A COMPARISON OF ADJUSTMENTS ON TWO TRAVERSES BY THE TRANSIT RULE, COMPASS RULE AND METHOD OF LEAST SQUARES

by

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INTRODUCTION

During the last decade there have been more technological advances in the surveying industry than during the preceding 90 years. A century ago angles were being measured with compasses and distances were measured with the Gunter chain. Fifty years ago transits and engineers chains were being used. Even in the past decade the transit and engineers tape were still being used. Today, surveyors are using compact theodolites to measure angles and electronic distance measuring devices to measure distances with more accuracy and precision than in the decade just past.

The increased accuracy and precision mentioned above has led to much smaller closure errors and to a desire for better techniques for adjustment of this error in achieving closure for either level or traverse circuits. The electronic computer is a major factor in the improvement of error adjustment procedures. Through the use of the computer, methods which were previously impractical or difficult to use are now easily applied.

The transit rule, the compass rule and the method of least squares, are methods of adjustment of traverse closure errors which will be compared in this study. Each method has its own advantages and disadvantages which are of interest. The reason for this study is to satisfy the curiosity of the author regarding the manner in which the error of closure is adjusted by each of these methods when applied to a given traverse.

PURPOSE AND SCOPE

Many articles have been written on the subject of traverse adjustments using the transit rule, the compass rule and the method of least squares. Each of these methods of adjustment partitions the closure error in a unique manner which results in the allocation of different portions of the closure error to given traverse segments by each method.

The purpose of this report is to apply the different methods of traverse adjustments to a set of field data to compare the results obtained. The field data were obtained from two traverses run on the KSU Campus, one North-South and one East-West, for which highly accurate information was available from earlier traverse work in which a precision of 1/150,000 was obtained.

REVIEW OF LITERATURE

In the adjustment of traverses, the errors are adjusted. Rainsford (9) (chapter 1) discusses four types of errors: blunders, constant errors, systematic errors, and accidental or random errors.

"Blunders (or mistakes) are a definite mis-reading of whatever scale is being used." Examples would be the reading of 60° 45' for an angle which actually is 61° 45' or reading 100.75 feet instead of 99.75 feet.

"Constant errors are those which do not vary throughout the particular work concerned. They always have the same sign." An example would be an uncalibrated tape 100.02 feet in length which would be assumed to be 100.00 feet.

"Systematic errors are those which follow some fixed law (possibly unknown) dependent on local circumstances." Examples would be failure to apply temperature and sag corrections to tapes.

"Accidental or random errors are the remaining small errors after all the others just mentioned have been eliminated. They are due to the imperfection of the instrument used, the fallibility of the observer, and the changing conditions, all of which affect the quality of the observation to a greater or lesser degree." These accidental of random errors are the only errors which are adjusted. Wolf (14) and Vreeland (13) discuss the source of these errors as either in distance or in angle.

The adjustment of the errors depends upon the type and source of the errors. Blunders are not acceptable whether in angle or distance and must be remeasured. The source of constant errors must be identified and corrections or adjustments made in instrumentation or technique so that this type of error may be eliminated. Systematic errors follow definite laws (2) which are used to compute their values in order that they may be eliminated. The accidental or random errors conform to the normal distribution (9) which, for a given set of data, have unique values of the mean, $\bar{X}$, and standard deviation, s.

These accidental or random errors are the errors which are to be adjusted. Several procedures have been devised to adjust these errors (10). Each of these procedures adjusts the observations or measured quantities to fulfill the conditions of angular closure, latitude closure, and departure closure. The procedures include the transit rule, the compass rule and the method of least squares.

The transit rule assumes that the angles are measured more precisely than the distances. "The corrections to be applied to the latitude/departure of any course is to the total error in latitude/departure as the latitude/departure of the course is to the sum of all latitude/departure (without regard to algebraic sign)."(3)

Goussinsky (5) explains that the adjustment by the compass rule (also called the Bowditch Method) assumes that the error in the angle is as great as the error in the distance. This was true 100 years ago when the compass and the Gunter chain were common surveying equipment, hence the name compass rule. "The corrections to be applied by this rule to the latitude/departure of any course is to the total error in latitude/departure as the length of the course is to the perimeter."(3) Richardus (10) explains that in applying this procedure, the bearings are adjusted before computing the latitudes and departures for the adjustment.

The method of least squares which makes no assumptions about the configuration of the traverse applies corrections to the angles and distances such that the cumulative sum of the squares of the corrections is a minimum. The method, although developed over 150 years ago by Gauss and Legendre, was not used extensively until recently and has gained general acceptance since the era of the computer. Adams (1) in 1924 presented the basic procedures for obtaining the least squares solution through the use of several simple examples.

Gale (4) indicates that least squares adjustments can be divided into three categories;

1. The observation equation or parametric method. In survey adjustments, this method is sometimes referred to as variation of coordinates and is discussed by Wolf (14). He states "Two observation equations (equations relating the observed quantities and their inherent random error to the most probable X and Y coordinates of the points involved), one for distance, and one for direction, may be written for each side of a traverse. Therefore the number of observation equations for any traverse is 2r, where r is the number of sides in the traverse. Each traverse point introduces two unknowns, an X and a Y coordinate, except that the initial and final traverse point coordinates are either known or assumed; hence the number of unknowns is 2r - 2. Normally, therefore, there are two redundant observation equations in the traverse adjustment."

Schmid (11), Gale (4) and Madkour (7) present the idea of writing the observation equations in the following matrix form.

$$ {}_{M}A_{N}\ {}_{N}X_{1} = {}_{M}L_{1} \qquad (M > N) $$

where

${}_{N}X_{1}$ = A column vector of parameters

${}_{M}A_{N}$ = A matrix containing the coefficients of ${}_{N}X_{1}$

${}_{M}L_{1}$ = A column vector of the correlated observations

M = The number of rows in the matrix

N = The number of columns in the matrix

No adjustment is necessary if the equations are consistent. If they are inconsistent an adjustment vector, ${}_{M}V_{1}$, is added to the correlated observations thus obtaining

$$ {}_{M}A_{N}\ {}_{N}X_{1} = {}_{M}L_{1} + {}_{M}V_{1} $$

To apply the method of least squares to this problem, the Gaussian Function, Φ, must be minimized. This minimization is accomplished by setting the matrix first derivative equal to zero.

$$ \Phi = {}_{1}V^{*}_{M}\ {}_{M}P_{M}\ {}_{M}V_{1} $$

where
${}_{1}V^{*}_{M}$ = The transpose of the adjustment vector
${}_{M}P_{M}$ = The observation weight matrix (discussed in Appendix A)

2. The condition equation or correlate method. Vreeland (13) presents a traverse adjustment by the condition equation method using the summation of latitude and departure as conditions. Gale (4) and Madkour (7) present their discussions of the condition equation method through the use of this matrix equation;

$$ {}_{M}B^{*}_{N}\ {}_{N}X_{1} + {}_{M}C_{1} = 0 $$

where
${}_{M}B^{*}_{N}$ = The transpose of a matrix containing the partial derivatives of the unknowns ${}_{N}X_{1}$
${}_{N}X_{1}$ = A column vector of the true values of the unknowns
${}_{M}C_{1}$ = A column vector of constants necessary to complete the condition equations

Then by introducing the undetermined Lagrangian Multipliers, ${}_1K_M^*$ (Adams (1) presents a simple example of Lagranian Multipliers), and minimizing the Gaussian Function,

$$\Phi = {}_1V_N^* \, {}_NP_N \, {}_NV_1 - 2 \, {}_1K_M^* \, ({}_MB_N \, {}_NX_1 + {}_MC_1)$$

the least squares solution is obtained. Further development of the algebraic formulas may be found in Appendix A.

3. The general or combined methods. Schmid (11) discusses the general category of least squares. In his discussion are presented the difficulties met when trying to write a general least squares computer program. This category is not discussed here because it is not used to adjust a traverse.

Since all of the least squares procedures allow for the use of weights, Meyer (8) discusses the use of the arithmetic mean of several observations to obtain a better value of the observation. He expands his discussion to include the case where an angle might be read 2 times with a theodolite and 3 times with a 30" transit. Obviously the angles read by the theodolite are more accurate than those read with the 30" transit. He therefore applies a weight to the angles to obtain a weighted arithmetic mean.

Vreeland (13) presents the precisions of the various surveying instruments used, but doesn't discuss weights. Wolf (14) on the other hand discusses methods which may be used to obtain the relative weights of the angles and distances (See Appendix A for more detail.).

Rainsford (9) and Meyer (8) discuss the use of the inverse of the variance as a weight. Rainsford presents a derivation which shows the weight of the observation is equal to 1 ÷ variance.

PROCEDURES AND APPARATUS

Recently work was begun to place the Kansas State University campus on a coordinate system. The initial phase of the project was to fix control points whose coordinates were accurately obtained on the perimeter of the campus. This was done by completing a loop traverse (see Fig. 1) using a Wild T-2 theodolite and the Tellurometer MRA-4 electronic distance measuring device. A precision of 1/150,000 was obtained from this traverse. (The U.S. Coast and Geodetic Survey considers 1/25,000 as first-order accuracy.)

In order to further subdivide the campus, field parties were sent out with Wild T-16 theodolites and 100-ft. engineers tapes to obtain the two traverses under study here. The two traverses--one in a North-South direction along Mid-Campus Drive (Fig. 2) and the other in an East-West direction along College Heights and Vattier Drive (Fig. 3)--yielded data which had angles estimated to the nearest 6 seconds and distances obtained to the nearest 0.01 foot.

The North-South traverse was completed by a field party from the KSU Campus Planning Department and the East-West traverse was completed by an advanced surveying class from the Department of Civil Engineering. Since the traverse is the particular case in surveying in which both angles and distances are measured, they naturally fall into these four categories depending on the type of closure:

1. No Closure
2. Angular Closure (i.e., the angles must meet a specified condition)
3. Coordinate Closure (i.e., the latitude and departure must meet specified conditions)
4. Coordinate and Angular Closure (i.e., the angles, latitude and departures must meet specified conditions)

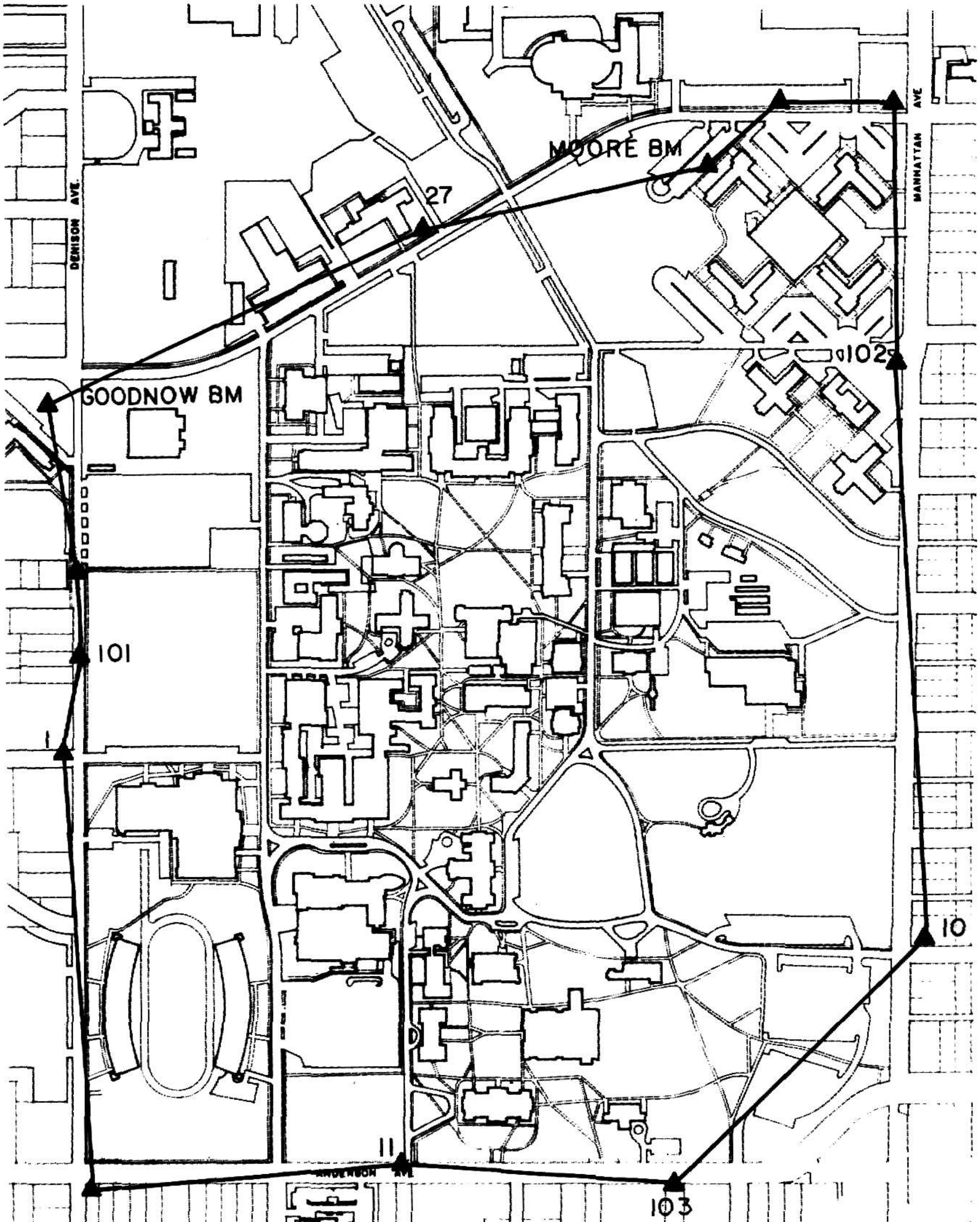

Fig. 1. A map of the KSU Campus indicating the location of the control traverse.

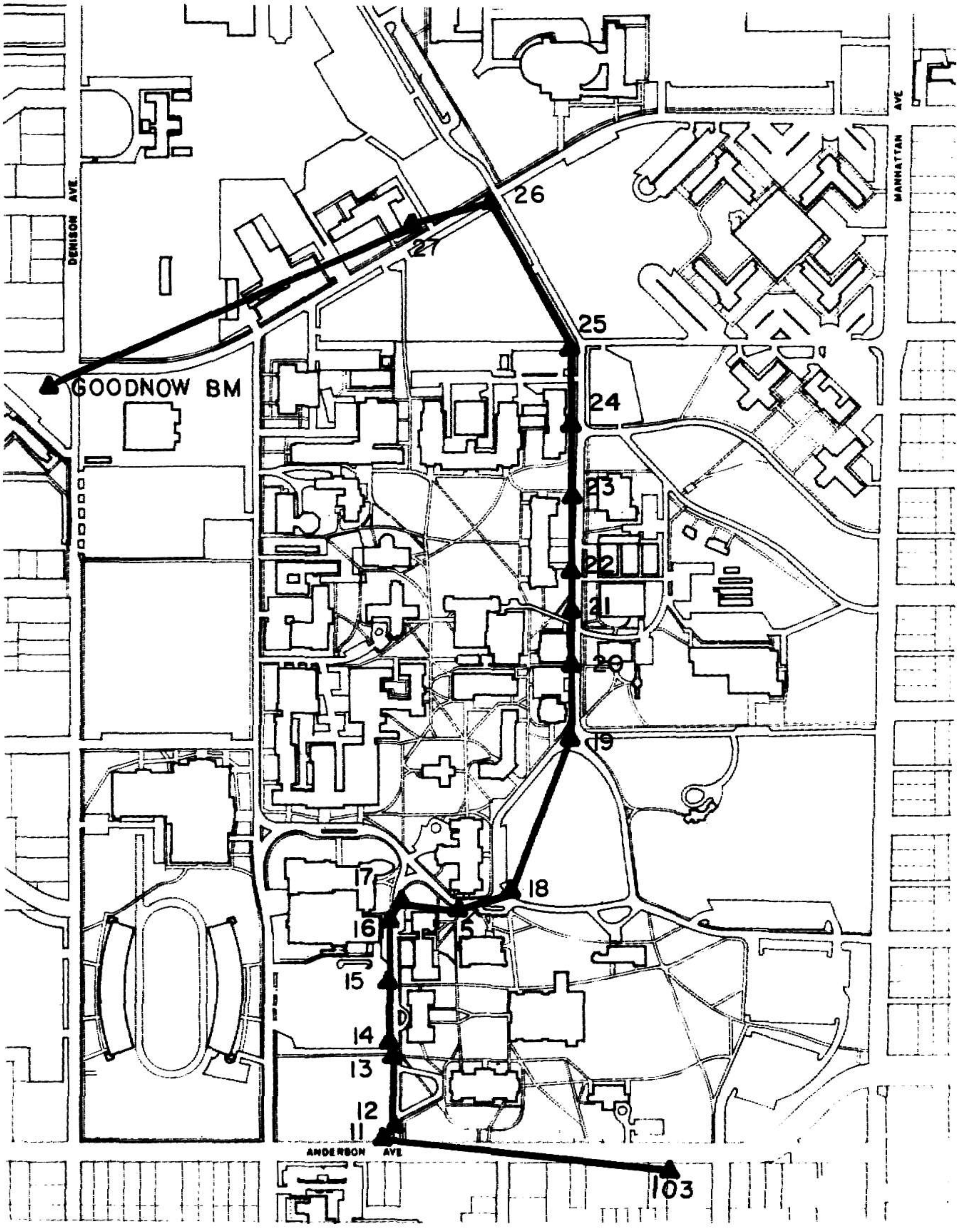

Fig. 2. A map of the KSU Campus indicating the location of the North-South traverse.

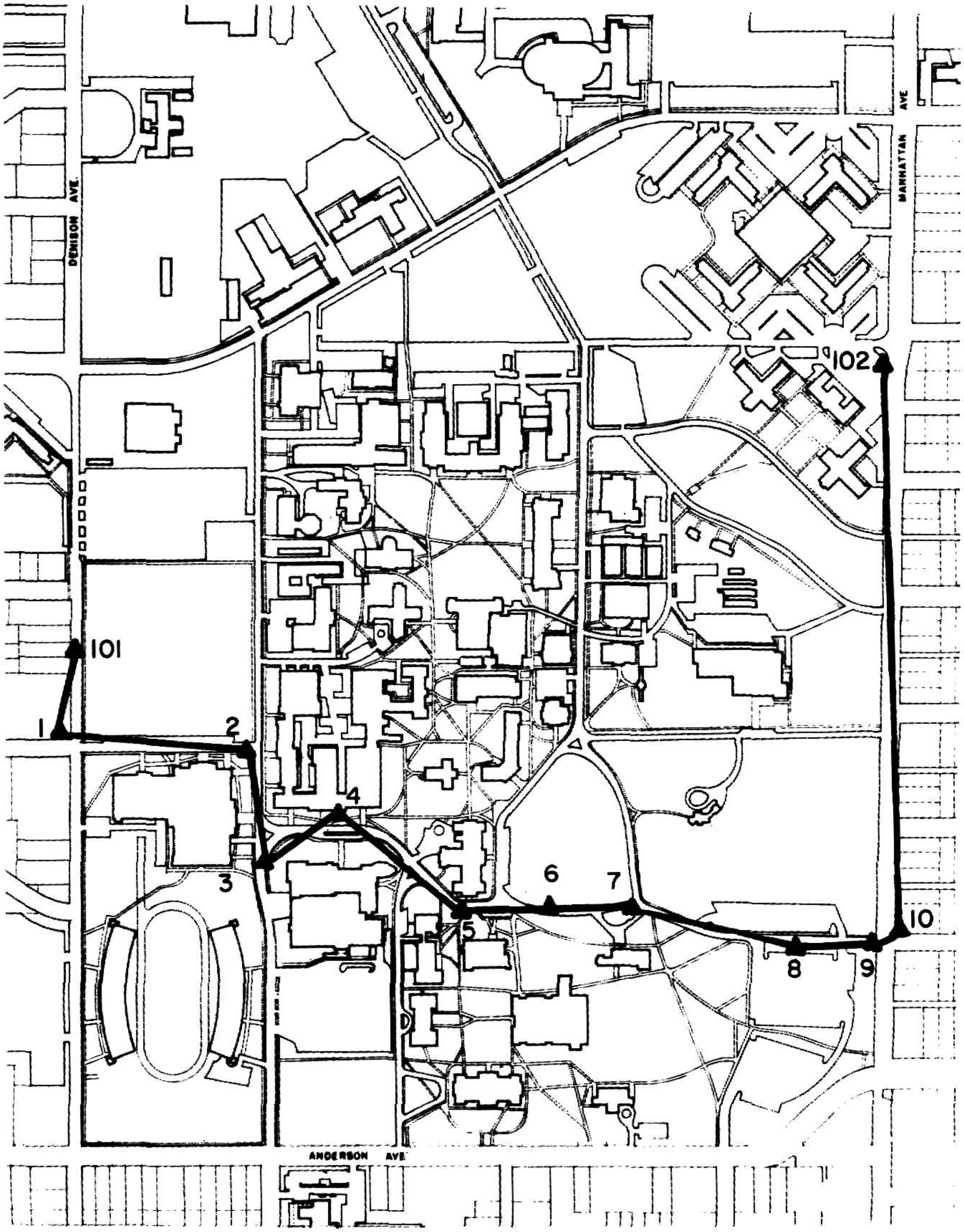

Fig. 3. A map of the KSU Campus indicating the location of the East-West traverse.

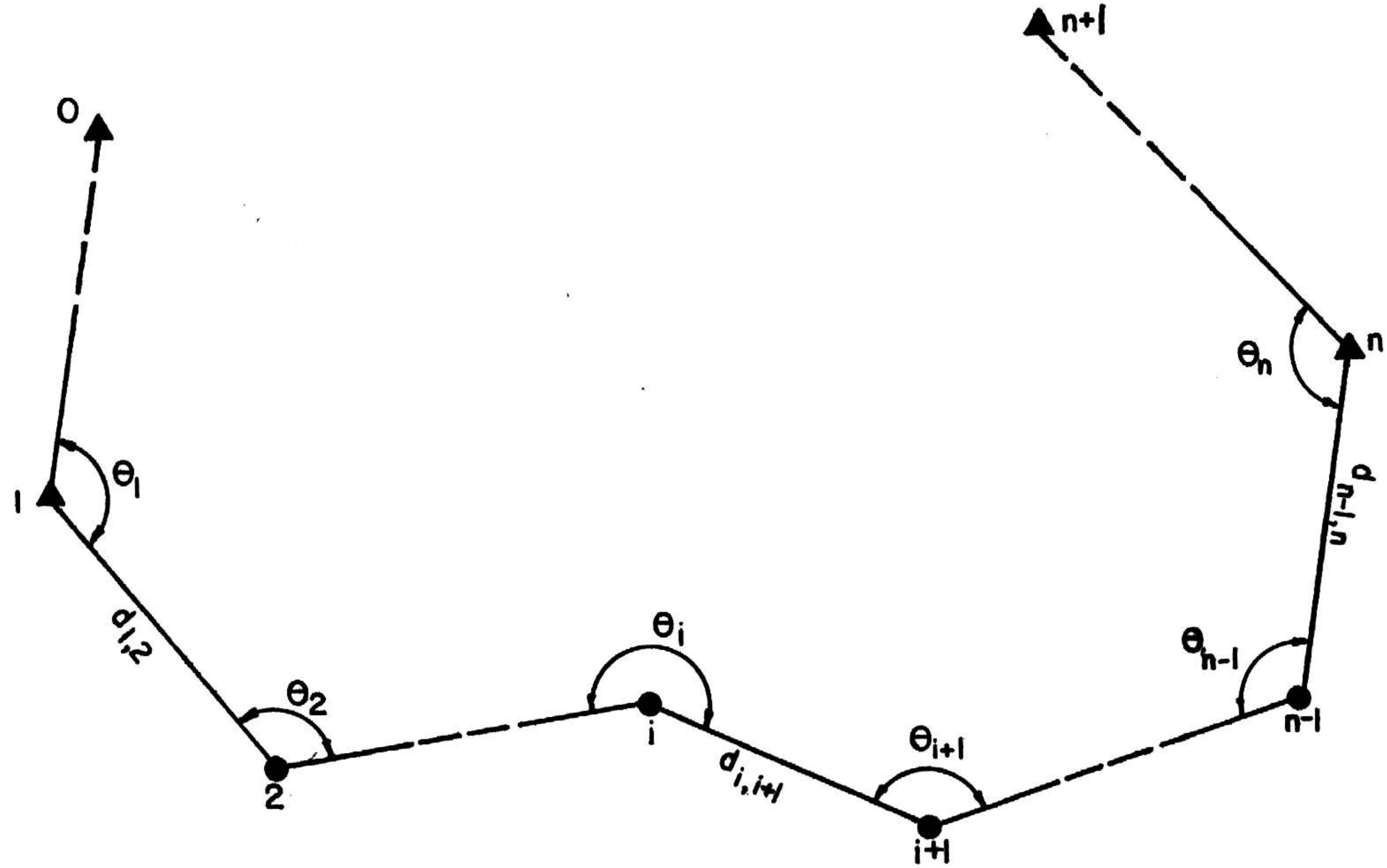

Fig. 4. A Generalized Traverse Showing the Notation Used in This Study.

The two traverses had both angle and coordinate closure and therefore fell into the fourth category.

The conditions referred to above are described below with reference made to Fig. 4. Stations 1 and n are the terminals. The observed angle on station 1 is θ_1, or in general θ_i on station i. The measured distance between i and i+1 is $d_{i,i+1}$ and the azimuth of the line is $Az_{i,i+1}$.

Stations 0 and n+1 are the orientation stations, the orienting azimuths are $Az_{0,1}$ and $Az_{n,n+1}$. If stations 0 and n, and n+1 and 1 are the same stations the traverse is a loop.

1. <u>The Angular Condition</u>. The sum of the first orientation azimuth $Az_{0,1}$ and the observed angles, θ_i, must equal the terminal azimuth $Az_{n,n+1} \pm n\ 180°$ which may be written as

$$Az_{0,1} + \sum_{i=1}^{n} \theta_i = Az_{n,n+1} \pm n\ 180° \qquad 1)$$

The relationship of any azimuth to any other azimuth should also be noted

$$Az_{i,i+1} = Az_{0,1} + \sum_{j=1}^{i} \theta_j \pm n\ 180° \qquad 1a)$$

2. <u>The Departure Condition</u>. The sum of the products of the distances and the sines of their azimuths should be equal to the differences in the abscissas of the terminal stations.

$$\sum_{i=1}^{n} d_{i,i+1} \operatorname{Sin}(Az_{i,i+1}) = X_n - X_1 \qquad 2)$$

3. <u>The Latitude Condition</u>. The sum of the products of the distances and the cosines of their azimuths must equal the differences in the ordinates of the terminal stations.

$$\sum_{i=1}^{n} d_{i,i+1} \operatorname{Cos}(Az_{i,i+1}) = Y_n - Y_1 \qquad 3)$$

The three methods of adjustment to be used in this study--the transit rule, the compass rule and the method of least squares--apportion the closure errors of these conditions differently. The transit rule and compass rule adjust the angles prior to computing the course latitudes and departures by adding the amount (angular closure error ÷ number of angles) to each angle whereas the method of least squares uses the unadjusted angles to compute the course latitudes and departures and adjusts the angles simultaneously.

The transit rule adjustment may be stated as follows: the correction to be applied to the latitude/departure of any course is to the total error in latitude/departure as the latitude/departure of that course is to the sum of all the latitude/departure (without respect to sign) (14). The compass rule adjustment of the latitude and departure of a traverse is simply stated as follows: the corrections to be applied to the latitude/departure of any course is to the total error in latitude/departure as the length is to the perimeter (14).

The method of least squares adjustment applies corrections which have been minimized to the observed values. The method uses the matrix condition

$$B\,X + C = 0$$

where
B = The partial derivatives of the conditions
X = The true values of the variables
C = The constants of the conditions

And its Gaussian Function, Φ, solved for a minimum.

$$\Phi = V^{*}\,P\,V - 2\,K^{*}\,(B\,X + C)$$

where
V = The corrections to the observed values
P = The weight of the variable
K = A Lagrangian Multiplier
(The * refers to a transposed matrix)

A more detailed algebraic explanation of the condition equation method of least squares is in Appendix A.

The various adjustments were computed through the use of the MIT ICES COGO--Geometric processor. This Processor did not perform the least squares adjustment using the unadjusted angles as described in Appendix A, but rather used the adjusted angles of the transit and compass rule. Although not correct, since the angles have been adjusted, this procedure is useful in that the latitude and departure errors are equal for all three thereby producing an easy comparison of error adjustment by the three methods.

As the data were being prepared for use in the ICES COGO--Geometric Processor it was learned that another error adjustment procedure, the Crandall Rule, was contained in the program package. Since the Crandall Rule could be examined with very little additional effort it was decided to apply this procedure of error adjustment to determine its applicability when compared with the three procedures originally included in the study. This rule holds the course bearing as correct and adjusts only the course lengths. The Crandall Rule is discussed in more detail in Appendix B.

PRESENTATION AND DISCUSSION OF DATA

The presentation of these data is divided into two sections--one for the North-South traverse and one for the East-West traverse.

The North-South traverse.

The initial point (Pt. 27) of the North-South traverse (Fig. 2) had coordinates (N 6687.05, E 2243.70) and an initial azimuth of 244° 46' 50" when sighting to the Civil Engineering Dept. Bench Mark in front of Goodnow Hall. The coordinates of the final point (Pt. 11) were (N 3756.26, E 2188.87) and an azimuth of 93° 16' 51" to Pt. 103. The computation of the traverse produced the following closure errors--N -0.012 ft. and E -0.434 ft. Tables 1, 2, and 3 show the computation of the latitude and departure adjustments and Table 4 shows the angle and distance adjustments produced by the three adjustment methods.

The Angle Adjustment. The first half of Table 4 shows adjustments which were made to the angles. A comparison of the arithmetic totals of the angle adjustments indicated that the method of least squares produced the smallest corrections (89 sec. total) followed by the compass rule (116 sec. total) and then the transit rule (375 sec.).

Comparing the individual adjustments, the method of least squares yielded many adjustments which were within one estimation unit (6 sec.) of the T16 theodolite and all within 2 units (12 sec.). The compass rule had 4 adjustments larger than 2 units (12 sec.) but smaller than 5 units (0.5 minute). The transit rule had 11 adjustments larger than 2 units with 7 of these larger than 5 units (0.5 minute).

The Distance Adjustment. The last half of Table 4 indicates the adjustments to the distance. A comparison of the arithmetic totals indicated a wide variation in adjustments. The method of least squares had 0.011 ft. adjustment, followed by the compass rule with 0.127 ft. adjustment and then the transit rule with 0.306 ft. adjustment.

A comparison of the individual adjustments indicate the method of least squares makes no adjustment to distance (The largest adjustment was 0.005 ft.) The compass rule yielded 5 adjustments which were more than 0.01 ft. with 4 of these adjustments between 0.025 ft. and 0.030 ft. These four adjustments were in the four courses with large departure components. The transit rule also yielded 5 adjustments more than 0.01 ft. These adjustments were in the same courses as the compass rule but were much larger--all between 0.036 ft. and 0.091 ft.

The Latitude Adjustment. No comparison was made due to the extremely small closure error of 0.012 ft.

The Departure Adjustment. The comparison of the departure adjustment (Fig. 5) yielded a transit rule adjustment with most of the error in the four courses with large departure components. The compass rule yielded the expected straight line (This was expected since the magnitude of adjustment is in direct proportion to the total length.) The method of least squares yielded adjustments which increased to -0.485 ft. for the next to the last point on the traverse and dropped to -0.434 ft. for the last point (This drop is not possible with the transit and compass rules.).

The Crandall Rule Adjustment. A discussion of the Crandall Rule Adjustment is in Appendix B.

Table 1. Transit Rule Adjustment of the North-South traverse.

Point on Traverse	Adjusted Angle ° ' "	Course Length (feet)	Course Latitude (feet)	Per Cent of Total Latitude	Latitude Adjustment (feet)	Adjusted Latitude (feet)	Course Departure (feet)	Per Cent of Total Departure	Departure Adjustment (feet)	Adjusted Departure (feet)
27	189 23 9									
		256.16	69.867	2.2	0.000	69.867	246.448	22.1	-0.095	246.353
26	256 9 1									
		481.76	-418.501	13.5	-0.002	-418.503	238.640	21.3	-0.091	238.549
25	209 1 2									
		290.02	-290.000	9.3	-0.001	-290.001	3.373	0.3	-0.002	3.371
24	179 16 1									
		250.05	-249.975	8.1	-0.001	-249.976	6.108	0.5	-0.002	6.106
23	179 59 54									
		250.01	-249.935	8.1	-0.001	-249.936	6.115	0.5	-0.003	6.112
22	179 59 54									
		100.00	-99.970	3.2	0.000	-99.970	2.449	0.2	-0.001	2.448
21	179 59 54									
		149.98	-149.935	4.8	-0.001	-149.935	3.677	0.3	-0.001	3.676
20	179 59 54									
		249.98	-249.905	8.1	-0.001	-249.906	6.137	0.5	-0.003	6.134
19	205 24 57									
		541.75	-494.907	15.9	-0.002	-494.909	-220.363	19.7	-0.085	-220.448
18	215 39 17									
		124.69	-62.994	2.0	0.000	-62.994	-107.607	9.6	-0.041	-107.648
5	215 17 49									
		217.45	18.811	0.6	0.000	18.811	-216.635	19.5	-0.083	-216.718
17	114 10 40									
		71.87	-62.780	2.0	0.000	-62.780	-34.985	3.1	-0.014	-34.999
16	149 38 4									
		199.08	-199.034	6.4	-0.001	-199.035	4.263	0.4	-0.001	4.262
15	180 2 36									
		213.96	-213.914	6.9	-0.001	-213.915	4.420	0.4	-0.001	4.419
14	158 59 32									
		17.88	-16.555	0.5	0.000	-16.555	6.755	0.5	-0.002	6.753
13	201 5 41									
		248.12	-248.075	8.0	-0.001	-248.075	4.745	0.4	-0.003	4.742
12	212 33 15									
		15.21	-12.976	0.4	0.000	-12.976	-7.936	0.7	-0.006	-7.942
11	69 49 19									
Algebraic Totals			-2930.778		-0.012	-2930.790	-54.396		-0.434	-54.830
Arithmetic Totals		3677.97	3108.134	100.0			1120.656	100.0		