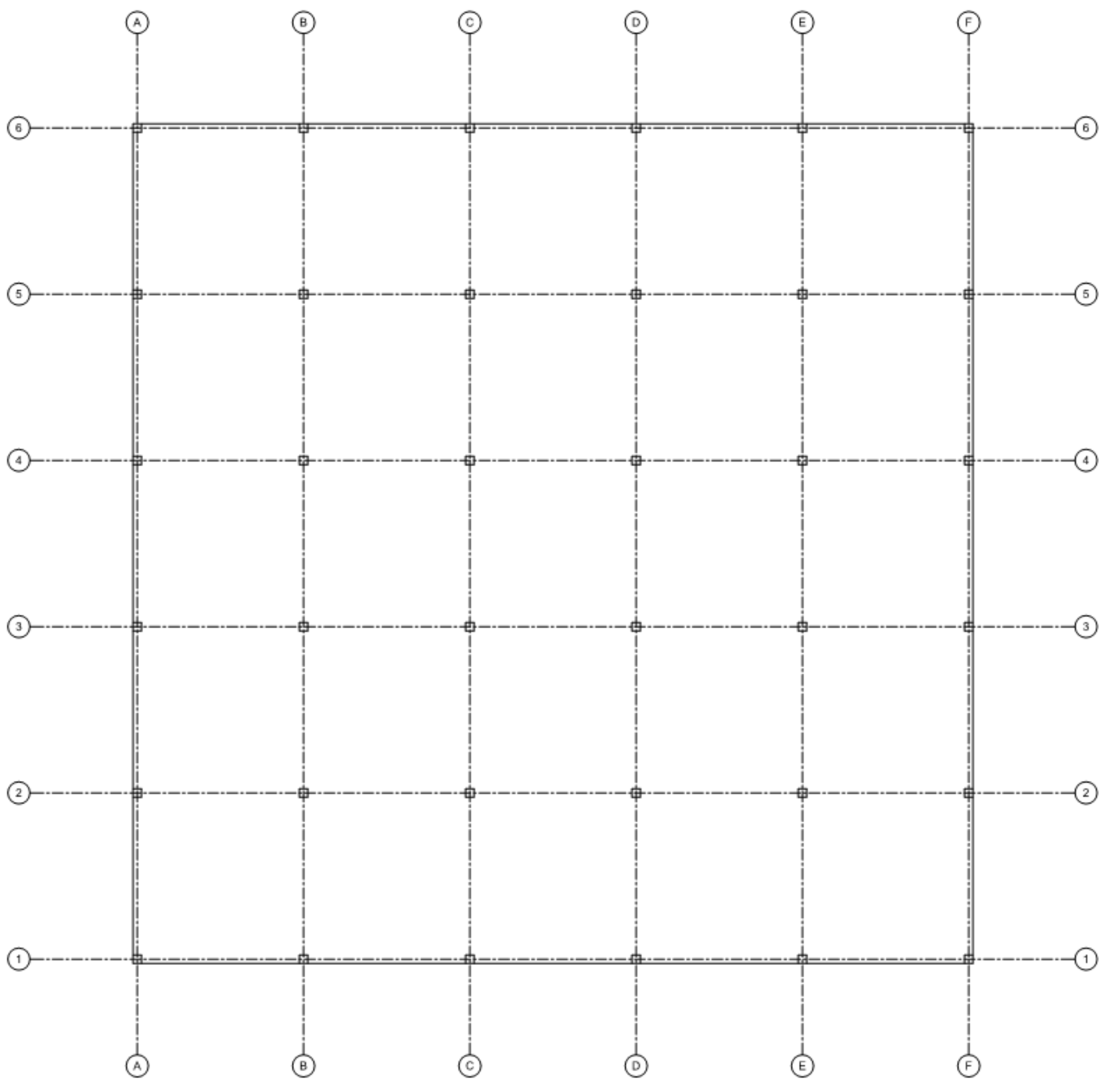


Figure 7-1: Parametric Study Framing Plan

The parametric study has consistent loading, concrete strength, reinforcing steel strength, and column sizes among all design examples. The superimposed dead load is 20 psf and the live load is 60 psf. A concrete compressive strength of 4,000 psi was chosen with a reinforcing steel yield strength of 60 ksi. The columns for each design example are 18 inch square concrete columns. For an efficient design, the concrete columns would not be the same size for the three different bay sizes chosen, but the columns were chosen to be consistent to not cause any more variables in the results. Normal-weight concrete is assumed for all slabs.

The results of the parametric study proved that flat plate voided concrete slabs substantially reduce the volume of concrete, weight of the structure, and the carbon dioxide emission. A summary of the results is shown in Table 7-1 below. The design calculations for the voided slabs can be seen in Appendix A and calculations for the solid slabs can be seen in Appendix B.

Table 7-1: Summary of Parametric Study Results

Bay Size	Type	Slab Thickness	Void Size	Self-Weight[1]	Concrete Reduction[2]	CO_2 Reduction
25' x 25'	Voided	12.5"	8.3"	112.0 PSF	10.4%	15.0 PSF
	Solid	10.0"	-	125.0 PSF		
30' x 30'	Voided	15.0"	10.8"	128.3 PSF	29.3%	15.3 PSF
	Solid	14.5"	-	181.3 PSF		
35' x 35'	Voided	16.5"	12.6"	137.5 PSF	45.0%	17.0 PSF
	Solid	20.0"	-	250.0 PSF		

[1] For the voided slabs, this is the average self-weight.
[2] Reduction of concrete in voided slabs compared to solid slabs.

The concrete reduction for each of the three spans shown in Table 7-1 is determined from the concrete reduction of the voided slab compared to the solid slab. The concrete reduction shown in the voided slab calculations in Appendix A is determined from the concrete reduction caused by the displacement of concrete from the void formers and is not compared to the volume of concrete in the solid slabs. The CO_2 reduction is also only calculated for the concrete reduction from the void formers in the voided slabs and does not factor in the concrete reduction compared to the solid slab. Therefore, the carbon dioxide emission reduction is actually less for the 25' x 25' bays and is more for the 35' x 35' spans. The carbon dioxide emission reduction is discussed further in Chapter 8.

The results for the 35' x 35' bay are slightly skewed due to the fact that the solid flat plate slab was not designed with any transverse reinforcement. An efficient design would design the two-way flat plate slab with transverse reinforcement using either shear studs or stirrups to increase the two-way punching shear strength of the slab. This would allow for a slightly thinner slab to be used. This would decrease the amount of concrete reduction, but there would still be a significant amount of concrete reduction in the voided slab.

Due to two-way punching shear requirements previously discussed, an area around the interior columns must be left unvoided in order to get adequate shear strength at these locations. The edge and corner columns usually do not require an unvoided area in the slab for punching shear strength, but there is typically no voids near the exterior of the slab. In the parametric study, a two-foot perimeter around the exterior of the slab is assumed to have no voids. Table 7-2 below summarizes the percent of the slab that contains voids for each design example and the amount of concrete displaced.

Table 7-2: Concrete Reduction in Slabs from Voids

Bay Size	Total Area	% of Slab Voided	Concrete Reduction
25' x 25'	62,500 ft^2	85%	177.9 yd^3
30' x 30'	90,000 ft^2	78%	836.5 yd^3
35' x 35'	122,500 ft^2	74%	2,792.5 yd^3

The volume of concrete reduced factors in the displacement of concrete from the void formers and the difference in slab thickness between the voided slabs and solid slabs. However, the reduction of concrete in the columns and foundations from the reduced self-weight of the slabs is not considered in this parametric study, which is a key advantage to the voided slab system.

The parametric study comparing the design of voided flat plate concrete slabs to solid flat plate concrete slabs proved that voided slabs are more effective for structures with large spans. Voided slabs effectiveness for long spans is due to the significant reduction in self-weight. Voided slabs are still an effective structural system for shorter spans, such as 25 feet, but the concrete savings for these spans could potentially not outweigh the additional labor costs associated with voided slabs.

Chapter 8 – Sustainability

One of the biggest challenges society is facing right now is climate change, which is partly the result of human activity since the Industrial Revolution. The rising temperature of the Earth is proceeding at an unprecedented rate, which is due to the rapid increase in carbon dioxide levels in the atmosphere. The heat-trapping nature of carbon dioxide and other greenhouse gases and their ability to affect the transfer of infrared energy through the atmosphere is the cause of global warming. Scientists and researchers all across the globe are working on new ways to limit greenhouse gas emissions from human activity.

In structural engineering, the biggest cause of carbon dioxide emissions stems from cement production, which is the binding ingredient in concrete. Cement manufacturing is very energy intensive due to the extreme heat required. Producing a ton of cement requires 4.7 million BTU of energy, which is the cause of indirect CO_2 emission. The production of a ton of cement results in nearly a ton of CO_2 emission into the atmosphere. The direct emissions of cement occur through a chemical process called calcination, which occurs when limestone is heated, breaking down into calcium oxide and carbon dioxide. (Rubenstein, 2012).

Cement production accounts for nearly 5% of the global carbon dioxide emissions from human activity and the production of cement is growing by 2.5% annually. The United States ranks third in cement production, only trailing China and India. Given its high carbon dioxide emissions and critical importance to the construction industry and society, cement is an obvious place to look to reduce greenhouse gas emissions. Figure 8-1 below shows how carbon dioxide emissions from cement production are exponentially rising.

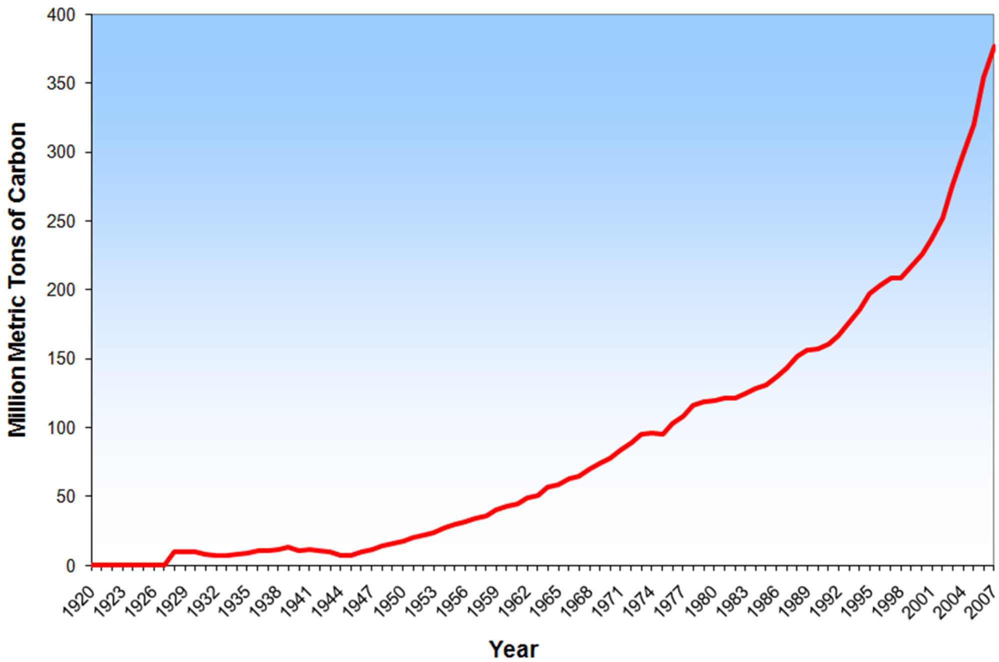


Figure 8-1: Global CO_2 Emissions from Cement Production Since 1920

The implementation of voided flat plate concrete slabs into structural design is one way to reduce the carbon footprint of the construction industry. Voided slabs have proven to be a viable structural system that can reduce the volume of concrete up to 35%. This decrease in concrete usage results in a reduction in CO_2 emissions, since cement makes up 7% to 15% of concrete according to the Portland Cement Association. According to Cobiax's Christian Roggenbuck, voided slabs reduce 210 kg of CO_2 per cubic meter of concrete. This value comes from Cobiax's Ecological Product Declaration and Life Cycle Analysis. The CO_2 emissions from the production of the plastic void formers is accounted for in this approximation.

Reducing the volume of concrete also reduces the amount of concrete trucks required for delivery and pouring of the concrete. An average concrete truck can deliver approximately 9.5 cubic yards of concrete. In the parametric study, the 30 foot bay design resulted in a reduction of 836.5 cubic yards of concrete by using voided slabs instead of traditional solid slabs. This would reduce the amount of concrete trucks for this project by nearly ninety, which would save the owner a significant amount of money and result in less fuel usage by the concrete trucks, which would indirectly help lower the carbon dioxide emissions.

The void formers manufactured by Cobiax and Bubbledeck are made of high-density polyethylene (HDPE) plastic. HDPE plastic is the most widely used plastic due to its versatility and strength. One of the key features of HDPE plastic is that it is lightweight, yet very strong, which can be interpreted to "less impact on the environment" since less material has to be used. An example of this is a milk jug. The HDPE plastic container of a milk jug weighs approximately 80 grams, yet it can support an entire gallon of milk, which weighs approximately 3900 grams. It is critical for the void formers in voided slabs to be lightweight, yet have a high strength capacity. The plastic void formers add minimal weight to the slab, yet can still keep its shape under the applied loads to the slab during construction.

High-density polyethylene plastic is also weather resistant and long lasting, which is an important element, since concrete structures typically have long lifespans. HPDE plastic is known for replacing other materials that are much heavier. In the case of voided flat plate concrete slabs, the void formers are acting as leave-in formwork. Formwork is typically made from wood, which is much more expensive and heavier than plastic.

The concept of putting recycled plastic inside the slab is helping cut down the amount of plastic waste in landfills and the ocean. This factor, as well as the reduction of concrete, has helped many of the projects that have implemented flat plate voided concrete slabs reach LEED accreditation. An increased use of voided slabs in structural engineering will reduce the carbon footprint of the construction industry and help fight climate change on Earth.

Chapter 9 – Conclusion

The concept of putting voids into concrete slabs is nothing revolutionary since engineers have been utilizing this idea to lighten structures for nearly 2000 years. However, the modern flat plate voided slab is an innovative structural system that has evolved from the original voided slab concept. This system has been well received by architects and engineers all across the globe and is gaining a lot of traction in the United States in recent years due to the many benefits from major concrete reduction to the flat soffit aesthetic that is favored by architects and occupants.

Voided flat plate concrete slabs can be a very efficient, cost-effective structural system for a wide range of projects when properly designed and constructed. Projects that require long spans or limited floor-to-floor heights are ideal for this system, but there are many other applications where this system is an economical choice. There are some projects where the economical advantage of a voided slab system is not significant, but the owners still choose to implement this system into the design in order to limit the carbon footprint of the structure and to help create a more environmentally friendly building.

Overall, flat plate voided concrete slabs are a viable structural system that are on the rise in the United States due to the concrete reduction, which has a trickle-down effect to its benefits from the lateral force resisting system to the foundation system. As American architects and engineers begin to learn more about this structural system, the implementation of voided slabs in buildings will begin to rise, which will have many positive impacts to the structural engineering industry.

Material and Labor Cost

One of the biggest concerns that American architects and engineers have with flat plate voided concrete slabs is the cost of the system. Cost is usually one of the leading factors in design, especially in structural engineering since most of the structural elements of the building are unexposed and not visible to the owners and occupants of a building. At first, most industry professionals assume the voided slab system will be a more expensive option than a traditional concrete system due to the extra cost of the void formers and the intricate labor involved with this construction. However, voided slabs can reduce the cost of the structural design for certain projects.

The material cost for flat plate voided slabs compared to traditional solid flat plate slabs is significantly reduced. According to the National Ready Mixed Concrete Industry Data Survey, the 2016 national average cost of concrete is $108 per cubic yard. The 30 foot span design example in the parametric study in Chapter 7 reduced the volume of concrete by 935 cubic yards, which would result in a material cost saving of $105,300. This does not even factor in the reduction in the column sizes and foundations due to the lighter floor system. The reduced self-weight of the slab also results in less required reinforcement.

The cost of labor widely varies by location and if the state of the project is unionized or not unionized. The labor cost associated with flat plate voided slabs is often higher than the labor cost of a traditional concrete system due to the additional construction steps involved and the unfamiliarity of the system to most contractors since it is a relatively new method in the United States. A conservative estimate of the additional labor cost involved with placing the void formers is an additional $0.75 per square foot compared to traditional two-way flat plate slabs. However, this additional cost is often offset due to the elimination of high chairs for the top layers of reinforcing steel since the void former cages act like chairs for the top layers of reinforcement. A final cost analysis that accounts for the material costs and labor costs for the parametric study is shown in Table 9-1 below. However, this cost analysis did not account for the cost of high chairs in the solid slabs, which would increase the cost reduction of the voided slabs even more.

Table 9-1: Final Cost Analysis

Bay Size	Total Area	Type	Concrete Cost	Rebar Cost	Labor Cost	Total Cost	Cost Reduction
25	62,500 ft^2	Voided	$137,032	$40,688	$580,272	$757,991	**$29,539**
		Solid	$208,333	$43,388	$535,810	$787,530	
30	90,000 ft^2	Voided	$329,657	$70,500	$840,657	$1,240,814	**$47,991**
		Solid	$435,000	$76,095	$777,710	$1,288,805	
35	122,500 ft^2	Voided	$657,993	$116,988	$1,153,218	$1,928,198	**$92,390**
		Solid	$816,667	$132,388	$1,071,535	$2,020,589	

The labor cost was estimated from the *2011 RSMeans Building Construction Data* and considered placing formwork, rebar, void formers, and concrete. The cost of concrete and rebar is significantly less in the voided slabs compared to the solid slabs, but the labor cost is higher for the voided slabs, which is expected. The overall cost reduction in the voided slabs is a significant amount of money for the four-story structure used for the parametric study. It can also be noted that the savings increase as the spans increase, which is also expected.

Future Research

There is currently a wide variety of research being conducted on advantages and disadvantages of voided flat plate concrete slabs. The biggest disadvantage of this system throughout the construction industry is the added complexity of the construction of voided slabs compared to solid slabs. Since there is major concern with this aspect of the system, this is the area of study that requires the most research. New construction methods need to be studied and analyzed in order to decrease labor cost and reduce the construction schedule in order to improve this structural system's viability.

References

ACI Committee 318. (2014). *Building Code Requirements for Structural Concrete (ACI 318-14) and Commentary on Building Code Requirements for Structural Concrete (ACI 318R-14)*. Farmington Hills, Michigan: American Concrete Institute.

Beres AB, Mota M. *Design guide for voided concrete slabs.* First ed. Concrete Reinforcing Steel Institute (CRSI); 2014.

BubbleDeck eliminates dead weight concrete on university project. For Construction Pros Web site. https://www.forconstructionpros.com/concrete/article/10415905/bubbledeck-technology-eliminates-dead-weight-concrete-on-university-project. Updated 2011. Accessed October 02, 2018.

Cobiax engineering manual. 2012 ed. Dedham, MA: Cobiax USA; 2010.

Environmental impact of cement. Geos Technologies Web site. http://geostechnologies.com/about-geos/environmental-impact-of-cement. Updated 2011. Accessed October 07, 2018.

Fanella DA, Mota M, Trygestad A. Fire resistance of flat plate voided concrete floor systems. *Structure Magazine*. 2018:12-13.

Furuto. 'BubbleDeck' technology at harvey mudd college / MATT construction. *Arch Daily*. 2012. https://www.archdaily.com/229105/bubbledeck-technology-at-harvey-mudd-college-matt-construction.

Kozen. Columbia university medical and graduate education building, new york. *Concrete Construction*. 2015. https://www.concreteconstruction.net/business/columbia-university-medical-and-graduate-education-building-new-york_o.

Krichels. Harvey mudd college BubbleDeck construction. *The Architects Newspaper*. 2012. https://archpaper.com/2012/04/harvey-mudd-college-bubbledeck-construction/.

Mota M. Innovative structural slab practices - voided slabs.

Mota M, Fanilla DA. Vibration excitations part 2: How to select a reinforced concrete floor system. *Structure Magazine*. 2017:23-25.

Pfeil S. Bubbledeck in the united states. Wisconsin Society of Professional Engineers Web site. http://www.wspe.org/PDF/2013.DC.Presentations/130502BubbleDeck.pdf. Updated 2013. Accessed October 02, 2018.

Plastics. High density polyethylene (HDPE): So popular. Plastics Make it Possible Web site. https://www.plasticsmakeitpossible.com/about-plastics/types-of-plastics/professor-plastics-high-density-polyethylene-hdpe-so-popular/. Updated 2017. Accessed October 07, 2018.

Rubenstein. Emissions from the cement industry. *State of the Planet*. 2012. https://blogs.ei.columbia.edu/2012/05/09/emissions-from-the-cement-industry/.

Sesil, Melrose, Hopper, Polimeni. Columbia medical center's vertical campus. *Structure Magazine*. 2016.

Two-way flat plate concrete floor system analysis and design. Structure Point; 2018.

Waier PR. *RS means building construction data.* 69th ed. Norwell, MA: RS Means; 2010.

2015 International Building Code. Country Club Hills, Illinois: International Building Code; 2014.

Appendix A - Voided Slab Design Examples

This appendix contains the calculations of the three different voided flat plate concrete slabs discussed in the parametric study.

STEP DESCRIPTION	COMPUTATION	REFERENCE
		ACI 318-14
Design Parameters	see Design Parameters table below	
Determine Minimum Slab Thickness for Serviceability	$h_{min} = \frac{l_n}{30} = \frac{(25 \ x \ 12) - 18}{30} =$ 9.4 in. ***Use a 12.5" Slab with 8.3" SlimLine Void Formers***	Table 8.3.1.1
Self-Weight of Voided Slab	$DL =$ **112.0 PSF** *(from Cobiax product catalog)* % Concrete Savings = **28.3 %** Equivalent Slab Thickness = $(12.5")(1 - 0.283) =$ 9.0 in.	
Determine Total Factored Static Moment in Each Span	$q_u = 1.2D + 1.6L = 1.2(112 + 20) + 1.6(60) =$ **254.4 PSF**	Table 5.3.1
	$l_n = 25' - 1.5' =$ 23.5 ft.	
	$M_0 = \frac{q_u l_2 l_n^2}{8} = \frac{(254.4 \ PSF)(25')(23.5')^2}{8} =$ 439.0 k-ft	Eq. 8.10.3.2
Direct Design Method	see Direct Design Method table below	Table 8.10.4.2
Required Reinforcement	d= 12.5" - 1.25" = 11.25 in. b = (25 ft/2)(12 in/ft) = 150 in.	

Design Parameters

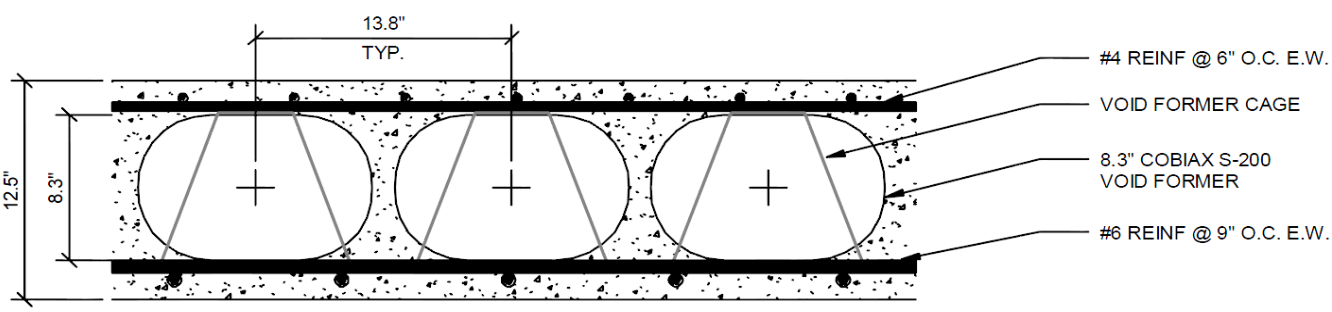
Concrete	$f'_c =$ 4000 psi
	$w_c =$ 150 pcf
Reinf. Steel	$f_y =$ 60 ksi
Loads	*Superimposed Dead Load* = 20 psf
	Live Load = 60 psf
Spans	Typical Bay: 25' x 25'
Columns	18" x 18"

Direct Design Method

Location			M_u (k-ft)	A_s (in^2)	Reinf.
End Span	Column Strip	Exterior Negative	-114.2	3.52	8 # 6
		Positive	136.1	3.40	**17 #4**
		Interior Negative	-232.7	4.84	**11 #6**
	Middle Strip	Exterior Negative	0.0	3.52	8 #6
		Positive	92.2	3.40	17 #4
		Interior Negative	-74.6	3.52	8 #6
Interior Span	Column Strip	Positive	92.2	3.40	17 #4
		Negative	-215.1	4.84	11 #6
	Middle Strip	Positive	61.5	3.40	17 #4
		Negative	-70.2	3.52	8 #6

STEP DESCRIPTION	COMPUTATION	REFERENCE
	$R_n = \frac{M_u}{\phi b d^2} = \frac{(232.7\ k-ft)(1000\frac{lb}{k})(12"/ft)}{0.9(150")(11.25")^2} =$ 163 psi	
	$\rho = \frac{0.85f'_c}{f_y}\left[1-\sqrt{1-\frac{2Rn}{0.85f'_c}}\right] = \frac{0.85(4)}{60}\left[1-\sqrt{1-\frac{2(163)}{0.85(4000)}}\right] =$ 0.0028	
	$A_s = \rho bd = (0.0028)(150")(11.25") =$ 4.71 in^2	
	$A_{s\ min} = 0.0018bh = 0.0018(150")(12.5") =$ 3.38 in^2	Table 8.6.1.1
Check Compression Block	$a = \frac{A_s f_y}{0.85f'_c b} = \frac{(4.71)(60)}{0.85(4)(150)} =$ 0.554 in.	
	$c = \frac{a}{\beta_1} = \frac{0.554"}{0.85} =$ 0.65 in. $< 1.50\ in.\ \therefore\ OK$	Eq. 22.2.2.4.1
	$\varepsilon_t = 0.003\left(\frac{d}{c}-1\right) = 0.003\left(\frac{11.25}{0.65}-1\right) =$ 0.048744 $> 0.004\ \therefore OK$	
Select Reinforcement	→ USE **(11) #6 BARS** *$A_s = 4.84\ in^2$*	Appendix A
Two-Way Shear Design Capacity	$\phi V_c = \phi 4\lambda\sqrt{f'_c}b_0 d$ $= 0.75(4)(1.0)\sqrt{4000}[4(18+11.25)](11.25)\left(\frac{1}{1000}\right)$ = 249.7 k	Table 22.6.5.2
Solid Area Around Column	$A_{solid} = Tributary\ Area\ of\ Column - \frac{(Shear\ Reduction\ Factor)(Allowable\ Direct\ Shear\ Force)}{Total\ Factored\ Uniformly\ Distributed\ Load}$ $= (25' x\ 25') - \frac{(0.55)(249.7\ k)}{(0.2544\ ksf)} =$ **85.07 ft^2**	
Factored Shear Force	$V_u = qu(A_t - b_1 b_2)$ $= (0.2544\ k)\left(331\ ft^2 - (23.6")(29.3")\left(\frac{1}{144}\right)\right) =$ 83.0 k $A_t = \left(\frac{25'}{2} + \frac{18"}{2(12"/ft)}\right)(25') =$ 331 ft^2 $b_1 = 18" + \frac{11.25"}{2} =$ 23.6 in. $b_2 = 18" + 11.25" =$ 29.3 in.	

STEP DESCRIPTION	COMPUTATION	REFERENCE
	$\gamma_v = 1 - \gamma_f = 1 - 0.625 =$ 0.375	Eq. 8.4.4.2.2
	$\gamma_f = \dfrac{1}{1 + \frac{2}{3}\sqrt{23.6''/29.3''}} =$ 0.625	Eq. 8.4.2.3.2
	$0.3M_0 = Mu = 0.3(439.0\ k - ft) =$ 131.7 k-ft	
Section Properties	$A_c = (2b_1 + b_2)d = (2(23.6'')+29.3'')(11.25'') =$ 860.6 in^2	
	$J_c = \dfrac{2b_1{}^2 d(b_1 + 2b_2) + d^3(2b_1 + b_2)}{6b_1} =$ 21158 in^3	R8.4.4.2.3
Total Factored Sheared Stress	$v_u = \dfrac{V_u}{A_c} + \dfrac{\gamma_v M_u}{J_c c_{AB}} = \dfrac{83.0\ k}{860.6\ in^2} + \dfrac{0.375(131.7\ k - ft)(12''/ft)}{21158\ in^3}$ = **98.8 psi**	R8.4.4.2.3
Allowable Shear Stress	$\phi v_c = \phi 4\lambda\sqrt{f'_c} = (0.75)(4)(1.0)\sqrt{4000} =$ **189.7 psi** **v_u < φv_c → OK**	Table 22.6.5.2



STEP DESCRIPTION	COMPUTATION	REFERENCE
		ACI 318-14

Design Parameters

Concrete	$f'_c =$ 4000 psi
	$w_c =$ 150 pcf
Reinf. Steel	$f_y =$ 60 ksi
Loads	*Superimposed Dead Load* = 20 psf
	Live Load = 60 psf
Spans	Typical Bay: 30' x 30'
Columns	18" x 18"

Determine Minimum Slab Thickness for Serviceability (Table 8.3.1.1)

$$h_{min} = \frac{l_n}{30} = \frac{(30\ x\ 12) - 18}{30} = \quad 11.4 \text{ in.}$$

Use a 15" Slab with 10.8" EcoLineVoid Formers

Self-Weight of Voided Slab

$$V_{void} = \frac{4}{3}\pi r^3 = \frac{4}{3}\pi(5.4")^3 = \quad 659 \text{ in}^2$$

$$V_{total} = (15")(11.8")(11.8") = \quad 2089 \text{ in}^2$$

% Concrete Savings = **31.6 %**

Equivalent Slab Thickness = $(15")(1 - 0.327) =$ 10.3 in.

$$DL = (150\ PCF)(1\ ft)\left(\frac{10.3}{12}ft\right) = \quad \mathbf{128.3\ PSF}$$

Determine Total Factored Static Moment in Each Span

$$q_u = 1.2D + 1.6L = 1.2(128.3 + 20) + 1.6(60) = \quad \mathbf{274\ PSF}$$ (Table 5.3.1)

$$l_n = 30' - 1.5' = \quad 28.5 \text{ ft.}$$

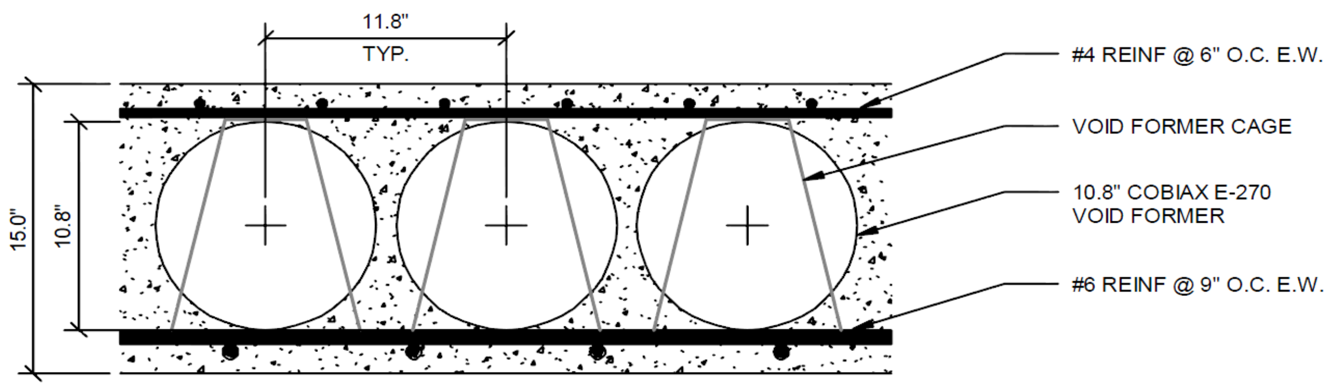
$$M_0 = \frac{q_u l_2 l_n{}^2}{8} = \frac{(274\ PSF)(30')(28.5')^2}{8} = \quad 834.5 \text{ k-ft}$$ (Eq. 8.10.3.2)

Direct Design Method (Table 8.10.4.2)

Location			M_u (k-ft)	A_s (in^2)	Reinf.
End Span	Column Strip	Exterior Negative	-217.0	4.84	11 # 6
		Positive	258.7	4.80	**24 #4**
		Interior Negative	-442.3	7.48	**17 #6**
	Middle Strip	Exterior Negative	0.0	4.84	11 #6
		Positive	175.3	4.80	24 #4
		Interior Negative	-141.9	4.84	11 #6
Interior Span	Column Strip	Positive	175.3	4.80	24 #4
		Negative	-408.9	7.04	16 #6
	Middle Strip	Positive	116.8	4.80	24 #4
		Negative	-133.5	4.84	11 #6

STEP DESCRIPTION	COMPUTATION	REFERENCE
Required Reinforcement	d = 15" - 1.25" = 13.75 in.	
	b = (30 ft/2)(12 in/ft) = 180 in.	
	$R_n = \frac{M_u}{\phi b d^2} = \frac{(442.3\ k-ft)(1000\frac{lb}{k})(12"/ft)}{0.9(180")(13.75")^2} =$ 173 psi	
	$\rho = \frac{0.85f'_c}{f_y}\left[1-\sqrt{1-\frac{2Rn}{0.85f'_c}}\right] = \frac{0.85(4)}{60}\left[1-\sqrt{1-\frac{2(173)}{0.85(4000)}}\right] =$ 0.0030	
	$A_s = \rho bd = (0.0030)(180")(13.75") =$ 7.34 in^2	
	$A_{s\ min} = 0.0018bh = 0.0018(180")(15") =$ 4.86 in^2	Table 8.6.1.1
Check Compression Block	$a = \frac{A_s f_y}{0.85f'_c b} = \frac{(7.42)(60)}{0.85(4)(180)} =$ 0.720 in.	
	$c = \frac{a}{\beta_1} = \frac{0.720"}{0.85} =$ 0.85 in. $< 1.50\ in.\ \therefore\ OK$	Eq. 22.2.2.4.1
	$\varepsilon_t = 0.003\left(\frac{d}{c}-1\right) = 0.003\left(\frac{13.75}{0.85}-1\right) =$ 0.045722 $> 0.004\ \therefore OK$	
Select Reinforcement	→ USE **(17) #6 BARS**	Appendix A
	$A_s = 7.48\ in^2$	
Two-Way Shear Design Capacity	$\phi V_c = \phi 4\lambda\sqrt{f'_c}b_0 d$	Table 22.6.5.2
	$= 0.75(4)(1.0)\sqrt{4000}[4(18+13.75)](13.75)\left(\frac{1}{1000}\right)$	
	= 331.3 k	
Solid Area Around Column	$A_{solid} = Tributary\ Area\ of\ Column - \frac{(Shear\ Reduction\ Factor)(Allowable\ Direct\ Shear\ Force)}{Total\ Factored\ Uniformly\ Distributed\ Load}$	
	$= (30' x\ 30') - \frac{(0.55)(331.3\ k)}{(0.274\ ksf)} =$ **234.9 ft^2**	
Factored Shear Force	$V_u = q_u(A_t - b_1 b_2)$	
	$= (0.274\ k)\left(473\ ft^2 - (24.9")(31.8")\left(\frac{1}{144}\right)\right) =$ 128.0 k	
	$A_t = \left(\frac{30'}{2}+\frac{18"}{2(12"/ft)}\right)(30') =$ 473 ft^2	

STEP DESCRIPTION	COMPUTATION	REFERENCE
	$b_1 = 18" + \frac{13.75}{2} =$ 24.9 in.	
	$b_2 = 18" + 13.75" =$ 31.8 in.	
	$\gamma_v = 1 - \gamma_f = 1 - 0.629 =$ 0.371	Eq. 8.4.4.2.2
	$\gamma_f = \frac{1}{1 + \frac{2}{3}\sqrt{24.9"/31.8"}} =$ 0.629	Eq. 8.4.2.3.2
	$0.3M_0 = Mu = 0.3(834.5\ k - ft) =$ 250.4 k-ft	
Section Properties	$A_c = (2b_1 + b_2)d = (2(24.9")+31.8")(13.75") =$ 1120.6 in^2	
	$J_c = \frac{2b_1{}^2d(b_1 + 2b_2) + d^3(2b_1 + b_2)}{6b_1} =$ 37588 in^3	R8.4.4.2.3
Total Factored Sheared Stress	$v_u = \frac{V_u}{A_c} + \frac{\gamma_v M_u}{J_c c_{AB}} = \frac{128\ k}{1121\ in^2} + \frac{0.37(250.4\ k - ft)(12"/ft)}{37588\ in^3}$ = **116.7 psi**	R8.4.4.2.3
Allowable Shear Stress	$\phi v_c = \phi 4\lambda\sqrt{f'_c} = (0.75)(4)(1.0)\sqrt{4000} =$ **189.7 psi** $\mathbf{v_u < \varphi v_c \rightarrow}$ **OK**	Table 22.6.5.2



STEP DESCRIPTION	COMPUTATION	REFERENCE
		ACI 318-14

Design Parameters

Concrete	$f'_c =$ 4,000 psi
	$w_c =$ 150 pcf
Reinf. Steel	$f_y =$ 60 ksi
Loads	*Superimposed Dead Load* = 20 psf
	Live Load = 60 psf
Spans	Typical Bay: 35' x 35'
Columns	18" x 18"

Determine Minimum Slab Thickness for Serviceability (Reference: Table 8.3.1.1)

$$h_{min} = \frac{l_n}{30} = \frac{(35\ x\ 12) - 18}{30} = \text{13.4 in.}$$

Use a 16.5" Slab with 12.6" EcoLine Void Formers

Self-Weight of Voided Slab

$$V_{void} = \frac{4}{3}\pi r^3 = \frac{4}{3}\pi(6.3")^3 = \text{1047 in}^2$$

$$V_{total} = (16.5")(13.8")(13.8") = \text{3142 in}^2$$

% Concrete Savings = **33.3 %**

$$\text{Equivalent Slab Thickness} = (16.5")(1 - 0.333) = \text{11.0 in.}$$

$$DL = (150\ PCF)(1\ ft)\left(\frac{11}{12}ft\right) = \textbf{137.5 PSF}$$

Determine Total Factored Static Moment in Each Span

$$q_u = 1.2D + 1.6L = 1.2(138 + 20) + 1.6(60) = \textbf{285 PSF}$$ (Reference: Table 5.3.1)

$$l_n = 35' - 1.5' = \text{33.5 ft.}$$

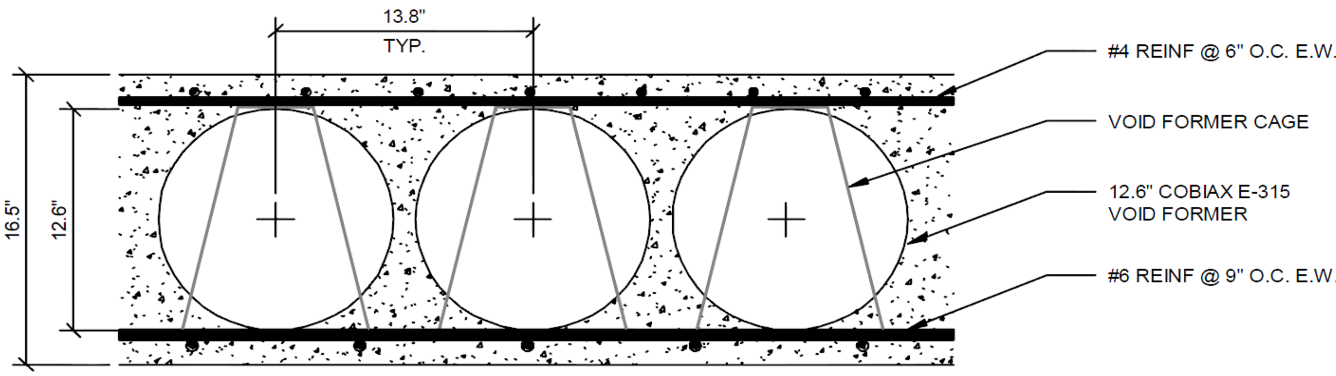
$$M_0 = \frac{q_u l_2 l_n^2}{8} = \frac{(285\ PSF)(35')(33.5')^2}{8} = \text{1399.5 k-ft}$$ (Reference: Eq. 8.10.3.2)

Direct Design Method (Reference: Table 8.10.4.2)

Location			M_u (k-ft)	A_s (in^2)	Reinf.
End Span	Column Strip	Exterior Negative	-363.9	6.60	15 #6
		Positive	433.9	6.60	**33 #4**
		Interior Negative	-741.7	11.44	**26 #6**
	Middle Strip	Exterior Negative	0.0	6.60	15 #6
		Positive	293.9	6.60	33 #4
		Interior Negative	-237.9	6.60	15 #6
Interior Span	Column Strip	Positive	293.9	6.60	33 #4
		Negative	-685.8	10.56	24 #6
	Middle Strip	Positive	195.9	6.60	33 #4
		Negative	-223.9	6.60	15 #6

STEP DESCRIPTION	COMPUTATION	REFERENCE
Required Reinforcement	d = 16.5" - 1.25" = 15.25 in. b = (35 ft/2)(12 in/ft) = 210 in. $R_n = \frac{M_u}{\phi b d^2} = \frac{(741.7\ k-ft)(1000\frac{lb}{k})(12"/ft)}{0.9(210")(15.25")^2} =$ 202.5 psi $\rho = \frac{0.85f'_c}{f_y}\left[1-\sqrt{1-\frac{2Rn}{0.85f'_c}}\right] = \frac{0.85(4)}{60}\left[1-\sqrt{1-\frac{2(203)}{0.85(4000)}}\right] =$ 0.0035 $A_s = \rho bd = (0.0035)(210")(15.25") =$ 11.15 in^2	
	$A_{s\ min} = 0.0018bh = 0.0018(210")(16.5") =$ 6.24 in^2	Table 8.6.1.1
Check Compression Block	$a = \frac{A_s f_y}{0.85f'_c b} = \frac{(11.15)(60)}{0.85(4)(210)} =$ 0.937 in.	
	$c = \frac{a}{\beta_1} = \frac{0.937"}{0.85} =$ 1.10 in. $< 1.50\ in.\ \therefore\ OK$	Eq. 22.2.2.4.1
	$\varepsilon_t = 0.003\left(\frac{d}{c}-1\right) = 0.003\left(\frac{15.25}{1.10}-1\right) =$ 0.038498 $> 0.004\ \therefore OK$	
Select Reinforcement	→ USE **(26) #6 BARS** $A_s = 11.44\ in^2$	Appendix A
Two-Way Shear Design Capacity	$\phi V_c = \phi 4\lambda\sqrt{f'_c}b_0 d$ $= 0.75(4)(1.0)\sqrt{4000}[4(18+15.25)](15.25)\left(\frac{1}{1000}\right)$ = 384.8 k	Table 22.6.5.2
Solid Area Around Column	$A_{solid} = Tributary\ Area\ of\ Column - \frac{(Shear\ Reduction\ Factor)(Allowable\ Direct\ Shear\ Force)}{Total\ Factored\ Uniformly\ Distributed\ Load}$ $= (35' x\ 35') - \frac{(0.55)(384.8\ k)}{(0.285\ ksf)} =$ **482.5 ft^2**	
Factored Shear Force	$V_u = qu(A_t - b_1 b_2)$ $= (0.285\ k)\left(639\ ft^2 - (25.6")(33.3")\left(\frac{1}{144}\right)\right) =$ 180.4 k $A_t = \left(\frac{35'}{2}+\frac{18"}{2(12"/ft)}\right)(35') =$ 639 ft^2	

STEP DESCRIPTION	COMPUTATION	REFERENCE
	$b_1 = 18" + \frac{15.25}{2} =$ 25.6 in.	
	$b_2 = 18" + 15.25" =$ 33.3 in.	
	$\gamma_v = 1 - \gamma_f = 1 - 0.63 =$ 0.369	Eq. 8.4.4.2.2
	$\gamma_f = \frac{1}{1 + \frac{2}{3}\sqrt{25.6"/33.3"}} =$ 0.631	Eq. 8.4.2.3.2
	$0.3M_0 = Mu = 0.3(1399.5\ k - ft) =$ 419.9 k-ft	
Section Properties	$A_c = (2b_1 + b_2)d = (2(25.6")+33.3")(15.25") =$ 1288.6 in^2	
	$J_c = \frac{2b_1^2 d(b_1 + 2b_2) + d^3(2b_1 + b_2)}{6b_1} =$ 51308 in^3	R8.4.4.2.3
Total Factored Sheared Stress	$v_u = \frac{V_u}{A_c} + \frac{\gamma_v M_u}{J_c c_{AB}} = \frac{180.4\ k}{1289\ in^2} + \frac{0.37(419.9\ k - ft)(12"/ft)}{51308\ in^3}$ = **143.0 psi**	R8.4.4.2.3
Allowable Shear Stress	$\phi v_c = \phi 4\lambda\sqrt{f'_c} = (0.75)(4)(1.0)\sqrt{4000} =$ **189.7 psi**	Table 22.6.5.2
	v_u < φv_c → OK	



Appendix B - Solid Slab Design Examples

This appendix contains the calculations of the three different solid flat plate concrete slabs discussed in the parametric study.

STEP DESCRIPTION	COMPUTATION	REFERENCE
		ACI 318-14
Design Properties	**Concrete**: $f'_c =$ 4000 psi; $w_c =$ 150 pcf **Reinf. Steel**: $f_y =$ 60 ksi **Loads**: *Superimposed Dead Load* = 20 psf; *Live Load* = 60 psf **Spans**: Typical Bay: 25' x 25' **Columns**: 18" x 18"	
Determine Minimum Slab Thickness for Serviceability	$h_{min} = \frac{l_n}{30} = \frac{(25\ x\ 12) - 18}{30} =$ 9.4 in. ***Use a 10" Solid Slab***	Table 8.3.1.1
Self-Weight of Voided Slab	$DL = (150\ PSF)(10")(1'/12") =$ **125.0 PSF**	
Factored Load	$q_u = 1.2D + 1.6L = 1.2(125 + 20) + 1.6(60) =$ **270 PSF**	Table 5.3.1
Clear Span	$l_n = 25' - 1.5' =$ 23.5 ft.	
Total Factored Static Moment in Each Span	$M_0 = \frac{q_u l_2 {l_n}^2}{8} = \frac{(270\ PSF)(25')(23.5')^2}{8} =$ 466.0 k-ft	Eq. 8.10.3.2
Direct Design Method	(see table below)	Table 8.10.4.2

Location			M_u (k-ft)	A_s (in^2)	Reinf.
End Span	Column Strip	Exterior Negative	-121.1	3.52	8 # 6
		Positive	144.4	3.96	**9 #6**
		Interior Negative	-247.0	7.04	**16 #6**
	Middle Strip	Exterior Negative	0.0	3.08	7 #6
		Positive	97.9	3.08	7 #6
		Interior Negative	-79.2	3.08	7 #6
Interior Span	Column Strip	Positive	97.9	3.08	7 #6
		Negative	-228.3	6.60	15 #6
	Middle Strip	Positive	65.2	3.08	7 #6
		Negative	-74.6	3.08	7 #6

Required Reinforcement

d = 10" - 1.5" = 8.50 in.

b = (25 ft/2)(12 in/ft) = 150 in.

$$R_n = \frac{M_u}{\phi b d^2} = \frac{(247\ k-ft)(1000\frac{lb}{k})(12"/ft)}{0.9(150")(8.5")^2} = \quad 304\ \text{psi}$$

STEP DESCRIPTION	COMPUTATION	REFERENCE
	$\rho = \frac{0.85f'_c}{f_y}\left[1-\sqrt{1-\frac{2Rn}{0.85f'_c}}\right] = \frac{0.85(4)}{60}\left[1-\sqrt{1-\frac{2(304)}{0.85(4000)}}\right] =$ 0.0053	
	$A_s = \rho bd = (0.0053)(150")(8.5") =$ 6.77 in^2	
	$A_{s\,\min} = 0.0018bh = 0.0018(150")(10") =$ 2.70 in^2	Table 8.6.1.1
Check Compression Block	$a = \frac{A_s f_y}{0.85f'_c b} = \frac{(6.77)(60)}{0.85(4)(150)} =$ 0.797 in.	
	$c = \frac{a}{\beta_1} = \frac{0.797"}{0.85} =$ 0.94 in. $< 1.50\,in.\ \therefore\ OK$	Eq. 22.2.2.4.1
	$\varepsilon_t = 0.003\left(\frac{d}{c}-1\right) = 0.003\left(\frac{8.5}{0.94}-1\right) =$ 0.024198 $> 0.004\ \therefore OK$	
Choose Reinforcement	→ USE **(16) #6 BARS** *$A_s = 4.84\ in^2$*	Appendix A
One-Way Shear Design Capacity	$\phi V_c = \phi 2\lambda\sqrt{f'_c}b_w d$ $= 0.75(2)(1.0)\sqrt{4000}(12)(8.5)\left(\frac{1}{1000}\right)$ = **9.7 k**	Eq. 22.5.5.1
	$V_u = q_u A_t = (270\ PSF)(11.04\ ft^2) =$ **3.0 k**	
	$A_t = \left(\frac{25'}{2} - \frac{18"}{2(12"/ft)} - \frac{8.5"}{12"/ft}\right) =$ 11.04 ft^2	
	V_u < φV_c → OK	
Two-Way Shear Design Capacity	$\phi V_c = \phi 4\lambda\sqrt{f'_c}b_0 d$ $= 0.75(4)(1.0)\sqrt{4000}[4(18+8.5)](8.5)\left(\frac{1}{1000}\right)$ = **171.0 k-ft**	Table 22.6.5.2
	$V_u = q_u A_t = (270\ PSF)(621\ ft^2) =$ **167.6 k**	
	$A_t = (25' * 25') - \left(\frac{18" + 8.5"}{12"/ft}\right)(2) =$ 621 ft^2	
	V_u < φV_c → OK	

STEP DESCRIPTION	COMPUTATION	REFERENCE
		ACI 318-14

Design Properties

Concrete	$f'_c =$ 4000 psi
	$w_c =$ 150 pcf
Reinf. Steel	$f_y =$ 60 ksi
Loads	*Superimposed Dead Load* = 20 psf
	Live Load = 60 psf
Spans	Typical Bay: 30' x 30'
Columns	18" x 18"

Determine Minimum Slab Thickness for Serviceability (Reference: Table 8.3.1.1)

$$h_{min} = \frac{l_n}{30} = \frac{(30\ x\ 12) - 18}{30} = 11.4\ \text{in.}$$

Use a 14.5" Solid Slab

Self-Weight of Voided Slab

$$DL = (150\ PSF)(14.5")(1'/12") = \underline{\underline{\textbf{181.3 PSF}}}$$

Factored Load (Reference: Table 5.3.1)

$$q_u = 1.2D + 1.6L = 1.2(181.3 + 20) + 1.6(60) = \textbf{337.5 PSF}$$

Clear Span

$$l_n = 30' - 1.5' = 28.5\ \text{ft.}$$

Total Factored Static Moment in Each Span (Reference: Eq. 8.10.3.2)

$$M_0 = \frac{q_u l_2 l_n^2}{8} = \frac{(337.5\ PSF)(30')(28.5')^2}{8} = 1028.0\ \text{k-ft}$$

Direct Design Method (Reference: Table 8.10.4.2)

Location			M_u (k-ft)	A_s (in^2)	Reinf.
End Span	Column Strip	Exterior Negative	-267.3	4.84	11 # 6
		Positive	318.7	5.72	**13 #6**
		Interior Negative	-544.8	9.68	**22 #6**
	Middle Strip	Exterior Negative	0.0	4.84	11 #6
		Positive	215.9	4.84	11 #6
		Interior Negative	-174.8	4.84	11 #6
Interior Span	Column Strip	Positive	215.9	4.84	11 #6
		Negative	-503.7	9.24	21 #6
	Middle Strip	Positive	143.9	4.84	11 #6
		Negative	-164.5	4.84	11 #6

Required Reinforcement

d = 14.5" - 1.5" = 13.00 in.

b = (30 ft/2)(12 in/ft) = 180 in.

$$R_n = \frac{M_u}{\phi b d^2} = \frac{(544.8\ k - ft)(1000\frac{lb}{k})(12"/ft)}{0.9(180")(13")^2} = 239\ \text{psi}$$

STEP DESCRIPTION	COMPUTATION	REFERENCE
	$\rho = \frac{0.85f'_c}{f_y}\left[1-\sqrt{1-\frac{2Rn}{0.85f'_c}}\right] = \frac{0.85(4)}{60}\left[1-\sqrt{1-\frac{2(239)}{0.85(4000)}}\right] =$ 0.0041	
	$A_s = \rho bd = (0.0041)(180")(13") =$ 9.67 in^2	
	$A_{s\,\min} = 0.0018bh = 0.0018(180")(14.5") =$ 4.70 in^2	Table 8.6.1.1
Check Compression Block	$a = \frac{A_s f_y}{0.85f'_c b} = \frac{(9.67)(60)}{0.85(4)(180)} =$ 0.948 in.	
	$c = \frac{a}{\beta_1} = \frac{0.948"}{0.85} =$ 1.11 in. $< 1.50\ in.\ \therefore\ OK$	Eq. 22.2.2.4.1
	$\varepsilon_t = 0.003\left(\frac{d}{c}-1\right) = 0.003\left(\frac{13}{1.11}-1\right) =$ 0.031982 $> 0.004\ \therefore OK$	
Choose Reinforcement	→ USE **(22) #6 BARS** *$A_s = 9.68\ in^2$*	Appendix A
One-Way Shear Design Capacity	$\phi V_c = \phi 2\lambda\sqrt{f'_c}b_w d$ $= 0.75(2)(1.0)\sqrt{4000}(12)(13)\left(\frac{1}{1000}\right)$ = **14.8 k**	Eq. 22.5.5.1
	$V_u = q_u A_t = (337.5\ PSF)(13.17\ ft^2) =$ **4.4 k**	
	$A_t = \left(\frac{30'}{2} - \frac{18"}{2(12"/ft)} - \frac{13"}{12"/ft}\right) =$ 13.17 ft^2	
	V_u < φV_c → OK	
Two-Way Shear Design Capacity	$\phi V_c = \phi 4\lambda\sqrt{f'_c}b_0 d$ $= 0.75(4)(1.0)\sqrt{4000}[4(18+13)](13)\left(\frac{1}{1000}\right)$ = **305.9 k-ft**	Table 22.6.5.2
	$V_u = q_u A_t = (337.5\ PSF)(895\ ft^2) =$ **302.0 k**	
	$A_t = (30' * 30') - \left(\frac{18" + 13"}{12"/ft}\right)(2) =$ 895 ft^2	
	V_u < φV_c → OK	

STEP DESCRIPTION	COMPUTATION	REFERENCE
		ACI 318-14

Material Properties

Concrete	$f'_c =$ 4000 psi
	$w_c =$ 150 pcf
Reinf. Steel	$f_y =$ 60 ksi
Loads	*Superimposed Dead Load* = 20 psf
	Live Load = 60 psf
Spans	Typical Bay: 35' x 35'
Columns	18" x 18"

Determine Minimum Slab Thickness for Serviceability (Reference: Table 8.3.1.1)

$$h_{min} = \frac{l_n}{30} = \frac{(35 \; x \; 12) - 18}{30} = \quad 13.4 \text{ in.}$$

Use a 20" Solid Slab

Self-Weight of Voided Slab

$$DL = (150 \; PSF)(20")(1'/12") = \underline{\underline{\textbf{250 PSF}}}$$

Factored Load (Reference: Table 5.3.1)

$$q_u = 1.2D + 1.6L = 1.2(225 + 20) + 1.6(60) = \quad \textbf{420 PSF}$$

Clear Span

$$l_n = 35' - 1.5' = \quad 33.5 \text{ ft.}$$

Total Factored Static Moment in Each Span (Reference: Eq. 8.10.3.2)

$$M_0 = \frac{q_u l_2 {l_n}^2}{8} = \frac{(420.0 \; PSF)(35')(33.5')^2}{8} = \quad 2062.1 \text{ k-ft}$$

Direct Design Method (Reference: Table 8.10.4.2)

Location			M_u (k-ft)	A_s (in^2)	Reinf.
End Span	Column Strip	Exterior Negative	-536.2	7.92	18 # 6
		Positive	639.3	7.92	**18 #6**
		Interior Negative	-1092.9	13.64	**31 #6**
	Middle Strip	Exterior Negative	0.0	7.92	18 #6
		Positive	433.0	7.92	18 #6
		Interior Negative	-350.6	7.92	18 #6
Interior Span	Column Strip	Positive	433.0	7.92	18 #6
		Negative	-1010.4	12.76	29 #8
	Middle Strip	Positive	288.7	7.92	18 #6
		Negative	-329.9	7.92	18 #6

Required Reinforcement

d = 20" - 1.5" = 18.50 in.

b = (35 ft/2)(12 in/ft) = 210 in.

$$R_n = \frac{M_u}{\phi b d^2} = \frac{(1093 \; k - ft)(1000 \frac{lb}{k})(12"/ft)}{0.9(210")(18.5")^2} = \quad 203 \text{ psi}$$

STEP DESCRIPTION	COMPUTATION	REFERENCE
	$\rho = \frac{0.85f'_c}{f_y}\left[1-\sqrt{1-\frac{2Rn}{0.85f'_c}}\right] = \frac{0.85(4)}{60}\left[1-\sqrt{1-\frac{2(203)}{0.85(4000)}}\right] =$ 0.0035	
	$A_s = \rho bd = (0.0035)(210")(18.5") =$ 13.54 in^2	
	$A_{s\,min} = 0.0018bh = 0.0018(210")(18.5") =$ 7.56 in^2	Table 8.6.1.1
Check Compression Block	$a = \frac{A_s f_y}{0.85f'_c b} = \frac{(13.54)(60)}{0.85(4)(210)} =$ 1.138 in.	
	$c = \frac{a}{\beta_1} = \frac{1.138"}{0.85} =$ 1.34 in. $< 1.50\ in.\ \therefore\ OK$	Eq. 22.2.2.4.1
	$\varepsilon_t = 0.003\left(\frac{d}{c}-1\right) = 0.003\left(\frac{18.5}{1.34}-1\right) =$ 0.03845 $> 0.004\ \therefore OK$	
Choose Reinforcement	→ USE **(31) #6 BARS** $A_s = 13.64\ in^2$	Appendix A
One-Way Shear Design Capacity	$\phi V_c = \phi 2\lambda\sqrt{f'_c}b_w d$ $= 0.75(2)(1.0)\sqrt{4000}(12)(18.5)\left(\frac{1}{1000}\right)$ = **21.1 k**	Eq. 22.5.5.1
	$V_u = q_u A_t = (390\ PSF)(15.38\ ft^2) =$ **6.4 k**	
	$A_t = \left(\frac{35'}{2} - \frac{18"}{2(12"/ft)} - \frac{16.5"}{12"/ft}\right) =$ 15.21 ft^2	
	$\mathbf{V_u < \varphi V_c}$ → **OK**	
Two-Way Shear Design Capacity	$\phi V_c = \phi 4\lambda\sqrt{f'_c}b_0 d$ $= 0.75(4)(1.0)\sqrt{4000}[4(18+16.5)](16.5)\left(\frac{1}{1000}\right)$ = **512.5 k-ft**	Table 22.6.5.2
	$V_u = q_u A_t = (390\ PSF)(1219\ ft^2) =$ **511.9 k**	
	$A_t = (35' * 35') - \left(\frac{18" + 16.5"}{12"/ft}\right)(2) =$ 1219 ft^2	
	$\mathbf{V_u < \varphi V_c}$ → **OK**	