A STUDY OF OPTIMUM ECONOMIC OPERATION OF ELECTRIC POWER SYSTEMS

by

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A MASTER'S REPORT

submitted in partial fulfillment of the

requirements for the degree

MASTER OF SCIENCE

Department of Electrical Engineering

KANSAS STATE UNIVERSITY
Manhattan, Kansas

1964

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INTRODUCTION

Methods of optimum operation of power systems have been defined in the literature in many different ways[10]. Optimization techniques, which vary from a consideration of the derivatives to the calculus of variations, Lagrangian multipliers, linear programming, integer programming, and dynamic programming, have been successfully applied to the solution of electric utility system operating and planning problems, and significant system improvements have been achieved.

Minimizing the total cost of system production, yet maintaining all the requirements such as loads, operating restrictions, is known as optimum economic operation. Choosing the technique to be used depends on the problem that exists, since no particular technique among those that are mentioned can be considered to be the best. According to this reason and the author's interest, this paper makes a study of the classical techniques applying to the solution of electric power system economic operating problems. Beginning with the relatively simple problems; i.e., all-thermal problems, the study then goes to the more complex one; i.e., hydrothermal problems.

ALLOCATION OF THERMAL PLANTS

Coordination Equations[7]

In viewing the problem of determining the allocation of generation among thermal plants that are currently operating and on the line in the area of system operation, it is necessary to recognize the different costs of fuels, the various thermodynamic characteristics, and the losses in the transmission network. It is desired that the total input to the system in dollars per hour be a minimum at each instant with the restriction that the load requirements be maintained. The method of Lagrangian multipliers[13] handles the solution of the problem as follows.

Let

F_t = total input to system in dollars per hour

F_n = input to plant n in dollars per hour

P_n = output of plant n in megawatts

P_L = total transmission losses to system in megawatts

P_R = given received load in megawatts

It is desired that

$$F_t = \sum_n F_n = \text{minimum}$$

subject to a constraint

$$\psi = P_R + P_L - \sum_n P_n = 0$$

Also, it must be noted that each plant has a certain minimum and maximum rating that must be observed.

Form the function

$$\mathscr{F} = F_t + \lambda \psi$$

where

$$\lambda = \text{Lagrangian multiplier}$$

the minimum input for a given received load is obtained when

$$\frac{\partial \mathscr{F}}{\partial P_n} = \frac{\partial F_t}{\partial P_n} + \lambda \frac{\partial \psi}{\partial P_n} = 0$$

Thus

$$\frac{\partial (\sum_n F_n)}{\partial P_n} + \lambda \frac{\partial}{\partial P_n} (P_R + P_L - \sum_n P_n) = 0$$

$$\frac{\partial F_n}{\partial P_n} + \lambda \left(0 + \frac{\partial P_L}{\partial P_n} - \frac{\partial P_n}{\partial P_n} \right) = 0$$

$$\frac{dF_n}{dP_n} + \lambda \frac{\partial P_L}{\partial P_n} = \lambda \tag{1}$$

or

$$\frac{dF_n}{dP_n} L_n = \lambda \tag{2}$$

where

$$L_n = \frac{1}{1 - \dfrac{\partial P_L}{\partial P_n}} \tag{3}$$

The terms involved in equations (1) and (2) were defined as follows

$\dfrac{dF_n}{dP_n}$ = incremental production cost of plant n in dollars per megawatt-hour

$\dfrac{\partial P_L}{\partial P_n}$ = incremental transmission loss at plant n in megawatts per megawatt

λ = incremental cost of received power in dollars per megawatt-hour

L_n = penalty facter of plant n

Thus it is seen that the optimum allocation of generation among thermal plants is given by the solution of a set of simultaneous nonlinear equations (1). This set of equations shows that the optimum economy is obtained when the incremental cost of received power at the system load is the same from each source. Note that if the incremental transmission loss at plant n is charged at a constant rate β instead of λ, the following set of linear simultaneous equations results:

$$\frac{dF_n}{dP_n} + \beta \frac{\partial P_L}{\partial P_n} = \lambda \tag{4}$$

β may be choosen as the average values of λ. If the incremental transmission loss of plant n is charged at a rate corresponding to the incremental production cost of plant n, equation (1) becomes

$$\frac{dF_n}{dP_n} + \frac{dF_n}{dP_n}\frac{\partial P_L}{\partial P_n} = \frac{dF_n}{dP_n}\left(1 + \frac{\partial P_L}{\partial P_n}\right) = \lambda \tag{5}$$

which is called the approximate penalty-factor equation, since L_n now is approximated by $(1 + \partial P_L/\partial P_n)$.

A number of computer-control arrangements[8] in service today maintain, on a continuous basis, economic allocation of generation according to solution of these equations while simultaneously maintaining frequency and the desired interchange. Interchange is defined as the summation of flows on the transmission lines forming the boundary of a given system.

Incremental Production Costs

The production cost of a given unit is made up of fuel cost plus the cost of such items as labor, supplies, maintenance, and water. Only fuel cost can be expressed accurately as a function of output, the cost of the latter items, however, may be assumed to be a fixed percentage of the fuel cost. In many systems, for purposes of scheduling generation, the incremental production cost is assumed to be equal to the incremental fuel cost. When the fuel cost is known, the incremental fuel cost values are obtained from the input-output curve of the unit without difficulty[12].

The dF_n/dP_n must not decrease as the output increases to assure that F_t attains a minimum value provided that F_n is a continuous function of P_n. Since approximate methods may prove helpful in solving engineering problems, the incremental fuel cost curve can be adjusted to meet the restriction above. Of course, there are many different shapes of such adjusted curves, but only two types, namely smooth incremental curves including connected straight line segments, curves and step incremental curves, are in general use. The input-output curve that corresponds to a smooth incremental curve is shown in Figure 1 and that corresponding to a step incremental curve is shown in Figure 2. Note that both of them are continuous functions of the output. The step incremental curves have been found to yield a lower cost schedule than the smooth incremental curve when effects of valve loops are considered[11]. The exact incremental curve when the effects of valve loops are considered is shown in Figure 3, accompanied with its input-output curve and step incremental curve representation.

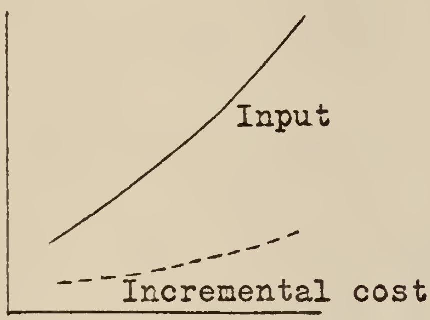

Figure 1

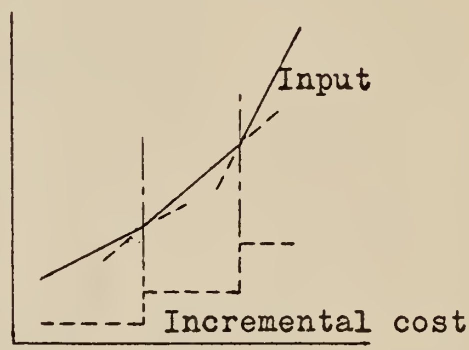

Figure 2

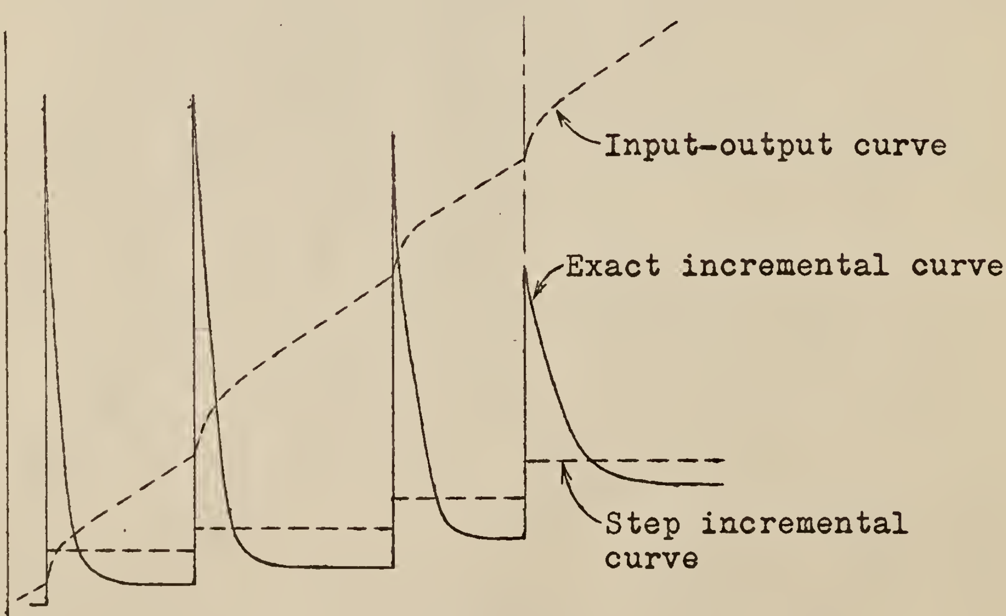

Figure 3

Incremental Transmission Losses

In order to complete the solution of the optimization problem, one must be able to compute the function $\partial P_L/\partial P_n$. This is an important interrelationship between modeling and optimization. For this particular modeling problem, an expression for transmission losses of the system in terms of source loading and a set of loss formula coefficients has been developed[7]. It is of the following form:

$$
\begin{aligned}
P_L &= B_{11}P_1^2 + B_{22}P_2^2 + B_{33}P_3^2 + \dots + B_{nn}P_n^2 \\
&\quad + 2B_{12}P_1P_2 + 2B_{13}P_1P_3 + \dots \\
&\quad + 2B_{23}P_2P_3 + \dots + 2B_{mn}P_mP_n \\
&= \sum_m \sum_n P_m B_{mn} P_n \qquad (6)
\end{aligned}
$$

where

P_m, P_n = output of plant m, n in megawatts

$B_{mn} = B_{nm}$ = transmission-loss-formula coefficients

The assumptions involved in deriving a loss formula of this form are

1. The equivalent load current at any bus remains a constant complex fraction of the total equivalent load current.

2. The voltage magnitudes and angles at all generator and tie points remain constant. (Usually the values of the base case load flow are used).

3. All shunt paths to neutral, line charging, and synchronous condenser reactive powers are lumped with the system loads.

4. Transformation ratios are unity around each closed loop of the network, that is the turn-ratios of transformers do not differ from the nominal turn ratios.

5. The ratios of source reactive power to source output, Q_m/P_m, remain constant. (The base case values are usually used).

In using this loss formula, nonconforming loads may be included properly as negative sources. Also, it is sometimes desirable to divide the loads at the various buses into a component which varies with the total and a component which remains constant. The constant components are treated as negative generations in the loss formula. The loss formula then includes linear terms and a constant term in addition to the quadratic terms. It is

$$P_L = \sum_m \sum_n P_m B_{mn} P_n + \sum_n P_n B_{no} + B_{oo} \qquad (7)$$

where

$B_{no} = 2P_j B_{nj}$

$B_{oo} = P_j B_{jk} P_k$

P_j, P_k = constant megawatt components of loads

B_{nj} = mutual loss-formula coefficients between constant components of load and generators.

B_{jk} = self and mutual loss-formula coefficients for constant components of loads

This form of loss formula allows more flexibility in the first assumption relating to the manner in which each individual load varies with the total load; that is, each individual load current is now assumed to be a linear complex fraction of the total load current.

When the flows in tie lines interconnecting the area studied with foreign areas are considered to be independent of generation allocation within the area studied, the incremental transmission losses are given by

$$\frac{\partial P_L}{\partial P_n} = \sum_m 2P_m B_{mn} \tag{8}$$

or

$$\frac{\partial P_L}{\partial P_n} = \sum_m 2P_m B_{mn} + B_{no} \tag{9}$$

However, for an area with several interconnecting ties, the individual tie-line flow may change even though the net interchange out of the area remains constant. The changes in these flows also contribute to the change in transmission loss and are included in the expression for $\partial P_L/\partial P_n$ as

$$\frac{\partial P_L}{\partial P_n} = \sum_m 2P_m B_{mn} + \sum_f \frac{\partial P_f}{\partial P_n} \sum_m 2P_m B_{mf} \tag{10}$$

or

$$\frac{\partial P_L}{\partial P_n} = \sum_m 2P_m B_{mn} + B_{no} + \sum_f \frac{\partial P_f}{\partial P_n} \left(\sum_m 2P_m B_{mf} + B_{fo}\right) \tag{11}$$

where

$\frac{\partial P_f}{\partial P_n}$ = ratio of the change in tie-line flow P_f to the change in P_n

B_{mf} is similar to B_{mn}

B_{fo} is similar to B_{no}

Figure 4 shows the tie-line flows.

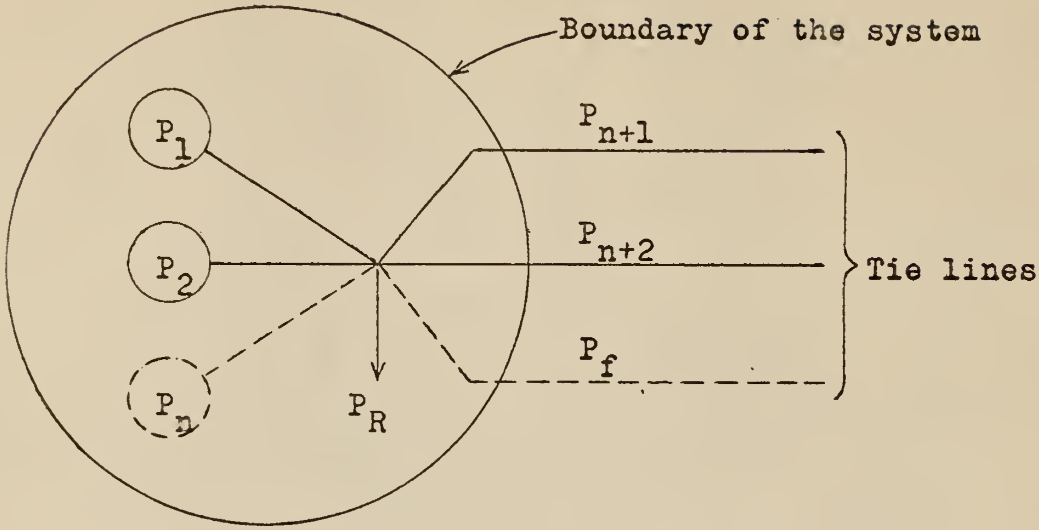

Figure 4

Because the system constants are absorbed into B_{mn} by the transformations and mathematical manipulations it is difficult to provide revised loss formulae coincident with each major system revision and temporary outage of lines or transformers. These formulae still give satisfactory results as long as the power system is operating at conditions similar to the base case. The future development of digital computers will overcome the problem of revisions in loss formulae and some or all of their assumptions may be eliminated[14].

Incremental Cost of Received Power

In Figure 5, the incremental production cost of a given

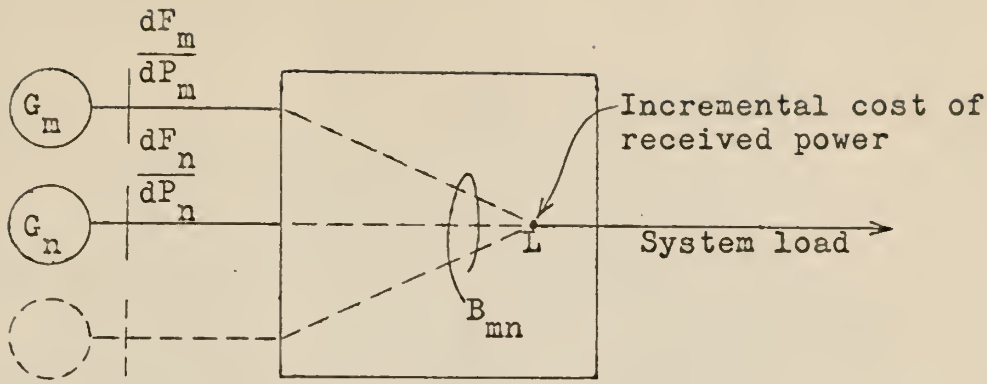

Figure 5

plant n is measured at the plant bus and is denoted by dF_n/dP_n. Suppose that the load increases by an amount ΔP_R, and assume that this load change is first taken up by plant n only by increasing the output of plant n by ΔP_n. Then the cost of this increment of power at the receiver point L is given by

$$\lambda_n \equiv \frac{dF_n}{dP_n} \frac{\Delta P_n}{\Delta P_R}$$

which may be rewritten as

$$\lambda_n \equiv \frac{dF_n}{dP_n} \frac{\Delta P_n}{\Delta P_n - \Delta P_L}$$

$$= \frac{dF_n}{dP_n} \frac{1}{1 - \dfrac{\Delta P_L}{\Delta P_n}}$$

As ΔP_n becomes progressively smaller, we have

$$\lambda_n \equiv \frac{dF_n}{dP_n} L_n$$

which is the same as the term on the left hand side of equation

(2) so that λ is called the incremental cost of received power. Equation (2) requires the incremental cost of the power received from each plant to be the same at the receiver point L.

Solutions of equation (2) (also equations (1), (4) and (5)) for different total loads are obtained by varying the magnitude of λ.

Penalty Factors

In general, the penalty factor of a given plant may be thought of as the reciprocal of the incremental efficiency of the transmission network with respect to supplying an increment of system load from that plant, since the limit of $\Delta P_n/\Delta P_R$ or $\Delta P_n/(\Delta P_n - \Delta P_L)$ as ΔP_n becomes progressively smaller is L_n. When the incremental efficiency of the transmission network is 100 percent or when the transmission loss of the system, P_L, is neglected and does not appear in the constraint, the penalty factors, L_n, become unity and equation (2) becomes

$$\frac{dF_n}{dP_n} = \lambda \qquad (12)$$

The generation schedule based on equation (12) is then one of equal incremental production cost, while the schedule based on equation (1) is one obtained by coordinating the incremental production costs and the incremental transmission losses. Evaluating annual savings of the latter schedule over the former one can be obtained by using a load-duration curve[7].

UNIT COMMITMENT SCHEDULING

The generation allocation methods discussed previously have been widely applied in the industry. However, the principle of scheduling to equal incremental costs of received power does not directly determine the units to be placed in operation at a given time[3,7]. Determination of the units is based upon such consideration as

1. Economic evaluation.
2. Reserve requirements.
3. Stability limitations.
4. Voltage limitations.
5. Ability to pick up load quickly.

Very frequently, and in particular in widespread systems, conditions 2 to 5 overrule condition 1.

The determination of the most economic combination of the units to be placed in operation at a given time must recognize the total costs involved; that is, in addition to incremental costs, the no-load costs and the costs of starting and stopping units must be included. This problem to date has been usually solved by successive trials in which various combinations of units are assumed, total costs evaluated, and the best of the alternatives chosen. Of course, for any assumed capacity in operation the economic allocation of generation is given by equal incremental-cost loading. The digital computer offers a great advantage over the analog computer for undertaking such calculations, since neither total fuel input nor total transmission losses are readily and economically obtainable with existing designs of analog computers.

In general, in a given station the units are placed in service in ascending order of their heat rates assuming the cost per Btu to be the same. To determine the most economic combination of units for a given station load it is necessary to plot total station heat-rate curves of successive combinations and to note the combination providing the lowest heat rate for a given station load.

Another problem of importance is to determine the economic advisability of taking units off the line for relatively short periods of time, such as between the morning and evening peaks. This determination is based upon calculating the total fuel input in dollars to the system during this period of time with the units in question both on and off the line. This calculation should include cost of restoring the units under consideration back in service and losses involved in banking the boilers.

More recently, the method of integer programming has been applied by Dr. Garver[3] to this problem for the case in which transmission losses may be neglected. It is necessary that the variables denoting start-up and shutdown be either zero or one.

In other words, whole numbers of generators are to be scheduled. For this reason, the problem cannot be solved by the usual methods of linear programming, but requires the use of integer programming. The application of this method to large systems may be limited by the dimensionality of the variables.

FUEL SCHEDULING

The allocation of generation methods discussed previously is predicated upon a known cost of fuel at each plant. This fuel cost is related to the manner in which fuel is purchased. For some companies a number of alternate sources of fuel are available, and thus the scheduling of fuel purchased is subjected to methods of optimization. Consider, for example, the complex of mines and stations shown in Figure 6[9].

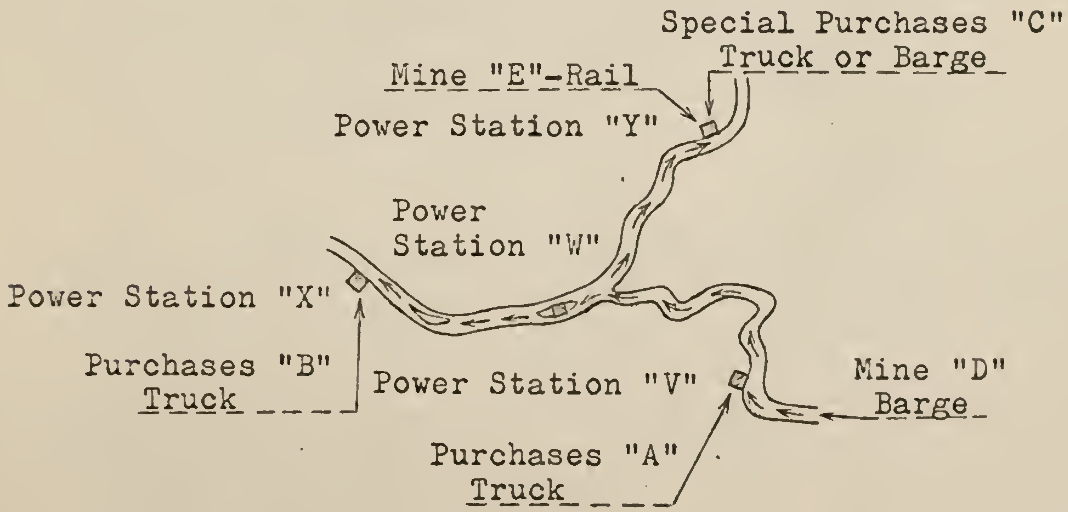

Figure 6

The purchase contracts establish minimum deliveries and provide for the purchase of variable amounts up to a maximum tonnage for the quoted price. The mine costs are based on a fixed annual charge and an incremental cost of fuel per ton between the minimum and maximum straight-time production. Additional production is available by overtime operation at an increased cost per ton.

For this problem, we desire to minimize

$$Z = \sum C_{ij} X_{ij} = \text{total cost of fuel}$$

where

X_{ij} = shipment from mine i to station j ≥ 0

C_{ij} = per unit cost of X_{ij}

$\sum_j X_{ij} \leq (\text{mine capacity})_i$

$\sum_i X_{ij} = (\text{station requirement})_j$

The problem can be solved by the method of linear programming and, in particular, most advantageously by the transportation technique. Significant assumptions here are that the cost function is linear and the variables are constrained by linear equalities or inequalities.

Furthermore, there exists an economic optimization problem in operation of the power plant under a variety of load and fuel conditions[5]. The objective is to find the operating level and the fuel mixture ratio of each boiler-turbine-generator combination to minimize fuel costs.

ECONOMIC OPERATION OF INTERCONNECTED AREAS[7]

The coordination equation (1) states that for optimum economy the incremental cost of received power should be the same from all sources. This equation would be applicable if all of the areas were treated as a single area and would involve the use of a computer representing the entire interconnected area.

Another approach would involve application of computer-controllers to the individual areas with means of determining automatically the most economic interchange between the areas. It would be desirable for each area to require only a knowledge of the plant loadings within the area and interconnection flows out of the areas in addition to control information which would determine whether the area should increase or decrease its delivery to the interconnected areas. The coordination equation (1) then can be extended to obtain coordination equations whose solution results in optimum economy for the pool formed by the interconnected companies. Multiple-area operation of the pool is defined as operation for which the interchanges between the areas are directly determined and controlled. This theory is first illustrated for two radially interconnected areas and then extended to three loop-interconnected areas.

Two Radially Interconnected Areas

Rigorous coordination equations for an interconnected system may be developed by setting the total differential of the fuel cost equal to zero while recognizing the constraining relations between the variables. Consider, for example, the two area system shown in Figure 7. For this two-area system the constraining equations may be written as

$$\psi_a = P_{Ra} + P_{La} + P_{ea} - \sum_a P_{Ga} = 0$$
$$\psi_b = P_{Rb} + P_{Lb} - P_{ea} - \sum_b P_{Gb} = 0$$

Where P_{ea} is the net interchange or excess flow out of area A.

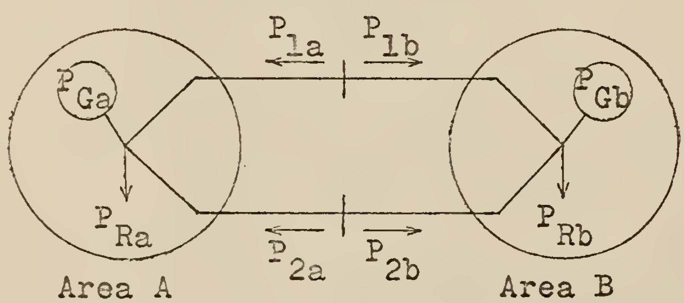

Figure 7

Thus

$$P_{ea} = P_{1b} + P_{2b} = - P_{1a} - P_{2a}$$

Also P_{Ra}, P_{Rb} = received load for area A and B, respectively

P_{La}, P_{Lb} = transmission loss in areas A and B, respectively

P_{Ga}, P_{Gb} = generating plants in areas A and B, respectively

It is desired to minimize the total fuel input to this interconnected system, which is given by F_t = total input in dollars per hour to interconnected system = $F_a + F_b$, where F_a, F_b = input in dollars per hour to areas A and B, respectively.

By the method of Lagrangian multipliers, we obtain the following equations for economic allocation of generation within the areas and interchange between the areas:

$$\frac{\partial \mathscr{F}}{\partial P_{Ga}} = 0$$

$$\frac{\partial \mathscr{F}}{\partial P_{Gb}} = 0$$

$$\frac{\partial \mathscr{F}}{\partial P_{ea}} = 0$$

where $\mathscr{F} = F_t + \lambda_a \psi_a + \lambda_b \psi_b$

The resulting coordination equations are

$$\frac{dF_a}{dP_{Ga}} + \lambda_a \frac{\partial P_{La}}{\partial P_{Ga}} + \lambda_b \frac{\partial P_{Lb}}{\partial P_{Ga}} = \lambda_a \tag{13}$$

$$\frac{dF_b}{dP_{Gb}} + \lambda_a \frac{\partial P_{La}}{\partial P_{Gb}} + \lambda_b \frac{\partial P_{Lb}}{\partial P_{Gb}} = \lambda_b \tag{14}$$

$$\lambda_a + \lambda_a \frac{\partial P_{La}}{\partial P_{ea}} + \lambda_b \frac{\partial P_{Lb}}{\partial P_{ea}} = \lambda_b \tag{15}$$

where $\frac{dF_a}{dP_{Ga}}$ = incremental production cost in dollars per mw-hr of a particular plant G_a in area A

$\frac{dF_b}{dP_{Gb}}$ is similar defined for area B

λ_a, λ_b = incremental cost of received power in area A and B, respectively

$\frac{\partial P_{La}}{\partial P_{Ga}}, \frac{\partial P_{Lb}}{\partial P_{Ga}}$ = ratio of change in transmission loss in area

A and B, respectively, to change in P_{Ga} when delivering an increment of power from P_{Ga} to the hypothetical load of area A

$\frac{\partial P_{La}}{\partial P_{Gb}}, \frac{\partial P_{Lb}}{\partial P_{Gb}}$ = ratio of change in transmission loss in areas A and B, respectively, to change in P_{Gb} when delivering an increment of power from P_{Gb} to hypothetical load of area B

$\frac{\partial P_{La}}{\partial P_{ea}}, \frac{\partial P_{Lb}}{\partial P_{ea}}$ = ratio of change in transmission loss in areas A and B, respectively, to change in net interchange P_{ea} when delivering an increment of power from the hypothetical load of area A to the hypothetical load of area B

The system of Figure 7 can be transformed to be the equivalent circuit shown in Figure 8. The loss formulas for each area then express the losses in terms of generators in both areas and the excess flow of area A.

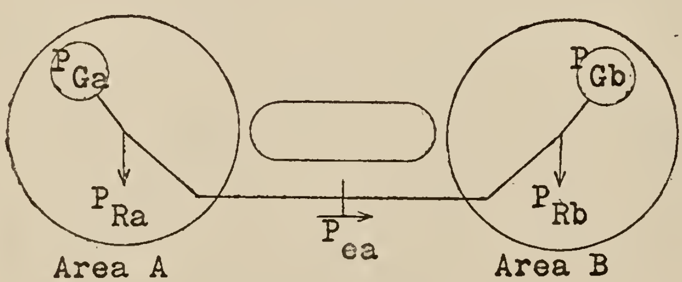

Figure 8

Equations (13) and (14) represent the form of the intra-area equations. The incremental cost of received power in each area is the sum of the incremental cost of production plus the incremental cost of losses in both areas. The cost of the several incremental losses are priced at the cost of the received load in the area in which the losses occur.

Consider equation (13) which corresponds to the following test:

An increment of power is sent from a particular generator in area A to supply an increment of received load in area A. The incremental loss in A is charged off at λ_a. As a result of this test, the power circulating through the parallel area will vary and cause an incremental loss. The resulting area-B incremental loss $\partial P_{Lb}/\partial P_{Ga}$ is charged at λ_b.

Equation (15) is the interarea coordination equation and defines the necessary condition for economic interchange. In effect, at the hypothetical load of area B be equal to the incremental cost of power received from the generating sources of area B. Similar to the intra-area equations, the cost includes the effects of losses in both areas. The incremental cost of received power in area A is analogous to a generation source. These three coordination equations, together with the two constraining equations on received loads, provide the necessary conditions for economic operation of the system.

Equation (15) may be thought of in terms of the following test: An incremental of power is transferred from the hypothetical load of area A to the hypothetical load of area B. As a consequence, the incremental loss $\partial P_{La}/\partial P_{ea}$ is incurred in area A; the incremental loss $\partial P_{Lb}/\partial P_{ea}$ in area B. The incremental loss in each area is charged at the incremental cost of received power in that area. Equation (15) may be written in the form of equation (16) to indicate the costs at the boundary between area A and area B:

$$\lambda_a + \lambda_a \frac{\partial P_{La}}{\partial P_{ea}} = \lambda_b - \lambda_b \frac{\partial P_{Lb}}{\partial P_{ea}} \qquad (16)$$

The quantity on the left-hand side of this equation corresponds to the incremental cost, referred to area A, at the boundary between the two areas for delivering an increment of power from the hypothetical load of area A to the hypothetical load of area B. Similarly, the term on the right-hand side of the equation corresponds to the incremental cost, referred to area B, at the boundary between the two areas for delivering an increment of power from the hypothetical load of area A to the hypothetical load of area B. For optimum economy the boundary cost referred to area A should be the same as the boundary cost referred to area B.

Until now the approach to the problem of economic scheduling has been made in terms of a single equivalent interconnection which carries the net interchange or excess flow. It is possible to express the coordination equations in terms of individual tie-line flows and incremental losses, provided due recognition is given to the dependence of tie-line flows on system generation. The coordination equations for the two-area sample system then take the form shown below:

$$\frac{dF_a}{dP_{Ga}} + \lambda_a \frac{\partial L_{Ta}}{\partial P_{Ga}} + \lambda_a \frac{\partial L_{Ta}}{\partial P_{1a}} \frac{\partial P_{1a}}{\partial P_{Ga}} + \lambda_a \frac{\partial L_{Ta}}{\partial P_{2a}} \frac{\partial P_{2a}}{\partial P_{Ga}} + \lambda_b \frac{\partial L_{Tb}}{\partial P_{1b}} \frac{\partial P_{1b}}{\partial P_{Ga}} + \lambda_b \frac{\partial L_{Tb}}{\partial P_{2b}} \frac{\partial P_{2b}}{\partial P_{Ga}} = \lambda_a \tag{17}$$

$$\frac{dF_b}{dP_{Gb}} + \lambda_b \frac{\partial L_{Tb}}{\partial P_{Gb}} + \lambda_b \frac{\partial L_{Tb}}{\partial P_{1b}} \frac{\partial P_{1b}}{\partial P_{Gb}} + \lambda_b \frac{\partial L_{Tb}}{\partial P_{2b}} \frac{\partial P_{2b}}{\partial P_{Gb}} + \lambda_a \frac{\partial L_{Ta}}{\partial P_{1a}} \frac{\partial P_{1a}}{\partial P_{Gb}} + \lambda_a \frac{\partial L_{Ta}}{\partial P_{2a}} \frac{\partial P_{2a}}{\partial P_{Gb}} = \lambda_b \tag{18}$$

$$\lambda_a + \lambda_a \left(\frac{\partial L_{Ta}}{\partial P_{1a}} \frac{\partial P_{1a}}{\partial P_{ea}} + \frac{\partial L_{Ta}}{\partial P_{2a}} \frac{\partial P_{2a}}{\partial P_{ea}} \right) + \lambda_b \left(\frac{\partial L_{Tb}}{\partial P_{1b}} \frac{\partial P_{1b}}{\partial P_{ea}} + \frac{\partial L_{Tb}}{\partial P_{2b}} \frac{\partial P_{2b}}{\partial P_{ea}} \right) = \lambda_b \tag{19}$$

where $\frac{\partial L_{Ta}}{\partial P_{Ga}}$ = ratio of change in transmission loss in area A to change in P_{Ga} when delivering an increment of power from P_{Ga} to the hypothetical load of area A, with all other variables assumed constant. Consequently, this expression does not include the change in loss that occurs because of a change in tie flows.

$\frac{\partial L_{Tb}}{\partial P_{Gb}}$ = same as above but with respect to area B

$\frac{\partial L_{Ta}}{\partial P_{1a}}$ = ratio of change in transmission loss in area A to change in tie flow P_{1a} when delivering an increment of power from bus 1 to the hypothetical load of area A, assuming no changes in the remaining variables occur.

$\frac{\partial L_{Ta}}{\partial P_{2a}}$ is similarly defined for P_{2a}

$\frac{\partial L_{Tb}}{\partial P_{1b}}$, $\frac{\partial L_{Tb}}{\partial P_{2b}}$ are similarly defined for area B

P_{1a}, P_{2a} = tie flows into area A measured at buses 1 and 2, respectively

$\frac{\partial P_{1a}}{\partial P_{Ga}}$ = ratio of change in tie flow into area A at bus 1 to the change in P_{Ga} when an increment of power is delivered from P_{Ga} to the hypothetical load of area A.

$\frac{\partial P_{2a}}{\partial P_{Ga}}$ = same as above but with respect to tie flow into area A at bus 2

Since

$$P_{1b} = - P_{1a} \qquad \text{and} \qquad P_{2b} = - P_{2a}$$

we may write

$$\frac{\partial P_{1b}}{\partial P_{Ga}} = - \frac{\partial P_{1a}}{\partial P_{Ga}} \tag{20}$$

and

$$\frac{\partial P_{2b}}{\partial P_{Ga}} = - \frac{\partial P_{2a}}{\partial P_{Ga}} \tag{21}$$

Also $\frac{\partial P_{1b}}{\partial P_{Gb}}$ = ratio of change in tie flow into area B at bus 1 to the change in P_{Gb} when an increment of power is delivered from P_{Gb} to the hypothetical load of area B

$$\frac{\partial P_{2b}}{\partial P_{Gb}}$$ = same as above but with respect to tie flow into area B at bus 2

$$\frac{\partial P_{1a}}{\partial P_{Gb}} = -\frac{\partial P_{1b}}{\partial P_{Gb}} \tag{22}$$

$$\frac{\partial P_{2a}}{\partial P_{Gb}} = -\frac{\partial P_{2b}}{\partial P_{Gb}} \tag{23}$$

$$\frac{\partial P_{1a}}{\partial P_{ea}}$$ = ratio of change in tie flow into area A at bus 1 to the change in excess flow out of area A when an increment of power is delivered from the hypothetical load of area A to the hypothetical load of area B

$$\frac{\partial P_{2a}}{\partial P_{ea}}$$ = same as above but with respect to the flow into area A at bus 2

$$\frac{\partial P_{1b}}{\partial P_{ea}} = -\frac{\partial P_{1a}}{\partial P_{ea}} \tag{24}$$

$$\frac{\partial P_{2b}}{\partial P_{ea}} = -\frac{\partial P_{2a}}{\partial P_{ea}} \tag{25}$$

Consider equation (17) and its similarity to equation (13). An increment of power is delivered from P_{Ga} to the hypothetical load of area A. An incremental loss $\partial L_{Ta}/\partial P_{Ga}$ occurs, since P_{Ga} has changed. Also, the incremental losses $(\partial L_{Ta}/\partial P_{1a})(\partial P_{1a}/\partial P_{Ga})$ and $(\partial L_{Ta}/\partial P_{2a})(\partial P_{2a}/\partial P_{Ga})$ occur because of changes in tie-flows P_{1a} and P_{2a}, respectively. These incremental losses are charged at λ_a, and the expression

$$\lambda_a \frac{\partial L_{Ta}}{\partial P_{Ga}} + \lambda_a \frac{\partial L_{Ta}}{\partial P_{1a}} \frac{\partial P_{1a}}{\partial P_{Ga}} + \lambda_a \frac{\partial L_{Ta}}{\partial P_{2a}} \frac{\partial P_{2a}}{\partial P_{Ga}}$$

corresponds to the expression $\lambda_a(\partial P_{La}/\partial P_{Ga})$ of equation (13). Also, the incremental loss $(\partial L_{Tb}/\partial P_{1b})(\partial P_{1b}/\partial P_{Ga}) + (\partial L_{Tb}/\partial P_{2b}) \times (\partial P_{2b}/\partial P_{Ga})$ occurs in area B because of the change in tie flows and corresponds to the expression $\partial P_{Lb}/\partial P_{Ga}$ of equation (13).

By evaluation of the various partial derivatives of tie-line flows with respect to the remaining variables[7] and noting equations (20) to (25), we obtain

$$\frac{\partial P_{1a}}{\partial P_{Ga}} = - \frac{\partial P_{1b}}{\partial P_{Ga}} = - \frac{\partial P_{2a}}{\partial P_{Ga}} = \frac{\partial P_{2b}}{\partial P_{Ga}} = \beta_{Ga} \tag{26}$$

$$\frac{\partial P_{1a}}{\partial P_{Gb}} = - \frac{\partial P_{1b}}{\partial P_{Gb}} = - \frac{\partial P_{2a}}{\partial P_{Gb}} = \frac{\partial P_{2b}}{\partial P_{Gb}} = \beta_{Gb} \tag{27}$$

$$\frac{\partial P_{1a}}{\partial P_{ea}} = - \frac{\partial P_{1b}}{\partial P_{ea}} = \beta_{ea} \tag{28}$$

$$\frac{\partial P_{2a}}{\partial P_{ea}} = - \frac{\partial P_{2b}}{\partial P_{ea}} = - (1 + \beta_{ea}) \tag{29}$$

Equations (17) to (19) may then be written

$$\frac{dF_a}{dP_{Ga}} + \lambda_a \frac{\partial L_{Ta}}{\partial P_{Ga}} + \lambda_a \beta_{Ga} \left(\frac{\partial L_{Ta}}{\partial P_{1a}} - \frac{\partial L_{Ta}}{\partial P_{2a}} \right) + \lambda_b \beta_{Ga} \left(- \frac{\partial L_{Tb}}{\partial P_{1b}} + \frac{\partial L_{Tb}}{\partial P_{2b}} \right) = \lambda_a \tag{30}$$

$$\frac{dF_b}{dP_{Gb}} + \lambda_b \frac{\partial L_{Tb}}{\partial P_{Gb}} + \lambda_b \beta_{Gb} \left(- \frac{\partial L_{Tb}}{\partial P_{1b}} + \frac{\partial L_{Tb}}{\partial P_{2b}} \right) + \lambda_a \beta_{Gb} \left(\frac{\partial L_{Ta}}{\partial P_{1a}} - \frac{\partial L_{Ta}}{\partial P_{2a}} \right) = \lambda_b \tag{31}$$

$$\lambda_a + \lambda_a\left(\beta_{ea}\frac{\partial L_{Ta}}{\partial P_{1a}} - (1+\beta_{ea})\frac{\partial L_{Ta}}{\partial P_{2a}}\right)$$
$$= \lambda_b - \lambda_b\left(-\beta_{ea}\frac{\partial L_{Tb}}{\partial P_{1b}} + (1+\beta_{ea})\frac{\partial L_{Tb}}{\partial P_{2b}}\right) \quad (32)$$

Consider equation (30) and Figure 9. If an increment of power is sent from plant G_a to the hypothetical load of area A,

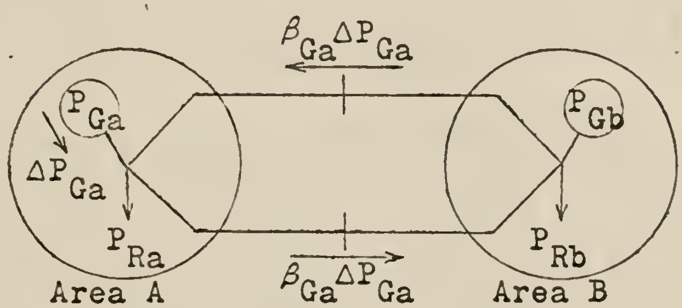

Figure 9

the loss $\partial L_{Ta}/\partial P_{Ga}$ occurs in area A because of the change in P_{Ga} only. However, as $\beta_{Ga}\,\Delta P_{Ga}$ circulates as indicated, the incremental loss $\beta_{Ga}(\partial L_{Ta}/\partial P_{1a} - \partial L_{Ta}/\partial P_{2a})$ occurs in area A and the incremental loss $\beta_{Ga}(-\,\partial L_{Tb}/\partial P_{1b} + \partial L_{Tb}/\partial P_{2b})$ occurs in area B. The incremental losses in each area are charged at the λ corresponding to that area.

Consider equation (32) and Figure 10. This equation determines the cost of sending an increment of power from the hypothetical load of area A to the hypothetical load of area B. The ΔP_{ea} that is sent is measured at the boundary between the areas. The components of flows are $\beta_{ea}\,\Delta P_{ea}$ over tie-line 1a and

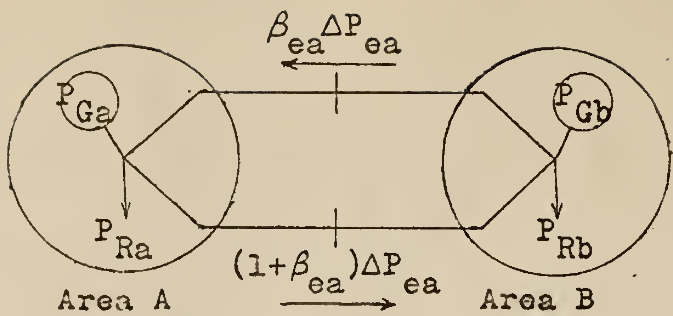

Figure 10

$(1 + \beta_{ea})\,\Delta P_{ea}$ over tie-line 2b. The incremental loss $\left[\beta_{ea} \times (\partial L_{Ta}/\partial P_{1a}) - (1 + \beta_{ea})(\partial L_{Ta}/\partial P_{2a})\right]$ incurred in A is charged at λ_a. Similarly, the incremental loss $\left[-\beta_{ea}(\partial L_{Ta}/\partial P_{1b}) + (1 + \beta_{ea}) \times (\partial L_{Tb}/\partial P_{2b})\right]$ incurred in B is charged at λ_b.

Note that in equation (30) $(\partial L_{Ta}/\partial P_{1a}) - (\partial L_{Ta}/\partial P_{2a}) - (\partial L_{Tb}/\partial P_{1b}) + (\partial L_{Tb}/\partial P_{2b})$ corresponds to the incremental loss around a closed loop. If the X/R ratios are uniform, this quantity is zero. Since β_{Ga} is usually small, the quantity

$$\lambda_a\,\beta_{Ga}\left(\frac{\partial L_{Ta}}{\partial P_{1a}} - \frac{\partial L_{Ta}}{\partial P_{2a}}\right) + \lambda_b\,\beta_{Ga}\left(-\frac{\partial L_{Tb}}{\partial P_{1b}} + \frac{\partial L_{Tb}}{\partial P_{2b}}\right)$$

may frequently be neglected. If the terms of this nature are assumed to have negligible values, the coordination equations corresponding to equations (30) and (31) simplify to

$$\frac{dF_a}{dP_{Ga}} + \lambda_a \frac{\partial L_{Ta}}{\partial P_{Ga}} = \lambda_a \tag{33}$$

$$\frac{dF_b}{dP_{Gb}} + \lambda_b \frac{\partial L_{Tb}}{\partial P_{Gb}} = \lambda_b \tag{34}$$

By use of equations (32), (33), and (34), each area requires only a knowledge of the plant loadings within that area and the tie flows or boundary conditions of that area.

Three Loop-Interconnected Areas

As an example of loop-interconnected operation, consider the three-area system shown in Figure 11. Each of the three areas has a typical generator and each area has one tie line to each other area. In general, there will be several ties between companies; but, for simplicity of illustration only single ties between the companies are treated here.

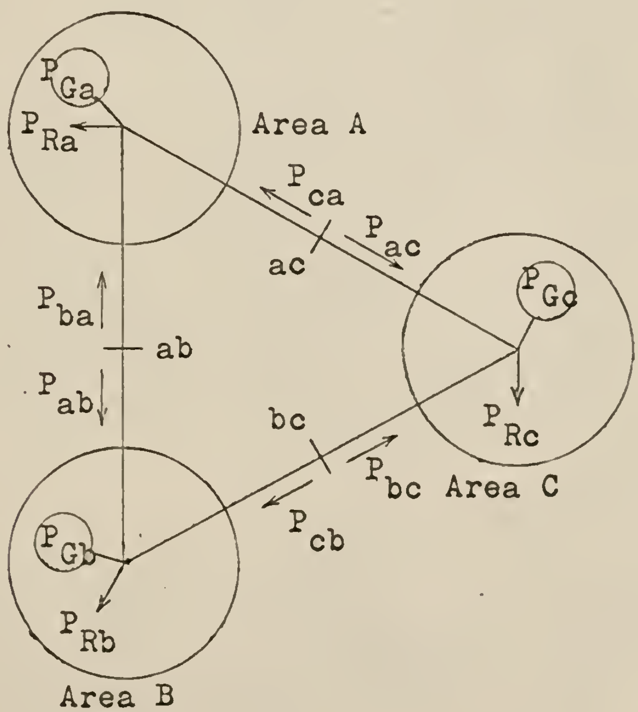

Figure 11

For the three-area system the three constraints (one of each area) may be written

$$\psi_a = P_{Ra} + P_{La} + P_{ea} - \sum_a P_{Ga} = 0$$

$$\psi_b = P_{Rb} + P_{Lb} - P_{ea} - P_{ec} - \sum_b P_{Gb} = 0$$

$$\psi_c = P_{Rc} + P_{Lc} + P_{ec} - \sum_c P_{Gc} = 0$$

where P_{ea} and P_{ec} are the net interchanges or excess flows out of areas A and C, respectively.

Thus

$$P_{ea} = P_{ab} + P_{ac} = - P_{ba} - P_{ca}$$

$$P_{ec} = P_{ca} + P_{cb} = - P_{ac} - P_{bc}$$

It is desired to minimize the total fuel input to this interconnected system which is given by $F_t = F_a + F_b + F_c$ = total input in dollars per hour, where F_a, F_b, F_c = input in dollars per hour to areas A, B, and C, respectively.

By the method of Lagrangian multipliers, the following equations are obtained for economic allocation of generation within the areas and interchange between the areas:

$$\frac{dF_a}{dP_{Ga}} + \lambda_a \frac{\partial P_{La}}{\partial P_{Ga}} + \lambda_b \frac{\partial P_{Lb}}{\partial P_{Ga}} + \lambda_c \frac{\partial P_{Lc}}{\partial P_{Ga}} = \lambda_a \tag{35}$$

$$\frac{dF_b}{dP_{Gb}} + \lambda_a \frac{\partial P_{La}}{\partial P_{Gb}} + \lambda_b \frac{\partial P_{Lb}}{\partial P_{Gb}} + \lambda_c \frac{\partial P_{Lc}}{\partial P_{Gb}} = \lambda_b \tag{36}$$

$$\frac{dF_c}{dP_{Gc}} + \lambda_a \frac{\partial P_{La}}{\partial P_{Gc}} + \lambda_b \frac{\partial P_{Lb}}{\partial P_{Gc}} + \lambda_c \frac{\partial P_{Lc}}{\partial P_{Gc}} = \lambda_c \tag{37}$$

$$\lambda_a + \lambda_a \frac{P_{La}}{P_{ea}} + \lambda_b \frac{P_{Lb}}{P_{ea}} + \lambda_c \frac{P_{Lc}}{P_{ea}} = \lambda_b \tag{38}$$

$$\lambda_c + \lambda_a \frac{\partial P_{La}}{\partial P_{ec}} + \lambda_b \frac{\partial P_{Lb}}{\partial P_{ec}} + \lambda_c \frac{\partial P_{Lc}}{\partial P_{ec}} = \lambda_b \tag{39}$$

The terms in these equations may be interpreted similarly to those for the preceding two-area discussion. The first three equations represent the intra-area equations and the last two are the interarea equations. The interarea equations require that the incremental cost of received interchange power at the hypothetical load of area B, whether received from area A or area C, be equal to the incremental cost of received load of area B. These five coordination equations, together with the three constraining equations on received loads, provide the necessary conditions for economic operation of the three-area system.

The interarea equation (38) may be rewritten to indicate the costs at the boundaries of the reference area, area B:

$$\lambda_a + \lambda_a \frac{\partial P_{La}}{\partial P_{ea}} + \lambda_c \frac{\partial P_{Lc}}{\partial P_{ea}} = \lambda_b - \lambda_b \frac{\partial P_{Lb}}{\partial P_{ea}}$$

The quantity on the left-hand side of this equation corresponds to the incremental cost at the boundary of area B referred to area A for delivering an increment of power from the hypothetical load of area A to the hypothetical load of area B. Similarly, the expression on the right-hand side of this equation corresponds to the incremental cost at the boundary of area B referred to area B for delivering an increment of power from the hypothetical load of area A to the hypothetical load of area B. For optimum economy the boundary cost referred to area A should be the same as the boundary cost referred to area B. Equation (39) may be similarly interpreted. Note that the term $\lambda_c(\partial P_{Lc}/\partial P_{ea})$ represents the cost of the increment of power from area A to area B.

The analysis presented has been made in terms of the quasi-radial model of the loop-interconnected system shown in Figure 12. It is possible to work with the actual circuit representa-

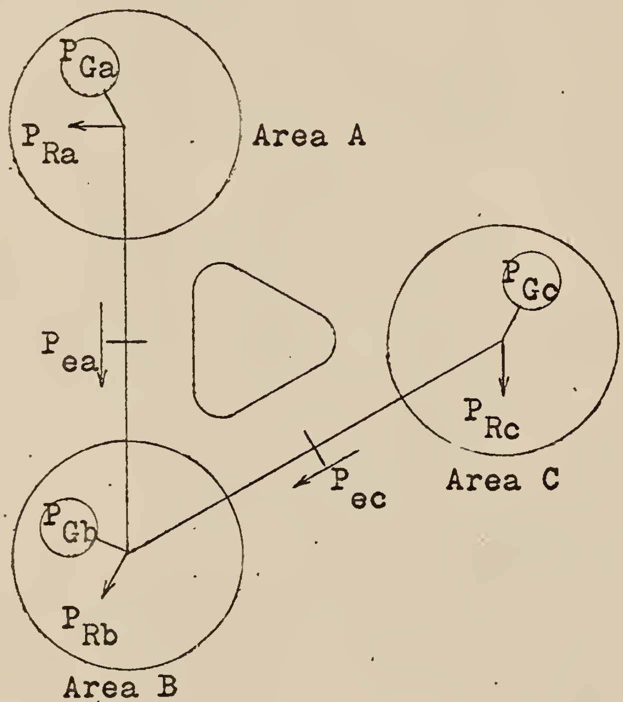

Figure 12

tion itself, provided due recognition is given to the dependence of tie-line flows on system generation. Equations (35) to (39) may, therefore, be rewritten.

$$\frac{dF_a}{dP_{Ga}} + \lambda_a\left(\frac{\partial L_{Ta}}{\partial P_{Ga}} + \frac{\partial L_{Ta}}{\partial P_{ca}}\frac{\partial P_{ca}}{\partial P_{Ga}} + \frac{\partial L_{Ta}}{\partial P_{ba}}\frac{\partial P_{ba}}{\partial P_{Ga}}\right)$$
$$+ \lambda_b\left(\frac{\partial L_{Tb}}{\partial P_{ab}}\frac{\partial P_{ab}}{\partial P_{Ga}} + \frac{\partial L_{Tb}}{\partial P_{cb}}\frac{\partial P_{cb}}{\partial P_{Ga}}\right)$$
$$+ \lambda_c\left(\frac{\partial L_{Tc}}{\partial P_{ac}}\frac{\partial P_{ac}}{\partial P_{Ga}} + \frac{\partial L_{Tc}}{\partial P_{bc}}\frac{\partial P_{bc}}{\partial P_{Ga}}\right) = \lambda_a \tag{40}$$

$$\frac{dF_b}{dP_{Gb}} + \lambda_a\left(\frac{\partial L_{Ta}}{\partial P_{ca}}\frac{\partial P_{ca}}{\partial P_{Gb}} + \frac{\partial L_{Ta}}{\partial P_{ba}}\frac{\partial P_{ba}}{\partial P_{Gb}}\right)$$
$$+ \lambda_b\left(\frac{\partial L_{Tb}}{\partial P_{Gb}} + \frac{\partial L_{Tb}}{\partial P_{ab}}\frac{\partial P_{ab}}{\partial P_{Gb}} + \frac{\partial L_{Tb}}{\partial P_{cb}}\frac{\partial P_{cb}}{\partial P_{Gb}}\right)$$
$$+ \lambda_c\left(\frac{\partial L_{Tc}}{\partial P_{ac}}\frac{\partial P_{ac}}{\partial P_{Gb}} + \frac{\partial L_{Tc}}{\partial P_{bc}}\frac{\partial P_{bc}}{\partial P_{Gb}}\right) = \lambda_b \tag{41}$$

$$\frac{dF_c}{dP_{Gc}} + \lambda_a\left(\frac{\partial L_{Ta}}{\partial P_{ca}}\frac{\partial P_{ca}}{\partial P_{Gc}} + \frac{\partial L_{Ta}}{\partial P_{ba}}\frac{\partial P_{ba}}{\partial P_{Gc}}\right)$$
$$+ \lambda_b\left(\frac{\partial L_{Tb}}{\partial P_{ab}}\frac{\partial P_{ab}}{\partial P_{Gc}} + \frac{\partial L_{Tb}}{\partial P_{cb}}\frac{\partial P_{cb}}{\partial P_{Gc}}\right)$$
$$+ \lambda_c\left(\frac{\partial L_{Tc}}{\partial P_{Gc}} + \frac{\partial L_{Tc}}{\partial P_{ac}}\frac{\partial P_{ac}}{\partial P_{Gc}} + \frac{\partial L_{Tc}}{\partial P_{bc}}\frac{\partial P_{bc}}{\partial P_{Gc}}\right) = \lambda_c \tag{42}$$

$$\lambda_a + \lambda_a\left(\frac{\partial L_{Ta}}{\partial P_{ca}}\frac{\partial P_{ca}}{\partial P_{ea}} + \frac{\partial L_{Ta}}{\partial P_{ba}}\frac{\partial P_{ba}}{\partial P_{ea}}\right)$$
$$+ \lambda_b\left(\frac{\partial L_{Tb}}{\partial P_{ab}}\frac{\partial P_{ab}}{\partial P_{ea}} + \frac{\partial L_{Tb}}{\partial P_{cb}}\frac{\partial P_{cb}}{\partial P_{ea}}\right)$$
$$+ \lambda_c\left(\frac{\partial L_{Tc}}{\partial P_{ac}}\frac{\partial P_{ac}}{\partial P_{ea}} + \frac{\partial L_{Tc}}{\partial P_{bc}}\frac{\partial P_{bc}}{\partial P_{ea}}\right) = \lambda_b \tag{43}$$

$$\lambda_c + \lambda_a \left(\frac{\partial L_{Ta}}{\partial P_{ca}} \frac{\partial P_{ca}}{\partial P_{ec}} + \frac{\partial L_{Ta}}{\partial P_{ba}} \frac{\partial P_{ba}}{\partial P_{ec}} \right)$$

$$+ \lambda_b \left(\frac{\partial L_{Tb}}{\partial P_{ab}} \frac{\partial P_{ab}}{\partial P_{ec}} + \frac{\partial L_{Tb}}{\partial P_{cb}} \frac{\partial P_{cb}}{\partial P_{ec}} \right)$$

$$+ \lambda_c \left(\frac{\partial L_{Tc}}{\partial P_{ac}} \frac{\partial P_{ac}}{\partial P_{ec}} + \frac{\partial L_{Tc}}{\partial P_{bc}} \frac{\partial P_{bc}}{\partial P_{ec}} \right) = \lambda_b \tag{44}$$

The incremental tie-line flows may be expressed in terms of β factors as previously discussed.

$$\frac{\partial P_{ca}}{\partial P_{Ga}} = \frac{\partial P_{ab}}{\partial P_{Ga}} = \frac{\partial P_{bc}}{\partial P_{Ga}} = - \frac{\partial P_{ac}}{\partial P_{Ga}} = - \frac{\partial P_{ba}}{\partial P_{Ga}} = - \frac{\partial P_{cb}}{\partial P_{Ga}} = \beta_{Ga} \tag{45}$$

$$\frac{\partial P_{ca}}{\partial P_{Gb}} = \frac{\partial P_{ab}}{\partial P_{Gb}} = \frac{\partial P_{bc}}{\partial P_{Gb}} = - \frac{\partial P_{ac}}{\partial P_{Gb}} = - \frac{\partial P_{ba}}{\partial P_{Gb}} = - \frac{\partial P_{cb}}{\partial P_{Gb}} = \beta_{Gb} \tag{46}$$

$$\frac{\partial P_{ca}}{\partial P_{Gc}} = \frac{\partial P_{ab}}{\partial P_{Gc}} = \frac{\partial P_{bc}}{\partial P_{Gc}} = - \frac{\partial P_{ac}}{\partial P_{Gc}} = - \frac{\partial P_{ba}}{\partial P_{Gc}} = - \frac{\partial P_{cb}}{\partial P_{Gc}} = \beta_{Gc} \tag{47}$$

$$\frac{\partial P_{ca}}{\partial P_{ea}} = \frac{\partial P_{bc}}{\partial P_{ea}} = - \frac{\partial P_{ac}}{\partial P_{ea}} = - \frac{\partial P_{cb}}{\partial P_{ea}} = \beta_{ea} \tag{48}$$

$$\frac{\partial P_{ab}}{\partial P_{ea}} = - \frac{\partial P_{ba}}{\partial P_{ea}} = 1 + \beta_{ea} \tag{49}$$

$$\frac{\partial P_{ca}}{\partial P_{ec}} = \frac{\partial P_{ab}}{\partial P_{ec}} = - \frac{\partial P_{ac}}{\partial P_{ec}} = - \frac{\partial P_{ba}}{\partial P_{ec}} = \beta_{ec} \tag{50}$$

$$\frac{\partial P_{cb}}{\partial P_{ec}} = - \frac{\partial P_{bc}}{\partial P_{ec}} = 1 - \beta_{ec} \tag{51}$$

If equations (45) to (51) are substituted into equations (40) to (44), we get

$$\frac{dF_a}{dP_{Ga}} + \lambda_a \frac{\partial L_{Ta}}{\partial P_{Ga}} + \beta_{Ga} \left[\lambda_a \left(\frac{\partial L_{Ta}}{\partial P_{ca}} - \frac{\partial L_{Ta}}{\partial P_{ba}} \right) \right.$$

$$\left. + \lambda_b \left(\frac{\partial L_{Tb}}{\partial P_{ab}} - \frac{\partial L_{Tb}}{\partial P_{cb}} \right) + \lambda_c \left(\frac{\partial L_{Tc}}{\partial P_{bc}} - \frac{\partial L_{Tc}}{\partial P_{ac}} \right) \right] = \lambda_a \tag{52}$$

$$\frac{dF_b}{dP_{Gb}} + \lambda_b \frac{\partial L_{Tb}}{\partial P_{Gb}} + \beta_{Gb}\left[\lambda_a\left(\frac{\partial L_{Ta}}{\partial P_{ca}} - \frac{\partial L_{Ta}}{\partial P_{ba}}\right) + \lambda_b\left(\frac{\partial L_{Tb}}{\partial P_{ab}} - \frac{\partial L_{Tb}}{\partial P_{cb}}\right)\right.$$
$$\left. + \lambda_c\left(\frac{\partial L_{Tc}}{\partial P_{bc}} - \frac{\partial L_{Tc}}{\partial P_{ac}}\right)\right] = \lambda_b \qquad (53)$$

$$\frac{dF_c}{dP_{Gc}} + \lambda_c \frac{\partial L_{Tc}}{\partial P_{Gc}} + \beta_{Gc}\left[\lambda_a\left(\frac{\partial L_{Ta}}{\partial P_{ca}} - \frac{\partial L_{Ta}}{\partial P_{ba}}\right) + \lambda_b\left(\frac{\partial L_{Tb}}{\partial P_{ab}} - \frac{\partial L_{Tb}}{\partial P_{cb}}\right)\right.$$
$$\left. + \lambda_c\left(\frac{\partial L_{Tc}}{\partial P_{bc}} - \frac{\partial L_{Tc}}{\partial P_{ac}}\right)\right] = \lambda_c \qquad (54)$$

$$\lambda_a + \lambda_a\left(\beta_{ea}\frac{\partial L_{Ta}}{\partial P_{ca}} - (1 + \beta_{ea})\frac{\partial L_{Ta}}{\partial P_{ba}}\right)$$
$$+ \lambda_b\left((1 + \beta_{ea})\frac{\partial L_{Tb}}{\partial P_{ab}} - \beta_{ea}\frac{\partial L_{Tb}}{\partial P_{cb}}\right)$$
$$+ \lambda_c\,\beta_{ea}\left(\frac{\partial L_{Tc}}{\partial P_{bc}} - \frac{\partial L_{Tc}}{\partial P_{ac}}\right) = \lambda_b \qquad (55)$$

$$\lambda_c + \lambda_a\left(\beta_{ec}\frac{\partial L_{Ta}}{\partial P_{ca}} - \frac{\partial L_{Ta}}{\partial P_{ba}}\right) + \lambda_b\left(\beta_{ec}\frac{\partial L_{Tb}}{\partial P_{ab}} + (1 - \beta_{ec})\frac{\partial L_{Tb}}{\partial P_{cb}}\right)$$
$$+ \lambda_c\left(-(1 - \beta_{ec})\frac{\partial L_{Tc}}{\partial P_{bc}} - \beta_{ec}\frac{\partial L_{Tc}}{\partial P_{ac}}\right) = \lambda_b \qquad (56)$$

where $\dfrac{\partial L_{Ta}}{\partial P_{Ga}}$ = ratio of change in transmission loss in area A to change in P_{Ga} when delivering an increment of power from P_{Ga} to the hypothetical load of area A, assuming that no changes in tie flows occur

$\dfrac{\partial L_{Tb}}{\partial P_{Gb}}$, $\dfrac{\partial L_{Tc}}{\partial P_{Gc}}$ are similarly defined with respect to areas B and C

$\dfrac{\partial L_{Ta}}{\partial P_{ca}}$ = ratio of change in transmission loss in area A to change in tie flow P_{ca} when delivering an increment of power from bus ac to the hypothetical load of area A, assuming that no changes in the remaining variables occur

$\frac{\partial L_{Ta}}{\partial P_{ba}}$ is similarly defined with respect to P_{ba}

$\frac{\partial L_{Tb}}{\partial P_{ab}}$, $\frac{\partial L_{Tb}}{\partial P_{cb}}$, $\frac{\partial L_{Tc}}{\partial P_{bc}}$, $\frac{\partial L_{Tc}}{\partial P_{ac}}$ are defined in a similar manner

β_{Ga} = rate of change of P_{ca}, P_{ab}, P_{bc} with respect to P_{Ga} when delivering an increment of power from a particular generator Ga in area A to the hypothetical load of area A

β_{Gb} = rate of change of P_{ca}, P_{ab}, P_{bc} with respect to P_{Gb} when delivering an increment of power from a particular generator Gb in area B to the hypothetical load of area B

β_{Gc} = rate of change of P_{ca}, P_{ab}, P_{bc} with respect to P_{Gc} when delivering an increment of power from a particular generator Gc in area C to the hypothetical load of area C

β_{ea} = rate of change of P_{ca} and P_{bc} with respect to P_{ea} when delivering an increment of power from the hypothetical load of area A to the hypothetical load of area B

β_{ec} = rate of change of P_{ca} and P_{ab} with respect to P_{ec} when delivering an increment of power from the hypothetical load of area C to the hypothetical load of area B

Equation (52) determines the incremental cost of delivering an increment of power to the hypothetical load of area A from a particular generator in area A. The incremental loss in area A resulting from a change in generator A only is $\partial L_{Ta}/\partial P_{Ga}$, which is charged at λ_a. However, as this increment is delivered, part of the power may flow through the parallel connected areas. As indicated in Figure 13, the amount that flows through the parallel loop is $\beta_{Ga}\,\Delta P_{Ga}$. The incremental loss in A due to this flow is $\beta_{Ga}(\partial L_{Ta}/\partial P_{ca} - \partial L_{Ta}/\partial P_{ba})$, which is charged at λ_a. Similar-

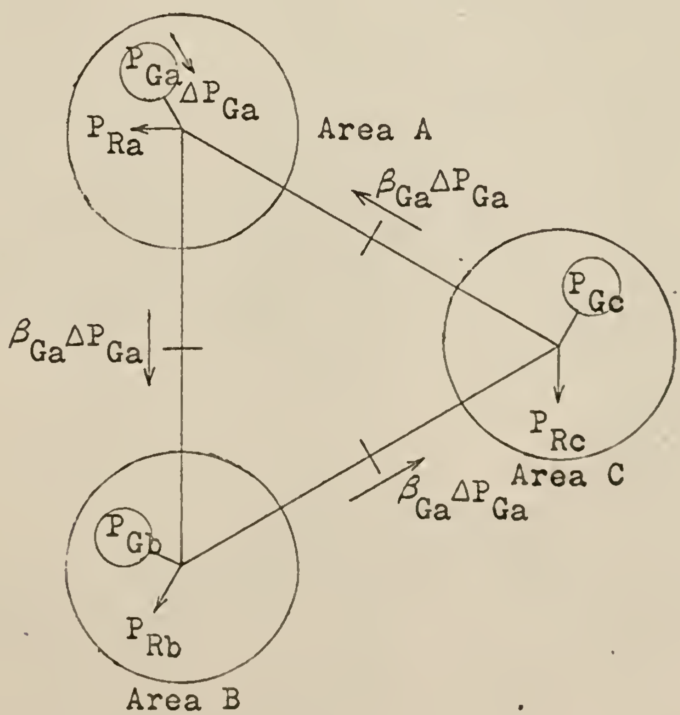

Figure 13

ly, the incremental loss in B is $\beta_{Ga}(\partial L_{Tb}/\partial P_{ab} - \partial L_{Tb}/\partial P_{cb})$, which is charged at λ_b. For area C the corresponding incremental loss is $\beta_{Ga}(\partial L_{Tc}/\partial P_{bc} - \partial L_{Tc}/\partial P_{ac})$, which is charged at λ_c.

Equation (55) determines the cost of sending an increment of power from the hypothetical load at A to the hypothetical load at B, as indicated in Figure 14. The ΔP_{ea} that is sent is measured at the boundaries of A. The components of flow are $(1 \div \beta_{ea})\Delta P_{ea}$ over the ab line and $\beta_{ea}\,\Delta P_{ea}$ flowing through the parallel path through area C. In equation (55) the incremental loss $\left[\beta_{ea} \times (\partial L_{Ta}/\partial P_{ca}) - (1 \div \beta_{ea})(\partial L_{Ta}/\partial P_{ba})\right]$ incurred in area A is charged at λ_a. Similarly, the incremental loss $\left[(1 \div \beta_{ea})(\partial L_{Tb}/\partial P_{ab}) -\right.$

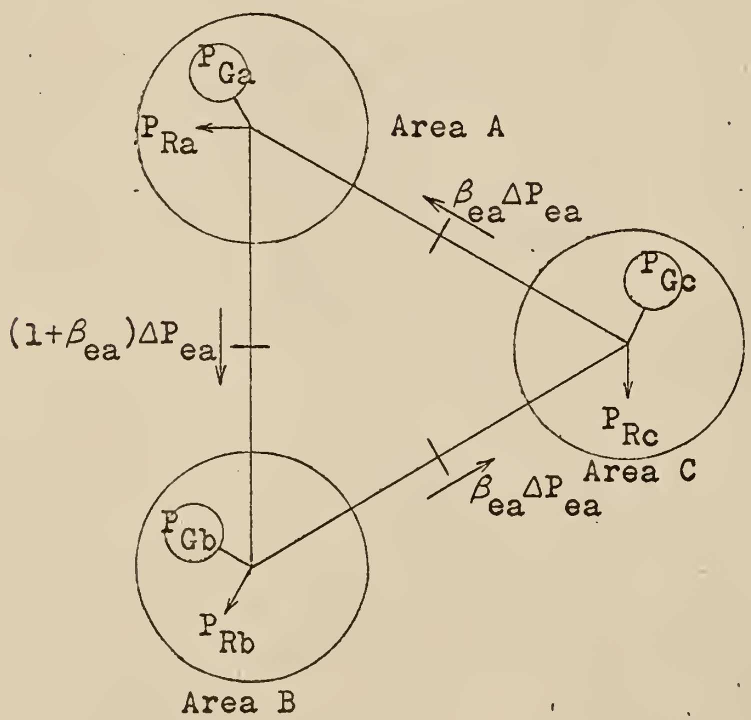

Figure 14

$\beta_{ea}(\partial L_{Tb}/\partial P_{cb})]$ incurred in area B is charged at λ_b. The incremental wheeling loss through the parallel interconnected area β_{ea} X $(\partial L_{Tc}/\partial P_{bc} - \partial L_{Tc}/\partial P_{ac})$ is charged at λ_c.

If the terms of the form

$$\beta_{Ga}\left[\lambda_a\left(\frac{\partial L_{Ta}}{\partial P_{ca}} - \frac{\partial L_{Ta}}{P_{ba}}\right) + \lambda_b\left(\frac{\partial L_{Tb}}{\partial P_{ab}} - \frac{\partial L_{Tb}}{\partial P_{cb}}\right) + \lambda_c\left(\frac{\partial L_{Tc}}{\partial P_{bc}} - \frac{\partial L_{Tc}}{\partial P_{ac}}\right)\right]$$

are considered negligible in equations (52), (53), and (54), these equations may be written

$$\frac{dF_a}{dP_{Ga}} + \lambda_a \frac{\partial L_{Ta}}{\partial P_{Ga}} = \lambda_a \tag{57}$$

$$\frac{dF_b}{dP_{Gb}} + \lambda_b \frac{\partial L_{Tb}}{\partial P_{Gb}} = \lambda_b \tag{58}$$

$$\frac{dF_c}{dP_{Gc}} + \lambda_c \frac{\partial L_{Tc}}{\partial P_{Gc}} = \lambda_c \tag{59}$$

Equations (55) and (56) remain unchanged. Equations (55), (56), (57), (58), and (59) require a minimum exchange of information between areas. The neglected terms are small, since the β factors are usually small, and the incremental loss around a closed loop approaches zero as the X/R ratios become similar. When the neglected terms are considered significant, they may either be carried in full or approximated by representation of the dominant terms.

HYDROTHERMAL SCHEDULING

The problem of optimizing hydroelectric and especially hydrothermal systems is more complex than the all-thermal problem previously presented. The hydro problem is difficult because of the many independent variables involved. These include availability of water at the hydro plants, the many project and operating limitations, and various contractual requirements[2] which are to be satisfied in addition to the ordinary economy-loading restrictions. For this reason, no method of optimization can ever be considered perfect, because only some of the numerous factors encountered can be included at a time. It is therefore recognized that many contributions are still very welcome.

The methods employing the calculus of variations[6, 15, 16] are among many methods developed. The major difficulty in adapting the calculus of variations approach to an existing physical system is that all variables must be made time-dependent. For a typical hydroelectric plant, this leads to complicated expressions, subject to many nonlinear constraints. For this reason some variational methods solve problems in a point-by-point manner; i.e., finding what the contribution of each plant should be for certain loads and transmission losses at any one time.

Since less reliable data and more uncertainties are involved in forecasting water inflow over a long period, the short-range problems are considered in the following. In fact, any short-range method can, without too much difficulty, be applied to long-range problems, provided the input data are reliable.

Coordination Equations Neglecting Head Variations

When using desired amounts of water from the hydro plants over a given period of time, it is desired to minimize the total system input in dollars for the operating area being studied. The length of time to be considered as short range is restricted to periods during which the variation in head is negligible. By applying the calculus of variations, a set of equations whose solution gives the optimum scheduling of the system is obtained.

Let

F_T = total input to system in dollars per hour

F_n = input to thermal plant n in dollars per hour

P_{Sn} = output of thermal plant n in megawatts

P_{Hj} = output of hydro plant j in megawatts

P_L = total transmission losses to system in megawatts

P_R = received load in megawatts

W_j = input of water to hydro plant j in cfs

K_j = desired amounts of water in cubic feet from hydro plant j over a fixed future interval t_1

α = number of thermal plants in the system

β = number of all plants in the system

The problem may now be stated mathematically as

$$\int_0^{t_1} F_T \, dt = \text{minimum} \tag{60}$$

with the restriction that

$$\int_0^{t_1} W_j \, dt = K_j \qquad j = \alpha + 1, \alpha + 2, \ldots, \beta \tag{61}$$

and

$$\sum_{n=1}^{\alpha} P_{Sn} + \sum_{j=\alpha+1}^{\beta} P_{Hj} - P_L = P_R \tag{62}$$

Conditions (60) and (61) are satisfied when

$$\delta \left(\int_0^{t_1} F_T \, dt + \sum_{j=\alpha+1}^{\beta} \gamma_j \int_0^{t_1} W_j \, dt \right) = 0$$

where γ_j are constant multipliers. The foregoing equation may be written next as

$$\int_0^{t_1} \left(\delta F_T \, dt + \delta \sum_{j=\alpha+1}^{\beta} \gamma_j W_j \, dt \right) = 0$$

$$\sum_{n=1}^{\alpha} \frac{\partial F_T}{\partial P_{Sn}} \delta P_{Sn} + \sum_{j=\alpha+1}^{\beta} \gamma_j \frac{\partial W_j}{\partial P_{Hj}} \delta P_{Hj} = 0 \tag{63}$$

From equation (62)

$$\sum_{n=1}^{\alpha} \delta P_{Sn} + \sum_{j=\alpha+1}^{\beta} \delta P_{Hj} - \sum_{n=1}^{\alpha} \frac{\partial P_L}{\partial P_{Sn}} \delta P_{Sn} - \sum_{j=\alpha+1}^{\beta} \frac{\partial P_L}{\partial P_{Hj}} \delta P_{Hj} = 0$$

Solving for a particular hydro power δP_{Hr} and obtaining

$$\left(1 - \frac{\partial P_L}{\partial P_{Hr}}\right) \delta P_{Hr}$$

$$= - \sum_{n=1}^{\alpha} \left(1 - \frac{\partial P_L}{\partial P_{Sn}}\right) \delta P_{Sn} - \sum_{\substack{j=\alpha+1 \\ j\neq r}}^{\beta} \left(1 - \frac{\partial P_L}{\partial P_{Hj}}\right) \delta P_{Hj} \tag{64}$$

By rewriting equation (63) and obtaining

$$\sum_{n=1}^{\alpha} \frac{\partial F_T}{\partial P_{Sn}} \delta P_{Sn} + \gamma_r \frac{\partial W_r}{\partial P_{Hr}} \delta P_{Hr} + \sum_{\substack{j=\alpha+1 \\ j\neq r}}^{\beta} \gamma_j \frac{\partial W_j}{\partial P_{Hj}} \delta P_{Hj} = 0$$

$$\gamma_r \frac{\partial W_r}{\partial P_{Hr}} \delta P_{Hr} = - \sum_{n=1}^{\alpha} \frac{\partial F_T}{\partial P_{Sn}} \delta P_{Sn} - \sum_{\substack{j=\alpha+1 \\ j\neq r}}^{\beta} \gamma_j \frac{\partial W_j}{\partial P_{Hj}} \delta P_{Hj} = 0$$

Multiply by $(1 - \partial P_L/\partial P_{Hr})$, and obtain

$$\left(1 - \frac{\partial P_L}{\partial P_{Hr}}\right) \gamma_r \frac{\partial W_r}{\partial P_{Hr}} \delta P_{Hr}$$

$$= \left(1 - \frac{\partial P_L}{\partial P_{Hr}}\right) \left(- \sum_{n=1}^{\alpha} \frac{\partial F_T}{\partial P_{Sn}} \delta P_{Sn} - \sum_{\substack{j=\alpha+1 \\ j\neq r}}^{\beta} \gamma_j \frac{\partial W_j}{\partial P_{Hj}} \delta P_{Hj}\right) \tag{65}$$

Substitute equation (64) into (65),

$$\sum_{n=1}^{\alpha} \left[\frac{\partial F_T}{\partial P_{Sn}} \left(1 - \frac{\partial P_L}{\partial P_{Hr}}\right) - \gamma_r \frac{\partial W_r}{\partial P_{Hr}} \left(1 - \frac{\partial P_L}{\partial P_{Sn}}\right)\right] \delta P_{Sn} +$$

$$\sum_{\substack{j=\alpha+1 \\ j\neq r}}^{\beta} \left[\gamma_j \frac{\partial W_j}{\partial P_{Hj}} \left(1 - \frac{\partial P_L}{\partial P_{Hr}}\right) - \gamma_r \frac{\partial W_r}{\partial P_{Hr}} \left(1 - \frac{\partial P_L}{\partial P_{Hj}}\right)\right] \delta P_{Hj} = 0 \tag{66}$$

The coefficient of each variation δP_{Sn}, δP_{Hj} must be identically zero. Hence,

$$\frac{\partial F_T}{\partial P_{Sn}}\left(1 - \frac{\partial P_L}{\partial P_{Hr}}\right) - \gamma_r \frac{\partial W_r}{\partial P_{Hr}}\left(1 - \frac{\partial P_L}{\partial P_{Sn}}\right) = 0$$

$$\gamma_j \frac{\partial W_j}{\partial P_{Hj}}\left(1 - \frac{\partial P_L}{\partial P_{Hr}}\right) - \gamma_r \frac{\partial W_r}{\partial P_{Hr}}\left(1 - \frac{\partial P_L}{\partial P_{Hj}}\right) = 0 \qquad (67)$$

It is to be noted that

$$\frac{\partial F_T}{\partial P_{Sn}} = \frac{\partial F_n}{\partial P_{Sn}} = \frac{dF_n}{dP_{Sn}}$$

and that

$$\frac{\partial W_j}{\partial P_{Hj}} = \frac{dW_j}{dP_{Hj}}$$

From equation (67),

$$\begin{aligned}\frac{dF_n}{dP_{Sn}} \frac{1}{[1 - (\partial P_L/\partial P_{Sn})]} &= \gamma_r \frac{dW_r}{dP_{Hr}} \frac{1}{[1 - (\partial P_L/\partial P_{Hr})]} \\ &= \gamma_j \frac{dW_j}{dP_{Hj}} \frac{1}{[1 - (\partial P_L/\partial P_{Hj})]} \\ &= \lambda \qquad (68)\end{aligned}$$

Then

$$\frac{dF_n}{dP_{Sn}} + \lambda \frac{\partial P_L}{\partial P_{Sn}} = \lambda \qquad (69)$$

$$\gamma_j \frac{dW_j}{dP_{Hj}} + \lambda \frac{\partial P_L}{\partial P_{Hj}} = \lambda \qquad (70)$$

Equations (69) and (70) are the scheduling equations.

Coordination Equations Including Head Variations

Let

F = hourly fuel cost

P_S = thermal output

P_H = hydro output

P_L = system losses

P_R = load

h = net head

q = flow

$\dot{q}$ = discharge (rate of flow)

S = surface area of reservoir

(a) Derivation of Ricard's Equation[4]. Consider the solid lines of Figure 15 to represent drawdown of the hydroelectric

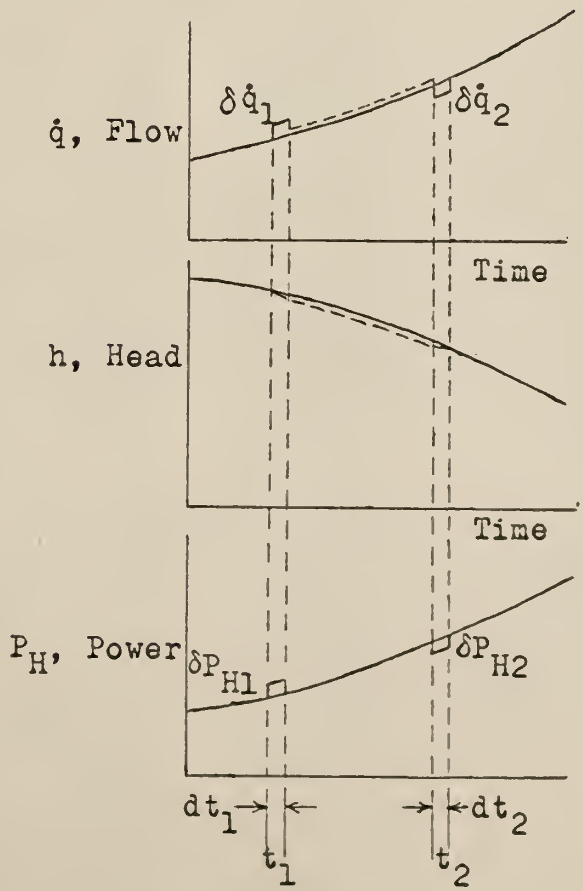

Figure 15

plant most economically. Then, at t_1, make an excursion δP_{H1} and hold it for time dt_1.

Since

$$\dot{q} = \dot{q}(h, P_H) \tag{71}$$

$$\delta\dot{q} = \frac{\partial\dot{q}}{\partial h}\,\delta h + \frac{\partial\dot{q}}{\partial P_H}\,\delta P_H$$

$$\delta\dot{q}_1 = \left.\frac{\partial\dot{q}}{\partial P_H}\right|_1 \delta P_{H1} \tag{72}$$

Consider

$$\delta h = \text{constant}, \quad t_1 < t < t_2$$

Since

$$S\delta h = -\,\delta\dot{q}\,dt$$

$$\delta h = -\,\delta\dot{q}\,\frac{dt}{S} = -\left.\frac{\partial\dot{q}}{\partial P_H}\right|_1 \delta P_{H1}\,\frac{dt_1}{S} \tag{73}$$

If P_H is now considered along the original curve, the flow $\dot{q}$ will be somewhat above its original curve because of δh. At t_2, make the excursion δP_{H2}, and hold this excursion for a long enough time dt_2 such that h returns to the most economic curve.

Now

$$\delta\int_0^T \dot{q}\,dt = 0 \tag{74}$$

is a fundamental requirement. Therefore, algebraically,

$$A + B + C = 0$$

where

$$A = - \left.\frac{\partial \dot{q}}{\partial P_H}\right|_1 \delta P_{H1} dt_1 = \delta \int_{t_0}^{t_1 + dt_1} \dot{q}\, dt \qquad (75)$$

$$B = - \frac{\partial \dot{q}}{\partial h} \delta h (t_2 - t_1) = \delta \int_{t_1 + dt_1}^{t_2} \dot{q}\, dt \qquad (76)$$

$$C = - \frac{\partial \dot{q}}{\partial P_H}{}_2\, \delta P_{H2} dt_2 = \delta \int_{t_2}^{t_2 + dt_2} \dot{q}\, dt \qquad (77)$$

$$\left.\frac{\partial \dot{q}}{\partial P_H}\right|_1 \delta P_{H1} dt + \frac{\partial \dot{q}}{\partial h} \delta h (t_2 - t_1) + \left.\frac{\partial \dot{q}}{\partial P_H}\right|_2 \delta P_{H2} dt_2 = 0$$

By use of equation (73), we obtain:

$$\left.\frac{\partial \dot{q}}{\partial P_H}\right|_1 \delta P_{H1} dt_1 \left(1 - \frac{\partial \dot{q}}{\partial h} \frac{(t_2 - t_1)}{S} \right) + \left.\frac{\partial \dot{q}}{\partial P_H}\right|_2 \delta P_{H2} dt_2 = 0 \qquad (78)$$

For the load to be met:

$$P_H + P_S - P_L = P_R$$

but

$$\delta P_R = 0 = \delta P_H + \delta P_S - \delta P_L$$

If

$$P_L = P_L(P_H, P_S)$$

$$\delta P_L = \frac{\partial P_L}{\partial P_H} \delta P_H + \frac{\partial P_L}{\partial P_S} \delta P_S$$

and

$$\delta P_R = 0 = \delta P_H \left(1 - \frac{\partial P_L}{\partial P_H} \right) + \delta P_S \left(1 - \frac{\partial P_L}{\partial P_S} \right)$$

whence

$$\delta P_S = - \frac{L_S}{L_H} \delta P_H$$

where

$$L_S = \frac{1}{1 - \dfrac{\partial P_L}{\partial P_S}}, \qquad L_H = \frac{1}{1 - \dfrac{\partial P_L}{\partial P_H}}$$

so that

$$\delta P_{S1} = - \frac{L_{S1}}{L_{H1}} \delta P_{H1} \tag{79}$$

and

$$\delta P_{S2} = - \frac{L_{S2}}{L_{H2}} \delta P_{H2} \tag{80}$$

We also require

$$\delta \int_0^T F_t dt = 0 \tag{81}$$

where

$$F_t = \text{operating fuel cost}$$

Therefore

$$\left.\frac{dF}{dP_S}\right|_1 \delta P_{S1} dt_1 \div \left.\frac{dF}{dP_S}\right|_2 \delta P_{S2} dt_2 = 0 \tag{82}$$

Using equations (79) and (80) in equation (82)

$$\left.\frac{dF}{dP_S}\right|_1 \frac{L_{S1}}{L_{H1}} \delta P_{H1} dt_1 \div \left.\frac{dF}{dP_S}\right|_2 \frac{L_{S2}}{L_{H2}} \delta P_{H2} dt_2 = 0 \tag{83}$$

Solving for $\delta P_{H1} dt_1$ in equations (78) and (83)

$$\delta P_{H1} dt_1 = \frac{\left.\dfrac{-\partial \dot{q}}{\partial P_H}\right|_2 \delta P_{H2} dt_2}{\left.\dfrac{\partial \dot{q}}{\partial P_H}\right|_1 \left(1 - \dfrac{\partial \dot{q}}{\partial h} \dfrac{(t_2 - t_1)}{S}\right)} = \frac{\left.\dfrac{dF}{dP_S}\right|_2 \dfrac{L_{S2}}{L_{H2}} \delta P_{H2} dt_2}{\left.\dfrac{dF}{dP_S}\right|_1 \dfrac{L_{S1}}{L_{H1}}}$$

$$\frac{\left.\dfrac{\partial \dot{q}}{\partial P_H}\right|_2 \dfrac{L_{H2}}{L_{S2}}}{\left.\dfrac{dF}{dP_S}\right|_2 \left(1 - \dfrac{\partial \dot{q}}{\partial h} \dfrac{(t_2 - t_1)}{S}\right)} = \frac{\left.\dfrac{\partial \dot{q}}{\partial P_H}\right|_1 \dfrac{L_{H1}}{L_{S1}}}{\left.\dfrac{dF}{dP_S}\right|_1}$$

Let

$$\frac{\dfrac{dF}{dP_S} L_S}{\dfrac{\partial \dot{q}}{\partial P_H} L_H} = \gamma$$

Then

$$\frac{1}{\gamma_2 \left(1 - \dfrac{\partial \dot{q}}{\partial h} \dfrac{(t_2 - t_1)}{S}\right)} = \frac{1}{\gamma_1}$$

and

$$\gamma_2 - \gamma_1 = \gamma_2 \frac{\partial \dot{q}}{\partial h} \frac{t_2 - t_1}{S} \tag{84}$$

For $t_2 - t_1 \longrightarrow dt$

$$d\gamma = \gamma \frac{\partial \dot{q}}{\partial h} \frac{dt}{S}$$

$$\gamma = \gamma_0 \epsilon^{\int_0^t \frac{\partial \dot{q}}{\partial h} \frac{dt}{S}}$$

Recall the definition of γ

$$\gamma = \frac{\dfrac{dF}{dP_S} L_S}{\dfrac{\dot{q}}{P_H} L_H} = \gamma_0 \epsilon^{\int_0^t \frac{\partial \dot{q}}{\partial h} \frac{dt}{S}}$$

whence

$$\gamma \frac{\partial \dot{q}}{\partial P_H} L_H = \frac{dF}{dP_S} L_S = \lambda \tag{85}$$

$$\gamma = \gamma_0 \epsilon^{\int_0^t \frac{\partial \dot{q}}{\partial h} \frac{dt}{S}} \tag{86}$$

These are used as one equation for scheduling

$$\frac{dF}{dP_S} = \frac{L_H}{L_S} \frac{\partial \dot{q}}{\partial P_H} \gamma_0 \epsilon^{\int_0^t \frac{\partial \dot{q}}{\partial h} \frac{dt}{S}} \tag{87}$$

(b) <u>Derivation of Kron's Equation.</u> It is desired to minimize

$$\int_0^T F dt$$

With the auxiliary relation

$$P_S + P_H - P_L = P_R$$

where

$$P_H = P_H(q, \dot{q})$$

Let

$$\phi = \lambda(P_R + P_L - P_S - P_H) = 0$$

then

$$\int_0^T (F + \phi) dt = \int_0^T F dt$$

and the extended integral to be minimized is

$$\int_0^T (F + \emptyset)dt$$

Let

$$J = F + \emptyset$$

The Euler equation is

$$\frac{\partial J}{\partial x_i} - \frac{d}{dt}\left(\frac{\partial J}{\partial \dot{x}_i}\right) = 0$$

For

$$x_i = P_S$$

$$\frac{\partial F}{\partial P_S} + \lambda \frac{\partial P_L}{\partial P_S} - \lambda = 0 \tag{88}$$

For

$$x_i = q$$

$$\lambda\left(\frac{\partial P_L}{\partial q} - \frac{\partial P_H}{\partial q}\right) - \frac{d}{dt}\left[\lambda\left(\frac{\partial P_L}{\partial \dot{q}} - \frac{\partial P_H}{\partial \dot{q}}\right)\right] = 0$$

But

$$\frac{\partial P_H}{\partial q} = \frac{\partial P_H}{\partial h}\left(- \frac{1}{S}\right)$$

and

$$\frac{\partial P_L}{\partial q} = \frac{\partial P_L}{\partial P_H} \frac{\partial P_H}{\partial q}$$

while

$$\frac{\partial P_L}{\partial \dot{q}} = \frac{\partial P_L}{\partial P_H} \frac{\partial P_H}{\partial \dot{q}}$$

Therefore

$$- \lambda \frac{\partial P_H}{\partial q}\left(1 - \frac{\partial P_L}{\partial P_H}\right) + \frac{d}{dt}\left[\lambda \frac{\partial P_H}{\partial \dot{q}}\left(1 - \frac{\partial P_L}{\partial P_H}\right)\right] = 0$$

or

$$\frac{\lambda}{S}\left(1 - \frac{\partial P_L}{\partial P_H}\right)\frac{\partial P_H}{\partial h} + \frac{d}{dt}\left[\lambda\left(1 - \frac{\partial P_L}{\partial P_H}\right)\frac{\partial P_H}{\partial \dot{q}}\right] = 0 \tag{89}$$

Equations (88) and (89) are the scheduling equations.

(c) <u>Proof of identity of Kron's and Ricard's Equations</u>[4].

It is understood that $\dot{q} = \dot{q}(h, P)$ relating flow, head, and power can be written

$$f = \dot{q} - \dot{q}(h, P) = 0 \tag{90}$$

Then

$$\frac{\partial f}{\partial \dot{q}} = 1 \neq 0 \tag{91}$$

Neglecting losses, Kron's equation is

$$\frac{\lambda}{S}\frac{\partial P}{\partial h} + \frac{d}{dt}\left(\lambda \frac{\partial P}{\partial \dot{q}}\right) = 0 \tag{92}$$

and Ricard's

$$\frac{d\gamma}{\gamma} = \frac{\partial \dot{q}}{\partial h}\frac{dt}{S} \tag{93}$$

where

$$\gamma = \frac{\lambda}{\frac{\partial \dot{q}}{\partial P}} \tag{94}$$

Substituting equation (94) in (93)

$$d\left(\frac{\lambda}{\frac{\partial \dot{q}}{\partial P}}\right) = \frac{\lambda}{\frac{\partial \dot{q}}{\partial P}}\frac{\partial \dot{q}}{\partial h}\frac{dt}{S} \tag{95}$$

$$-\frac{\lambda}{S}\frac{\frac{\partial \dot{q}}{\partial h}}{\frac{\partial \dot{q}}{\partial P}} + \frac{d}{dt}\left(\frac{\lambda}{\frac{\partial \dot{q}}{\partial P}}\right) = 0 \tag{96}$$

Equations (90) and (91) permit the derivation of

$$\frac{-\frac{\partial \dot{q}}{\partial h}}{\frac{\partial \dot{q}}{\partial P}} = \frac{\partial P}{\partial h} \quad \text{and} \quad \frac{\partial \dot{q}}{\partial P} = \frac{1}{\frac{\partial P}{\partial \dot{q}}}$$

Substitution of these quantities in equation (96) yields Kron's equation.

(d) Arismunandar and Noakes's Equations[1]. The following derivation makes use of time-dependent variables. With a system of one hydro plant and one thermal plant as sample system, the object is to minimize the integral I of the total fuel cost C, which is a function of the thermal power output P_T, over a fixed future short-time interval T

$$I = \int_0^T C(P_T, t)dt = \text{minimum}$$

The problem admits two sets of restrictions, for energy and for load requirements, respectively,

$$J = \int_0^T P_H(Q, h, t)dt = \text{constant } B$$

and

$$\phi = \phi(P, Q, h, t)$$

$$= P_H(Q, h, t) + P_T(t) - P_L(P_H, P_T, t) - P_D(t) = 0$$

where P_H, P_L, and P_D are hydro-plant output, loss, and load, respectively. Using more suitable variables, flow F and storage S, instead of more conventional discharge Q and head h, this transformation can be made:

$$P_H(Q, h, t) = P_H(F, S, \frac{dS}{dt}, t)$$

Eliminating the uncontrollable and indeterminable alien variable F, the load and hydro restraints now become

$$J = \int_0^T P_H(S, S', t)dt = \text{constant } B$$

and

$$\begin{aligned} \emptyset = {} & P_H(S, S', t) + P_T(t) \\ & - P_L(P_H(S, S', t), P_T, t) - P_D(t) = 0 \end{aligned}$$

where the prime indicates differentiation with respect to time t.

There are, henceforth, two independent variables P_T and S, and their derivatives. Thus, the dependent variables C, P_H and $\emptyset$, together with their integrals, each are functions of P_T and S.

The general equations of the problem of minimizing I, where

$$I = \int_0^T Hdt = \int_0^T (C + \lambda_1 P_H + \lambda_2 \emptyset)dt$$

are given by Euler equations

$$\frac{\partial H}{\partial q_u} - \frac{d}{dt}\frac{\partial H}{\partial q_u'} = 0 \qquad u = 1, 2$$

where q_u are time functions.

For the one hydro plant with $q_1 = S$; $q_1' = S'$, the Euler equation becomes

$$g(t) - \frac{d}{dt}\left(g(t)\,\frac{\partial S}{\partial S'}\right) = 0 \qquad (97)$$

where

$$g(t) = \left[\left(\frac{\partial C}{\partial P_T} + \lambda_2\right)\frac{\partial P_T}{\partial P_H} + (\lambda_1 + \lambda_2) - \lambda_2\left(\frac{\partial P_L}{\partial P_H} + \frac{\partial P_D}{\partial P_H}\right)\right]\frac{\partial P_H}{\partial S} \qquad (98)$$

Similarly, for the thermal plant, with $q_2 = P_T$; $q_2' = P_T'$

$$f(t) - \frac{d}{dt}\left(f(t)\,\frac{\partial P_T}{\partial P_T'}\right) = 0 \qquad (99)$$

where

$$f(t) = \left(\frac{\partial C}{\partial P_T} + \lambda_2\right) + (\lambda_1 + \lambda_2)\,\frac{\partial P_H}{\partial P_T} - \lambda_2\left(\frac{\partial P_L}{\partial P_T} + \frac{\partial P_D}{\partial P_T}\right) \qquad (100)$$

Equations (97) and (99) can be combined into one differential equation

$$\frac{\partial^2 H}{\partial P_T \partial S'} - \frac{\partial^2 H}{\partial S \partial P_T'} + H_1(P_T'S'' - P_T''S') = 0 \qquad (101)$$

where

$$H_1 = \frac{1}{(S')^2}\,\frac{\partial^2 H}{\partial P_T'^2}$$

$$= \frac{1}{(P_T')^2}\,\frac{\partial^2 H}{\partial S'^2}$$

$$= -\frac{1}{(S')(P_T')}\,\frac{\partial^2 H}{\partial S' \partial P_T'}$$

Differential equation (101) is of the second order. Its general solution, known as the extremal C_o, hence contains two arbitrary constants of integration α and β, and two isoperimetric constants λ_1 and λ_2

$$\left.\begin{aligned} P_T &= P_T(\alpha, \beta, \lambda_1, \lambda_2, t) \qquad (102) \\ S &= S(\alpha, \beta, \lambda_1, \lambda_2, t) \qquad (103) \end{aligned}\right\} C_o$$

From equation (103), $S' = dS/dt$ can be found. This, together with the water inflow $F(t)$, will determine the scheduled hydro-plant output

$$P_H = P_H(\alpha, \beta, \lambda_1, \lambda_2, F, t) \qquad (104)$$

(e) Comparison with Previously Developed Formulas[1]. The previously developed formulas involve only scheduling of generations or load allocations among plants at any one time. Such is the case in practice, since the curve of load demand does not usually follow a pattern which is presentable in the form of a simple, continuous, and differentiable function of time. Therefore, in order to make the comparison feasible, it is necessary to reduce the general Euler equations into a simplified form without considering the time variations. The thermal equation then becomes, similar to all other formulae

$$\frac{\partial C}{\partial P_T} + \lambda_2\left(1 - \frac{\partial P_L}{\partial P_T}\right) = 0 \qquad (105)$$

and the hydro equation is

$$\lambda_1\left(\frac{\partial P_H}{\partial S} - \frac{d}{dt}\frac{\partial P_H}{\partial S'}\right) + \lambda_2\left(1 - \frac{\partial P_L}{\partial P_H}\right)\frac{\partial P_H}{\partial S} - \frac{d}{dt}\left[\lambda_2\left(1 - \frac{\partial P_L}{\partial P_H}\right)\frac{\partial P_H}{\partial S'}\right] = 0 \qquad (106)$$

The following shows the comparison with Kron's equation. Kron's problem is not restricted by the energy requirement, B. Hence, the Lagrangian multiplier will vanish, and equation (106) will reduce to

$$\lambda_2\left(1 - \frac{\partial P_L}{\partial P_H}\right)\frac{\partial P_H}{\partial S} - \frac{d}{dt}\left[\lambda_2\left(1 - \frac{\partial P_L}{\partial P_H}\right)\frac{\partial P_H}{\partial S'}\right] = 0 \qquad (107)$$

This is to be compared with Kron's equation, which, with changing the notations used, is given in equation (89)

$$\frac{\lambda}{A}\left(1 - \frac{\partial P_L}{\partial P_H}\right)\frac{\partial P_H}{\partial h} \div \frac{d}{dt}\left[\lambda\left(1 - \frac{\partial P_L}{\partial P_H}\right)\frac{\partial P_H}{\partial q'}\right] = 0 \qquad (89)$$

where A is the surface area of the reservoir.

Kron's variables q and q' are related analytically by

$$q = q(t) = q(0) \div \int_0^t q'dt \qquad (108)$$

If leakage and evaporation are ignored, the inflow to a reservoir equals the outflow—which includes the amount being discharged $q'(t)$ and spilled $\sigma(t)$—plus the time rate of change of storage. Hence

$$q'(t) = F(t) - \sigma(t) - S'(t) \qquad (109)$$

Since the storage at any time t can be expressed as

$$S(t) = S(0) \div \int_0^t S'dt \qquad (110)$$

equation (108) can now be given as

$$q(t) = q(0) \div \int_0^t Fdt - \int_0^t \sigma dt - S(t) \div S(0) \qquad (111)$$

By letting

$$\int_0^t F dt = \Gamma(t) \div \Gamma(0) \qquad (112)$$

and

$$\int_0^t \sigma dt = \pi(t) \div \pi(0) \qquad (113)$$

Kron's q(t) can be further equalized with the negative of the storage S(t) if

$$\pi(t) = \Gamma(t)$$

$$q(0) = -\Gamma(0) \div \pi(0) - S(0) \qquad (114)$$

The latter equation stipulates the actual practical condition that the volume of storage to start each planning period depends on the integrated flow and spillage during the previous time interval.

Substitution of equations (112) through (114) in equation (111) gives the desired equality

$$q(t) = -S(t) \qquad (115)$$

from which the equivalence of equations (107) and (89) can be observed, with

$$\lambda = -\lambda_2 \qquad (116)$$

and

$$\frac{\partial P_H}{\partial q} = \frac{\partial P_H}{h}\left(-\frac{1}{A}\right) = -\frac{\partial P_H}{\partial S} \qquad (117)$$

for a vertical-sided reservoir.

The proof of equivalence between Kron's equation and Ricard's has been made and the equation of neglecting head variation can be observed to be the reduction form of Ricard's equation. Therefore no further proof is required to indicate the identity between the general time-dependent equations and those others.

(f) Effect of Head Variations. When head variations are significant, the coefficient γ becomes a function of time. Examine the form

$$\left[\gamma_{j0}\, \epsilon^{\int_0^t \frac{\partial W_j}{\partial h_j} \frac{dt}{S_j}} \right]$$

which takes over the function of γ_j for the fixed head case, according to Ricard's equations. The quantity $\partial W_j / \partial P_{Hj}$ is a negative number, for the required flow of water for a fixed power output decreases as the head increases. Thus the quantity

$$\int_0^t \frac{\partial W_j}{\partial h_j} \frac{dt}{S_j}$$

is negative and becomes increasingly negative with time. Consequently, the quantity

$$\left[\gamma_{j0}\, \epsilon^{\int_0^t \frac{\partial W_j}{\partial h_j} \frac{dt}{S_j}} \right]$$

decreases with time. Compared to the fixed head equations with a fixed value γ_j, Ricard's equation leads to scheduling less hydro power early in the time period under consideration and more power later in the period, since in general, increasing γ reduces the volume of water used; decreasing γ increases the volume of water used.

SUMMARY

Economic loading of an all-thermal system is obtained by a method based on equal incremental cost of received power as a necessary condition. Since a sufficient condition for a minimum depends on the characteristics of the units in the system, the non-decreasing smooth characteristics and step characteristics are used satisfactorily as the approximations of the characteristics of the units. The incremental cost of received power is equal to the incremental production cost of a unit times the reciprocal of incremental efficiency of the transmission network, called penalty factor, of the unit. In order to minimize the total production cost of the system, an evaluation of economic combination of the units to be placed on operation at the time must be made before load scheduling. The scheduling of fuel purchases may be also considered.

The necessary conditions for economic operation of interconnected areas are similar to those for the single area with the additional conditions for economic interchanges. The economic interchanges between the areas may be obtained by comparison of

appropriate incremental costs at the boundaries of the areas.

The necessary conditions of obtaining the short-range optimum economic schedule of the steam and hydroelectric plants within a given area are of the following forms

$$\frac{dF_n}{dP_{Sn}} \div \lambda \frac{\partial P_L}{\partial P_{Sn}} = \lambda \tag{69}$$

$$\gamma_j \frac{dW_j}{dP_{Hj}} \div \lambda \frac{\partial P_L}{\partial P_{Hj}} = \lambda \tag{70}$$

where $\frac{dF_n}{dP_{Sn}}$ = incremental production cost of steam plant n in dollars per mw-hr

$\frac{\partial P_L}{\partial P_{Sn}}$ = incremental transmission loss of steam plant n

$\frac{dW_j}{dP_{Hj}}$ = incremental water rate at hydro plant j in cfs per megawatt

$\frac{\partial P_L}{\partial P_{Hj}}$ = incremental transmission loss of hydro plant j

λ = incremental cost of received power in dollars per mw-hr

γ_j = water-conversion coefficient which converts incremental rate into equivalent incremental plant cost

These equations result in the scheduling of generation only at any one time which is the case in practice. The first conditions are again similar to those for the all-thermal system. The second conditions define the economic use of water in which γ_j's are constant when the effect of head variations are neglected and γ_j's are functions of time when head variations are significant.

Additional research is required to define optimum long-range reservoir drawdown mathematically in order to apply the short-range method to long-range problems.

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A STUDY OF OPTIMUM ECONOMIC OPERATION OF ELECTRIC POWER SYSTEMS

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B. E., Chulalongkorn University, 1958
Bangkok, Thailand

AN ABSTRACT OF
A MASTER'S REPORT

submitted in partial fulfillment of the

requirements for the degree

MASTER OF SCIENCE

Department of Electrical Engineering

KANSAS STATE UNIVERSITY
Manhattan, Kansas

1964

Development of the methods employing the classical techniques concerning optimum economic operation of electric power system is studied. In the area of system operation, the problem of determining the allocation of generation among thermal plants that are currently operating and on the line is viewed first. The equations resulting as a necessary condition are

$$\frac{dF_n}{dP_n}\left(\frac{1}{1 - \partial P_L/\partial P_n}\right) \equiv \frac{dF_n}{dP_n} L_n = \lambda \tag{1}$$

which state that incremental production cost of plant n, dF_n/dP_n, times penalty factor for plant n accounting for effect of transmission losses, L_n, is equal to incremental cost of received power, λ, and is the same for every plant. The allocation of generation so derived is predicated upon a known cost of fuel at each plant. The principle of scheduling to equal incremental costs of received power does not directly determine the units to be placed in operation at a given time. This determination to date has been usually solved by successive trials.

The coordination equations (1) are extended to obtain coordination equations whose solution results in optimum economy for the pool formed by the interconnected companies. The intra-area equations for each area are similar to those for the single- area system problem.

Another problem of optimization of system operation relates to the integration of the scheduling of hydro plants in a combined hydro and thermal power system so as to obtain the minimum fuel expenditures over the time period of interest. This problem is more complex than the all-thermal problem, as we must now be concerned with operation over a given period of time. We obtain equations of the form

$$\frac{dF_n}{dP_n} L_n = \lambda \tag{2}$$

$$\gamma_j \frac{dW_j}{dP_{Hj}} L_j = \lambda \tag{3}$$

where

$\frac{dW_j}{dP_{Hj}}$ = incremental water rate of plant j

γ_j = water conversion coefficient for plant j

When the effect of head variations upon the plant characteristics may be neglected, γ_j's are constants whose values are determined by an iterative procedure until the desired volume releases are obtained. When the effect of head variation is significant, γ_j's are functions of time and equation (3) leads to scheduling less hydro power early in the time period under consideration and more power later in the period.

However, the coordination equations (2) and (3) result in the scheduling of generation only at any one time, since the variables are made time-independent during the time period of interest. When true variational calculus procedures are pursued, all variables must be made time-dependent. This approach solves for whole intervals to be optimized as integral units.