PROPERTIES OF THE MAXIMUM LIKELIHOOD AND BAYESIAN ESTIMATORS OF AVAILABILITY

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1. INTRODUCTION

Increasing complexity of modern equipment, both in the military and commercial areas, has brought with it new engineering problems involving high performance, reliability, and maintainability. Reliability has long been considered, during system design, as a measure of system effectiveness. However, it has proved to be an incompleted measure because it does not consider maintainability, another important aspect of system performance. With increasing complexity and the resulting high operational and maintenance costs, greater emphasis has been placed on reducing system maintenance while improving reliability. In this regard availability, which is a combined measure of maintainability and reliability, has received wide usage as a measure of maintained systems effectiveness.

Availability is defined as the probability that the system is operating satisfactorily at any point in time under stated conditions. Lie et al. [8] surveyed and classified the literature related to the availability of various systems. Depending on the time interval considered, availability can be classified into: (1) instantaneous availability, (2) average uptime availability, and (3) steady-state availability.

Instantaneous availability, $g(t)$ is defined as the probability that the system is operational at any time t, where $0 \le t < \infty$. Average uptime availability, $A(T)$, is the proportion of time in a specified time $(0,T)$ that the system is available for use and is expressed as

$$g(T) = \frac{1}{T} \int_0^T g(t)dt$$

Steady-state availability is then the instantaneous availability at time ∞, which is the limiting case of the instantaneous availability. Since both steady-state availability and average uptime availability are special cases of instantaneous availability, the derivation and evaluation of instantaneous availability is fundamental and of interest.

Availability estimation is no more than a typical statistical estimation problem. Two typical procedures can be used, namely, non-Bayesian inference such as the maximum likelihood estimate technique and Bayesian inference. Kuo [7] recently reported on the maximum likelihood estimator of availability and the Bayesian estimator of availability for gamma distributed system cycle time and on time. Properties of these estimators have not been studied.

It is controversial to use Bayesian approach in statistical estimation. However, Bayesian apprach has its merits in reliability/availability problems especially when (1) the sample size is small due to the expensive or time consuming testing procedure, and (2) prior information is available in practical engineering problem from past experience.

This study is an extension of a previous study [7, 11]. Some properties of the maximum likelihood estimator of availability and the Bayesian estimator of availability for negative exponential distributed system on time and off time are investigated through simulation. It can be concluded that: (1) Both the maximum likelihood estimator of availability and the Bayesian estimator of availability are biased, (2) the maximum likelihood estimator of availability has a larger variance, a wider range, and a wider 90% C.I. than those of the Bayesian estimator of availability, and (3) Bayesian estimator of availability is insensitive to the prior information within at least a certain range.

Future study in the availability estimation problems is directed toward the use of the nonparametric Bayesian estimation techniques. Some preliminary study of the nonparametric Bayesian estimation of life distributions, which can be applied to system on time and system off time, has been investigated.

2. THE SYSTEM AND THE ASSUMPTIONS

Statement of the System:

Consider a system which can be in one of two states, "on" or "off", when in the "on" state, the system is operating and in the "off" state, the system is failing and under repair. We assume that at time 0, it is "on". The system is then in service until it fails at a random time T_{on} with the distribution function $F_{on}(t)$. When it fails, it is then in the "off" state and under repair for a random time T_{off} with the distribution function $F_{off}(t)$. Then the cycle repeats by being operative for a random time and then being inoperative for another random time. Successive times to breakdown and to repair are assumed to be independent.

It is assumed that the events of either operative or inoperative are independent of time. A complete cycle time T is also a random variable which is equal to the addition of random variables T_{on} and T_{off}. T then is a random variable of the time from 0 to the time at which the system failed, was repaired and just restored to the operative state. (See Fig. 1.)

The Assumptions:

Assume that a system cycle time T, and on time, T_{on}, are gamma distributed with pdf's:

$$f_T(z) = \frac{\lambda}{(k-1)!} (\lambda z)^{k-1} e^{-\lambda z} \qquad (1)$$

and

$$f_{T_{on}}(y) = \frac{\beta}{(\alpha-1)!} (\beta y)^{\alpha-1} e^{-\beta y} \qquad (2)$$

where α, $\beta>0$, x, $y>0$, k and α are positive integers.