GRADING CRITERIA OF COLLEGE ALGEBRA TEACHERS

by

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Abstract

The purpose of my research is to identify what features of a graph are important for college teachers with the intention of eventually developing a system by which a machine can recognize those features. In particular, I identify the features that college algebra teachers look at when grading graphs of lines and how much disagreement there is in the relative importance graders assign to each feature.

In the process, eleven students from college algebra classes were interviewed and asked to graph six linear functions of varying difficulty. Eleven experienced college algebra graders were asked to grade the selected graphs, and interviewed to clarify what features of the graphs were important to them in grading.

Altogether, a general grading rule appears to be: slope is worth 4 points, y-intercept is worth 4 points, labeling of intercepts, points and graph is worth 1 point. After that, add 1 point if everything is correct. All graders considered slope and y-intercept to be very important. Only some of them considered labeling to be important. Anything else was a matter of a single point adjustment. Furthermore, the graders judged slope and intercept from two points(the y-intercept and the first point to the right). Returning to the students' work, I saw that the students also placed extra importance on points to the right of the y-axis. I conclude that this grading style may have a role in students' learning to think only about two points in a line (but nothing else), and that replicating human grading may not be the best use of machine grading.

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Lastly, I attribute all my success to my wonderful family whom I love very much.

Chapter 1 - Introduction

The purpose of my research is identifying what features of a graph are important for college teachers so that in the future a machine can recognize or improve on those features. If a successful system is developed, future students and teachers may benefit from quicker and more uniform assessments. Developing an automated grading system for graphs is difficult for hand-drawn graphs because there are no standards of precision. Hand drawn graphs are sketches based on the cultural expectations of students and teacher. In order to develop a grading system, we first have to identify what features of a graph are important and how a machine can recognize them.

College algebra is a basic math course and enrolls over 2000 students each year at our university. Therefore, doing research on the grading criteria of college algebra teachers will benefit many college students and teachers.

We will have students produce digitized images representing solutions to common algebra problems and will interview experienced algebra graders to see which features of the proposed solutions are important to their grading. This information can be used to develop automated programs that identify and evaluate student graphs based on these features.

It is important to understand what features of a graph are crucial to algebra teachers in grading student work so that this process can be mechanized and made more efficient in a world where students do their work on digital tablets rather than paper. Because the technology to allow students to draw graphs on a screen is so recent, almost nothing has been done concerning how students perform graphing tasks in such an environment. After searching for all articles with both keywords “grading” and “graph” in the U.S. Department of Education's ERIC database, I only found one relevant article, “Automatically assessing graph-based diagrams” by Thomas, Smith, and Waugh, (2008), which dealt with students drawing box diagrams rather than graphing lines. In order to provide effective instruction in a modern setting, there is a need to develop ways to accurately assess student generated graphs in mathematics, which is the ultimate goal of this research.

The questions that drive this are: (1) What are the features that college algebra teachers look at when grading graphs of lines; (2) How much disagreement is there in the relative importance graders assign to each feature.

Chapter 2 - The Process of the Research

This study was based on a three part structure. First students from college algebra classes were asked to graph six linear functions of varying difficulty. Second experienced college algebra graders were asked to grade the selected graphs, and interviewed to clarify what features of the graphs were important to them in grading. Lastly, in order to see how much of an influence accuracy has on grading, these scores given by the teachers based on those features were be compared to scores determined by applying linear regression to the graphs.

2.1 Interviews of the Students

In order for the instructors to have something to grade and talk about, I needed examples of student work. Students were recruited for the project from college algebra classes and paid $5 for their time. Eleven volunteers were selected based on their time availability and first exam grade in the course. This selection emphasized a broad spectrum of first exam grades for a greater variety of graphing styles. The one-on-one interviews took 15 to 30 minutes and were voice recorded.

The eleven selected students were given six different linear equations and asked to draw the corresponding lines on x-y graphs using an iPad and stylus. The equations were chosen such that in some cases it is easy to find the slope, in some cases it is easy to find the y-intercept, and in some cases all of the useful information was non-obvious. The equations were given on three different backgrounds, which gave different amounts of help in drawing the graphs precisely.

The six different linear equations:

A: $y=2x-3$	D: $y+3=x-2$
B: $y-3=2(x+1)$	E: $-3x+4y=2$
C: $-4y=2x+1$	F: $y=-2x+1$

To graph linear equations, I anticipated that the students would use plotted points, slope, x-intercept and y-intercept. For example: we need only two points (such as the x- and y-intercepts) or a point and the slope to draw a line. The six linear equations given above have different features and difficulties.

A: $y=2x-3$

- Equation A is in a form that students are familiar with. I expected that students would see that the slope is 2, the y-intercept is (0, -3), and would easily be able to find many points on the line – for example, (1, -1), (2, 1), (3, 3), etc.

B: $y-3=2(x+1)$

- Equation B looks like a complicated equation for students; however, I expected that students would be able to perform some simple calculations to get a form that is easier to work with, such as equation A above. For example, one could perform the calculations as follows:

  $y-3=2(x+1)$

  $y-3=2x+2$

  $y=2x+5$.

C: $-4y=2x+1$

- In equation C, dividing by -4 on both sides gives us $y=-(1/2)\,x-(1/4)$; students who did this would be able to see that the y-intercept is (0,-1/4). For additional information, the students would need to do calculations with fractions. I designed this equation to check the students' abilities in making calculations with fractions and scaling the fraction in the graph.

D: $y+3=x-2$

- In equation D I anticipated that, by means of subtraction, students would get $y=x-5$, would see that the slope is 1, the x- intercept is (5, 0), the y-intercept is (0, 1), and would be able to find many points that lie on this line easily, for example, (1, -4), (2,-3), (3, -2), etc.

E: $-3x+4y=2$

- I expected Equation E to be the most difficult equation for students to draw because its slope, intercepts, and other points are the most difficult to find. After some calculations, students would find that it has a negative-valued fraction as its slope, giving it a similar form to equation C.

F: $y=-2x+1$

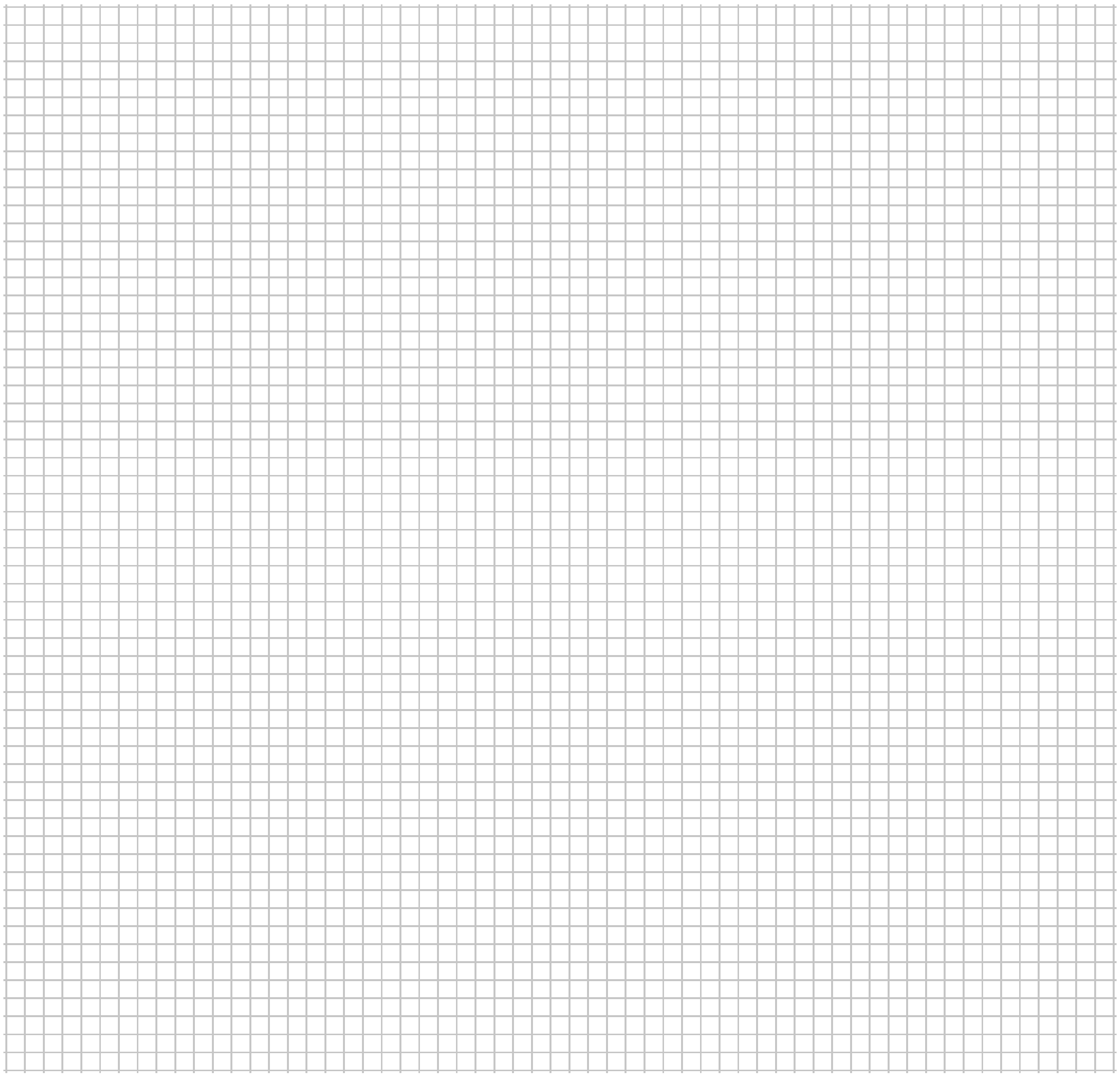
- Equation F is similar in form with equation A. I anticipated that students would see that the slope is -2, the x-intercept is (1/2, 0), and the y-intercept is (0, 1), and would find many points which lie on this line easily, for example, (1, -1), (2,-3), (-1, 3), etc. With equation F, I was namely interested in finding whether students have a clear idea of how a negative slope would appear, in general.

The students drew these six different equations on three different backgrounds. The backgrounds and equations were paired differently for different students. These three different backgrounds from (1) to (3) gave increasing information to help students in drawing the graphs precisely. Having different backgrounds, the students would have different reactions in drawing. Different graders might have different standards of precision in their own classes. Having different backgrounds gave both students and graders information about how much precision was expected. This helped to control for the variety in the level of precision that graders initially expected.

Three backgrounds on which to draw the graphs:

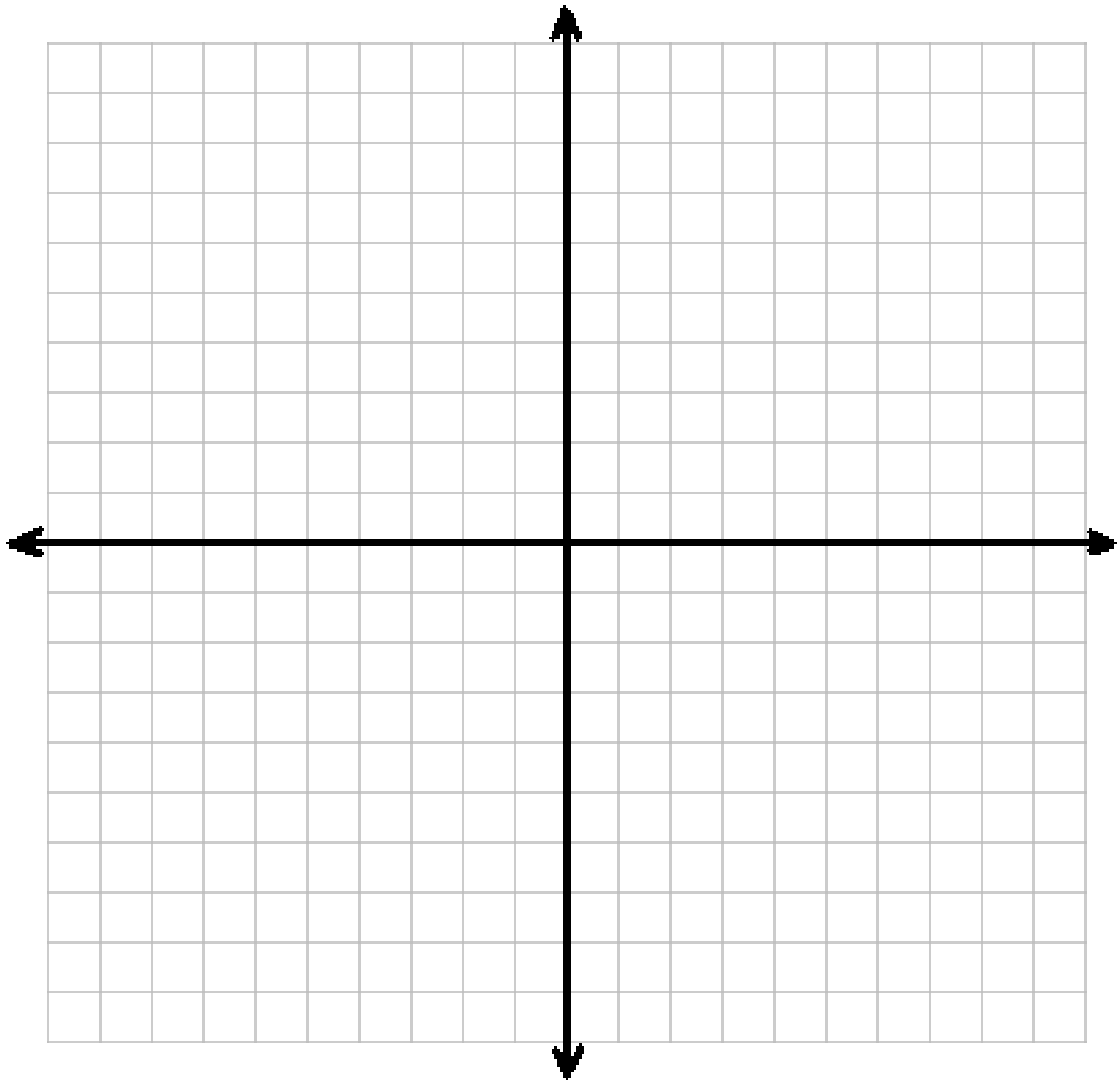
(1) Grid graph paper.
(2) Coordinate grid graph paper without numbers.
(3) XY coordinate graph paper.

Figure 2.1 Grid graph paper.



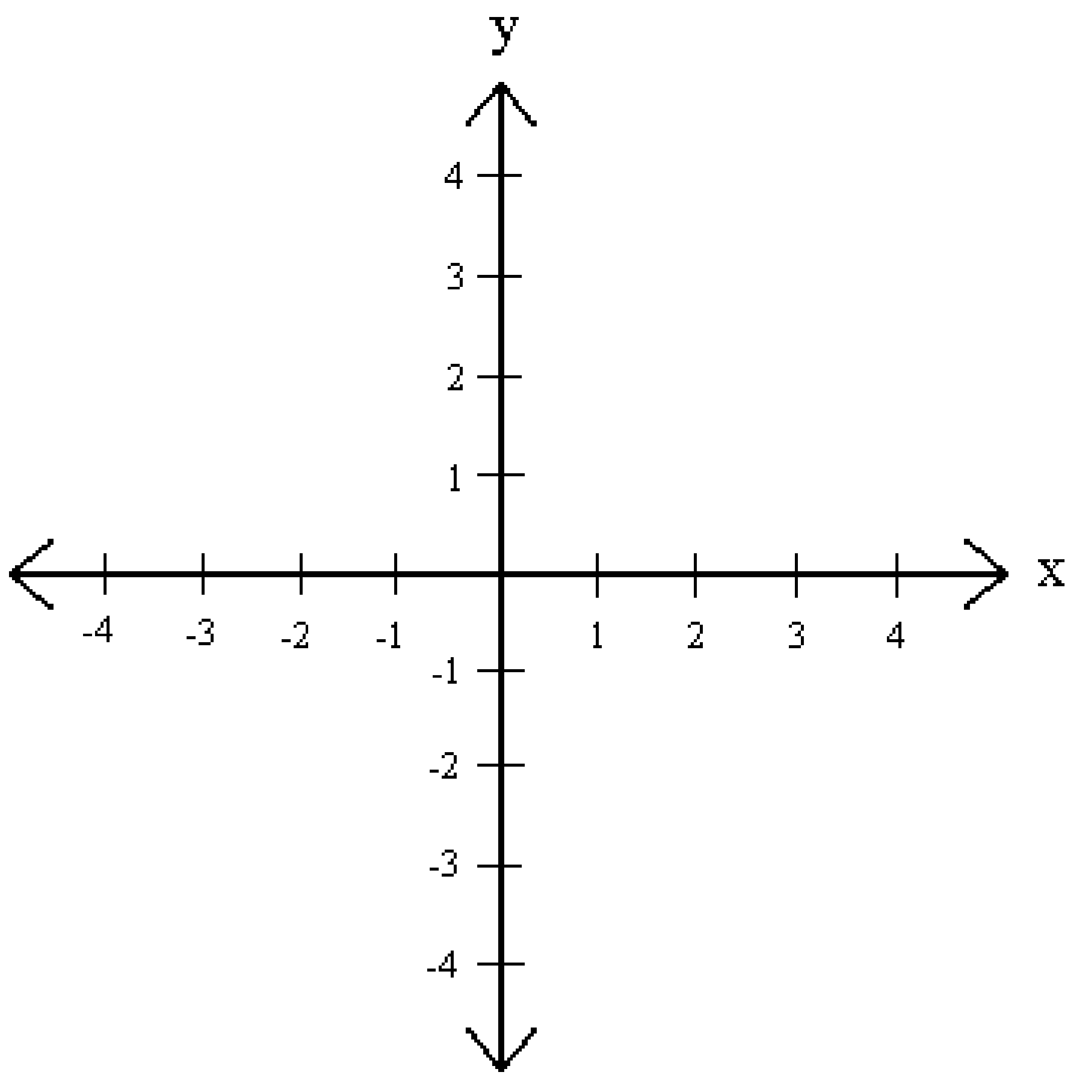
This grid graph paper was light grey color and the squares of the grid were 0.10 inches on a side. Since this grid graph paper was a light color with very small squares, I did not expect it to help students to draw the graph with accuracy. Drawing on this grid graph paper could be similar to drawing on a blank sheet of paper for students. On the other hand, this grid paper did provide a scale that students might use. An established scale is also very useful for a computer when that analyzes the graphs.

Figure 2.2 Coordinate grid graph paper without numbers



This coordinate grid graph paper without numbers was grey color and the squares of the grid were 0.25 inches on a side. This graph paper had clear blocks by which to measure, and the x- and y- axes could help students to scale and draw the graph. This graph did not give a scale for the markings on the x- and y- axes, so the students themselves could declare the scales. This graph could also gives clear measurement information to a computer.

Figure 2.3 XY coordinates graph paper



The XY coordinate graph paper had a defined scale for the x- and y-axes. With this graph paper, students had to draw with the given scale, and the graph also established clear measurements for computer grading.

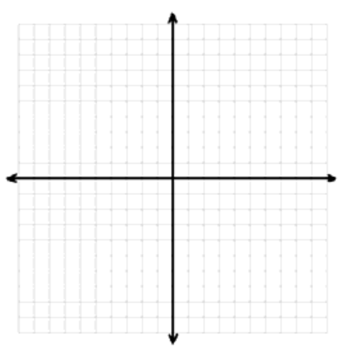
From the backgrounds (1) through (3), increasingly clearer scale information is given, making it increasingly easier to draw a precise graph. Students were asked to draw two linear equations on each of the backgrounds in the order (1), (2), and then (3). This order was given so that students would not transfer precision standards from (3) to a graph on background (1). The pairing of equations and types of backgrounds were changed for each student. Using this method, the students had different initial qualities in their set of graphs (Figure 2.4).

Figure 2.4 An example of one of the problem sets

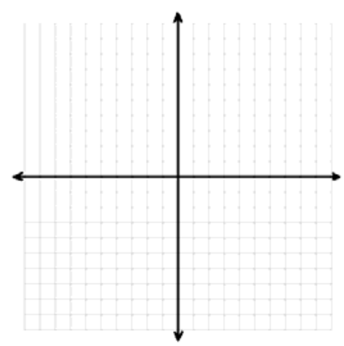
A: $y=2x-3$

D: $y+3=x-2$

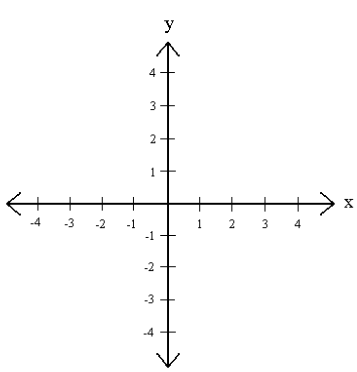
B: $y-3=2(x+1)$



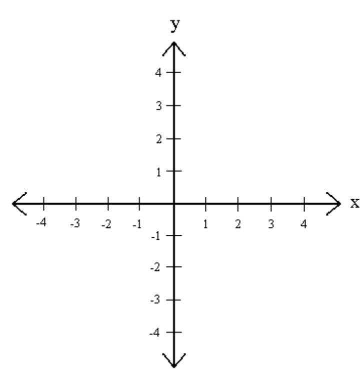
E: $-3x+4y=2$



C: $-4y=2x+1$



F: $y=-2x+1$

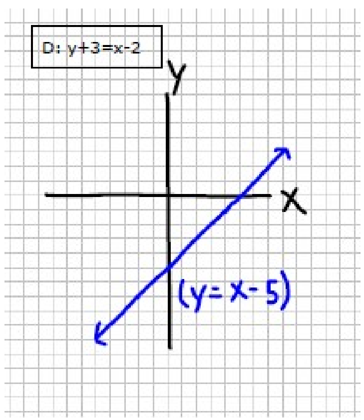


In the interview, students drew the graphs without being given any instruction by the interviewer. They drew each graph based upon their own outside knowledge. Therefore, we have the actual work of these college algebra students. This actual work was necessary to provide a realistic set of conditions for the instructor interviews.

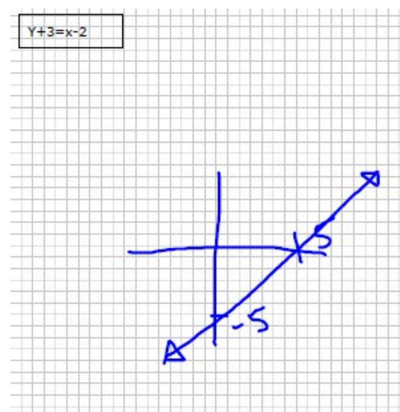
The eleven students produced 66 graphs. Each student had his or her own way of graphing lines, and all 66 graphs were unique. The next task was to choose a smaller set of graphs to talk with instructors about. In these 66 graphs I found that different students made mistakes common to each other. After comparing all the graphs, I chose the twenty most interesting graphs of the linear equations C and D as below:

Figure 2.5 Student solution set for equation D: y+3=x-2:

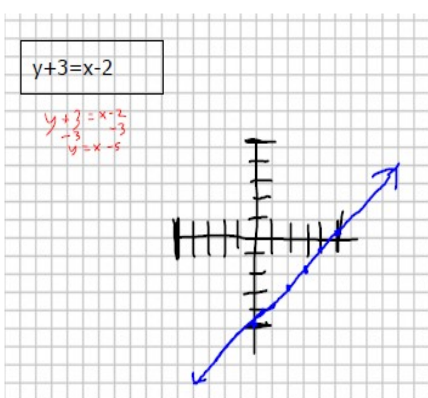
(A)



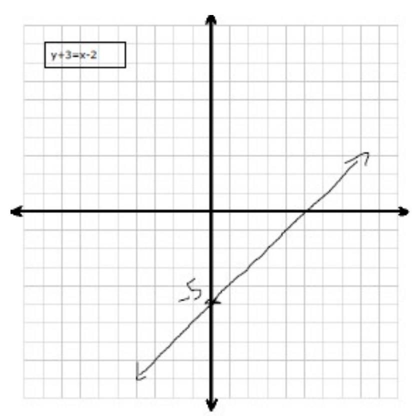
(B)



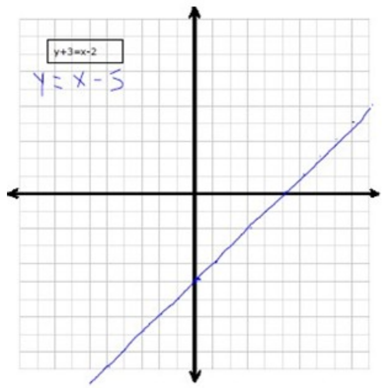
(C)



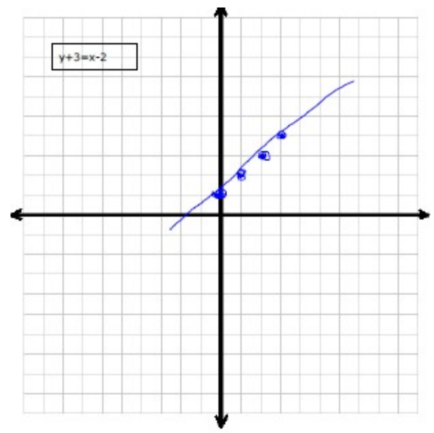
(D)



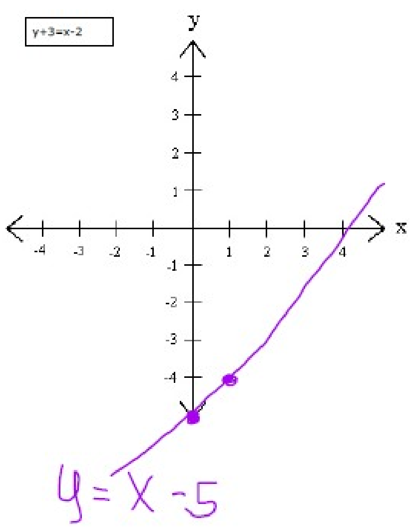
(E)



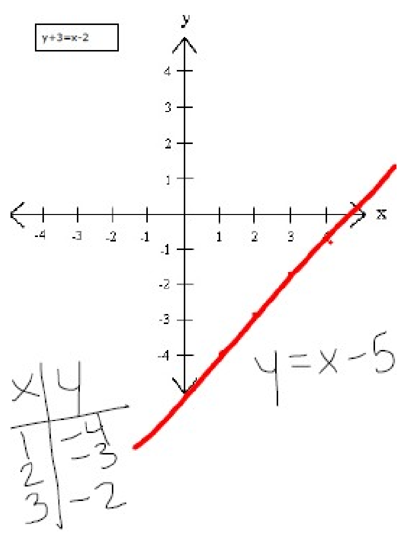
(F)



(G)



(H)



(I)

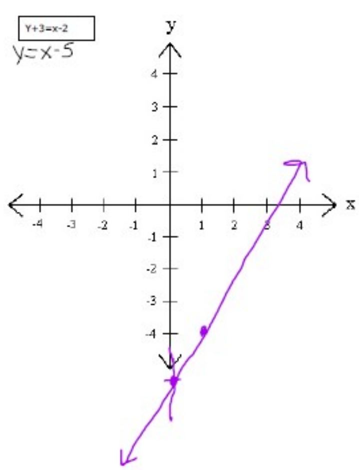
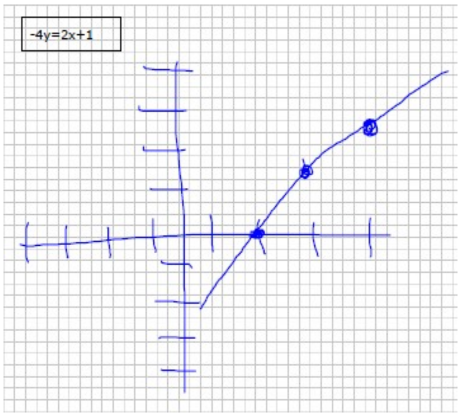
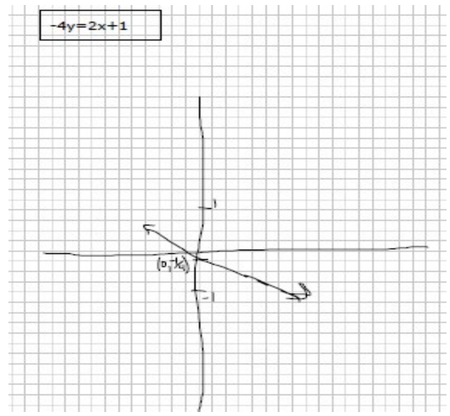


Figure 2.6 Student solution set for equation C: -4y=2x+1:

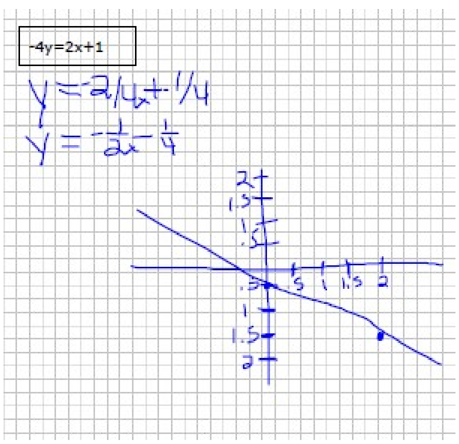
(1)



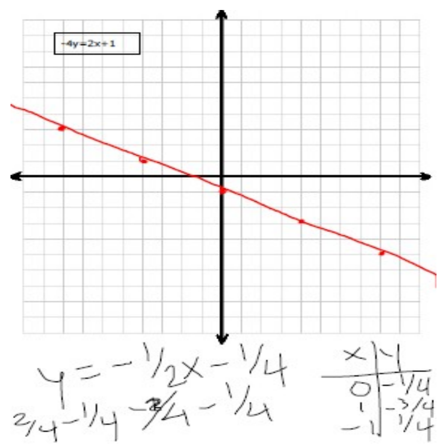
(2)



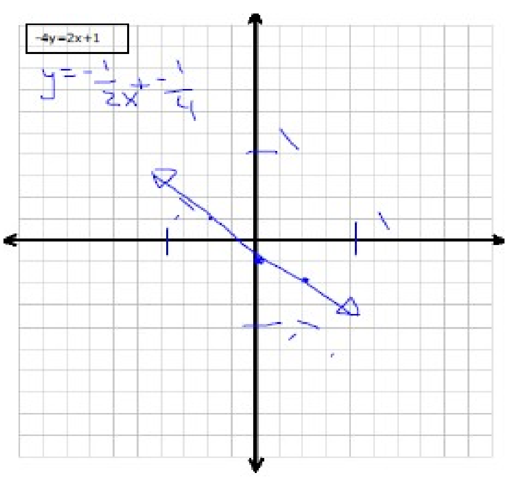
(3)



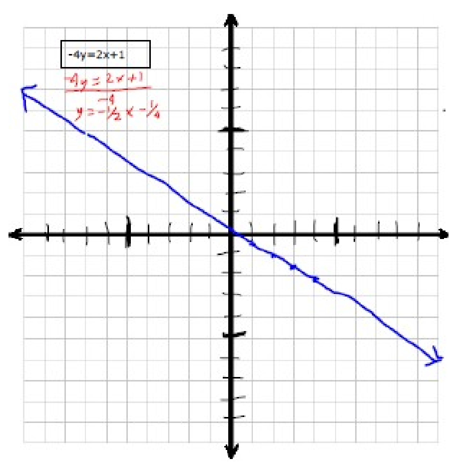
(4)



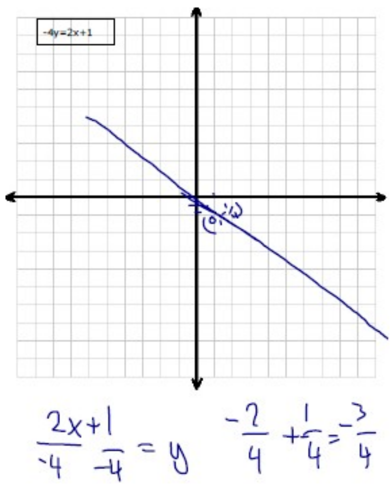
(5)



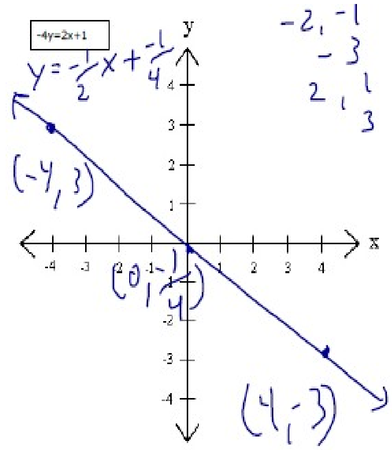
(6)



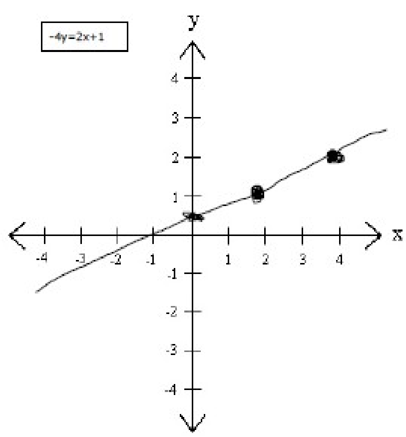
(7)


-4y=2x+1
$\frac{2x+1}{-4}\ \frac{}{-4} = y$
$-\frac{2}{4} + \frac{1}{4} = -\frac{3}{4}$

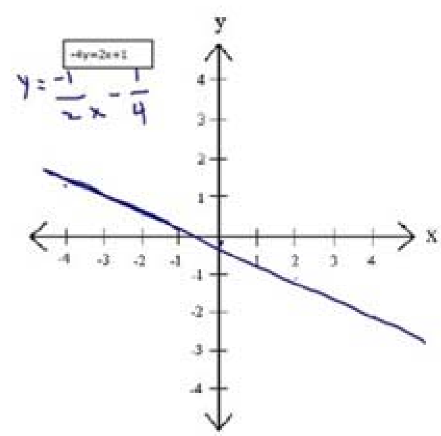
(8)


-4y=2x+1
y
x
$y = -\frac{1}{2}x + \frac{-1}{4}$
(-4, 3)
$(0, -\frac{1}{4})$
(4, -3)

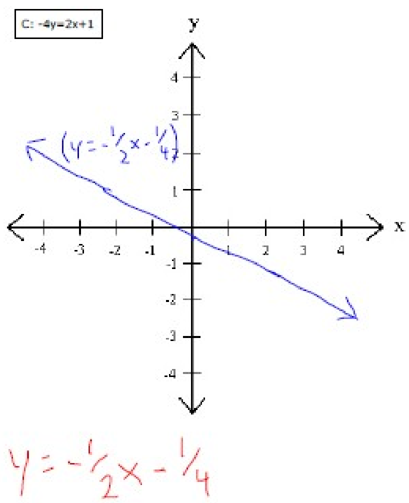
(9)


-4y=2x+1
y
x

(10)


-4y=2x+1
y
x
$y = \frac{-1}{2}x - \frac{1}{4}$

(11)


C: -4y=2x+1
y
x
$(y = -\frac{1}{2}x - \frac{1}{4})$
$y = -\frac{1}{2}x - \frac{1}{4}$

The graphs of the linear equations C and D carried the largest number of interesting situations for grading. Some graphs had unlabeled axes or unlabeled points, for example Figure 2.5, (A), (D), (E), and (F). Some lines had a noticeable curve, for example, Figure 2.5 (G). Some lines had sections with correct slope but other sections not, for example Figure 2.6 (2), (5) and (1). Some lines had incorrect intercepts or had correct y-intercepts but incorrect x-intercepts, for example Figure 2.6 (G), (I), Figure 2.6 (2), (5). Some graphs had some incorrect points, Figure 2.6 (8). Some graphs had written algebra work with errors, Figure 2.5 (F), Figure 2.6(3). Lastly, the line in Figure 2.6 (9) was drawn with an opposite slope by missing the negative sign.

There were in total 22 graphs of the linear equations C and D, and 20 of them have already represented all the typical mistakes of all the graphs that the students produced in interviews, and reflected the general graphing skills of the interviewed students. Therefore, I chose all the 20 graphs of the linear equations C and D for the college algebra graders' interviews. The remaining 46 graphs were set aside for future research.

2.2 Interviews of College Algebra Graders

In order to clarify what features of the graphs were important to college algebra teachers in grading, I had to interview some experienced college algebra graders. The grader pool included one undergraduate grader, nine graduate teaching assistants in mathematics, and one mathematics professor, which represent the whole grading system for the college algebra course in KSU. Although there were differences in grading styles between the graduate students, the professor and the undergraduate grader were very similar to the majority. In order to capture the culture of mathematics education in the United States, all these chosen college algebra graders were native English speakers and had been educated in America.

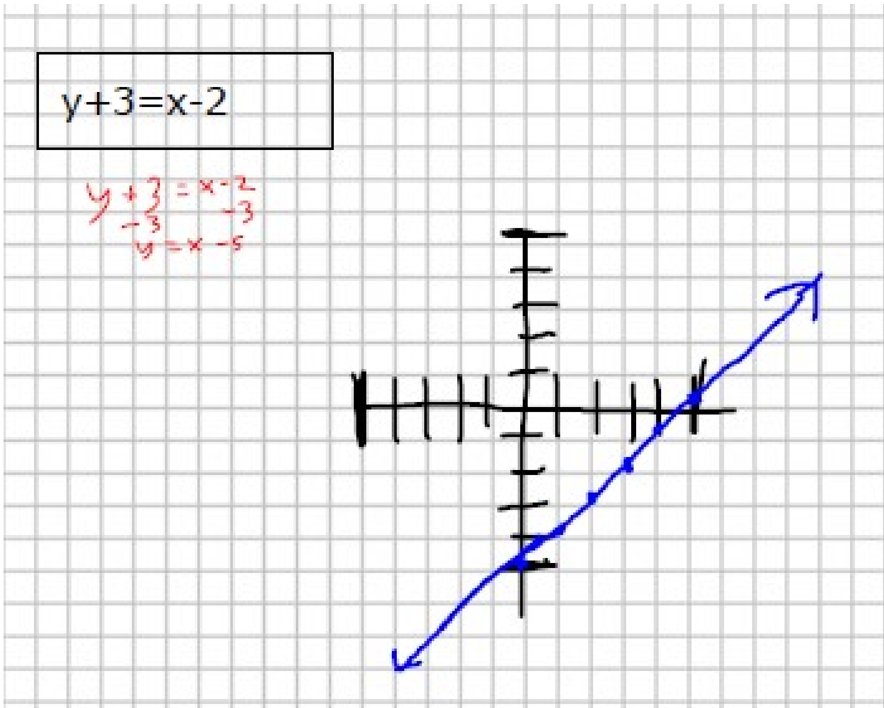
Each grader was asked to grade twenty graphs by using the grading scale of 0 to 10. These graphs were complete solutions sets of the two linear equations C and D, so that the graders could grade more efficiently during the interview, as well as enter the rhythm that they had when grading a real problem set. All the graphs are shown to the graders in the same order as Figure 2.5, Figure 2.6. The graph set of equation D was presented before the graph set of equation C because it was simpler and easier to grade. Each linear equation set was showed with the backgrounds in the order (1), (2), and then (3). This order was chosen so that graders would not transfer precision grading standards from the background (3) to a graph on background (1).

All the chosen graders were given the grading problems (two linear equations C and D) a week before their interviews so that they could have some time to think about how to grade the problems. Each one-on-one interview took 30 to 45 minutes and was voice recorded. During the interview it was suggested that the graders talk aloud while they were grading so that I could hear what they were thinking. The graders set their own grading standards and after they finished all their grading, I had a short discussion with them.

Before the interviews I had graded all the graphs myself, and during these discussions, I asked the graders about any graph where my grade and their grade disagreed by a certain amount. I also chose in advance some seven graphs that I considered most interesting. During the discussion, I talked about the features and tried to convince graders to change their grade, in order to see how important each feature was to them.

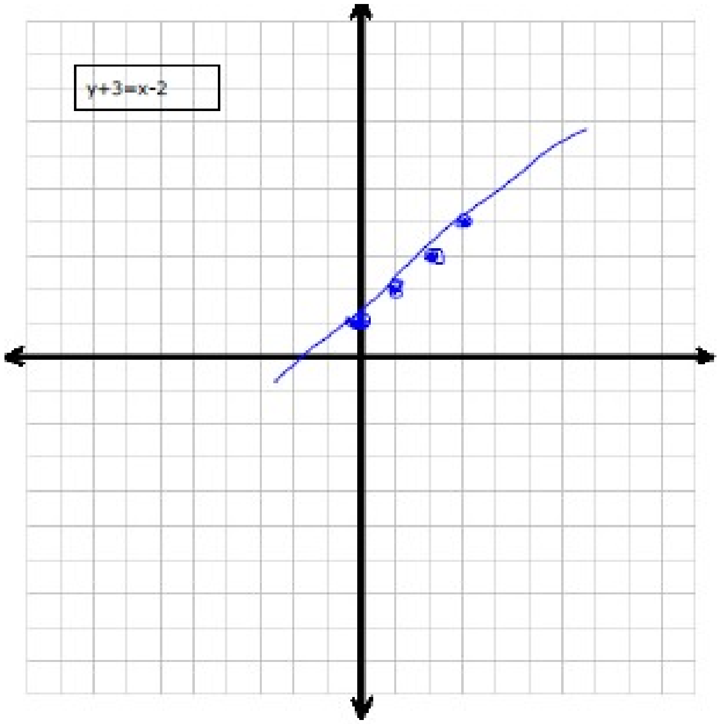
Let's look at the graphs I considered most interesting:

Figure 2.7 Student solution C, for y+3=x-2



In this graph, the student simplified the equation correctly and got y=x-5. The points (2,-3) (3,-2) and (4,-1) which the student plotted were correct, so that the slope was correct. However, the line did not pass through the intercepts (0,-5) (5, 0) correctly. I was interested in what was more important in graphing a line, correct intercepts or the majority of points plotted. Plotting more points correctly means the student was able to find the correct intercepts because if the students can find points at x=1,2,3…that they have the ability to use the same method to find point at x=0. However, I wanted to know how much the graders cared about the inaccuracy of the intercepts.

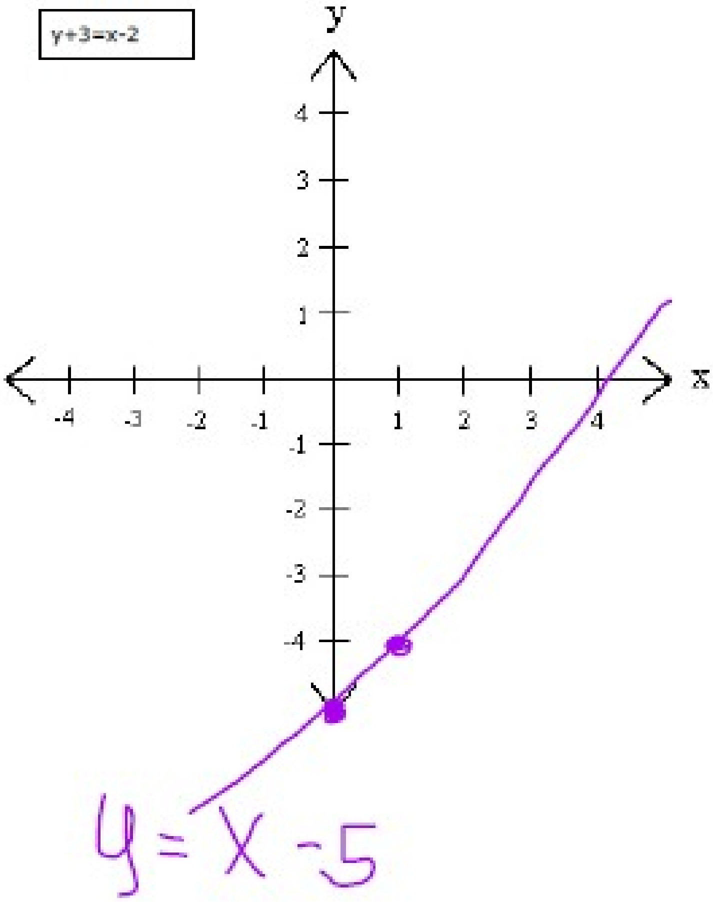
Figure 2.8 Student solution F, for y+3=x-2



In this graph, the line was not drawn through the points and none of the points plotted were correct corresponding to the equation. However, the slope was correct. Therefore, it was a very useful graph for eliciting the graders' ideas of how important the slope is in graphing.

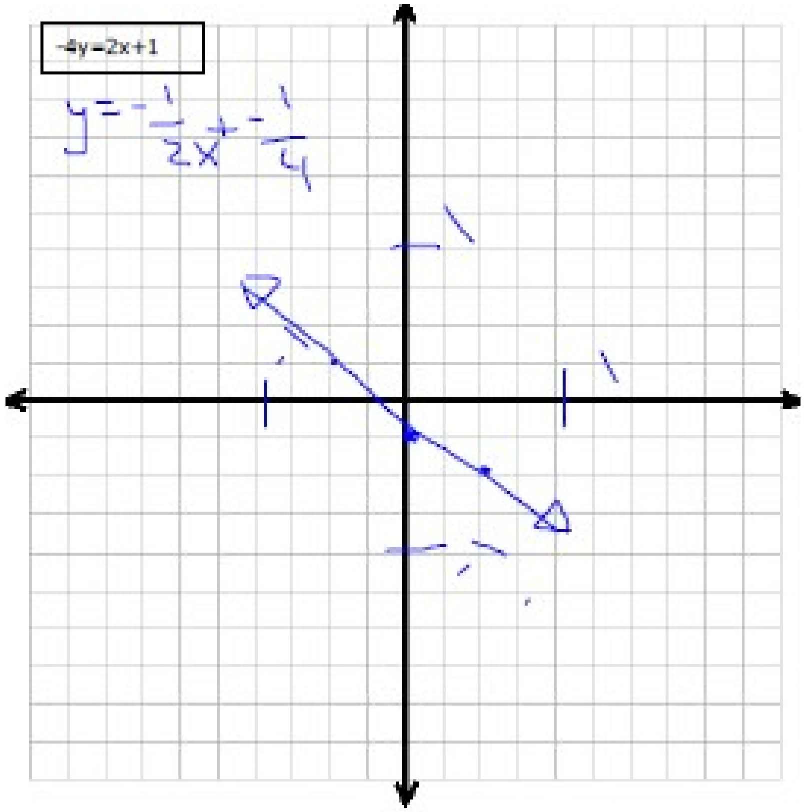
In this graph, the student did not show any algebra work. However, if we paid more attention to the line we found that the student drew a line of the equation y= x+1. It was possible that the student had made a mistake when he or she simplified the equation, such as getting y=x-2+3=x+1 instead of y=x-2-3=x-5. In such a case, the student did have the skill to graph line but he or she only had an error in the manipulation of the equation. I was interested in whether the graders would think deeply about the student's reasoning in creating his graph when they were grading, and how much time they would spend in grading a graph.

Figure 2.9 Student solution G, for y+3=x-2



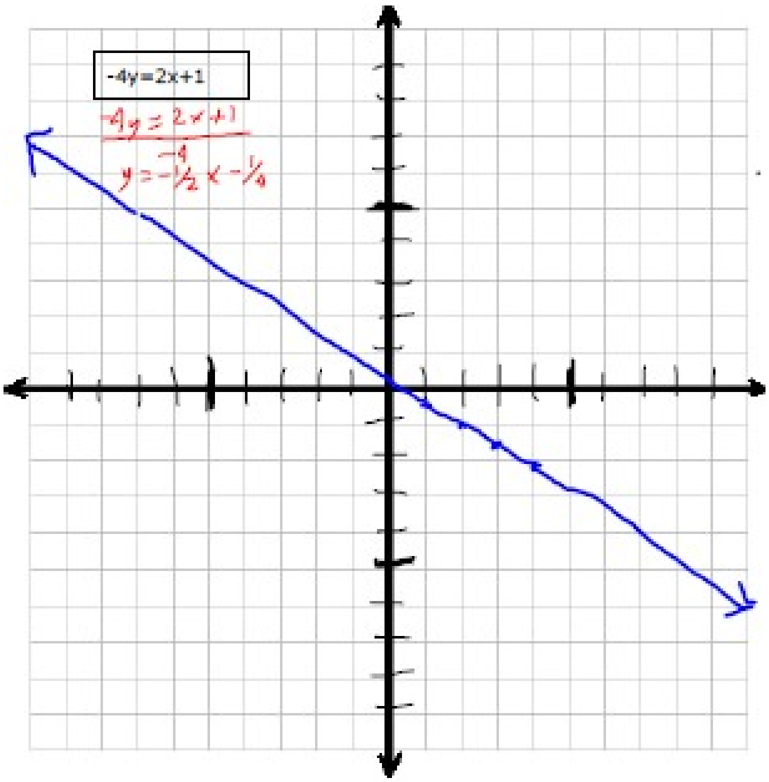
In this graph, the line had two points (0, 5) and (1,-4) correct. However, if we look at the x-intercept, it should have been at (5, 0), but it was instead at about (4, 0) on this graph. This problem was caused by curving up of the line. I was interested in whether graders would consider two points (one y-intercept and one close-by point) as being enough to define a correct line, whether the graders cared about either x- and y- intercepts, or only the y-intercept, and whether they cared about the line being curved.

Figure 2.10 Student solution 5, for -4y=2x+1



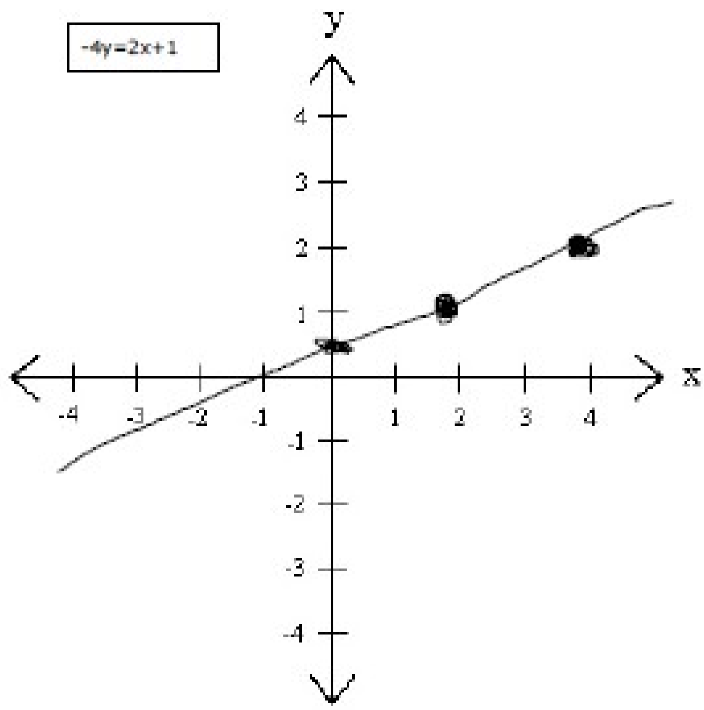
In this graph, there were many intersting details. First, the student sloppily wrote the equation y= - (1/2) x-(1/4) as y= - 1/ (2x)-(1/4). Second, the student set 4 small squares as a unit 1. Third, the two points (0, -1/4), (1/2,-1/2) on the right side were correct and so the slope was correct. However, on the left side, the student had the x-intercept at (-1/4, 0), but it should be (-1/2, 0), and as a result, the slope was also incorrect in the left side. As a result, the line is not straight. I was interested in whether the graders would notice all the problems, and if they did, how much they cared about each problem.

Figure 2.11 Student solution 6, for -4y=2x+1



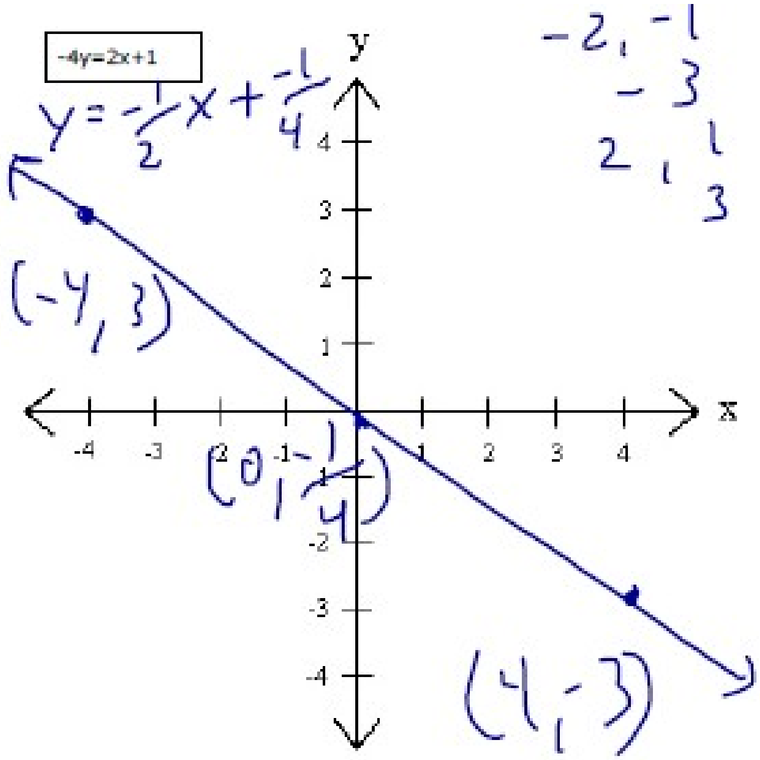
For this equation, the intercepts should be (0, -1/4) and (-1/2, 0). On the graph, both intercepts should be negative instead of positive. The error in the intercepts was additively small, the distinction between positive and negative could be considered important. I wondered if the grader would think this was a pretty big error, or would they think it was not a big deal when it was hard to plot the points exactly with values this small. The student simplified the equation correctly and the slope was pretty close to -1/2.

Figure 2.12 Student solution 9, for -4y=2x+1



In this graph, all of the points were wrong and the slope is incorrect. However, if you compare the line in the graph and the correct line for the equation, they are reflections of each other. Therefore, it looks like this graph resulted from a sign error, slope and intercept are positive when they should be negative. I was interested in whether the grader would notice this easily, and give some points despite an incorrect slope and y-intercept.

Figure 2.13 Student solution 8, for -4y=2x+1



I used this graph to investigate the relative importance of labeled points, slope, and intercept. In this graph, it is very clear the student plotted three points (-4, 3), (0, -1/4), (4, 3). However, when *x* equal to 4, *y* should equal to -9/4, and when *x* equal to -4, *y* should equal to 7/2. So two of the three points plotted are wrong. Would the graders think this was a big problem? On the other hand, the student simplified the equation correctly and the labeled point (0, -1/4) is the correct y-intercept in the graph, and the slope is pretty close. Would the grader think the student got a large portion of this problem right?

Chapter 3 - Results and Analysis

Below are the original grading results from which I discussed the graph with the graders. For privacy, the teachers were all given pseudonyms.

Table 3.1 Original Grading Results

Graph\Grader	Daniel	Ray	Frank	Gerry	Jimmy	Sandra	Dolly
NO.A	8	10	10	10	8	10	10
NO.B	6	10	10	10	8	9	9
NO.C	2	8	10	10	7	9	10
NO.D	5	9	10	10	8	10	10
NO.E	8	10	10	10	6	10	10
NO.F	1	3	5	3	2	4	6
NO.G	3	7	10	10	7	9	9
NO.H	9	9	10	10	8	10	10
NO.I	2	6	10	9	6	9	7
NO.1	1	2	0	0	0	0	0
NO.2	7	10	7	9	5	7	8
NO.3	9	9	5	10	8	6	9
NO.4	10	10	10	10	8	10	9.5
NO.5	5	8	10	10	6	7	10
NO.6	3	8	5	5	4	7	6
NO.7	7	10	7	9	7	10	5
NO.8	6	8	7	3	5	6	6
NO.9	1	3	0	6	1	0	1
NO.10	6	10	7	9	8	9	10
NO.11	6	10	7	9	8	10	10
Mean	5.25	8	7.5	8.1	6	7.6	7.775

Table 3.1 continued

Graph\Graders	Kara	Sam	Alice	Michael	Mean	Stdev
NO.A	8	10	10	10	9.45	0.93
NO.B	8	10	10	9	9.00	1.26
NO.C	5.5	9	9	10	8.14	2.49
NO.D	7	10	10	8	8.82	1.66
NO.E	6	10	10	9	9.00	1.61
NO.F	4	7	5	5	4.09	1.76
NO.G	8	8	8	7	7.82	1.94
NO.H	9	10	10	9	9.45	0.69
NO.I	8	8	6	6	7.00	2.19
NO.1	0	0	0	0	0.27	0.65
NO.2	6	8	6	8	7.36	1.43
NO.3	5.5	5	7	5	7.14	1.92
NO.4	6	8	9	8	8.95	1.31
NO.5	7	8	10	4	7.73	2.15
NO.6	5	3	2	7	5.00	1.90
NO.7	7	7	10	6	7.73	1.74
NO.8	8	6	6	2	5.73	1.85
NO.9	3	0	0	0	1.36	1.91
NO.10	9	7	8	9	8.36	1.29
NO.11	9	7	8	10	8.55	1.44
Mean	6.45	7.05	7.2	6.6		

Table 3.2 Grading Results after discussion

Graph/Grader	Daniel	Ray	Frank	Gerry	Jimmy	Sandra	Dolly
NO.A	8	10	10	10	**10**	10	10
NO.B	6	10	10	10	8	9	9
NO.C	2	8	10	10	7	9	10
NO.D	5	9	10	10	8	10	10
NO.E	8	10	10	10	**8**	10	10
NO.F	1	3	5	3	2	4	6
NO.G	3	7	10	**9**	7	9	**7**
NO.H	9	9	10	10	**10**	10	10
NO.I	2	6	10	9	6	9	7
NO.1	1	2	0	0	**1**	0	0
NO.2	7	10	7	9	5	7	8
NO.3	9	**6**	5	**7**	8	6	**5**
NO.4	10	10	10	10	8	10	9.5
NO.5	**8**	8	10	**9**	6	7	10
NO.6	3	8	5	5	4	7	6
NO.7	7	10	7	9	7	10	5
NO.8	6	**6**	7	3	5	6	6
NO.9	1	3	0	6	1	0	1
NO.10	6	10	7	9	8	9	10
NO.11	6	10	**10**	9	8	10	10
Mean	5.4	7.75	7.65	7.85	6.35	7.6	7.475

Remark:

---the grade changed after discussion

Table 3.2 continued

Graph/Grader	Kara	Sam	Alice	Michael	Mean	Stdev
NO.A	8	**8**	10	10	9.45	0.93
NO.B	8	10	10	9	9.00	1.26
NO.C	5.5	9	9	**9**	8.05	2.43
NO.D	7	10	10	8	8.82	1.66
NO.E	6	10	10	9	9.18	1.33
NO.F	**2**	**6**	5	**4**	3.73	1.68
NO.G	8	8	8	7	7.55	1.80
NO.H	9	10	10	**10**	9.73	0.47
NO.I	8	8	**7**	**7**	7.18	2.14
NO.1	0	0	0	0	0.36	0.67
NO.2	6	8	6	8	7.36	1.43
NO.3	5.5	**6**	7	5	6.32	1.31
NO.4	6	**10**	**10**	**9**	9.32	1.27
NO.5	7	8	**8**	**6**	7.91	1.38
NO.6	5	**5**	**7**	**5**	5.45	1.44
NO.7	7	7	10	**8**	7.91	1.64
NO.8	**7**	6	6	**4**	5.64	1.21
NO.9	3	**1**	**1**	0	1.55	1.81
NO.10	9	7	8	9	8.36	1.29
NO.11	9	7	8	10	8.82	1.40
Mean	6.3	7.15	7.5	6.85		

Remark:

[orange box] **---the grade changed after discussion**

From the grading results above, we could see that there was not a big difference among these graders in their grading. Looking at the standard deviation, most of them were lower than 2 in the original grades. The only three standard deviations bigger than 2 were NO.C: 2.49, NO.I: 2.19, and NO.5: 2.15. It is easy to see that in all these 3 graphs, Daniel gave much lower grade than other graders, which induced the standard deviation large than normal. And after I discussed with the graders, only NO.C and NO.I had standard deviations bigger than 2, which was not a big difference from the original grading standard deviations. Generally speaking, the grading criteria of all these graders were close.

From the table, the graphs with particularly high grades the graders considered were drawn properly, and graphs with particularly low grades the graders considered were drawn poorly. The graphs where lots of graders changed their minds were graphs where the graders had different grading standards for some features of the graphs, and the graphs for which fewer graders changed their minds were graphs that satisfied most graders' requirements. The graphs with similar characteristics had similar grades, meaning that the graders had pretty stable grading standards.

After the interviews, I looked over the details of each graph the graders had graded, listened to the voice recorded audio of each graders' interview, and analyzed the tables above. I found some important features of the grader's grading. In the interview, I found that the graders graded the graphs depending on a couple of different features, and among the 20 different graphs, the Graphs A, E, F, G, 5, 9 and 11 all had some special problems that can be used to address the importance of certain individual features. Discussing these 7 graphs was the most effective way to find important features of the graders' grading and the relative importance of each feature. Therefore, I chose to focus on the analysis of graphs A, E, F, G, 5, 9 and 11 instead of the graphs C, F, G,5, 6, 9, 8 which I had highlighted as interesting in Chapter 2 before.

I found 6 important features of the graders' grading highlighted in graphs A, E, F, G, 5 9 and 11.

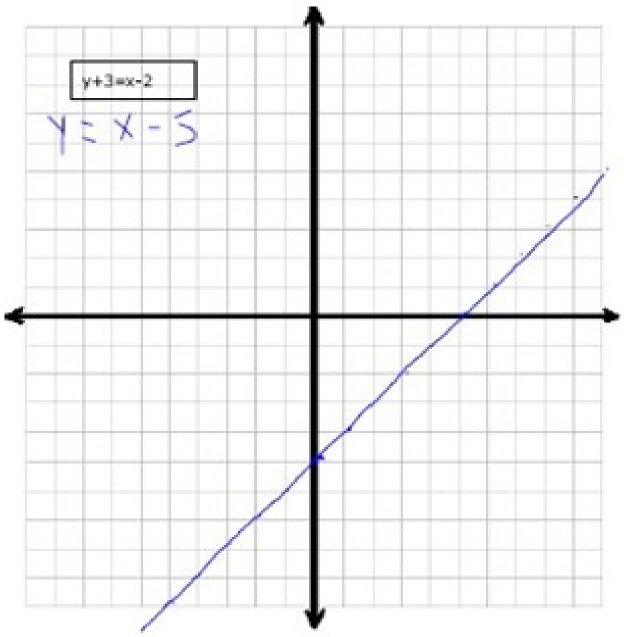
Important features:

(1) Labeling intercepts, points and graph

Labeling points or labeling axes on a graph defines the scale of the graph and is part of establishing the graph as a relationship between quantities. The teachers were divided about the importance of labeling points with values. Many graders did not take off any points at all, but some graders took off 40% of a student's score.

For example:

Figure 3.1 Student solution E, for y+3=x-2



Graph E was a pretty proper graph, the slope and intercepts were correct, and the line drawn straightly. However, it lacked any labeling. That meant any points the grader took off would be due to the lack of labeling.

In the interview, Kara said, "I would take away 2 points for not labeling axes and 2 points for not labeling the scales because I don't want to have to guess what they are doing." So she gave this graph 6 points and labeled the axes and intercepts for the graph. Jimmy had the same grading standard as Kara. Sandra said, "Grading the labeling depends on the instruction. In this case, it depends on the communication of the graph; this line can tell what exactly the points are. We can see it is using the grid for measure. And we can be sure this student understands what the graph is.” So she gave full credit for this graph. Dolly said, “It’s better if students do the labeling, but it is okay if they are absent.” She did not take off any points for the absence of labeling. Ray also gave full credit and said, “If the graph is difficult to read they have to label the points, but in this graph, it is very clear where the intercepts are so it is fine without labeling.”Daniel would like the student to show more work in labeling so he took off 2 points. Michael took off 1 point for the absence of labeling. Frank and Gerry did not look at or talk about the labeling at all when they were grading, and so they gave full credits for this graph.

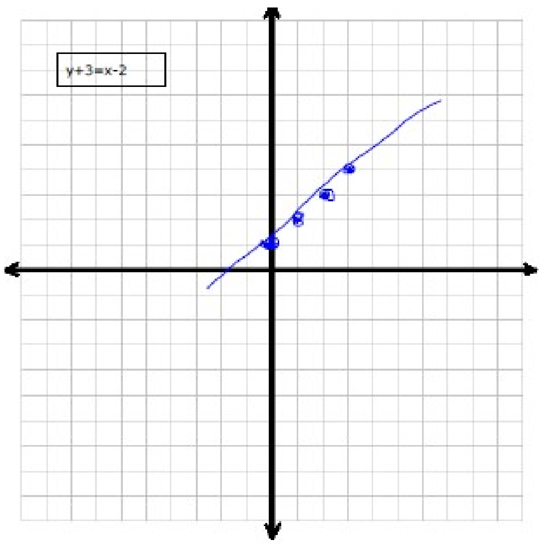
In the table 3.1, we would see 7 of the 11graders gave full points, 1 grader took off 1 point, 2 graders took off 2 points, and 2 graders took off 4 points. The mean grade for the graph is 9. When I discussed the graph with the Jimmy, I prompted him “The graph was drawn on graph paper and one would consider the 1 grid was a unit, so it should not be necessary to label the axes.”And Jimmy changed his grade from 6 to 8. The mean grade of the graph E is still very close to 9.Therefore, in general the labeling of intercepts, points and graph was valued at about 1 point.

(2) Slope

The slope of a line describes its steepness. The slope is a measurement of the (quality of) steepness of a line, and is calculated as the ratio of the "rise" divided by the "run" between two points on a line, or in other words, the ratio of the altitude change to the horizontal distance between any two points on the line.

Example:

Figure 3.2 Student solution F, for y+3=x-2



Student solution F was a very useful graph from which to obtain the graders' ideas of how important the slope is in graphing, because only the slope was correct in this graph.

In the interview, Daniel gave the lowest grade, 1 point for this graph. He said, "The graph is completely wrong and no information telling me what they did. When I was in class I told my students what I expected of them; if you only gave me a graph completely wrong I only gave them 1 point." After I told him the slope was very close to correct, he said, "If the student labels the x and y intercepts so that I would see what exactly is the slope I may gave them 3 to 4 points."

Ray gave 3 points for this graph, and judged the slope of the line to be correct. However he would have liked the student to show more work in order to get the full points from a correct slope. Gerry also gave this graph 3 points, he said, "even if is a right slope but he did not write down what the equation was y equals x minus 5; if they write that down, we know the slope should be 1." We see that for Ray and Gerry a correct slope alone is worth 3 points, with potentially more for showing work.

Frank and Alice had clear grading standards for slope and gave 5 points for this graph.

Sandra said, "A correct slope is awarded 4 points."Jimmy said, "The student can have 2 point for having the correct slope." Dolly said, "I will give about half of the points for the slope, and I would give 5 or 6 for this slope depending on the student's actual work." She gave 6 points to this graph.

Kara awarded 4 points to this graph originally, but after I mentioned that only the slope was correct in this graph, she said, "I usually gave 2 points for a correct slope only, so I would like to change the grade to 2 points."

Michael gave 5 points for a correct slope, and after I discussed with him and he realized that the graph did not label axes, he changed to 4 points.

Normally Sam gave a correct slope 5 points, but he gave 7 points for this graph, because he judged that the student had made an algebra mistake of putting y=x-2-3=x-5 into y=x-2+3=x+1, and he thought the student understood how to graph and he could not take off a lot points. After I discussed with him he said, "This graph awarded 5 to 7 points". And I recorded the average 6 points as his grade for this graph after discussion. He was the only grader to mention this potential algebra mistake.

Before I discussed with the graders, the mean was 4.09, the highest score was 7 and the lowest score was 1. After I discussed with the graders the mean was 3.74, the highest score was 6 and the lowest score was 1. It looked like the grade of this graph dropped down a little bit. However, from all the conversation with the graders, we would see that the value given to a correct slope was higher than the average grade given to this. First, Daniel gave 1 point for this graph which was very low compared to other graders, and as he said would give 3 or 4 points for a correct slope with more clear descriptions to show the exactness of the slope. Both Ray and Gerry said that the value of a correct slope should be higher than the 3 points that they gave for this graph. Michael and Sam gave a clear signal that they would give 5 points for a correct slope and they changed the grade after discussion because the other errors of this graph, but not the slope, such as lack of labeling.

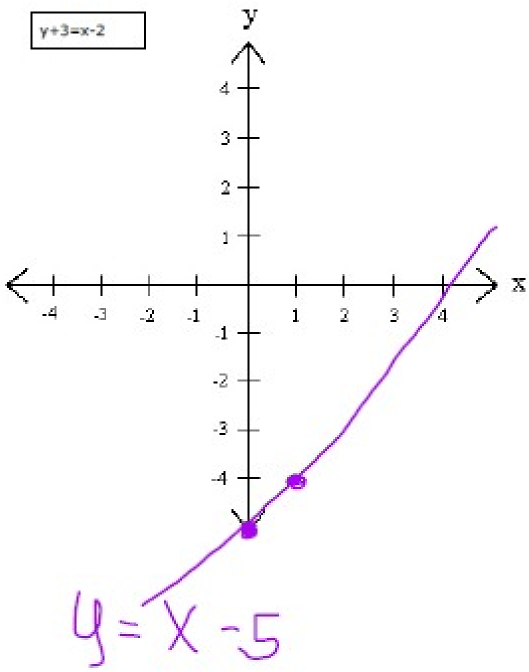
If we assume Ray and Gerry would give 4 points for a fully correct slope, and Daniel gave 4 points for a correct slope, we had 5 graders who gave 5 points, 4 graders who gave 4 points, and 2 graders who gave 2 points for a correct slope. That means the mean of the correct slope is 4.09. Therefore, roughly speaking, a correct slope was worth 4 points.

(3) Intercepts: y-intercept; both x- and y-intercepts

The y-intercept of this line is the value of *y* at the point where the line crosses the y axis. Analogously, the x-intercept of this line is the value of *x* at the point where the line crosses the x axis. If a student can determine the x- and y-intercepts, that's often enough information to make a pretty accurate sketch of the graph. Or if a student can find the y-intercepts and any other point of the line, that's often good enough to make a pretty accurate line because two points determine a line.

For example:

Figure 3.3 Student solution G, for y+3=x-2



This graph had the y-intercept correct but the x-intercept is wrong due to the curving of the line. This graph was on the background 3 and so labeling was provided. Therefore, it was a nice graph for researching the importance of intercepts to graders.

Gerry and Frank gave 10 points for this graph because they thought getting the right y-intercepts and a point (or slope) is good enough to graph the equation and claimed it is more important that we can see this student knows how to graph it. After I discussed with Gerry, he took off 1 point for curving the straight line.

Sandra said "I gave 4 points for a correct slope , 4 points for the correct y-intercept, and 2 points for the whole correct graph, and I gave 1 point for the correct graph here because the curving of the line."So Sandra gave 9 points for this graph.

Kara said "Two points can make up a line but it curved up and got the totally wrong x-intercept, and the x-intercept is important." She paid attention to both the x and y intercepts, and took off 2 points for the incorrect x-intercept.

Sam said, "Some people said getting x and y intercepts, some people said getting the y-intercept and a point. Finding the y-intercept and a point are basically fine for me." He took off 2 points because of the line was not being drawn perfectly, not because of the x-intercept being wrong.

Alice gave 5 points for a correct y-intercept. She said, "In this graph the y- intercept is correct, but the student makes a mistake of the slope by having the x-intercept go to 4 instead of 5. I will give the student 8 points because it looked the student start with a correct slope by a point but somehow curving the line." She took off 2 points for the graph.

Michael said "The student got the y-intercept correct but he drew the line poorly and the x-intercept was wrong, I gave 7points."I asked him whether he agreed that 2 points can determine a line; he said he agreed but that he also paid attention to the x-intercept.

Ray and Jimmy paid attention to both x- and y-intercepts all the time. And they both took off 3 points for the incorrect x-intercept.

Dolly said "I gave 5 points for the correct y-intercept, and the x-intercept here could be placed more accurately." So she gave 9 points for this graph. After I discussed with her, she decided the line was curved and she changed her grade to 7 points.

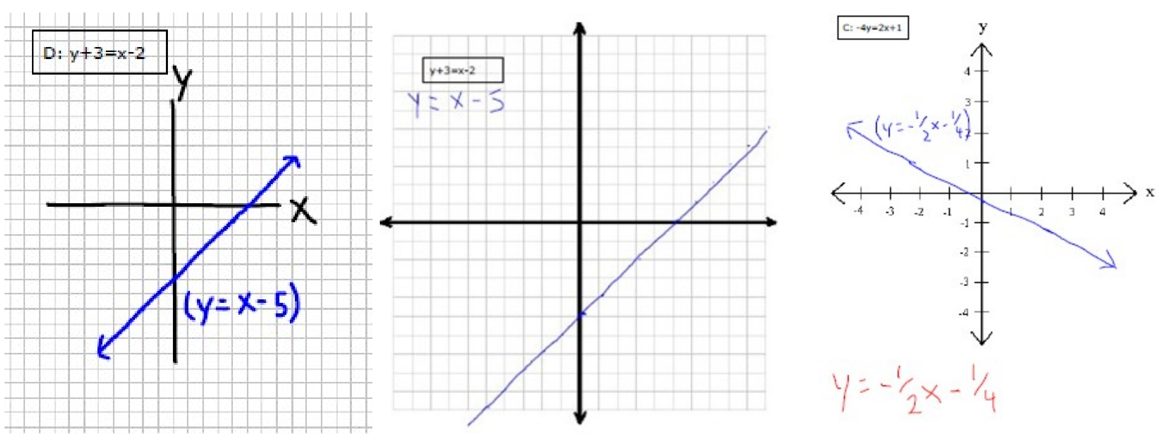
Daniel gave 3 points for this graph and the other graders gave 7 to 10 points; that was a big difference between him and other graders. In order to get more precise grade from the whole grading pool, I omitted Daniel's grade for this graph. From Table 3.1 and Table 3.2, we can see that the mean score of this graph was 7.82, and the mean score was 7.55 after I discussed with them. If we omit the grade of Daniel, the mean scores were 8.30 and 8.00. From these data, we would see the graders only took off 1 to 2 points for an incorrect x-intercept, and/or 1 to 2 points for the curve of the sketch. It is also possible that taking off points for the x-intercept could be considered taking off some points for the slope, because if both the x and y intercepts are correct, the slope of the line must be correct. All the graders cared strongly about the y-intercept, and most of them had a clear grading standard of giving 2 to 5 points for getting a correct y-intercept. Of the graders that had a clear standard, Gerry, Frank, Alice, Dolly said that they would give 5 points for getting the correct y-intercept, Sandra gave 4 points, Ray and Jimmy gave 3 points, and Kara gave 2 points. Therefore, a correct y-intercept was generally awarded about 4 points.

(4) No difference when the graders graded the graph in different backgrounds

I designed the problem sets for the interview in three different backgrounds, and which gave increasing information to help students in drawing the graphs precisely. Progressing from the backgrounds (1) through (3), clearer scale information is available, making it easier to draw a precise graph. However, after the interview, I found there was no difference when the graders graded the graph in different backgrounds.

For example,

Figure 3.4 Student solution A, E, 11



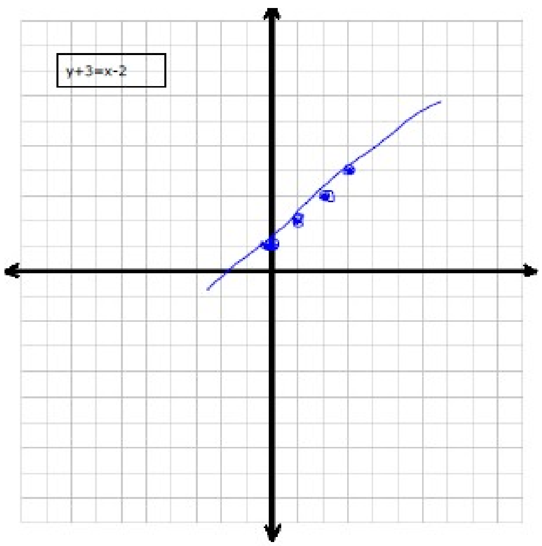
The graph A, E and 11 had the backgrounds (1) through (3). And these graphs had the same quality: the slope and intercepts were correct and the line was drawn straight, but the graphs lack labeling. Labeling includes labeling of intercepts, points and graph. In graph 11 with background 3, the background came with labeled axes and had clear scale information, but the student did not label anything else (such as points) to make the graph clearer. This was also considered a lack of labeling by the graders. The mean grades of all these three grades were close to 9. The graders took off 1 point for the absence of labeling for all these graphs no matter what background they had. Therefore, there was no difference when the graders graded the graph in different backgrounds.

(5) Only grading depending on the work shown on the graph

If a student draws an improper graph, maybe the student does not have the skill being graded, or maybe the student has other deficiencies besides missing the graded skill. In graphing lines, students frequently make algebra mistakes that result in drawing the wrong graph. However, if the student with a graphing error does not show their algebra work, do the graders spend time analyzing the reason for the error in the graph and assign a grade based on that analysis?

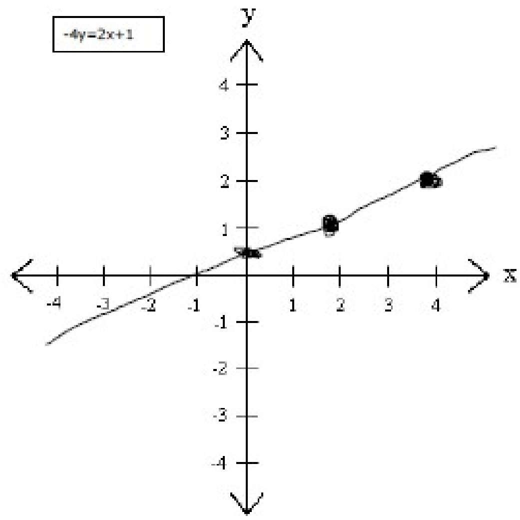
Example:

Figure 3.5 Student solution F, for y+3=x-2



In the graph F, the student did not show any algebra work. However, if we examine the line graphed, we find that the student drew a line of the equation y= x+1. It was possible that the student made a mistake when he or she simplified the equation, such as solving for y as y=x-2+3=x+1. If this was the case, the student did have the skill to graph a line but he or she only had an error in the manipulation of the equation. Only Gerry hypothesized about this student's reasoning in creating the graph when he was grading. Gerry gave the student 7 points. The other graders only gave the points for the correct slope – which is a feature of the appearance of the graph. Danny said, "I will only look at what the students wrote or drew on the graph." After I discussed with the graders and mentioned the possible reason why the student drew this line, none of the graders gave more points for this student. We can see that most graders only graded depending on the work shown on the graph.

Figure 3.6 Student solution 9, for -4y=2x+1



In the graph 9, all of the points are wrong and the slope is incorrect. However, if we compare the line in the graph and the correct line for the equation, they are reflections of each other. Therefore, it may be that this graph resulted from a sign error, slope and intercept are positive when they should be negative. The mean grades for this graph were 1.36 before discussion. Only Kara, Ray, and Gerry judged the graph resulted from a sign error; Kara and Ray gave 3 points while Gerry gave 6 points for the graph. Gerry said, “I think the student put the negative sign on the right side when he simplified the equation. And the student knew how to graph so that I gave him 6 points.” The other graders gave the graph 0 or 1 points. After each grader gave the grade, I discussed the possible reasons why the student created this particular graph. Only Alice and Sam changed their grades, both from 0 to 1. And the mean changed to 1.55 after the discussion. We can see that most graders graded based on the incorrect slope and intercepts shown on the graph and did not give any points based on the possible reasoning behind the graph.

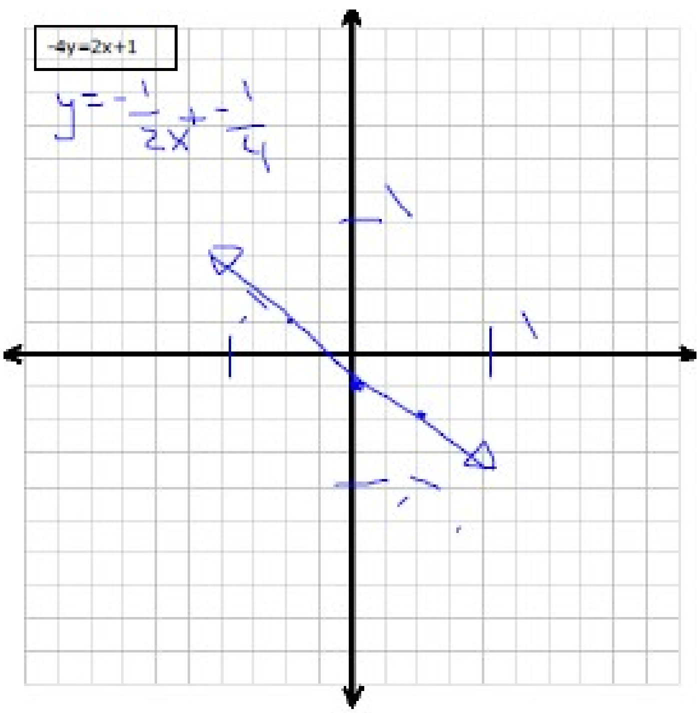
From the graph F and graph 9, we see that most graders only graded depending on the work shown on the graph and did not venture to think about the students’ reasons for creating the particular grape that they did.

(6) Paying attention to the right side of the graph

Every straight line can be represented by an equation: $y = mx + b$, where m is the slope, b is the y-intercept. As I discussed above, the graders gave greatest importance to the slope and the y-intercept in their grading. In grading these graphs, many graders used the y-intercept and a close-by point to determine whether the slope was correct. It is a very general idea that if you know the y-intercept and find any other point, you can calculate its slope as the rise over the run of these two points. Choosing a close-by point can make it easier to calculate the slope. I found that in judging slope, the graders preferred to look at a close-by point on the right side of the y-intercept. In other words, the graders paid more attention to the right side of the graph.

For example:

Figure 3.7 Student solution 5, for -4y=2x+1



In this graph, there were many intersting details. First, the student sloppily wrote the equation y= - (1/2) x-(1/4) as y= - 1/ (2x)-(1/4). Second, the student set 4 small squares as a unit 1. Third, the two points (0, -1/4), (1/2,-1/2) on the right side were correct and so the slope was correct. However, on the left side, the student marked an incorrect point at (-1/2, 1/4) and had the x-intercept at (-1/4, 0), where it should be (-1/2, 0), and as a result, the slope was incorrect on the left side.

Only Michael mentioned the algebra mistake and took off 1 point. Michael gave 4 points for this graph because of the correct y-intercept, and he thought the slope was totally wrong. After I discussed with him the two dark points on the right side he realized the slope was correct, and then I reminded him that the x-intercept was incorrect by going up the line on the left side. Finally he changed 4 point to 6 points for the graph.

The student set 4 small squares as a unit 1, but Daniel did not realize this when he graded the graph and he gave 5 points for the graph only, and after I discussed with him he changed the grade to 8.

Sam took off 2 points, not because of the incorrect x-intercept, but because of the lack of labeling.

Frank, Gerry, Dolly and Alice gave full credit for this graph, and they all had the grading standard of 5 points for the correct slope and 5 points for the correct y- intercept. It was clear that they only looked at the close point on the right side to determine that the slope was correct. After I pointed out to them that the marked point to the left was incorrect, Gerry and Alice changed the grade to 9 and 8 points, respectively. However, Frank and Dolly considered the correct y-intercept and a close-by point were good enough to show the student understood how to graph a line.

Ray, Jimmy and Kara checked the x-intercept all the time. They judged the x-intercept incorrect and took off 2 to 4 points. Sandra judged that the line was curved up and so the graph did not have a correct slope, and she took off 3 points. When Alice, Daniel, Dolly, Frank, Gerry, Michael, Sandra and Sam were grading the graph, they did not look at and talk about the left side of the graph. Therefore, most graders only looked at the y-intercept and the other point on the right side before discussion.

From Table 3.1 and Table 3.2, the mean grades for this graph were 7.73 (before discussion) and 7.91 (after). Michael, Daniel and Ray took off points for problems other than the incorrect slope and x-intercept on the left side. Therefore, from the grade, we would see the graders did not take off many points for mistakes in the left side of the graph. Only 4 out of 11 graders took off any points based on the left side of the graph. And 3 of these 4 graders looked at the left side because the x-intercept was on the left side. If the x-intercept had also been on the right side of the graph, I could not be sure whether they would have looked at the left side. Therefore, most graders only paid attention to the right side of the graph.

Chapter 4 - Conclusion and Future work

4.1 Conclusion

Research questions that I set out to answer in this thesis are:

(1) What are the features that college algebra teachers look at when grading graphs of lines?

(2) How much disagreement is there in the relative importance graders assign to each feature?

Altogether, the general grading rule appears to be: slope is worth 4 points, y-intercept is worth 4 points, labeling of intercepts, points and graph is worth 1 point. After that, add 1 point if everything is correct. I designed the problem set for the interview in three different backgrounds to control for the variety in the level of precision that graders initially expected. However, there was no difference when the graders graded the graph in different backgrounds. So that we can see our general grading rule can apply to different standards of precision.

Sandra said, "I gave 4 points for a correct slope, 4 points for a correct y-intercept, and 2 points for the graph as whole being correct." Her grading standard is very close to the grading rule I summarized. From the Table 3.1 and Table 3.2, we can see Sandra is the only grader who never changed any grade after the discussion. This reflects that this grading rule is stable for her when applied to all kinds of graphs.

Our general grading rule was constructed from the features the graders considered important, which I identified from the graders interviews' and the grading results. Except for Sandra, this general rule does not express each individual grader's grading accurately. There were some disagreements in the relative importance graders assigned to each feature.

First, in Graph F, in which only the slope was correct, the mean score was 4.09 before discussion and 3.74 after discussion. Both means were pretty close to our general grading rule: 4 points for the slope. However, the highest score for the graph was 7(before discussion) and 6(after discussion) and the lowest score was 1. From these numbers it appears that there were big disagreements in the relative importance graders assign to the slope. In fact, there were not. From the interviews, I knew that the extreme high and low grades were affected by other concerns. They were not solely the grade those graders gave for a correct slope. Roughly speaking, 5 graders gave 5 points, 4 graders gave 4 points, and 2 graders gave 2 points for a correct slope. That means the mean score for the correct slope is 4.09, and 82% of the graders had close grading standard for the slope, and only 18% graders have a grading standard 3 or 2 points lower. Therefore, the disagreement in the relative importance graders assign to the slope was small.

Second, in the graph G, the y-intercept was correct but the x-intercept was wrong, due to the curving of the line. This made it a nice graph for researching the importance of intercepts. There was a big difference (up to 7 points) between Daniel and other graders. In order to get more precise evaluation of the whole grading pool, I omitted Daniel's grade for this graph. After we omitted his grade, the mean scores were 8.30 and 8.00. From these data, we would see the graders only took off 1 to 2 points for an incorrect x-intercept, and/or 1 to 2 points for the curve of the sketch. It is also possible that taking off points for the x-intercept could be considered as taking off some points for the slope, because if both the x and y intercepts are correct, the slope of the line must be correct. All the graders cared strongly about the y-intercept, and 8 of them had a clear grading standard for the y-intercept. For those graders, 4 gave 5 points, 1 gave 4 points, 2 gave 3 points, and 1 gave 2 points for a correct y-intercept. That means that the mean score of the correct y-intercept is 4, and 88% of graders had a grading standard for the slope close to this, and only 12% of graders had a grading standard 2 points lower. Therefore, the disagreement in the relative importance graders assign to the y-intercept was also small.

Third, labeling was not very important for most of the graders, but some of them considered labeling to be quite important. In the graph E which only lacked labeling,7 of the 11graders gave full points, 1 grader took off 1 point, 2 graders took off 2 points, and 2 graders took off 4 points. Only 1 grader had exactly the same grading rule as our general grading rule and took off 1 point for the lack of labeling. However, of the remaining graders, 70% of them did not take off any points for lack of labeling, 30% of them considered labeling was quite important and in total may took off an additional 12 points. As a result, the mean grade for the graph is 9 and the graders generally took off 1 point for the lack of labeling. We can see there was a great deal of disagreement in the relative importance graders assigned to the labeling.

Lastly, in addition to the slope, y-intercept and labeling, some of the graders also scored based on the algebra, quality of the sketch or some other detail of the graph. They subtracted one point for each thing they noticed that was incorrect or added 1 point if everything seemed correct to them. Other graders only graded based on slope, y-intercept and labeling. There was not a big disagreement.

Overall, there were some disagreements in the relative importance graders assigned to each feature. All graders considered slope and y-intercept to be very important. Only some of them considered labeling to be important. Anything else was a matter of a single point adjustment.

We also know the graders graded depending on the work shown on the graph and did not venture to think about the student's reasons for creating graph as they did. Instead, they graded almost entirely from the important features identified in Chapter 3: y-intercept, slope, and labeling. More strikingly, graders paid attention to the right side of the graph only.

All together, this means that grading the slope doesn't really mean judging the overall steepness of the line. Instead, graders judged the slope from two points. A correct slope was judged from the y-intercept and the first point to the right (usually). If these two points are correct, then the graders judged the slope to be correct. Moreover, the whole graph is then judged to be correct. Although the graders know a graph is more than two points, in this experiment they only graded two points. I suspect this is partly because they have to grade a lot of problems at once, and two points is an efficient way to do it.

However, this type of grading may also have an effect on student learning.

In all the 20 graphs of the linear equations C and D used for the college algebra graders' interviews, 6 of them did not have any points plotted, 5 of them had points plotted on both left and right sides, none of them had points plotted only on the left side, and 9 of them had points plotted only on the right side. We can see the students had a big tendency to only plot points on the right side of the graph. Feedback from graders may play a large role in this: the reason why the students thought the right side of the graph was more important was that they knew from education and experience that the graders would look at the right side of the graphs. There is a danger that from this type of grading, the students learn not to think about anything else in a line. Students learn from this type of grading that the only important features of a graph are the y-intercept and the first point.

There is a danger that – as a result of this type of grading -- students understand graphing a line as just plotting two points and then connecting them. Beginning at the y intercept, students then go "over one and up m" to graph a line with slope m. Therefore, students may consider a line as being composed of two discrete points instead of an infinite number of points. This method is good enough to generate graphs of lines, but begins to fail as students begin dealing with any type of non-linear function, where changes smaller than one become more and more important, leading into limits and Calculus. In summary, the grading feature of “A correct graph (slope) was judged from two points(the y-intercept and the first point to the right)” may have had a role in students’ learning to think only about two points in a line (but nothing else), which is not not good for students’further mathematical studies

After ascertaining the features that college algebra teachers look at when grading graphs of lines, the disagreement in the relative importance graders assign to each feature, and the effect that choices of feature have on student learning, our future work will be directed to designing an automated grading system for student-generated graphs. The type of grading used by the graders in this study is pretty easy for a computer to do. We only have to teach the computer to identify the (roughly two) points a teacher would look at to grade from and calculate the slope. However, computer grading also opens up new possibilities. It can mimic teacher grading and only look at two points, but it doesn’t have to. The two-point grading method seems to exist for efficiency. If a computer is doing the grading, the computer can look at more than just those two points without making more work for the teacher. That might be better for the students than the current system. Therefore, it may be beneficial to design an automated grading system for student-generated graphs. After all, we can compare these scores given by the teachers based on those features to scores determined by applying linear regression to the graphs to see how much of an influence accuracy has on grading.

4.2 Future work

In the future, I will design a computational statistical analysis of the graphs. First, I will write a program to teach the computer how to identify and evaluate student graphs based on these features we found from the graders. Second, I will apply linear regression to design an automated grading system for student-generated graphs. Finally, I can compare these scores given by the teachers based on those features to scores determined by my automated grading system to see how much of an influence accuracy has on grading. Before I teach the computer how to identify and evaluate student graphs, I shall have identified some graphs by hand. Here are some preliminary results:

Because I had students produce digitized images in the iPad, I had some digital data. I converted the digital data to an Excel document. In order to reduce the difficulty, I chose to work with the graphs on the background 3, so that I did not have to identify the x- and y-axis computationally

The digital data was expressed by two lists of digits, the first list is the x-value, and the second list is the y-value. And each stroke was separated by the x, y value 0, 0. I identified the lines and dots of the student-drawn graph by:

(1) Finding the averages and the standard deviation of each stroke.

(2) Making a scatter plot with straight line and markers for each stroke.

(3) Adding a linear trend line to the scatter plot with the regression equation and the R-squared value of the regression.

Usually the students would do some algebra calculations before they drew the dots and lines, so I had to pay attention to the first part of the data and compare every scatter chart and identify the algebra work.

Besides comparing the scatter chart and actual graph, I used the R-squared value of the regression to identify the dots and the lines. R-squared measures how well the scattered data fit the linear regression trend line. I found the rule to be: if R-squared is bigger than 0.8, it is a line; if R-squared is less than 0.1, it is a dot.

After identifying the dots and lines, I found some dots were drawn by more than 1 stroke. Roughly speaking, if the averages of the strokes of the dots were close and the standard deviations were pretty small, they were the same dot.

Now, I convert the screen coordinates to the graph coordinates. The rule is:

Origin: (2540, 2745)

A unit for x and y: 345

Graph x= (Screen x -2540)/345

Graph y= (2745-Screen y)/345

Often a student will draw multiple “lines” in making his graph, with some strokes being helping lines (e.g., axes, tick marks, etc.) and one main stroke for the line which is to be graphed. After converting all dots and strokes drawn by the student to the graph coordinates, I first have to identify the stroke which the student drew to represent the actual graph of the linear equation. I will then use linear regression to fit a line to the points that comprise this stroke. I refer to this as line 1. Second, I have to identify which dots people would use to judge the graph. As the conclusion I got from the research, both graders and students look at the right side of the graph; especially if they judged the graph from two points (the y-intercept and the first point to the right). Therefore, I can simply identify these two dots. I can compute the centroid of these dots. Students in this experiment drew all their dots with integer or half-integer coordinates. Therefore, I will pick point 1 and point 2 to be the points with integer or half-integer coordinates nearest each centroid. I define the line through these two points as line 2.

After that, I will Compare the slopes and intercepts of these two lines to see how close line 2, which is what the graders’ primarily use, is to line 1, derived from the linear regression. Then I will convert the line 2 into a 10 point grade of the general grading rule I found for the graders, and then compare those grades with the actual grades I got from the graders. With this information, one could experiment with varying grading methods based on either line 1 or line 2 to determine what effects grading based on just two points vs. grading based on the complete line has on student learning. Determining different algorithms for grading such work would be part of this future work.

References

Thomas, P., Smith, N., and Waugh, K. 2008). Automatically assessing graph-based diagrams. *Learning, Media and Technology*, 33 (3), pp 249-267

Appendix A - Student Protocol

Each one-on-one interview took 15 to 30 minutes and was voice recorded. In the interview, students were asked to draw graphs in the iPad without being given any instruction by the interviewer. They drew each graph based upon their own outside knowledge. The student protocol is based on difficulties that I anticipated students might have:

Possible questions from students:

- (1)Students: I do not know how to start to draw? I have no idea how to draw this graph?
- Answer: I will give them a sample question: draw "y= 2x" in background (1), and give some simple direction. I will draw a draft graph for them if necessary.

- (2) Student: Do I always have to find the y value first?
- Answer: That is a good point to draw the graph. However, you do not have to do that if you would find some other easier way to draw.

- (3) Student: (After finishing her or his drawing) do you think my graph is correct?
- Answer: Although I am not allowed to inform you if it correct, it adds effective data for my research. Once your graphs are completed, then I can give you the general ideas on your work if you want.

- (4) Student: I think this question is too difficult for me to do, can I skip it?
- Answer: It does not matter whether you draw it correctly or not, we only need some graph data, if you do not know how to do this, you can just guess and do your best.

- (5) Student: I found that I have some mistakes on my last graph, can I go back to fix it?
- Answer: We can go back to the last graph and then you can tell me what mistake you think you have, but you cannot fix on the graph. By the way, there is no grade for your work, but it is more important for you to strengthen your graph skills.

- (6)Student: Can I label something on the graph when I draw the line?

- Answer: Yes, you can write whatever you think is useful for your drawing on the graph.

- (7)Student: Do I have to declare my measurement on the graph?
- Answer: You do not have to do it, but it will be more effective if you do.

Appendix B - Grader Protocol

Each one-on-one interview took 30 to 45 minutes and was voice recorded . Each grader was asked to grade twenty graphs by using the grading scale of 0 to 10. All graders were given the grading problems (two linear equations C and D) a week before their interviews. During the interview the graders were suggested to talk aloud while they were grading.The graders set their own grading standards and after they finished all their grading, I had a short discussion with them.

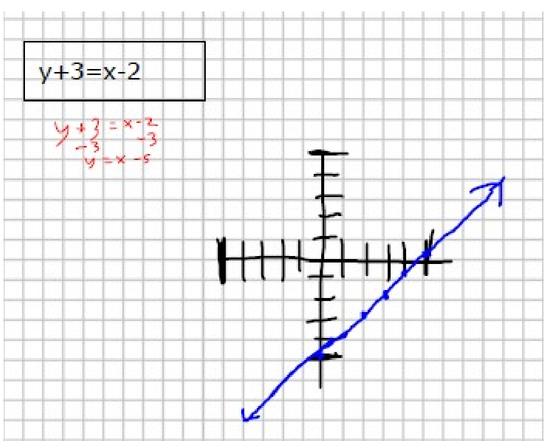
Before the interviews I had graded all the graphs myself, and during these discussions, I asked the graders about any graph where my grade and their grade disagreed by a certain amount(2 points different or more). I also asked pre-prepared interview questions about seven graphs that I found particularly interesting for isolating certain features or standards of precision.

- **Universal Questions:**

1) Why did you give this graph the grade that you did?
2) What do you look for when you're grading a graph like this?

- **The seven graphs I chose in advance that I considered most interesting:**

Figure 4.1 Appendix B -Student solution C, for y+3=x-2



If the grader gives a grade higher than my grade I will ask:

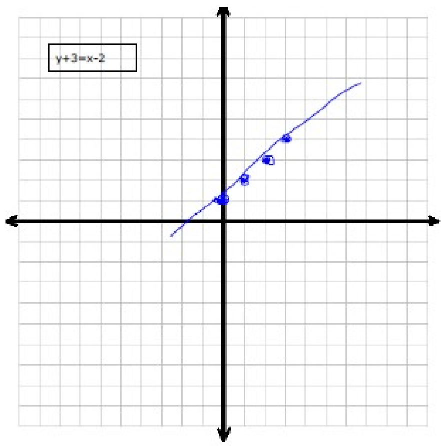
- Is that matter the line does not pass the intercepts (0,-5) (5, 0) correctly?

If the graders give a grade lower than my grade I will ask:

- I noticed that the points (2,-3) (3,-2) and (4,-1) which the student plotted are correct. What do you think about that?
- The slope is correct, because those points are correct. Should that matter?

My grade: 8

Figure 4.2 Appendix B -Student solution F, for y+3=x-2



If the grader gives a grade higher than my grade I will ask:

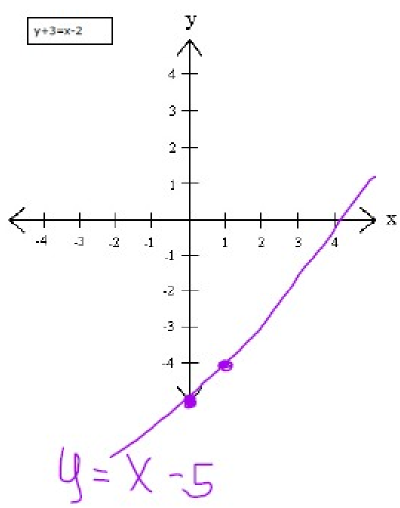
- Does it matter that the line is not drawn through the points?
- I've noticed that none of the points are correct. Isn't that a big problem?

If the graders give a grade lower than my grade I will ask:

- The slope is correct. Should that be worth something?

My grade: 3

Figure 4.3 Appendix B -Student solution G, for y+3=x-2



If the grader gives a grade higher than my grade I will ask:

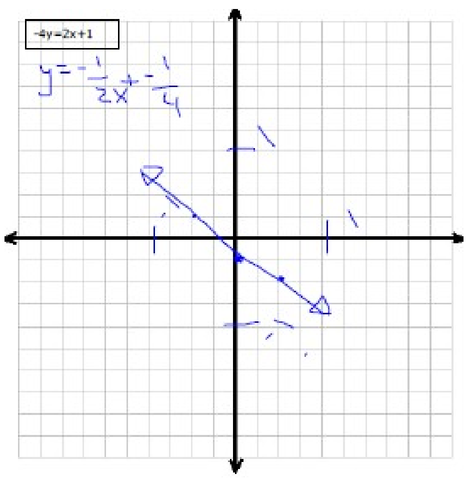
- Does it matter the line is curved?
- If we look at the x-intercept, it should be at (5, 0), but it is about (4, 0) on this graph. Do you think it is a big problem?

If the graders give a grade lower than my grade I will ask:

- The line has two points that are correct (0, 5) and (1,-4). Shouldn't that be enough to define the correct line?

My grade: 8

Figure 4.4 Appendix B -Student solution 5, for -4y=2x+1



If the grader gives a grade higher than my grade I will ask:

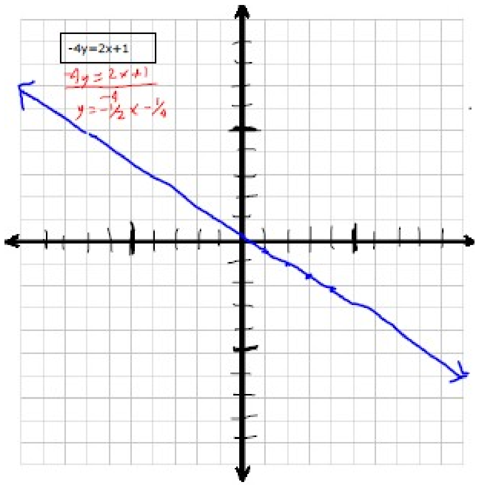
- What do you think about the unlabeled axes?
- Did you notice that the line isn't straight? Is that important?
- The x-intercept for this line should be (-1/2, 0), but if you look at the placement of the -1, this student has it at (-1/4, 0). What do you think about that?

If the graders give a grade lower than my grade I will ask:

- If you look at this [gesture] point and this [gesture] point, the slope and the intercept are correct. Isn't that good enough?

My grade: 8

Figure 4.5 Appendix B -Student solution 6, for -4y=2x+1



If the grader gives a grade higher than my grade I will ask:

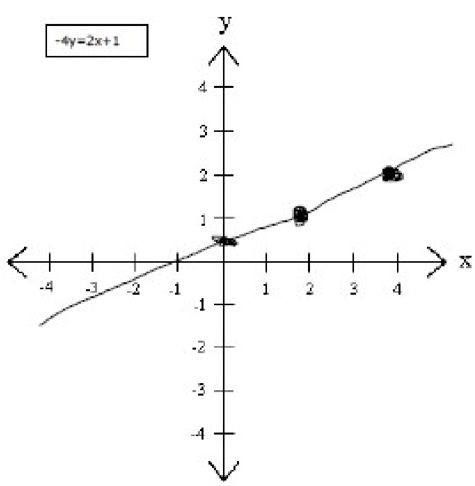
- If we look at intercepts (0, -1/4) and (-1/2, 0), the line is not drawn properly. Both intercepts should be negative instead of positive. That seems like a pretty big error to me. What do you think?

If the graders give a grade lower than my grade I will ask:

- With values this small, it's hard to plot the points exactly. Do you think they're close enough?
- I think the student simplified the equation correctly and the slope it draws is pretty close to -1/2. What do you think?

My grade: 6

Figure 4.6 Appendix B -Student solution 9, for -4y=2x+1



If the grader gives a grade higher than my grade I will ask:

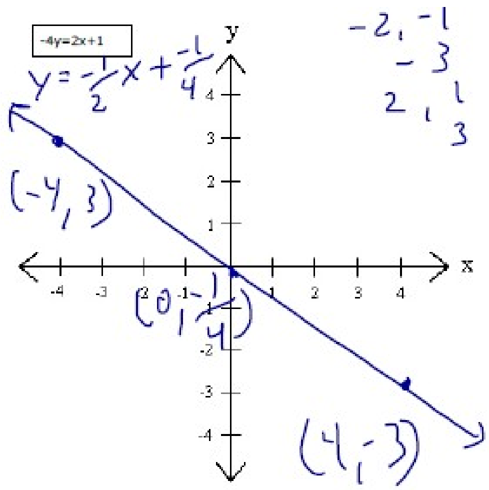
- All of the points are wrong

If the graders give a grade lower than my grade I will ask:

- It looks to me like this is just a sign error. The slope and intercept are positive when they should be negative. Should the student get some points for that?

My grade: 3

Figure 4.7 Appendix B -Student solution 8, for -4y=2x+1



If the grader gives a grade higher than my grade I will ask:

- When x is equal to 4, y should equal -9/4, and when x is equal to -4, y should equal 7/2. So two of the three points it plots in are wrong. Do you want to change your mind?

If the graders give a grade lower than my grade I will ask:

- Do you notice that it simplifies the equation correctly and the point (0, -1/4) is correct in the graph? I think the student got a large portion of this problem right.
- It seems to me that the slope is pretty close.

My grade: 6