MULTIOBJECTIVE DECISION MAKING IN WATER RESOURCES SYSTEM

by

YEN-TUNG CHANG

B.S. (Industrial Engineering)

NATIONAL TAIWAN INSTITUTE OF TECHNOLOGY

TAIPEI, TAIWAN, 1976

A MASTER'S THESIS

submitted in partial fulfillment of the

requirements for the degree

MASTER OF SCIENCE

Department of Industrial Engineering

KANSAS STATE UNIVERSITY

Manhattan, Kansas

1979

Approved by:

Major Professor

ACKNOWLEDGMENTS

The author wishes to express his sincere appreciation to Dr. E. S. Lee for his inspiring guidance, valuable criticisms and constant encouragement during the preparation of this thesis.

The author also thanks Dr. C. A. Bennett and Dr. Chi-lung Huang for serving on his committee.

The author is especially grateful to his parents for their constant encouragement and financial assistance. It is to them that this thesis is dedicated.

TABLE OF CONTENTS

LIST OF TABLES

LIST OF FIGURES

INTRODUCTION

In recent years, many problems concerning water resources have been solved by means of mathematical programming. The methods used in solving these problems include linear, dynamic, nonlinear, stochastic and simulation techniques. A common feature of these approaches is a basic assumption on the decision-making structure of the system. Specifically, it is postulated that there is only one decision maker with a single objective function to summarize all the economic goals to be pursued. If there exists a multiplicity of decision makers or there are multiple objectives, it becomes much more complex to obtain the optimization.

Multiobjective mathematical programming techniques have been widely used in the recent years in water resources systems to solve the trade-off problems such as environmental quality versus income in an agricultural area [3], income production versus income distribution in a developing country [4], reservoir evaporation losses versus cost and capacity [32], and other conflicting objectives such as hydroelectric generating, recreation, water quality, flooding control, prevention for drought seasons, etc.. These approaches are characterized by the existence of one decision maker but consider the real situation where the unique manager has no complete knowledge of the economic framework and hence must take a set of alternative targets into account.

Another case is the existence of many decision makers, each pursuing an individual objective and hierarchically subjected to a supervisor. Haims has assumed this case [11], and so did Hall et. al. [14]. Such an upper-level supervisor removes the infeasibility implied by the separate choice of lower-level decision makers and hence binds them to a set of cooperation. A case which often occurs in practice is the one with each of many decision makers pursuing an individual objective in conflict with the others and not been subjected to any supervision. This situation is described by Cohon et. al. [4]. The problem in establishing the yearly contract volumes of a two-purpose reservoir has been approached by Moran [29]. There is also a situation in which no "water sellers" or supervisors exist but the solution results from a trade-off between the two users.

In essence, water resources problems always create special problems which make the application of optimization methodologies quite difficult and unless treated with considerable insight quite meaningless. Most of these difficulties stem from three important characteristics of water resources systems. First of all, there is a large number of quasi-independent decision makers, each of them may make decisions or influence decisions according to his own, but different version of the desired goals. Second, even for any one decision maker, there is a large number of non-commensurable objectives to be optimized. Third, there is a very large uncertainty and risk in virtually all water resources decisions. This element is due to the high degree of irreversibility of these decisions coupled with both hydrologic uncertainty and inability to predict the future with reasonable accuracy.

The recent trend in water resources and other real world problems, however, is to elevate many of the non-commensurable objectives to parity with economic efficiency as criteria for excellence. The inclusion of a vector of objective functions introduces a new dimension in the fields of modeling, mathematical programming and optimal control.

Non-commensurability is one of the important characteristics in large scale water resources problems. For example, in water resources planning, one wants to maximize both economic efficiency, which is measured in monetary units, and environmental quality, which is measured in units of pollutant concentration. Traditionally, only one objective (economic efficiency) was considered, with the other objectives being included as constraints. However, society is placing an increasing importance on non-monetary objectives which are difficult to quantify monetarily. Multiple objective analysis has been applied to a wide variety of problems including transportation, project selection for research activities, economic production, the quality of life, managing an academic department, game theory and many others.

A fundamental characteristic of decision process is the development of logical bases for eliminating further considerations of large number of otherwise possible decisions, with assurance that the most desirable decision is not inadvertently lost. The reduced field of possibilities can then be more easily analyzed by a decision maker in order to arrive at a final decision.

If two or more objectives are not commensurable, then there is generally no single optimum decision. Despite this assertion, decisions which involve apparently non-commensurable objectives are reached every day by millions of people. A substantial percentage of these individuals are quite sure they made the best decision - best in the sense that no other could be demonstrated as superior. Thus it would appear that the problem is one of finding the means of reducing non-commensurable objectives to an appropriate common denominator.

Much of pricing theory in economics is devoted to this question. Physically noncommensurable quantities which are traded in large numbers in a "free", non-coercive market appear to have been rather well commensurable in monetary units. This has encouraged the development of strategies to create, by law, the institutional equivalent of a market for the remaining non-commensurable objectives of water resources. Pollution certificates, effluent charges, scarcity based pricing, etc. are examples of this approach. While very attractive in some respects, it is clear from other institutionally managed markets that these economic artifices in many ways may not be adequate to create pseudo-market conditions which would in fact represent even the important objectives to any satisfactory degree.

The reluctance of the political system to adopt such pseudo-market institutions, and the acceptance of direct political allocation suggests that for the immediate future, at least, it will be necessary to seek other alternatives for treating the non-commensurable objective problem.

Thus the development of mathematical techniques for the solution of multiple objective problem is quite important.

Water resources projects are generally constructed to serve multi-objectives. This fact is inherent in the nature of almost any large-scale project, e.g., reservoirs, dams, aqueducts, the development of ground water systems, and so on. A large reservoir created by a high dam may supply water for irrigation, municipal and industrial needs, provide for fishing and recreation facilities, improve navigation and flood control capabilities, generate hydroelectric power, maintain suitable water quality for both ground and surface water, provide a buffer for drought years and ground water recharge, improve related land use and prevent damages from run-off, and enhence the regional development in terms of a better economy and quality of life. In regional planning of water and related land resources, the simultaneous consideration of more than one project is often essential due to the interactions and coupling that exist among them. Clearly, multiobjective planning becomes truly large scale and complex.

Proposed methods: The first method to be introduced in this thesis for nonlinear multiobjective programming problems is "surrogate worth trade-off method." This method is one of the most powerful methods for nonlinear multiobjective water resources problems. However, it becomes complex and trivial for multi-variable and high dimension problems when applying lagrange multipliers to obtain the trade-off ratios of non-dominated solutions. This difficulty can be avoided by directly applying GRG to obtain the trade-off ratios of non-dominated solutions rather than using lagrange multipliers.

The second method to be used in this study is goal programming. In spite of the difficulty of deciding the goals and ordinal number of each objectives, this method has been successfully applied to solve multi-objective linear/nonlinear programming problems.

The third method to be used in this study is SEMOPS, it is also one of the best methods among all the techniques. The fourth method is STEM method, and the fifth method is the method of Geoffrion.

The Generalized Reduced Gradient Technique (GRG) will be used to handle the nonlinear programming problems for those methods included in this study. Furthermore, this technique can be used to solve large scale non-linear programming problems without dimensionality difficulty.

The proposed methods are applied to the Bow River Basin Problem in which water qualities are to be improved. The Reidy-Vermuri model deal with the problem in which dam height is to be decided.

An IBM 360/50 computer with a Fortran complier is used in the computations.

CHAPTER 2

LITERATURE SURVEY

In this chapter, a literature survey is made on the application of multiobjective programming method to water resources systems. Those references having direct bearing on the subject are considered. Mathematical derivations proofs on detailed explanations are excluded from this chapter.

This survey includes articles that have been published on or before July 1978. In reviewing the literature, it is found convenient to classify the field into the following categories:

1. Goal Programming Method
2. Surrogate Worth Trade-off Method
3. STEM Method
4. SEMOPS Method
5. Method of Geoffrion

Goal Programming

Because of the complex, incommensurable nature between conflict goals of water resources systems, various mathematical have been proposed to obtain optimal or even satisfied solutions. Goal programming is one of the most practical and wide-spread used techniques on solving the large-scale real world water resources problems. It requires both cardinal and ordinal information to use goal programming methods. The goal programming was originated by Charms and Cooper [3] for a linear model. Further developments were furnished by Ijiri [20], Lee [21-24], and Ignizio [19], among others. Ignizio proposed some special computer codings for linear models. It is basically a modified simplex algorithm for linear programming problems. However, as the Model size becomes larger and to some extent, the modified simplex algorithm approach is trivial and time consuming. Dauer and Krueger presented iterative goal programming method to solve this problem easily. Hwang [15-18] solved the problem by iteratively using single objective nonlinear optimization technique. Griffith and Steward [9] solved the nonlinear goal programming problems by linear goal programming using linear approximation method. It should be noted that goal programming method requires goals for the objectives be set by the decision maker and achievement functions be minimized in the order they are formed. For goal programming method, the decision maker needs not to give the numerical weights for the objects. The only thing needed to be done is to give an ordinal rank of these objectives.

It is found that iterative nonlinear goal programming is a powerful procedure for solving a set of minimization problems sequentially in one attempt. The reason is that there exist more than one solution which satisfy the constraints at each priority level. And as the number of priority levels of the achievement functions increases, the solutions become fewer and fewer.

Surrogate Worth Trade-off Method

Haimes, Hall and Freedman [13] presented the surrogate worth trade-off method. The major characteristics and advantages of the surrogate worth trade-off method are as follows:

1. Noncommensurable objective functions can be handled quantitatively.
2. The surrogate worth functions, which relate the decision maker's references to the noninferior solutions through the trade-off functions,

are constructed in the functional space. It is the easiest way to help decision maker to make precise decisions.

3. The decision maker makes decisions based on his subjective preference according to the functional space rather than in the decision space. It is more familiar and meaningful to him since the dimensionality of decision space is much larger than the dimensionality of the functional space.

4. The surrogate worth trade-off method encourages the system modeling and the pattern of thinking in multiobjective function terms.

5. The surrogate worth trade-off method provides an outstanding contribution in the field of higher-level coordination in hierarchical multilevel structures.

Note that there are two steps in performing the SWT method: (1) Calculating the set of non-dominated solutions (2) Searching for a preferred solution from the non-dominated solutions.

STEM (The Step Method)

Benayoun [1,2] developed this method for solving multiobjective linear programming problems. It is one of the most powerful methods for MOLP problems. The decision maker can learn to recognize good solutions and the relative importance of the objectives through STEM.

SEMOPS (Sequential Multiobjective Problem Solving Technique)

SEMOPS was proposed by Monarchi, Kisiel, and Duckstein [27]. It is an interactive programming technique that dynamically involves the decision maker in a search process that attempts to locate a satisfactory course of action, that is, a 'satisfactum'. The concept of a satisfactum expresses the idea that we cannot define precisely a multiobjective optimum because we may not know how to trade off one objective versus another except in a subjective way. SEMOPS allows the decision maker to trade off one objective versus another in an interactive manner. SEMOPS cyclically uses a surrogate objective function based on goals and the decision maker's aspirations toward achieving these goals.

Operationally, SEMOPS is a three-step algorithm involving set up, iteration, and termination. The set up step involves transforming the original problem into the format. The iteration step is the truly interactive segment of the algorithm and involves a cycling between an optimization phase and an evaluation phase until a satisfactum is reached, which terminates the algorithm.

Method of Geoffrion

This method proposed by Geoffrion, Dyer, and Feinberg [8] is an interactive mathematical programming approach to multi-objective optimization programming. The difficulties of the mathematical programming technique to real world problems are the presence of more than two objectives. Under the circumstances, the trade-off ratios are very hard to obtain. However, the decision maker can assess the trade-off between any two objectives on a specific achievement level of the objectives. Hence the improvement of an overall utility function through successive trade-offs can be made by using Geoffrion method.

The method demonstrates that a large step gradient algorithm can be used for solving the vector maximum problems if the decision maker is able to specify an overall utility function defined on the values of the objectives. However, the method does not require this function to be identified explicitly. Instead, it asks only for local informations to be used to perform the computation.

Geoffrion et. al. proposed the approach in the context of the Frank-

Wolfe algorithm because of its simplicity, its robust convergence properties, and because it is embodied in the computer program used to obtain the numerical results.

The greatest advantage of this method is that decision makers can provide the required information without significant difficulty. Based on the favorable responses and suggestions of the decision maker, we may refined the model. The refine model can then be used for further approach. Geoffrion suggested further research based on this method such as the development of reliable experimental procedures for estimating trade-offs (marginal rates of substitution) between objectives, and the comparative study of initial rates of convergence for various mathematical programming algorithms with potential for interactive implementation.

CHAPTER 3

ALGORITHMS OF THE PROPOSED METHODS

A taxonomy of the MODM was developed by Hwang et. al. [18] shown in Fig. 3.1. The classification has been made in three steps: Step 1: the stage at which the preference information is needed; Step II: the type of information needed; Step 3: the major methods in any branch formed from Step I and II. All these methods have the common characteristics that they possess: (1) a set of quantifiable objectives; (2) a set of well defined constraints; (3) a process of obtaining some trade-off information, implicit or explicit, between the stated quantifiable objectives and also between stated or unstated nonquantifiable objectives. Thus MODM is associated with design problems (in contrast to selection problems for the MADM).

Among all these MODM methods, we selected five methods because of its powerful and nonlinear-handled functions. They are: Nonlinear Goal Programming; Surrogate Worth Trade Off Method (SWT); STEM; SEMOPS; and the Method of Geoffrion. Algorithms for these methods will be carried out accordingly.

3.1 Nonlinear Goal Programming

In the NLGP approach, the problem is decomposed into a set of single objective nonlinear optimization problems and solved iteratively. Hwang et. al. [16] introduced the following algorithm:

Algorithm of an iterative nonlinear goal programming problem

To find $\underline{x} = (x_1, x_2, \ldots, x_n)$ so as to

	I. Stage at which information is needed	II. Type of information	III. Major classes of methods
Multiple Objective Decision Making	1. No Articulation of Preference Information		1.1.1. Global Criterion Method
	2. A Priori Articulation of Preference Information	2.1. Cardinal Information	2.1.1. Utility Function 2.1.2. Bounded Objective Method
		2.2. Ordinal and Cardinal Information	2.2.1. Lexicographic Method 2.2.2. Goal Programming 2.2.3. Goal Attainment Method
	3. Progressive Articulation of Preference Information (Interactive Methods)	3.1. Explicit Trade-off	3.1.1. Method of Geoffrion and Interactive Gaol Programming 3.1.2. Surrogate Worth Trade-off Method 3.1.3. Method of Satisfactory Goals 3.1.4. Method of Zionts-Wallenius
		3.2. Implicit Trade-off	3.2.1. STEM and Related Methods 3.2.2. SEMOPS and SIGMOP Methods 3.2.3. Method of Displaced Ideal 3.2.4. GPSTEM Method 3.2.5. Method of Steuer (Interactive MOLP Method)
	4. A Posteriori Articulation of Preference Information (Nondominated Solutions Generation Method)	4.1. Implicit Trade-off	4.1.1. Parametric Method 4.1.2. ε-constraint Method 4.1.3. MOLP Methods 4.1.4. Adaptive Search Method

Fig. 3.1 A taxonomy of methods for multiple objective decision making.

$$\min \quad \underline{a} = \{a_1(\underline{d}^-,\underline{d}^+),\ a_2(\underline{d}^-,\underline{d}^+),\ \ldots,\ a_\ell(\underline{d}^-,\underline{d}^+)\}$$

$$\text{subject to} \quad g_i(\underline{x}) + d_i^- - d_i^+ = c_i, \ i = 1, \ldots, m$$

$$f_i(\underline{x}) + d_{m+i}^- - d_{m+i}^+ = b_i, \quad i = 1, \ldots, k$$

$$\underline{d}^-, \underline{d}^+ > 0, \quad d_i^- \cdot d_i^+ = 0 \qquad V_i$$

Each achievement function, $a_j(\underline{d}^-, \underline{d}^+)$, is a linear function of the appropriate deviational variables. Each deviational variable is determined "independently" from the corresponding constraint equation as follows:

$$d_i^- = \begin{cases} d_i^- & \text{if } d_i^- \geq 0 \\ 0 & \text{if } d_i^- \leq 0 \end{cases}$$

where $\quad d_i^- = C_i - g_i(\underline{x})$

or $\quad d_i^- = b_i - f_i(\underline{x})$

similarly

$$d_i^+ = \begin{cases} d_i^+ & \text{if } d_i^+ \geq 0 \\ 0 & \text{if } d_i^+ < 0 \end{cases}$$

where $\quad d_i^+ = g_i(\underline{x}) - C_i$

or $\quad d_i^+ = f_i(\underline{x}) - b_i$

By an iterative approach, the GP model can be decomposed into ℓ number of single objective problems ($\ell \leq k + 1$) as follows:

Problem 1: To find $\underline{x}$ - $(x_1, x_2, \ldots, x_n)$ so as to

$$\min \quad a_1(\underline{d}^-, \underline{d}^+)$$

$$\text{s.t.} \quad z_i(\underline{x}) + d_i^- - d_i^+ = C_i, \qquad i = 1, 2, \ldots, m$$

$$\underline{d}^-, \underline{d}^+ \geq 0 \quad \text{and} \quad d_i^- \cdot d_i^+ = 0 \quad \forall_i$$

Let a_1^* be the optimal solution for Problem 1, i.e., $a_1^* = \min a_1(\underline{d}^-, \underline{d}^+)$. a_1^* is usually zero, since the absolute constraints must be satisfied.

Problem 2. To find $\underline{x}$ so as to

$$\min \quad a_2(\underline{d}^-, \underline{d}^+)$$

$$\text{s.t.} \quad g_i(\underline{x}) + d_i^- - d_i^+ = C_i, \qquad i = 1, 2, \ldots, m$$

$$a_1(\underline{d}^-, \underline{d}^+) \leq a_1^*$$

$$f_1(\underline{x}) + d_{m+1}^- - d_{m+1}^+ = b_1$$

$$\underline{d}^-, \underline{d}^+ \geq 0 \quad \text{and} \quad d_i^+ \cdot d_i^- = 0 \quad \forall_i$$

Notice that the first two constraints imply that in trying to achieve goal 1, we will not sacrifice our previously determined attainment of Problem 1.

Let a_2^* be the solution to this problem. We can then proceed to goal 2.

Problem 3: To find $\underline{x}$ so as to

$$\min \quad a_3(\underline{d}^-, \underline{d}^+)$$

$$\text{s.t.} \quad g_i(\underline{d}^-, \underline{d}^+) \leq a_1^*$$

$$f_1(\underline{x}) + d^-_{m+1} - d^+_{m+1} = b_1$$

$$a_2(\underline{d}^-, \underline{d}^+) \leq a^*_2$$

$$f_2(\underline{x}) + d^-_{m+2} - d^+_{m+2} = b_2$$

$$\underline{d}^-, \underline{d}^+ \geq \underline{0} \quad \text{and} \quad d^+_i \cdot d^-_i = 0 \qquad \forall_i$$

Let a^*_3 be the solution for Problem 3.

We can now write a general goal attainment problem (j+1) for attaining goal j, $0 \leq j \leq \ell - 1$ as follows:

Problem (j+1) = To find $\underline{x}$ so as to

$$\min \quad a_{j+1}(\underline{d}^-, \underline{d}^+)$$

$$\text{s.t.} \quad g_i(\underline{x}) + d^-_i - d^+_i = C_i, \quad i = 1, 2, \ldots, m$$

$$a_i(\underline{d}^-, \underline{d}^+) \leq a^*_i, \qquad i = 1, 2, \ldots, j$$

$$f_i(\underline{x}) + d^-_{m+i} - d^+_{m+i} = b_i, \quad i = 1, 2, \ldots, j$$

$$\underline{d}^-, \underline{d}^+ \geq 0 \qquad \text{and } d^+_i \cdot d^-_i = 0 \quad \forall_i$$

Computational procedures of the iterative nonlinear goal programming

The proceeding "ℓ" simple objective decision making problems can be solved by any proper nonlinear programming method. Also, GRG will be used here as a tool to aporoach the compromised solution of NLGP problems.

3.2. Surrogate Worth Trade Off Method

There are five steps in the SWT method

Step 1. Set up the multiple objective problem in the form:

$$\text{Max } [f_1(\underline{x}),\ f_2(\underline{x}),\ \ldots,\ f_k(\underline{x})]$$

$$\text{s.t. } g_i(\underline{x}) \leq 0, \quad i = 1, 2, \ldots, m$$

Determine the ideal solution for each of the objective in the problem. Select a primary objective $[f_\ell]$ arbitrarily.

Step 2. Identify and generate a set of nondominated solutions by varying ε's parametrically in the problem as follows:

$$\max f_1(\underline{x})$$

$$\text{s.t. } f_j(\underline{x}) \geq \varepsilon_j, \quad j = 2, 3, \ldots, k$$

$$g_i(\underline{x}) \leq 0, \quad i = 1, 2, \ldots, m$$

$$\text{where } \varepsilon_j = \bar{f}_j - \bar{\varepsilon}, \quad j = 2, 3, \ldots, k$$

$$\bar{\varepsilon}_j > 0, \quad j = 2, 3, \ldots, k$$

$\bar{f}_j$ are the feasible ideal solutions of each single objective problems.

The nondominated solutions are the ones which have non-zero values for the trade-off functions, $\lambda_{\ell j} = -\partial f_\ell / \partial f_j$.

Step 3. Interact with the DM to assess the surrogate worth function $W_{\ell j}$, provided that the values of $\underline{f}$ and $\lambda_{\ell j}$ are presented to the DM.

Step 4. Isolate the indifference solutions. The solutions, which have $W_{\ell j} = 0$ for all j, are said to be indifference solutions. Any one of such solutions is a preferred solution to the DM and hence the optimal solution to the multiple objective problem. However, if there exists no indifference solution, develop approximate relations for all worth functions $W_{\ell j} = \hat{W}_{\ell j}(f_j, V_j, j \neq \ell)$. Solve the simultaneous equations $\hat{W}_{\ell j}(\underline{f}) = 0$ for all j to obtain $\underline{f}^*$ ($\underline{f}^*$ does not contain f_ℓ^*). This would possibly form an indifference solution. Solve the problem.

$$\max \quad f_1(\underline{x})$$

$$\text{s.t.} \quad f_j(\underline{x}) \geq f_j^*(\underline{x}), \qquad j = 2,3, \ldots, k$$

$$g_i(\underline{x}) \leq 0 \quad , \qquad i = 1,2, \ldots, m$$

with $\underline{f}^*$ for ε's to obtain f_ℓ^*, $\underline{X}^*$ and $\lambda_{\ell j}^*$ for all j. Present this solution to the DM, and ask if this is an indifference solution. If yes, it is a preferred solution; proceed to Step 5. Otherwise, repeat the process to generate more non-dominated solutions around $\hat{W}_{\ell j} = 0$ and refine the estimated $\underline{f}^*$ until it results in an indifference solution.

Step 5. If the solution in the decision space $\underline{x}$ corresponding to the preferred solution of Step 4 is not obtained already, solve problem (Step 4) with $\underline{\varepsilon} = \underline{f}^*$ (note that $\underline{f}^*$ does not contain f_ℓ^*). The optimal solution f_ℓ^* along with $\underline{f}^*$ and $\underline{x}^*$ would be the optimal solution to the multiple objective problem.

3.3. STEM METHOD

Step 1. Calculation phase:

At the mth cycle, the feasible solution to the following LP problem

$$\max \quad f_j(\underline{x}) = \underline{C}_j^T \underline{x}$$

$$\text{s.t.} \quad A\,\underline{x} \leq b, \quad \underline{x} \geq 0$$

$$j = 1,2, \ldots, k$$

is sought which is the nearest, in the MINIMAX sense, to the ideal solution f_j^*:

$$\min_{\{\underline{x},\lambda\}} \quad \lambda$$

$$\text{s.t.} \quad \lambda \geq \{f_j^* - f_j(\underline{x})\} \cdot \Pi_j, \quad j = 1,2, \ldots, k$$

$$\underline{x} \in X^m$$

$$\lambda \geq 0$$

where X^m includes $A\,\underline{x} \leq b$, $\underline{x} \geq 0$ plus any constraint added in the previous (m-1) cycles; Π_j give the relative importance of the distance to the optima, but they are only locally effective.

Let f_j^* be the maximum value, f_j^{min} be the minimum value then

$$\Pi_j = \frac{\alpha_j}{\sum_i \alpha_i}$$

$$\text{where} \quad \alpha_j = \frac{f_j^* - f_j^{min}}{f_j^*} \left(\frac{1}{\sum_{i-1}^{n} (C_{ji})^2} \right) \quad \text{if} \quad f_j^* > 0$$

$$\alpha_j = \frac{f_j^{min} - f_j^*}{f_j^{min}} \left(\frac{1}{\sum_{i=1}^{n} (C_{ji})^2} \right) \quad \text{if} \quad f_j^* \geq 0$$

Step 2. Decision phase:

If the DM is satisfied with some objectives and not satisfied with the others, then he must relax a satisfctory objective f_j^m enough to allow an improvement of the unsatisfactory objectives in the next iterative cycle. The DM gives Δf_j as the amount of acceptable relaxation. Then, for the next cycle the feasible region is modified as

$$X^{m+1} = \begin{cases} X^m \\ f_j \geq f_j(x^m) - \Delta f_j \\ f_i(\underline{x}) \geq f_i(\underline{x}^m); \quad i \neq j, \quad i = 1,2, \ldots, k \end{cases}$$

3.4. SEMOPS METHOD

Let $\underline{AL} = (AL_1, AL_2, \ldots, AL_T)$ be the DM's aspiration levels, and $\underline{f}(\underline{x}) = (f_1(\underline{x}), f_2(\underline{x}), \ldots, f_T(\underline{x}))$ be the multiple objective functions. A relevant range of $\underline{f}(\underline{x})$ for each objective is choosen as $[f_{jL}, f_{jU}]$. By transforming the original response surface $f_i(\underline{x})$ to $y_i(\underline{x})$, we have:

$$y_i(\underline{x}) = \frac{f_i(\underline{x}) - f_{iL}}{f_{iU} - f_{iL}} + \varepsilon$$

where ε is a small positive value added to circumvent the division by zero while defining the dimensionless indicator of attainment.

The DM's aspiration levels $\underline{AL}$ is transformed into $\underline{A}$ by the same procedure as follows:

$$A_i = \frac{AL_i - f_{jL}}{f_{jU} - f_{jL}}$$

$\underline{d}$, the corresponding dimensionless indicator of attainment, are

(1) at most

$$f_i(\underline{x}) \leq AL_i; \quad d_i = \frac{f_i(\underline{x})}{AL_i} = \frac{y_i(\underline{x})}{A_i} \quad .$$

(2) at least

$$f_i(\underline{x}) \geq AL_i; \; d_i = \frac{AL_i}{f_i(\underline{x})} = \frac{A_i}{y_i(\underline{x})}$$

Operationally, SEMOPS is a three-step algorithm:

(1) Set up: Involves transforming the original problem into a principal problem and a set of auxiliary problems with surrogate objective functions.

(2) Iteration:

The principal problem

$$\min \quad S_1 = \sum_{t=1}^{T} dt$$

$$\text{s.t.} \quad \underline{x} \in X$$

The set of auxiliary problems, $\ell = 1,2, \ldots, T =$

$$\min \quad s_{1\ell} \sum_{\substack{t=1 \\ t \neq \ell}}^{T} dt$$

$$\text{s.t.} \quad \underline{x} \in X$$

$$f_\ell(\underline{x}) \geq AL_\ell$$

The resulting vector will be sent to the DM for use in the evaluation phase. The impact of an action on the attainment of the other objective is assessed, and a new aspiration level for an objective is set.

(3) Termination.

3.5 Method of Geoffrion

Geoffrion et. al. [7,8] developed this method. It consists of the following steps:

Step 0. Choose an initial point $\underline{x}^i \in X$. Set $i = 1$

Step 1. Determine an optimal solution $\underline{y}^i$ of the direction - finding problem:

$$\max \quad \nabla_{\underline{x}} U \; (f_1(\underline{x}^i), \; f_2(\underline{x}^i), \; \ldots, \; f_k(\underline{x}^i)) \cdot \underline{y}^i$$

$$\underline{y} \in X$$

Set $\underline{z}^i = \underline{y}^i - \underline{x}^i$

Step 2. Determine an optimal t^i of the step-size problem:

$$\max U \left(f_1(\underline{x}^i + t^i\underline{z}^i), \ldots, f_k(\underline{x}^i + t^i\underline{z}^i)\right)$$

$$0 \leq t^i \leq 1$$

Set $\underline{x}^{i+1} = \underline{x}^i + t^i\underline{z}^i$, $i = i + 1$, and return to Step 1.

The theoretical termination criterion is satisfied if the solution $\underline{x}^i$ and $\underline{x}^{i+1}$ are equal.

CHAPTER 4

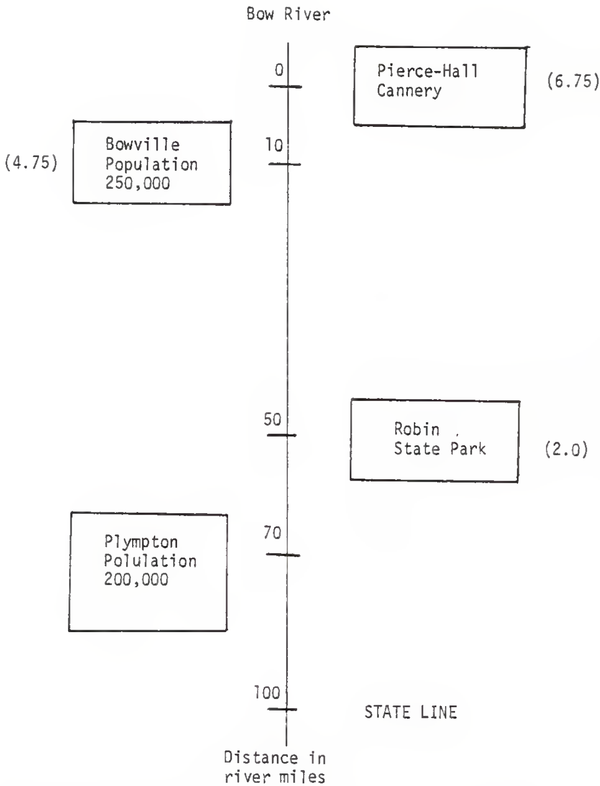
BOW RIVER BASIN

4.1 DESCRIPTION OF THE SYSTEM

To illustrate the use of the proposed computational methods, the Bow River Basin is chosen for demonstration. It is a hypothetical case developed by Dorfman and Jacoby [5] and modified by Monarchi et. al. [27]. It centers around the pollution problems of an artificial river basin, the Bow River Valley, whose main features are shown in Fig. 4.1. Industrial pollution is represented by the Pierce-Hall Cannery, located near the head of the valley, and by two sources of municipal waste at Bowville and Plympton. A state park is located between the cities, and the lower end of the valley is part of the state boundary line.

The specification of water quality has been reduced to a single dimension: dissolved oxygen concentration (DO), whose elements are in part of the following: floating solids, color, turbidity, coliform bacteria, taste and odor, temperature, pH, and radioactivity. The waste content of the municipal and industrial effluents is assumed to be described by the number of pounds of biochemical oxygen demanding material (BOD) that they carry. BDO is separated into carbonaceous (BOD_c) and nitrogenous (BOD_n) material.

The cannery and the two cities already have primary treatment facilities which reduce BOD by 30% of their gross discharge. To reduce waste further, additional treatment facilities would have to be installed. The costs of the additional treatment facilities will reduce the return of investment from the cannery and increase the tax rate in Bowville and Plympton.



(Values in parentheses at the sides are current DO levels in milligram per liter)

Fig. 4.1 Main features of the Bow River Valley

Streeter and Phelps [33] have proposed a model in which a stream reduces the DO level at a rate proportional to the concentration of waste in the stream. Although more involved models of oxygen dynamics can be used, the Streeter-Phelps model is adequate for this case. If the DO level is 0, anaerobic decomposition takes place. As the DO level falls below the saturation level g_s, additional oxygen is absorbed into the water from the atmosphere. The two opposing processes, deoxygenation and reoxygenation, determine the actual DO level.

The proportion of carbonaceous substances to nitrogenous substances affects the decomposition process because the oxidation of the nitrogenous material begins some time after that of the carbonaceous. The effects of the two water wastes are assumed to be additive, so that the impact of the nitrogenous component can be approximated as a dummy waste source downstream. The downstream distance used in this case is 20 miles.

The effects of the three different waste sources are also additive, so that the changes in the DO level for any point on the river are the sum of changes caused by variations in waste reduction at each source.

In the Street-Phelps model, the impact of a change in waste load at point i as measured at point j downstream is given by

$$d_{ij} = - \alpha \frac{K_1}{F(K_2-K_1)} \left(10^{-K_1 m/v} - 10^{-K_2 m/v}\right)$$

where

- d_{ij}, transfer coefficient (mg/L)/(LB/day) ;
- α, dimensionality constant, equal to 10^6 mg/L;
- F, flow rate of the stream, LB/day (800 ft^3/sec);
- m, distance between points i and j, miles;
- v, velocity of the stream, mi/day (0.5 ft/sec);
- K_1, K_2, constants characteristic of the stream,

(K_1 = 0.2/day for BOO_c, K_1 = 0.3/day for BOD_n,

K_2 = 0.4/day for both)

For a derivation see Fair et al. [Ch 6]. The transfer coefficients for the points of interest are given datum.

On the basis of our additivity assumptions the water quality g_i at point j is calculated from

$$q_j = - \sum_{V_i} [d^c_{ij} L^c_i (x_i - 0.3)$$

$$+ d^n_{ij} L^n_i (W_i - 0.3)] + \bar{q}_j$$

where

d^c_{ij}, carbonaceous transfer coefficient between point i and j;

d^n_{ij}, nitrogenous transfer coefficient between points i and j

L^c_i, carbonaceous BOO load for source i;

L^n_i, nitrogenous BOD load for source i;

x_i, proportionate reduction in L^c_i;

W_i, proportionate reduction in L^n_i;

$\bar{q}_j$, current DO level at point j;

i,j, points (point i = 1, 2, or 3 represents Pierce-Hall cannery, Bowville, or Plympton, respectively, and point j = 1,2,3, or 4 represents Bowville, Robin State Park, Plympton, or the state line).

The cannery and the two cities have primary treatment facilities in place that reduce both the BOD_c and BOD_n by 30% of their gross untreated values. To reduce waste further, additional treatment facilities would have to be installed. A consulting firm has developed the figures for various specific alternatives. The respective gross additional annual costs in thousands of dollars are:

for Pierce-Hall Cannery

$$C_1 = [50/(1.09 - x_1^2)] - 59 \qquad 1000\$/year$$

for Bowville

$$C_2 = [532/(1.09 - x_2^2)] - 532 \qquad 1000\$/year$$

for Plympton

$$C_3 = [450/(1.09 - x_3^2)] - 450 \qquad 1000\$/year$$

where x is the corresponding proportional reduction in $BOD(L_i^c)$.

However, there are mitigating factors that reduce the gross cost for each institution. The federal corporation tax reduces the Cannery's cost by 40%. The gross costs to Bowville and Plympton are reduced because the Federal Water Pollution Control Act provides a grant to municipolities that covers 50% of the construction costs. These costs are about one-half the total costs, so that each city pays about 75% of the total cost.

The cannery's average net profit has been \$375,000 per year, a return of 7.5% on the stockholders' equity of \$5 million. We assume that sales will remain stable over the foreseeable future and that the cannery is technologically bound to the production costs it now incurs. In addition, we assume that the firm is unable to raise its prices because of the pressures of competition. Consequently, any increase in treatment costs will reduce net profits. The relationship between costs C and the percent of return on investment r is

$$r = \frac{100}{5,000,000}(374,000 - 0.6\ C_1)$$

Additional waste treatment costs at Bowville will affect the city's tax rate. On the basis of the consultants' report the city planning division has developed a relationship between the change in the tax rate Δt and costs C:

$$\Delta t_2 = (2.4 \times 10^{-3})\ (0.75\ C_2)$$

where Δt is the change per thousand dollars assessed valuation.

Plympton is smaller than Bowville and somewhat less affluent with a lower value of taxable property per capita, so that, although treatment costs are lower in Plympton, the effect of the additional costs is accentuated. The relationship between cost and the change in the tax rate is given by

$$\Delta t_3 = (3.33 \times 10^{-3})(0.75\ C_3)$$

Plympton has no recreational facilities of its own and is completely dependent on the facilities of Robin State Park. Consequently, Plympton must bear its share of cost of cleaning up the river. In addition, the city is more dependent than Bowville on tourism for revenues and for this reason would like to have Robin State Park improved. Finally, maintenance of an adequate DO level at the state line is principally Plympton's responsibility.

The Bow Valley Water Pollution Control Commission is made up of representatives from all three waste dischargers together with members of the state and federal government. It is responsible for setting waste reduction requirements for the entire valley, but it must act with awareness of the effect of any additional effluent treatment costs on the economic health of the valley. We view this group as a composite individual who represents all his constituents.

We will refer to the abstract governmental decision-making body as "Decision maker" (OM). The first problem of OM is to determine a policy vector x satisfying the constraint that the 00 level at the state line greater than 3.5 mg/L. The components of $x(x_1, x_2, x_3)$ are the proportional reduction in gross BOD_c to be imposed on Pierce-Hall Cannery, Bowville, and Plympton, respectively. We assume that the relationship between BOO_c and BOO_n is defined by

$$W = 0.39/(1.39 - x^2)$$

where x is the proportionate amount of gross BOD_c removed and W is the corresponding value for BOD_n. Thus only one set of waste reduction requirements must be specified. Initially, x = 0.3.

Then the OM is checking the six goals: the 00 levels at Bowville, Robin State Park, and Plympton. (goals 1,2, and 3 respectively); the percentage of return on investment at the Pierce-Hall cannery (goal 4); and the addition to the tax rate for Bowville and Plympton (goal 5 and 6, respectively).

4.2 MODELING

Objectives and constraints:

Objective 1 = DO level at Bowville

$$f_1 = q_1 = \bar{q}_1 - [d^c_{12}L^c_1(x_1 - 0.3) + d^n_{12}L^n_1(W_1 - 0.3)]$$

$$= 5.0 + (5.68 \cdot 10^{-5}) \cdot (4.0 \cdot 10^4) \cdot (x_1 - 0.3)$$

Objective 2: OO level at Robin State Park

$$f_2 = q_2 = 2.5 + [(1.31 \cdot 10^{-5}) \cdot (4.0 \cdot 10^4) \cdot (x_1 - 0.3) + (3.15 \cdot 10^{-5}) \cdot (2.8 \cdot 10^4) \cdot (W_1 - 0.3)]$$

$$+ [(2.18 \cdot 10^{-5}) \cdot (1.28 \cdot 10^5) \cdot (x_2 - 0.3) + (5.53 \cdot 10^{-5}) \cdot (4.8 \cdot 10^4) \cdot (W_2 - 0.3)]$$

Objective 3 = OO level at Polympton

$$f_3 = q_3 = 5.3 + [(0.442 \times 10^{-5})(4.0 \times 10^4)(x_1 - 0.3) + (0.764 \times 10^{-5})(1.28 \times 10^5)(x_2 - 0.3) + (0.771 \times 10^{-5})(2.8 \times 10^4)(w_1 - 0.3) + (1.60 \times 10^{-5})(4.8 \times 10^4)(w_2 - 0.3)]$$

Objective 4: Percentage of return on equity at Pierce-Hall Cannery

$$f_4 = r = \frac{10^2}{5.10^6}[(3.75 \times 10^5) - 0.6 \cdot (\frac{59}{1.09 - x_1^2} - 59) \times 10^3]$$

Objective 5: Addition to the tax rate at Bowville

$$f_5 = \Delta t_2 = (2.4 \times 10^{-3})(\frac{532}{1.09 - x_2^2} - 532)(0.75)$$

Objective 6: Addition to the tax rate at Plympton

$$f_6 = \Delta t_3 = (3.33 \times 10^{-3})\left(\frac{450}{1.09 - x^2} - 450\right)(0.75)$$

Constraints: DO level at the state line

$$g_1 = 1.0 + [(8.3 \times 10^{-7})(4.0 \times 10^4)(x_1 - 0.3) + (7.3 \times 10^{-7})$$

$$- (2.8 \times 10^4)(W_1 - 0.3)]$$

$$+ [(1.45 \times 10^{-6}) \cdot (1.28 \times 10^5)(x_2 - 0.3)$$

$$+ (1.62 \times 10^{-6})(4.8 \times 10^4)(W_2 - 0.3)]$$

$$+ [(3.49 \times 10^{-5})(9.57 \times 10^4)(x_3 - 0.3)$$

$$+ (7.33 \times 10^{-3})(3.57 \times 10^4)(W_3 - 0.3)] \geq 3.5$$

Bounds: Proportionate reduction in gross BOD_c

$$0.3 < x_i < 1.0 \qquad i = 1,2,3$$

$$W_i = \frac{0.39}{1.39 - x_i} \qquad i = 1,2,3$$

4.3 SEMOPS method

The DM's aspiration levels for the six goals are

$$\underline{AL} = (AL_1, AL_2, \ldots, AL_6)$$

$$= (6mg/\ell,\ 6mg/\ell,\ 6mg/\ell,\ 0.065,\ 1.5,\ 1.5)$$

The relevant range for each objective is:

$$0 \leq f_i \leq 8.5, \quad \text{i.e., } f_{i\ell} = 0, \quad f_{iu} = 8.5, \ i = 1,2,3$$

$$0 \leq f_4 \leq 7.5$$

$$0 \leq f_5 \leq 10$$

$$0 \leq f_6 \leq 12$$

The six goals are:

$$f_i \geq 6.0, \qquad i = 1,2,3$$

$$f_4 \geq 6.5,$$

$$f_5 \leq 1.5,$$

$$f_6 \leq 1.5$$

The initial aspiration levels of the DM are the same as the goals, they are:

$$AL_i = 6.0 \qquad i = 1,2,3$$

$$AL_4 = 6.5$$

$$AL_5 = 1.5$$

$$AL_6 = 1.5$$

The objectives and the corresponding dimensionless indicator of attainment, $\underline{d}$, are:

$$f_i \geq AL_i; \; d_i = \frac{AL_i}{f_i(\underline{x})}, \qquad i = 1,2,3,4$$

$$f_i \leq AL_i; \; d_i = \frac{f_i(\underline{x})}{AL_i}, \qquad i = 5,6$$

The first cycle in the solution of the case study consists of a principal problem and six auxiliary problems. The principle problem is:

$$\min \quad s_{1k} = \sum_{\substack{t=1 \\ t \neq k}} d_t$$

$$\text{s.t.} \quad \underline{x} \in X$$

$$f_k(\underline{x}) \geq AL_k \qquad (\text{if } k = 1,2,3, \text{ or } 4)$$

of

$$f_k(\underline{x}) \leq AL_k \qquad (\text{if } \ell = 5 \text{ or } 6)$$

This is a nonlinear programming problem. Generalized Reduced Gradient (GRG) Technique was used to obtain the results of these optimizations. The results are shown in Table 4.1.

In the examination of these numbers, it is apparent that the change in the tax rate at Plympton (objective 6) is relatively independent of attainment or nonattainment of the other objectives because the necessary reduction in BOO_c at Plympton is heavily influenced by the DO constraint at the state line. It seems reasonable to choose an aspiration level for that objective and enter it as a constraint.

Using Table 4, DM can assess the impact of such action on the attainment of the other objectives. This assessment is made as follows. The results of the principal problem and the sixth auxiliary problem are:

Objective	Principal Problem	Auxiliary Problem
1	5.95	6.10
2	4.02	5.87
3	5.73	5.40
4	6.40	5.68
5	0.62	2.55
6	1.58	1.50

Table 4.1 Results of the first cycle for the problem, where $\underline{AL}$ = (6.0,6.0,6.0,6.5,1.5,1.5)

Kind of problem	$s'=\sum_{t \in T, d_t}$	$\underline{x}$	$\underline{f}$
Principal	$s'=d_1+d_2+d_3+d_4+d_5+d_6=6.10$	x_1 =(0.831,0.690,0.800)	f_1 =(6.02,4.02,5.73,6.40,0.62,1.58)
Auxiliary problem 1	$s_1'=d_2+d_3+d_4+d_5+d_6$ =5.04	$x_{1.1}$ =(0.845,0.670,0.800)	$f_{1.1}$ =(<u>6.01</u>,4.04,5.75,6.25,0.60,1.58)
Auxiliary problem 2	$s_2'=d_1+d_3+d_4+d_5+d_6$ =7.35	$x_{1.2}$ =(0.942,0.946,0.791)	$f_{1.2}$ =(6.03,<u>6.05</u>,6.31,4.25,4.24,1.43)
Auxiliary problem 3	$s_3'=d_1+d_2+d_4+d_5+d_6$ =5.62	$x_{1.3}$ =(0.875,0.826,0.801)	$f_{1.3}$ =(5.99,4.71,<u>6.00</u>,6.03,1.94,1.49)
Auxiliary problem 4	$s_4'=d_1+d_2+d_3+d_5+d_6$ =5.06	$x_{1.4}$ =(0.821,0.709,0.800)	$f_{1.4}$ =(5.93,4.00,5.72,<u>6.55</u>,0.65,1.62)
Auxiliary problem 5	$s_5'=d_1+d_2+d_3+d_4+d_6$ =5.51	$x_{1.5}$ =(0.805,0.812,0.824)	$f_{1.5}$ =(5.88,4.53,5.95,6.56,<u>1.50</u>,1.59)
Auxiliary problem 6	$s_6'=d_1+d_2+d_3+d_4+d_5$ =5.24	$x_{1.6}$ =(0.895,0.910,0.810)	$f_{1.6}$ =(6.10,5.87,5.40,5.68,2.55,<u>1.50</u>)

where objectives 1-3 are in milligrams per liter, objective 4 is the percentage of return on investment, and objective 5 is the change in the tax rate in dollars per $1000 of assessed valuation.

DM can try out values of A_6' until he believes that he can accept the estimated effect on the other goals. Where A_6' means the new aspiration level for f_6 on the next cycle. Set b equals to the proportion

$$b = \frac{A_6 - 1.5}{1.58-1.5}$$

Then the approximate effects of adding this constraint are:

Objective	Approximate Effects
1	b . (5.85 - 6.10) + 6.10
2	b · (4.02 - 5.87) + 5.87
3	b · (5.73 - 5.40) + 5.40
4	b · (6.40 - 5.68) + 5.68
5	b · (1.58 - 1.50) + 1.50

where objective 1-3 are in milligrams per liter. Objective 4 is the percentage of return on investment, and objective 5 is the change in the tax rate in dollars per $1000 of assessed valuation.

After trying out the values of A_6', DM sets a new aspiration level of 1.55 for objective 6 and estimates the results as:

Objective	Results
1	6.10
2	4.20
3	5.78
4	6.22
5	0.92

where objectives 1-3 are DO concentrations in milligrams per liter, objective 4 is the percentage of return on investment, and objective 5 is the increase in the tax rate in dollars per $1000 of assessed valuation. The DM argues that the increase to 1.55 is not greatly different from his original objective of 1.50.

Proceeding to the second cycle, we have only five auxiliary problems to solve because one objective has been added to the constraint set for this principal problem. The results are shown in Table 4.2. The DM notes that goal 2 is closely related to objectives 1 and 3 and decides to enter it as a constraint on the third cycle. Auxiliary problem 2 in Table 4.2 shows the effects of satisfying this goal. It is apparent that the addition to the tax rate at Bowville is going to be affected drastically if a DO levle of 6 mg/ℓ is set rather arbitrarily without regard to accepted water quality use standards. The state standards indicate that 5 mg/ℓ is suitable for bathing and recreational. We will make this change to enter the third cycle.

For the third cycle, the principal problem is

$$\min \quad s_{3,62} = \sum_{\substack{t=1 \\ t \neq 2}}^{5} dt$$

$$\text{s.t.} \quad \underline{x} \in X$$

$$f_6(\underline{x}) \leq 1.55$$

$$f_2(\underline{x}) \geq 5.0$$

For the auxiliary problems, k = 1,2,4,5 are

Table 4.2 Results of the second cycle for the problem, where $\underline{AL} = (6.0,6.0,6.0,6.5,1.5,1.55)$

Kind of problem	$s^2 = \sum_{t \epsilon T, dt}$	$\underline{x}$	$\underline{f}$
Principal	$s_6^2 = d_1+d_2+d_3+d_4+d_5 = 5.10$	$x_{2,6} = (0.859,0.810,0.815)$	$f_{2,6} = (6.03,4.32,5.87,6.13,1.02,1.53)$
Auxiliary problem 1	$s_{61}^2 = d_2+d_3+d_4+d_5 = 4.18$	$x_{2,61} = (0.855,0.802,0.814)$	$f_{2,61} = (\underline{6.00},4.28,5.87,6.25,1.17,1.53)$
Auxiliary problem 2	$s_{62}^2 = d_1+d_3+d_4+d_5 = 6.87$	$x_{2,62} = (0.941,0.948,0.816)$	$f_{2,62} = (6.21,\underline{6.05},6.45,6.02,5.08,1.53)$
Auxiliary problem 3	$s_{63}^2 = d_1+d_3+d_4+d_5 = 4.35$	$x_{2,63} = (0.874,0.823,0.867)$	$f_{2,63} = (5.92,4.64,\underline{6.15},5.73,1.32,1.53)$
Auxiliary problem 4	$s_{64}^2 = d_1+d_2+d_3+d_5 = 4.07$	$x_{2,64} = (0.792,0.839,0.798)$	$f_{2,64} = (5.94,4.55,5.93,6.42,\underline{1.30},1.55)$
Auxiliary problem 5	$s_{65}^2 = d_1+d_2+d_3+d_4 = 4.24$	$x_{2,65} = (0.864,0.835,0.816)$	$f_{2,65} = (5.85,4.64,5.96,6.62,1.52,\underline{1.55})$

$$\min \quad s_{3.62k} = \sum_{\substack{t=1 \\ t\neq 2 \\ t\neq k}}^{5} d_t$$

$$\text{s.t.} \quad \underline{x} \in X$$

$$f_6(\underline{x}) \leq 1.55$$

$$f_2(\underline{x}) \geq 5.0$$

$$f_k(\underline{x}) \geq AL_k \text{ (if } k = 1,3,4)$$

$$f_k(\underline{x}) \leq AL_k \text{ (if } k = 5)$$

The vector of the aspiration level is

$$\underline{AL} = (6.0, \underline{5.0}, 6.0, 6.5, 1.5, \underline{1.55})$$

The results of the third cycle are presented in Table 4.3. The continued economic existence of the Pierce-Hall cannery is important to the welfare of the whole valley but particularly to Bowville. If the cannery goes out of business, all the 800 people will be out of the job and create a burden on the city. The DM recognizes this situation and decides that a firm bound on the return on investment for the cannery must be entered as a constraint. This is objective 4. The examination of Table 4.3 enables DM to adopt 6.0% as an acceptable level of return. This constraint is added to the principal problem for tne fourth cycle. DM is informed by the results of auxiliary problem 3 that it informs an inconsistent constraint set (see Table 4.4). After several unsuccessful attempts by the optimization algorithm to find a feasible point with alternate starting points, he concludes that AL_5 will indeed have to be

Table 4.3 Results of the third cycle for the problem, where $\underline{AL}$ = (6.0,6.0,6.0,6.5,1.5,1.55)

Kind of problem	$s^3=\sum_{t\epsilon T,dt}$	$\underline{x}$	$\underline{f}$
Principal	$s^3_{62} = d_1+d_2+d_4+d_5 = 4.24$	$x_{3,62}$ =(0.893,0.812,0.808)	$f_{3,62}$=($\underline{6.04}$,5.06,6.01,4.08,1.65,1.59)
Auxiliary problem 1	$s^3_{621} = d_2+d_4+d_5 = 3.33$	$x_{3,621}$ =(0.885,0.861,0.800)	$f_{3,621}$=($\underline{6.05}$,6.05,6.01,4.07,1.64,1.59)
Auxiliary problem 2	$s^3_{623} = d_1+d_4+d_5 = 3.30$	$x_{3,623}$ =(0.910,0.850,0.800)	$f_{3,623}$=(6.06,$\underline{6.00}$,6.00,4.09,1.57,1.59)
Auxiliary problem 3	$s^3_{624} = d_1+d_3+d_5 = 3.37$	$x_{3,624}$ =(0.802,0.881,0.800)	$f_{3,624}$=(5.90,5.06,$\underline{6.00}$,5.54,1.40,1.59)
Auxiliary problem 4	$s^3_{625} = d_1+d_3+d_4 = 3.44$	$x_{3,625}$ =(0.944,0.830,0.800)	$f_{3,625}$=(6.21,5.06,6.00,4.35,$\underline{1.50}$,1.59)

Table 4.4 Results of the forth cycle for the problem where $\underline{AL}$ = (6.0,5.0,6.0,6.0,1.5,1.55)

Kind of problem	$s^4 = \sum_{t\epsilon T,dt}$	$\underline{x}$	f
Principal	$s^4_{624} = d_1+d_3+d_5 = 3.21$	$x_{4,624}$ =(0.874,0.860,0.818)	$f_{4,624}$ =(6.06,5.00,6.04,5.00,1.65,1.55)
Auxiliary problem 1	$s^4_{6241} = d_3+d_5 = 2.22$	$x_{4,6241}$ =(0.874,0.860,0.818)	$f_{4,6241}$ =(6.04,5.00,6.04,6.00,1.65,1.55)
Auxiliary problem 2	$s^2_{6243} = d_1+d_5 = 2.22$	$x_{4,6243}$ =(0.874,0.866,0.818)	$f_{4,6243}$ =(6.04,5.00,6.05,6.00,1.65,1.55)
Auxiliary problem 3	$s^4_{6245} = d_1+d_3$	Apparent inconsistent constraint set	Apparent inconsistent constraint set

modified, or he will have to revise the previous aspirations that he has developed and satisfied. Fortunately, raising AL_5 is not logically at odds with the development up to this point. Note that Bowville already has an advantage over Plympton in the realm of tax rates. The OM decides to modifiy his aspirations regarding objective 5. As DM examines the values of f_5 (1.65, 1.65, 1.65) on Table 4.4, he decides to raise AL_5 up to 1.65 and enter into the next cycle.

The results of the fifth cycle are shown in Table 4.5. From this information, DM decides that he has reached a satisfactum. DM's policy decision is to impose waste reduction requirements of 85% on the cannery, 87% on Bowville, and 80% on Plympton. The values of objective functions $\underline{f}$ = (6.05, 5.01, 6.01, 1.88, 1.56).

4.4 The Surrogate Worth Trade-off Method

Suppose both objectives corresponding to DO levels at Bowville and Plympton are changed to constraints for $\geq$ 6.0, the tax rate increase at Bowville to be $\leq$ 1.5, then the problems is modified as:

Objective 1: maximization DO level at state park

$$\max \; f_1 = 2.0 + [(1.31 \times 10^{-5})(4.0 \times 10^4)(x_1 - 0.3) + (3.15 \times 10^{-5})(2.8 \times 10^4)(W_1 - 0.3)] + [(2.18 \times 10^{-5})(1.28 \times 10^5)(x_2 - 0.3) + (5.53 \times 10^{-5})(4.8 \times 10^4)(W_2 - 0.3)]$$

Objective 2: maximization percentage of return on equity at Pierce-Hall cannery

$$\max \; f_2 = \frac{10^2}{5 \times 10^6} [(3.75 \times 10^5) - 0.6 \left(\frac{59}{1.09 - x_1^2} - 59\right) \times 10^3]$$

Table 4.5 Results of the fifth cycle for the problem, where $\underline{AL}$ = (6.0,5.0,6.0,1.65,1.55)

Kind of problem	$s^5 = \sum_{t \in T, dt}$	$\underline{x}$	f
Principal	$s_{5,6245} = d_1+d_2 = 0.95$	$x_{5,6245}$ =(0.841,0.862,0.800)	$f_{5,6245}$ = (6.06,5.00,6.00,1.85,1.55)
Auxiliary problem 1	$s_{5,62451} = d_3 = 0.94$	$x_{5,62451}$ =(0.841,0.862,0.800)	$f_{5,62451}$ = (6.06,5.00,6.00,1.85,1.55)
Auxiliary problem 2	$s_{5,62453} = d_1 = = 0.94$	$x_{5,62453}$ =(0.841,0.862,0.860)	$f_{5,62453}$ = (6.06,5.00,6.00,1.85,1.55)

Objective 3: addition to the tax rate at Bowville

$$\min \quad f_3 = (2.4 \times 10^{-3}) \left(\frac{532}{1.09 - x_2^2} - 532\right)(0.75)$$

subject to the following constraints:

Constraint 1: DO level at Bowville must be larger than 6.0

$$g_1(\underline{x}) = 4.75 + (5.68 \times 10^{-5})(4.0 \times 10^4)(x_1 - 0.3) \geq 6.0$$

Constraint 2: DO level at Plympton must be greater than 6.0

$$g_2(\underline{x}) = 5.1 + [(4.42 \times 10^{-6})(4.0 \times 10^4)(x_1 - 0.3) + (7.71 \times 10^{-6})$$
$$(12.8 \times 10^4)(W_1 - 0.3)] + [(7.64 \times 10^{-6})(1.28 \times 10^5)$$
$$(x_2 - 0.3) + (1.60 \times 10^{-5})(4.8 \times 10^4)(W_2 - 0.3)] \geq 6.0$$

Constraint 3: The addition to the tax rate at Plympton must be less than 1.5

$$g_3(\underline{x}) = (3.33 \times 10^{-3}) \left(\frac{450}{1.09 - x^2} - 450\right)(0.75) \leq 1.5$$

Constraint 4: DO levle at the state line must be greater than 3.5

$$g_4(\underline{x}) = 1.0 + [(8.3 \times 10^{-7})(4.0 \times 10^4)(x_1 - 0.3) + (7.3 \times 10^{-7})$$
$$(2.8 \times 10^4)(W_1 - 0.3)] + [(1.45 \times 10^{-7})(1.28 \times 10^5)$$
$$(x_2 - 0.3) + (1.62 \times 10^{-6})(4.8 \times 10^4)(W_2 - 0.3)]$$
$$+ [(3.49 \times 10^{-5})(9.57 \times 10^4)(x_3 - 0.3) + (7.33 \times 10^{-5})$$
$$(3.57 \times 10^4)(W_3 - 0.3)] \geq 3.5$$

Bounds: Proportionate reduction in gross BOD_c

$$0.3 \leq x_i \leq 1 \qquad i = 1,2,3$$

Let the above constraints be denoted by $\underline{x} \in X$.

W_i is related to x_i by

$$W_i = \frac{0.39}{1.39 - x_i^2} \qquad i = 1,2,3$$

Solution

Step 1 Ideal solutions: The ideal solutions are obtained by solving the following nonlinear programming problems (GRG was used):

$$\max \quad f_i(\underline{x})$$

$$\text{s.t.} \quad \underline{x} \in X \qquad i = 1,2$$

and

$$\min \quad f_3(\underline{x})$$

$$\text{s.t.} \quad \underline{x} \in X$$

they are

$$f_1^* = 6.782 \quad \text{at} \quad \underline{x} = (1.0,\ 1.0,\ 0.7414)$$

$$f_2^* = 6.273 \quad \text{at} \quad \underline{x} = (0.84,\ 0.843,\ 0.810)$$

$$f_3^* = 0.00 \quad \text{at} \quad \underline{x} = (0.3,\ 0.3,\ 0.3)$$

Step 2. Nondominated solutions: The single objective optimization for generating a set of nondominated solutions can be formulated by using ε-constraint method:

$$\max \quad f_3'(\underline{x}) = -(2.4 \times 10^{-3}) \left(\frac{532}{1.09 - x_2^2} - 532\right) (0.75)$$

s.t.

$$f_1(\underline{x}) = 2.0 + [(1.31 \times 10^{-5}) (4.0 \times 10^4) (x_1 - 0.3) + (3.15 \times 10^{-5}) (2.8 \times 10^4) (W_1 - 0.3)] + [(2.18 \times 10^{-5}) (1.28 \times 10^5) (x_2 - 0.3) + (5.53 \times 10^{-5}) (4.8 \times 10^4) (W_2 - 0.3)] \geq \varepsilon_2$$

$$f_2(\underline{x}) = \frac{10^2}{5 \times 10^6} [(3.75 \times 10^5) - 0.6 \left(\frac{59}{1.09 - x_1^2} - 59\right) \times 10^3] \geq \varepsilon_3$$

and $\underline{x} \in X$, where $f_3'(\underline{x}) = 1/f_3(\underline{x})$

objective f_3 is arbitrarily chosen as the primary objective. ε_2 and ε_3 are bounds for f_1 and f_2, respectively. Haim and Hall [10,13,14] used Kuhn-Tucker conditions for SWT method, but it is too complex to use Kuhn-Tucker conditions for deriving trade-off ratios (Lagrange multipliers) as explicit functions of the decision variables. Hwang et. al. [18] used Generalized Reduced Gradient Technique to derive the trade-off ratios. ε_2 and ε_3 are altered parametrically to obtain a lot of solutions, all dominated solutions will be discarded. The nondominated solutions are those with nonzero lagrange multipliers. Table 4.6 presents a set of non-dominated solutions for the problem.

Step 3. Surrogate worth functions: The values of the surrogate worth function assessed by the DM for the trade-off ratios λ_{12} and λ_{13} are tabulated in the columns 5 and 7 of Table 4.6. Note that there exists a solution $\underline{f} = (5.10, 5.00, 1.68)$ for which both W_{12} and W_{13} are zero, hence

Table 4.6 Non-dominated solution set and DM's response

f_1	f_2	f_3^*	λ_{31}	W_{31}	λ_{32}	W_{32}
00 level at park	% rate	tax rate				
4.80	4.15	1.33	0.711	+10	0.731	+6
4.80	4.35	1.41	0.711	+ 9	0.731	+4
4.80	4.45	1.44	0.711	+ 9	0.736	+4
4.90	5.50	1.63	0.701	+ 5	0.724	-2
4.90	5.25	1.57	0.701	+ 6	0.717	+2
4.90	5.00	1.53	0.701	+ 6	0.709	+2
5.10	6.00	1.84	0.689	- 8	0.717	-5
5.10	5.75	1.78	0.682	+ 2	0.709	-2
5.10	5.50	1.74	0.673	+ 2	0.703	-2
5.10	5.00	1.68	0.668	0	0.695	0
5.20	6.25	1.91	0.597	- 8	0.717	-9
5.20	6.00	1.97	0.591	- 8	0.709	-9
5.20	5.75	2.01	0.545	-10	0.703	-10

this solution belongs to the indifference band.

Step 4. The indifference solutions: We know that $\underline{f}$ = (5.10, 5.00, 1.68) is a nondominated solution in the indifference band. The corresponding trade-off ratios are λ_{31} = 0.668 and λ_{32} = 0.695. The corresponding solution in the decision space is $\underline{x}$ (0.85, 0.87, 0.80).

However, it is usually the case that no solution W_{31} and W_{32} are zero simultaneously. Under the cases, a linear approximation may be used to obtain the approximated worth function. The solution obtained through linear approximation is an indifference solution, but this must be validated by the OM since the worth functions developed are approximations of the OM's actual preference (which is, certainly not explicitly).

The Bow River Valley problem is solved with $\varepsilon_2 = f_1^* = (5.10)$ and $\varepsilon_3 = f_2^* = (5.00)$, which yields the following solution:

$\underline{x}$ = (0.85, 0.87, 0.80)

$\underline{f}$ = (5,10, 5,00, 1.68)

$\lambda_{31}^* = (0.668)$ and $\lambda_{32}^* = (0.695)$

When these trade-off ratios and the solution $\underline{f}^*$ are presented for validation, the OM has assigned values $W_{12} = W_{13} = 0$. Note that if the OM does not indicate this as an indifference solution, more solutions have to be generated around $W_{31} = 0$ and $W_{32} = 0$; the DM assesses the new solutions for W_{12} and W_{13} until an indifference solution (preferred solution) is reached.

Step 5. Transformation to decision space: This problem is already solved in step 4 for the indifference vector $\underline{f}$ = (5.10, 5.00, 1.68); the solution in terms of the decision variables is

$\underline{x}$ = (0.85, 0.87, 0.80)

Discussion and the Results: The solution procedure used to solve the problem is the generalized reduced gradient (GRG) algorithm for nonlinear optimization. The FORTRAN IV computer code GREG is implemented on ITEL AS/5 system.

The SWT method is moderately suitable for this problem. Note that once the number of objective increases, the required set of nondominated solutions increases enormously.

CHAPTER 5

REID-VERMURI MODEL

5.1 Description of the system

Reid and Vermuri [32] introduced this problem of multiobjective function in water resources planning. It considers the problem of determining the optimum storage capacity of a reservoir subject to a specified set of release rules. Also, it assumes that a dam of finite height impounds water in the reservoir and that water is required to be released for various purposes such as flood control, irrigation, industrial and urban use, and power generations. The reservoir may also be used for fish and wild-life enhancement, recreation, salinity and pollution control, mandatory release to satisfy the riparian rights of downstream users, and so forth. The problem is essentially one of determining the storage capacity of the reservoirs so as to maximize the net benefits accrued.

It is not always straightforward, nor is it desirable, to express the benefits in terms of net income, because the procedure for comparing the economical and social dislocations is not very clear at the outset. Under these circumstances the concept of utility, as used by economists, is far more useful than scalar-valued performance criteria.

5.2 MODELING

To demonstrate the point and the computational reasons this problem is simplified as follows. Let f_1 be an indicator of the capital cost of the project which depends on the total man hours x_1 devoted to building the dam and also on the mean radius x_2 of the lake impounded in some fashion. The height k of the proposed dam can be related to the variable x_1 by an equation of the form

$$k = [e^{x_1}(x_1)^2]^{1/a} \quad ; \quad a \text{ constant} > 0 \tag{5.1}$$

and the surface area of the reservoir A is

$$A = K_1\Pi(x_2)^2; \quad K_1 \text{ constant} > 0 \tag{5.2}$$

capital cost f_1 may be denoted by

$$f_1 = K_2h^2A; \; K_2 \text{ constant} > 0,$$

$$f_1 = K_1K_2\Pi[e^{2x_1/a} \; (x_1)^{4/a} \; (x_2)^2]. \tag{5.3}$$

Similarly, let f_2 represent the water loss (volume/year) due to evaporation. This water loss is proportional to the surface area of the lake, so

$$f_2 = K_3 \, A \quad ; \quad K_3 \text{ constant} > 0, \tag{5.4}$$

$$f_2 = K_1K_3\Pi \, (x_2)^2 \, .$$

The total volume capacity V of the reservoir is vital to the realization of the various goals set forth previously. This reservoir volume may be approximated by

$$V = hA,$$

$$V = K_1\Pi[e^{x_1/a} \; (x_1)^{2/a} \; (x_2)^2] \tag{5.5}$$

Since one is interested in formulating performance indexes to be minimized the third index f_3 will be taken as the reciprocal of V. The physical situation, in this case, leads to the qualitative assumption that f_1 and f_2 are quantities to be decreased, whereas V must be increased, in order to improve the system performance. Therefore, let

$$f_3 = \frac{1}{V} \tag{5.6}$$

$$f_3 = \frac{1}{K_1 \pi} [e^{-x_1/a} (x_1)^{-2/a} (x_2)^{-2}]$$

The scalar indexes given by Eqs. 5-3, 5-4, and 5-6 represent the elements of the three-dimensional vector f.

In order to deal with a specific numerical form of the problem described above, the constants K_1, K_2, K_3 and a are chosen so that the three scalar performance indexes become

$$f_1 = e^{0.01x_1} (x_1)^{0.02} (x_2)^2, \tag{5.7}$$

$$f_2 = \frac{1}{2}(x_2)^2, \tag{5.8}$$

$$f_3 = e^{-0.005x_1} (x_1)^{-0.01} (x_2)^{-2} . \tag{5.9}$$

Note that $1/f_3$, the reciprocal of f_3, represents the reservoir volume and thus has more physical significance than the quantity f_3.

5.3 Surrogate Worth Trade Off Method

Haimes et. al. [13] obtained the nonlinear solution set by the so called SWT (surrogate worth trade off method). However, we have previously mentioned the inconvenience of the method, the GRG technique will be used here to obtain the nondominated solution set. To be more clearly, we describe the method Haimes developed in detail at first, then introduce the GRG technique.

Haimes' Surrogate Worth Trade Off Method

The first step is to find the minimum values for each objective function. Clearly, $f_1^* = 0$, $f_2^* = 0$ at $x_2 = 0$, and $f_3^* = 0$ at $x_1 = \infty$. The ε constraint formulation is now adopted to generate λ_{12} and λ_{13}

$$\min \quad \exp\,(0.01x_1)\,(x_1)^{0.02}\,x_2^2 \tag{5.10}$$

$$\text{s.t.} \quad \frac{1}{2}\,x_2^2 \leq \varepsilon_2 \tag{5.11}$$

$$\exp\,(-0.005x_1)(x_1)^{-0.01}\,x_2^{-2} \leq \varepsilon_3 \tag{5.12}$$

$$x_1 \geq 0, \quad x_2 \geq 0$$

Then the Lagrangian L is formed,

$$L = \exp\,(0.01x_1)\,(x_1)^{0.02}\,x_2^2 + \lambda_{12}(\tfrac{1}{2}\,x_2^2 - \varepsilon_2)$$

$$+ \lambda_{13}[\exp(-0.005x_1)\,(x_1)^{-0.01}\,x_2^{-2} - \varepsilon_3]$$

The Kuhn-Tucker necessary conditions for a minimum are

$$x_i\,\frac{\partial L}{\partial x_i} = 0 \qquad x_i \geq 0, \qquad i = 1,2 \tag{5.13}$$

$$\frac{\partial L}{\partial x_i} \geq 0 \qquad x_i \geq 0, \qquad i = 1,2 \tag{5.14}$$

$$\lambda_{ij}\,\frac{\partial L}{\partial \lambda_{ij}} = 0 \qquad \lambda_{ij} \geq 0, \qquad j = 1,2 \qquad i = 1,2 \tag{5.15}$$

$$\frac{\partial L}{\partial \lambda_{ij}} \leq 0 \qquad \lambda_{ij} \geq 0, \qquad j = 1,2 \qquad i = 1,2 \tag{5.16}$$

The above conditions were solved for various values of ε_2 and ε_3 via the Newton-Raphson method. The results are presented in Table 5.1. Note that

$$f_3(x_1,x_2) = \frac{1}{\hat{f}_3(x_1,x_2)} \quad . \tag{5.17}$$

Since λ_{13} corresponds to $f_3(x_1,x_2)$ and yet the decision maker is rather familiar with $\hat{f}_3(x_1,x_2)$, which is the volume of the reservoir, a trade-off function $\bar{\lambda}_{13}$ is needed, i.e.,

$$\bar{\lambda}_{13} = - \frac{\partial f_1}{\partial \bar{f}_3} = - \frac{\partial f_1}{\partial f_3} \frac{df_3}{d\bar{f}_3}$$

$$= \lambda_{13}\left(- \frac{1}{(\bar{f}_3)^2}\right) = - \frac{\lambda_{13}}{(\bar{f}_{13})^2} \tag{5.18}$$

A multiple regression analysis for the construction of λ_{12} and λ_{13} as functions of f_2 and f_3 by using the wide band of noninferior points (Table 5.1) resulted in a correlation coefficient of only 0.80. This is attributed to the exponential nature of the objective functions. Consequently, the second approach was adopted, where the decision maker provide the surrogate-worth values W_{12} and W_{13} for those values of λ_{12} and λ_{13} given in Table (5.1). Clearly, for each λ_{12} and λ_{13} the corresponding f_1, f_2, and f_3 can also be found in Table 5.1. Should the decision maker need additional information in the neighborhood of λ_{12}^* and λ_{13}^*, then a multiple regression analysis can be conducted to yield the needed information.

The values of the surrogate worth functions generated with a decision maker are tabulated as W_{12} and W_{13} in Table 5.1. More than one set of trade

Table 5.1 Non-dominated solutions and DM's response

f_2	f_3	x_1	x_2	f_1	λ_{12}	$\hat{\lambda}_{13}$	w_{12}	w_{13}
250.00	500.00	0.69	22.38	499.93	2.00	-2.00	+9	+6
250.00	1000.00	128.92	22.38	2000.00	8.01	-4.00	+2	+2
250.00	1750.00	239.57	22.38	6120.75	24.51	-6.59	-2	-2
250.00	2500.00	310.40	22.38	12473.00	50.05	-10.00	-6	-6
250.00	3750.00	391.05	22.38	28132.00	112.40	-14.93	-10	-10
250.00	5000.00	448.28	22.38	49974.00	197.38	-18.76	-10	-10
750.00	1750.00	24.45	38.75	2039.05	2.74	-2.15	+7	+4
750.00	2500.00	93.05	38.75	4143.40	5.46	-3.08	+5	+3
750.00	3750.00	172.90	38.75	9370.42	12.05	-5.42	0	0
750.00	5000.00	229.90	38.75	16547.68	22.17	-6.27	-3	-3
100.00	1750.00	421.72	14.25	15308.00	153.00	-17.38	-10	-10
500.00	1750.00	102.65	30.05	3053.10	6.13	-3.12	+5	-3
100.00	3750.00	573.53	14.25	70280.00	703.07	-36.24	-10	+2
500.00	3750.00	253.27	30.05	14048.24	28.10	-6.79	-4	+1

offs resulted in an indifference band, namely, $W_{ij} = 0$. The corresponding values of λ_{12}, λ_{13}, f_1, f_2, and f_3 can be read directly from Table 5.1, rows 9, 25, 30, and 32 are optimal in the sense defined in the section on the derivation of the trade off function, namely, they are noninferior solutions that belong to the indifference band.

The decision variables corresponding to the above optimal solutions can be obtained in several ways. The simple way in this problem is the use of Table 5.1. Thus, for example, row 9 provides the following optimal decisions and values of the objective functions: $x_1 = 172.90$, $x_2 = 38.75$, $f_1 = 9374.95$, $f_2 = 750.00$, $f_3 = 3750.00$. By using Table 5.1 to generate the optimal decisions x_1 and x_2 one may need to make an additional analysis in the case where there is no row with both W_{12} and W_{13} equal to zero.

Surrogate Worth Trade-off Method on Quadratic Programming Problems

The Reid-Vermuri dam model is simplified here to become a quadratic form. The SWT method has its advantages in solving such small scale, simple problems.

Objective 1. maximum reservoir storage capacity (1000 ton)

$$\max \quad f_1(\underline{x}) = \frac{1}{2} x_1 x_2$$

Objective 2. minimum capital cost ($10,000)

$$\min \quad f_2(\underline{x}) = (2x_1 - 3)^2 - \frac{1}{2} x_2^2$$

subject to the constraint

$$g_1(\underline{x}) = x_2 \leq 20$$

of which, x_1 = labor hours

x_2 = height of the dam

Step 1

The ideal solution for f_1 is $f_1^* = \infty$ at $x_1 = \infty$ and $x_2 = 20$, for f_2 is $f_2^* = 0$ at $x_1 = 1.5$ and $x_2 = 20$.

Step 2 Nondominated solutions

The problem in the form required by the SWT method is:

$$\max \quad f_1(\underline{x}) = \frac{1}{2} x_1 x_2$$

subject to

$$-(2x_1 - 3)^2 - \frac{1}{2} x_2^2 \geq -\varepsilon$$

$$x_2 \leq 20$$

$$x_1, x_2 \geq 0$$

where ε is the capital cost. The lagrangian then becomes:

$$L = \frac{1}{2} x_1 x_2 + u_1(x_1 - 20) + \lambda_{12}[-(2x_1 - 3)^2 - \frac{1}{2} x_2^2 + \varepsilon]$$

Then the Kuhn-Tucker necessary conditions become:

$$\frac{\partial L}{\partial x_1} = \frac{1}{2} x_2 - 2\lambda_{12}(2x_1 - 3)(2) \leq 0$$

$$\frac{\partial L}{\partial x_2} = \frac{1}{2} x_1 - \lambda_{12} x_2 + u_1 \leq 0$$

$$\frac{\partial L}{\partial \lambda_{12}} = -(2x_1 - 3)^2 - \frac{1}{2}x_2^2 + \varepsilon \geq 0$$

$$\frac{\partial L}{\partial u_1} = x_2 - 20 \leq 0$$

$$u_1 (x_2 - 20) = 0$$

$$\lambda_{12} [-(2x_1 - 3)^2 - \frac{1}{2}x_2^2 + \varepsilon] = 0$$

We can see clearly that when $x_2 \leq 20$, $u_1 = 0$, and $x_1, x_2 \geq 0$, then

$$\frac{\partial L}{\partial x_1} = \frac{\partial L}{\partial x_2} = 0$$

$$\lambda_{12} = \frac{\frac{1}{2}x_2}{2(2x_1-3)(2)} - \frac{x_2}{(8)(2x_1-3)}$$

also

$$\lambda_{12} = \frac{x_1}{2x_2}$$

Consequently

$$2x_2^2 = (8)(2x_1^2 - 3x_1)$$

$$x_2^2 = (4)(2x_1^2 - 3x_1)$$

$$x_2 = 2\sqrt{x_1(2x_1 - 3)}$$

and $(2x_1 - 3)^2 + \frac{1}{2}x_2^2 = \varepsilon$

for all $\lambda > 0$, it implies that $x_1 > \frac{3}{2}$ and $x_2 > 0$. Then the relation $x_2 = 2\sqrt{x_1(2x_1 - 3)}$ defines all the nondominated solution set. Table 5.2 shows all this nondominated solutions.

Step 3. Surrogate Worth Functions

Table 5.2 also shows the DM's preference among the presented solutions. For example, given the storage capacity 2.828 units (1000 Ton/unit) and the capital cost of 1000 units ($10000/unit), the DM greatly prefers $W = +10$ on -10 to +10 scale to trade 0.354 units of increased storage capacity for 1 unit increase in capital cost. Note that $\lambda = -\partial f_1/\partial f_2 = 0.354$. Similarly, when $f_1 = 130,384$ units and $f_2 = 289$ units, the DM is completely indifferent ($W = 0$) to a trading of 0.192 units increased storage capacity for one unit increased capital cost.

Step 4. The indifference solution(s)

Any two or more solutions from Table 5.2 can be used to estimate λ^* at which $W(\lambda^*) = 0$ as described in Figure 5.1. Assuming that the DM's assessment is consistent and accurate, we take this solution as the preferred solution.

Step 5. Transformation to decision space

From Table 5.2, we note that the solution in decision space is $\underline{x} = (10, 26.08)$.

Modified Surrogate Worth trade-off method

GRG technique enables us to apply the GRG subroutines directly to the nonlinear multiobjective problems. By putting in different ε values, we obtain a set of noninferior solutions. The advantages of this method in

Table 5.2 The nondominated solution set for SWT method

x_1	x_2	f_1	f_2	λ_{12}	W_{12}
2.00	2.83	2.83	1.00	0.36	+10
3.00	6.00	9.00	9.00	0.25	+10
4.00	8.95	17.89	25.00	0.23	+9
5.00	11.83	29.58	49.00	0.21	+8
6.00	14.70	44.09	81.00	0.21	+7
7.00	17.55	61.43	121.00	0.20	+5
8.00	20.40	81.59	169.00	0.20	+4
9.00	23.24	104.57	225.00	0.20	+2
10.00	26.08	130.39	289.00	0.19	0
11.00	28.92	159.03	361.00	0.19	-5
12.00	31.75	190.50	441.00	0.19	-8
13.00	34.58	224.80	529.00	0.19	-10

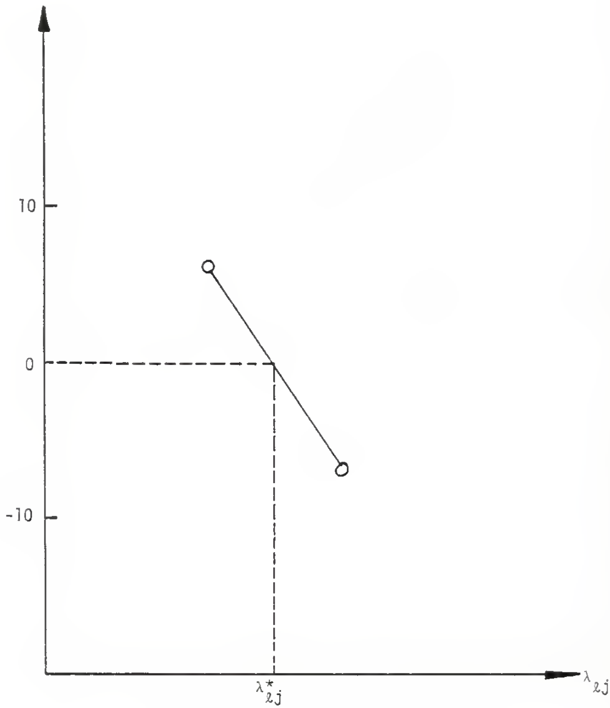


Fig. 5.1. Determination of the indifference band.

comparison with the original SWT method is its applicability to the large-scale complex problems.

The rest parts of this modified method in choosing the preference solutions are just like the original SWT method.

Passy's method

Passy [30] found that some multiobjective problems can be solved by so-called Cobb-Douglas production type functions provided that the problems are characterized by the following properties:

(1) The decision space x is positive orthant; i.e., if x is a decision vector, then $x_j > 0$, where $j = 1,2, \ldots, n$

(2) The criterion functions are of the Cobb-Douglas type,

$$f_i(\underline{x}) = C_i \prod_{j=1}^{n} x_j^{\alpha_{ij}}, \quad i = 1, \ldots, q \tag{5.19}$$

where q is the number of different criteria and $C_i > 0$ and α_{ij} are given numbers. In mathematical programming literature, such functions are called single-term polynomials.

These functions are similar to Cobb-Douglas production functions, where $\underline{x}$ is the vector of production factors and f is the vector of the various outputs.

If any $\underline{x} \varepsilon X$ is noninferior, then a vector of weights $\underline{W}^*$ that satisfies the following three conditions does exist:

$$\sum_{i=1}^{q} W_i^* = 1 \tag{5.20}$$

$$\sum_{i=1}^{q} W_i^* \alpha_{ij} = 0, \quad j = 1,2, \ldots, n \tag{5.21}$$

$$W_i^* > 0, \qquad i = 1,2, \ldots, q \tag{5.22}$$

Consider the following program

$$\min_{\underline{x} \in X} \ell n(f_1(\underline{x})) + \sum_{i=2}^{q} \frac{W_i^*}{W_1^*} \ell n(f_i(\underline{x})) \tag{5.23}$$

For notational convenience, denote a given solution by $\underline{x}^*$ and consider the following program:

$$\min_{\underline{x} \in X} \ell n\ (f_1(\underline{x})) + \sum_{i=2}^{q} \frac{W_t^*}{W_1^*} [\ell n(f_i(\underline{x})) - \ell n(f_i(\underline{x}^*))] \tag{5.24}$$

Since the difference between (5.23) and (5.24) is the constant term $-\sum_{i=2}^{q} W_i^*/W_1^* \ \ell n(f_i(\underline{x}^*))$, the point $\underline{x}^*$ is also a solution to this problem .

Now consider the Lagrangian function:

$$\min \{\ell n(f_1(\underline{x})) + \sum_{i=2}^{q} u_{1i}\ [\ell n\ f_i(\underline{x}) - \varepsilon_i]\} \tag{5.25}$$

where

$$\varepsilon_i = \ell n\ (f_i(\underline{x}^*)$$

A solution to this Lagrangian problem is given by

$$x_j = x_j^*, \qquad j = 1,2, \ldots, n$$

$$u_{1i} = W_i^*/W_1^*, \quad i = 2, \ldots, q$$

Recall from the Kuhn-Tucker theory that

$$-\mu_{1i} = \left.\frac{\partial(\ell n\ f_1(\underline{x}))}{\partial(\ell n\ f_i(\underline{x}))}\right|_{x=x^*} \tag{5.26}$$

Or equivalently,

$$- u_{1i} = \frac{\partial f_1(\underline{x})}{\partial f_i(\underline{x})} \frac{f_i(\underline{x})}{f_1(\underline{x})} \Bigg|_{x=x^*} \tag{5.27}$$

Finally, one obtains

$$- \lambda_{1i} = \frac{\partial f_1}{\partial f_i} = - u_{1i} \frac{f_1(x^*)}{f_i(x^*)} = - \frac{W_i^*}{W_1^*} \frac{f_1(\underline{x}^*)}{f_i(\underline{x}^*)} \tag{5.28}$$

Given a vector $W^* > 0$ that satisfies (5.20,21,22), the trade-off at a point x is given by

$$\lambda_{1i} = \frac{W_i^*}{W_1^*} \frac{f_1(\underline{x})}{f_i(\underline{x})} \tag{5.29}$$

where x is any positive vector. Since any x is efficient, a minimization for generating efficient points is not required. And for a given point x the lagrange multiplier is calculated directly through (5.29), and a minimization of the lagrangian function is not required.

<u>Numerical Example</u>

We continue to use the Reid and Vermuri model in the following examples. Capital cost of the project

$$f_1(\underline{x}) = \exp\,(0.01x_1)\; x_1^{0.02}\; x_2^2 \tag{5.30}$$

water cost due to evaporation

$$f_2(\underline{x}) = \frac{1}{2}\, x_2^2 \tag{5.31}$$

Total volume capacity of the reservoir ($1/f_3$)

$$f_3(\underline{x}) = \exp(-0.05x_1)\ x_1^{-0.01}\ x_x^{-2}$$

Define a new set of variables

$$y_1 = \exp(0.01x_1)x_1^{0.02} \tag{5.32}$$

$$y_2 = x_2 \tag{5.33}$$

In terms of this set of variables the objectives can be written as

$$f_1(\underline{y}) = y_1 y_2^2,\ y_1 > 0,\ \ y_2 > 0 \tag{5.34}$$

$$f_2(\underline{y}) = \frac{1}{2}\, y_2^2,\ y_2 > 0 \tag{5.35}$$

$$f_3(\underline{y}) = y_1^{-1/2}\ y_2^{-2},\ y_1 > 0,\ y_2 > 0 \tag{5.36}$$

These functions are Cobb-Douglas functions, and the corresponding g are given by

$$g_1(\underline{y}) = \ell n\ y_1 + 2\ \ell n\ y_2 \tag{5.37}$$

$$g_2(\underline{y}) = \ell n\ \frac{1}{2} + 2\ \ell n\ y_2 \tag{5.38}$$

$$g_3(\underline{y}) = -\frac{1}{2}\ \ell n\ y_1 - 2\ \ell n\ y_2 \tag{5.39}$$

Let $z_1 = \ell n\ y_1$ and $z_2 = \ell n\ y_2$, and the three linear objective functions are given by

$$g_1(\underline{z}) = z_1 + 2z_2 \tag{5.40}$$

$$g_2(\underline{z}) = \ell n\ \frac{1}{2} + 2z_2 \tag{5.41}$$

$$g_3(\underline{z}) = -\frac{1}{2}\ z_1 - 2z_2 \tag{5.42}$$

where z_1 and z_2 are any real number. The parametric programming to be solved is

$$\min \; W_1(z_1 + 2z_2) + W_2(\ln \frac{1}{2} + 2z_2) + W_3(-\frac{1}{2} z_1 - 2z_2) \tag{5.43}$$

or

$$\min \; (W_1 - \frac{1}{2} W_3)z_1 + (2W_1 + 2W_2 - 2W_3)\, z_2 + W_2 \ln \frac{1}{2} \tag{5.44}$$

This problem has a solution only if

$$W_1 - \frac{1}{2} W_3 = 0 \tag{5.45}$$

$$2W_1 + 2W_2 - 2W_3 = 0 \tag{5.46}$$

Recall that $W_1 + W_2 + W_3 = 1$, and the value of $\underline{W}^*$ is therefore $W_1^* = \frac{1}{4}$, $W_2^* = \frac{1}{4}$, and $W_3^* = \frac{1}{2}$. Since a vector of weights W^* exits, every point $\underline{z} \in E^2$, $\underline{y} > 0$, or $\underline{x} > 0$ is noninferior.

The Lagrange multipliers for this example are given by (5.29):

$$\lambda_{12} = \frac{f_1(\underline{x})}{f_2(\underline{x})}, \quad \lambda_{13} = \frac{\frac{1}{4} f_1(\underline{x})}{\frac{1}{2} f_3(\underline{x})} = \frac{2 f_1(\underline{x})}{f_3(\underline{x})} \quad . \tag{5.47}$$

5.4. Nonlinear Iterative Goal Programming

The previously mentioned Reid-Vermuri model is modified as follows:

Goal 1 = Limiting the capital cost to 10 (unit: \$10000.)

$$f_1(\underline{x}) = (x_1 - 6)^2 + (x_2 - 4)^2 \leq 10 \tag{5.48}$$

Goal 2: Water cost due to evaporation must less than 7.5 (100 volumes/year)

$$f_2(\underline{x}) = \frac{1}{2}(x_2)^2 \leq 7.5 \quad (5.49)$$

Goal 3: The reservoir storage volume must not greater than 20 (1000 volume)

$$f_3(\underline{x}) = 2x_1x_2 \leq 20 \quad (5.50)$$

Goal 4: The height of the dam must be greater than 6 (10 ft)

$$f_4(\underline{x}) = x_2 \geq 6 \quad (5.51)$$

Absolute constraint:

$$x_1 \geq 0 \quad (5.52)$$

$$x_2 \geq 0$$

We summarize all the above conditions into the following model:

$$f_1(\underline{x}) = (x_1 - 6)^2 + (x_2 - 4)^2 \leq 10$$

$$f_2(\underline{x}) = \frac{1}{2}(x_2)^2 \leq 7.5$$

$$f_3(\underline{x}) = 2x_1x_2 \leq 20$$

$$f_4(\underline{x}) = x_2 \geq 6$$

subject to

$$x_1 \geq 0, \quad x_2 \geq 0$$

The nonlinear iterative Goal Programming formulation for this model becomes:

$$\min \ \underline{a} = [(a_1, a_2), a_3, a_4, a_5]$$

$$= [(d_1^- + d_2^-), (d_3^+), (d_4^+), (d_5^+), (d_6^-)]$$

subject to

$$x_1 + d_1^- - d_1^+ = 0$$

$$x_2 + d_2^- - d_2^+ = 0$$

$$(x_1 - 6)^2 + (x_2 - 4)^2 + d_3^- - d_3^+ = 10$$

$$\frac{1}{2}(x_2)^2 + d_4^- - d_4^+ = 7.5$$

$$\partial x_1 x_2 + d_5^- - d_5^+ = 20$$

$$x_2 + d_6^- - d_6^+ = 6$$

$$\underline{x}, \underline{d}^-, \underline{d}^+ \geq 0, \quad d_i^- \cdot d_i^+ = 0 \quad \forall_i$$

<u>Problem 1</u> (for the absolute constraints)

$$\min \quad a_1 = d_1^- + d_2^-$$

subject to

$$x_1 + d_1^- - d_1^+ = 0$$

$$x_2 + d_2^- - d_2^+ = 0$$

$\underline{x}, \underline{d}^-, \underline{d}^+ \geq 0, \qquad d_i^- \cdot d_i^+ = 0, \ \forall_i$

This first problem is a linear case, the ordinary simplex procedure gives solutions for decision variables ($\underline{x}$) and $d_1^- = d_2^- = 0$ as shown in figure 5.2 for deviational variables. Now we proceed to the 2nd priority.

Problem 2 (for the 2nd priority)

$$\min \quad a_2 = d_3^+$$

subject to

$$x_1 \geq 0$$

$$x_2 \geq 0$$

$$f_1(\underline{x}) = (x_1 - 6)^2 + (x_2 - 4)^2 + d_3^- - d_3^+ = 10$$

$$\underline{x}, d_3^-, d_3^+ \geq 0, \quad d_3^- \cdot d_3^+ = 0$$

Let starting point = $\underline{x}$ = (9,10)

By using GRG compliers, its solution gives

$$x_1 = 8.021, \quad x_2 = 6.337$$

$$d_3^+ = 0 \quad , \quad d_3^- = 0.0004$$

as an result $a_2^* = 0$ is shown in figure 5.2.

Problem 3 (for the third priority)

$$\min \quad a_3 = d_4^+$$

subject to

$x_1 \geq 0$

$x_2 > 0$

$(x_1-6)^2 + (x_2-4)^2 \leq 10$

$f_2(\underline{x}) = \frac{1}{2} (x_2)^2 + d_4^- - d_4^- = 7.5$

Its solution gives

$x_1 = 8.021, \quad x_2 = 3.676$

$d_4^+ = 0 \quad , \quad d_4^- = 0.125$

as a result, $a_3 = d_4^+ = 0$ is shown in figure 5.2

Problem 4 (for the forth priority)

$\min \quad a_4 = d_5^+$

subject to

$x_1 \geq 0$

$x_2 \geq 0$

$(x_1 - 6)^2 + (x_2 - 4)^2 \leq 10$

$1/2 \, (x_2)^2 \leq 7.5$

$f_3(\underline{x}) = 2x_1x_2 + d_5^- - d_5^+ = 20$

Its solution gives

$$x_1 = 7.248, \qquad x_2 = 1.274$$

$$d_5^+ = 0 \quad , \qquad d_5^- = 0.136$$

Problem 5 (for the fifth priority)

$$\min \quad a_5 = d_6^-$$

subject to

$$x_1 \geq 0$$

$$x_2 \geq 0$$

$$(x_1 - 6)^2 + (x_2 - 4)^2 \leq 10$$

$$1/2\ (x_2)^2 \leq 7.5$$

$$2x_1x_2 \leq 20$$

$$x_2 + d_6^- - d_6^+ = 6$$

Its solution gives

$$x_1 = 2.854, \qquad x_2 = 3.329$$

$$d_6^+ = 0 \quad , \qquad d_6^- = 2.654$$

The final solution for $\underline{f}$:

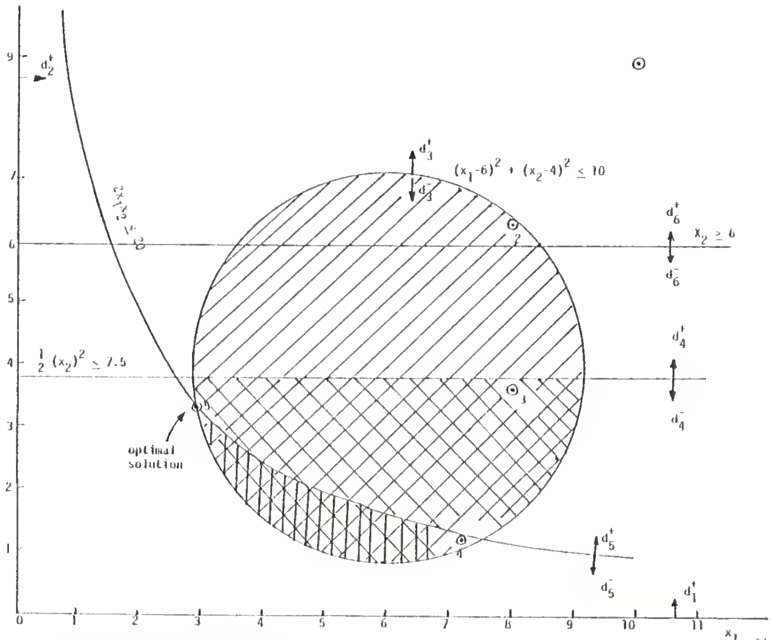


Fig. 5.2. Solution to the NLGP problem.

$f_1(\underline{x}) = 10.348$

$f_2(\underline{x}) = 5.541$

$f_3(\underline{x}) = 19.002$

$f_4(\underline{x}) = 3.329$

$x_1 = 2.854, \quad x_2 = 3.329$

5.5 STEM METHOD

The Reid-Vermuri model is modified and converted into the following linear problem:

$$\min f_1(\underline{x}) = 0.203x_1 + 1.98x_2 + 0.728x_3 + 0.08x_4 + 0.045x_5 + 0.234x_6$$

$$\min f_2(\underline{x}) = 9,5x_1 + 18.5x_2 + 95x_3 + 15x_4$$

$$\max f_3(\underline{x}) = -21.6x_1 - 24.57x_2 - 1.5x_3 - 13.65x_4 - 1.001x_5 - 47.0x_6$$

s.t.

$$g_1(\underline{x}) = 86.4x_1 + 12.84x_2 + 849.6x_3 + 1.5x_4 + 16.08x_5 + 124.5x_6 \geq 605$$

$$g_2(\underline{x}) = 0.066x_1 + 3.34x_2 + 4.36x_3 + 0.25x_4 + 0.05x_5 + 0.40x_6 \geq 4.13$$

$$g_3(\underline{x}) = 79.12x_1 + 335.8x_2 + 239.2x_3 + 17.25x_4 + 4.35x_5 + 55.2x_6 \geq 575$$

$$g_4(\underline{x}) = 2.34x_1 + 19.63x_2 + 10.14x_3 + 0.325x_4 + 0.026x_5 + 0.52x_6 \geq 8.19$$

$$x_1 \leq 7, \quad x_2 \leq 1.5, \quad x_3 \leq 0.25, \quad x_4 \leq 10.0, \quad x_5 \leq 10.0, \quad x_6 \leq 4.0$$

$$x_i \geq 0, \quad i = 1,2, \ldots, 6$$

Let this constraint set be denoted by $\underline{x} \varepsilon X$

Step 0. Construction of a pay-off table

The three LP problems for ideal solutions are:

(1) $\min \ f_1(\underline{x})$

s.t. $\underline{x} \in X$

(2) $\min \ f_2(\underline{x})$

s.t. $\underline{x} \in X$

(3) $\max \ f_3(\underline{x})$

s.t. $\underline{x} \in X$

we can construct the following pay-off table from the three ideal solutions.

	f_1	f_2	f_3	x_1	x_2	x_3	x_4	x_5	x_6
f_1	-2.23	-65.6	-272.7	3.75	0	0.22	10.0	0	0.774
f_2	-7.97	-8.32	-484.38	0	0.43	0	10.0	10.0	3.7
f_3	-7.87	-83.75	-81.01	1.75	1.00	0.25	0.74	10.0	0

Iteration No. 1:

Step 1. Calculation Phase

(a) Calculate weights

$$\alpha_1 = \frac{f_1^{min} - f_1^*}{f_1^{min}} \left(\frac{1}{\sqrt{\sum_{i=1}^{6} (c_{1i})^2}} \right)$$

$$= \frac{-7.87 + 2.23}{-7.87} \quad \frac{}{\sqrt{(0.203)^2+(1.98)^2+(0.728)^2+(0.08)^2+(0.045)^2+(0.234)^2}}$$

$$= 0.3358$$

$$\alpha_2 = \frac{f_2^{min} - f_2^*}{f_2^{min}} \left(\frac{1}{\sum_{i=1}^{6} (C_{2i})^2} \right)$$

$$= \frac{-83.75 + 8.32}{-83.75} \quad \frac{1}{(9.5)^2+(18.5)^2+(95)^2+(15)^2}$$

$$= 0.0092$$

$$\alpha_3 = \frac{f_3^{min} - f_3^*}{f_3^{min}} \left(\frac{1}{\sum_{i=1}^{6} (C_{3i})^2} \right)$$

$$= \frac{-484.38 + 81.01}{-484.38} \quad \frac{1}{(-21.6)^2+(24.57)^2+(1.5)^2(13.65)^2+(1.001)^2+(47.0)^2}$$

$$= 0.0144$$

$$\Pi_1 = \frac{\alpha_1}{\alpha_1+\alpha_2+\alpha_3} = 0.9344$$

$$\Pi_2 = \frac{\alpha_2}{\alpha_1+\alpha_2+\alpha_3} = 0.0256$$

$$\Pi_3 = \frac{\alpha_3}{\alpha_1+\alpha_2+\alpha_3} = 0.0401$$

b) solve the following LP problem

$$\min \quad \lambda$$

$$\text{s.t.} \quad \underline{x} \in X, \quad \lambda \geq 0$$

$$\lambda + 0.9344\,(-0.203x_1-1.98x_2-0.728x_3-0.08x_4-0.045x_5-0.234x_6) \geq 0.9344(-2.23)$$

$$\lambda + 0.0256(-9.5x_1-18.5x_2-95x_3-15x_4) \geq 0.0256(-8.32)$$

$$\lambda + 0.0375(-21.6x_1-24.57x_2-1.5x_3-13.65x_4-1.00x_5-47.0x_6) \geq 0.041(-81.01)$$

The solution is:

$$\underline{x}^1 = (x_1^1,\ x_2^1,\ x_3^1,\ x_4^1,\ x_5^1,\ x_6^1) = (1.55,\ 0.97,\ 0.25,\ 0.12,\ 0,\ 1.18)$$

$$\underline{f}^1 = (f_1^1,\ f_2^1,\ f_3^1) = (-3.00,\ -64.9,\ -128.5)$$

Step 2. Decision phase

After comparing $\underline{f}^1$ with the ideal solution set $\underline{f}^*$, the DM says that f_3^1 is satisfactory and can be relaxed by $\Delta f_3 = 40.5$ for the improvement of the other objectives. This decision forms the following feasible region:

$$X^2 \begin{cases} x^1 = X \\ f_3(\underline{x}) > f_3(\underline{x}^1) - \Delta f_3 = -128.5 - 40.5 = -169 \\ f_1(\underline{x}) > f_1(\underline{x}^1) = -3.00 \\ f_2(\underline{x}) > f_2(\underline{x}^1) = -64.9 \end{cases}$$

Iteration No. 2:

a) calculate weights.

$$\Pi_1 = \frac{0.3214}{0.3214 + 0.0074} = 0.9775$$

$\Pi_2 = 1 - 0.9775 = 0.0225$

$\Pi_3 = 0$

b) Solve the following LP problem.

$$\min_{\{\underline{x},\lambda\}} \quad \lambda$$

$$\text{s.t.} \quad \underline{x} \in X, \quad \lambda \geq 0$$

$$\lambda + 0.9775\,(-0.203x_1-1.98x_2-0.728x_3-0.08x_4-0.045x_5-0.234x_6) \geq 0.9775\,(-2.23)$$

$$\lambda + 0.0225\,(-9.5x_1-18.5x_2-95x_3-15x_4) \geq 0.0225(-8.32)$$

The solution is

$$\underline{x}^2 = (x_1^2,\ x_2^2,\ x_3^2,\ x_4^2,\ x_5^2,\ x_6^2)$$

$$= (1.19,\ 0.89,\ 0.25,\ 0.005,\ 0,\ 2.24)$$

$$\underline{f}^2 = (f_1^2,\ f_2^2,\ f_3^2) = (-3.01,\ -59.7,\ -169.15)$$

Step 2. Oecision Phase

Comparing $\underline{f}^2$ with the ideal solution set $\underline{f}^*$, the OM says both f_2 and f_3 are satisfactory and f_2 can be relaxed by $\Delta f_2 = 4.0$, the new feasible region, x^3, is formed as follows:

$$
X^3 \begin{cases} X^2 \\ f_2(\underline{x}) \geq f_2(\underline{x}^2) - \Delta f_2 = -59.7-4 = -63.7 \\ f_1(\underline{x}) \geq f_1(\underline{x}^2) = -3.01 \\ f_3(\underline{x}) \geq f_3(\underline{x}^2) = -169.15 \end{cases}
$$

Iteration No. 3:

Step 1. Calculation phase

a) Calculate weights

$$\Pi_1 = 1.0$$

$$\Pi_2 = 0$$

$$\Pi_3 = 0$$

b) Solve the following LP problem

$$\min_{\{\underline{x},\lambda\}} \lambda$$

s.t. $\underline{x} \in X, \quad \lambda \geq 0$

$$\lambda + (-0.203x_1-1.98x_2-0.728x_3-0.08x_4-0.045x_5-0.234x_6) \geq -2.26$$

The solution is

$$\underline{x}^3 = (x_1^3, x_2^3, x_3^3, x_4^3, x_5^3, x_6^3)$$

$$= (3.21, 0.58, 0.17, 5.12, 0, 0)$$

$$\underline{f}^3 = (f_1^3, f_2^3, f_3^3) = (-2.65, -64.1, -100.3)$$

Step 2. Decision Phase

The DM is satisfied with compromise solution, $\underline{x}^3$, which becomes the final solution.

5.6. Method of Geoffrion

The same linear problem used in (5.35) will also be solved by the method of Geoffrion.

$$\min \; f_1(\underline{x}) = 0.203x_1+1.98x_2+0.728x_3+0.08x_4+0.045x_5+0.234x_6$$

$$\min \; f_2(\underline{x}) = 9.5x_1+18.5x_2+95x_3+15x_4$$

$$\max \; f_3(\underline{x}) = -21.6x_1-24.57x_2-15.3x_3-13.65x_4-1.001x_5-47.0x_6$$

s.t.

$$g_1(\underline{x}) = 86.4x_1+12.84x_2+849.6x_3+1.5x_4+16.08x_5+124.5x_6 \geq 605$$

$$g_2(\underline{x}) = 0.066x_1+3.34x_2+4.36x_3+0.25x_4+0.05x_5+0.40x_6 \geq 413$$

$$g_3(\underline{x}) = 79.12x_1+335.8x_2+239.2x_3+17.25x_4+4.35x_5+55.2x_6 \geq 575$$

$$g_4(\underline{x}) = 2.34x_1+19.63x_2+10.14x_3+0.325x_4+0.026x_5+0.52x_6 \geq 8.19$$

$$x_1 \leq 7, \; x_2 \leq 1.5, \; x_3 \leq 0.25, \; x_4 \leq 10.0, \; x_5 \leq 10.0, \; x_6 \leq 4.0$$

$$x_i \geq 0, \qquad i = 1,2, \ldots, 6$$

Let this constraint set be denoted by $\underline{x} \in X$.

Iteration No. 1:

Step 0. Choose an initial point $\underline{x}^1$

Let an initial point $\underline{x}^1 \varepsilon X$ be

$$\underline{x}^1 = (3.0, 0.5, 0.15, 5.0, 5.0, 3.0)$$

set i = 1.

Step 1. Determine the direction of improvement

a) Calculate the weights.

with the initial point, we have

$$\underline{f}^0 = (f_1(\underline{x}^1), f_2(\underline{x}^1), f_3(\underline{x}^1)) = (-3.035, -127, -293.64)$$

to determine weights, W_i^1, i = 1, 2, 3. The DM indicates the following trade-off to be equivalent.

For f_1 and f_2:

$$(-3.035, -127, -293.64) \sim (-3.035 + 0.4, -127 - 15, -293.64)$$

For f_1 and f_3:

$$(-3.035, -127, -293.64) \sim (-3.035 + 0.4, - 127, -322 - 25)$$

That is,

$$W_2^1 = - \frac{\Delta f_1}{\Delta f_2} = - \frac{0.4}{-15} = 0.027$$

$$W_3^1 = - \frac{\Delta f_1}{\Delta f_3} = - \frac{0.4}{-25} = 0.016$$

b) Compute an optimal solution $\underline{y}^1$ and $\underline{z}^1$.

$$\sum_{j=1}^{3} W_j^1 \nabla_{\underline{x}} f_j(\underline{x}^1) \cdot \underline{y}^1$$

$$= 1 \cdot (-0.203, -1.98, -0.728, -0.08, -0.045, -0.234,) \cdot \underline{y}^1$$

$$+ 0.027 (-9.5, -18.5, -95, -15) \cdot \underline{y}^1$$

$$+ 0.016 (-21.6, -24.57, -15.3, -13.65, -1.001, -47.0) \underline{y}^1$$

$$= f_1(\underline{y}^1) + 0.027 f_2(\underline{y}^1) + 0.016 f_3(\underline{y}^1)$$

where $\underline{y}^1 = (y_1^1, y_2^1, y_3^1, y_4^1, y_5^1, y_6^1)^T$

The value of $\underline{y}^1$ can be obtained by solving the following LP problem.

$$\max \quad f_1(\underline{y}^1) + 0.027 f_2 (\underline{y}^1) + 0.016 f_3(\underline{y}^1)$$

$$= -0.203y_1-1.98y_2-0.728y_3-0.08y_4-0.045y_5-0.23y_6$$

$$-0.257y_1-0.50y_2-2.565y_3-0.405y_4$$

$$-0.346y_1-0.393y_3-0.245y_3-0.219y_4-0.016y_5-0.752y_6$$

$$= -0.806y_1-2.873y_2-3.538y_3-0.704y_4-0.061y_5-0.986y_6$$

$$\max\{-0.806y_1-2.873y_2-3.538y_3-0.704y_4-0.061y_5-0.986y_6\}$$

s.t. $\underline{y}^1 \varepsilon X$

The solution is:

$$\underline{y}^1 = (2.95, 0.721, 0.25, 4, 7.43, 0)$$

$$\underline{z}^1 = \underline{y}^1 - \underline{x}^1 = (-0.05, 0.221, 0.1, -1, 2.43, -3.0)$$

Step 2. Determine the step-size, t^1

To determine an optimal t^1, we calculate $f_j(\underline{x}^1 + t\underline{z}^1)$ values for the 0.2 interval of t:

t	0	0.2	0.4	0.6	0.8	1.0
f_1	-3.035	-3.068	-3.101	-3.134	-3.167	-3.200
f_2	-127	-131.719	-136.438	-141.157	-145.876	-150.595
f_3	-293.64	-256.159	-218.678	-181.197	-143.716	-106.235

The above results are presented to the DM, and he choose $\underline{f}^1$ = (-3.068, -131.719, -256.159) as the best solution for the first iteration, then $t' = 0.2$.

Set

$$\underline{x}^2 = \underline{x}^1 + t^1\underline{z}^1 = (2.99,\ 0.544,\ 0.35,\ 4,\ 5.486,\ 2.4)$$

$$i = i + 1 = 2$$

Iteration No. 2:

Step 1. Determine the direction of improvement

a) Calculate the weights.

The DM took the advise from analyst to change the reference objective from f_1 to f_3 because f_3 conflicts with both f_1 and f_2 at the neighborhood of $\underline{f}^1$.

For f_3 and f_1

$$(-3.068,\ -131.719,\ -256.159) \sim (-3.068 + 0.7,\ -131.719,\ -256.159 - 20)$$

For f_3 and f_2

$$(-3.068,\ -131.719,\ -256.159) \sim (-3.068,\ -131.719 + 15,\ -256.159 - 20)$$

That is

$$W_1^3 = - \frac{\Delta f_3}{\Delta f_1} = - \frac{-20}{0.7} = 28.57$$

$$W_2^3 = - \frac{\Delta f_3}{\Delta f_2} = - \frac{-20}{+15} = 1.33$$

$$W_3^3 = 1$$

Hence the weight vector is:

$$\underline{W}^1 = (28.57,\ 1.33,\ 1)$$

which is also equivalent to

$$\underline{W}^1 = (28.57/28.57,\ 1.33/28.57,\ 1/28.57)$$

$$= (1,\ 0.047,\ 0.035)$$

b) Calculate $\underline{y}^2$ and $\underline{z}^2$

The value of $\underline{y}^2$ is obtained by solving the following LP problem.

$$\max \quad f_1(\underline{y}^2) + 0.047\ f_2(\underline{y}^2) + 0.035\ f_3(\underline{y}^2)$$

$$\text{s.t.} \quad \underline{y}^2 \in X$$

The solution is

$$\underline{y}^2 = (3.03,\ 0.596,\ 0.45,\ 3,\ 5.506,\ 5.12)$$

Set $\underline{z}^2 = \underline{y}^2 - \underline{x}^2 = (0.04,\ 0.052,\ 0.10,\ -1,\ 0.002,\ 2.72)$

Step 2. Determine t^2

The value of $f(\underline{x}^2 + t\underline{z}^2)$ is given below for 0.2 interval of t

t	0	0.2	0.4	0.6	0.8	1.0
f_1	-3.668	-3.217	-2.766	-2.315	-1.864	-1.413
f_2	-131.719	-130.899	-130.079	-129.259	-128.439	-127.619
f_3	-256.159	-279.784	-298.409	-317.034	-335.659	-354.284

The OM choose $t^2 = 0.2$,

and $\underline{f}^2 = (-3.217, -130.899, -279.784)$

The ratio of improvement on the second iteration to that on the first iteration is:

$$\frac{\Delta^2}{\Delta^1} = \frac{(1, W_2^2, W_3^2)(\underline{f}^2 - \underline{f}^1)}{(1, W_1^3, W_2^3)(\underline{f}^1 - \underline{f}^0)}$$

$$= \frac{(1, 0.147, 0.035)(-3.217+3.068, -130.899+131.719, +256.59)}{(1, 0.027, 0.016)(-3.068+3.035, -131.719+127, -256.59+293.64)}$$

$$= \frac{-0.92}{0.55} = -1.67$$

If we select $\alpha = 0.6$, the termination criteria is not satisfied. Set

$$\underline{x}^3 = \underline{x}^2 + t^2\underline{z}^2 = (2.998,\ 0.555,\ 0.37,\ 3.8,\ 5.972,\ 2.944)$$

and $i = i+1 = 3$. Go to the next iteration.

Iteration No. 3

Step 1. Oetermine the direction of improvement

a) Calculate the weights

The DM makes the following trade-offs with f_3 as the reference objective:

For f_3 and f_1:

$(-3.217, -130.899, -279.784) = (-3.217 - 0.025, -130.899, -279.784 + 8)$

For f_3 and f_2:

$$(-3.217, -130.899, -279.784) = (-3.217, -130.899 - 4, -279.784 + 8)$$

That is

$$W_1^3 = \frac{8}{-0.025} = 320$$

$$W_2^3 = -\frac{8}{-4} = 2$$

$$W_3^3 = 1$$

The weight vector is equivalent to

$$\underline{W}^3 = (320/320,\ 2/320,\ 1/320)$$

$$= (1,\ 0.006,\ 0.003)$$

b) Calculate $\underline{y}^3$ and $\underline{z}^3$

$\underline{y}^3$ is obtained by solving the following LP problem.

$$\max \quad f_1(\underline{y}^3) + 0.006\ f_2(\underline{y}^3) + 0.003\ f_3(\underline{y}^3)$$

$$\text{s.t.} \quad \underline{x} \in X$$

The solution is

$$\underline{y}^3 = (3.021,\ 0.49,\ 0.40,\ 3.91,\ 1.899,\ 1.54)$$

and

$$\underline{z}^3 = (0.023,\ -0.065,\ 0.03,\ 0.11,\ -4.028,\ -1.404)$$

Step 2. Oetermine t^3

The value of $\underline{f}(x^3 + t\underline{z}^3)$ is gven below for 0.2 interval of t

t	0	0.2	0.4	0.6	0.8	1.0
f_1	-3.217	-3.117	-3.017	-2.917	-2.817	-2.717
f_2	-130.899	-131.60	-132.30	-133.00	-133.70	-134.41
f_3	-279.784	-266.44	-253.10	-239.75	-226.41	-213.07

The OM choose $t^3 = 0.4$

and $\underline{f}^3 = (-3.017, -132,30, -253.10)$

The ratio of improvement on the third iteration is

$$\frac{\Delta 3}{\Delta 1} = \frac{(1, W_2^3, W_3^3)\ (\underline{f}^3 - \underline{f}^2)}{(1, W_2^1, W_3^1)\ (\underline{f}^1 - \underline{f}^0)}$$

$$= \frac{(1,0.006,0.003)(-3.017+3.217,-132.30+130.899,-253.1+279.79)}{(1,0.027,0.016)(-3.068+3.035,-131.719+127,-256.159+293.64)}$$

$$= \frac{0.272}{0.432} = 0.56$$

which satisfies the given criteria $\alpha = 0.6$. We terminate the iteration and have the final solution

$\underline{x} = (3.00, 0.542, 0.376, 3.822, 5.17, 2.66)$

$\underline{f} = (-3.017, -132.30, -253.10)$

CHAPTER 6

CONCLUSIONS AND FURTHER RESEARCH

Conclusion: The two models presented in this work illustrate the use of the five proposed MODM methods in the planning, design and operation of water resources systems.

The nonlinear programming technique GRG used in nonlinear model presented no convergence difficulties. While the MPS/360 was used for the linear cases. All the computer works are implemented on ITEL AS/5 system. During the approaching process, the roles of the DM and analyst were played by the same person. The best compromise solutions obtained by each method are not in good agreement. The reason is that in each method, different kinds of information were sent to the DM. The DM's response wouldn't be the same with the information with different characteristics. Besides, the convergence property, the information needed from the DM, and the confidence the DM has are quite different. However, we could make the evaluation of each method according to our experience in this work:

Evaluation:

Since all of the above mentioned five MODM methods have been developed only in recent years. The evaluation of them is scarce.

The criterias for evaluating the performance of MODM methods are complex and have not been explicitly defined and discussed in literature. However, based on our research, we think the following five criteria used in Wallenius' paper are most suitable:

Criteria 1: DM's confidence in the best compromise solution.

Criteria 2: Ease of use of the method.

Criteria 3: Ease of understanding the logic of the method.

Criteria 4: Usefulness of the information provided to aid the DM.

Criteria 5: Rapidity of convergence, measured by the number of cycles and the total time for solving the problem.

Based on the above criterias, we shall use five levels of grade - excellent, very good, good, fair and poor to evaluate the results of various methods.

[1] Goal Programming

Criteria 1: Excellent

The DM does not need to give the numerical weights for the objectives. He needs to give only an ordinal ranking of them.

Criteria 2: Excellent

The goals can easily be set by the DM. For minimization problem, it can be set to zero. For maximization problem, it can be set to a large number. The analyst only assigns under-achievement and over-achievement associated with each goal and forms the formulation.

Criteria 3: Very good

The basic logic of this method is that a lower ranking achievement function can not satisfy the detriment of a higher ranking achievement function.

Criteria 4: Fair

The method only requires the DM to give some information before implementation. It does not generate so much information for DM as the interactive method does.

Criteria 5: Good

Even though this method is powerful to use iterative way to solve complex problem, it spends relatively longer time than using other methods.

[2] Surrogate Worth Trade-Off Method

Criteria 1: Good

The DM is not so confident in this method because of:

1) The trade-off values between any two objectives are effective only in a narrow range.

2) The linear interpolation technique does not guarantee a best compromise solution.

Criteria 2: Excellent

This method can handle both linear and nonlinear models. The analyst interactive with the DM toward the compromise solution.

Criteria 3: Very good

This method employs the ε-constraint method and the Kuhn-Tucker theory to generate the nondominated solution set associated with trade off values, with the DM's preference or with the linear interpolation. Therefore, the best compromise solution could be obtained.

Criteria 4: Good

Even there are trade-off values associated with each nondominated solution, it is effective only within a narrow range. It could not guarantee a best compromise solution.

Criteria 5: Good

For nonlinear model, it is too complex to employ Kuhn-Tucker theory to generate nondominated solutions.

[3] STEM Method

Criteria 1: Excellent

This method allows the DM to learn to recognize good solutions and the relative importance of the objectives.

Criteria 2: Very Good

In this method, the DM is presented with subset of non-dominated extreme points. If the subset contains an acceptable solution, the procedure is terminated. Otherwise the DM chooses a best subset of solutions, which is used to determine a new set of nondominated extreme points.

Criteria 3: Very Good

If the DM is satisfied with all the objectives or no objectives, the procedure is terminated. If some objectives are satisfactory and others are not, the DM must relax the satisfactory one to improve the others.

Criteria 4: Very Good

This method provides the DM all the needed information such as ideal solution, minimum solution of the objective function. The weighted values of objectives (Π_j) are also provided.

Criteria 5: Very Good

The best compromise solution is usually obtained within three iterations. Convergence of the problem is guaranteed.

[4] SEMOPS Method

Criteria 1: Fair

The DM can hardly select the upper and lower bounds of the objective function values. The goal level and the aspiration level are set by the DM without any preliminary solution.

Criteria 2: Good

The analyst must cyclically uses a surrogate objective function which is based on goals and the DM's aspirations toward the achieving of objectives.

Criteria 3: Fair

The SEMPOS method requires that the DM understand the whole method and make right decision all the time. Actually, this method includes too many cases and rules that it becomes impossible for all the DM to handle it.

Criteria 4: Very Good

This method provides the DM with the resulting policy vector and objectives for the principal problem and the set of auxiliary problems. Then the DM could set the new aspiration level.

Criteria 5: Fair

When the DM couldn't set a proper aspiration level and goals, it is probable to find incosistent constraint set when solving auxiliary problems.

[5] Method of Geoffrion

Criteria 1: Very Good

The DM need only assess the trade-off between two objectives on a specific achievement level of the objectives.

Criteria 2: Excellent

This method can treat both linear and nonlinear problems. It has powerful convergence property as well as its simplicity.

Criteria 3: Very Good

Using the well-known Frank-Wolfe algorithm, this method determined an optimal weighted coefficient for each objective function. When the utility function is maximized, we may calculate the direction of improvement, $\underline{z}$, then provide the DM with the best compromise solution.

Criteria 4: Very Good

The complete information needed is provided to the DM. Also, this method interactively informs the DM and help him to correct the decision.

Criteria 5: Excellent

With the robust convergence property, this method usually guarantees a compromise solution within three or four iterations.

Further Research: The MODM models, methods and their applications were developed in recent years. Only very few methods have been tested with the real-world problems. The MODM models are surprisingly few existing in the literature.

From a methodological view, there are two areas which have not been studied extensively: MODM problems with uncertainty and stochastic process, and the MODM dynamic control problems.

To make MODM methods more useful in analytical sense, further studies are needed to develop a methodology to provide the DM with rigid information and precisely convert the DM's response into the procedure.

Table 6.1 Evaluation of five proposed methods

	Method / Criteria	Goal Programming	Surrogate Worth Trade-off Method	STEM Method	SEMOPS Method	Method of Geoffrion
1	DM's confidence in the best compromise	Excellent	Good	Excellent	Fair	Very Good
2	Ease of the use	Excellent	Excellent	Very Good	Good	Excellent
3	Ease of the under-standing the logic	Very Good	Very Good	Very Good	Fair	Very Good
4	Usefulness of the information	Fair	Good	Very Good	Very Good	Very Good
5	Rapidity of convergence	Good	Good	Very Good	Fair	Excellent

REFERENCES

1. Benayoun, R., O. Larichev, J. de Montgolfier, and J. Tergny, "Linear Programming with Multiple Objective Functions: The Method of Constraints" Automatic and Remote Control, Vol. 32, No. 8, pp. 1257-1264, 1971.

2. Benayoun, R., J. de Montgolfier, J. Tergny, and O. Larichev, "Linear Programming with Multiple Objective Functions: Step Method (STEM)", Mathematical Programming, Vol. 1, No. 3, pp. 366-375, 1971.

3. Charnes, A. and W. W. Cooper, Management Models and Industrial Applications of Linear Programming, Vol. 1, Wiley, N.Y., 1961.

4. Cohon, J. L., and Marks, D. H., "Multiobjective Screening Models and Water Resources Investment" Water Resources Research, Vol. 9, No. 4, 1973.

5. Dorfman, R., and H. Jacoby, "A Model for Public Decision Illustrated by a Water Pollution Policy Problem, The Analysis and Evaluation of Public Expenditures: The PPB Systems", pp. 226-276, Joint Economic Committee, 91st Congress, 1st Session, 1969.

6. Fair, G., J. Geyer, and D. Okum, Water and Waste Water Engineering, Vol. 2, pp. 123-151, John Wiley, N.Y., 1968.

7. Geoffrion, A. M. and W. Hogan, "Coordination of Two-level Organizations with Multiple Objectives", in Balakrisknan (ed.), Techniques of Optimization, pp. 455-466, Academic Press, N.Y., 1972.

8. Geoffrion, A. M., J. S. Dyer, and A. Feinberg, "An Interactive Approach for Multicriteria Optimization, with an Application to the Operation of an Academic Department," Management Science, Vol. 19, No. 4 (Part 1), pp. 357-368, 1972.

9. Griffith, R. E., and P. A. Stewart, "A Nonlinear Programming Technique for the Optimization of Continuous Processing Systems", Management Science, Vol. 7, No. 4, pp. 379-392, 1961.

10. Haimes, Y. Y. and W. A. Hall, "Multiobjectives in Water Resources Systems Analysis: The Surrogate Worth Trade Off Method", Water Resources Research, Vol. 1D, No. 4, pp. 615-623, 1974.

11. Haimes, Y. Y., L. S. Lasdon and D. A. Wismer, "On a Bicriterion Formulation of the Problems of Integrated System Identification and System Optimization," IEEE Trans. on Systems, Man, and Cybernetics, Vol. 1 SMC-1, No. 3, pp. 296-297, 1971.

12. Haimes, Y. Y., and D. A. Wismer, "A Computational Approach to the Combined Problem of Optimization and Parameter Identification," Automatica, Vol. 8, No. 3, pp. 337-347, 1972.

13. Haimes, Y. Y., W. A. Hall and H. T. Freedman, Multiobjective Optimization in Water Resources Systems, The Surrogate Worth Trade-off Method, Elsier Scientific, N.Y., 1975.

14. Hall, W. A. and Y. Y. Haimes, "The Surrogate Worth Trade-off Method with Multiple Decision-makers", Multiple Criteria Decision Making: Kyoto 1975, in Zeleny (ed.), Springer-Verlag, N.Y., 1976.

15. Hwang, C. L. and S. Paidy, "Regional Water Quality Management by Nonlinear Goal Programming", to be published.

16. Hwang, C. L., S. Paidy, K. Raju, and A. Masud, "Nonlinear Goal Programming Method - An Iterative Approach", to be published.

17. Hwang, C. L., F. A. Tillman, K. Yoon, and A. Masud, "Optimum System Reliability by Nonlinear Goal Programming", to be published.

18. Hwang, C. L., and S. M. Masud, "Multiobjective Decision Making - Methods and Applications, Springer-Verlag, N.Y., 1978-79.

19. Ignizio, J. P., "An Approach to the Capital Budgeting Problem with Multiple Objectives", The Engineering Economist, Vol. 21, No. 4, pp. 259-272, 1976.

20. Ijiri, Y., "A Historical Cost Approach to Aggregation of Multiple Goals", in Zeleny (ed.), Multiple Criteria Decision Making, University of South Carolina Press, Columbia, South Carolina, 1973.

21. Lee, S. M., "Goal Programming for Decision Analysis", Auerbach Publishers, Philadelphia, Pennsylvania, 1972.

22. Lee, S. M. and E. R. Clayton, "A Goal Programming Model for Academic Resource Allocation", Management Science, Vol. 18, No. 8, pp. B395-B408, 1972.

23. Lee, S. M. and V. Jaaskelainen, "Goal Programming: Management's Math Model, Industrial Engineering, Vol. 3, No. 2, pp. 30-35, 1971.

24. Lee, S. M. and L. J. Moore, "Multi-criteria School Busing Models", Management Science, Vol. 23, No. 7, pp. 703-715, 1977.

25. Masud, A. and C. L. Hwang, "A Multiple Objective Energy Planning and Allocation Model", Dept. of Industrial Engineering, Kansas State University, Manhattan, Kansas.

26. Monarchi, D. E., "Interactive Algorithm for Multiple Objective Oecision Making", Technical Reports on Hydrology and Water Resources, (No. 6), University of Arizona, 1972.

27. Monarchi, D. E., C. C. Kisiel, and L. Duckstein, "Interactive Multi-objective Programming in Water Resources: A Case Study", Water Resources Research, Vol. 9, No. 4, pp. 837-85D, 1973.

28. Monarchi, D. E., J. E. Weber, and L. Duckstein, "An Interactive Multiple Objective Decision-Making Aid using Nonlinear Goal Pro-gramming", in Zeleny (ed.), Multi-criteria Decision Making-Kyoto, 1975, Springer-Verlag, N.Y., 1976.

29. Moran, P. A. P., "A Probability Theory of a Dam with Continuous Release", Quart. J. Mathematics, Dxford, Second series, Vol. 7, No. 26, pp. 13D-137, June, 1956.

30. Passy, U., "On the Cobb-Oouglas Functions in Multiobjective Dpti-mization", Water Resources Research, Vol. 6, pp. 688-69D, 1978.

31. Reid, R. W., and V. Vemuri, "Dn the Non-inferior Index Approach to Large Scale Multi-Criteria Systems", Journal of the Franklin Institute, Vol. 291, No. 4, pp. 241-254, 1971.

32. Reid, R. W., and S. J. Citron, "On Non-inferior Performance Index Vectors," Journal of Optimization Theory and Applications, Vol. 7, No. 1, pp. 11-28, 1971.

33. Streeter, H. W. and E. B. Phelps, "A Study of the Pollution and Natural Purification of the Ohio River, Illinois Factors Concerned in the Phenomenon of Oxidation and Reaeration", Public Health Service Bulletin, Vol. 146, 1925.

34. Vermuri, V., "Multiple Objective Optimization in Water Resources Systems", Water Resources Research, Vol. 10, No. 1, pp. 44-48, 1974.

35. Wallenius, J., "Comparative Evaluation of Some Interactive Approaches to Multi-criterion Optimization", Management Science, Vol. 21, No. 12, pp. 1387-1396, 1975.

36. Yoon, K. and C. L. Hwang, "A Goal Programming Model for Development Planning: A Case of Korea", Dept. of Industrial Engineering, Kansas State University, Manhattan, Kansas.

APPENOIX

The generalized reduced gradient algorithm is coded in FORTRAN IV in the GREG program. This program was developed by Abadie and his associates of France. The GREG program consists of a main program, nine permanent or internal subroutines, and four user supplied, temporary or external subroutines. A typical source deck consists of about 2500 cards, requires approximately 150K (K = 1024 bytes) of storage and 18 minutes of CPU time to compile by the IBM FORTRAN IV (G) compiler. It needs 120K of storage for execution.

The GREG program will handle problems involving up to 50 inequality and/or equality constraints. The maximum number of variables a problem may have depends on the type of constraints involved. The program automatically provides slack and artificial variables. There is one slack variable added for each inequality constraint. For a given constraint, if the starting point is not feasible, an artificial variable is supplied with an appropriate penalty attached to the objective function.

Use of the GREG program is approached in the following four steps:

(1) The developing of the four user supplied subroutines.

The user supplied subroutines must define the optimization problem in the form of

$$\max \quad f_0(\bar{x}), \ \bar{x} = (x_j | j = 1, \ldots, NV)$$

subject to the constraints

$$f_i(\bar{x}) \leq \text{ or } = 0, \qquad i = 1, \ldots, NC$$

$$a_j \leq x_j \leq b_j \qquad j = 1, \ldots, NV$$

where NV = Number of variables

NC = Number of constraints

Any nonlinear programming problem may be put into this form. The four subroutines that describe this problem set-up to the GREG are PHIX, CPHI, JACOB, and GRADF. PHIX defines the objective function, CPHI the constraint function, JACOB the gradients of the constraint functions, and GRADFI the gradient of the objective function. Each of the four subroutines performs a unique task. They are referred to many times during the execution of the program and warrant careful programming considerations.

Each of the user supplied subroutines must contain a set of "common" statements that are commensurate to the internal GREG subroutines. Figure A.1 shows a list of the GREG common block definition statements.

PHIX

The external, user-supplied subroutine PHIX defines the objective function to the GREG program. This value is stored in FORTRAN variable PHI, and is described in terms of the FORTRAN vector array, XC(J). Only the original problem variables are used. That is, J ranges from one to NV. The penalties due to the artificial variables are added to PHI automatically in an internal subroutine.

CPHI

CPHI defines the constraint functions as previously defined ($\leq$ or = 0). The values are stored in the vector array VC(I), I = 1, ..., NC, and in terms of the original problem variable, XC(J), J = 1, ..., NV. The constraints must be ordered with inequalities first and equalities second.

JACOB

The subroutine JACOB defines the gradients of the constraint functions. The partial derivative $\partial f_i/\partial x_i$ is stored in the matrix array A(i,j). The rows of the matrix represent each constraint function, $f_i(\bar{x})$, i = 1, ..., NC, in the same order as sequenced in CPHI. The partial derivatives are represented in terms of the FORTRAN variable XC(j), j = 1,2, ..., NV.

Constant values may also be initialized in JACOB. It is the third subroutine referred to and may be used to initialize values in GRADFI.

GRADFI

The fourth and final user-supplied subroutine is GRADFI. This subroutine defines the gradient of the objective function in terms of the array XC(J), J = 1, ..., NV. The component values are stored in the vector array C(J), J = 1, ..., NV.

Like the other subroutines, initialization may be accompanied in GRADFI, but only for this subroutine since it is the last one called for initialization purpose.

INPUT DATA FOR THE GREG PROGRAM

To use the GREG program, values for nineteen parameters, a starting point, a lower bound, and an upper bound must be established. The parameter input has been programmed in such a way that a minimum of two may be read in as input while the remaining ones take on default values. Besides this, the user may have designed some input for initialization purpose in PHIX, CPHI, JACOB, and GRADFI. It is the purpose of this section to discuss the above three types of input data. The GREG program is extremely sensitive to the values given to the parameters. This allows only seven significant digits and stopping criteria smaller than 1.0×10^{-7} can yield erroneous results.

MULTIOBJECTIVE DECISION

MAKING IN WATER RESOURCES SYSTEM

by

YEN-TUNG CHANG

B.S. (Industrial Engineering)

NATIONAL TAIWAN INSTITUTE OF TECHNOLOGY

TAIPEI, TAIWAN, 1976

AN ABSTRACT OF A MASTER'S THESIS

submitted in partial fulfillment of the

requirements for the degree

MASTER OF SCIENCE

Department of Industrial Engineering

KANSAS STATE UNIVERSITY

Manhattan, Kansas

1979

ABSTRACT

This work involves finding the best compromise solution over the multiobjective water resources systems. The decision-maker, analyst and machine work interactively toward the achieving of the best compromise solution.

Two models of water resources systems were demonstrated. The one is Bow River Valley water quality problem. It concerns with the pollution problems of an artificial river basin. The other model involves the reservoir storage capacity problem.

Five multiobjective decision making methods were demonstrated and applied to approach the best compromise solutions in this work. All these methods are mathematical programming which help the decision maker make proper decision and consequently obtain the desired solution among all the nondominated solutions.

Generalized Reduced Gradient technique and its computer code were used as a tool to solve the nonlinear programming cases.

Evaluation of each method has been done based on five selected criterias which indicate the performance of these five methods.