ESSAYS IN MACROECONOMIC ECONOMETRICS

by

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B.S., Emporia State University, 2003
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AN ABSTRACT OF A DISSERTATION

submitted in partial fulfillment of the
requirements for the degree

DOCTOR OF PHILOSOPHY

Department of Economics
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Abstract

This dissertation consists of three essays in macroeconomic econometrics. The first essay investigates industry level production functions. Part of the interest in doing this is to contribute to the ongoing improvements in dynamic macroeconomic models which are increasingly disaggregating economies into industrial sectors. This paper provides useful production function parameter values for this endeavour. In addition, the paper shows that there are differences across industry level production functions, so model disaggregation cannot rely on a generic scaled down aggregate production function. Futhermore, evidence of these differences is provided in several ways. First, it is shown that some, but not all, industry level production functions exhibit constant returns to scale. Second, conducted pairwise tests show whether government capital production elasticities are the same for different pairs of industries. In the majority of these tests, the null hypothesis was rejected.

In the second essay, the relevance of wage rigidities for understanding the effect of oil price shocks on output and inflation is examined. The theoretical framework of Blanchard and Gali (2007) is adopted and modified in two important ways. First, an empirically estimated wage adjustment cost function is incorporated following work by Kim and Ruge-Murcia (2009). Second, a realistic monetary policy function is incorporated into the model to be consistent with the current macroeconomic literature. The paper provides evidence that the degree of wage stickiness has little effect on the oil price-macroeconomy relationship. We find that the only way to generate large changes in the variances of output and inflation is to increase the wage adjustment cost by an extreme amount.

The third essay assesses the statistical adequacy of the Cobb-Douglas aggregate production function with public capital as an input. The paper tests the statistical adequacy of the models proposed by Aschauer (1989*a*) and Tatom (1991) and finds that both models are

misspecified. Furthermore, the paper finds that Tatom's model suffers from the same criticism he levels against Aschauer's model, non-stationarity in the data series used to estimate the model. Using Aschauer's framework, a properly specified model is found that models both deterministic heterogeneity and serial autocoreelation. Model results find that public capital is positive and significant. The results are in contrast to a large body of literature that discredits Aschauer's findings claiming his model is incorrect. Finally, an additional specification of the model using the student's t linear regression model is explored to capture potential heteroskedasticity.

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Dedication

To my family for their endless love and support.

Chapter 1

Government Capital and Production: Industry Level Estimates

1.1 Introduction

A large body of empirical work has been devoted to estimating the contribution of government capital to production.[1] Almost all of this work has been carried out on aggregate production functions. However, questions remain about the local or micro implications of government capital. Some work has entertained this possibility with investigations of the regional or state effects of government capital.[2,3]

There are several reasons that a sectoral estimation can be useful. First, a sectoral estimation can provide evidence, either for or against, whether using a single aggregate production function with constant returns to scale to represent all economic sectors, as is so common in macroeconomic work today, is appropriate.[4] Second, should evidence be found

[1]Some of the earliest work was done by Landau (1983) and Ratner (1983). Interest in this topic increased when papers by Aschauer (1989*a*,*b*) and Munnell (1990); Munnell and Cook (1990) connected public capital investment to productivity. Numerous studies followed, including Cassou and Ai (1995); Holtz-Eakin (1994); Lynde and Richmond (1992), and others that sought to improve upon the earlier estimation techniques. See Romp and de Haan (2007) for a recent survey of the literature.

[2]See for instance, Costa, Ellson and Martin (1987); Holtz-Eakin (1994); Munnell and Cook (1990) or Garcia-Mila, McGuire and Porter (1996). Other studies have provided a focus on the manufacturing sector effects of policy.

[3]See for instance, Nadiri and Mamuneas (1994) or Mullen and Williams (1990) But so far, nobody has investigated the effects of government capital on all the different sectors of the economy. This paper takes up this investigation.

[4]One of the earliest and best known works to use this style of model is Kydland and Prescott (1982). More recent work with a public capital feature include Alonso-Carrera and Raurich (2010); Barro (1991);

that production functions are different, it can provide useful parameter estimates that will be useful to a variety of deeper economic analysis which need sectoral production function parameter estimates.

We find that there are significant differences across sectors in their need for government capital. Surprisingly, the education sector has the lowest public capital elasticity. This result probably arises because the sector is dominated by education providers (schools), that make decisions that are likely different than those coming from the optimizing agent assumption that is behind our estimation approach. For the other sectors, which have a greater proportion of optimizers, we find public capital estimates ranging from a low of 0.226 for Manufacturing to a high of 0.379 for Finance & Insurance. Interestingly, all but the Education elasticity are larger than the Aggregate Production elasticity. Furthermore, in testing pairwise whether two industry have the same public capital elasticity, we almost always find that the elasticities are different.[5] This evidence, shows that assuming a single aggregate production function is not appropriate.

There are numerous empirical approaches within the empirical literature on how to estimate the elasticity of government capital. Most rely on single equation methods which estimate production or cost functions using time series for output, labor and private and public capital stocks. A number of problems have been pointed out with the different methods, such as the high degree of multicollinearity or reverse causality. In this paper, we make use of a GMM method used in Cassou and Ai (1995) which does not suffer from these problems. This method makes use of several different, jointly estimated equations that come from basic optimization decisions or other identities from an optimizing agent model, including the production function and those relating capital stocks and investment over time.

To present our results in a clear format, the paper has been organized as follows. Section

Cassou and Lansing (1998, 1999); de Hek (2006); Glomm and Ravikumar (1997); Hashimzade and Myles (2010); Marrero (2008); Turnovsky (1997).

[5] Tests of three or higher numbers of industrial sectors having the same elasticities, which are reported in section 1.6, almost never find that the elasticities are the same value.

1.2 describes the estimation approach and the testing methodology. The results of the estimation procedures, along with a discussion of the various testing results are provided in Section 1.3. Section 1.4 sums up the paper.

1.2 The Empirical Model

The empirical method follows the GMM method used in Cassou and Ai (1995). This approach augments the production function used in many other studies with several other equations. Two of these equations are the Euler equations from a dynamic optimization model and two others are the intertemporal capital good relationships. One of the attractions of this approach is that some of these relationships are able to tightly identify parameters that appear in several equations. This tight identification of a parameter in one equation, then allows the other equations to better estimate the remaining parameters and escape the multicollinearity issue that resulted in such larger public capital estimates in the early work of Aschauer (1989*a*,*b*) and Munnell (1990). Another attraction of this approach is that there is no casualty issues that can arise in linear models where the right-hand side variables are interpreted as causing the left-hand side variables.

Since the method was fully described in Cassou and Ai (1995), here we provide only a brief recap of the approach. The dynamic optimization model is a standard growth model in which agents choose $\{i_t^c, n_t, d_t, k_{t+1} : 0 \leq t\}$ to maximize

$$E_0 \left\{ \sum_{t=0}^{\infty} \left(\frac{1}{1+r} \right)^t d_t \right\} \tag{1.1}$$

subject to

$$d_t = A\mu^t (n_t)^{\alpha_1} (k_{t-1}^c)^{\alpha_2} (k_{t-1}^g)^{\alpha_3} (u_t)^{\alpha_4} \theta_t - w_t n_t - i_t^c$$

and

$$k_t^c = (1 - \delta^c) k_{t-1}^c + i_t^c \tag{1.2}$$

given k_0 and the sequence $\{k_{t+1}^g : 0 \leq t\}$, where $\left(\frac{1}{1+r}\right)$ is the discount factor the firm uses to discount future dividends d_t, $A\mu^t$ is the exogenous level of technology at time t, n_t is labor input at time t, k_t^c is the corporate capital stock that is decided upon at time t and is available for production at time $t+1$, k_t^g is the government capital stock that is decided upon at time t and is available for production at time $t+1$, u_t is the capital utilization rate at time t, θ_t is a random shock to production at time t, w_t is the wage rate at time t and i_t^c is the amount invested in corporate capital at time t. The Euler equations from this optimization problem are

$$E_t\left\{\alpha_1 \frac{y_t}{w_t n_t} - 1\right\} = 0 \tag{1.3}$$

and

$$E_t\left\{\left(\frac{1}{1+r}\right)\left(\alpha_2 \frac{y_{t+1}}{k_t^c} + (1-\delta^c)\right) - 1\right\} = 0. \tag{1.4}$$

These equations can be used in a GMM vector of moments.

Following the notation in Cassou and Ai (1995), we define two moments using (1.2) and a government capital analogue as

$$M1_{t+1} = \begin{bmatrix} 1 - \frac{k_{t+1}^c - i_{t+1}^c}{k_t^c} - \delta^c \\ 1 - \frac{k_{t+1}^g - i_{t+1}^g}{k_t^g} - \delta^g \end{bmatrix}, \tag{1.5}$$

where it is assumed $E_t\{M1_{t+1}\} = 0$. These two moments are particularly well suited for identifying δ^c and δ^g. Next using (1.3) and (1.4), we define

$$M2_{t+1} = \begin{bmatrix} \alpha_1 \frac{y_t}{w_t n_t} - 1 \\ \left(\frac{1}{1+r}\right)\left(\alpha_2 \frac{y_{t+1}}{k_t^c} + (1-\delta^c)\right) - 1 \end{bmatrix} \tag{1.6}$$

for our second two moments. The Euler equations imply $E_t\{M2_{t+1}\} = 0$. These two moments are particularly well suited for identifying α_1 and α_2. Finally, we make use of the production function for the third set of moments. Here we allow the possibility that the production function variables are not cointegrated, by taking the log difference of the production function. This is analogous to the single equation approach used in Tatom (1991). Following notation in Cassou and Ai (1995), we let

$$F_{t+1} = \Delta\log(y_{t+1}) - \log(\mu) - \alpha_1\Delta\log(n_{t+1}) - \alpha_2\Delta\log(k_t^c) - \alpha_3\Delta\log(k_{t-1}^g) - \alpha_4\Delta\log(u_t),$$

$$H_{t+1} = F_{t+1} - \rho F_t,$$

and

$$M3_{t+1} = H_{t+1} \times Z_t$$

where

$$Z_t = \begin{bmatrix} 1 \\ \Delta \log(k_t^c) \\ \Delta \log(k_t^g) \\ \Delta \log(u_{t+1}) \\ \Delta \log(k_{t-1}^c) \\ \Delta \log(k_{t-1}^g) \\ \Delta \log(u_t) \end{bmatrix}$$

are instruments that are in the information set at time t. Under the assumption that

$$E_t\{\Delta \log(\theta_{t+1}) - \rho \Delta \log(\theta_t)\} = 0,$$

it follows that $E_t\{M3_{t+1}\} = 0$. Because the instruments are in the information set, we will obtain consistent estimates even when θ_t is serially correlated and the specification allows us to test if there is even higher order integration which would occur if $\rho = 1$.

These moments are next stacked to get $M_{t+1} = [M1'_{t+1} M2'_{t+1} M3'_{t+1}]'$ and the GMM objective function is to choose $\gamma = (\log(\mu), \alpha_1, \alpha_1, \alpha_1, \alpha_1, \delta^c, \delta^g, \rho)'$ so as to minimize

$$M'V^{-1}M \qquad \text{where} \qquad M = \frac{1}{T}\sum_{t=1}^{T} M_t,$$

V is a positive definite weighting matrix and T is the sample size. In our application, we undertake a two step optimization procedure which results in asymptotically efficient estimates. This approach begins by using V equal to the identity matrix which results in consistent estimates of γ. These estimates are then used to construct an optimal weighting matrix given by

$$\widehat{V} = \frac{1}{T}\sum_{t=1}^{T} \widehat{M_t}\widehat{M_t'},$$

where $\widehat{M_t}$ is M_t evaluated at the consistent parameter estimates from the first step. A consistent estimate of the parameter covariance matrix is given by

$$\Omega = (\widetilde{S}'\widehat{V}^{-1}\widetilde{S})^{-1} \qquad \text{where} \qquad \widetilde{S} = \frac{1}{T}\sum_{t=1}^{T}\frac{\partial \widetilde{M_t}}{\partial \gamma}$$

and $\frac{\partial \widetilde{M_t}}{\partial \gamma}$ is $\frac{\partial M_t}{\partial \gamma}$ evaluated at the second step parameter estimates.

We also perform a number of different tests on the estimated parameters. For ordinary t-tests, we use standard formulas. For more sophisticated tests, such as testing for constant returns to scale, or testing whether government capital parameters in different production functions are equal, we use the DM test described in Newey and McFadden (1994) on page 2222. The DM test statistic is given by

$$DM_T = -2T[Q_T(\overline{\gamma}_T) - Q_T(\widetilde{\gamma}_T)],$$

where T is the sample size, $\overline{\gamma}_n$ is the constrained estimator, $\widetilde{\gamma}_n$ is the unconstrained estimator and

$$Q_T(\gamma_n) = -\frac{1}{2}M'\widehat{V}^{-1}M.$$

McFadden and West show that asymptotically this test statistic has a distribution of χ_r^2 where r is the number of restrictions.

1.3 Estimation Results

The model was estimated in numerous different ways, but to keep the exposition short, we will present only our final approach and summarize the alternatives in our discussion. All estimation approaches used a return on investment of $r = 4.0$, which is a popular rate of return used in the macroeconomic literature. The first estimation approach imposed no restrictions on the parameters and was used to test $H_0 : \rho = 0$, $H_0 : \alpha_1 + \alpha_2 + \alpha_3 = 1$, and both restrictions simultaneously using the DM test. The results of these tests for the different industries along with the aggregate economy are summarized in Table 1.1 below.

The DM test for the test statistics with one restriction have χ^2 distributions with one degree of freedom. These test statistics have critical values of 3.84 at the 95%, 5.02 at the 97.5% and 6.63 at the 99% confidence levels. The joint test DM statistic has a χ^2 distribution with two degrees of freedom and has critical values at the 5.99 at the 95%, 7.38 at the 97.5 and 9.21 at the 99% levels.

Table 1.1 is organized so that each row provides information on the various tests for first the aggregate production function in the top row and then the nine industry groups in the next rows.[6] Reading across the columns, the second and third column give results for $H_0 : \rho = 0$, with the second column giving the test statistic value and the third column a statement of yes or no as to whether the test statistic is significant at the 95% level. Columns four and five have a similar format, only here the focus is on $H_0 : \alpha_1 + \alpha_2 + \alpha_3 = 1$ while columns six and seven have the same format with a focus on the joint test.

Focusing on the aggregate production function, we see that none of the restrictions are rejected at the 95% level. This is consistent with findings in Cassou and Ai (1995). Next, focusing on the individual industries, we see that the test of $H_0 : \rho = 0$ is mostly insignificant at the 95% level, but there are a few cases where it is not. However, for these industries, the test statistic is not greater than 6.63, which is the 99% critical level. Taken as a whole, these tests indicated that there is little evidence that second differencing is necessary.

Next, focusing on the constant returns to scale test, we see fewer insignificant test statistics at the 95% level. Two of the test statistics that are significant at the 95% level are not larger than 5.02, which is the 97.5% confidence value, while three of the test statistics remain significant even at the 99% level. For the joint test, we see that there are four industries that are significant at the 95% level, but two of the test statistics are somewhat marginally significant as they do not maintain significance at the 97.5% or 99% level. The

[6] These include, Manufacturing, Mining, Construction, Transportation & Utilities, Trade, Finance & Insurance, Education, Healthcare and Lodging. We use "&" rather than "and" when it is part of an industry name.

All industries, except Transportation & Utilities and Lodging, use yearly data from 1948 to 2009. Due to data limitations, Lodging and Transportation & Utilities use yearly data from 1977 to 2008. A complete description of the data is contained in section 1.5.

Table 1.1: *Initial Tests*
Tests of $H_0 : \rho = 0$, $H_0 : \alpha_1 + \alpha_2 + \alpha_3 = 1$ and
$H_0 : \rho = 0$ and $\alpha_1 + \alpha_2 + \alpha_3 = 1$

	$H_0 : \rho = 0$		$H_0 : \alpha_1 + \alpha_2 + \alpha_3 = 1$		Joint Test	
	DM Stat	Reject	DM Stat	Reject	DM Stat	Reject
Aggregate Production	0.037	no	0.373	no	0.386	no
Manufacturing	0.729	no	3.262	no	3.268	no
Mining	0.574	no	1.539	no	1.559	no
Construction	4.444	yes	7.376	yes	8.425	yes
Transport & Utilities	0.092	no	2.560	no	4.314	no
Trade	4.980	yes	9.580	yes	10.894	yes
Finance & Insurance	0.539	no	20.641	yes	25.418	yes
Education	1.758	no	0.279	no	1.761	no
Healthcare	0.063	no	4.400	yes	5.803	no
Lodging (1977-2008)	0.003	no	4.187	yes	6.197	yes

fact that these restrictions do not hold for all industries is our first evidence that there are differences in the production functions across industries.

Our next investigation into the differences in the industry production functions is to formally test whether the industries have the same government capital production elasticity. To do this, we need to construct restricted and unrestricted estimation algorithms that are nested in such a way that we can compute the DM statistic. This requires that the individual industry production functions are estimated under the same constraints. Since the majority of the firms indicated that the constraints were not binding, we will focus on industry estimates in which both the constraints are imposed.

Table 1.2 presents the production function estimates under the restrictions where $\rho = 0$ and $\alpha_1 + \alpha_2 + \alpha_3 = 1$. The aggregate production function shows rather common elasticity estimates, with the labor elasticity equaling 0.640, the private capital elasticity equaling 0.233 and the government capital elasticity equaling 0.127. Next focusing on the industry estimates, we see that the elasticities vary considerably. Education has the highest labor elasticity of 0.866. This seems reasonable, since labor inputs are relatively larger than capital inputs in this industry. Next, Manufacturing and Construction come in with labor elasticities of 0.705 and 0.699. Again these numbers likely reflect the high labor intensity. At the other extreme are Transportation & Utilities and Mining with considerably smaller labor elasticities. These likely reflect the rather capital intensive nature of these industries.

Table 1.2 also shows that the private capital elasticities reflect more or less opposite patterns from the labor elasticities. So Education, Manufacturing and Construction, with their large labor elasticities show smaller private capital elasticities while Transportation & Utilities and Mining with their small labor elasticities show larger private capital elasticities.

The government capital elasticities are fairly consistent across industries, with the exception of Education which has a very low value. Outside of Education these elasticity estimates range from a low of 0.226 for Manufacturing to a high of 0.379 for Finance & Insurance. Interestingly, all but the Education elasticity are larger than the Aggregate Production elasticity.[7]

[7] This could arise for a number of reasons. It could be an artifact of the way the industry level data is collected. Alternatively, it may reflect an economy of scale substitution feature of the Aggregate Production function that is unavailable to individuals. For instance, individual farmers may be more dependent on roadway capital for shipping their output than large Agribusiness that can use private railways for shipping.

Table 1.2: *Parameter Estimation ($\rho = 0$ and $\alpha_1 + \alpha_2 + \alpha_3 = 1$)*

	$\ln(\mu)$	α_1	α_2	α_3	α_4	δ^c	δ^g
Aggregate Production	0.017	0.640	0.233	0.127	0.236	0.079	0.017
	(1.722)	(26.265)	(18.702)		(1.054)	(7.182)	(2.283)
Manufacturing	0.01	0.705	0.069	0.226	0.257	0.121	0.028
	(0.64)	(12.530)	(2.462)		(0.665)	(4.779)	(1.283)
Mining	0.011	0.389	0.259	0.351	0.057	0.106	0.028
	(0.138)	(5.557)	(3.252)		(0.030)	(3.364)	(1.129)
Construction	0.009	0.699	0.04	0.261	0.165	0.175	0.027
	(0.479)	(21.62)	(4.224)		(0.47)	(6.179)	(1.091)
Transportation & Utilities	0.002	0.463	0.213	0.324	-0.171	0.091	0.028
	(0.098)	(18.105)	(5.937)		(-0.228)	(5.610)	(2.149)
Trade	0.004	0.640	0.039	0.322	0.382	0.121	0.028
	(0.209)	(14.107)	(2.249)		(0.573)	(4.858)	(1.114)
Finance & Insurance	0.02	0.566	0.056	0.379	0.017	0.144	0.028
	(0.767)	(23.148)	(2.115)		(0.332)	(5.822)	(1.184)
Education	0.016	0.866	0.113	0.021	0.148	0.066	0.027
	(0.582)	(37.375)	(3.297)		(0.239)	(2.617)	(1.088)
Healthcare	0.018	0.581	0.073	0.347	0.257	0.094	0.030
	(0.776)	(4.30)	(4.734)		(0.651)	(3.724)	(1.242)
Lodging (1977-2008)	0.016	0.590	0.102	0.308	0.251	0.062	0.028
	(0.683)	(17.547)	(7.238)		(0.35)	(3.35)	(1.179)

Our next investigation is to formally test whether the public capital elasticity estimates are the same across industries. There are a number of different ways one can do this. For example, at one extreme, one could construct a DM test statistic that jointly tests whether all the industry coefficients are the same simultaneously, or, at the other extreme, one could construct simple DM test statistics which test only two industries at a time, or one could construct something in between. We carried out all the variations of these tests, but the results are strong enough at the two industry level that here we only provide those results.[8]

Table 1.3 shows the DM test statistics for all of the pairwise industry tests. The table is constructed so that an entry in a particular row and column shows the DM statistic for the null hypothesis that the government capital elasticity for the industry in that row is equal the government capital elasticity for the industry in that column. So in particular, looking

[8] Results for greater numbers of industries are reported in section 1.6

Table 1.3: *DM test - 2 Sectors*

	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)
(1) Aggregate Production	-								
(2) Manufacturing	207.00	-							
(3) Mining	334.39	88.02	-						
(4) Construction	711.91	24.27	50.95	-					
(5) Transportation & Utilities	408.74	110.97	0.75	17.75	-				
(6) Trade	821.46	122.44	4.74	67.52	9.29	-			
(7) Finance & Insurance	1435.67	315.32	3.58	268.22	0.01	45.41	-		
(8) Education	249.14	562.15	596.15	1141.97	754.11	1264.81	1845.27	-	
(9) Healthcare	88.40	18.36	2.03	7.86	147.97	0.01	6.39	200.14	-
(10) Lodging	493.71	106.51	26.78	8.35	2.44	3.51	3.44	883.79	144.67

down the first (numerical) column to the second row, we find the DM statistic value of 88.02. This DM statistic is computed for the null hypothesis that the government capital elasticity for Manufacturing is the same as the government capital elasticity for Mining. Since all of the pairwise tests impose one restriction, they all have a χ^2 distribution with one degree of freedom. These test statistics have critical values of 3.84 at the 95%, 5.02 at the 97.5% and 6.63 at the 99% confidence levels. In the case of the Manufacturing and Mining test, we see that the value of 88.02 is far above all of the critical values and thus the null hypothesis is easily rejected.

Now that the structure of Table 1.3 is understood, one can look through the table and see that the vast majority of the DM statistics are very large, indicating that the public capital elasticities are quite different across the industries. There are only six DM statistics that are insignificant at the 95% level, and another one at the 99% level. These statistics provide strong evidence that the contribution of government capital to industry production is considerably different across industries. Furthermore, these pairwise tests are sufficient to recognize what would happen if greater numbers of industries where tested simultaneously. In those tests, which are reported in the section 1.6, the DM statistics are almost always significant indicating that the industry level elasticities are different.

1.4 Conclusion

This paper investigated industry level production functions. Part of the interest in doing this is to contribute to the ongoing improvements in dynamic macroeconomic models which are increasingly disaggregating the economies into industrial sectors. This essay provides useful production function parameter values for that endeavour.

Second we show that there are differences across industry level production functions, so model disaggregation cannot rely on a generic scaled down aggregate production function. We have shown evidence of these differences in several ways. First, we showed that some, but not all, industry level production functions exhibit constant returns to scale. Second, we conducted pairwise tests as to whether the government capital production elasticity was the same for a pair of industries. In the majority of these tests, this null hypothesis was easily rejected.

1.5 Data Sources

Table 1.4: *Output*

Industry	Dates	Source
(1) Private Industries	1948-2009	BEA: Value Added by Industry (Release Date - Sep 30, 2010)
(2) Manufacturing	1948-2009	BEA: Value Added by Industry (Release Date - Sep 30, 2010)
(3) Mining	1948-2009	BEA: Value Added by Industry (Release Date - Sep 30, 2010)
(4) Construction	1948-2009	BEA: Value Added by Industry (Release Date - Sep 30, 2010)
(5) Transportation & Utilities[1]	1977-2008	BEA: Value Added by Industry (Release Date - Sep 30, 2010)
(6) Trade[2]	1948-2009	BEA: Value Added by Industry (Release Date - Sep 30, 2010)
(7) Finance & Insurance	1948-2009	BEA: Value Added by Industry (Release Date - Sep 30, 2010)
(8) Education	1948-2009	BEA: Value Added by Industry (Release Date - Sep 30, 2010)
(9) Healthcare	1948-2009	BEA: Value Added by Industry (Release Date - Sep 30, 2010)
(10) Lodging/Accommodation	1977-2008	BEA: Value Added by Industry (Release Date - Sep 30, 2010)

[1]Series was obtained by adding four subseries: transportation & warehousing, waste management, broadcasting & telecommunications and utilities

[2]Series was obtained by adding two subseries: wholesale trade and retail trade

Table 1.5: *Private Sector Capital*

Industry	Dates	Source
Private Industries	1948-2009	BEA: Fixed Assets Tables: Table 3.3ES: Stock of Private Fixed Assets by Industry
Manufacturing	1948-2009	BEA: Fixed Assets Tables: Table 3.3ES: Stock of Private Fixed Assets by Industry
Mining	1948-2009	BEA: Fixed Assets Tables: Table 3.3ES: Stock of Private Fixed Assets by Industry
Construction	1948-2009	BEA: Fixed Assets Tables: Table 3.3ES: Stock of Private Fixed Assets by Industry
Transportation & Utilities[1]	1977-2009	BEA: Fixed Assets Tables: Table 3.3ES: Stock of Private Fixed Assets by Industry
Trade[2]	1948-2009	BEA: Fixed Assets Tables: Table 3.3ES: Stock of Private Fixed Assets by Industry
Finance & Insurance	1948-2009	BEA: Fixed Assets Tables: Table 3.3ES: Stock of Private Fixed Assets by Industry
Education	1948-2009	BEA: Fixed Assets Tables: Table 3.3ES: Stock of Private Fixed Assets by Industry
Healthcare	1948-2009	BEA: Fixed Assets Tables: Table 3.3ES: Stock of Private Fixed Assets by Industry
Lodging/Accommodation	1948-2009	BEA: Fixed Assets Tables: Table 3.3ES: Stock of Private Fixed Assets by Industry

Table 1.6: *Private Sector Investment*

Industry	Dates	Source
Private Industries	1948-2009	BEA: Fixed Assets Tables: Table 3.7ES: Investment in Private Fixed Assets by Industry
Manufacturing	1948-2009	BEA: Fixed Assets Tables: Table 3.7ES: Investment in Private Fixed Assets by Industry
Mining	1948-2009	BEA: Fixed Assets Tables: Table 3.7ES: Investment in Private Fixed Assets by Industry
Construction	1948-2009	BEA: Fixed Assets Tables: Table 3.7ES: Investment in Private Fixed Assets by Industry
Transportation & Utilities[1]	1977-2009	BEA: Fixed Assets Tables: Table 3.7ES. Investment in Private Fixed Assets by Industry
Trade[2]	1948-2009	BEA: Fixed Assets Tables: Table 3.7ES: Investment in Private Fixed Assets by Industry
Finance & Insurance	1948-2009	BEA: Fixed Assets Tables: Table 3.7ES: Investment in Private Fixed Assets by Industry
Education	1948-2009	BEA: Fixed Assets Tables: Table 3.7ES: Investment in Private Fixed Assets by Industry
Healthcare	1948-2009	BEA: Fixed Assets Tables: Table 3.7ES: Investment in Private Fixed Assets by Industry
Lodging/Accommodation	1948-2009	BEA: Fixed Assets Tables: Table 3:7ES. Investment in Private Fixed Assets by Industry

Table 1.7: *Hours*

Industry	Dates	Source
(1) Private Industries	1948-2009	NIPA: GDP: Table 6.9: Hours Worked by FT and PT Employees by Industry
(2) Manufacturing	1948-2009	NIPA: GDP: Table 6.9: Hours Worked by FT and PT Employees by Industry
(3) Mining	1948-2009	NIPA: GDP: Table 6.9: Hours Worked by FT and PT Employees by Industry
(4) Construction	1948-2009	NIPA: GDP: Table 6.9: Hours Worked by FT and PT Employees by Industry
(5) Transportation & Utilities	1977-2009	NIPA: GDP: Table 6.9: Hours Worked by FT and PT Employees by Industry
(6) Trade[3]	1948-2009	NIPA: GDP: Table 6.9: Hours Worked by FT and PT Employees by Industry
(7) Finance & Insurance[4]	1948-2009	NIPA: GDP: Table 6.5: Full-Time Equivalent Employees by Industry
(8) Education[4]	1948-2009	NIPA: GDP: Table 6.5: Full-Time Equivalent Employees by Industry
(9) Healthcare[4]	1948-2009	NIPA: GDP: Table 6.5: Full-Time Equivalent Employees by Industry
(10) Lodging/Accommodation[4]	1948-2009	NIPA: GDP: Table 6.5: Full-Time Equivalent Employees by Industry

Table 1.8: *Wages*

Industry	Dates	Source
Private Industries	1948-2009	NIPA: GDP: Table 6.2 - Compensation of Employees by Industry
Manufacturing	1948-2009	NIPA: GDP: Table 6.2 - Compensation of Employees by Industry
Mining	1948-2009	NIPA: GDP: Table 6.2: Compensation of Employees by Industry
Construction	1948-2009	NIPA: GDP: Table 6.2: Compensation of Employees by Industry
Transportation & Utilities	1977-2009	NIPA: GDP: Table 6.2: Compensation of Employees by Industry
Trade[3]	1948-2009	NIPA: GDP: Table 6.2: Compensation of Employees by Industry
Finance & Insurance	1948-2009	NIPA: GDP: Table 6.2: Compensation of Employees by Industry
Education	1948-2009	NIPA: GDP: Table 6.2: Compensation of Employees by Industry
Healthcare	1948-2009	NIPA: GDP: Table 6.2: Compensation of Employees by Industry
Lodging/Accommodation	1948-2009	NIPA: GDP: Table 6.2: Compensation of Employees by Industry

Table 1.9: *Government Capital, Investment and Industry Utilization Rates*

Series	Dates	Source
Government Capital Stock[5]	1948 - 2009	BEA: Fixed Assets Tables: Table 7.2 : Chain-Type Quantity Indexes for Net Stock of Government Fixed Assets
Government Capital Investment[5]	1948 - 2009	BEA: Fixed Assets Tables: Table 7.6 : Chain-Type Quantity Indexes for Investment in Government Fixed Assets
Utilization Rate	1948 - 2009	Board of Governors: Table G.17: Industrial Production and Capacity Utilization

1.6 Other elasticity tests

In this section we show the results of the DM tests for larger numbers of government capital elasticities being equal than those given in Table 1.3. In particular, Table 1.10 shows the DM test statistics for the null that three industries have the same elasticity, Table 1.11 shows the DM test statistics for the null that four industries have the same elasticity, Table 1.12 ...

Table 1.10: *DM test - 3 Sectors (critical value = 5.99)*

	(1) & (2)	(1) & (3)	(1) & (4)	(1) & (5)	(1) & (6)	(1) & (7)	(1) & (8)	(1) & (9)
(1) Aggregate Production	-							
(2) Manufacturing	-	-						
(3) Mining	449.52	-	-					
(4) Construction	720.10	865.30	-	-				
(5) Transportation & Utilities	416.39	434.36	784.06	-	-			
(6) Trade	857.04	1003.86	1136.58	536.10	-	-		
(7) Finance & Insurance	1442.83	1584.41	1641.48	786.25	1815.53	-	-	
(8) Education	568.97	651.15	1324.59	767.19	1334.99	2080.45	-	-
(9) Healthcare	254.00	385.17	723.27	409.44	847.56	1453.69	364.16	-
(10) Lodging	494.38	512.42	788.96	682.87	587.14	805.07	950.27	500.06

Table 1.11: *DM test - 4 Sectors (critical value = 7.81)*

	(1)&(2)&(3)	(1)&(2)&(4)	(1)&(2)&(5)	(1)&(2)&(6)	(1)&(2)&(7)	(1)&(2)&(8)	(1)&(2)&(9)
(1) Aggregate Production	-						
(2) Manufacturing	-	-					
(3) Mining	-	-	-				
(4) Construction	893.87	-	-	-			
(5) Transportation & Utilities	438.63	810.32	-	-	-		
(6) Trade	1044.68	1147.81	536.21	-	-	-	
(7) Finance & Insurance	1601.99	1644.66	794.66	1819.10	-	-	-
(8) Education	905.01	1428.41	826.48	1483.40	2183.92	-	-
(9) Healthcare	508.95	759.92	418.26	901.23	1478.51	658.23	-
(10) Lodging	513.91	819.04	686.48	588.29	818.86	981.74	501.82

Table 1.12: *DM test - 5 Sectors (critical value = 9.49)*

	(1) & (2) & (3) & (4)	(1) & (2) & (3) & (5)	(1) & (2) & (3) & (6)	(1) & (2) & (3) & (7)	(1) & (2) & (3) & (8)	(1) & (2) & (3) & (9)
(1) Aggregate Production	-					
(2) Manufacturing	-	-				
(3) Mining	-	-	-			
(4) Construction	-	-	-	-		
(5) Transportation & Utilities	815.24	-	-	-	-	
(6) Trade	1287.25	553.12	-	-	-	-
(7) Finance & Insurance	1767.06	805.41	1941.42	-	-	-
(8) Education	1565.36	866.46	1736.65	2408.05	-	-
(9) Healthcare	930.24	441.59	1084.22	1634.06	985.53	-
(10) Lodging	823.03	697.37	602.72	827.42	1014.18	521.29

Table 1.13: *DM test - 6 Sectors (critical value = 11.1)*

	(1) & (2) & (3) (4) & (5)	(1) & (2) & (3) (4) & (6)	(1) & (2) & (3) (4) & (7)	(1) & (2) & (3) (4) & (8)	(1) & (2) & (3) (4) & (9)
(1) Aggregate Production	-				
(2) Manufacturing	-	-			
(3) Mining	-	-	-		
(4) Construction	-	-	-	-	
(5) Transportation & Utilities	-	-	-	-	-
(6) Trade	841.19	-	-	-	-
(7) Finance & Insurance	972.85	2137.54	-	-	-
(8) Education	1645.52	2244.33	2805.03	-	-
(9) Healthcare	869.05	1403.56	1882.10	1728.21	-
(10) Lodging	905.55	846.31	972.31	1679.15	880.31

Table 1.14: *DM test - 7 Sectors (critical value = 12.6)*

	(1) & (2) & (3) (4) & (5) & (6)	(1) & (2) & (3) (4) & (5) & (6)	(1) & (2) & (3) (4) & (5) & (6)	(1) & (2) & (3) (4) & (5) & (6)
(1) Aggregate Production	-			
(2) Manufacturing	-	-		
(3) Mining	-	-	-	
(4) Construction	-	-	-	-
(5) Transportation & Utilities	-	-	-	-
(6) Trade	-	-	-	-
(7) Finance & Insurance	986.95	-	-	-
(8) Education	1705.60	1909.32	-	-
(9) Healthcare	902.23	1047.18	1670.02	-
(10) Lodging	922.58	1033.94	1814.70	974.04

Table 1.15: *DM test - 8 Sectors (critical value = 14.1)*

	(1) & (2) & (3) & (4) (5) & (6) & (7)	(1) & (2) & (3) & (4) (5) & (6) & (7)	(1) & (2) & (3) & (4) (5) & (6) & (7)
(1) Aggregate Production	-		
(2) Manufacturing	-	-	
(3) Mining	-	-	-
(4) Construction	-	-	-
(5) Transportation & Utilities	-	-	-
(6) Trade	-	-	-
(7) Finance & Insurance	-	-	-
(8) Education	1948.95	-	-
(9) Healthcare	1064.82	1735.60	-
(10) Lodging	1043.33	1857.61	997.26

Table 1.16: *DM test - 9 Sectors (critical value = 15.5)*

	(1) & (2) & (3) & (4) (5) & (6) & (7) & (8)	(1) & (2) & (3) & (4) (5) & (6) & (7) & (9)
(1) Aggregate Production	-	
(2) Manufacturing	-	-
(3) Mining	-	-
(4) Construction	-	-
(5) Transportation & Utilities	-	-
(6) Trade	-	-
(7) Finance & Insurance	-	-
(8) Education	-	-
(9) Healthcare	1995.12	-
(10) Lodging	2060.42	1132.25

Table 1.17: *DM test - 10 Sectors (critical value = 16.9)*

	(1) & (2) & (3) & (4) (5) & (6) & (7) & (8) & (9)
(1) Aggregate Production	-
(2) Manufacturing	-
(3) Mining	-
(4) Construction	-
(5) Transportation & Utilities	-
(6) Trade	-
(7) Finance & Insurance	-
(8) Education	-
(9) Healthcare	-
(10) Lodging	1610.56

Chapter 2

Do Sticky Wages Affect the Oil Price-Macroeconomy Relationship?

2.1 Introduction

A large literature has attempted to understand the effects of an oil shock on the economy (Ferderer, 1996; Hamilton, 1983, 1996; Kilian and Vigfusson, 2011; Lee, Ni and Ratti, 1995). Several mechanisms have been proposed to explain the transmission of oil shocks to the macroeconomy. One mechanism views an increase in the price of oil as a supply shock that raises the marginal cost of production (Blanchard and Gali, 2007). The higher oil price leads to higher inflation and less demand for output. Lee, Ni and Ratti (1995), and Ferderer (1996) have argued that the uncertainty caused by big fluctuations in the price of oil is more important than the change in the price of oil. Firms have an incentive to postpone investment and consumers have an incentive to postpone purchases of vehicles during periods of high oil price volatility. Hamilton (1988) demonstrated that changes in the price of oil in either direction can be recessionary. A decrease in the price of oil will cause unemployment of workers in the oil sector, but those workers cannot immediately move to non-oil sectors, so the overall unemployment rate rises. Oil shocks can also affect the economy indirectly if the central bank changes its monetary policy instrument in response to oil price changes (Bernanke, Gertler and Watson, 1997, 2004; Hamilton and Herrera, 2004; Leduc and Sill, 2004).

Blanchard and Gali (2007, hereafter BG) have emphasized the importance of wage rigidities for the transmission of oil shocks to the economy. They showed that the response of output and inflation to a change in the price of oil will be much greater in the presence of wage rigidities than they are in a flexible wage (no wage rigidity) equilibrium. This important result suggests that one cannot hope to have a good understanding of the effects of oil shocks without understanding how wages are determined. One limitation of their work was that wage rigidities were introduced in an ad hoc fashion. This essay builds on their work by deriving wage rigidities in a microfounded model that uses parameters taken from published papers.

BG imposed wage rigidities by adding to their model a parameter, γ, that they describe as "an index of the degree of real wage rigidities". To interpret the parameter γ, let rw_t be the deviation from the steady state of the observed real wage at time t and $\widetilde{rw}_t$ be the deviation from the steady state of the real wage at time t in the absence of wage rigidities, with the relationship $rw_t = (1-\gamma)\,\widetilde{rw}_t$. As $\gamma \to 0$, there are no wage rigidities, but as $\gamma \to 1$, the wage does not respond to shocks to the economy. They explain, "While clearly ad-hoc, equation (3) is meant to capture in a parsimonious way the notion that real wages may not respond to labor market conditions as much as implied by the model with perfectly competitive markets." While this specification of wage rigidities suffices for their goal of demonstrating that labor market frictions can affect the oil price-macroeconomy relationship, it leaves unanswered the question of how important labor market frictions actually are for the US economy. Bachmeier and Cha (2011) found no evidence that the labor market plays a role in the response of inflation to oil shocks.

This chapter solves the model of BG with two important modifications. First, we add a wage adjustment cost function as in Kim and Ruge-Murcia (2009). Doing so allows us to solve for the equilibrium values of output and inflation in terms of the parameters of the wage adjustment cost function. We then plug in the estimates of the wage adjustment cost function parameters reported by Kim and Ruge-Murcia (2009). Simulations along the lines of BG are used to compute the effect of wage stickiness on the responses of output and inflation to oil shocks. Second, we impose a monetary policy rule that is consistent with the published literature. Monetary policy in BG responds only to the contemporaneous value of inflation. The monetary policy rule in our model sets the interest rate as a function of expected output and inflation, incorporates interest rate smoothing, and uses the coefficients estimated by Clarida, Gali and Gertler (2000). The transmission of oil shocks to the economy was shown by BG to be sensitive to the monetary policy rule, so it is desirable to work with a monetary policy rule that is consistent with the practice of the Federal Reserve.

There are (at least) two potential criticisms of our choice of theoretical framework. The first is that we are evaluating the ability of only one type of model with wage rigidities to affect the oil price-macroeconomy relationship. As noted by Kim and Ruge-Murcia (2009), there are two approaches to adding wage rigidities to macroeconomic models, with one being the wage adjustment cost as in this chapter, and the other a Calvo-style staggering of wage decisions. There are good reasons to prefer the former approach. Calvo-style staggered wage setting implies symmetry in wage stickiness - there is the same resistance to a wage increase as there is to a wage decrease of the same magnitude. It is not clear why wage stickiness should be symmetric. Economic principles tell us that workers will be upset with nominal wage reductions but happy with nominal wage increases, and there is strong empirical evidence in favor of asymmetric wage stickiness (Kim and Ruge-Murcia, 2009). A Calvo mechanism would have to be supplemented with some other source of asymmetry. In addition, we are able to pull estimates of the parameters of the wage adjustment cost function from the literature, but are not aware of a published paper with estimates of the parameters of Calvo-style staggered wage setting in a DSGE setting.[1]

[1] Obviously, there are other ways to estimate those parameters. A simple approach, given the proper dataset, would just be to use the average amount of time between wage negotiations to infer the probability a worker's wage changes in each time period. This simple approach could lead to misleading results, however, because it would only capture those cases in which the wage changed, ignoring times that the wage could have changed but did not. A time period with large oil shocks would cause wages to change more often than normal. Note that this is not an argument that the parameters of the Calvo model are time-varying. Even if the probability δ that a worker's wage will change is constant, looking at the duration between wage changes will not properly capture this, because it excludes all of the times that it was optimal to leave

The second criticism is that we are studying a log-linearized solution of our model. Given the asymmetry in the wage adjustment cost function, it is perhaps more natural to take a higher-order approximation around the steady state. That was the approach taken by Kim and Ruge-Murcia (2009). Higher-order approximations of DSGE models are now routinely used in published papers (Faia and Monacelli, 2007; Gomme and Klein, 2011). The default solution algorithm for the Dynare (Adjemian et al., 2012) program for solution of rational expectations models is to take a second-order approximation as in Sims (2002). BG analyzed a log-linearized model, and our goal is to see if the effect of wage rigidities on the oil price-macroeconomy relationship that they found is consistent with empirical estimates of wage adjustment costs, so an apples-to-apples comparison can only be done if we use a log-linearized model.

We believe that there is merit to both criticisms. However, both of those extensions to our model, while important and interesting, are beyond the scope of this chapter. We believe they would be fruitful topics for future research.

The essay is structured in the following order. Section 2.2 describes the theoretical model, log-linearization around the steady state, and solution using the generalized Schur method. Section 2.3 explains the choice of parameters and compares the results of simulations from our model to those in BG. Section 2.4 demonstrates the magnitude of the change in wage adjustment costs that would be necessary to generate large changes in the effects of oil shocks. Section 2.5 is the conclusion.

2.2 The Model

The model is a modified version of Blanchard and Gali (2007). The two important differences, discussed in detail below, are the presence of wage adjustment costs, and a more realistic monetary policy rule. Throughout this section, upper case variables with time sub-

the wage unchanged. One reason that wages might not change would be a fixed cost associated with the negotiation. If the data come from a low inflation time period, as is the case for the US over the last two decades, it is possible that it was optimal to not change the wage in most cases when they could have done so. That would lead to a large upward bias in the estimate of δ.

script denote levels, while lower case variables with time subscripts are percentage deviations their steady state values.

2.2.1 Households

The economy consists of a continuum of infinitely lived households. Following Kim and Ruge-Murcia (2009), households are assumed to have differentiated job skills, but are otherwise identical, giving them monopolistic bargaining power over their labor supply. Each household maximizes

$$max \ \ E_0 \sum_{t=0}^{\infty} \beta^t \left(ln\left(C_t\right) - \frac{N_t^{1+\phi}}{1+\phi} \right)$$

subject to the constraints

$$(i) \quad P_{q,t}C_{q,t} + P_{m,t}C_{m,t} + Q_t^B = W_t N_t \left(1 - \Phi_t\right) + B_{t-1} + \Pi_t \tag{2.1}$$

$$(ii) \quad \Phi_t = \left(\frac{\delta}{\psi^2}\right)\left[e^{-\psi\left(\frac{\sigma W_t}{W_{t-1}}-1\right)} + \psi\left(\frac{W_t}{W_{t-1}} - 1\right) - 1\right] \tag{2.2}$$

$$(iii) \quad C_t = \Theta_\chi C_{m,t}^{\chi} C_{q,t}^{1-\chi} \tag{2.3}$$

$$(iv) \quad C_{q,t} = \left[\int_0^1 C_{q,t}(i) di^{\frac{\epsilon-1}{\epsilon}}\right]^{\frac{\epsilon}{\epsilon-1}} \tag{2.4}$$

$$(v) \quad \Theta_\chi = \chi^{-\chi}(1-\chi)^{-(1-\chi)} \tag{2.5}$$

$$(vi) \quad P_{q,t} = \left[\int_0^1 P_{q,t}(i) di^{1-\epsilon}\right]^{\frac{1}{1-\epsilon}} \tag{2.6}$$

$$(vii) \quad P_{c,t} = P_{m,t}^{\chi} P_{q,t}^{1-\chi} \tag{2.7}$$

where C_t is consumption; N_t is employment; $C_{q,t}$ is a constant elasticity of substitution (CES) index of domestic goods; $C_{m,t}$ is consumption of (imported) oil; χ is the share of oil in consumption; $P_{q,t}$ is a price index of domestically produced goods; $P_{m,t}$ is the price of oil (in domestic currency); B_t is the quantity of risk-free domestic bonds purchased in period t; Q_t^B is the price of that bond; W_t is the nominal wage.

While the household's problem here is similar to that in BG, there is an important difference with respect to wage setting, as can be seen in equations (2.1) and (2.2). Following

Kim and Ruge-Murcia (2009), households face a cost when wages are changed (Φ_t), where σ, δ and ψ are cost parameters that determine the shape of the adjustment cost function. Households choose the wage and labor supply accounting for firms' demand for labor.

The function in equation (2.2) was called a linex function by Varian (1974). The advantage of the linex function is that it is extremely flexible. Several special cases illustrate the flexibility of the wage adjustment cost function. As $\psi \to 0$, Φ_t becomes a quadratic function in W_t/W_{t-1}. The shape of Φ_t implies that a wage decrease is more costly to households than a wage increase of the same magnitude when $\psi > 0$. As $\psi \to \infty$, the cost of a wage decrease goes to infinity, while the cost of a wage increase goes to zero, which would be a case of fully flexible wages subject to the constraint that wages cannot fall. As $\delta \to 0$, there is no wage adjustment cost, and wages are fully flexible in either direction. As $\delta \to \infty$, wages do not change in either direction in response to shocks. These special cases demonstrate the ability of the choice of Φ to capture many types of symmetric or asymmetric wage stickiness.

First order conditions for households

The household's optimization problem is solved by substituting equations (2.2)-(2.7) into the budget constraint (2.1), setting up the Lagrangian, and taking derivatives. The first order conditions are:

$$\frac{\partial L}{\partial N_t} : \; N_t^{\phi} = \lambda_t W_t \left(1 - \Phi_t\right) \tag{2.8}$$

$$\frac{\partial L}{\partial C_{q,t}} : \; (1 - \chi) = \lambda_t P_{q,t} C_{q,t} \tag{2.9}$$

$$\frac{\partial L}{\partial C_{m,t}} : \; \chi = \lambda_t P_{m,t} C_{m,t} \tag{2.10}$$

$$\frac{\partial L}{\partial B_t} : \; Q_t^B = \beta \left(\frac{\lambda_{t+1}}{\lambda_t}\right) \tag{2.11}$$

$$\frac{\partial L}{\partial W_t}:\ \lambda_t N_t \left\{ \left[\sigma\psi \left(\frac{W_t}{W_{t-1}} \right) - 1 \right] e^{-\psi\left(\frac{\sigma W_t}{W_{t-1}}-1\right)} - 2\psi \left(\frac{W_t}{W_{t-1}} \right) + \psi \left(1 + \frac{\psi}{\delta} \right) + 1 \right\} =$$

$$= \beta\lambda_{t+1} N_{t+1} \left(\frac{\psi}{W_t^2} \right) \left\{ \sigma e^{-\psi\left(\frac{\sigma W_t}{W_{t-1}}-1\right)} - 1 \right\} \quad (2.12)$$

where λ_t is the Lagrange multiplier. The equilibrium behavior of households is summarized by the following three equations. The first is an intertemporal condition on consumption

$$c_t = E_t c_{t+1} - (i_t - E_t \pi_{c,t+1}) \quad (2.13)$$

where i_t is the nominal interest rate, and $\pi_{c,t} = p_{c,t} - p_{c,t-1}$ is CPI inflation. Equation (2.13) corresponds to equation (A.13) in the appendix.

The second equation describes the labor supply

$$\left[1 + \frac{\varphi\Phi}{1-\Phi} \right] w_t = p_{c,t} + c_t + \phi n_t + \left(\frac{\varphi\Phi}{1-\Phi} \right) w_{t-1} \quad (2.14)$$

with

$$\varphi = \frac{\psi\left[\sigma e^{-\psi(\sigma-1)} - 1\right]}{\left[e^{-\psi(\sigma-1)} - 1\right]}$$

and

$$\Phi = \left(\frac{\delta}{\psi^2} \right) \left[e^{-\psi(\sigma-1)} - 1 \right],$$

where w_t is the nominal wage and ϕ is the inverse of the Frisch elasticity of labor supply. Equation (2.14) corresponds to equation (A.12) in the appendix.

The third equation is a log-linearized version of household's first order condition with respect to the nominal wage (W_t)

$$(E_t p_{c,t-1} - p_{c,t}) + (c_{t+1} - c_t) - (E_t n_{t+1} - n_t) + (\varphi_1 w_t + \varphi_2 w_{t-1} + \varphi_3 E_t w_{t+1}) = 0. \quad (2.15)$$

Households will choose a value for the wage such that equation (2.15) holds.

Oil and Consumption

The price of oil affects households because oil is a consumption good. Total consumption is given by

$$c_t = (1 - \chi)\, c_{q,t} + \chi c_{m,t} \tag{2.16}$$

where c_t is total consumption; $c_{q,t}$ is the consumption of domestically produced goods (gross output); and $c_{m,t}$ is the consumption of imported oil. Equation (2.16) is the log-linearized version of the third constraint in the household's optimization setup.

Let $p_{q,t}$ and $p_{c,t}$ be the prices of domestic output and consumption respectively. Let $p_{m,t}$ be the price of oil, and $[s_t = p_{m,t} - p_{q,t}]$ be the real price of oil. The relation between the domestic output price and the consumption price is given by

$$p_{c,t} = p_{q,t} + \chi p_{q,t} \tag{2.17}$$

One can see that an increase in the real price of oil will increase consumption price relative to the domestic output price. Equation (2.17) corresponds to equation (A.10) in the appendix.

2.2.2 Firms

Intermediate goods firm

Each firm produces a differentiated good indexed by $i \in [0, 1]$ using the following production function

$$Q_t(i) = A_t M_t(i)^{\alpha_m} N_t(i)^{\alpha_n} \tag{2.18}$$

where M_t and N_t are the quantities of imported oil and labor used in production. Using the assumption that firms take input prices as given, the firm's problem becomes:

$$min \;\; W_t N_t(i) + P_{m,t} M_t(i) + \lambda_t^f \left[A_t M_t(i)^{\alpha_m} N_t(i)^{\alpha_n} - Q_t(i) \right].$$

Firms minimize cost, captured by the two terms on the left, taking the production function as a constraint.

First order conditions for intermediate goods firms

Taking derivatives of the objective function, we obtain the following equations

$$\frac{\partial L}{\partial N_t(i)} : \; W_t = \lambda_t^f(\alpha_n)\left[\frac{Q_t(i)}{N_t(i)}\right] \tag{2.19}$$

$$\frac{\partial L}{\partial M_t(i)} : \; P_{m,t} = \lambda_t^f(\alpha_m)\left[\frac{Q_t(i)}{M_t(i)}\right] \tag{2.20}$$

$$\frac{\partial L}{\partial \lambda_t^f} : \; Q_t(i) = A_t M_t(i)^{\alpha_m} N_t(i)^{\alpha_n} \tag{2.21}$$

where λ_t^f can be interpreted as marginal cost. Let Ψ_t denote marginal cost. From equations (2.19) and (2.20)

$$\Psi_t(i) = \frac{W_t}{\alpha_n\left[\frac{Q_t(i)}{N_t(i)}\right]} = \frac{P_{m,t}}{\alpha_m\left[\frac{Q_t(i)}{M_t(i)}\right]} \tag{2.22}$$

Letting $\left[M_t^p(i) = \frac{P_{q,t}(i)}{\Psi_t(i)}\right]$ denote a gross markup by firm i, we have

$$M_t^P(i) \times S_t \times M_t(i) \;\; = \;\; \alpha_m Q_t(i)\left[\frac{P_{q,t}(i)}{P_{q,t}}\right], \tag{2.23}$$

where $S_t = \frac{P_{m,t}}{P_{q,t}}$ is the real price of oil. Equation (2.23) corresponds to equation (A.19) in the appendix.

Final goods firms

Let aggregate gross output be given by the formula

$$Q_t = \left[\int_0^1 Q_t(i)^{1-\frac{1}{\epsilon}} di\right]^{\frac{\epsilon}{\epsilon-1}} \tag{2.24}$$

The final goods firms then solve

$$\max_{Q_{t(i)}} P_{q,t}Q_t - \int_0^1 P_{q,t}(j)Q_t(j)dj$$

Plugging (2.24) into the objective function, the final goods firms solve

$$max \quad P_{q,t} \left[\int_0^1 Q_t(i)^{1-\frac{1}{\epsilon}} di \right]^{\frac{\epsilon}{\epsilon-1}} - \int_0^1 P_{q,t}(j) Q_t(j) dj \tag{2.25}$$

First order conditions for final goods firms

Taking derivatives of the objective function, we get

$$Q_t(i) = \left(\frac{P_{q,t}(i)}{P_{q,t}} \right)^{-\epsilon} \tag{2.26}$$

Plugging (2.26) into (2.23),

$$\left[\int_0^1 M_t(i) di \right] \left[\int_0^1 M_t^p(j) dj \right] = \frac{\alpha_m Q_t}{S_t}$$

Letting $M_t = \left[\int_0^1 M_t(i) di \right]$ and $M_t^p = \left[\int_0^1 M_t^p(j) dj \right]$, we can re-write the above equation as

$$M_t = \frac{\alpha_m Q_t}{S_t M_t^p} \tag{2.27}$$

Log-linearizing equation (2.27) gives us the demand for oil as an input in production.

$$m_t = -\mu_t^p - s_t + q_t \tag{2.28}$$

where μ_t^p is the price markup, s_t is the real price of oil, and q_t is domestic output. The demand for oil is negatively correlated with its price and positively correlated with output. It is negatively correlated with the markup because a higher markup requires that firms reduce output.

Log-linearizing equation (2.24) around the steady state, we get

$$q_t = a_t + \alpha_m m_t + \alpha_n n_t \tag{2.29}$$

Plugging (2.28) into (2.29), we obtain the reduced form production function

$$q_t = \left(\frac{1}{1-\alpha_m}\right)\left[a_t + \alpha_n n_t - \alpha_m \mu_t^p - \alpha_m s_t\right] \tag{2.30}$$

where q_t is the gross domestic output; a_t is the level of technology (exogenously given); n_t is hours worked; m_t is the quantity of oil used in production; s_t is the real price of oil; and $\alpha_n + \alpha_m \leq 1$.

Combining the cost minimization conditions for oil and labor with the aggregate production function, we obtain the factor price frontier[2]

$$(1-\alpha_m)(w_t - p_{c,t}) + [\alpha_m + (1-\alpha_m)\chi] s_t + (1-\alpha_n-\alpha_m) n_t - a_t + \mu_t^p = 0. \tag{2.31}$$

Staying on the factor price frontier requires that an increase in the real price of oil is accompanied by at least one of lower real wages, lower employment, or lower markup.

Calvo Price Setting

Firms set prices in a staggered fashion. Calvo's specification assumes that firms changes their prices infrequently. Each period there is a probability $(1-\theta)$ that a firm can adjust its price. Thus price adjustments occur randomly. For derivation of Calvo's model, please refer to Walsh (2003), pp. 225 - 228.

The optimal rule for price setting is

$$E_t \left\{ \sum_{k=0}^{k\infty} \theta^k \Lambda_{t,t+k} Q_{t,t+k|t} \left[P_t^* - M^P \Psi_{t+k|t}\right] \right\} = 0 \tag{2.32}$$

where P_t^* is the new price set at time t, $Q_{t,t+k}$ is the level of output in period $(t+k)$ for a firm that last set price in period t, $\Psi_{t+k|t}$ is the marginal cost in period $(t+k)$ for a firm that last set price in period t, and $M^p = \frac{\epsilon}{\epsilon-1}$ is the desired gross markup in the steady state.

Domestic price evolution is given by

$$P_{q,t} = \left[\theta (P_{q,t-1})^{1-\epsilon} + (1-\theta)(P_t^*)^{1-\epsilon}\right]^{\frac{1}{1-\epsilon}} \tag{2.33}$$

Log-linearizing (2.32) and (2.33) and rearranging terms

[2] We have adopted the terminology in Blanchard and Gali (2007).

$$\pi_{q,t} = \beta E_t \{\pi_{q,t+1}\} - \lambda_p \, \mu_t^p \tag{2.34}$$

where $\lambda_p = \left[\frac{(1-\theta)(1-\beta\theta)}{\theta}\right]\left[\frac{\alpha_m+\alpha_n}{1+(1-\alpha_m-\alpha_n)(\epsilon-1)}\right]$; $\pi_{q,t} = p_{q,t} - p_{q,t-1}$; β is the discount factor; θ is the fraction of firms that leave prices unchanged each time period, and ϵ is the elasticity of substitution between domestic goods in consumption.

2.2.3 Equilibrium

Combining equations (2.14) and (2.31), we get

$$c_t = q_t - \chi s_t + \eta \mu_t^p \tag{2.35}$$

where

$$\eta = \frac{\alpha_m}{M^p - \alpha_m}$$

and M^p is the steady state gross markup. Combining equation (2.35) with the reduced form production function (2.30) gives consumption as a function of productivity, employment, the real price of oil, and the markup

$$c_t = \left(\frac{1}{1-\alpha_m}\right)[a_t + \alpha_n n_t] - \left(\frac{\alpha_m}{1-\alpha_m} + \chi\right) s_t + \left(\eta - \frac{\alpha_m}{1-\alpha_m}\right) \mu_t^p. \tag{2.36}$$

Combining equations (2.36) and (2.14) with the factor price frontier, we obtain an expression for the markup

$$\Gamma_n n_t - \Gamma_w (w_t - w_{t-1}) = \mu_t^p \tag{2.37}$$

where $\Gamma_n = \frac{(1-\alpha_m)(1+\phi)}{1+\alpha_m-\eta(1-\alpha_m)}$ and $\Gamma_w = \frac{(1-\alpha_m)\left[\frac{\varphi\Phi}{1-\Phi}\right]}{1+\alpha_m-\eta(1-\alpha_m)}$. Finally, replacing the expression for the markup in equation (2.34) with equation (2.37) gives an expression for domestic inflation

$$\pi_{q,t} = \beta E_t \pi_{q,t+1} - \lambda_p \Gamma_n n_t + \lambda_p \Gamma_w (w_t - w_{t-1}). \tag{2.38}$$

2.2.4 Value Added and GDP Deflators

The value added deflator is given by

$$p_{y,t} = p_{q,t} - \left(\frac{\alpha_m}{1-\alpha_m}\right) s_t, \tag{2.39}$$

implying that the real price of oil has a negative effect on output, holding the domestic output price constant. Combining the definition of value added with the demand for oil, we come up with the following relationship between value added and output

$$y_t = q_t + \left(\frac{\alpha_m}{1-\alpha_m}\right) s_t + \eta \mu_t^p. \tag{2.40}$$

The relationship between consumption and value added is given by

$$y_t = c_t + \left(\frac{\alpha_m}{1-\alpha_m} + \chi\right) s_t \tag{2.41}$$

which implies that an increase in the real price of oil reduces consumption, holding value added constant. Finally, the relationship between value added and employment is described by

$$y_t = \left(\frac{1}{1-\alpha_m}\right) [a_t + \alpha_n n_t] \tag{2.42}$$

The relation between value added and employment is independent of the real price of oil. To obtain equations (2.42) and (2.41) see equations (A.31) and (A.32) in the appendix.

2.2.5 Monetary Policy

The central bank is assumed to implement monetary policy by choosing a level of the interest rate in response to expectations of output and inflation, using the rule

$$i_t = \alpha_1 E_t y_{t+1} + \alpha_2 E_t \pi_{q,t+1} + \alpha_3 i_{t-1} + \alpha_4 i_{t-2}. \tag{2.43}$$

The lagged interest rate values are included to account for the well-documented practice of "interest rate smoothing" that has been found to be a standard practice of the Federal

Reserve (Rebelo and Xie, 1999; Rudebusch, 2002). The values of the coefficients in (2.43) are taken from the estimates in the careful empirical study of Clarida, Gali and Gertler (2000). Specifically, the baseline monetary policy rule used in the simulations is

$$i_t = 0.93 E_t y_{t+1} + 2.15 E_t \pi_{q,t+1} + 0.5 i_{t-1} + 0.29 i_{t-2}.$$

Clarida, Gali and Gertler (2000) did not report their estimates of α_3 and α_4, so we have chosen values that sum to the 0.79 that they reported for the sum of coefficients on the lagged interest rate terms.

In contrast, BG assumed an interest rate rule of the form

$$i_t = \phi_\pi \pi_{q,t}.$$

A notable difference between that interest rate rule and the one that we use is that their rule assumes the central bank does not attempt to stabilize output. Clarida, Gali and Gertler (2000) found that the Federal Reserve assigns a large weight to the goal of output stabilization, a point which matters in our simulations, for which one of our primary concerns is the effect of oil shocks on output. Another difference is that the BG rule is not forward-looking. The central bank waits until after the inflationary effects of an oil shock are realized before acting to offset the inflation. Our interest rate rule allows the central bank to act to prevent inflation from occurring. This is an important component of the recent literature on monetary policy (Clarida, Gali and Gertler, 2000).

2.2.6 Oil Price Dynamics

We follow BG in assuming that the price of oil follows the AR(1) process:

$$s_t = \rho_s s_{t-1} + \varepsilon_t.$$

The error term is normally distributed with mean zero and variance equal to 0.16.

2.2.7 Solution

Equations (2.17), (2.13), (2.15), (2.38), (2.41), and (2.42) describe the equilibrium dynamics of prices and quantities, given exogenous processes for technology (a_t) and the real price of oil (s_t). The following equations are solved to obtain the rational expectations equilibrium:

$$\begin{gathered} s_t = \rho_s s_{t-1} + \varepsilon_{s,t} \\ \pi_{q,t} = \beta E_t \pi_{q,t+1} + k_1 y_t + \lambda_p \Gamma_w (w_t - w_{t-1}) \\ y_t = E_t y_{t+1} - [i_t - E_t \pi_{q,t+1}] + k_2 s_t \\ i_t = \alpha_1 E_t y_{t+1} + \alpha_2 E_t \pi_{q,t+1} + \alpha_3 i_{t-1} + \alpha_4 i_{t-2} \\ k_3 E_t y_{t+1} - k_3 y_t - E_t \pi_{q,t+1} - k_2 s_t = \varphi_1 w_t + \varphi_2 w_{t-1} + \varphi_3 E_t w_{t+1} \end{gathered} \tag{2.44}$$

The system of equations (2.44) is solved using the generalized Schur method in R (R Core Team, 2012). The solution takes the form of a vector autoregressive (VAR) model for s, π_q, y, w, and i.

The first step to find the equilibrium is to specify values of all the parameters. Table 1 describes the values and the sources of the calibrated parameters that appear in the model. The parameters of the wage adjustment cost function are the estimates of Kim and Ruge-Murcia (2009), the parameters of the monetary policy rule are the estimates of Clarida, Gali and Gertler (2000), and all other parameters are those used by BG. The reason for taking parameters from BG when possible is to allow for comparison of our simulation results with those of BG.

Table 2.1: *Calibrated Values*

Variable	Value	Source
β	0.99	Blanchard and Gali (2007)
θ	0.75	Blanchard and Gali (2007)
ψ	3844	Kim and Ruge-Murcia (2009)
δ	280.4	Kim and Ruge-Murcia (2009)
ϕ	1	Blanchard and Gali (2007)
ρ_s	0.97	Blanchard and Gali (2007)
χ	0.017	Blanchard and Gali (2007)
α_n	0.7	Blanchard and Gali (2007)
α_m	0.012	Blanchard and Gali (2007)
α_1	0.93	Clarida, Gali and Gertler (2000)
α_2	2.15	Clarida, Gali and Gertler (2000)
α_3	0.5	Clarida, Gali and Gertler (2000)
α_4	0.29	Clarida, Gali and Gertler (2000)
M^p	1.1	Blanchard and Gali (2007)
ϵ	11	Blanchard and Gali (2007)
σ	1.5	

2.3 Can Wage Adjustment Costs Explain BG's Results?

BG evaluated the importance of wage rigidities by simulating the variances of output and inflation when $\gamma = 0.9$ and $\gamma = 0.6$. The case of completely inflexible wages is given by $\gamma = 1$, so their baseline case assumes extreme wage rigidities. The choice of a baseline case is not relevant for the simulations of our model because we are using an estimated wage adjustment cost function. The estimates in Kim and Ruge-Murcia (2009) serve as our baseline.

An accommodation needs to be made for the fact that our equilibrium, based on a log-linearization of the first order conditions, may not be accurate once we move far away from the steady state. We therefore compare the simulated variances of output and inflation following a 5% increase in wage rigidity in BG's model against a 5% increase in wage adjustment costs in our model. To see the motivation for increasing both the wage stickiness parameter and the wage adjustment cost by 5%, when they measure different things, our goal is to capture the difference in equilibrium when the wage is fully flexible and when there are wage rigidities. We get the same information by moving the parameters of both models 5% in the direction of fully flexible wages.

Table 2.2: *Simulation Results*

			Variance	ΔVariance (%)
Model	$\psi = 3844$	$\pi_{q,t}$	1.741623e-08	-
		y_t	4.210282e-08	-
	$\psi = 3651.8$	$\pi_{q,t}$	1.741943e-08	0.018 %
		y_t	4.217797e-08	0.018 %
Model w/ BG monetary rule	$\psi = 3844$	$\pi_{q,t}$	2.85524e-07	-
		y_t	7.137252e-07	-
	$\psi = 3651.8$	$\pi_{q,t}$	2.855312e-07	0.003 %
		y_t	7.137429e-07	0.002 %
BG	$\gamma = 0.855$	$\pi_{q,t}$	4.788527e-07	-
		y_t	0.0001820876	-
	$\gamma = 0.9$	$\pi_{q,t}$	6.081898e-07	27.010 %
		y_t	0.0002746878	50.855 %

Table 2.2 shows the simulated variances of output and inflation in the two models for the experiments. Many significant digits need to be shown in the table because the changes are so small with our model. The message that emerges is that, for all practical purposes, a 5% increase in wage adjustment costs has no effect on the variances of output and inflation for our model and baseline parameterization. That is radically different from what we find for BG's model. A 5% rise in wage stickiness causes the variance of output to increase by 51%, and the variance of inflation increases by 27%. The results are the same for our model when there is a change in δ, the other wage cost parameter, or when we use the estimated quadratic adjustment cost function from Kim and Ruge-Murcia (2009), so those results are not reported.

We have also experimented with changes in the monetary policy function. BG demonstrated that monetary policy had considerable effects on the variances of output and inflation in their model. The monetary policy rule in our model is forward-looking and puts weight on stabilization of both output and inflation. We replaced the monetary policy rule in our model with that of BG. The results can also be viewed in Table 2.2. There are slight changes in the results, but they do not in any way change the conclusion that small changes in wage adjustment costs have only trivial effects on the simulated variances. Changing the coefficients on output or inflation in the policy rule does not change our conclusions either.

The last experiment we did was to change the persistence of the oil shock. There are two ways that a wage adjustment cost can affect the response of wages to oil shocks. The first is by causing firms to adjust wages smoothly in response to a shock, reflecting the fact that the cost of adjusting the wage grows with the magnitude of the change in the wage. The second is that it may be optimal to not change the wage at all, or change it only a small amount, if oil shocks are short-lived. A short-lived increase in the price of oil would require that the adjustment cost initially be paid to decrease the wage, and then when the price of oil returns to its mean, a second adjustment cost be paid to raise the wage. The importance of the second effect will depend on the persistence of oil shocks. Oil shocks are extremely persistent for the BG parameterization ($\rho = 0.97$) and as such may make wage adjustment costs irrelevant. Table 2.3 reports the results for a range of smaller values of ρ. The message is the same, however. Moving 5% in the direction of fully flexible wages has almost no effect on the variances of output and inflation.

Table 2.3: *Change in Oil Shock Persistence*

			Variance	ΔVariance (%)
$\rho = 0.97$	$\psi = 3844$	$\pi_{q,t}$	1.741623e-08	-
		y_t	4.210282e-08	-
	$\psi = 3651.8$	$\pi_{q,t}$	1.741943e-08	0.018 %
		y_t	4.217797e-08	0.018 %
$\rho = 0.5$	$\psi = 3844$	$\pi_{q,t}$	2.516347e-07	-
		y_t	2.537062e-07	-
	$\psi = 3651.8$	$\pi_{q,t}$	2.536986e-07	0.014 %
		y_t	2.516709e-07	0.003 %
$\rho = 0.3$	$\psi = 3844$	$\pi_{q,t}$	2.146095e-07	-
		y_t	4.591836e-07	-
	$\psi = 3651.8$	$\pi_{q,t}$	2.14634e-07	0.013 %
		y_t	4.592465e-07	0.011 %

These results make clear that one cannot interpret a change in the *wage adjustment cost* as though it is the same as a change in *wage stickiness*. Small changes in wage adjustment costs, at the point estimates of the wage adjustment cost function for the US, have very little effect on the oil price-macroeconomy relationship. Figure 2.1 provides an explanation for why this might be the case. It plots the variances of output and inflation for BG's model as the wage stickiness parameter changes. The simulated variances are very sensitive to a change in wage stickiness when wages are very sticky, but at low levels of wage stickiness the equilibrium is close to the fully flexible wage equilibrium, and it makes little difference if γ changes. It is possible to get either a large or a small effect of wage stickiness depending on which region of the graph one chooses to analyze. The simulation results discussed above tell us that the estimates of Kim and Ruge-Murcia (2009) put us on the right side of the graph.

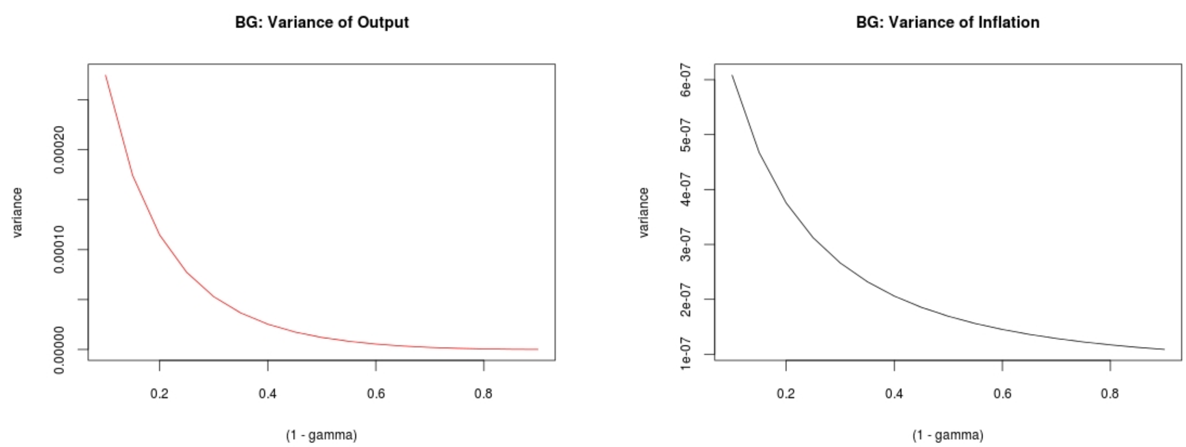

Figure 2.1: *Blanchard and Gali (2007) - Volatility and Changes in Wage Rigidity (γ)*

2.4 Generating Large Effects of Wage Adjustment Costs

We have found that a model with empirically estimated wage adjustment costs imply that wage stickiness has only a trivial effect on the oil price-macroeconomy relationship. This section discusses the results from simulations using the estimated parameters of the quadratic special case ($\psi = 0$) of the wage adjustment cost function Φ_t from Kim and Ruge-Murcia (2009). We report the ratio of the variances of output and inflation when the wage adjustment cost parameter δ is multiplied by 1000.

We stress that we are not claiming that the US economy has experienced such an extreme change in wage adjustment costs. Our goal is to provide information about the change in wage adjustment costs that would be necessary to generate large effects on the variances in output and inflation. It is our belief that it is extremely unlikely that the US has seen a thousandfold change in wage adjustment costs.[3] As above, we check the sensitivity of our findings to changes in the monetary policy rule and the persistence of oil shocks.

The results are found in Table 2.4. For the baseline simulation, where all parameters other than δ and ψ are set equal to the values reported in Table 1, the variance of inflation increases by 135%. The variance of output rose 46%. This demonstrates that a model with wage adjustment costs has no difficulty generating a large effect of wage stickiness on the oil price-macroeconomy relationship, provided one increases the wage adjustment cost by a sufficiently large amount.

[3] We are not aware of any published empirical papers that have found such large changes in wage adjustment costs in the US. Unfortunately, a literature review did not find any empirical papers that were relevant to this question.

Table 2.4: *Changes in δ*

			Variance	ΔVariance (%)
Model	$\delta = 1033.4$	$\pi_{q,t}$	2.0250e-09	-
		y_t	3.1684e-08	-
	$\delta = 1033400$	$\pi_{q,t}$	4.7610e-09	135.11 %
		y_t	4.6225e-08	45.89 %
Model w/ $\alpha_1 = 0.1$	$\delta = 1033.4$	$\pi_{q,t}$	7.569e-09	-
		y_t	1.28881e-07	-
	$\delta = 1033400$	$\pi_{q,t}$	1.4161e-08	87.09 %
		y_t	1.58404e-07	22.91 %
Model w/ $\alpha_2 = 1.15$	$\delta = 1033.4$	$\pi_{q,t}$	3.136e-09	-
		y_t	4.1616e-08	-
	$\delta = 1033400$	$\pi_{q,t}$	4.9e-09	56.25 %
		y_t	5.3824e-08	29.33 %
Model w/ $\alpha_1 = 0.1$ $\alpha_2 = 1.15$	$\delta = 1033.4$	$\pi_{q,t}$	2.0164e-08	-
		y_t	2.6010e-07	-
	$\delta = 1033400$	$\pi_{q,t}$	2.8224e-08	39.97 %
		y_t	3.31776e-07	27.56 %
Model w/ BG monetary rule	$\delta = 1033.4$	$\pi_{q,t}$	3.32929e-07	-
		y_t	3.26041e-07	-
	$\delta = 1033400$	$\pi_{q,t}$	3.721e-07	11.77 %
		y_t	5.65504e-07	73.45 %

Table 2.4 shows the simulation results when the parameters in the monetary policy rule are changed to $\alpha_1 = 0.1$ and $\alpha_2 = 1.15$. This corresponds to the case where the Federal Reserve puts little weight on output stabilization and the response to inflationary pressures is barely strong enough to rule out sunspots. This change in monetary policy rule represents a change from aggressively stabilizing output and inflation to an almost passive regime. The results are consistent with our prior expectations that the monetary policy rule matters - the variances of both output and inflation are much higher with the less aggressive policy rule. The change in monetary policy nevertheless had little effect on the response to an increase in delta. It is still necessary to multiply the adjustment cost by a very large value to get the reduction in variances that BG found.

The final set of simulations demonstrate the effect of changes in persistence of the oil shock. The reason the persistence of oil shocks matters was discussed in the previous section.

We see that the parameter ρ is indeed a critical part of the story for the effects of wage stickiness. Setting $\rho = 0.3$, and leaving all the other parameters at their baseline values, the variance of inflation rises by 524% and the variance of output rises by 43%. The persistence of the oil shock is very important to any story about wage stickiness and the effects of oil shocks. The oil shock may simply be too persistent for wage adjustment costs to be relevant.

Table 2.5: *Changes in Oil Shock Persistence*

			Variance	ΔVariance (%)
$\rho = 0.97$	$\delta = 1033.4$	$\pi_{q,t}$	1.741623e-08	-
		y_t	4.210282e-08	-
	$\delta = 1033400$	$\pi_{q,t}$	1.741943e-08	0.018 %
		y_t	4.217797e-08	0.018 %
$\rho = 0.5$	$\delta = 1033.4$	$\pi_{q,t}$	8.6436e-08	-
		y_t	8.5849e-06	-
	$\delta = 1033400$	$\pi_{q,t}$	5.12656e-07	493.1 %
		y_t	1.239744e-05	44.41 %
$\rho = 0.3$	$\delta = 1033.4$	$\pi_{q,t}$	1.33225e-07	-
		y_t	1.56025e-05	-
	$\delta = 1033400$	$\pi_{q,t}$	8.31744e-07	524.32 %
		y_t	2.225952e-05	42.67 %

2.5 Conclusion

This essay has evaluated the relevance of wage rigidities for understanding the effect of oil price shocks on output and inflation. We took the theoretical framework of Blanchard and Gali and modified it in two important ways. First, we add an empirically estimated wage adjustment cost function as in Kim and Ruge-Murcia (2009). Second, we introduce a realistic monetary policy function to be consistent with the current macroeconomic literature. For a parametrization of the model that uses an empirically-estimated wage adjustment cost function, the degree of wage stickiness has little effect on the oil price-macroeconomy relationship. Monetary policy is not an important factor with respect to the effect of changes in wage stickiness on the effects of oil shocks on inflation and output. Large changes in the variances of output and inflation require very large changes in wage adjustment costs.

As pointed out by Kim and Ruge-Murcia (2009), the other common way to model wage stickiness is to assume Calvo-type staggered wage setting. That is worthy of future investigation. Doing so will require specifying and estimating a complete model of the economy, as we are unaware of any published papers that have estimated the parameters of a Calvo specification in a DSGE model. Another avenue of future research is to use a higher-order approximation, because log-linearization removes the asymmetry of the wage adjustment cost function from our model.

Chapter 3

Statistical Adequacy of Aggregate Production Function and Growth Econometric Models

3.1 Introduction

Colander (2008) argues that empirical modeling of macroeconomic processes in the United States is dominated by a "theory comes first" paradigm. Spanos (2008) supports this view that there is a "pre-eminence of theory" in empirical macroeconomic modeling and data serve a subordinate role that helps to quantify theories that are already presumed accurate. Thus, much of applied macro-econometric modeling in the literature does not fully test whether the macro-econometric models are statistically adequate. That is, if the underlying statistical assumptions of the estimated regression model being estimated are satisfied (e.g. Edwards, Institute and University (2003)). Potential model misspecification may lead to biased and inconsistent estimates, making any substantive inferences or policy recommendations based on the model inappropriate (McGuirk, Driscoll and Alwang, 1993). Thus, to ensure that inferences being made provide substantive validity, the applied macroeconomic modeler must ensure that the underlying statistical assumptions of the model being estimated are supported by the observed data, as the inference is dependent on the model's statistical validity or adequacy.

Many macroeconomic models for aggregate production function analysis are linear regression models, derived through appropriate transformations of the underlying theoretical model (i.e. a log linearization, see Aschauer (1989*a*); Tatom (1991) for examples). Spanos and McGuirk (2001) show that the testable assumptions of the linear regression model include: normality of the underlying conditional distribution; linearity of the conditional mean or regression function; homoskedasticity of the conditional variance; stability of the parameters (homogeneity); and independence in the error term. A variety of misspecification tests have been developed to test these assumptions individually and jointly (Spanos, 1999). While a number of studies have examined the influence of government capital in aggregate production function models (Cassou and Lansing, 1998; Glomm and Ravikumar, 1997; Lynde and Richmond, 1992), many of these studies do not methodically check for model misspecification beyond trend stationarity and unit roots. Even when examining these models for potential trends and underlying unit roots in the data series (or cointegration), model misspecification may skew the results (Andreou and Spanos, 2003). Thus, it is imperative that the underlying statistical adequacy of the models being estimated be determined before any substantive inference is made using the coefficient estimates from the model.

The purpose of this paper is to assess the statistical adequacy of the Cobb-Douglass aggregate production model with public capital as a factor of production and examine the impact on substantive inference from statistical misspecification. More specifically, the paper will (i) test the statistical adequacy of the models proposed by Aschauer (1989*a*) and Tatom (1991) using a battery of misspecification tests for the time periods used by the authors and a sample including more current events; (ii) if the models are misspecified, then respecify the models to arrive at a statistically adequate model and assess the impact on inferences made using the prior models; and (iii) examine the use of the student's t linear heteroskedastic regression model as a potential modeling alternative for capturing heteroskedasticity. The debate about the role of public capital in aggregate production or capital productivity is still undecided today. This paper will examine if model misspecification has potentially resulted in biased and/or inconsistent estimates and conclusions in the applied macroeconomic literature on the subject by going back to the foundational papers.

3.2 Literature Review

The significance of public capital on macroeconomic variables (GDP in particular) has been debated for several decades. In 1989, David Aschauer, while trying to explain what caused the productivity slowdown in the US starting in the mid-1970s, examined the influence of government capital on post WWII GDP in the United States. He found that "...the decrease in productive government services may be crucial in explaining the general decline in the rate of growth of productivity which apparently arose in the early 1970s" (Aschauer (1989*a*), p.179). In order to prove his case Aschauer (1989*a*) expanded on the Cobb-Douglas aggregate production function literature by incorporating public capital as an additional variable. He proposed an empirical model in log linearized form which is given by:

$$y_t = \alpha_0 + \alpha_1 t + \alpha_2 n_t + \alpha_3 g_t + \alpha_4 u_t + \varepsilon_t, \tag{3.1}$$

where

$y_t = \ln(Y_t) - \ln(K_t)$,

$n_t = \ln(L_t) - \ln(K_t)$,

$g_t = \ln(G_t) - \ln(K_t)$,

$u_t = \ln(U_t)$,

Y_t is aggregate output (value added),

L_t is hours worked,

K_t is the stock of private capital,

G_t is the stock of non-military public capital, and

U_t is the utilization rate, which is included to capture the effects of business cycles (Aschauer, 1989*a*). The utilization rate measures what fraction of maximum production capacity is utilized.

Table 3.1 shows some of the regression results from Aschauer (1989*a*). The key finding is that α_3 is positive and significant, implying that a one percent increase in the public-to-private capital ratio leads to an increase in the productivity of capital by 0.39 percent. This is higher than the impact of the labor-capital ratio, which increases capital productivity by 0.35 percent for every 1 percent increase in the labor-capital ratio. Aschauer's findings have significant policy implications, by advocating an increase in government spending on infrastructure as a means to stimulate economic growth in the United States.

Table 3.1: *Estimation Results with Public Capital (Aschauer, 1989*a*)*

	α_0	α_1	α_2	α_3	α_4	$\bar{R}^2$	S^2
Aschauer (1949-1985)	-2.42 (−21.58)	0.008 (4.62)	0.35 (4.85)	0.39 (16.23)	0.43 (12.28)	0.976	0.0078

Numerous authors have challenged Aschauer's results. Researchers have used different econometric models to support or challenge his findings: systems of simultaneous equations and single equation models using both time series and panel data. Some researchers found his results to be upwardly biased, but significant (Cassou and Ai, 1995; G. Duggal, Saltzman and Klein, 1999). Others have found Aschauer's results insignificant or significant but with the wrong sign (Holtz-Eakin and Schwartz, 1995). In this paper, we chose to focus on single equation time series models following Aschauer (1989*a*). Tatom (1991) challenged Aschauer's findings based on the following reasons: (i) omission of an energy series in the aggregate production function, (ii) omission of significant nonlinear time trends and (iii) the use of non-stationary variables in the regression.

The first change Tatom (1991) advocates is the addition of an energy series as a variable in the Cobb-Douglas aggregate production function. He then makes an assumption that the energy series can be represented by the ratio of output to a relative energy price series. Unfortunately the author does not provide the source of the price of energy used in the paper. Private sector energy use data is readily available, making it unclear why Tatom makes such a strong assumption.

Tatom finds unit roots in the explanatory variables using Dickey-Fuller tests and accordingly takes the log-differences of the variables to estimate the following model:

$$\Delta \ln \left(\frac{Y_t}{K_t}\right) = \beta_0 + \beta_1 t + \beta_2 \Delta \ln \left(\frac{N_t}{K_t}\right) + \beta_3 \Delta \ln \left(\frac{G_t}{K_t}\right) + \beta_4 \Delta \ln \left(p_t^e\right) + \epsilon_t \tag{3.2}$$

where p_t^e is the price of energy relative to the price of business output. Estimation results for this transformed model are provided below in Table 3.2.

Table 3.2: *Estimation Results with Public Capital (Tatom, 1991)*

	β_0	β_1	β_2	β_3	β_4	$\bar{R}^2$	S^2
Tatom (1949-1989)	0.025 (6.30)	−0.0005 (−3.05)	0.737 (14.34)	0.042 (0.33)	−0.058 (−3.23)	0.85	0.0015

Table 3.2 shows that public capital is positive, but statistically insignificant. The author states that the log-difference of the model causes public capital to become insignificant and the inclusion of the relative energy price series to be significant. Tatom (1991) estimates equation (3.2) without energy and finds that public capital is insignificant, as well.

3.3 Data

For our analysis we use annual data for three time periods: (i)1949 to 1985; (ii)1949 to1989; and (iii)1949 to 2009. The first two time periods provide coverage of the original time periods examined by Aschauer (1989*a*) and Tatom (1991). We expand these time periods to include more recent macroeconomic events up to 2009, expanding on the knowledge concerning the processes underlying these models.

Output, private capital and public capital data were taken from the Bureau of Economic Analysis (BEA). Public capital excludes any military fixed assets. Hours worked were taken from the National Income and Product Accounts (NIPA). We used the hours worked by full time and part time employees in the private sector. The utilization rate was taken from the Board of Governors table G.17. Table 3.3 lists the specific tables the dataset was compiled from. Please note that the data has undergone multiple revisions since Aschauer and Tatom published their papers. "Since then, the data has been revised numerous times, including changing the base period for computing constant-dollar output and capital stock data" (Tatom (1991), p.6). It is unlikely we would be able to replicate the exact results of Aschauer (1989*a*) and Tatom (1991). However, we do expect to get similiar results. Since we have additional observations of data in our expanded dataset from 1949 to 2009, the larger sample will provide a test of the robustness of their models over time.

Table 3.3: *Data Sources*

Series	**Source**
Output	BEA: Value Added by Industry (Release Date - Sep 30, 2010)
Hours	NIPA: GDP: Table 6.9: Hours Worked by FT and PT Employees by Industry
Private Capital	BEA: Fixed Assets Tables: Table 3.3ES:Stock of Private Fixed Assets by Industry
Public Capital	BEA: Fixed Assets Tables: Table 7.2: Net Stock of Government Fixed Assets
Utilization Rate	Board of Governors: Table G.17: Industrial Production and Capacity Utilization
Energy	US Energy Information Administration: Energy Consumption by Sector

3.4 Initial Estimation

In this section we estimate both Aschauer's (1989) and Tatom's (1991) models for the time periods both authors used in their papers and for the full sample of data collected. Table 3.4 presents the original estimation results by Aschauer (1989*a*) and our estimation for the subsample (1949-1985) and the full sample (1949-2009).

Table 3.4: *OLS Estimation Results (Aschauer, 1989*a*)*

	α_0	α_1	α_2	α_3	α_4	$\bar{R}^2$	S^2
Aschauer (1989) [1949-1985]	-2.42 (-21.58)	0.008 (4.62)	0.35 (4.85)	0.39 (16.23)	0.43 (12.28)	0.976	0.0078
Estimated [1949-1985]	-0.96 (-1.59)	0.01 (4.62)	0.39 (3.51)	0.27 (3.76)	0.38 (8.38)	0.9344	0.0001
Estimated [1949-2009]	0.06 (0.09)	0.01 (7.00)	0.50 (4.74)	0.24 (2.24)	0.23 (3.17)	0.9301	0.0005

Our estimates for the same period used by Aschauer show that private capital is positive and significant. The difference is that we obtain a lower coefficient estimate than Aschauer.

For the full sample, the estimate for α_3 is marginally lower than for the partial sample estimate. For all the models the coefficient on the public-to-private capital ratio is statistically significant at the 5 percent level.

Next, Tatom's model (Tatom, 1991) with the relative price of energy and log-differenced variables given by equation (3.2) is estimated. Table 3.5 shows published estimates and our results. For the model estimated here a variable representing energy used by the non-residential private sector was included as a proxy for energy in the model (based on the discussion in section 2). The estimation results indicate that labor productivity has a coefficient larger than one and public capital has an insignificant impact on output productivity for both time periods examined.

Table 3.5: *OLS Estimation Results (Model 3.2)*

	β_0	β_1	β_2	β_3	β_4	$\bar{R}^2$	S^2
Tatom (1991) [1949-1989]	0.025 (6.30)	−0.0005 (−3.05)	0.737 (14.34)	0.042 (0.33)	−0.058 (−3.23)	0.85	0.0015
Estimated [1949-1989]	0.04 (5.42)	−0.001 (−2.46)	1.21 (10.96)	−0.10 (−0.33)	0.24 (2.97)	0.7890	0.0002
Estimated [1949-2009]	0.02 (4.53)	−0.00 (−0.52)	0.99 (10.54)	0.18 (0.74)	0.17 (2.36)	0.6941	0.0002

The stark difference between Tatom's results and ours can be attributed to data revisions and to the fact that an alternative energy series was used as an input. We believe that Tatom made several very strong assumptions: (i) he assumed that energy consumption is a linear function (fraction of) output, and (ii) he imposed constant returns to scale (CRS) over labor, public capital, private capital and energy without testing for it. While CRS has been tested and proven to hold on the aggregate level for the first three factors of production (Cassou and Ai, 1995), there has yet to be published evidence that CRS holds with energy included in the aggregate production function. Furthermore, recall that Tatom did not explain how he obtained his energy price series. As a result we are unable to directly replicate his results. Thus, it was decided to utilize an energy series that is more reflective of the nature of the aggregate production function and energy usage. In addition, Tatom assumes that the relative price of energy is a constant proportion of the output series. Figure 3.1 plots output and the energy series used in this paper. Visual examination of the plots would seem to indicate that the relative energy series is not a constant proportion of output. The slopes of the two series are not a constant proportion to each other over time, and substantially deviate from each other after the oil price shocks and macroeconomic policies of the 1970s.

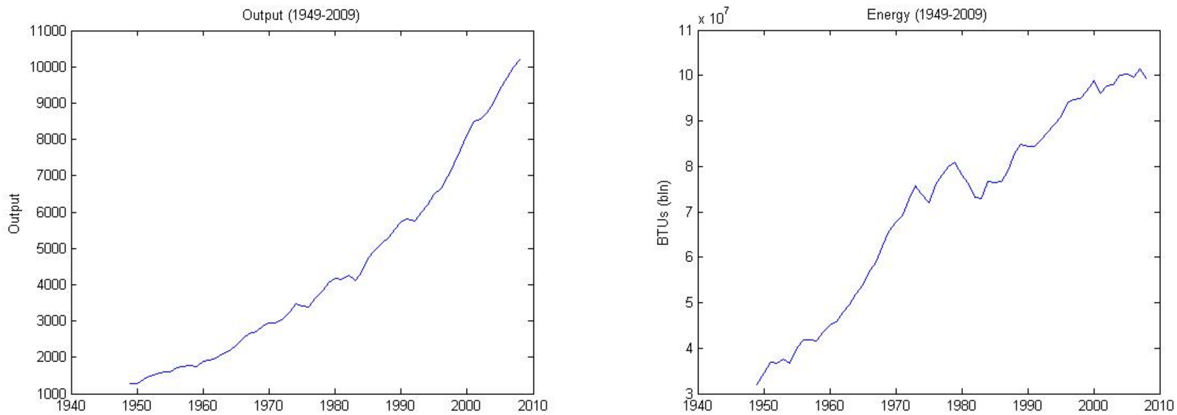

Figure 3.1: *Plots of Output and Energy Series (1949-2009)*

3.5 Testing for Statistical Adequacy

This section we will describe the tests used for misspecification testing and the results of the battery of tests for both Aschauer's (1989) and Tatom's (1991) models.

3.5.1 Misspecification Tests

Following Spanos (1986) the normal linear regression model has 5 testable assumptions:

1. Distributional: Normality of the underlying conditional distribution($f\left(y_t|n_t, k_t, u_t\right)$) and error distribution ($f\left(\varepsilon_t\right)$)

2. Functional Form: Linearity of the conditional mean or regression function ($E\left[y_t|n_t, k_t, u_t\right] = \alpha_0 + \alpha_2 n_t + \alpha_3 g_t + \alpha_4 u_t$)

3. Homoskedasticity: The conditional variance does not vary over time and is not a function of the regressors ($Var\left[y_t|n_t, k_t, u_t\right] = Var\left(\varepsilon_t\right) = \sigma^2$)

4. Homogeneity: The parameters of the model are stable over time (α is not a function of t)

5. Independence: The stochastic process $\{y_t|n_t, k_t, u_t, t \epsilon T\}$ is independent over t.

Based on these assumptions, a number of misspecification tests have been developed to test if these assumptions are satisfied by the observed data. These tests and the assumptions they address are outlined below.

- **Bera-Jarque normality test** (Jarque and Bera, 1981) is a parametric normality test assessing if the data series has a skewness of zero and kurtosis of three.

- **D'Agostino-Pearson skewness test** (D'Agostino and Stephens, 1986) tests the null hypothesis that the data series has a skewness of zero. This is a parametric distributional test.

- **D'Agostino-Pearson Kurtosis test** (D'Agostino and Stephens, 1986) tests the null hypothesis that the data series has an excess kurtosis of zero. This is a parametric distributional test.

- **Ramsey RESET test** (Ramsey, 1969) assesses the functional form of the model to assess if the conditional mean is non-linear. This test is performed by adding squared and cubic fitted residuals as additional covariates and testing their significance using a Wald test.

- **Homogeneity tests** are performed to check for the presence of non-linear trends and structural breaks. These tests are based on auxiliary regression tests following Spanos (1986). To conduct theses tests, quadratic and cubic trends are added as additional covariates and Wald tests are performed to see if the non-linear trends are significant.

- **Independence tests** are conducted to test for serial correlation in the error term. To perform the test, lagged values of the fitted residuals are included as additional regressors in an auxiliary regression with the fitted residuals as the dependent variable following Spanos (1986). A Wald test is used to assess if the lagged residuals are significant.

- **Joint Conditional Mean test** is performed following McGuirk, Driscoll and Alwang (1993) and allows the modeler to test jointly for linearity, homogeneity and independence. This test provides a robust check of the individual tests as it allows the modeler to test each individual assumption in the framework while relaxing the other assumptions. This may be important as heterogeneity may be masked as dependence and vice versa. The joint test gives us greater confidence in the individual test results and can help to isolate the true departure(s) when multiple assumptions fail. The joint testing framework uses an auxiliary regression approach by incorporating non-linear trends, lags of the fitted residuals and RESET terms.

- **Breusch-Pagan test** (Breusch and Pagan, 1979) tests for heteroskedasticity. This test assesses if the variance from the linear regression is independent of the values of the regressors. The test is performed by regressing the squares of the residuals on the original independent variables. If the coefficients are significant, the null hypothesis of homoskedasticity can be rejected.

- **Augmented Dickey Fuller test** (Greene, 2002) is an augmented version of the Dickey-Fuller test and allows one to test for higher order unit root processes. This is incorporated to assess if the underlying data series contain unit roots.

3.5.2 Statistical Adequacy of Aschauer' Model

First, the variables used by Aschauer (1989*a*) are tested for the presence of unit roots. Table 3.6 shows the results of the Augmented Dickey-Fuller unit root tests performed on the variables in equation (3.1). Given that Augmented Dickey Fuller (hereafter ADF) tests have a relatively low power, if any of the three versions of ADF tests (random walk, random walk with drift, and random walk with drift and trend) cannot reject the presence of unit root process, then the variable is assumed to be non-stationary and has to be dealt with accordingly.

Table 3.6: *Augmented Dickey Fuller Unit Root Test Results for Aschauer Model*

	1948 - 1985 Unit Root	1948 - 2009 Unit Root
y_t	Yes	Yes
n_t	Yes	Yes
g_t	Yes	Yes
u_t	No	No
ε_t	No	Yes

It is important to notice that for the sample size used by Aschauer three variables are non-stationary, but the residuals are stationary. On the other hand when testing the variables over the entire sample we observe that the only stationary variable is the utilization rate. Thus, for the full sample it would seem that first differencing may be in order, but recall that these tests may be biased if the model is not correctly specified. This will be further explored in the next section of the paper.

Table 3.7 shows the results of misspecification tests preformed on equation 3.1 for the period of 1948-1985. The model fails the normality, independence and homogeneity tests. The sample kurtosis of the fitted residuals is 1.95, which is significantly lower than 3. The homogeneity test results provide evidence that non-linear time trends should be included in the model. In addition, the independence tests provide evidence of the presence of serial autocorrelation. However, the joint conditional mean tests show that the model in its current form suffers only from trend homogeneity.

Next we we run the same battery of tests over the entire sample of data. Table 3.8 shows that we fail homogeneity, independence, homoskedasticity and the joint conditional mean

Table 3.7: *Misspecification Tests (Aschauer, 1948 - 1985)*

	Test Statistic	P-Value
Normality		
Bera-Jarque Normality test	1.6634	0.435
D-Agostino-Pearson Normality test	4.0274	0.133
D-Agostino-Pearson Skewness test	0.0418	0.967
D-Agostino-Pearson Kurtosis test	-2.01	0.0448
Skewness	0.0148	
Kurtosis	1.95	
Linearity		
Ramsey RESET - Quadratic	0.0000	0.9934
Ramsey RESET - Cubic	0.0441	0.957
Homogeneity		
Second Order	15.1146	0.0005
Third Order	6.3542	0.0173
Independence		
One Period Lag	6.9251	0.0137
Two Period Lag	3.7686	0.036
Homoskedasticity		
Breusch-Pagan	3.0480	0.5498
White Test		0.1149
Joint Conditional Mean		
Homogeneity	6.6370	0.0053
Linearity	0.4602	0.8030
Independence	0.2749	0.7621
Overall	4.1295	0.0059

tests. For the larger sample, the joint conditional mean test points to serial autocorrelation as the problem in the regression function and the individual homogeneity tests are failing due to this. Thus Aschauer's model in its current form is statistically misspecified for both time periods.

3.5.3 Statistical Adequacy of Tatom's Model

As it was mentioned earlier, Tatom (1991) log-differenced the variables used by Aschauer based on unit root test results and added a relative price of energy as a proxy for energy. Given that we are unable to obtain the original energy price series, a direct energy series is used instead (as described earlier). Tatom's model is tested for statistical adequacy using the same battery of tests.

Table 3.9 shows that the model passes all the tests conducted. Table 3.10 presents the results of the ADF tests performed on the variables used in Tatom's regressions. The table presents test results for the variables in levels and in log-differences. Tatom used log-

Table 3.8: *Misspecification Tests (Aschauer, 1948 - 2009)*

	Test Statistic	P-Value
Normality		
Bera-Jarque Normality test	1.474	0.479
D-Agostino-Pearson Normality test	1.9211	0.383
D-Agostino-Pearson Skewness test	-0.647	0.518
D-Agostino-Pearson Kurtosis test	-1.23	0.22
Skewness	-0.186	
Kurtosis	2.34	
Linearity		
Ramsey RESET - Quadratic	0.2281	0.6348
Ramsey RESET - Cubic	1.4919	0.2341
Homogeneity		
Second Order	48.7747	0.0000
Third Order	24.0730	0.0000
Independence		
One Period Lag	197.38	0.0000
Two Period Lag	98.83	0.0000
Homoskedasticity		
Breusch-Pagan	12.4116	0.0145
White Test		0.0001
Joint Conditional Mean		
Homogeneity	0.2501	0.7797
Linearity	0.3911	0.6784
Independence	34.7451	0.0000
Overall	30.7888	0.0000

Table 3.9: *Misspecification Tests (Tatom, 1949 - 1989)*

	Test Statistic	P-Value
Normality		
Bera-Jarque Normality test	0.3179	0.853
D-Agostino-Pearson Normality test	0.1843	0.912
D-Agostino-Pearson Skewness test	-0.415	0.678
D-Agostino-Pearson Kurtosis test	-0.11	0.912
Skewness	-0.142	
Kurtosis	2.67	
Linearity		
Ramsey RESET - Quadratic	0.0013	0.9711
Ramsey RESET - Cubic	0.4049	0.6703
Homogeneity		
Second Order	0.00185	0.8926
Third Order	0.0181	0.8938
Independence		
One Period Lag	2.0414	0.1628
Two Period Lag	1.0053	0.3776
Homoskedasticity		
Breusch-Pagan	5.5193	0.2380
White Test		0.3395
Joint Conditional Mean		
Homogeneity	0.1845	0.8325
Linearity	0.0533	0.9482
Independence	0.9821	0.3875
Overall	0.3795	0.8856

differenced variables in his analysis. However, the log-differenced g_t is non-stationary, while every other variable including the residuals are stationary. Thus, the underlying model is misspecified with respect to the transformations taken to correct for unit roots in the log-differenced g_t data series. While Tatom's model is much closer to being properly specified from a statistical viewpoint, the results do not provide much empirical meaning.

Table 3.10: *Augmented Dickey Fuller Unit Root Test Results for Tatom's Model*

	Log-Differenced Variables		Variables in Levels	
	1948 - 1989	1948 - 2009	1948 - 1989	1948 - 2009
	Unit Root	Unit Root	Unit Root	Unit Root
y_t	No	No	Yes	Yes
n_t	No	No	Yes	Yes
g_t	Yes	Yes	Yes	Yes
$(en)_t$	No	No	Yes	Yes
ε_t	No	No	No	No

Next we test Tatom's model over the entire sample. The results are presented in table 3.11. For the full sample, the model fails homogeneity and one of the homoskedasticity tests. Thus, for the entire sample the model is not statistically adequate. It is of interest that Tatom tests for a nonlinear trend in the nondifferenced model and finds both a trend and squared trend are statistically significant. In addition, the public-to-private capital ratio is statistically significant in that model, as well.

3.6 Model Respecification

Test results in section 3.5.3 show that in Tatom's (1991) model one of the log-differenced variables remains non-stationary, and in its current form, the model is misspecified. Furthermore, further log-differencing the variables to achieve stationarity would likely lead to nonsensical results theoretically. As a result, we will begin by considering Aschauer's (1989) specification.

Andreou and Spanos (2003) suggest correcting for deterministic heterogeneity in a data series before testing for a unit root, as the results may be biased otherwise. Deterministic

Table 3.11: *Misspecification Tests (Tatom, 1949 - 2009)*

	Test Statistic	P-Value
Normality		
Bera-Jarque Normality test	1.2025	0.548
D-Agostino-Pearson Normality test	1.8979	0.387
D-Agostino-Pearson Skewness test	0.205	0.837
D-Agostino-Pearson Kurtosis test	1.36	0.173
Skewness	0.0586	
Kurtosis	3.68	
Linearity		
Ramsey RESET - Quadratic	1.1734	0.2834
Ramsey RESET - Cubic	0.7332	0.4851
Homogeneity		
Second Order	5.3248	0.0248
Third Order	5.3066	0.0250
Independence		
One Period Lag	0.0000	0.9987
Two Period Lag	1.7637	0.1815
Homoskedasticity		
Breusch-Pagan	8.7502	0.0677
White Test		0.6178
Joint Conditional Mean		
Homogeneity	0.5768	0.5656
Linearity	0.1903	0.8274
Independence	1.0274	0.3657
Overall	0.8360	0.5483

heterogeneity can result from trends, structural breaks, or a combination/interaction of both. Thus, we first examine structural breaks in the data series. We tested for structural breaks over time by examining the significance of time dummies for each year in the dataset. It was found that, structural breaks should be modeled for the following two periods of time: (i) 1969-1985 and (ii) 1992-2004. Next we estimate the following regression for the dependent and each explanatory variable.

$$Z_t = \delta_0 + \delta_1 D_{69} + \delta_2 D_{92} + \delta_3 t + \delta_4 D_{69} t + \delta_5 D_{92} t + \delta_6 t^2 + \delta_7 t^3 + \omega_t \tag{3.3}$$

where D_{69} and D_{92} are dummy variables equal to 1 for the two time periods indicated above. We added quadratic and cubic trends to capture higher order trends. Interactions between the time dummies and linear time trend where the only significant interactions found after testing each data series for higher order interactions. We then used the detrended mean deviation form (using ω_t) of the variables $(\widetilde{n}_t, \widetilde{g}_t, \widetilde{u}_t)$ to test for unit roots. We found that there are no unit roots processes in the data series. These results contrast with Tatom's (1991) results and further supports the use of Aschauer's model specification.

Given that we have already corrected for deterministic heterogeneity in the data, we run the following regression using the detrended mean deviation data series:

$$\widetilde{y}_t = \gamma_1 \widetilde{n}_t + \gamma_2 \widetilde{g}_t + \gamma_3 \widetilde{u}_t + \eta_t \tag{3.4}$$

Table 3.12: *Model (3.4) Estimation Results*

	γ_1	γ_2	γ_3	$\bar{R}^2$	S^2
Estimated (1949 - 2009)	0.26 (2.79)	0.30 (3.90)	0.42 (9.19)	0.8476	0.0001

Table 3.12 shows that all variables are significant at the 5 percent level. Before making further inference, we test for statistical adequacy of the model by running the same set of misspecification tests on model (3.4) as in the previous section. Table 3.13 shows that the model fails the independence test. This implies that there is potential serial autocorrelation in the error term. We do not test for homogeneity because the data has already been detrended

Table 3.13: *Model (3.4) Misspecification Tests*

	Test Statistic	P-Value
Normality		
Bera-Jarque Normality test	1.3782	0.502
D-Agostino-Pearson Normality test	1.5798	0.454
D-Agostino-Pearson Skewness test	-1.24	0.213
D-Agostino-Pearson Kurtosis test	0.175	0.861
Skewness	-0.364	
Kurtosis	2.88	
Linearity		
Ramsey RESET - Quadratic	0.3547	0.5538
Ramsey RESET - Cubic	1.0694	0.3501
Independence		
One Period Lag	6.6868	0.0124
Two Period Lag	3.5350	0.0361
Homoskedasticity		
Breusch-Pagan	0.4324	0.8056
White Test		0.8715
Joint Conditional Mean		
Linearity	1.0733	0.3493
Independence	3.2580	0.0153
Overall	2.3090	0.0368

We correct for serial autocorrelation by modeling the error as $[\eta_t = \rho\eta_{t-1} + v_t]$. Following Greene (2002), we estimate the above regression model with the detrended mean deviation form of the variables and the serially autocorrelated error using the method of maximum likelihood. The regression takes the following form

$$\widetilde{y}_t = \gamma_1\widetilde{n}_t + \gamma_2\widetilde{g}_t + \gamma_3\widetilde{u}_t + \eta_t,\ \eta_t = \rho\eta_{t-1} + v_t. \tag{3.5}$$

Results are provided in Table 3.14.

Table 3.14: *Model (3.5) Estimation Results*

	γ_1	γ_2	γ_3	ρ	$\bar{R}^2$	S^2
Estimated (1949 - 2009)	0.24 (2.26)	0.30 (3.19)	0.41 (8.30)	0.32 (2.56)	0.8246	0.0001

Again, all of the coefficient estimates and ρ are statistically significant at the 5 percent level, indicating the presence of first order serial autocorrelation. However, this model is a restricted form of the dynamic linear regression model as presented by Spanos (1986) that explicitly incorporates lags of the dependent and explanatory variables directly into the regression model. Thus, the serially autocorrelated model is nested within the dynamic linear regression model via a set of nonlinear restrictions known as common factors (Greene, 2002). A Lagrange Multiplier test was used to test if the observed data provides support for the common factor restrictions. A test statistic of 2.6683 with p-value equal to 0.6148 was obtained, indicating strong support for the common factor restrictions and serially autocorrelated correction undertaken. Given the new specification, the misspecification tests are re-ran to test the statistical adequacy of the new serially autocorrelated model. Results of the tests are provided in Table 3.15 .

Table 3.15: *Model (3.5) Misspecification Tests*

	Test Statistic	P-Value
Normality		
Bera-Jarque Normality test	1.3028	0.521
D-Agostino-Pearson Normality test	1.9321	0.381
D-Agostino-Pearson Skewness test	-1.17	0.242
D-Agostino-Pearson Kurtosis test	0.75	0.453
Skewness	-0.341	
Kurtosis	3.22	
Linearity		
Ramsey RESET - Quadratic	1.3640	0.2478
Ramsey RESET - Cubic	0.9997	0.3746
Independence		
One Period Lag	0.1319	0.7179
Two Period Lag	0.0854	0.9183
Homoskedasticity		
Breusch-Pagan	0.4323	0.8056
White Test		0.8715
Joint Conditional Mean		
Linearity	1.0450	0.3587
Independence	0.2498	0.6192
Overall	0.7407	0.5325

This time the battery of misspecification tests conducted indicates that the underlying assumptions of the regression model specified are supported by the observed data. That is, a statistically adequate model has been obtained and it shows that public capital is highly significant. A one percent increase in the public-to-private capital ratio will increase output productivity by 0.30 percent. This number is less than the 0.39 estimated by Aschauer, however it is statistically significant.

3.7 Student's t Linear Heteroskedastic Regression Model

It may be of interest to examine other distributional assumptions for completeness and as an extension to the above framework. Other distributions in the symmetric elliptical family, of which the normal distribution is a member, can provide similar shapes as the normal distribution, but have conditional variances that are heteroskedastic. One of particular interest is the Student's t regression model, which has a linear conditional mean, which would be specified similarly to the respecified regression model for the Aschauer model given by equation (3.4). The conditional variance would take the form:

$$Var(y_t|\mathbf{X}_t = \mathbf{x_t}) = \left(\frac{v}{v+k-2}\right) \times \sigma^2 \left(1 + \left[\frac{1}{v}\left(\mathbf{X_t} - \mu_\mathbf{t}\right)' \boldsymbol{\Sigma}_{\mathbf{22}}^{-1}\left(\mathbf{X_t} - \mu_\mathbf{t}\right)\right]\right) \qquad (3.6)$$

where v is the degrees of freedom for the conditional Student's t distribution, k is the number of regressors, σ^2 is a shape parameter, $\mathbf{X}_t$ is the vector of explanatory variables, μ_t is the mean vector of the explanatory variables, and Σ_{22} is the covariance matrix of the explanatory variables Spanos (1999).

The model is estimated using the method of maximum likelihood and the likelihood function must include the multivariate distribution of the explanatory variables, given the parameters of the conditional distribution being estimated are not weakly exogenous with respect to the parameters of the multivariate distribution of the explanatory variables (see Spanos (1999); i.e. hence the inclusion of the parameters μ_t and Σ_{22} in the conditional variance). Following Spanos (1999), we take the mean deviation form of the variables so that $\mu_t = 0$ in the model, making the intercept α_0 in the conditional mean function equal to 0. Furthermore, we remove trends and structural breaks when taking the mean deviation form. This is done by estimating the regression: $Z_t = \delta_0 + \delta_1 D_{69} + \delta_2 D_{92} + \delta_3 t + \delta_4 D_{69} t + \delta_5 D_{92} t + \delta_6 t^2 + \delta_7 t^3 + \omega_t$, and using ω_t as the detrended mean deviation form of the variables. This transformation corrects for any heterogeneity in the data prior to model estimation. In addition, during estimation v is set to a known value. We examine different values of v up to 10 and find that $v = 8$ provides the maximum log-likelihood value. In addition, examination of a quantile-quantile plot provides evidence that $v = 8$ provides the best fit. The model was estimated using the procedures outlined in Spanos (1999) in MATLAB.

The estimated model is given below with asymptotic t-statistics in parentheses.
Conditional Mean Function:

$$\widetilde{y}_t = \underset{(2.51)}{0.27}\widetilde{n}_t + \underset{(3.26)}{0.29}\widetilde{k}_t + \underset{(8.24)}{0.43}\widetilde{u}_t + \eta_t$$

Conditional Variance Function:

$$\underset{(182.77)}{0.0000012}\left(1 + 0.25\left[\underset{(9.76)}{0.00061} - \underset{(-2.56)}{0.00017}n_t k_t + \underset{(5.29)}{0.00087}n_t u_t + \underset{(5.85)}{0.00050}k_t^2 + \underset{(0.81)}{0.00011}k_t u_t + \underset{(9.31)}{0.0024}u_t^2\right]\right)$$

with a log-likelihood value of 442.77. The coefficient estimates for the conditional mean are very similar to the those in the respecified Aschauer model, but there the conditional variance is assumed to be homoskedastic. The regression results here would seem to indicate that this assumption may not be adequate enough and that a heteroskedastic model, as the one estimated above is possibly more appropriate. In addition, the conditional variance provides additional information about the variability of capital productivity as a function of labor, public capital and the utilization rate. For example, if the public to private capital ratio is increased by one unit, then the conditional variance will change by $\partial Var(y_t|n_t, k_t, u_t)/\partial k_t = 0.000000076k - 0.0000000055n + 0.000000000025u < 0$ within the range of the observed data used to estimate the model. The implications from this model for macroeconomic theory and policy need to be explored further and are beyond the scope of this paper. In addition, the model would need to be expanded to incorporate potential dynamics.

3.8 Conclusion

In this paper we assess the statistical adequacy of the Cobb-Douglas aggregate production function as specified in Aschauer (1989) and Tatom (1991); and examine the impact on substantive inference from statistical misspecification. We test the statistical adequacy of the models proposed by Aschauer and Tatom and find that both models are misspecified. Aschauer's model fails to satisfy the underlying assumptions of the linear regression model. Tatom's model suffers from the same criticism he levies against Aschauer's model, non-stationarity in the data series used to estimate the model. Tatom, who used first differencing to achieve stationary variables, still has variables that remain nonstationary after first differencing. It is found that the unit root testing in Tatom's paper may be misleading, given any deterministic heterogeneity not taken into account prior to testing may bias unit root testing results.

To correct for potential misspecification of the model we detrend the variables used and utilize the mean deviation form of the variables to test for unit roots. To detrend we regress each variable on the linear and non-linear time trends plus the interaction of linear time trend with dummy variables. Dummy variables are included to capture structural breaks in the data. Unit root testing results find that the data actually support the assumption of trend stationarity over a unit root. Thus, Aschauer's specification of the model is utilzied for the remainder of the paper. To correct for serial autorcorrelation, an AR(1) error process is assumed and common factor restrictions are tested for, which are both supported by the data. The final model is fully tested and passes the full battery of misspecification tests. The final statistically adequate model shows that public capital is statistically significant with a coefficient of 0.3, which is lower then Aschauer's estimate of 0.39.

For a robustness check, we decide to further explore the assumption of homoskedaticity by estimating a Student's t linear heteroskedastic model where the conditional variance is heteroskedastic. Once again we find estimates similar to those from the homoskedastic model, but the terms in the conditional variance are found to be significant, pointing toward potential heteroskedasticity. In general we can confirm Aschauer's results that public capital is an important input in the aggregate production function. Our results also refute a large body of literature that discredits Aschauer's findings claiming that his model is statistically misspecified. We can confirm that his model was misspecified but the general results Aschauer obtained are valid. Future research will further explore the statistical adequacy of macroeconomic models using Cobb-Douglas aggregate production functions; and the implications of using a linear heteroskedastic regression model as an empirical modeling framework.

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Appendix A

Model Derivation - Essay 2

A.1 Households

$$max\ E_0 \sum_{t=0}^{\infty} \beta^t \left(ln\,(C_t) - \frac{N_t^{1+\phi}}{1+\phi} \right)$$

s.t

$$(i)\ \ P_{q,t}C_{q,t} + P_{m,t}C_{m,t} + Q_t^B = W_t N_t \left(1 - \Phi_t\right) + B_{t-1} + \Pi_t$$

where $C_{q,t}$ is a CES index of domestic goods; $C_{m,t}$ is the consumption of (imported) oil; $P_{q,t}$ is a price index of domestically produced goods; $P_{m,t}$ is the price of oil (in domestic currency); B_t is the quantity of risk-free domestic bonds purchased in period t; Q_t^B is the price of that bond; W_t is the nominal wage;

$$(ii)\ \ \Phi_t = \left(\frac{\delta}{\psi^2}\right) \left[e^{-\psi\left(\frac{\sigma W_t}{W_{t-1}} - 1\right)} + \psi \left(\frac{W_t}{W_{t-1}} - 1 \right) - 1 \right]$$

Φ_t is the wage adjustment cost, where σ and ψ are the asymmetric wage rigidity parameters.

$$(iii)\ \ C_t = \Theta_\chi C_{m,t}^{\chi} C_{q,t}^{1-\chi}$$
$$(iv)\ \ C_{q,t} = \left[\int_0^1 C_{q,t}(i) di^{\frac{\epsilon-1}{\epsilon}} \right]^{\frac{\epsilon}{\epsilon-1}}$$
$$(v)\ \ \Theta_\chi = \chi^{-\chi}(1-\chi)^{-(1-\chi)}$$
$$(vi)\ \ P_{q,t} = \left[\int_0^1 P_{q,t}(i) di^{1-\epsilon} \right]^{\frac{1}{1-\epsilon}}$$
$$(vii)\ \ P_{c,t} = P_{m,t}^{\chi} P_{q,t}^{1-\chi}$$

A.1.1 FOCs

$$\frac{\partial L}{\partial N_t}: \; N_t^{\phi} = \lambda_t W_t \left(1 - \Phi_t\right) \tag{A.1}$$

$$\frac{\partial L}{\partial C_{q,t}}: \; (1-\chi) = \lambda_t P_{q,t} C_{q,t} \tag{A.2}$$

$$\frac{\partial L}{\partial C_{m,t}}: \; \chi = \lambda_t P_{m,t} C_{m,t} \tag{A.3}$$

$$\frac{\partial L}{\partial B_t}: \; Q_t^B = \beta(\frac{\lambda_{t+1}}{\lambda_t}) \tag{A.4}$$

$$\frac{\partial L}{\partial W_t}: \; \lambda_t N_t \left\{ \left[\sigma\psi \left(\frac{W_t}{W_{t-1}}\right) - 1 \right] e^{-\psi\left(\frac{\sigma W_t}{W_{t-1}} - 1\right)} - 2\psi \left(\frac{W_t}{W_{t-1}}\right) + \psi \left(1 + \frac{\psi}{\delta}\right) + 1 \right\} = \\ = \beta \lambda_{t+1} N_{t+1} \left(\frac{\psi}{W_t^2}\right) \left\{ \sigma e^{-\psi\left(\frac{\sigma W_t}{W_{t-1}} - 1\right)} - 1 \right\} \tag{A.5}$$

Divide (A.2) by (A.3) we get the following:

$$(1-\chi) P_{m,t} C_{m,t} = \chi P_{q,t} C_{q,t} \tag{A.6}$$

$$P_{m,t} C_{m,t} = \chi P_{c,t} C_t \tag{A.7}$$

$$P_{q,t} C_{q,t} = (1-\chi) P_{c,t} C_t \tag{A.8}$$

Note, plug (A.7) and (A.8) into (A.6) to check that (A.7) and (A.8) hold.

From (A.3) solve for λ_t and plug it into (A.4) to get $Q_t^B = \beta(\frac{P_{m,t} C_{m,t}}{P_{m,t+1} C_{m,t+1}})$.

Now plug (A.7) and (A.8) to get the following:

$$Q_t^B = \beta E_t \left(\frac{C_t}{C_{t+1}} \frac{P_{c,t}}{P_{c,t+1}} \right) \tag{A.9}$$

$$P_{c,t} = P_{m,t}^{\chi} P_{q,t}^{1-\chi} = P_{q,t} S_t^{\chi}$$

where $\left\{S_t = \frac{P_{m,t}}{P_{q,t}}\right\}$ is the real price of oil expressed in the domestic goods

Log-linearizing

$$p_{c,t} = p_{q,t} + \chi s_t \tag{A.10}$$

Marginal rate of substitution between N_t and C_t is

$$(-1)\frac{\frac{\partial L}{\partial N_t}}{\frac{\partial L}{\partial C_t}} = (-1)\left[\frac{N_t^{\phi} - \lambda_t W_t\left(1-\Phi_t\right)}{\frac{1}{C_t}}\right] = MRS_{C,N}$$

Plug $\left\{\lambda_t = \frac{\chi}{P_{m,t}C_{m,t}}\right\}$ and $\{P_{m,t}C_{m,t} = \chi P_{c,t}C_t\}$ into $\{MRS_{C,N} = 1\}$ to get the following labor supply condition under perfect competition

$$N_t^{\phi}C_t = \frac{W_t\left(1-\Phi_t\right)}{P_{c,t}} \tag{A.11}$$

Log-linearizing equation (A.11)

$$\left[1+\frac{\varphi\Phi}{1-\Phi}\right]w_t - p_{c,t} = c_t + \phi n_t + \left(\frac{\varphi\Phi}{1-\Phi}\right)w_{t-1} \tag{A.12}$$

where $\varphi = \frac{\psi\left[\sigma e^{-\psi(\sigma-1)}-1\right]}{\left[e^{-\psi(\sigma-1)}-1\right]}$ and $\Phi = \left(\frac{\delta}{\psi^2}\right)\left[e^{-\psi(\sigma-1)}-1\right]$. Equation (A.12) shows the relationship between wage in period t and consumption, consumer price level, and employment in period t as well as the wage in the previous period.

Log-linearizing equation (A.9)

$$c_t = E_t c_{t+1} - \left[i_t - E_t\pi_{c,t+1}\right] \tag{A.13}$$

where $\frac{Q_t^B}{Q^B} - 1 = -i_t$ and $E_t\pi_{c,t+1} = p_{c,t+1} - p_{c,t}$

A.2 Firms

A.2.1 Intermediate goods firm

Each firm produces a differentiated good indexed by $i \in [0,1]$ using the following production function

$$Q_t(i) = A_t M_t(i)^{\alpha_m} N_t(i)^{\alpha_n} \tag{A.14}$$

where M_t and N_t are the quantities of imported oil and labor used in production. Using the assumption that firms take input prices as given, firm's problem becomes:

$$min \;\; W_t N_t(i) + P_{m,t} M_t(i) + \lambda_t^f \left[A_t M_t(i)^{\alpha_m} N_t(i)^{\alpha_n} - Q_t(i)\right]$$

FOCs

$$N_t(i):\;\; W_t = \lambda_t^f(\alpha_n)\left[\frac{Q_t(i)}{N_t(i)}\right] \tag{A.15}$$

$$M_t(i):\;\; P_{m,t} = \lambda_t^f(\alpha_m)\left[\frac{Q_t(i)}{M_t(i)}\right] \tag{A.16}$$

$$\lambda_t^f:\;\; Q_t(i) = A_t M_t(i)^{\alpha_m} N_t(i)^{\alpha_n} \tag{A.17}$$

λ_t^f is the marginal cost. Following the approach by Blanchard and Gali redefine marginal cost as

$$\Psi_t(i) = \frac{W_t}{\alpha_n\left[\frac{Q_t(i)}{N_t(i)}\right]} = \frac{P_{m,t}}{\alpha_m\left[\frac{Q_t(i)}{M_t(i)}\right]} \tag{A.18}$$

Letting $\left\{M_t^p(i) = \frac{P_{q,t}(i)}{\Psi_t(i)}\right\}$ denote a gross markup by firm i, we have

$$M_t^P(i) \times S_t \times M_t(i) \;\; = \;\; \alpha_m Q_t(i)\left[\frac{P_{q,t}(i)}{P_{q,t}}\right] \tag{A.19}$$

In order to show that (A.19) holds, plug $\left\{S_t = \frac{P_{m,t}}{P_{q,t}}\right\}$, $\left\{M_t^p(i) = \frac{P_{q,t}(i)}{\Psi_t(i)}\right\}$ and $\left\{\Psi_t(i) = \frac{P_{m,t}}{\alpha_m[\frac{Q_t(i)}{M_t(i)}]}\right\}$ into the left hand side of that equation to get

$$Q_t = \left[\int_0^1 Q_t(i)^{1-\frac{1}{\epsilon}} di\right]^{\frac{\epsilon}{\epsilon-1}} \tag{A.20}$$

A.2.2 Final goods firm

Let aggregate gross output be given by the following formula

$$Q_t = \left[\int_0^1 Q_t(i)^{1-\frac{1}{\epsilon}} di\right]^{\frac{\epsilon}{\epsilon-1}} \tag{A.21}$$

The final goods firm

$$\underset{Q_{t(i)}}{max}\ P_{q,t}Q_t - \int_0^1 P_{q,t}(j)Q_t(j)dj$$

Plug (A.21) to get

$$max \quad P_{q,t}\left[\int_0^1 Q_t(i)^{1-\frac{1}{\epsilon}} di\right]^{\frac{\epsilon}{\epsilon-1}} - \int_0^1 P_{q,t}(j)Q_t(j)dj \tag{A.22}$$

FOCs

$$Q_t(i) = \left(\frac{P_{q,t}(i)}{P_{q,t}}\right)^{-\epsilon} \tag{A.23}$$

plug (A.23) into (A.19)

$$\left[\int_0^1 M_t(i)di\right]\left[\int_0^1 M_t^p(j)dj\right] = \frac{\alpha_m Q_t}{S_t}$$

Let $M_t = \left[\int_0^1 M_t(i)di\right]$ and $M_t^p = \left[\int_0^1 M_t^p(j)dj\right]$, we can re-write the above equation as

$$M_t = \frac{\alpha_m Q_t}{S_t M_t^p} \tag{A.24}$$

Log-linearizing equation (A.24)

$$m_t = -\mu_t^p - s_t + q_t \tag{A.25}$$

where $\mu_t^p = log(M_t^p)$

Log-linearizing equation (A.21)

$$q_t = a_t + \alpha_m m_t + \alpha_n n_t \tag{A.26}$$

Plug (A.25) into (A.26) to get reduced gross output production function:

$$q_t = \left(\frac{1}{1-\alpha_m}\right)[a_t + \alpha_n n_t - \alpha_m \mu_t^p - \alpha_m s_t] \tag{A.27}$$

A.3 Consumption and Gross Output

Note that in an equilibrium with the balanced trade (and hence $B_t = 0$) the following relation must hold

$$P_{c,t}C_t = P_{q,t}Q_t + P_{m,t}M_t$$

Plug (A.24) to get

$$P_{c,t}C_t = \left(1 - \frac{\alpha_m}{M_t^p}\right) P_{q,t}Q_t$$

Plug $\{P_{c,t} = P_{q,t}S_t^{\chi}\}$to get

$$S_t^{\chi}C_t = \left(1 - \frac{\alpha_m}{M_t^p}\right) Q_t$$

Log-linearizing the equation above we have

$$c_t = q_t - \chi s_t + \eta \mu_t^p \tag{A.28}$$

where $\eta = \frac{\alpha_m}{M^p - \alpha_m}$.

Plugging (A.28) into (A.27) and using the fact $\left(\frac{\alpha_m}{M^p-\alpha_m} - \frac{\alpha_m}{1-\alpha_m}\right)\mu_t^p \simeq 0$[1] we get

$$c_t = \frac{a_t}{1-\alpha_m} + \left(\frac{\alpha_m}{1-\alpha_m}\right) n_t - \left(\frac{\alpha_m}{1-\alpha_m} + \chi\right) s_t \tag{A.29}$$

[1]See Blanchard and Gali (2007) p.69

A.4 Gross Output, Value Added, and the GDP deflator

$$P_{q,t} = (P_{y,t})^{1-\alpha_m} (P_{m,t})^{\alpha_m}$$

where $P_{y,t}$ is the GDP deflator.

Log-linearize and rearrange terms

$$p_{y,t} = \left(\frac{1}{1-\alpha_m}\right) p_{q,t} - \left(\frac{\alpha_m}{1-\alpha_m}\right) p_{m,t}$$

Plug $\{s_t = p_{m,t} - p_{q,t}\}$ to get the formula for the GDP deflator

$$p_{y,t} = p_{q,t} - \left(\frac{\alpha_m}{1-\alpha_m}\right) s_t \tag{A.30}$$

Value added Y_t is defined by

$$P_{y,t}Y_t = P_{q,t}Q_t - P_{m,t}M_t = \left(1 - \frac{\alpha_m}{M_t^p}\right) P_{q,t}Q_t$$

Log-linearizing

$$p_{y,t} = \eta\mu_t^p + p_{q,t} + q_t - y_t$$

Plug (A.30)

$$y_t = q_t + \eta\mu_t^p + p_{q,t} - p_{q,t} + \left(\frac{\alpha_m}{1-\alpha_m}\right) s_t$$

Plug (A.27)

$$y_t = \left(\frac{1}{1-\alpha_m}\right) [a_t + \alpha_n n_t] \tag{A.31}$$

Plug (A.31) into (A.29) to get the relationship between consumption and value added

$$c_t = y_t - \left(\frac{\alpha_m}{1-\alpha_m} + \chi\right) s_t \tag{A.32}$$

A.5 Price Setting

The firms set prices in a staggered fashion[2]. The optimal rule for price setting is

$$E_t \left\{ \sum_{k=0}^{k\infty} \Theta^k \Lambda_{t,t+k} Q_{t,t+k|t} \left[P_t^* - M^P \Psi_{t+k|t} \right] \right\} = 0 \tag{A.33}$$

where

$P_t^*$is the new price set at time t

$Q_{t,t+k}$ is the level of output in period $(t+k)$ for a firm that last set price in period t

$\Psi_{t+k|t}$ is the marginal cost in period $(t+k)$ for a firm that last set price in period t

$M^p = \frac{\epsilon}{\epsilon - 1}$ desired gross markup (in steady state)

Domestic price evolution equation is given by

$$P_{q,t} = \left[\theta (P_{q,t-1})^{1-\epsilon} + (1-\theta) \left(P_t^* \right)^{1-\epsilon} \right]^{\frac{1}{1-\epsilon}} \tag{A.34}$$

Log-linearizing (A.33) and (A.34) and rearranging terms

$$\pi_{q,t} = \beta E_t \left\{ \pi_{q,t+1} \right\} - \lambda_p \hat{\mu_t^p} \tag{A.35}$$

where

$\hat{\mu_t^p} = \mu_t^p - \mu^p$; $\lambda_p = \left[\frac{(1-\theta)(1-\beta\theta)}{\theta} \right] \left[\frac{\alpha_m + \alpha_n}{1 + (1 - \alpha_m - \alpha_n)(\epsilon - 1)} \right]$ and $\pi_{q,t} = p_{q,t} - p_{q,t-1}$

[2]See Calvo (1983)