A STUDY OF SAMPLING AND SCALE-UP IN SOLIDS MIXING

by 1050 710

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CHAPTER I

INTRODUCTION

1.1 GENERAL

Solids mixing may be described as any operation in which energy is applied to a particulate solid system such that the inhomogeneity and concentration gradients tend to diminish. It is a critical operation in many process industries, such as agricultural, pharmaceutical, and ceramic industries. However, it has been much less developed both theoretically and practically compared to other unit operations. Recently there has been a spurt of activity to further develop solids mixing.

Unlike liquid mixing, research on solids mixing has been relatively limited. Probably the statistical nature and discontinuity property of the solid particles hinder the development of this field. For example, the categories of the sampling technique used in a particulate process are far more complex than those used in a liquid process.

The degree of mixedness is a fundamental state of a system and is always evaluated from the sampling results. Fan, et al., (1970) reviewed over thirty different definitions of the degree of mixedness. The difference in the definitions for the criterion reveals the complexity of the mixing process and the uncertainty of various concepts and notions in the field of solids mixing.

Since the mixing action is very complex, it is extremely difficult to formulate an adequate mathematical model describing the action. The practicality and experience still predominate in the design and operation of the

mixing equipment and in the assessment of the quality of a mixture.

1.2 PREVIOUS WORK

The literature on solids mixing has been thoroughly reviewed by Weidenbaum (1953), Gren (1967), Klothen (1969), Fan, et al. (1970), and Fan, et al. (1972a). A brief review of the recent pertinent literature is given below.

Several researchers (Valentin, 1965; Rose, et al., 1965; and Fan, et al., 1970) stressed the following needs in this field.

a. Unification of the mixing index
b. Clarification of the different mixing mechanisms
c. Measurement and control of segregation
d. Systematic study of mixers
e. Modelling and simulation of the mixing process
f. Rules for scale-up and design
g. Synthesis of the mixing process

1.2.1 Statistical Approach to Solids Mixing

Statistical analysis has become the approach most frequently used among investigators because of the random nature of the mixing process. Probably the point at which most analyses begin is that of defining a suitable measure of the degree of mixing. This measure indicates how the composition of the bed being mixed varies from point to point. Most authors (for example, Lacey, 1943; Bourne, 1965; Weidenbaum, 1969) have agreed that the best way to express this degree of mixing is through statistical methods, namely some form of variance which is based upon samples taken from various points in the bed.

Lacey (1943) has shown that for a completely random mixture, the variance in composition among a group of samples drawn from it is given by

$$s_r^2 = \frac{P(1 - P)}{n} \tag{1}$$

where P = overall fraction of a particular type of trace particle

n = number of particles in the sample.

Also, for a completely unmixed system, he has shown that

$$s_0^2 = P(1 - P) \tag{2}$$

This leads to his definition of the degree of mixing any mixture

$$M = \frac{s_0^2 - s^2}{s_0^2 - s_r^2} \tag{3}$$

where

$$s^2 = \frac{1}{n-1} \sum_{i=1}^{n} (x_i - \bar{x})^2$$

Fan, et al. (1970) reviewed over thirty different definitions of the degree of mixedness, which differ with the systems used and the experimental procedures followed, especially the sample size. Nevertheless, the relationship between the variance and the sample size is unknown so that mixing indices based upon the variance are dependent upon the sample size, and comparisons among mixing studies in which different sample sizes have been used are therefore of limited value (Williams, 1969). To overcome these difficulties a theoretical description of the relationship between variance and sample size for non-random mixtures must be deduced. Bourne (1965, 1967) gave an interpretation of the results obtained by Poole, et al. (1964), using a statistical theory developed by Landry (1944). Danckwerts (1963) proposed a description of the correlation by correlograms, i.e., the relationship between

the coefficient of correlation of point samples and the distance between the samples. Schofield (1968) showed that the description mentioned can be used to elucidate the mechanism involved in the mixing process. Williams (1969) made a theoretical approach assuming mixing components of uniform particle size. Furthermore, Harnby (1971) has discussed the application of social survey statistical techniques to mixing problems. He mentioned the possibility of describing variance-sample size relationships by correlation theories. Recently, Kristensen (1973) derived a general expression for the variance of the composition of samples drawn from random or non-random mixtures.

Buslik (1973) proposed the negative log of the sample weight required to obtain a standard deviation of 1% as a simple numerical homogeneity index for expressing varying degrees of homogeneity quantitatively. The proposed method is of universal applicability, and a spectrum of index values for homogeneity has been computed for certain mixtures over a very wide range. With a different viewpoint, Akao, et al. (1971) proposed a degree of mixedness for binary mixtures of uniform size particles in regular and random arrangements based on the coordination number. An imaginary or hypothetical particle model was proposed by Akao (1969) in evaluating the distribution of coordination numbers for fine-coarse mixtures.

For convenience of converting each of the degree of mixedness to other forms, conversion formulae are tabulated in TABLE 1. Several converted numerical values for different sizes of samples are presented in Appendix 1.5. It can be seen that M_1, M_4, and M_5 are more dependent upon the size of the sample than others. The form $M_3 = \sigma_0^2 - \sigma^2/\sigma_0^2 - \sigma_r^2$ approaches unity more rapidly than does the expression $M_8 = \sigma_0 - \sigma/\sigma_0 - \sigma_r$, while the latter form is more convenient in practical application. The comparison of various forms of expression may be the first step in unifying the definition of the degree of

TABLE 1

EQUIVALENT FORMS OF DEGREE OF MIXEDNESS FOR BINARY SYSTEMS

	M_1	M_2	M_3	M_4
M_1	$1 - \frac{\sigma}{\sigma_0}$	$2M_1 - M_1^2$	$\frac{2M_1 - M_1^2}{1 - \frac{1}{N}}$	$\frac{1}{\sqrt{N}\ (1 - M_1)}$
M_2	$1 - \sqrt{1 - M_2}$	$1 - \frac{\sigma^2}{\sigma_0^2}$	$\frac{M_2}{1 - \frac{1}{N}}$	$\frac{1}{\sqrt{N}\ (1 - M_2)}$
M_3	$1 - \sqrt{1 - M_3\ (1 - \frac{1}{N})}$	$M_3\ (1 - \frac{1}{N})$	$\frac{\sigma_0^2 - \sigma^2}{\sigma_0^2 - \sigma_r^2}$	$\frac{1}{\sqrt{N}\ (1 - M_3) + M_3}$
M_4	$1 - \frac{1}{\sqrt{N M_4}}$	$1 - \frac{1}{M_4^2 N}$	$\frac{NM_4^2 - 1}{M_4^2\ (N - 1)}$	$\frac{\sigma_r}{\sigma}$
M_5	$\frac{M_5\ (\sqrt{N} - 1)}{M_5\ (\sqrt{N} - 1) + 1}$	$1 - \frac{1}{[1+M_5(\sqrt{N}-1)]^2}$	$\frac{N[M_5^2(\sqrt{N}-1)+2M_5]}{(\sqrt{N}+1)[M_5(\sqrt{N}+1)+1]^2}$	$\frac{M_5\ (\sqrt{N} - 1) + 1}{\sqrt{N}}$

TABLE 1--Continued

	M_1	M_2	M_3	M_4
M_6	$1 - M_6$	$1 - M_6^2$	$\frac{1 - M_6^2}{1 - \frac{1}{N}}$	$\frac{1}{\sqrt{N} M_6}$
M_7	$1 - \exp\left[M_7^2 \ln \frac{1}{\sqrt{N}}\right]$	$1 - \exp\left[M_7^2 \ln \frac{1}{N}\right]$	$\frac{1 - \exp\left[M_7^2 \ln \frac{1}{N}\right]}{1 - \frac{1}{N}}$	$\frac{1}{\sqrt{N} \exp\left[M_7^2 \ln \frac{1}{\sqrt{N}}\right]}$
M_8	$M_8 \left(1 - \frac{1}{\sqrt{N}}\right)$	$1 - \left[1 - M_8\left(1 - \frac{1}{\sqrt{N}}\right)\right]^2$	$\frac{1 - \left[1 - M_8\left(1 - \frac{1}{\sqrt{N}}\right)\right]^2}{1 - \frac{1}{N}}$	$\frac{1}{\sqrt{N} - M_8 (\sqrt{N} - 1)}$
M_9	$1 - \sqrt{M_9}$	$1 - M_9$	$\frac{1 - M_9}{1 - \frac{1}{N}}$	$\frac{1}{\sqrt{N M_9}}$

TABLE 1--Continued

	M_5	M_6	M_7	M_8	M_9
M_1	$\dfrac{M_1}{(1 - M_1)(\sqrt{N} - 1)}$	$1 - M_1$	$\sqrt{\dfrac{\ln (1 - M_1)}{\ln \frac{1}{\sqrt{N}}}}$	$\dfrac{M_1}{1 - \frac{1}{\sqrt{N}}}$	$(1 - M_1)^2$
M_2	$\dfrac{1 - \sqrt{1-M_2}}{\sqrt{1-M_2}\ (\sqrt{N}-1)}$	$\sqrt{1 - M_2}$	$\sqrt{\dfrac{\ln (1 - M_2)}{\ln \frac{1}{N}}}$	$\dfrac{1 - \sqrt{1 - M_2}}{1 - \frac{1}{\sqrt{N}}}$	$1 - M_2$
M_3	$\dfrac{1-\sqrt{1-M_3(1-\frac{1}{N})}}{\sqrt{1-M_3(1-\frac{1}{N})}\ (\sqrt{N}-1)}$	$\sqrt{1 - M_3\ (1 - \frac{1}{N})}$	$\sqrt{\dfrac{\ln[1-M_3(1-\frac{1}{N})]}{\ln \frac{1}{N}}}$	$1 - \sqrt{1-M_3(1-\frac{1}{N})}$	$1 - M_3\ (1 - \frac{1}{N})$
M_4	$\dfrac{M_4\ \sqrt{N} - 1}{\sqrt{N} - 1}$	$\dfrac{1}{\sqrt{N}\ M_4}$	$\sqrt{\dfrac{\ln \sqrt{N}}{\ln \sqrt{N} \cdot M_4}}$	$\dfrac{1}{1 - \frac{\sqrt{N} \cdot M_4}{\sqrt{N}}}$	$\dfrac{1}{NM_4^2}$
M_5	$\dfrac{\frac{\sigma_0}{\sigma} - 1}{\frac{\sigma_0}{\sigma_r} - 1}$	$\dfrac{1}{M_5\ (\sqrt{N} - 1) + 1}$	$\sqrt{\dfrac{\ln \sqrt{N}}{\ln\ [M_5(\sqrt{N}-1)+1]}}$	$\dfrac{\sqrt{N}\ M_5}{M_5\ (\sqrt{N} - 1) + 1}$	$\dfrac{1}{(M_5\ (\sqrt{N} - 1) + 1)^2}$

TABLE 1--Continued

	M_5	M_6	M_7	M_8	M_9
M_6	$\frac{1 - M_6}{M_6\ (\sqrt{N} - 1)}$	$\frac{\sigma}{\sigma_0}$	$\sqrt{\frac{\ln M_6}{\ln \frac{1}{\sqrt{N}}}}$	$\frac{1 - M_6}{1 - \frac{1}{\sqrt{N}}}$	M_6^2
M_7	$\frac{\exp[M_7^2 \ln \sqrt{N}] - 1}{\sqrt{N} - 1}$	$\exp\ [M_7^2 \ln \frac{1}{\sqrt{N}}]$	$\frac{\ln \sigma_0^2 - \ln \sigma^2}{\ln \sigma_0^2 - \ln \sigma_r^2}$	$\frac{1 - \exp\ [M_7^2 \ln \frac{1}{\sqrt{N}}]}{1 - \frac{1}{\sqrt{N}}}$	$\exp\ [M_7^2 \ln \frac{1}{\sqrt{N}}]^2$
M_8	$\frac{M_8}{\sqrt{N} - M_8\ (\sqrt{N} - 1)}$	$1 - M_8\ (1 - \frac{1}{\sqrt{N}})$	$\sqrt{\frac{\ln[1-M_8(1-\frac{1}{\sqrt{N}})]}{\ln \frac{1}{\sqrt{N}}}}$	$\frac{\sigma_0 - \sigma}{\sigma_0 - \sigma_r}$	$[1 - M_8(1 - \frac{1}{\sqrt{N}})]^2$
M_9	$\frac{1 - \sqrt{M_9}}{\sqrt{M_9}\ (\sqrt{N} - 1)}$	$\sqrt{M_9}$	$\sqrt{\frac{\ln \sqrt{M_9}}{\ln \frac{1}{\sqrt{N}}}}$	$\frac{1 - \sqrt{M_9}}{1 - \frac{1}{\sqrt{N}}}$	$\frac{\sigma^2}{\sigma_0^2}$

mixedness.

The principle of uncertainty is introduced in a probabilistic or stochastic model. A stochastic process is a random phenomenon that is controlled by statistical laws. This approach seems to be more fundamental and helpful than the deterministic approach in analyzing and understanding the complex mechanisms of solids mixing processes. With this type of model, mathematical intractability can be avoided.

Oyama and Ayaki (1956) proposed a Markov chain model to describe the mixing of particles in a drum mixer but did not conduct experiments to verify the model. Oleniczak (1962) postulated a Poisson process to interchange particles between a volume element and the rest of the mixture. He obtained a stochastic model for the V-type mixer. The distribution of tracer particles was found to be bimodel at a low number of revolutions.

Makarov and Gorbushin (1970) used the Markov process technique to describe the mechanisms of transition of particles in a circular cell model. They proposed this model for the preliminary design of a batch mixer for free flowing materials with closed loop internal circulation. It is assumed that the termination of convective mixing is the determining factor in obtaining the optimum time of mixing because at some time $t_{cov} = t_{opt}$, and the mixing process achieves an equilibrium with the segregation process. The main idea is to divide the internal operating volume of the mixer into a number of zones, each of which has a characteristic particle flow pattern. Assuming that the laws governing the movement of particles through each zone are known, Makarov and Gorbushin (1970) determined the average residence time of particles in each zone and the standard deviation of the residence time distribution in any zone. If the system as a whole is linear, the total average residence time of a particle and the standard deviation for the entire mixer can be

calculated. Experimental verification of the method was presented.

Fan, et al. (1972b) employed a Markov chain model to model the axial mixing of solid particles in a motionless mixer. One-step transition probabilities were determined experimentally for the model. A fairly good agreement with the experimental data was obtained for up to seven steps of the Markov chain, or what was equivalent to seven consecutive passes of the mixture through the mixer.

1.2.2 Scale-up and Design of Solids Mixers

While solids mixing is widely employed and considerable progress has been made in understanding its mechanisms, sufficiently reliable 'design' formulae are not available that permit an engineer to design industrial scale mixers or scale-up small mixers based on the results of laboratory experiments. Comparatively little has been reported on the design and scale-up of solids mixers.

Muller (1967) and Rumpf and Muller (1962) evaluated different mixing elements for a paddle mixer. Muller compared the amount of material lifted by the differently shaped elements across the mixer diameter. He demonstrated that the mixing rate is directly dependent on mixer speed and on the effective surface area of the mixing element. The effective surface is a function of the angle between the shaft and the mixing blade. No generalizations were offered.

Luterek and Cachia (1971) used the Froude number as a criterion for scale-up of V-type mixers. Their method was verified by experiments where two different dry powders were mixed in V-type batch mixers of four different sizes. The scale-up procedure is based on the principle that Froude numbers for the laboratory scale mixer and the full scale mixer must be equal, i.e.:

$$\left(\frac{N_1^2\ kD_1}{g}\right)_{\text{lab. scale}} = \left(\frac{N_2^2\ kD_2}{g}\right)_{\text{full scale}} \tag{4}$$

Lynch and Ho (1973) presented a standard design procedure for determining the power requirements for double cone and ribbon blenders.

Sawahata (1969) employed the relationship between the circulation and mixing times to estimate the mixing time of a large-scale mixer. The circulation time of the particulate solids in a drum mixer was related to the operating variables as (Sawahata, 1967):

$$T_{HC} = \frac{R^2(F/V)}{(N/60)h(2R - h)}, \text{ if } \left(\frac{N^2R}{g}\right)_{HC} < 25 \times 10^{-3} \tag{5}$$

where

T_{HC} = average circulation time of solid particles

R = the radius of the mixer

F/V = filling ratio of particles

h = thickness of the transportation zone

The thickness of the transportation zone, h, in equation (5) can be related to the filling ratio and Froude number as (Sawahata, 1968, and Sawahata, 1969):

$$\frac{h}{R} = \left(\alpha - \beta \frac{N^2R}{g}\right)\left(\frac{F}{V}\right) \tag{6}$$

where constants, α and β, were determined by experiments (Sawahata, 1969). For the V-type mixer the following equations hold.

$$T_V = \eta \frac{\sqrt{R}}{(N/60)}\left(\frac{F}{V}\right)^{2/3}, \text{ if } \left(\frac{N^2R}{g}\right)_V < 7.6 \times 10^{-3} \tag{7}$$

where η is the correction factor.

The lengths of time needed to attain a satisfactory mixed state for the

drum mixer and V-type mixer, respectively (Sawahata, 1968), are

$$\theta_{HC} = 20\ T_{HC} \tag{8}$$

$$\theta_{V} = 10\ T_{V} \tag{9}$$

If the dynamic similarity exists between two geometrically similar drum mixers, i.e., by holding the Froude number as constant,

$$\frac{N^2 R}{g} = \text{constant} \tag{10}$$

Let

$$\frac{F}{V} = \text{constant} \tag{11}$$

Then, from equation (6), we have

$$\frac{h}{R} = \text{constant} \tag{12}$$

Relating equations (12) and (5), we have

$$N\ T_{HC} = \text{constant} \tag{13}$$

i.e., for two geometrically similar drum mixers,

$$N_I T_I = N_{II} T_{II} \tag{14}$$

Substituting equation (13) into equation (10)

$$T_I / \sqrt{R}_I = T_{II} / \sqrt{R}_{II} \tag{15}$$

If $(\theta_{HC})_I$ represents the mixing time of a small-scale mixer, then the mixing time $(\theta_{HC})_{II}$ of a large-scale mixer loaded with mixtures of the same concen-

tration as the small scale is

$$(\theta_{HC})_{II} = \frac{T_V}{T_I} (\theta_{HC})_I = \frac{\sqrt{R_{II}}}{\sqrt{R_I}} (\theta_{HC})_I \tag{16}$$

A similar result can be obtained by correlating equations (7), (9), and (10) for the V-type mixer as follows:

$$(\theta_V)_{II} = \frac{R_V}{R_I} (\theta_V)_I = \frac{N_I \sqrt{R_{II}}}{N_{II} \sqrt{R_I}} (\theta_V)_I \tag{17}$$

where

$(\theta_V)_{II}$ = the mixing time for a large-scale V-type mixer.

$(\theta_V)_I$ = the mixing time for a small-scale V-type mixer.

Sawahata (1968) presented experimental verification of this method.

1.3 OBJECTIVES

The purposes of the present study are threefold. The first series of studies sought to obtain further information on the statistical nature of the samples in solids mixing by a nonparametric statistical approach. Most of the previous works on the evaluation of the sampling results are parametrically oriented (Harby, 1971; Shinnar and Noar, 1961; and Miles, et al., 1960). They have to assume that the population is distributed with some parameters. The application of nonparametric statistics has its merit in testing hypotheses when we do not assume, or even care about, the normality. Many of the nonparametric tests and other nonparametric procedures are simpler than the usual parametric procedures, and have high power to detect true differences.

The second phase of investigation studied a microscopic and geometric mixing index-contact number. Most of the definitions of the degree of mixed-

ness concern primarily the measurement of the standard deviation or the variance of the spot samples taken from a mixture. Such a viewpoint always neglects the structure inside the spot samples, i.e., it assumes that a completely mixed state exists in any spot sample. This mixing index, first used by Akao, et al. (1973), has the merit of not depending on this assumption. The first part of this second phase was a computer simulation of the distribution of the contact number for a binary system at the completely mixed state. Results were obtained for the two dimensional cubic and hexagonal packings at different concentrations of key components. In the second part of this phase, mixing index was employed to the radial mixing of particles of the same size in a motionless mixer. The results were compared with those made by spot sampling.

The third phase of work investigated the scale-up and design procedures for tumbling mixers. The principle of similarity (Johnstone and Thring, 1957) was exploited to study this category of mixers. If the physical properties of the particles are not far different, it can be reasonably stated that the criteria derived are applicable to the scale-up procedures.

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1.5 APPENDIX

Computer Program and Numerical Comparison of Some Degrees of Mixedness


```
      PN=0.0
    4 PN=PN+10.
    7 WRITE (6,12) PN
   12 FORMAT (1X,//,1X,'NUMBER OF PARTICLES PER SAMPLE = ',F5.0)
      WRITE(6,11)
   11 FORMAT(3X,'M1',7X,'M2',7X,'M3',7X,'M4',7X,'M5',7X,'M6',7X,'M7',7X,
     1'M8',7X,'M9')
      DM8=0.0
    6 DM3=(1.-(1.-DM8*(1.-1./SQRT(PN)))**2)/(1.-1./PN)
      DM1=1.-SQRT(1.-DM3*(1.-1./PN))
      DM2=DM3*(1.-1./PN)
      DM4=1./(SQRT(PN*(1.-DM3)+DM3))
      DM5=(DM1*1.)/((1.-DM1)*(SQRT(PN)-1.))
      DM6=1.-DM1
      DM7=SQRT(ALOG(1.0-DM2)/ALOG(1./PN))
      DM9=1.-DM2
      WRITE(6,22) DM1,DM2,DM3,DM4,DM5,DM6,DM7,DM8,DM9
   22 FORMAT(1X,9(1X,F8.5))
      IF(DM8-1.0) 3,3,2
    2 DM8=DM8+0.1
      GO TO 6
    3 IF(PN-100.) 4,5,5
    5 IF (PN-1000.) 8,8,9
    8 PN=PN+100.
      GO TO 7
    9 STOP
      END
```

```
*ENTRY
```

NUMBER OF PARTICLES PER SAMPLE = 10.

M1	M2	M3	M4	M5	M6	M7	M8	M9
0.00000	0.00000	0.00000	0.31623	0.00000	1.00000	0.00000	0.00000	1.00000
0.06838	0.12209	0.14675	0.33944	0.03394	0.93162	0.24803	0.10000	0.86792
0.12675	0.25491	0.28312	0.36632	0.07326	0.86325	0.35747	0.20000	0.74519
0.20513	0.36819	0.40909	0.39784	0.11935	0.79487	0.44655	0.30000	0.63182
0.27351	0.47221	0.52468	0.43528	0.17411	0.72649	0.52682	0.40000	0.52779
0.34189	0.56699	0.62987	0.48051	0.24025	0.65811	0.60282	0.50000	0.43311
0.41026	0.65221	0.72468	0.53622	0.32173	0.58974	0.67726	0.60000	0.34779
0.47864	0.72819	0.80969	0.60654	0.42458	0.52136	0.75215	0.70000	0.27182
0.54702	0.79481	0.88312	0.69810	0.55848	0.45298	0.82936	0.80000	0.20519
0.61540	0.85209	0.94675	0.82221	0.73999	0.38460	0.91103	0.90000	0.14792
0.68377	0.90000	1.00000	1.00000	1.00000	0.31623	1.00000	1.00000	0.10000

NUMBER OF PARTICLES PER SAMPLE = 20.

M1	M2	M3	M4	M5	M6	M7	M8	M9
0.00000	0.00000	0.00000	0.22361	0.00000	1.00000	0.00000	0.00000	1.00000
0.07764	.14925	0.15711	0.24243	0.02424	0.92236	0.23228	0.10000	0.85075
0.15528	0.29645	0.30152	0.26471	0.05294	0.84472	0.33565	0.20000	0.71355
0.23292	0.41159	0.43325	0.29157	0.08745	0.76708	0.42274	0.30000	0.58841
0.31056	0.52467	0.55228	0.32433	0.12973	0.68944	0.49826	0.40000	0.47533
0.38820	0.62570	0.65863	0.36549	0.18274	0.61180	0.57274	0.50000	0.37430
0.46584	0.71467	0.75228	0.41861	0.25117	0.53416	0.64702	0.60000	0.28533
0.54349	0.79159	0.83325	0.48940	0.34286	0.45652	0.72352	0.70000	0.20841
0.62111	0.85645	0.90152	0.59017	0.47214	0.37889	0.81494	0.80000	0.14355
0.69875	0.90925	0.95711	0.74227	0.66805	0.30125	0.89500	0.90000	0.09075
0.77639	0.95000	1.00000	1.00000	1.00000	0.22361	1.00000	1.00000	0.05000

NUMBER OF PARTICLES PER SAMPLE = 30.

M1	M2	M3	M4	M5	M6	M7	M8	M9
-0.00000	0.00000	0.00000	0.18257	0.00000	1.00000	0.00000	0.00000	1.00000
0.08174	0.15680	0.16221	0.19883	0.01988	0.91826	0.22393	0.10000	0.84320
0.16349	0.30024	0.31060	0.21826	0.04365	0.83651	0.32399	0.20000	0.69976
0.24523	0.43032	0.44516	0.24189	0.07257	0.75477	0.40674	0.30000	0.56968
0.32697	0.54793	0.56589	0.27127	0.10851	0.67303	0.48253	0.40000	0.45297
0.40971	0.65038	0.67231	0.30877	0.15439	0.59129	0.55586	0.50000	0.34962
0.49046	0.74036	0.76589	0.35831	0.21499	0.50954	0.62966	0.60000	0.25964
0.57220	0.81699	0.84516	0.42677	0.29874	0.42780	0.70661	0.70000	0.18301
0.65394	0.88024	0.91060	0.52758	0.42206	0.34606	0.78993	0.80000	0.11976
0.73568	0.93014	0.96221	0.69074	0.62167	0.26432	0.88455	0.90000	0.06986
0.81743	0.96667	1.00000	1.00000	1.00000	0.18257	1.00000	1.00000	0.03333

NUMBER OF PARTICLES PER SAMPLE = 40.

M1	M2	M3	M4	M5	M6	M7	M8	M9
-0.00000	0.00000	0.00000	0.15811	0.00000	1.00000	0.00000	0.00000	1.00000
0.08419	0.16129	0.16543	0.17265	0.01726	0.91581	0.21836	0.10000	0.83871
0.16838	0.30840	0.31631	0.19013	0.03803	0.83162	0.31617	0.20000	0.69160
0.25257	0.44134	0.45266	0.21154	0.06346	0.74743	0.39728	0.30000	0.55866
0.33675	0.56011	0.57447	0.23939	0.09536	0.66325	0.47183	0.40000	0.43989
0.42094	0.66469	0.68174	0.27305	0.13653	0.57906	0.54426	0.50000	0.33531
0.50513	0.75511	0.77447	0.31951	0.19170	0.49487	0.61757	0.60000	0.24489
0.58932	0.83134	0.85266	0.38501	0.26950	0.41068	0.69462	0.70000	0.16866
0.67351	0.89340	0.91631	0.48428	0.38743	0.32649	0.77902	0.80000	0.10660
0.75770	0.94129	0.96543	0.65255	0.58729	0.24230	0.87668	0.90000	0.05871
0.84189	0.97500	1.00000	1.00000	1.00000	0.15811	1.00000	1.00000	0.02500

NUMBER OF PARTICLES PER SAMPLE = 50.

M1	M2	M3	M4	M5	M6	M7	M8	M9
-0.00000	0.00000	0.00000	0.14142	0.00000	1.00000	0.00000	0.00000	1.00000
0.08586	0.16434	0.16770	0.15470	0.01547	0.91414	0.21423	0.10000	0.83566
0.17172	0.31395	0.32035	0.17074	0.03415	0.82828	0.31035	0.20000	0.68605
0.25757	0.44880	0.45796	0.19149	0.05715	0.74243	0.39021	0.30000	0.55120
0.34343	0.56892	0.58053	0.21539	0.08616	0.65657	0.46378	0.40000	0.43108
0.42929	0.67429	0.68805	0.24780	0.12390	0.57071	0.53548	0.50000	0.32571
0.51515	0.76492	0.78053	0.29168	0.17501	0.48485	0.60835	0.60000	0.23508
0.60101	0.84080	0.85796	0.35444	0.24811	0.39899	0.68537	0.70000	0.15920
0.68686	0.90195	0.92235	0.45163	0.36130	0.31314	0.77046	0.80000	0.09805
0.77272	0.94834	0.96770	0.62224	0.56001	0.22728	0.87031	0.90000	0.05166
0.85858	0.98000	1.00000	1.00000	1.00000	0.14142	1.00000	1.00000	0.02000

NUMBER OF PARTICLES PER SAMPLE = 60.

M1	M2	M3	M4	M5	M6	M7	M8	M9
-0.00000	0.00000	0.00000	0.12910	0.00000	1.00000	0.00000	0.00000	1.00000
0.08709	0.16660	0.16942	0.14142	0.01414	0.91291	0.21097	0.10000	0.83340
0.17419	0.31832	0.32341	0.15633	0.03127	0.82582	0.30575	0.20000	0.68198
0.26127	0.45428	0.46198	0.17476	0.05243	0.73873	0.39461	0.30000	0.54572
0.34836	0.57537	0.58512	0.19811	0.07925	0.65164	0.45738	0.40000	0.42463
0.43545	0.68128	0.69283	0.22868	0.11434	0.56455	0.52947	0.50000	0.31872
0.52254	0.77293	0.78512	0.27039	0.16223	0.47746	0.60093	0.60000	0.22797
0.60963	0.84761	0.86198	0.33071	0.23150	0.39037	0.67786	0.70000	0.15239
0.69672	0.90802	0.92341	0.42568	0.34054	0.30328	0.76342	0.80000	0.09198
0.78381	0.95326	0.96942	0.59716	0.53744	0.21619	0.86496	0.90000	0.04674
0.87090	0.98333	1.00000	1.00000	1.00000	0.12910	1.00000	1.00000	0.01667

NUMBER OF PARTICLES PER SAMPLE = 70.

M1	M2	M3	M4	M5	M6	M7	M8	M9
-0.00000	0.00000	0.00000	0.11952	0.00000	1.00000	0.00000	0.00000	1.00000
0.08805	0.16834	0.17078	0.13106	0.01311	0.91195	0.20830	0.10000	0.83166
0.17610	0.32118	0.32584	0.14507	0.02901	0.82390	0.30197	0.20000	0.67882
0.26414	0.45851	0.46516	0.16243	0.04873	0.73586	0.37999	0.30000	0.54149
0.35219	0.58034	0.58875	0.18450	0.07380	0.64781	0.45209	0.40000	0.41966
0.44024	0.68667	0.69662	0.21352	0.10676	0.55976	0.52264	0.50000	0.31333
0.52829	0.77749	0.78875	0.25338	0.15203	0.47171	0.59474	0.60000	0.22251
0.61633	0.85280	0.86516	0.31153	0.21807	0.38367	0.67155	0.70000	0.14720
0.70438	0.91261	0.92584	0.40431	0.32345	0.29562	0.75743	0.80000	0.08739
0.79243	0.95691	0.97078	0.57582	0.51824	0.20757	0.86033	0.90000	0.04309
0.88048	0.98571	1.00000	1.00000	1.00000	0.11952	1.00000	1.00000	0.01429

NUMBER OF PARTICLES PER SAMPLE = 80.

M1	M2	M3	M4	M5	M6	M7	M8	M9
-0.00000	0.00000	0.00000	0.11180	0.00000	1.00000	0.00000	0.00000	1.00000
0.08882	0.16975	0.17190	0.12270	0.01227	0.91118	0.20604	0.10000	0.83025
0.17764	0.32372	0.32782	0.13595	0.02719	0.82236	0.29877	0.20000	0.67628
0.26646	0.46192	0.46776	0.15242	0.04572	0.73354	0.37607	0.30000	0.53808
0.35528	0.58433	0.59173	0.17341	0.06937	0.64472	0.44759	0.40000	0.41567
0.44410	0.69097	0.69972	0.20112	0.10056	0.55590	0.51767	0.50000	0.30903
0.53292	0.78183	0.79173	0.23937	0.14362	0.46708	0.58944	0.60000	0.21817
0.62174	0.85692	0.86776	0.29557	0.20690	0.37826	0.66611	0.70000	0.14308
0.71056	0.91622	0.92782	0.38627	0.30902	0.28944	0.75223	0.80000	0.08378
0.79938	0.95975	0.97190	0.55728	0.50155	0.20062	0.85624	0.90000	0.04025
0.88820	0.98750	1.00000	1.00000	1.00000	0.11180	1.00000	1.00000	0.01250

NUMBER OF PARTICLES PER SAMPLE = 90.

M1	M2	M3	M4	M5	M6	M7	M8	M9
-0.00000	0.00000	0.00000	0.10541	0.00000	1.00000	0.00000	0.00000	1.00000
0.08946	0.17092	0.17284	0.11577	0.01158	0.91054	0.20409	0.10000	0.82908
0.17892	0.32582	0.32949	0.12838	0.02568	0.82108	0.29600	0.20000	0.67418
0.26838	0.46473	0.46995	0.14468	0.04322	0.73162	0.37268	0.30000	0.53527
0.35784	0.58763	0.59423	0.16415	0.06566	0.64216	0.44369	0.40000	0.41237
0.44730	0.69452	0.70232	0.19072	0.09536	0.55270	0.51336	0.50000	0.30548
0.53675	0.78540	0.79423	0.22755	0.13653	0.46325	0.58482	0.60000	0.21460
0.62621	0.86028	0.86995	0.28200	0.19740	0.37379	0.66135	0.70000	0.13972
0.71567	0.91916	0.92949	0.37173	0.29659	0.28433	0.74764	0.80000	0.08084
0.80513	0.96203	0.97284	0.54093	0.48693	0.19487	0.85258	0.90000	0.03797
0.89459	0.98889	1.00000	1.00000	1.00000	0.10541	1.00000	1.00000	0.01111

NUMBER OF PARTICLES PER SAMPLE = 100.

M1	M2	M3	M4	M5	M6	M7	M8	M9
-0.00000	0.00000	0.00000	0.10000	0.00000	1.00000	0.00000	0.00000	1.00000
0.09000	0.17190	0.17364	0.10989	0.01099	0.91000	0.20238	0.10000	0.82810
0.18000	0.32760	0.33091	0.12195	0.02439	0.82000	0.29357	0.20000	0.67240
0.27000	0.46710	0.47182	0.13699	0.04110	0.73000	0.36970	0.30000	0.53290
0.36000	0.59040	0.59636	0.15625	0.06250	0.64000	0.44025	0.40000	0.40960
0.45000	0.69750	0.70455	0.18182	0.09091	0.55000	0.50955	0.50000	0.30250
0.54000	0.78840	0.79636	0.21739	0.13043	0.46000	0.58073	0.60000	0.21160
0.63000	0.86310	0.87182	0.27027	0.18919	0.37000	0.65711	0.70000	0.13690
0.72000	0.92160	0.93091	0.35714	0.28571	0.28000	0.74353	0.80000	0.07840
0.81000	0.96390	0.97364	0.52632	0.47368	0.19000	0.84926	0.90000	0.03610
0.90000	0.99000	1.00000	1.00000	1.00000	0.10000	1.00000	1.00000	0.01000