Creating a Cognitively Guided Instruction guidebook: The research and development of an educative curriculum for teachers

by

Sarah Rand

B.A., Ottawa University, 2009
M.A., University of Southern California, 2011

AN ABSTRACT OF A DISSERTATION

submitted in partial fulfillment of the requirements for the degree

DOCTOR OF EDUCATION

Department of Curriculum and Instruction
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Abstract

The purpose of this study was to create an educative guidebook for teachers based on the principles of Cognitively Guided Instruction (CGI), a constructivist approach to teaching mathematics through contextual word problems. The guidebook is designed to be used as a First-Grade mathematics supplemental curriculum resource with complete, daily lessons. The development of the handbook began by identifying the problem of lack of support and professional development for teachers to enact the instructional strategies aligned with CGI. CGI is an impactful instructional method as it allows a teacher the opportunity to enact all of the National Council of Teachers of Mathematics' "Effective Teaching Practices." A Qualitative Content Analysis of seven of the most widely used elementary mathematics curriculum resources was conducted using a coding framework that included 11 key elements of a CGI lesson. The findings showed a strong lack of these 11 features of CGI. In some cases, they were entirely or nearly entirely absent. This data was used to inform the development of the CGI Guidebook. A first draft of five lessons was implemented by two First-Grade teachers. Revisions were made based on feedback from the teachers. The final guidebook provided First-grade teachers with a module of CGI lessons that is designed for both student and teacher learning, making it an "educative curriculum for teachers". The "5 Practices for Orchestrating Productive Mathematics Discussions" (Smith & Stein, 2018) were also used to inform the guidebook and all 5 practices are embedded through each lesson. The guidebook provides a contextual word problem for each daily lesson, a rationale for the problem type and numbers in the problem, directions for selecting and sequencing student strategies and facilitating a discussion about the student strategies. The guidebook shows where standards are embedded in the lessons and provides, in every lesson, drawn example student strategies, rationales for the selection and sequencing of

strategies as well as discussion guidance that will support the teacher to make connections between student strategies and to illuminate mathematical connections for students. The guidebook fulfills a need for access to Cognitively Guided Instruction professional development through a medium that is not currently available.

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My first introduction to progressive mathematics teaching practices was in Dr. Gary Scott's course at the University of Southern California. He showed me that my mathematics instruction had the potential to be much different than what I had seen growing up and this planted a seed.

I was very fortunate to start my teaching career at Success Academy Charter Schools in New York City. It was there that I was immersed in Cognitively Guided Instruction and received professional development and coaching from Stephanie Smith and my principals Michele Caracappa, Liz Vandlik and Kaitlin McDermott. Before practicing CGI myself, I had a unique opportunity to observe master teachers at work. I could not have learned without hours of observing Britney Weinberg-Lynn and Tara Stant, two exceptional 2nd grade teachers.

I eventually took CGI back home to Kansas and was only able to further hone my craft because of the instructional liberty and support given to me by principal Ann Collins at Central Heights Elementary School. She believed in me and allowed me teach students in ways that I felt were best, even it was different than what had been done before.

Before becoming an educator, I had temporarily left college and didn't quite know what was next in life and my husband Alex has been along for the journey since then. His endless support, patience, flexibility and understanding was paramount in my own education, career, personal growth and this dissertation process.

Finally, this project would not be possible without all the students who have counted cubes, solved problems, made sense of numbers, persevered, presented their strategies, offered conjectures, proved and disproved ideas (including that their teacher, Mrs. Rand, is definitely wrong) and shared their thinking with me. Your ideas matter to me and I think you are going to change the world!

Dedication

For Hank, Lucille and Camille - Pedagogy was merely an interest before I met you, only then did it become a passion. May you have knowledgeable, skillful and inspiring teachers who provide lessons, materials and discussions that spark curiosity and build understanding. And, when you don't, remember to never give up. Math is for everyone.

Chapter 1 - Introduction

The curriculum materials that school districts and individual teachers purchase for use with students hold significant weight in the classroom. The mathematics curriculum, or planned sequence of activities that lead to student mastery of the content, is most often delivered via these materials and teachers, administrators, among other stakeholders, often associate the word "curriculum" with these resources. In my school district when we say, "math curriculum," we generally mean Math Expressions (Fuson, 2018). These curriculum materials guide the learning of students by taking apart concepts and building them up for students using a planned series of activities and experiences. These activities and experiences help students accomplish the many discrete skills that culminate in ultimately achieving the standards, or not. Additionally, these materials may determine the amount of time spent on concepts and the models used to help students comprehend new learning and make connections to prior learning. They also influence the extent to which instruction is teacher or student centered. With all of this in mind, the selection of curriculum materials matters.

The core curriculum products adopted by districts for all teachers to use play a prominent role in if and how learning takes place. However, teachers have always supplemented these core materials with outside resources that they believe will meet particular needs of their students. This is relevant because the implemented curriculum, or what is actually taught and learned in classrooms, is influenced by this blend of materials that teachers curate and the quality and intended outcomes of the materials are important.

1.1 Statement of Problem

The National Council of Teachers of Mathematics (NCTM) calls on today's K-12 math teachers to meet ambitious teaching goals. To guide teachers in meeting these goals, NCTM suggests eight effective mathematics teaching practices (NCTM, 2014) that are key to providing instruction that helps students build a deep understanding of the content and what it means to be a true problem solver and mathematician, even as kindergarteners. The eight practices are as follows:

- Establish mathematics goals to focus learning
- Implement tasks that promote reasoning and problem solving
- Use and connect mathematical representations
- Facilitate meaningful mathematical discourse
- Pose purposeful questions
- Build procedural fluency from conceptual understanding
- Support productive struggle in learning mathematics
- Elicit and use evidence of student thinking

These high-leverage practices "provide a framework for strengthening the teaching and learning of mathematics" (NCTM, p. 9). The practices are essential because learning mathematics is an active process that involves students engaging in experiences that help them build onto what they already know through interactions with their peers and actively working with problems rather than watching an adult demonstrate procedures. Unfortunately, these practices are also in sharp contrast to many traditional teaching practices and views on how mathematics teaching and learning should take place.

Currently, teachers are being called on to implement the eight practices in their classrooms, and, for some, this creates the need for a major instructional shift in order to meet these demands. However, there is a lack of resources (e.g., physical resources, professional development opportunities, and time) to support teachers in this shift as many traditional curriculum resources do not provide teachers with the tools and knowledge to put these practices into action. Without more support and explicit direction, these practices could remain an aspiration for many teachers. Thus, teachers would be well served by meaningful opportunities for learning that are embedded in their expected daily practice and curriculum materials of which they are using with their students on a daily basis. In my view, there are two instructional frameworks in the area of mathematics education that I propose as an avenue for carrying out the 8 practices effectively by embedding these within the curriculum. They are Cognitively Guided Instruction (CGI; Carpenter et al., 1999) and the “5 Practices for Orchestrating Productive Mathematics Discussions” (Smith & Stein, 2018).

Therefore, the purpose of this dissertation is to understand the strengths and weaknesses of curriculum materials currently used in elementary mathematics and to develop a first-grade mathematics guidebook for teachers that is designed for supporting both teacher and student learning. The guidebook will encompass a series of complete lessons that can be implemented with students, including embedded features to enhance the teachers’ mathematical knowledge for teaching and knowledge of the students in their classroom.

1.2 Significance of the Study

This study has significance for elementary teachers as well as professional development and curriculum staff. For my project, I will create educational curriculum materials based on supporting teachers with implementing 1st grade mathematics, CGI, and the “5 Practices for Orchestrating Productive Mathematics Discussions”. In general, CGI is a method of teaching where teachers pose word problems that students solve using their own intuition and solution strategies, prior to formal instruction on the underlying concept and/or strategy in the word problem. While students work on solving the word problem, the teacher selects particular students to share their strategy with the class at the end of the lesson and facilitates a whole-class discussion, both during and after students share their strategies.

In turn, the curriculum product that will result from this dissertation will be educative curriculum materials for 1st grade teachers to use with their students and will include lesson plans for daily CGI word problems. Each lesson plan will include educative features that help the teacher learn the process of the CGI method of teaching as well as how to have a rich and meaningful class discussion about the particular problem and student strategies using the "5 Practices for Orchestrating Productive Mathematics Discussions".

To further describe the significance, I will explain how CGI makes each of NCTM's effective teaching practices actionable through educative features that support the teacher to implement the practices and facilitate productive whole-class discussions. I will then further describe the challenges around a lack of access to professional development on CGI and facilitating whole-class discussions, including the ways in which the curriculum product from this dissertation intends to address these issues.

1.2.1 How CGI addresses the 8 Effective Teaching Practices

Teachers should "implement tasks that promote reasoning and problem solving" (NCTM, 2014, p. 17). During CGI lessons, the teacher helps the class read one word problem. The teacher does not initiate a search for keywords, a very common word problem teaching strategy. Student strategies are not discussed at this time. Instead, students are questioned about the context and actions in the problem to ensure and strengthen understanding of the story. They reason about the problem and decide for themselves the best way to solve the problem. Teachers write CGI problems with their students in mind. CGI problems should push students to think about numbers in a particular way.

Next, they get to work on solving the problem using their own strategies. The students reason about both the actions and numbers in the problem and make decisions on their own for how to go about solving. The students do the thinking. The teacher moves through the room and watches how they solve, observing the operation they chose, how they are counting, and the tools they are using. At this time, the teacher confers with individuals, asking questions for clarification on their strategies.

Teachers should “use and connect mathematical representations” (NCTM, 2014, p. 24), “facilitate meaningful mathematical discourse” (p. 29) and “pose purposeful questions” (p. 35). This happens in the final part of a CGI lesson. While students were working, the teacher questioned them about their strategies and thought about how the strategies connect to one another and to the mathematics goals of the lesson. Using this information, the teacher selects particular students to share their strategy for the class. The class gathers back together to engage in a meaningful discussion. The teacher poses questions about the strategies students are seeing as she draws the students’ strategies on chart paper for all to see. Students are led to make connections between the representations they are seeing, ask questions about the strategies and discuss a wide variety of relationships that connect to the teacher’s goals.

For example, to solve 24 -15, one student may have used Unifix cubes and separated 15 from the total one by one. The next student might have performed the same task by removing a ten stick and then 5 more. The teacher might then ask students to compare those strategies thereby drawing their attention to the use of tens and ones on the subtrahend. A third student might have chosen to count up from 15 and the teacher might emphasize the difference between the equation that matches his work compared to the first two strategies that were shared. All of this is “building procedural fluency from conceptual understanding.”

Teachers must “support productive struggle in learning mathematics” (NCTM, 2014, p. 48). During CGI lessons, teachers do this by allowing the students to make decisions about how to go about solving. The structure of CGI lessons is consistent, but students are constantly exposed to nonroutine problems. They must consider for themselves the actions and details of the story to decide what operation to use. They look at the numbers and decide in what ways they want to work with them. If their first idea is not working, it is up to them how they will “fix up” errors and try again. The teacher supports them through strategic questioning, not demonstration. Sometimes students are not able to solve, and they will learn strategies from their peers who share. But they will see that problem type again soon and have an opportunity to apply their new understanding.

CGI teachers “elicit and use evidence of student thinking” (NCTM, 2014, p. 53) immediately in lessons as well as to inform their ongoing instruction. The invaluable knowledge about how students think about numbers and concepts can permeate the rest of their math instruction. This knowledge about how they solve problems could be the tools they use, how they count, the way they break apart numbers and the operations they use. Teachers who get to see how students think will find a very wide range of problem-solving possibilities, many they likely have never considered if they are new to CGI. Knowing how students think can impact the way they use the curriculum provided by their school and hopefully lead to more thoughtful questions, activities for students and class discussions.

It is important that teachers “establish mathematical goals to focus learning” (NCTM, 2014, p. 12) and this can be particularly challenging for teachers new to CGI. Problem types, context and numbers should never be selected haphazardly. Instead, these decisions should be made carefully with the students, standards and mathematical goals in mind. Experienced CGI teachers have acquired knowledge about how students think that help them to write these problems, seek strategies and facilitate meaningful learning through a discussion. A teacher trying CGI for the first time might find this challenging and overwhelming, which is why a CGI curriculum is necessary for a teacher new to this method.

Opportunities for students to problem solve freely are not present in many traditional curriculum resources adopted by schools. For example, in *Math Expressions 2018* which is used by my school, there is not a lesson plan in the 1st grade teacher manual that explicitly asks for students to solve using their own intuitive strategies. Instead, they are asked to solve after seeing the teacher demonstrate "circle drawings" which is drawing circles to represent the objects in the story and then "direct modeling" (Carpenter et al., 1999) the actions in the story. If 6 of the 8 bunnies hopped away, the teacher would demonstrate crossing out 6 bunnies drawn as circles and counting those not crossed out. These experiences do not open opportunities for students to, for example, think about the difference between 6 and 8 and count up or back, among many other strategies that they might want to use. It is limiting and procedural.

Open-ended problem solving that is present in CGI lessons is more difficult to plan and manage than many of the lessons found in traditional curriculum resources. Teachers must be ready to address possibly unexpected student thinking that they are seeing during the lesson. The product I will create for teachers is a guide on how to do this that includes the questions, possible student strategies as drawings, questions to ask and discussion examples and a rationale that explains the numbers, context, problem type and what they might expect during the lesson and discussion and how to capitalize on the thinking of their students particular to the problem.

1.2.2 Facilitating Meaningful Discussions

One critical aspect of CGI lessons that also can help teachers meaningfully address the "8 Practices" is rich class discussions. The discussion that occurs after students have solved the problem is really the heart of the lesson and it is the teacher's job to use this discussion to highlight the ideas that will support the overall lesson objectives and will steer student thinking for future CGI lessons all the while deepening their understanding and number sense. Therefore, a clear framework is needed to support teachers in understanding how to facilitate a rich and meaningful discussion in a CGI lesson. According to Smith and Stein (2018), there are many steps teachers must take, beginning in the lesson planning stage, to be able to facilitate this discussion. The "5 Practices" are (1) Anticipating, (2) Monitoring, (3) Selecting, (4) Sequencing, and (5) Connecting. These 5 Practices are explained in detail in their book, "5 Practices for Orchestrating Productive Mathematical Discussions." CGI teachers must learn to anticipate the many different ways students might solve the problem, monitor while they are solving and ask questions to better understand their thinking, strategically select students who will show their strategy to the whole class, sequence those strategies in a way that will further the goal of the lesson and connect those strategies by asking particular questions and drawing students' attention to various aspects of the strategies shared. This is no easy task, and it requires a cycle of new learning and ongoing practice for the teacher.

1.3 Purpose of the Study

The purpose of this study is to research the impact of Cognitively Guided Instruction, especially relevant to NCTM's 8 Effective Teaching Practices, to understand educative curriculum features and to illustrate the extent to which principles of CGI and the "5 Practices for Orchestrating Productive Mathematics Discussions" are present in common, large elementary math curriculum textbooks. This information is pertinent to the development of a CGI guidebook for teachers that will help them learn the CGI method through the structure of the 5 practices and embedded educative features.

1.4 Qualitative Content Analysis

The purpose of the Qualitative Content Analysis (QCA) is to further illustrate the need for curriculum materials that help teachers fully address the 8 Effective Teaching Practices. In order to advance my understanding of the presence of CGI principles and educative features in common classroom curriculum materials, I conducted a QCA on seven textbook teacher's manuals. The information gleaned from this study informed the development of my guidebook by illuminating the features and learning opportunities that were typically present as well as those that were less common or lacking entirely.

1.5 Positionality Statement

Teachers lack time for both planning and professional development (PD) and do not have much time built into their days with students to independently study a math curriculum or instructional practice. My district, USD 497 in Lawrence, KS, includes 6 professional development days into our calendar each year. This time includes hours of mandatory training that are non-instructional (e.g., school safety). My district also does PD on Wednesday afternoons but how this time is used is also planned out by the district or principal with very little choice time for teachers. When planning a sequence of elementary teacher training, some schools also tend to overemphasize literacy instruction with reading being the primary focus. There have been years that I was not offered a single minute of mathematics professional development by my school. Therefore, I argue that most professional development can and should happen in the classroom with students present.

Part of the definition of CGI is that it is not a prescribed curriculum. One of the tenets of the program is that teachers are the decision makers and they decide the next course of action based on how their students are solving problems and talking about problems. It has always been a point of consideration for me that I am creating a curriculum product for a program that is not necessarily supposed to be a curriculum. However, what I am creating is what I would call a "guidebook" and it provides a starting point for teachers and a map of how this could look and could be done. It will be made clear in the product that this is support for beginning the program. The word problems provided are to ease the burden on the teacher of knowing some beginning problems aligned to their grade level expectations. The solution strategies provided may or may not be used by their students. The discussion guide will be more likely used as a suggestion than a script as there is no way to predict what thoughts students will share about another's work. I want to give a teacher everything they need to try out the method and feel successful. I hope they do and that at the end of that year, they can hand the guidebook to another teacher because they have come to know and understand the principles of the program for themselves.

1.6 Professional Development Guidebook with Educative Features

A CGI Curriculum Guidebook with embedded educative features will be significant to elementary teachers because it can provide new learning and ongoing practice opportunities that are embedded into daily instruction with students without adding hours to their workweek. Educative features are text features in a teacher's manual that are intended to support the learning of the teachers in addition to student learning (Land et al., 2015). Educative features could include examples of exemplary student work, common student misconceptions and explanations and/or rationales for instructional strategies or any other feature that is intended to support the teacher to learn new instructional strategies or deepen their content knowledge.

Teachers need resources that will provide cognitively demanding, standards-aligned activities for students that will also help them to understand their students' thinking and improve their ongoing instructional practices. My CGI Guidebook will include many educative features including those found to be lacking in the curriculum resources included in the Content Analysis.

1.7 Research Questions

Most teachers are provided with curriculum materials to use for teaching mathematics. These materials vary in their presentation of concepts, the theories that underlie their program and their level of support and guidance for teachers to implement new instructional strategies. In order to determine the need for a CGI Guidebook with educative features that help teachers to implement CGI within the framework of the "5 Practices," the following questions were identified:

- To what extent does the resource provide student learning opportunities consistent with the principles of CGI and the 5 Practices?"
- To what extent does the resource support teachers to understand the principles of CGI and the 5 Practices?

These questions informed the development of the coding framework used to code the textbook teacher's manuals. Further, this data was used to inform the development of the guidebook. These questions guided the research objectives which were to (1) examine the teacher's manuals for the presence of principles related to CGI, (2) examine the teacher's manuals for the presence of the 5 Practices, (3) gauge the extent to which teachers were provided with support to understand and implement these principles.

1.8 Limitations

Supplemental teaching materials are very widely available to teachers and even include lessons and resources found online on websites such as Pinterest and Teachers Pay Teachers. It would not be possible in the scope of this project to do QCA on the myriad resources available digitally and otherwise. Therefore, I chose to analyze some of the largest and most popular core curriculum products that many districts have adopted knowing those are in the classrooms of many teachers.

My coding framework was designed to find features of Cognitively Guided Instruction, the 5 Practices for Orchestrating Productive Mathematics Discussions intended for students and educative features to help the teacher better understand those teaching strategies and concepts. The publishers of the curriculum products analyzed made no claim to be promoting those programs in their product or to be an educative curriculum for teachers. Despite this, I found many lesson features aligned with those ideas, but this was not necessarily the intent of the authors.

1.9 Definition of Terms

“5 Practices”: Book, “The 5 Practices for Orchestrating Productive Mathematical Discussions” (1) anticipating (2) monitoring (3) selecting (4) sequencing (5) connecting (Smith & Stein, 2018)

“8 Practices”: Document, “8 Effective Mathematics Teaching Practices” (NCTM, 2014).

Cognitively Guided Instruction (CGI): A professional development program and approach to teaching mathematics through posing word problems to students and asking them to solve without direct instruction (Carpenter et al., 1999).

CGI Guidebook: A curriculum product influenced by CGI with lessons to be used with students and educative features to aid the learning of the teacher.

Curriculum: The sequence of planned experiences where students learn concepts, practice skills and achieve proficiency in grade level standards (Rhode Island Department of Education, 2022).

Curriculum Product: A product created for teachers to assist them in teaching learning objectives.

Curriculum Resource: A product teachers have been provided by their school or that teachers purchase to use for teaching students.

Educative Curriculum Materials: Curriculum product or resource that contains educative features for the teacher (Land et al., 2015).

Educative Features: Features embedded in a curriculum product or resource that are designed by the author to help the teacher learn instructional strategies, content knowledge or knowledge about their students.

Qualitative Content Analysis: "A method of describing the meaning of qualitative material in a systematic way" (Schreier, 2012, p.1).

1.10 Summary

The purpose of this dissertation is to research the relevance and impact of Cognitively Guided Instruction, educative curriculum materials and the extent to which principles of CGI and the "5 Practices for Orchestrating Productive Mathematics Discussions" are present in common, large elementary math curriculum textbooks. The problem is that those principles can be used to address NCTM's 8 Effective Teaching Practices, but they are not always present in the math curriculum resources. This dissertation will culminate in a CGI guidebook for teachers that addresses this problem by providing teachers with lessons to use with students that include CGI word problems with educative features that help teachers implement the “5 Practices” and learn the CGI method while they teach.

Chapter 2 - Literature Review and Theoretical Framework

2.1 Review of Literature

In the following sections I discuss the theoretical framework and literature that informs this dissertation study. I begin the chapter by defining aspects of the theoretical framework, including: teacher knowledge, Cognitively Guided Instruction (CGI), educative curriculum materials and student engagement. Each aspect of the theoretical framework is defined and a description of how each part is operationalized in this study is explained. Finally, I provide a review of relevant literature, largely focusing on literature regarding CGI and educative curriculum materials.

2.2 Theoretical Framework

2.2.3 Teacher Knowledge

2.2.3.1 Pedagogical Content Knowledge

Shulman (1986) described Pedagogical Content Knowledge (PCK) as "subject matter knowledge *for teaching" (p. 9).* The emphasis on "*for teaching*" implies that a teacher must not only have knowledge of the subject they are teaching but also knowledge of how to deliver the specific content in a way that is comprehensible for students. For example, elementary mathematics teachers must have the knowledge to represent topics in ways that are most useful for students and to have alternative representations that would best match the learning level and current thinking of students. PCK also includes an understanding of how to uncover student thinking and respond constructively, taking into consideration what topics are more difficult for students to comprehend and specifically what makes these topics difficult.

2.2.3.2 Mathematical Knowledge for Teaching

In practice, teaching mathematics requires teachers to make a series of instructional decisions before, during and after lessons. Before a lesson, teachers plan problems, activities and experiences that they believe best address grade level standards and lesson objectives and will further student thinking and understanding. At this stage, teachers also plan which numbers students will work with in a problem and plan the questions they'll ask, with considerations for which students the questions will be asked to and when. Additionally, teachers must make decisions about procedures to teach (or not) and models they will use to explain concepts, including additional models if an initial model fails to support understanding.

During a lesson, teachers make in-the-moment decisions when they respond to students' errors through their deep understanding of those mistakes and their students' misconceptions. Deborah Ball and colleagues (2005) described the specialized knowledge used to make these decisions as Mathematical Knowledge for Teaching (MKT), an expansion of Shulman's (1986) PCK. In this work, she and her colleagues explained the critical need for teachers to have knowledge of both mathematics and their students. In order to anticipate and plan for misconceptions and develop children's number sense, teachers must have a deep understanding of their particular students' thinking (i.e., how students individually think about mathematical concepts and work with numbers and problems). This is important as students enter all elementary classrooms with prior knowledge (i.e., background knowledge and ways of thinking about mathematics concepts) and it is critical that teachers have strategies to uncover and utilize this prior knowledge throughout instruction.

2.3 Cognitively Guided Instruction (CGI)

2.3.4 Definition of CGI

Cognitively Guided Instruction (CGI) is one useful method for teachers to develop MKT. CGI in and of itself is not a traditional curriculum, as it does not ask teachers to follow a scope and sequence or curriculum of any kind. Instead, CGI is a pedagogical practice where teachers pose word problems to students and, rather than teaching or demonstrating solution strategies, ask students to solve the problems in any way they choose, often making available relevant manipulatives (e.g., counters or Unifix cubes) for students to use. After students have solved the problem, sometimes in multiple ways, the teacher selects students to share their own solution strategy with the rest of the class. The teacher's role is to then facilitate a class discussion by posing questions that will help the class make sense of students' shared strategies and draw mathematical connections between the various strategies shared in the session. CGI does not involve explicit instruction on the part of the teacher and students are not taught number facts or strategies to use for specific problems. Furthermore, students are not taught to look for keywords, instead they are asked to make sense of the action in the problems and then decide for themselves the operations that make most sense to use. This is what makes CGI a unique and nontraditional approach to teaching and learning mathematics.

The learning and knowledge construction during CGI is not just for the students. It is also meant to develop teachers' knowledge of student thinking. CGI began as research on the ways in which mathematics instruction is impacted by the teacher's knowledge of student thinking (Carpenter et al., 1989). By watching their students solve the problems and posing questions about student strategies, teachers come to understand how their students think about the problems and how they make sense of and use the numbers involved in the problem. Based on these experiences, teachers are then able to pose future problems that extend the thinking of their students.

2.3.5 The Role of Social Constructivism in CGI

Social Constructivism is a theory of learning that highlights the role of social context in students' knowledge construction. Further, a classroom attending to social constructivism has students working as a community of learners, building and deepening their understanding of concepts through autonomy and engagement in classroom discussions with each other and the teacher (Watson, 2001). Specifically, a social constructivist teacher might have students problem solving together, participating in problem-based learning, and participating in small- and whole-class discussions (Akpan & Beard, 2016).

CGI is a daily, highly social constructivist teaching method that allows students to come together, see how others think, and draw connections between their own thinking and the thinking of others through the teacher facilitation of a discussion. In elementary school, for example, these experiences with CGI allow students to build their own understanding of our number system and use knowledge to solve future problems. Additionally, CGI classrooms invite all students to be part of a classroom mathematics community by trying out their own solution strategies, presenting them to other students, and engaging with the ideas of others instead of watching and repeating a teacher's demonstration.

2.3.6 CGI Problems and Problem Types

When most children come to school in kindergarten or first grade, they can typically solve Join-Result-Unknown problems consisting of numbers less than 10 (Figure 2.1) without being taught a specific procedure or skill. In fact, young children tend to first directly model the relationships found in the problem using drawings, counters, or their fingers. This self-modeling (i.e., intuition and drawing on current understanding of number rather than adding strategies or number facts) is how students are able to solve problems without instruction. In this way, word problems with an understandable and relatable context affords children the autonomy needed to build an understanding of mathematical concepts. Furthermore, by allowing students to solve problems like this in their own way teachers are provided insight into student thinking which can inform next steps for instruction with students and furthers their own knowledge of student thinking.

Figure 2.1. Cognitively Guided Instruction: Story Problems. Smith & Smith (2008-2010).

Cognitively Guided Instruction: Story Problems

Join Result Unknown **I** Leon had 9 colored pencils. His mother gave him 5 more. How many colored pencils does Leon have now? 9 + 5 = $\square$	**Join Change Unknown** **III** Kevin has 7 dollars. How many more dollars does Kevin need to have 11 dollars altogether? 7 + $\square$ = 11	**Join Start Unknown** **IV** Eve had some pebbles in her pocket. She found 8 more pebbles and put them in her pocket. Now she has 14 pebbles in her pocket. How many pebbles did Eve have to start with? $\square$ + 8 = 14
Separate Result Unknown TJ had 13 chocolate chip cookies. At lunch she ate 5 of them. How many cookies does TJ have left? 13 - 5 = $\square$	**Separate Change Unknown** Eleven children were playing in the sandbox. Some children went home. Now there are 3 children left playing in the sandbox. How many children went home? 11 - $\square$ =3	**Separate Start Unknown** Max had some money. He spent $9 on a video game. Now he has $7 left. How much money did Max have to start with? $\square$ - 9 = 7
Part-Part-Whole- Whole Unknown Fourteen girls and 6 boys were playing soccer. How many children were playing soccer altogether? 14 + 6 = $\square$	**Part-Part-Whole Part Unknown** Connie has 12 marbles. 5 are red and the rest are blue. How many blue marbles does Connie have? 5 + $\square$ = 12 OR $\square$ + 5 = 12	
Compare Difference Unknown **III** Willy has 15 crayons. Lucy has 7 crayons. How many more crayons does Willy have than Lucy? 15 - 7 = $\square$	**Compare Quantity Unknown** Coleman has 11 books. Kevin has 6 more books than Coleman. How many books does Kevin have? 11 + 6 = $\square$	**Compare Referent Unknown** Juan has 13 stickers. He has 4 more stickers than Angie. How many stickers does Angie have? 13 - 4 = $\square$
Multiplication Ms. Jenkins bought 7 boxes of cupcakes. There were 4 cupcakes in each box. How many cupcakes did Ms. Jenkins buy? 7 x 4 = $\square$	**Measurement Division** Bart has 24 pencils. They are packed 6 pencils to a box. How many boxes of pencils does he have? $\square$ x 6 = 24	**Partitive Division** Megan has 15 cookies. She put the cookies into 5 bags with the same number of cookies in each bag. How many cookies are in each bag? 5 x $\square$ = 15

K: Levels I, II **G1: Levels: I, II, III** **G2: Levels I, II, III, IV** S.Z. Smith & M. E. Smith (2008-2010); Adapted from Carpenter, et al. (1999)

2.3.7 CGI as Teacher Professional Development

As well as being an instructional strategy, CGI is also a teacher professional development program. It began as a series of studies where researchers sometimes worked directly with teachers, providing them insight into student thinking at the age level they worked with and measuring the extent to which this new knowledge was applied to their instruction (Carpenter et al., 1989). As teachers pose word problems (Figure 2.1) and watch students solve, they uncover how students think and use their students' thinking to drive further instruction. For example, with knowledge of student thinking, teachers can write more cognitively appropriate problems for students that are tailored to students' current level of understanding. Then, over time, students develop mathematical knowledge and number sense to solve different and increasingly complex problems. During this process, teachers build their knowledge of student thinking and develop their ability to pose future problems and questions that will help their students continue to build onto their existing knowledge. Thus, teachers engaged in the enactment of CGI are experiencing a construction of knowledge (e.g., PCK and MKT) right along with their students' development of mathematical knowledge. Overall, teachers build knowledge about the thinking of their students, mathematical concepts and the myriad ways problems can be solved that can be used in-the-moment with a current group of students and year-after-year with future groups of students.

2.3.8 How CGI is Operationalized in this Study

There are a few CGI resources available to help teachers learn about this method, but they are likely insufficient for many teachers to begin regularly implementing CGI in their classrooms. Thus, there is a need for a more detailed and structured presentation of the program for a teacher brand new to this type of teaching. The most common and first book on CGI is

Carpenter and colleagues (1999), *Children's Mathematics: Cognitively Guided Instruction*. This work is written for teachers and explains the thesis of CGI–that children come to school with lots of intuitive knowledge about numbers and they can construct workable solutions to problems without the instruction in algorithms and number facts common in some of the most utilized curriculums and teaching resources. The book explains the role of the teacher and students in CGI, different problem types, solution strategies with examples for various operations, and tips for getting started in the classroom. It also includes a DVD of classroom episodes (i.e., clips of classroom learning segments where CGI is implemented) to further illustrate the ideas discussed in the book. While it is full of incredibly useful information (and I have purchased every book they have written), it is not likely to be enough to get a teacher to incorporate this into their daily instruction, especially when we consider that this type of teaching might require a fundamental change in pedagogical orientation on the part of the teacher. In fact, I did not open this book until I completed a year of teaching CGI, when I understood the structure of CGI and was already interested in diving a bit deeper into the details and theory of the practice.

Through this dissertation, I am creating a mathematics curriculum product for First Grade teachers that could be used daily to supplement the core curriculum likely provided by their school district. My product will provide a CGI problem per day. The problems will be written based on the Common Core Standards for Grade 1 as well as CGI research that has demonstrated that young children can often work with larger numbers and more challenging problem types than typically presented at their grade level (Carpenter et al., 1993). Many core curriculum products introduce operations in a sequential order, with students learning about addition before learning about subtraction in first grade. Additionally, Join-Result-Unknown problems are typically focused on for long periods of time before the unknown is presented in other places, as in the case for current math curriculums like Math Expressions (Fuson & Houghton Mifflin Harcourt Publishing Company, 2018). However, as in CGI classrooms, the problems in the curriculum product I will develop will not be presented in a sequence and students will be introduced to a wide variety of problems right away in the beginning of the year, allowing teachers to see their ability and thinking right away.

2.4 Educative Curriculum Materials for Teachers

2.4.9 Definition of Educative Materials

In addition to the daily problems intended for CGI instruction, my curriculum product will include educative features for the teacher that are based on CGI and the 5 Practices. Educative Curriculum Materials (ECM) are curriculum materials that include educative features intended to support teacher learning (Land et al., 2015). These educative features are embedded into lessons and may include, for example, questions to consider posing to students and examples of student thinking or misconceptions that may arise in response to the question posed. According to Land and colleagues (2015), it is important that developers of ECMs "strongly communicate their intent" (p. 26) because educative features may not always be read in educative ways. Teachers need to be made aware of exactly what features are meant to be educative and how they should go about reading and using those features to inform their learning and practice.

Furthermore, Quebec-Fuentes & Ma (2018) conducted a comprehensive literature review of educative curricular materials which resulted in their framework for Teacher Learning Opportunities in Mathematics Curriculum Materials. This framework includes an emphasis on supporting teachers' understanding of student thinking and will be an integral part of the curriculum product from this dissertation. When teachers pick up curriculum material, it is often with the intent to make lesson plans for students. As discussed, rich, educative features targeted for the teacher may be present in the text but teachers' attention must be explicitly drawn to those and their intention to read the material in an educative way must be set prior to reading. According the Quebec-Fuentes & Ma, the material must also contain the following "dimensions" that will support teachers in developing these discipline specific types of knowledge: "(1) mathematics content knowledge for teaching, (2) teacher knowledge of student thinking in mathematics, (3) teacher knowledge of disciplinary discourse in mathematics, (4) teacher knowledge of assessment in mathematics, (5) teacher knowledge of differentiated instruction in mathematics, (6) teacher knowledge of technology use in mathematics, and (7) teacher knowledge of mathematical community" (p. 357).

2.4.10 How the features will be used in this study

In the curriculum product resulting from this dissertation, I will first clearly explain the intent of ECMs and explicitly state why and where educative features are embedded throughout the product. Teachers will see features in each lesson that will help them understand each of these components of CGI and the 5 practices embedded in the lessons. My ultimate goal is that after a year of using this curriculum product, teachers will no longer need it because they have come to understand how enact CGI by writing problems, observing student thinking, selecting students to share their thinking, and prepare for the next day's CGI session with a new problem tailored to the current thinking of their students.

My curriculum product will also include a rationale for each problem that is selected to drive a lesson. This rationale will briefly explain the problem type, context and chosen numbers, including why the problem is appropriate for 1st grade students at this particular point in the year. The product will include a routine to check for student understanding by having students retell the problem as well as answer a comprehension question about the numbers being used in the problem. I will give many examples of strategies that teachers might see their students using to solve the problems (e.g., direct modeling, counting, derived facts; Carpenter et al., 1999). I will present the student strategies in order of sophistication and explain the sequence. I will also provide suggestions and explanations for which strategies teachers might choose to share, in what order and why. Finally, I will provide questions teachers can use when facilitating a discussion about the problem and student strategies.

2.5 Student Engagement

2.5.11 Definition of Student Engagement

The enactment of CGI described thus far is also designed for supporting student engagement in mathematics learning. Bodovski and Farkas (2007) described student engagement as having three components: learning-related work habits (e.g., active participation, persistence at tasks), cognitive behaviors (e.g., attention, problem solving), and emotions (e.g., enthusiasm, interest). This aligns with the enactment of CGI as students are called on to actively participate by doing the math themselves and to participate in a class discussion, where they are presenting their thinking and engaging with the thinking of others. Students have opportunities to think creatively, which may enhance classroom equity (Luria, Sriraman & Kaufman, 2017), and are expected to persist in solving each problem. This is evident as students are choosing their own solution strategy, following through until they believe they have the answer and then preparing to explain their thinking and process to their peers. Additionally, the teacher also works to ensure that students are paying attention, actively listening, and comprehending the problem context before beginning to solve. Furthermore, during CGI enactment, students work to solve the problems while the teacher acts as facilitator, increasing students' positive emotions through feelings of math competence and confidence (Moscardini, 2010).

2.5.12 Why student engagement matters

It is critical that all students have access to rigorous, engaging, high-quality mathematics instruction that pushes students to think deeply about the content. Furthermore, "differential access to high-quality teachers, instructional opportunities to learn high-quality mathematics, opportunities to learn grade-level mathematics content, and high expectations for mathematics achievement are the main contributors to differential learning outcomes among individuals and groups of students" (NCTM, 2012). Allexsaht-Snider and Hart (2001) also suggest that students feeling a sense of belongingness and engagement leads to equitable outcomes and one practice that may promote this is allowing students to develop their own strategies for problem solving. Carpenter and colleagues (1999) emphasized that all students are capable of developing their own solution strategies, which implies that all students, regardless of race, gender, language or family income, must be given the opportunity to think creatively and problem solve independently (i.e., given the opportunity to engage with high-quality mathematics and mathematics instruction). This type of engagement, which is coupled with meaningful involvement in class discussions, is a way to achieve the belongingness and engagement necessary to foster equity.

2.5.13 Supporting Engagement Through Discourse and the 5 Practices

The Common Core Standards for Mathematical Practice call for students to be able to construct viable arguments and critique the reasoning of others (National Governors Association Center for Best Practices & Council of Chief State School Officers, 2010), which is supported during CGI when students solve problems and share their thinking and solution strategies during class discussions. Additionally, productive class discussions are associated with student achievement, especially through high-level dialogue where students make sense of and build off of other students' ideas (Webb et al., 2014). However, the key here is that teachers must notice and consider who is engaging in discussions and how they are participating. Thus, teachers must think about how they can create opportunities for students with varying needs to fully participate in the lesson and discussion. Teachers must also notice who is most active and implement strategies that will support students who are less engaged. All of which is key to successful implementation of CGI. Bishop (2021) refers to this kind of noticing and intervening as *responsiveness*– "the extent to which student thinking is acknowledged, elicited, taken up, and used as the basis for subsequent mathematics instruction" (p. 470). However, responding to student thinking in-the-moment and using it to push student thinking further towards pedagogical goals (the highest level of responsiveness in Bishop's framework) is no simple task.

To address this, the "5 Practices for Orchestrating Productive Mathematics Discussions" (5 practices; Smith & Stein, 2018) provides a detailed framework to help guide teachers in planning for and implementing lessons that provide a space for responsive teaching and facilitating meaningful discussions around mathematics problem solving tasks. Next, I will briefly outline the 5 practices (anticipating, monitoring, selecting, sequencing, connecting) and illustrate how teachers might use these practices to structure CGI enactment.

Anticipating. During the planning of a lesson about a mathematical task, Smith and Stein (2018) call on teachers to actively envision how their students might approach the problem(s) they are being asked to solve. This allows teachers to anticipate student strategies and plan questions they might ask to check for understanding and uncover potential misconceptions. Anticipating strategies before the lesson also allows the teacher to consider the many ways that the problem can be solved. One way to do this is by solving the problem themselves in as many ways as they can think of and keeping logs of previous solution strategies used by students. The curriculum product from this dissertation will include many examples of possible solution strategies that can be used by the teacher in different ways and one of those is to review in advance and consider which strategies they believe their students might use based on their knowledge of student thinking. Another reason to present teachers with these strategies is so that they can see a variety of student developed strategies that they likely have never been exposed to if they are not familiar with CGI or other student-centered teaching methods.

Monitoring. Monitoring occurs after the teacher has presented the problem with the class and instructed them to begin solving the problem. In this phase of the 5 practices, the teacher moves around the classroom observing students as they work and asking them assessing and advancing questions about their work. Teachers may notice some strategies that are similar or the same as those they considered in the anticipation phase. Teachers may also notice unexpected strategies and are encouraged to question students to uncover their thinking and make sense of the strategy students are using. For instance, when using word problems targeting addition and subtraction strategies, teachers will pay attention to which students are using direct modeling, counting, derived fact strategies, or invented strategies. They may also observe misunderstandings and counting errors which will need to be addressed. During this time, teachers may use a monitoring chart or anticipatory framework to note which students are using particular strategies.

Selecting. During and after monitoring, the teacher makes decisions about which students are going to share their strategy with the whole class. The teacher has likely already planned for which types of strategies they intend for students to share but may also have noticed some other strategies that are important to bring to the whole class in order to meet the goal of the lesson. However, when selecting student strategies, teachers must think critically about who to ask to share and why as this allows them to not only manage the outcome of the lesson but to call attention to certain ways of thinking that are highly likely to be repeated by other students during future lessons. To show teachers how to select strategies, my product will highlight 2-4 of the strategies and explain why those are appropriate for 1st graders at a given point in the year and how they support the objective of the lesson.

Sequencing. Not only must teachers select appropriate strategies to share with the whole class, but it is also best to present them in a strategic order that helps attend to the goal of a given lesson. For instance, a teacher may consider having students present a more concrete strategy (e.g., derived facts) before a more abstract strategy (e.g., invented strategies or compensation) to help facilitate students making connections between approaches. This is because having some concrete support for understanding can often lead to more sophisticated strategies. For example, in the beginning of 1st grade I will highlight a "count all" strategy followed by a "counting on" strategy for solving addition word problems. This is because the "counting on" may be less familiar for many students and the "count all" validates the more efficient strategy of "counting on" and provides the support necessary for facilitating a discussion that will build understanding. Each lesson in the curriculum product will include a suggested sequence as well as brief rationale like that given for "counting on" and "counting all."

Connecting. Once the selected students have explained their strategies and the teacher has drawn or displayed these strategies large enough for the whole class to see, it is critical that the teacher helps facilitate student conversations to draw mathematical connections between the different strategies. Being able to facilitate these connections is why it was so important to have a clear goal, anticipate strategies, and thoughtfully select strategies that would be shown to the whole class. Additionally, during the monitoring phase of the lesson, the teacher likely heard ideas and misconceptions that will be addressed during this connecting discussion and may have specific students in mind to call on to explain certain concepts or to consider their own misunderstanding by answering a question about another student's strategy. The curriculum product from this dissertation includes support for teachers to think beyond having students just demonstrate different ways of solving, but rather to make this conversation purposeful for a particular goal of the lesson. Furthermore, each lesson includes a series of potential questions and prompts to pose to students that will help guide the discussion around 2-4 strategies highlighted. This helps illuminate for teachers the similarities and differences between student strategies that may look quite different at a glance but in fact have mathematical connections that help deepen student understanding.

2.6 Literature Review

While 4th grade U.S. mathematics scores did increase slightly on the NAEP exam between 2017 and 2019, still only 41% of 4th graders were considered proficient in 2019 (National Center for Education Statistics, 2019). According to achievement levels from the National Center for Education Statistics' website, "NAEP Proficient represents solid academic performance for each grade assessed. Students reaching this level have demonstrated competency over challenging subject matter, including subject-matter knowledge, application of such knowledge to real world situations, and analytical skills appropriate to the subject matter" (Ward et al., 2006).

In Kansas, the average 4th grade score was 239, which is not significantly different from the national average of 240. On Kansas' own annual mathematics assessment, created annually by the Kansas Assessment Program (KAP) and better known as the *state test*, 50.25% of 3rd graders, 36.08% of 4th graders and 31.91% of 5th graders received a score of 3 or above on a 4-point scale (Kansas Report Card, 2019). According to KAP, "Level 3 indicates that a student shows an effective ability to understand and use the mathematics skills and knowledge needed for Postsecondary Readiness" (Kansas Report Card, 2019). More locally, in Lawrence, Kansas, 52.74% of 3rd graders, 40.65% of 4th graders and 38.81% of 5th graders achieved a performance level 3 or higher (Performance Level Reports, 2019-2020). This indicates that roughly 40 to 50 percent of local third, fourth and fifth graders understand and apply mathematics at grade level. This is a problem that indicates instruction needs to change in order to help many more students achieve a depth of understanding that will give them access to grade level mathematics concepts. These scores indicate a need for more teachers to meaningfully implement NCTM's 8 practices, of which CGI can assist in doing as was detailed previously. The following sections will further illustrate CGI's effectiveness as a mathematics teaching method and describe how educative curriculum materials can support teachers' self-directed learning.

2.6.14 Questions Guiding the Literature Review

The purpose of this dissertation is to create a CGI Guidebook so that teachers will have an opportunity to learn new instructional strategies, improve their Mathematical Knowledge for Teaching and become better equipped to give all students access to the high-quality instruction needed to raise the level of achievement in mathematics. In order to determine how CGI can support teachers to implement effective mathematics teaching practices, the following questions are used to guide this review of literature:

- To what extent can CGI help to close achievement gaps, increase learning and engagement for *all* students?

- To what extent can CGI act as a tool for teacher change in beliefs and instructional practices?
- To what extent can teachers experience self-directed learning through educative curriculum materials?

2.6.15 Search Parameters

To begin this literature review, I initially conducted a search for the term, "Cognitively Guided Instruction." I only included journal articles and books that included all three words in that order and in the title or abstract. I wanted to limit this search specifically to the CGI program and research related to the original research by the team of CGI founders (Carpenter et al., 1999) and others who have researched CGI subsequently. Other search terms related to this first part of the literature review were "Cognitively Guided Instruction Effectiveness" and "Cognitively Guided Instruction Equity," as well as "Cognitively Guided Instruction Differentiation." Again, this part of my search was peculiar to CGI. A search for connections between CGI and equity and differentiation was conducted because of a foundational goal of implementing NCTM's 8 practices is that *all* students achieve high levels of mathematical understanding.

Additionally, given the nature of the curriculum product to also support teacher learning, I was also interested in "self-directed learning for mathematics teachers", the next search term used to review literature. Through this search, I learned the terms "educative mathematics curriculum(a/ar) materials" and used those as well, searching for those specific terms in titles and abstracts. Further narrowing the search, I chose to limit the search only to articles about educative curriculum materials for mathematics teachers. One of the main ideas of CGI and the intended curriculum product from this dissertation is that mathematics teaching has specific Pedagogical Content Knowledge requirements for teachers. I feel that educative curriculum materials for literacy, for example, could be structured much differently and would not be relevant to the product that I will develop.

The research cited below was found by using Kansas State University library's general search of all journals and Google Scholar. I also more specifically searched the "What Works Clearinghouse" website and the "Journal of Mathematics Teacher Education" for relevant literature.

2.6.16 Effectiveness of Cognitively Guided Instruction

CGI is an approach to teaching that I use in my classroom, but as I explained previously, it was first designed as a professional development program to support teacher learning. While there is limited evidence on professional development effectiveness in general, as it is extremely difficult to isolate the effects of any program on teacher learning (Wilson, 2013), CGI is one professional development program that may have a significant impact on student achievement. For example, in a multi-site cluster randomized trial, Schoen and colleagues (2018) found that after one year of CGI professional development, students in grades 3-5 showed significant gains in their understanding of fractions. Traditional fraction instruction in Grade 3 focuses on placing fractions on a number line; however, fraction instruction using CGI is different than traditional instruction in that students are taught through solving "equal sharing" problems, such as the following, *3 kids want to share 4 brownies so that each child gets the same amount. How much brownie will each child get?* (Empson & Levi, 2011). This problem represents an extension of whole number computation that students would have had experience with in lower grades to help students make sense of fractional parts.

Nationally, student achievement declines in mathematics between grades 4 and 8 (Mullis et al., 2016) and one possible factor contributing to that could be a lack of understanding of fractions. Though fraction language is used starting in early grades (e.g., partitioning half of a shape), fractions do not appear formally in standards until 3rd grade, under Numbers and Operations, and continues to develop in 4th and 5th grades (National Governors Association Center for Best Practices & Council of Chief State School Officers, 2010). Furthermore, students understanding fractions is an early predictor of success in high school mathematics (Siegler et al., 2012). Booth and Newton (2012) reiterate this point in their study of students' fractions understanding, finding that students' specific knowledge of fraction magnitude is critical to algebra readiness. More specifically, these authors found that students' understanding of *unit* fractions (i.e., fractions with a numerator of 1) is of critical importance and is a primary focus in 3rd grade fraction instruction.

Through the enactment of CGI, a deep understanding of the meaning of unit fractions can be built. In the example problem with 3 kids sharing 4 brownies, students might be led to give out a whole brownie and then split the last one into 3 equal parts. Most 3rd graders will call these "pieces" or something other than *thirds*. This is the opportunity to connect their inventive thinking with the mathematical content of the meaning of *thirds.* Students also are likely to compare the "pieces" to the whole brownie and understand that 3 of this sized piece is always equal to one whole. They will then build upon this understanding as they work with more unit fraction pieces in subsequent word problems and eventually compare different unit fractions to one another and understand that fractions are numbers that can be placed on a number line in a certain order. Thus, CGI enactment with fraction content allows students to start by using their own knowledge to make sense of unit fractions by splitting an object into pieces that they see as "fair" shares. This gives all students access to the work with fractions.

Another important gatekeeper for future math success, particularly algebraic reasoning, is understanding the meaning of the equal sign. However, enacting CGI has been shown to have a statistically significant positive effect on student understanding of the equal sign in 1st through 5th grades (Jacobs et al., 2007). For instance, many elementary (and even older) children struggle to find the unknown quantity in a problem like this:

$$8 + 4 = \_\_\_\_ + 5$$

It is common for students to see addition signs and add all 3 numbers together or to write 12 because they believe that the equal sign is a direction symbol that basically means *put down the answer* (Faulkner, 2009). In contrast, it is critical that students develop a relational understanding of the equal sign, rather than viewing it as an operational signal to carry out the calculation that comes before it (Jacobs et al., 2007). Understanding this is necessary for students to be successful in learning mathematical concepts related to algebraic reasoning.

Given that in CGI a teachers' attention is drawn to student thinking, it can be specifically drawn to their algebraic reasoning. In a study of professional development focused on children's algebraic thinking, Jacobs and colleagues (2007) engaged participating teachers in a professional development that focused on how algebraic reasoning could support students' understanding of arithmetic. One large component of this professional development was asking students to solve problems using their own strategies and engaging them in conversation to elicit their thinking about the numbers embedded in the problems. Students of teachers from this professional development scored significantly higher on the final written mathematics test than students in the classrooms of nonparticipating teachers at the same schools.

2.6.17 Creating Equity through CGI

In their Equity Position Statement, NCTM (2014) states that, "Creating, supporting, and sustaining a culture of access and equity require being responsive to students' backgrounds, experiences, cultural perspectives, traditions, and knowledge when designing and implementing a mathematics program and assessing its effectiveness." This implies that one way to address equity in mathematics education is by ensuring that all students, regardless of background, are presented with rigorous, high-quality instruction. NCTM (2014) goes on to state that, "Educators must have the knowledge, skills, and disposition necessary to support effective, equitable mathematics teaching and learning." One way to gain the knowledge and skills necessary to address the specific needs of the students in a classroom is to focus on *their* thinking and to plan instruction based on what *they* know and are able to do at present. Continuing, NCTM (2014) states that, "A firm commitment to this work requires that all educators operate on the belief that all students can learn." Allowing students to do the thinking and trusting them to have the knowledge and skills to apply their own solution strategies is one way to operate on this belief through teachers' instructional practice.

A critical first step in providing this type of instruction is that teachers are noticing their students' thinking and that they learn to plan instruction that is based on that thinking. Given that understanding students' mathematical thinking is different from understanding students' math ability, instruction that is focused on a students' approach to a specific task allows teachers to look for what a student *did* rather than their deficits and to focus on how students' current thinking could be strengthened and broadened (Jacobs et al., 2007). Jacobs and colleagues (2010) studied teachers' examination of student work in a collaborative group. In their study, the authors asked teachers to pose problems to students and bring the written work to the group session for group analyses. The goal of this professional development was for the teachers to work together to make sense of the mathematical ideas present in their students' strategies and to respond to children's ideas in ways that will support their understanding. They found, especially in their analysis of the prospective teachers group, that "professional noticing" of children's thinking is both challenging and not something that adults automatically know how to do. However, they also found that increased experience with children's thinking was associated with increased engagement with children's thinking (p. 181).

Further, mathematics classrooms consist of students with a wide range of abilities and teachers must plan and teach in ways that address their varying needs in efficient ways. The structure of CGI truly epitomizes the idea of differentiation that will support addressing these various needs. For example, Baker and Harter (2015) examined six CGI studies and found that, in all of them, teachers first considered their students' current thinking before teaching. This focus on using student thinking to drive instructional decisions is a different approach than teaching "the next lesson" in a prescribed textbook series and is one of the tenets of CGI, where teachers consider what the students are able to do and know to then create problems that will help students expand on their thinking. For instance, word problems are written specifically for the students in a particular class, based on their current knowledge and, in CGI classrooms, the emphasis is on strategies and processes rather than "the answer." Instead of collecting papers and marking them correct or incorrect, teachers are asked to look at the work of the student and ask themselves: *What did the student do to show understanding of the problem? Was the student able to represent the numbers? How did they go about using the counting strategy?* Thus, teachers enacting CGI in the classroom know each student as individual mathematicians and gain a strong sense of where they are personally in their math understanding to inform instruction (Battey & Chan, 2010).

Further, teachers who enact CGI assess and use assessments of students in untraditional ways. A more traditional model of assessment consists of teachers seeking out what a child does not know or cannot do in order to use as a starting point for remediation before new learning. However, CGI is an asset-based approach to assessment that draws teachers' attention to what the child *did* as a solution strategy rather than what they lack in understanding, meaning teachers enacting CGI understand that children come into the classroom with prior math knowledge and prior intuitive and informal mathematical knowledge that should be used as a starting point to build from (i.e., meeting students where they are) (Carpenter et al., 1999). For example, teachers may notice that a child *can* count on and count back with concrete materials, but they are not yet able to count in groups of ten. This knowledge may lead to an offering of problems that encourage the child to use tens when counting (Moscardini, 2014, p. 75).

CGI also involves a great deal of classroom conversation that presents opportunities to make connections between student thinking and academic mathematics vocabulary (Musanti & Celedón-Pattichis, 2009). This is of particular importance to non-English speaking students who are trying to acquire a new language as well as the necessary vocabulary to be successful in specific academic domains. Additionally, word problems can also be written using a context that students in a particular classroom understand and can be adapted using a context that students from other backgrounds can understand. This is of particular importance for non-English speaking students and students who are unfamiliar with a specific U.S. context, such as immigrants (Musanti et al., 2009).

Classrooms that employ CGI also *engage* students in math learning. Given that lessons planned for CGI are not structured for having students watch a teacher demonstrate a mathematical concept or skill, students are instead learning actively and through their deep involvement in the learning process. In a study with students with moderate learning disabilities, Moscardini (2010) found that all 24 participating students were actively engaged in the lessons using CGI. For instance, students remarked that, *"I like it instead of maths. It's good you get to use stuff*," and, "*It's easy 'cos I did a bus and I put windows in*" (Moscardini, 2010, p. 133). In this study, the students used strategies like counting with manipulatives and direct modeling using their drawings of the story context to deepen their understanding of mathematical concepts. Fifteen of the students in this study used only direct modeling to solve a given problem. Direct modeling, just like it sounds, is modeling the exact actions in a story using drawings or manipulatives (Carpenter et al., 1999). For instance, in a word problem a child may have 18 cookies and they get 13 more. If students are asked how many total cookies they have now, a direct modeler would get out 18 counters or draw 18 cookies, then draw or use 13 more counters and count the entire group, modeling the actions that happened in the story. This is quite a different approach from a student who combines the tens and the ones, because in the story no such thing happened. However, having the option to directly model gives students, including those with disabilities, a seat at the table knowing their strategy is valid. This is important because when we teach a very limited range of strategies to students and require them to use those specific strategies, many students no longer have the opportunity to solve this problem at all as they do not understand a specific strategy and the teacher is not valuing their methods. Thus, CGI is a real opportunity to give all students a voice to share where they are and by facilitating connections between strategies to expand their understanding for future problems.

Overall, creating truly equitable classrooms requires teachers to differentiate instruction in various ways and the structure of CGI allows for this to happen more easily when teachers consider assessment in a way that focuses on building from students' current understanding. Doing so allows for more students to have access to problems if teachers are looking for what they *can* do and a student who used a less efficient strategy or miscounted still may have shown certain mathematical understanding related to the task that can contribute to the overall lesson goal. An even more intentional differentiation approach within the CGI framework is to offer students choices about the numbers used within the same story problem (Empson & Levi, 2011; Phelps, 2012). The numbers for two (or more) similar problems can be chosen strategically, at various levels, and students may decide which problem they would like to solve. In mathematics, this is often referred to as a *parallel task* (Small & Lin, 2010). Alternatively, a teacher may assign different numbers to different students based on the teachers' knowledge of student thinking. This gives all students access to the same problem and allows them all to work within the same problem type while working with numbers that are more suited for their current needs, making the whole class discussion something all students can contribute to and make connections during.

2.6.18 CGI as a professional development tool and agent of change for teachers' beliefs

Mathematics teachers' beliefs are related to their instructional decisions. The goal of CGI is to provide professional development for teachers by helping them to notice, understand and utilize their students' thinking (Fennema, 2000). With this goal in mind, this section examines studies on teachers' changed instruction and beliefs after implementing CGI in the classroom. Most CGI studies involve assessments of teachers' who underwent a CGI professional development. In most of these studies, teachers collaborated with the researchers whose roles were both the researcher and guide for participants learning about CGI. As a guide the researcher might have been modeling CGI lessons (e.g., Bowman et al., 1998, Musanti & Celedón-Pattichis, 2009) or supporting groups of teachers as they analyzed student work, engaged in discussions and planned instruction based on their students' thinking (e.g., Fennema et al., 1996; Jacobs, 2007). CGI is a powerful tool for making student thinking visible because students are working through problems without first learning or seeing any solution strategies. The work teachers see is all student thinking. This presents a lot of material for teacher analysis and learning.

Teachers with a range of experiences, skills and beliefs can be taught to attend to children's thinking and use it for instruction and for some teachers this can alter both their instruction and their beliefs. For example, Fennema and colleagues (1996) conducted a longitudinal study of teacher beliefs and measured teachers' instruction on a 4-point scale indicating the extent to which the teacher's instruction attended to CGI (p. 412). At Level 1, teachers provided very few or no opportunities for children to problem solve on their own or to share their strategies. At level 2, teachers provided limited opportunities for children to problem solve but attend to their thinking and utilize it in limited ways. At level 3, teachers gave students opportunities to problem solve and they did elicit student thinking and attended to their ideas but did not yet use it to drive instruction. At level 4, teachers allowed students to solve a variety of problems, they elicited students' thinking, and they gave students opportunities to share with others. Further, level 4 is differentiated by 4A and 4B, where 4A indicates teachers use more whole class knowledge to drive instruction and 4B indicates teachers use knowledge of individual students to drive instruction. The authors also developed and used a *Cognitively*

Guided Beliefs scale to measure beliefs that correspond to the 4-point scale of attending to CGI. By the final year of this study (year 4), 18 out of 21 teachers' instruction had become more "cognitively guided," where the beliefs of the 18 teachers were also shifted to be more cognitively guided in the final year compared to the first. In fact, the beliefs of over half the teachers were at level 4 by the end of the study.

The work of Fennema and colleagues (1996) indicates that learning about the principles of CGI and using those principles in the classroom had a significant impact on the instructional decisions and beliefs of teachers. Bowman and colleagues (1998) extended this research by expressing the need for long-term support for teachers as they try to implement CGI. Using a previously validated beliefs scale (Carpenter et al., 1989), these authors measured the beliefs of the teachers before and after they participated in a three-day CGI workshop. Although the overall belief scores increased from the beginning to end of the initial workshop, the authors re-measured teachers' beliefs and found that they declined after one year of teaching. This decline occurred despite extensive support from university educators visiting participants monthly and project staff visiting with participants once per semester. It took two years of implementation for belief scores to fully recover, highlighting that CGI training and implementation are a process not achieved quickly or without ongoing support.

Research has shown that CGI is not designed as a one-time workshop training. Rather, to achieve generative change requires a deep acquisition of knowledge and teaching skills that are developed over time through extensive practice, reflection, and ongoing adjustment. Furthermore, teacher change is more than just learning a discrete instructional practice and using it with students. Instead, teacher change is an ongoing process that can be adjusted, grown and applied with a wide variety of students, where teachers build onto their understanding of student thinking through practice (Franke et al., 1998). This suggests that teachers would benefit from professional development that happens in their classrooms, with continual opportunities for practice and this is a reason I propose an educative CGI Guidebook. Additionally, many CGI studies suggest that groups of teachers working together to discuss student solution strategies and the process of teaching through CGI is important, as there is evidence that this type of collaboration leads to long-term instructional change if teachers are engaged in inquiry on their students' mathematical thinking together. Thus, teachers must have opportunities to see and analyze their students' thinking beyond examining student work for correctness, which implies that talking with students and eliciting details about their thinking is critical for teachers to engage in practical inquiry (Steinberg et al., 2004; Franke et al., 1998). However, just as teacher change and learning can occur with other teachers, the components of CGI provide opportunities for teachers to engage students in rich discussion about their thinking and to learn more about how they think about problems and work with numbers. Through practice, teachers can learn to listen for details that help them understand the nuances of how children think and problem solve. They can bring this knowledge back to a teacher group to further unpack and make sense of it.

2.6.19 Self-directed learning through CGI Educative Materials

So far, I have discussed CGI studies that mostly occurred through a CGI professional development with groups of teachers, but schools may not necessarily offer this type of training nor may these professional developments be available locally. However, as previously noted, CGI training that impacts teachers' beliefs and practices requires ongoing engagement that is highly connected to teacher practice. Thus, another option for teacher professional learning about CGI could be through educative curriculum materials. This type of self-directed learning can be more accessible to teachers, as it can be done independently.

Educative curriculum materials are designed to support teachers as well as students in their learning (Land, Tyminski & Drake, 2015). Collopy (2003) found that curriculum materials can be an effective professional development tool for some teachers. In her study, teachers used TERC *Investigations (1995-1996)* which is a curriculum with some strong similarities to CGI, as it emphasizes problem situations and student-invented solution strategies. To support teachers in their own learning, TERC units, lessons, and activities include sections informed by questions teachers had during field tests of the TERC curriculum, such as, “About the Mathematics in This Unit” and “Teacher Notes.” It also has “Dialogue Boxes" for each activity which are a sample dialogue of a class discussion. Additionally, in TERC, class discussions are a major component. One of the two participant teachers in Collopy’s (2003) study experienced a major shift in instruction after closely following the materials for a school year, where the teacher became much more open to multiple means of solving problems and allowing the students to come up with those strategies. The other teacher in the study did not experience such a shift and the contrast highlighted the impact of beliefs as teachers undergo any structure of professional learning. The teacher who changed sought to make her mathematics teaching more fun and “relaxed” (p. 304) and to provide activities with less reading while the teacher who did not experience as much instructional change had very strong viewpoints on procedures and quick, correct answers as the meaning of mathematics (p. 295).

Both educative and reform-oriented curriculum materials can also prompt teachers to try out instructional strategies that they otherwise may not have been exposed to. “Reform-oriented” texts are similar to educative curriculum materials in that they are intended to help teachers make instructional shifts albeit usually specifically to student-centered practices. Remillard (2000) also studied the effect of reform-oriented textbook adoption and found that for teachers who tried something new, if they persevered even when it was not immediately successful, they often adopted new, more progressive teaching strategies. For example, one 4th grade teacher tried a “Problem of the Day” that was found in the new text. Implementing the problem of the day was a beginning of class activity that students were to do independently without first seeing teacher instruction. At first, the teacher found the problems to be unusual, as they felt they were too difficult for the students who were not used to problems structured in these ways. The teacher often ended the problem of the day by showing the students their method, typically involving a computational algorithm. But by halfway through the year, their perspective on the problems of the day change significantly. The teacher continued using the problems despite the lack of success in the beginning, as they described wanting to give the text a “fair try” (Remillard, 2000, p. 336) and found that students improved and gained confidence and eventually wanted to share and compare strategies. While both teachers in the study changed either their instruction, understanding of mathematical concepts, or both, it should be noted that they also missed opportunities to experience reform-oriented instruction. How teachers read the new curriculum materials and what they chose to read in the text was shaped by their personal views about teaching and learning mathematics, again calling attention to the importance of *how* teachers’ read ECMs.

Land, Tyminski, & Drake (2015) found that pre-service teachers most likely will not read ECMs in educative ways (p. 24) and went onto suggest that the educative features in curriculum materials could be made more explicit through curriculum developers “strongly” communicating “their intent” (p. 26). In fact, in my first 3 years of teaching I was a user of TERC *Investigations* and I did not know that it was intended to be educative for the teacher until I undertook this literature review. This implies that teachers may benefit from the author’s purpose being more explicitly stated. Teachers would also learn more from highly specific examples. Ball (1996) suggests "concrete examples of what student work might look like" as well as potential reasons for that work (e.g., rationales for student thinking) and ideas about how other teachers have addressed this type of student work (i.e., next steps with students) (p. 8). The TERC curriculum did not necessarily supply the "specific prompts and structured engagement" suggested by Drake, Land and Tyminski (2014). Thus, we know teachers are not likely to naturally read educative curriculum materials for professional development, but these curriculum materials need to be explicitly structured to engage teachers in educative ways.

Self-directed learning is one type of professional development that has certain benefits that are not necessarily present in traditional district directed or mandated PD. Self-directed learning could be networking online (e.g., participating in educator specific social media), online research, reading educative materials or any means of seeking out skills and information relating to improving one's teaching practice. A feature of self-directed learners is that they are personally motivated and this motivation is internal and occurs for a variety of reasons. For instance, Mushayikwa & Lubben (2009) found that when engaging in self-directed learning, teachers are motivated by seven main concerns: perceived professional identity, career development needs, theoretical and content knowledge, practical knowledge and professional skills, pedagogical content knowledge, professional networking and benefits to themselves and their students. The reality of many schools is that it is not always possible to provide meaningful professional development that will help teachers to address these concerns (e.g., Collopy, 2003; Davis & Krajcik, 2005; Mushayikwa & Lubben, 2009). School districts do not have unlimited funds and some districts, especially rural ones, are not located in close proximity to a wide variety of experts and training opportunities. Many districts rely on staff within their district to provide much of the needed PD making teachers' opportunity for professional development dependent on the expertise of their local staff, which can often vary as employees come and go from a given school district. However, to combat this, educative curriculum materials can spark teacher learning and change without the support of outsiders such as professional trainers (Davis & Krajcik, 2005).

2.7 Summary

CGI is an effective professional development program and instructional strategy that can improve both students' mathematics achievement levels as well as teachers' mathematical knowledge for teaching. Additionally, CGI is an excellent way to provide equitable opportunities for all students to develop deep conceptual and procedural knowledge and for teachers to meet a wide variety of learning needs in their class. In the *Catalyzing Change* webinar presented by the National Council of Teachers of Mathematics, Wilkerson & Berry (2020) stated that, "Mathematics instruction in early childhood and elementary school places too much emphasis on memorizing basic number facts and following procedures at the expense of developing deep conceptual understanding." The 8 practices emphasize having students engage in meaningful discourse, build procedural fluency from conceptual understanding and really do the thinking and work for themselves. CGI is one way to take action on these practices.

At present, there is not a CGI text or curriculum product that is both a curriculum for use with students *and* that contains the elements present in Quebec & Ma's (2018) framework as well as other educative features shown to be effective for teachers. Furthermore, because CGI is something that takes a great deal of ongoing support through the year that teachers may not have access to, a text like this would be beneficial for the field of mathematics teaching and learning. This curriculum product would provide a series of word problems that are aligned with the goals of a specific grade level but also contain detailed information about how to elicit, notice and utilize student thinking throughout the lesson. Additionally, a curriculum product like this would allow teachers to build knowledge of disciplinary discourse through clear discussion prompts and rationales and would be designed to help teachers build deeper content knowledge in mathematics as they closely examine student exemplars of grade-level standards. Through frequent practice, with the support provided by the curriculum product developed in this dissertation, teachers will have an opportunity to see their students problem solving in ways that surprise them and will ideally gain enough knowledge through the year to be able to continue CGI and adapt problems to fit their students' needs for years to come.

Chapter 3 - Methodology

The purpose of this chapter is to describe the process of qualitative content analysis used to gain a deeper understanding of the most common elementary mathematics teaching curriculum materials that school districts make available for teachers to deliver grade level content. This is an important step in developing a curriculum product as it is important to know the strengths and weaknesses of what teachers have access to and to understand what may be missing in terms of CGI and discussion-based alignment and resources. This qualitative content analysis will focus on K-5 mathematics *core resources* (i.e., a full curriculum including teacher's manuals and student materials) that a school might adopt with the expectation that it is utilized with all students. It is not possible in the scope of this project to evaluate the myriad materials available online to teachers (e.g. education blogs and websites, Pinterest, Teachers Pay Teachers, Scholastic Teachables and other paid subscription programs), so the purpose of this analysis is to create a picture of the materials teachers are likely to be provided with from their school district and what educative supports those include to implement the principles of Cognitively Guided Instruction and the 5 Practices of Orchestrating Productive Mathematics Discussions.

3.1 Research Questions that Guided Development of Coding Framework

Therefore, the purpose of this dissertation is to understand the strengths and weaknesses of curriculum materials currently used in elementary mathematics and to develop a first-grade mathematics guidebook for teachers that is designed for supporting both teacher and student learning. With this purpose, I need to know what kind of support they are provided with in the teacher's manuals that explain how to implement the programs their schools are using and it is necessary to identify how extensive the textbook guidance is (i.e., does it include examples, drawings, scripts, possible student strategies, misconceptions, how to address misconceptions, how to facilitate a discussion or other supportive features?). In order to understand these materials and how they might support a teacher to understand and integrate the principles of CGI and productive mathematical discussions, the following research questions were identified:

- To what extent do widely used elementary mathematics curriculums provide student learning opportunities consistent with the principles of CGI and the 5 Practices?
- To what extent do widely used elementary mathematics curriculum support teachers in understanding the principles of CGI and the 5 Practices?

Findings from these research questions will drive the overall purpose of this dissertation which is to identify the extent to which teachers are supported to implement CGI and the principles of CGI through their district's adopted mathematics curriculum and to create a CGI Guidebook that serves as both a curriculum resource for use with students and a resource for teacher learning. In essence, these questions seek to find out how frequently students experience learning opportunities such as open-ended problem solving by using their own thinking and strategies and also to understand the extent of the support provided to teachers to understand and utilize student thinking during these problem solving lessons. It is important to understand how much guidance teachers are provided throughout lessons to not only allow students to solve problems independently but to bring student ideas back together and use them to facilitate a whole-class discussion that furthers mathematical understanding. These research questions help to understand what is currently present in common mathematics curriculum products in order to inform the design of the CGI Guidebook. The Guidebook will require more supports such as examples and explanations in areas where common curriculum products lacked support.

3.2 Resource Selection Process

The state of Kansas recommends using EdReports during its mathematics curriculum adoption process (Kansas State Department of Education, 2012). EdReports is an independent non-profit whose mission is to, "increase the capacity of teachers, administrators, and leaders to seek, identify, and demand the highest quality instructional materials" (Our Process, n.d.). There is very little research on exactly how school districts go about adopting a new curriculum series and the factors that drive their final decision; however, EdReports is a free resource open to all and is widely used to aid in the selection process.

EdReports Process

In determining the quality of a given mathematics curriculum, EdReports trains reviewers to use a process of three *gateways* to analyze how well a product meets their expectations. The *gateways* are as follows:

- Gateway 1: Does the product assess grade level content and give students ample work with grade level problems? Is it well aligned to the common core state standards?
- Gateway 2: Is it aligned with the CCSS for rigor and mathematical practices?
- Gateway 3: Do the materials support teachers to use the curriculum fully, understand the skills and learning of their students and support the learning of a wide range of students?

For a curriculum product to be reviewed in Gateway 2, it must first meet or "partially meet" Gateway 1. To be reviewed in Gateway 3, the curriculum product must "meet" both Gateway 1 and Gateway 2.

3.2.20 Resources Selected for Analysis

Nine elementary mathematics curriculum resources that met expectations in all three gateways were selected for analysis in this study. The following resource list captures a snapshot of the commonly used products in elementary classrooms, which is important to be able to look at what most teachers are very likely to have in their classrooms currently and be able to gain a sense of what is missing. The nine selected curriculum resources include:

- enVision Mathematics Common Core (2020-2021)
- Eureka Math (2015)
- HMH Into Math (2020)
- i-Ready Classroom Mathematics (2020)
- Zearn (2018)
- Eureka Math (2013-2014)
- Ready (2017) (Curriculum Associates)
- Math Expressions (2018)
- Bridges In Mathematics (2015)

Two resources were then eliminated from this list. First, Zearn is a fully online, interactive curriculum with teachers on video. Because classrooms employing CGI attend to social constructivism, classroom discussions, and in-the-moment instructional decisions, an online curriculum that is more independent on the student's part does not align with the principles of CGI, the practices for engaging students through orchestrating productive discussions, nor educative features that support teacher learning. Additionally, there are two editions of Eureka Math that passed the EdReports analysis - their 2015 version, as well as 2013-2014 edition. Eureka Math is a free, open educational resource that any teacher may access online. The 2015 edition is freely available to Eureka users and ostensibly updated and improved from the 2013-2014 version, so I did not see the value in analyzing the 2013-2014 edition and eliminated it as well. However, the 2015 edition of Eureka Math was included for analysis.

3.3 Research Design

3.3.21 Resource Collection and Overview of Analysis

I procured the teacher's manuals from all seven curriculum resources covering the full 1st grade curriculum. Bridges and Eureka Math were PDF copies that I read online, while the rest were physical books. I read each curriculum resource using the same, iterative process. First, I began by reading the introductions and going through a few lessons to understand the structure of a lesson. In the introductions, I read about the various components of the curriculum, how they are to be used, when and for which students. To understand the structure of a lesson I examined how a few lessons were intended to be implemented in a classroom, specifically looking for what the teacher was being asked to do with the whole class. Once I felt I understood which parts of the lesson were core, whole-class instruction, I read each resource, lesson-by-lesson as presented, looking for a word problem and following the procedure outlined in the *coding framework* and *coding process* described below. I scored each lesson individually and then will use the results of this *coding process* to interpret strengths and weaknesses of the common curriculum resources.

3.3.22 Qualitative Content Analysis and Coding Framework

Qualitative Content Analysis (QCA) was used to analyze the texts. QCA is "a method of describing the meaning of qualitative material in a systematic way" (Schreier, 2012, p. 1). It is a flexible approach that can be applied in different ways. Here I used the systematic method of coding individual lessons to make sense of the curriculum resources as a whole. Specifically, I used three elements to guide my analysis, searching for evidence of CGI, student engagement through whole class discussions, and educative curriculum features (Figure 3.1). The educative features searched for within the texts are those that support teachers to lead an engaging discussion and to implement the principles of CGI problem-solving.

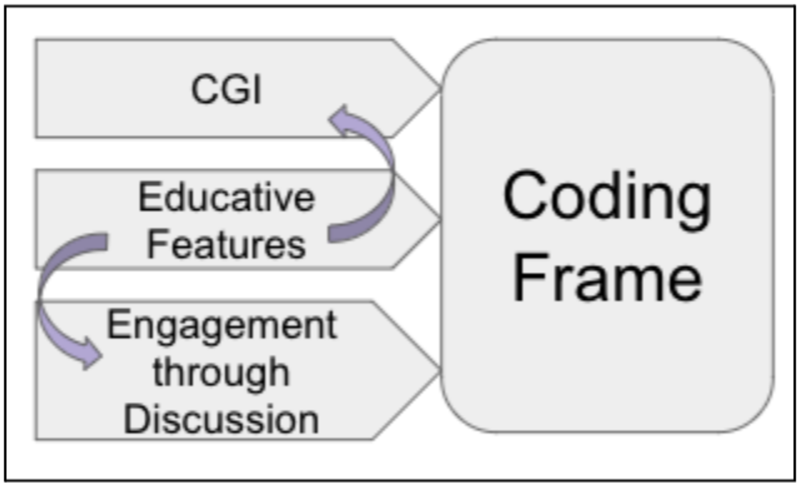


Figure 3.1. Elements of coding framework

3.3.23 Implementing Coding Framework

The following procedure was used to fully read each curriculum resource, one lesson at a time. Figure 3.2 is a sample after coding a set of lessons. I began using the far left cell and wrote the name of the lesson from the resource. Next, I worked across the sheet, left-to-right, coding each lesson for eleven elements broken into three groups. These three groups are influenced by CGI and the 5 practices for facilitating discussions. To examine evidence of CGI, I looked for word problems and for the extent to which students are solving the word problem using their own ideas, rather than after direct instruction (e.g., practice word problems after a skill is introduced and practiced). I also looked for evidence of student thinking being elicited, for evidence of the teacher attending to and using student thinking (i.e., monitoring, selecting, sequencing), and I looked for evidence of a class discussion around student thinking about a word problem (connecting). This framework also helps to identify educative features for the teacher throughout the text. For example, I looked for examples of possible student strategies, descriptions of connections between those strategies as well as questions or sentence starters to help the teacher lead a discussion about the strategies that would also highlight the mathematical goals of the lesson.

To what extent are the principles of CGI present in the textbook?

What supports and educative features are present to help a teacher implement the principles of CGI?

Notes: The word "elicit" used frequently but not dire

	Student thinking is elicited			Teacher attends to student thinking			Class discussion revolves around student thinking				
	Students are invited to solve contextual (word) problem that is provided in text	Teacher **asks students questions** to assess and support understanding of the action and context of the problem	Students are not directed to solve in any particular way. Students are told to solve in any way they choose.	**Workbook text features -** Students represent problem in their own way (The problem includes no feature that would aid students in solving such as a picture or partial number sentence, **not including directions**)	Teacher is provided with instruction on student strategies to look for with the purpose of sharing those with the rest of the class	Support is provided to help teacher make mathematical connections between different student strategies **while looking for and selecting strategies**	Teacher is instructed to bring students together for a class discussion about the strategies	Teacher is instructed to share only student strategies during this discussion	The students share how they solved to the whole class while **the teacher represents students' strategies** large enough for the whole class to see 2-Drawing 1-Explanation only	Teacher is provided with **support to lead a connecting discussion about the student strategies**: questions to ask class, questions to ask student sharing, sample dialogue, etc	There is no instruction to the **teacher** to **share their own strategy** or a strategy provided in the text.
Unit 2, Lesson 1	2	0	2	2	0	0	0	0	0	0	2
Unit 2, Lesson 2	2	0	1	2	0	0	0	0	0	0	0
Unit 2, Lesson 3	2	0	0	0	0	0	0	0	0	0	2
Unit 2, Lesson 4	2	0	0	2	0	0	2	0	0	1	2
Unit 2, Lesson 5	0										
Unti 2, Lesson 6	2	0	0	2	0	0	0	0	0	0	0

Figure 3.2. Sample of outcome from application of coding framework.

3.3.24 The Coding Process Step-by-Step

The following step-by-step process will explain how I read and coded each of the seven curriculum resources for evidence of the eleven elements. These same steps were repeated for each individual lesson and in the same order each time. Figure 3.3 briefly outlines the process and each step in the coding process, along with each element, is described below. After finishing reading a lesson, I started back at the beginning of the process for the next lesson in the curriculum resource and continued until all lessons in the resource were coded.

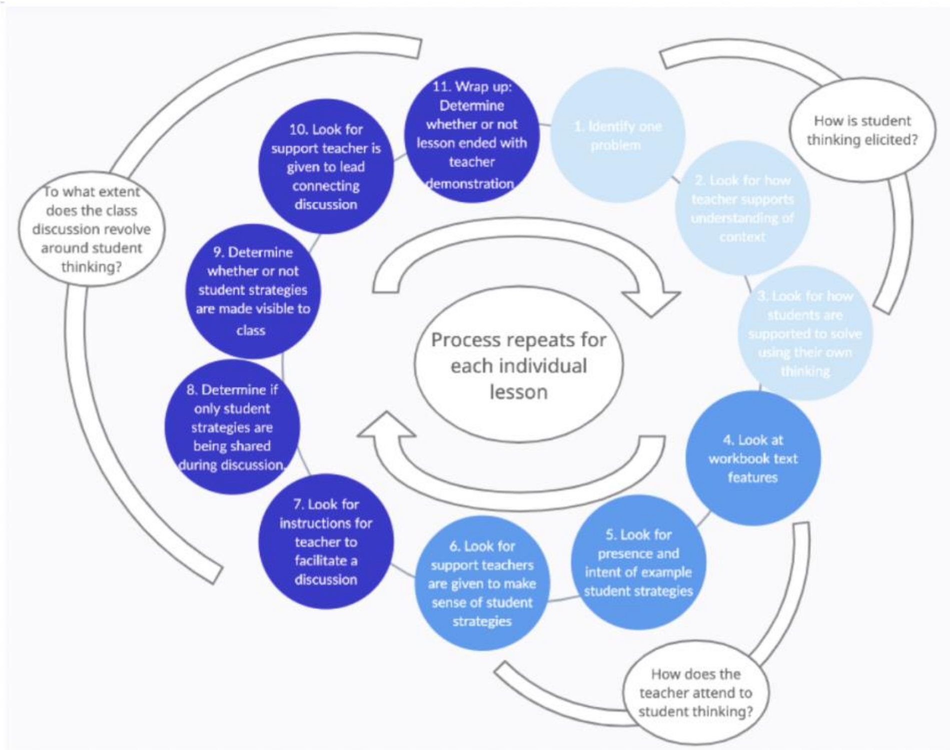


Figure 3.3. Steps in the coding process

How is student thinking elicited?

Step 1: Look at the whole-class lesson and identify one problem that most aligns to CGI. To code the problems, first, key features were identified that tied back to CGI: (1) context that "involves situations that students can easily imagine and relate to" and (2) will lead to "complex problem solving" (Carpenter et al., 2015, p.71). In first grade, these problems involve situations that lead students to consider joining, separating, comparing or creating groups to solve addition and subtraction problems (Carpenter et al., 2017). If a lesson contained a problem with these features, it was scored as 2, if it did not, it was scored 0. Lessons could be coded 0 or

2 in this step only, never 1. The purpose of this step was to determine if there was a CGI-type problem in the lesson in order to determine the need for further coding. For this reason, the problems were either included or excluded (0 or 2). Figure 3.4 shows sample word problems appropriate for CGI. These problems include supports such as contexts that students can relate to (cookies and dollars), actions that they understand (eating and buying) and

Eliz had 12 cookies. She ate 5 of them. How many cookies does Eliz have left?

Eliz has 5 dollars to buy cookies. How many more dollars does she need to earn to have 12 dollars?

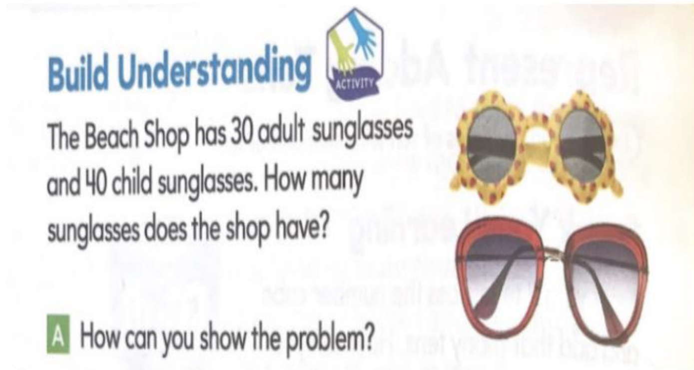
Eliz has 5 dollars. Tom has 12 dollars. How many more dollars does Tom have than Eliz?

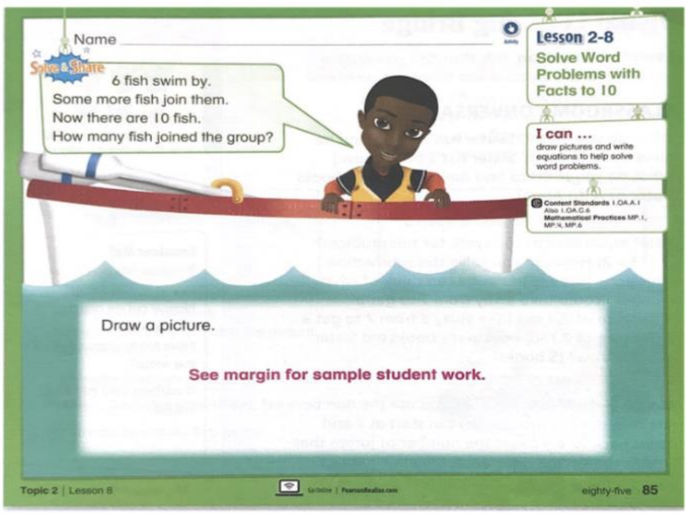
Figure 3.4. CGI word problems (Carpenter et al., 2015)

people that they can imagine. I suggest in my guidebook that teachers use the names of students in the class. Tables 3.1 and 3.2 show problems from some of the textbooks that were coded as a contextual word problem, similar to those in the CGI book, and examples of problems that were not coded as contextual problems to illustrate the differences and how these problems were selected for further coding.

Table 3.1. Examples of contextual word problems.

Problem	Reason for Inclusion

<table>
<tr>
<td>

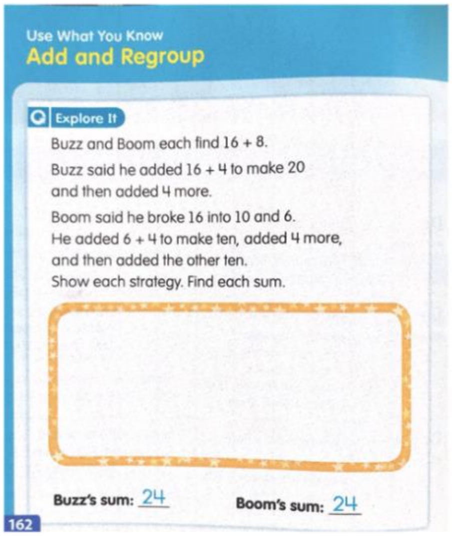
Into Math, Module 12, Lesson 1</td>
<td>Has relatable, real-world context and numerical answer, no picture to inhibit complex problem solving.</td>
</tr>
<tr>
<td>

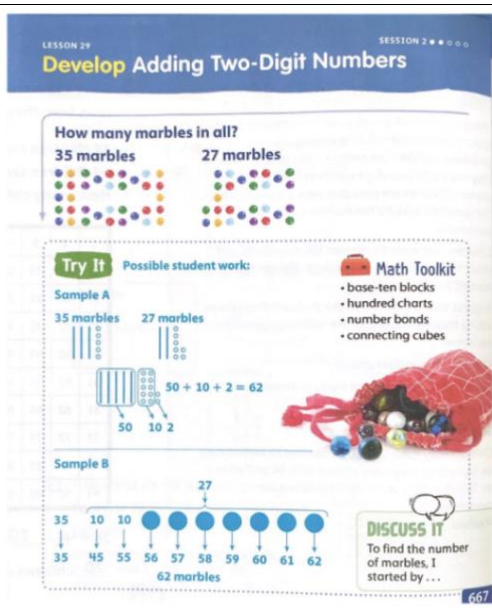
Envision, Lesson 2-8</td>
<td>Has relatable, real-world context and numerical answer, no picture to inhibit complex problem solving.</td>
</tr>
</table>

To be coded 2, the students must have been presented with a contextual problem that sought a numerical answer so I excluded problems with a question such as "Who has more?" if the numbers were already presented to them. I also excluded problems that included a picture

Table 3.2. Examples of problems not coded as contextual word problems

Problem	Reason for Exclusion

<table>
<tr>
<td>

Ready Common Core, Lesson 25, Day 1</td>
<td>Does not involve a context students can easily imagine and relate to.</td>
</tr>
<tr>
<td>

Ready Classroom, Lesson 29, Session 2</td>
<td>Picture inhibits complex problem solving</td>
</tr>
</table>

that must or could be relied on to answer the question. For example, "*6 red beads and 3 yellow beads. How many beads in all?*" with a picture of the red and yellow beads that could be counted to solve (Ready Common Core, Lesson 1, Day 3). Or larger quantities such as the marbles that can all be counted in the figure above. These types of pictures may hinder the complex problem solving that CGI teachers need in order to select from a variety of strategies and lead an engaging discussion because many students will simply count the objects on the page and will not think for themselves about more efficient and creative ways of putting together the numbers.

I also excluded problems that were presented as a riddle. For example, “Julio is thinking of a number larger than 26 but smaller than 28. What number is he thinking of?” This also ties back to the “5 Practices.” The problem needed to initiate a rich discussion in a CGI classroom and teachers must present a problem with a wide variety of possible solution strategies. In the *anticipate* stage of the 5 practices, the teacher thinks of as many strategies as they can to solve the one problem. If there truly are not very many, this is not a great problem for a CGI lesson. Problems that called on students to analyze math errors were also excluded as in the example from Envision, lesson 3-9 above. Some students may have known that Lidia was wrong because the pet store only has 9 frogs and did not directly need to join numbers. In this case, students are thinking about someone else’s strategy and misunderstanding, but CGI discussions are really about the joining, separating and comparing strategies of the students in the classroom.

Searching and selecting problems

In the core, whole group lessons, I first searched for any specifically problem-based section of the lesson. For example, EnVision began each lesson with “Problem-Based Learning” and Into Math frequently included an open problem based section called “Spark Your Learning.” If I found that that section did include a context problem that met the criteria, the first part of my coding frame, I only looked at that section because it was dedicated to the type of lesson I am looking for while the rest of the lesson is more focused on a standards objective and content development around that objective. The “Problem Based Learning” section was more closely aligned with my research question. If I found that it actually did not include a context problem as the basis for the section, I looked at the rest of the lesson to find a context problem, again, excluding centers, enrichment or small group tasks and homework and assessments.

Step 2: Look for how the teacher supports students to understand the context and action in the problem. What did the teacher’s guide provide to teachers to help them understand student thinking and support students in understanding the context and action/math in the problem? I first looked for whether dthere was any discussion that would support students in comprehending the word problem. Were they asked to retell the problem or asked questions about the context of the problem? The following is an example of teacher language that fully supported the students in understanding the context: “What is the problem about? What do each of the numbers describe? What math questions could you ask about the problem?” (Into Math). Some lessons included language that partially supported the students’ understanding. For example, students were only asked to retell the problem.

Table 3.3. Codes for support to understand the problem

Supporting Students to Understand the Context	
Score	Description
0	No support for students to understand the problem was provided
1	Some support was provided: Students asked to retell the problem
2	Support is problem specific: “Would there be more or less than 7 black petals? (Envision, Mod 1, Lesson 8)

Step 3: Look for whether students were directed to solve using a particular strategy or encouraged to use their own strategy. During this step, I looked for the extent to which

students were encouraged to solve in ways they thought of on their own. I looked for whether students were instructed to use a particular strategy that they had learned during that lesson or previous lessons. I looked for instructions to encourage students more indirectly by suggesting strategies. I also looked for if students were not given any directions for how to solve or explicitly told to solve the problem in whichever way they choose.

Table 3.4. Codes for students being asked to solve using their own ideas

Score	Description	Example
0	Students were told to use a specific strategy to solve	*Teacher Today, we are going to be solving some Popsicle problems. Let's do the first problem together. I'll read it aloud, and you listen carefully. Ready to listen? "Sage has 2 green Popsicles in her left hand and 4 purple Popsicles in her right hand. How many Popsicles does she have in all?"* *Before we solve this, let's write an equation to match the story. Any ideas?* Ask students to model and solve the problem on their number racks, and record the answer on their whiteboards when they're finished. After they've had a little while to work, invite a few students to share their solutions and strategies with the class. Bridges, Unit 1, Module 3, Session 1
1	Students may be led to a particular strategy but without being explicitly directed	**Sample Guided Discussion:** **Q What information are you asked to find?** how many fish are green **Q How do the counters help organize the information in this problem?** Possible answer: They show the total number of fish, 14. The 5 counters that are crossed out show the fish that are orange. The other counters show the fish that are green. *Attention is drawn to counters but explicit direction to use them is not present.* Into Math, Module 4, Lesson 6

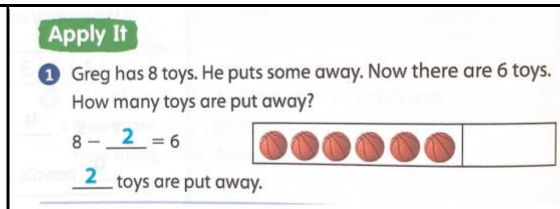
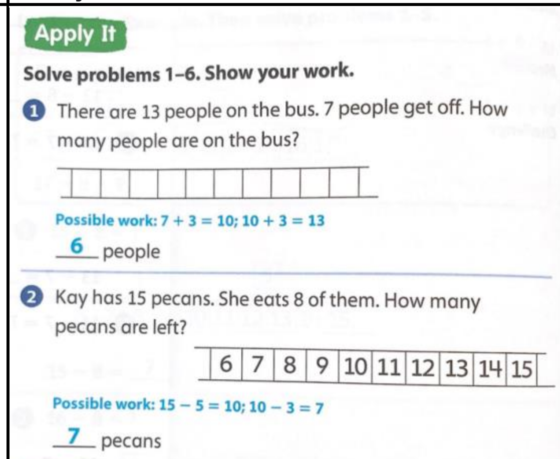
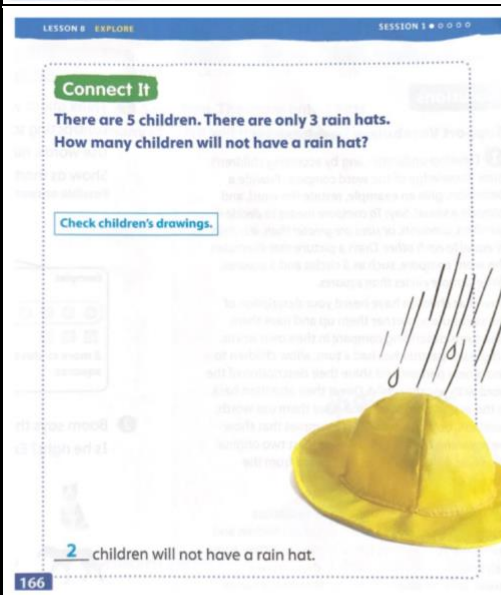
2	Students are asked to solve the problem with no instruction on how to do so. Could be encouraged to use multiple strategies or to use the tools available to them however they choose.	Stacy made 6 drawings. Matthew made 2 drawings. Tim made 4 drawings. How many drawings did they make altogether? Use a drawing, a number sentence, and a statement to match the story. *“Math Drawing” is quite open-ended. However many lessons coded as 2 had no directions for solving at all.* Eureka Math, Module 2, Lesson 7

How is the teacher instructed to attend to student thinking?

Step 4: Look at the workbook text features. Look for what is provided to the teacher and how open or empty the student workspace is. How much student thinking is elicited or afforded? In the previous example of the marbles that can all be counted, everything a student would need to count and solve that problem is already present on the page, so I did not count that as a CGI problem and go further into the coding process. Getting a “0” as a Workbook Text Feature code was slightly different. In this case, there was a picture that could be used to solve some of the problem but still more of the thinking had to be done by the student. In the “Greg” problem in the table below, the student must still think about the difference between 6 and 8. Those 2 basketballs are not present to be counted. To be coded 1, a tool was present such as an empty 10 frame or a number line (with or without numbers) but there was no indication of how the student should use that tool. Or, there was an empty equation, indicating an operation but with no numbers present.

Table 3.5. Codes and examples of student workbook text features

Workbook text features did not lead		
Score	Description	Example

0	Image can be used to solve the problem. Or, the problem cannot be solved without the image. Or, directions for solving or example was provided. Or, incomplete number sentence is provided.	 Apply It 1 Greg has 8 toys. He puts some away. Now there are 6 toys. How many toys are put away? 8 − 2 = 6 2 toys are put away. Ready Classroom, Lesson 7, Session 4
1	A tool is provided but without enough support to solve problem, e.g. a number line with no indicator of where to start or stop counting	 Apply It Solve problems 1–6. Show your work. 1 There are 13 people on the bus. 7 people get off. How many people are on the bus? Possible work: 7 + 3 = 10; 10 + 3 = 13 6 people 2 Kay has 15 pecans. She eats 8 of them. How many pecans are left? 6 7 8 9 10 11 12 13 14 15 Possible work: 15 − 5 = 10; 10 − 3 = 7 7 pecans
2	Students are working on an empty (open) page.	 LESSON 8 EXPLORE SESSION 1 Connect It There are 5 children. There are only 3 rain hats. How many children will not have a rain hat? Check children's drawings. 2 children will not have a rain hat. 166 Ready Classroom, Lesson 8, Session 1

Step 5: Look for the presence and intent of student strategies. Are examples of possible student strategies drawn or described? What was the intent of the provided strategies - to help the teacher identify misconceptions, become aware of types of strategies to share out with

the class, or some other purpose?

Table 3.6. Codes for presence of example student strategies

Example Student Strategies	
Score	Description
0	No potential student strategies present.
1	Teacher is provided with strategies, but their intent is unclear. Or, the intent is for teachers to understand misconceptions or to check for student understanding.
2	Teacher is provided with drawing or description of possible student strategies with the intent for the teacher to share these types of strategies.

Step 6: Look for support teachers are given to make sense of the student strategies. If there were student strategy examples present, I looked for the extent to which the text explained the connection between the strategies - how they are similar or different or mathematically related. I also looked at whether the text gave suggestions for which types of strategies might be best to share out and in what order (selecting and sequencing).

Table 3.7. Codes for teacher support to connect strategies.

Table 3.7. Codes for teacher support to connect strategies

Support for teacher to make connections between strategies	
Score	Description
0	No connection is made between potential student strategies in the previous section.
1	Minimal connection is made, e.g., it is pointed out that two strategies are similar but does not explain in what ways they are similar.
2	Text explains a strong sequence to share out these strategies and why, referring to the mathematical similarities between the different strategies.

To what extent is the teacher provided with support to lead a class discussion that revolves around student thinking?

Step 7: The teacher is instructed to bring the students together for a whole class discussion. If this step received a 0 because there was no class discussion, the next 4 steps also received a 0 because they are all parts of the class discussion. A 1 was coded in this section for instances where there was a class discussion about the problem, but it was not about the strategies. For example, after students solved, the teacher was instructed to draw their attention back to the context of the problem or compare the problem to a previous problem the class had solved.

Table 3.8. Codes for class discussion

Teacher is instructed to hold a class discussion	
Score	Description
0	Teacher is not instructed to hold a class discussion after students solve.
1	Class comes together for discussion about a topic related the problem but not student strategies
2	Class discussion revolves around students sharing their strategies.

Step 8: The teacher is instructed to share only student strategies during this discussion. The purpose of this step was to further code how focused the discussion was on student strategies. To be coded 2, the entire discussion was student-centered and focused on students sharing how they solved the problem with opportunities to answer questions from their teacher and peers. To be coded 1, it was somewhat focused on comparing and understanding student strategies but was also used for the teacher to introduce or discuss a tool (number rack,

number line, 10 frame, etc.) or strategy not presented by students.

Table 3.9. Codes for student strategies as focus of discussion

Class Discussion focuses on student strategies	
Score	Description
0	Class discussion is not about student thinking/strategies OR there was no discussion per the previous step.
1	A tool is discussed (e.g. rekenrek, number line, hundred chart) but *how* it was used is demonstrated/explained by students
2	Discussion revolved around students sharing strategies that they chose on their own.

Step 9: The student shares their strategy while the teacher represents their work for the whole class to see. Was the student's strategy drawn or somehow displayed for the rest of the class to view? One job of a CGI teacher is to make student thinking visible to others. In this step, I coded a 0 for instances when student work was not shared, 1 when students explained their thinking to others only orally and 2 when the teacher was directed to visibly display the work digitally or on the board or chart paper while the student explained their work.

Table 3.10. Codes for representation of student work during discussion.

Table 3.10. Codes for representation of student work during discussion

Student Work Representation	
Score	Description
0	Student work was not shared.
1	Students explained their strategy, but it was not made visible to others.
2	Student explained strategy while teacher represented the strategy large enough for class to see (by drawing or showing their work using document camera, etc.)

Step 10: The teacher was provided with support to lead a discussion that makes mathematical connections between the strategies and helps make thinking more visible to other students. Were questions provided to ask the students sharing as well as the class? Were these questions intended to draw students' attention to the similarities and differences between the strategies?

Table 3.11. Codes for teacher support to connect strategies during discussion

Support for teacher to lead a connecting discussion	
Score	Description
0	No discussion questions or guidance present.
1	Discussion questions to ask student sharing or the class are present.
2	Discussion questions for both the presenting student and the class are present. The questions draw students' attention to the mathematical connections between the strategies presented.

Step 11: The teacher was not instructed to end the lesson by demonstrating their own solution strategy. I coded the "wrap up" of the lesson separately from the discussion. Regardless of how the discussion was coded, I looked at the ending of the lesson to see how the lesson objectives were tied together and highlighted by the teacher and if the wrap up was more or less student-centered than the rest of the lesson. I hypothesized that some lessons that had been focused on student strategy solutions may still be wrapped up with teacher led instruction.

Table 3.12. Codes for lesson wrap up

Teacher demonstration during lesson wrap up	
Score	Description
0	Lesson ends with teacher demonstrating a strategy to solve problem
1	Additional ideas are suggested (e.g. "How could Avery have used counting on?")

2	Lesson is wrapped up with discussion of student strategies and/or the problem type, context or equation to match the word problem. Teacher does not share or suggest their own or textbook strategies.

3.4 Reliability

To test for reliability of this method, I recoded 5% of these lessons myself. I did this process 11 months after coding the first time, so my memory of individual lessons was very limited. I numbered each lesson in the order that they were originally coded and used a random number generator online (randomlists.com) to select 47 lessons which represents 5% of the 939 lessons originally coded. These 47 lessons were a wide, random variety sampling all 7 of the products. I then created a new list using those 47 lessons on a spreadsheet that was an exact copy of the coding framework originally used and went through the same 11 step process to "recode" the lessons.

The Kappa Statistic or "Cohen's Kappa" is a measure of interrater reliability which can also be used when the same rater has rated twice and that is what I did in this case. One purpose of Cohen's Kappa is to measure the extent to which multiple raters assign the same score to the same variable but it also accounts for chance agreement (McHugh, 2012). It can also be used to measure reliability of a single rater - *intrarater* reliability.

Cohen's Kappa produces a value that ranges from -1 to 1 where 0 represents chance agreement and 1 represents perfect agreement. Table 3.13 shows how Cohen suggested the result be interpreted according to McHugh (2012) who also notes that scores below 0 are rare in practice.

Table 3.13. Kappa agreement levels.

Value of Kappa	Level of Agreement
$\leq$ **0**	No Agreement
0.01–0.20	None to Slight
0.21–0.40	Fair
0.41– 0.60	Moderate
0.61–0.80	Substantial
0.81–1.00	Almost Perfect

I used Stata, a software program, to calculate Cohen's Kappa by entering the randomly selected lessons' scores for both the first coding and the second. The Kappa statistic is 0.7753 indicating substantial agreement.

3.5 Summary

In this chapter, I described the process of how qualitative content analysis was used to gain a deeper understanding of the most common elementary mathematics teaching curriculum materials. I detailed my coding framework and the steps of this process with examples and explained how I checked for reliability of my method. This was an important step in developing a curriculum product as it is necessary to know the strengths and weaknesses of what teachers have access to and to understand what may be missing in terms of CGI and discussion-based alignment and resources. In the next chapter, I will discuss my findings, present my CGI Guidebook and explain how the Guidebook helps to address some of the shortcomings found for the currently available resources.

Chapter 4 - Results

In this chapter, I will first describe the results of coding the 7 curriculum resource products. I will then explain how my CGI Guidebook addresses a lack of some of the features. As stated in Chapter 3, the following questions guided the development of the coding framework used to obtain this data:

- To what extent do widely used elementary mathematics curriculums provide student learning opportunities consistent with the principles of CGI and the 5 Practices?
- To what extent do widely used elementary mathematics curriculum support teachers in understanding the principles of CGI and the 5 Practices?

Table 4.1 shows the percentage of lessons in each resource that contained the given feature. The purpose of this chart is to illustrate the percentage of lessons or days that a student might experience each lesson feature. I also calculated an average for all resources for each of the 11 elements I looked for during the coding process. This allowed me to see, on average, features that were more often present than others. The data gleaned from the content analysis process informed the development of my CGI Guidebook. By becoming aware of elements that are extremely lacking, for example, "support to lead connecting discussion," I provided more extensive support in the Guidebook, knowing that this instructional strategy may not be something that a teacher is used to seeing. Teachers may not be accustomed to being directed to hold a connecting class discussion and will therefore require a lot of support to do that well.

Table 4.1. Average percentage of lessons containing each element in coding framework.

Average Percentage of Lessons Containing Each Element in Coding Framework

	Student thinking is elicited	Teacher attends to student thinking	Class discussion revolves around student thinking

Name of Curriculum Resource	Story problem is present	Comprehension support is present	Students choose how to solve	Workbook text features do not lead	Presence of example student strategies to share	Support to find connections between strategies for share-out	Instruction to hold class discussion is present	Only student strategies are shared during discussion	Teacher represents these student strategies	Support to lead connecting discussion is present	Teacher does not end discussion with their own or textbook strategy
Math Expressions	43.16%	2.11%	18.95%	27.37%	5.26%	1.05%	16.84%	13.68%	2.11%	4.21%	30.53%
Ready Common Core	31.33%	0.40%	10.44%	3.61%	3.21%	0.00%	5.22%	4.02%	0.00%	0.00%	14.46%
Bridges	10.63%	0.00%	5.63%	8.75%	1.88%	0.00%	3.75%	3.13%	3.13%	0.00%	6.25%
Envision	67.92%	3.77%	38.68%	34.91%	26.42%	0.00%	30.19%	17.92%	30.19%	0.94%	25.47%
Ready Classroom	48.19%	0.00%	40.96%	19.88%	19.88%	0.00%	25.30%	24.70%	0.00%	0.60%	26.51%
Into Math	67.35%	24.49%	45.92%	31.63%	0.00%	0.00%	25.51%	27.55%	0.00%	0.00%	60.20%
Eureka Math	92.11%	0.66%	62.50%	90.79%	4.61%	0.00%	11.84%	8.55%	4.61%	0.00%	88.82%
Average (Mean)	51.53%	4.49%	31.87%	30.99%	8.75%	0.15%	16.95%	14.22%	5.72%	0.82%	36.03%

4.1 Most Present Features of the Resources

First, I will discuss some of the elements that had the highest percentages of presence in the resources. On average, over 50% of lessons coded did have a contextual word problem. On the high end, Eureka Math provided students and teachers with this type of problem in roughly 92% of lessons while Bridges included contextual problems in only 10.63% of lessons. This is a wide range. A student in a classroom that has adopted Eureka Math might be solving this type of problem nearly daily while in other classrooms, the teacher might be more in need of additional resources. My CGI Guidebook provides a problem in every lesson. This problem is the focus of each lesson.

On average, in 31.87% of lessons had a contextual word problem where students were directed to solve in whatever way they choose or there were no directions for the teacher to tell students to solve in a particular way. This is a key feature of my guidebook and CGI in general. In fact, I would call this the most important element of CGI, that students choose how to solve the problem. This uncovers their thinking for the teacher allows the teaching to be “cognitively guided”. This is one feature that was present more frequently than others but still nowhere near enough. In my Guidebook, students are provided with this opportunity every day.

In 30.99% of lessons, on average, included a problem using paper resources like a worksheet that did not include features that might alter a student’s thinking. In these lessons, the student solved on a blank sheet of paper, a whiteboard, or a worksheet or workbook that was very simple and in no way supported them in solving the problem in a particular way. A counterexample might be a worksheet with the objects in the story drawn and crossed out to indicate subtraction. An “open” or blank sheet for solving is another key feature present in every lesson in my guidebook. This way, students can use their own thinking and draw or represent that thinking without any guidance. The only thing present on their paper will be their own ideas.

In 36.03% of lessons, on average, the teacher was not directed to wrap up the lesson by presenting a strategy of their own or the textbook. I hypothesized that there might be some open-ended problem-solving lessons that were wrapped up with more teacher-centered demonstration. That scenario happened infrequently. More commonly, the whole lesson was centered around the teacher telling students how to solve or showing them how to solve in some way and that just continued through the wrap-up.

4.2 Most Absent Features of the Resources

There were also some features that were significantly absent from the resources. The first was support for the students to comprehend the story problem. On average, only 4.49% of lessons included a story problem with full support in this area. Full support, in my coding framework, means “problem-specific” so the teacher should have been provided with at least one question to ask the class that would prompt them to consider and discuss the context and the size of the number that would make sense given the specific context of the problem. To address this, my CGI Guidebook includes a problem-specific comprehension question for every lesson. After the teacher poses the problem, they ask the students whether they think the answer will be smaller or larger than a number in the story. This prompts students to discuss why it must be larger or smaller without discussing strategies.

Only 8.75% of lessons, on average, included example student strategies that were intended as examples of strategies the teacher might share with the rest of the class. The resource with the most of these, Envision, included this feature in 26.42% of lessons. I address this in my CGI Guidebook by including multiple examples of possible student strategies in every lesson. This way, the reader has examples in front of them that are specific to the problem and the numbers within the problem.

Another feature notably lacking from the resources coded was “support to find connections between the strategies for the share out.” Only 0.15% of lessons contained this feature, on average. When coding, I was looking for explanations for how students’ strategies connected to one another. I looked for two example strategies where students counted in similar ways, counted on or back, made numbers “friendly” in similar ways, among other things. It was extremely rare to see these connections explicitly stated for teachers. As previously explained, it was very rare to see student strategy examples at all and this goes a step further with support related to those examples. My CGI Guidebook includes drawings of multiple student strategies for every lesson. It also includes suggestions for which strategies to select with a rationale for how they connect to one another, the order they could be shared in and a sample dialogue showing how the connections between them could be clear to students.

Also notably lacking were directions for teachers to represent the students’ strategies for all to see (average 5.72% of lessons) and resource-provided support to lead a connecting discussion about those strategies (average 0.82% of lessons). In my CGI Guidebook, teachers are instructed to select strategies that they represent for the class to see on chart paper. A photo example is provided for this, and every lesson includes an image of what this could look like along with discussion points and questions that teachers can ask both the sharing student and the rest of the class as they are representing these strategies. The student strategies and discussion guidance are specific to the problem so a teacher can see exactly what representing strategies and holding a discussion might look like using the same problem they will be presenting to their class.

Overall, there was no curriculum resource that consistently provided daily opportunities for students to be presented with a problem that they solved on their own without textbook or teacher strategy suggestions and provided the teacher extensive guidance to be able to select and sequence strategies and ultimately facilitate a rich discussion about student ideas that illuminates connections between their ideas and highlights important grade-level math concepts. Eureka Math came the closest to doing this but still provided very little support, or no support, in key areas. Teachers need resources that provide strong, consistent support to implement the instructional strategies that are most important for developing student understanding. This content analysis illustrated the need for a resource that provides teachers with the support to enact CGI lessons daily.

4.3 Teacher Feedback

Other data that informed the development of the CGI Guidebook is feedback from teachers. I received IRB approval to have two 1st grade teachers implement the first draft of the first five lessons in the book which I called "CGI Mini Unit" (Appendix A). The teachers taught the lessons and provided feedback in a survey. Both teachers reported that they knew what CGI was but had never used it before in their classrooms.

Both teachers reported that they did not read the entire lesson. One read the whole lesson on the first day and then parts of the lesson on subsequent days. The other teacher read only parts of the lesson every day. In my Guidebook, I included a “How to Read the Guidebook” section where I define and explain the importance of intellectual preparation. It is essential that teachers read the lessons in their entirety, especially in those first five lessons. There are elements to CGI that are new to many teachers and must be fully understood before beginning. For example, one teacher reported that she discussed student strategies during the “launch” portion of the lesson in the beginning and had students begin sharing them out then. This is more aligned to a Number Talk and is a very different process than CGI. Teachers must understand how important it is to not give anything away in the beginning and instead allow students to work independently. Teachers need to see their thinking in order to select and sequence strategies successfully. I added a portion in the directions of the Guidebook to explain this. I also added a section explicitly explaining to teachers how to pose the problem without accepting student strategy ideas and this direction is unique and specific to each problem.

Teachers reported that the most helpful parts of the CGI Mini Unit were the directions for posing the problem, student independent work time and directions for how to facilitate the discussion while the discussion questions themselves were reported to be unclear by one teacher. Considering this as well as the results of the content analysis, the directions for facilitating the discussion were made much more robust and related to the images of potential student strategies. This way, teachers can see exactly when to ask questions and draw students’ attention to aspects of the math while representing student strategies. The discussion questions are now more tied into the directions for facilitating the discussion.

4.4 CGI Guidebook

The CGI Guidebook (Link to CGI Guidebook) consists of an introduction, overview, and Module 1 for the beginning of 1st grade. Ultimately, I will develop this into a full school year's resource that would include six modules. Module 1 is a series of 25 lessons. Lessons 10 and 20 are assessments. There are "chapters" placed throughout the module and these chapters are meant for teacher learning since this is also an educative curriculum product. The chapters are placed strategically within the module so that they will be relevant to teachers and applicable starting in the upcoming lesson. For example, a chapter that teaches how assessments can be used is placed before the first assessment (Lesson 10). As teachers have likely gained proficiency with certain elements of the lessons, those directions are eliminated over time. For example, in Lesson 7, teachers are told for the final time how to prepare the chart paper for the lesson. This module includes many initial directions that are necessary to get started but will not be included in future modules. Instead, teachers will be learning in future chapters about more advanced CGI instructional strategies such as supporting students to take more of a leadership role in the discussion and how to utilize the "parallel tasks" strategy (Small & Lin, 2010).

Standards are placed in the discussion so that teachers can become aware of the exact types of questions and discussion prompts that can highlight important mathematics concepts for students. This allows teacher to really see how standards can be taught by asking questions and drawing students attention to the mathematical connections between student strategies rather than more teacher-centered instruction. In this module, every First-Grade standard under

“Operations and Algebraic Thinking” is addressed as well as 1.NBT.2a and 1.NBT.2b (National Governors Association Center for Best Practices, Council of Chief State School Officers, 2010). In future modules, these will continue to be taught. All but five first grade standards will ultimately be addressed by using this Guidebook. The ones that will not are part Measurement and Data or Geometry and do not lend themselves well to CGI lessons. This will be made explicit to teachers.

4.5 Summary

In this chapter, I described the results of the Qualitative Content Analysis of seven mathematics curriculum resources. I explained how those results informed the development of a GGI Guidebook. Areas that were found to be extremely lacking were given special attention in the development process, providing more explicit support and guidance for teachers in those areas. I also explained how teacher feedback was used to inform the finished product. Finally, I presented a link to the CGI Guidebook and briefly discussed some its features.

Chapter 5 - Discussion

5.1 Summary

The purpose of this study was to address the problem of lack of professional development and support for teachers to implement Cognitively Guided Instruction (CGI) by creating a CGI Guidebook. At present, there is no curriculum product or guidebook that is structured for a teacher to utilize as a daily lesson resource and learn about the CGI program as they implement the lessons. A Qualitative Content Analysis was conducted to determine the extent to which the principles of CGI are present within common elementary mathematics core curriculum resources. These features were found to be significantly absent from the resources. A draft of the CGI guidebook was developed and trialed by two First-Grade teachers. Feedback was collected and this feedback, along with the content analysis results were used to inform the final product – a CGI Guidebook for first-grade teachers.

The theoretical framework that supported this study included three main components – Mathematical Knowledge for Teaching (MKT), Cognitively Guided Instruction and the theory of Social Constructivism. CGI is a highly social constructivist teaching method. Students come together with their classmates to discuss each other's strategies and ideas. They have opportunities to share out their thinking and discuss comparisons to the thinking of others. They come to understand how their peers thought about the context and the numbers and how they chose to represent the situation and work through it. During CGI, students learn how to notice various components of the work of their peers and how to engage in a discussion about that work. During this same process, teachers have an opportunity to build their own knowledge. MKT is not just knowledge of mathematics. Teachers must be able to think beyond their own understanding of how to solve a problem. They must understand the different ways their students will think about the problem, how those ideas connect to one another and how the teacher can use those connections to illuminate the mathematical goals of the lesson. The teacher must also be able to plan a story problem that will lend itself to the goals of the lesson. The knowledge required to do all of this can be acquired through teaching CGI lessons. CGI is an opportunity to give students autonomy to solve in ways that make sense to them, and this process uncovers their thinking – one essential component to building MKT.

I previously argued that a teacher both acquires MKT and becomes proficient at teaching CGI through just that – the teaching of CGI. But of course, they will not achieve proficiency right away. A great deal of support is required. I presented the CGI Guidebook as an answer to the lack of support that teachers currently have access to. The Guidebook allows teachers to learn authentically through their day-to-day practice. Thus, the Guidebook presents a unique professional development opportunity whereby teachers learn by applying what they read and doing CGI, during the school day, with their students.

To use the Guidebook, teachers first read short chapters to introduce the program and to understand the basic components and structure of the lessons which are structured in the same way each day. Each lesson is broken into three parts – posing the problem, independent work time while the teacher selects strategies and finally a whole-class discussion. Early in the Guidebook, there is also a fourth part, which is a "Post-Lesson Reflection" for the teacher. Each of these components of the Guidebook are considered features present in educative curriculum materials (Land et al., 2015) and are designed and included in the Guidebook for the purpose of supporting *teacher* learning.

After teachers have read the introductory chapters, they will then read Part 1 of their first lesson before implementing it with their class. Each lesson begins by stating the objective for both the students and the teacher. The teacher has their own learning objective because the Guidebook is designed to be educative for the teacher as well as students. The “Lesson Focus and Goals” for students frequently include the Common Core State Standard(s) that will be addressed if the lesson plan is followed entirely. In early lessons, teachers will read instructions for how to set up their materials. Every lesson then includes a story problem and an “About the Problem” section explaining the problem type, rationale for the numbers in the problems and any additional information a teacher may need to know about the particular problem. The next component of the lesson is a comprehension question which allows the teacher to ensure that students understand the problem. This is especially important in First Grade because not all students are proficient readers. The comprehension questions are always specific to the story problem (i.e., “Did Landon eat more or less that 25 chips?”).

Teachers will then read Part 2 of the lesson which is all about the student independent work time when students solve the problem using their own intuitive strategies. The teacher is provided with an explanation about what to do and not to do during this time. The teacher is also instructed to engage in the Select and Sequence practices (Smith & Stein, 2018) where they will walk through the room, observing how students solve and asking clarifying questions. Each lesson plan includes a strategies chart for the teacher to use. The strategies on the chart are common First-Grade strategies with drawings that a student might use. They are specific to the problem in each lesson, so the numbers used and the strategies are related to the story problem of the lesson and are not just general strategies to look for. It should be noted that the Qualitative Content Analysis revealed that in the curriculum products analyzed, this type of support was extremely rare. Sample strategies are selected and sequenced in the lesson plans and numbered in green so that teachers can easily see the selected strategies that will be presented in the next part of the lesson. A rationale is then provided for why they were selected and sequenced in that particular order. Even if a teacher does not select and sequence the exact same strategies in their own classroom (and it is very likely that they will not be able to), reading the rationale is a highly educative experience that will support them to be able to select and sequence potentially similar strategies in their own classroom. Educative features like the rationales teach teachers how to think like CGI practitioners.

In Part 3 of each lesson, teachers are presented with a sample dialogue aligned with the selected strategies. Teachers can think of reading this portion of the lesson as an opportunity to observe in a CGI classroom. The strategies and specific questions that the teacher asks may not be exactly what they bring to their own lesson with that problem, but it will help them come to understand the kinds of questions CGI teachers ask to facilitate connection making and understanding amongst students. For example, in many lessons, the dialogue includes the teacher asking, "What is similar about these two strategies?" This is a general question that could be used for any set of student strategies. Although it is a simple question, it prompts both students *and* teachers to think deeply. Sometimes strategies are quite different but the equation to match them is the same. The students dealt with the numbers involved in the problem using the same order. Alternatively, sometimes they used different tools, but their counting is the same. One student might have counted on their fingers while another one wrote out those same numbers. There are often many connections that could be made between student strategies. Both teachers and students need to hear this question often to improve at noticing how different student thinking and strategies connect. Further, this is important for students to develop and expand their repertoire of strategies and for their strategies to become more efficient and sophisticated over time.

The sample classroom dialogue in Part 3 also includes full examples of what a whole class discussion could sound like. There is sample dialogue between the teacher and both the class and the sharing student. These sample dialogues frequently include examples of how to push students to explain more as they are sharing and to explain why they did certain steps in their strategy. There are also suggestions for classroom management and engaging all learners in the discussion. Two main strategies are presented for this – “cold calling” and “turn and talks” (Lemov, 2015). Cold calling is calling on students who do not have their hands raised. Students quickly become accustomed to this and they come to understand that they could be called on at any time and so they must always be actively listening. As one example, in Lesson 2 (CGI Guidebook, p. 16), it is suggested that the teacher cold call a student who was not able to solve the problem to retell the first strategy. By cold calling, the teacher is targeting the retell at a student who really needs the practice. There are a variety of cold calling situations throughout Part 3 of the lessons. Teachers are also instructed throughout Part 3 to have students turn and talk to a partner about a question that was posed before they share out with the class. This gives more students an opportunity to talk and share their ideas. It also allows the teacher to listen in to the partner discussions and select students to share out specific ideas that the teacher was looking for. This allows the teacher more control over the general direction of the discussion. It also allows the teacher to engage more students in the big mathematical ideas of the lesson. One example of this is in Lesson 6 (CGI Guidebook, p. 36). One sharing student added 9 + 4 by adding 1 and then 3 more so the 4 is not obviously present on the chart paper. Students are instructed to talk to their partner about where the 4 is in the work. This turn and talk opportunity allows all students the time to look for the 4 and share their thinking with someone about where the 4 is present in the work and why. The teacher also then has the opportunity to listen to student thinking and select students who noticed the 4 is broken into 1 and 3 and further, that she used the benchmark number 10 to help her count from 9 and add a total of 4.

Finally, the first 4 lessons of the Guidebook include a Part 4 – "Post Lesson Reflection for the Teacher." This section includes questions for reflection about the strategies that students used, what surprised the teacher about these strategies, how similar or different they were than the strategies presented in the Guidebook, what tools students used, and it also prompts them to look ahead into the following lesson and use the thinking and strategies saw in their classroom on the current lesson to predict how students might solve the next one. The teachers who provided feedback on the sample Guidebook reported that this was a less useful part of the Guidebook, so it was limited to just these initial lessons in future drafts. The questions also repeat, and the first four reflections show teachers what they should be considering post-lesson and can be transferred to any future CGI lesson that they teach.

In using this Guidebook, teachers will have the support and learning necessary to begin CGI lessons daily in their First-Grade classrooms. The lessons, as they enact them, will not look exactly like the lesson presented in the Guidebook, but they will learn how to pose a problem, allow students to solve, uncover their thinking and use their thinking to facilitate a productive discussion that will support them in responding to the student thinking and strategies that emerge in their classroom. The sample dialogue allows them to see a daily example of what a student-centered and social-constructivist discussion sounds like. The teacher will have the opportunity to become more flexible in their own thinking by listening to and watching their students solve problems. This daily uncovering and utilization of student thinking is one way to potentially help teachers build the MKT necessary to facilitate deep understanding and mastery of First Grade mathematics standards in their classroom.

5.2 Interpretation

The following questions guided the development of the coding framework that was used to analyze the curriculum products discussed in Chapter 3:

- To what extent do widely used elementary mathematics curriculums provide student learning opportunities consistent with the principles of CGI and the 5 Practices?
- To what extent do widely used elementary mathematics curriculum support teachers in understanding the principles of CGI and the 5 Practices?

It is important to note that the publishers of the commonly used curriculum products analyzed made no claim to have created resources specifically aligned with CGI (Carpenter et al., 1999) or the 5 Practices (Smith & Stein, 2018). The goal of the Qualitative Content Analysis was to better understand the potential mathematical experiences of teachers and students utilizing these resources across the course of their school year. If the analysis showed that teachers were provided with extensive support to engage students in open-ended problem-solving and productive discussions about student approaches to problem solving, the CGI Guidebook may not be necessary. However, it was found that the commonly used curriculum resources contained very little support in these areas and therefore there is likely a need for this type of lesson and this kind of professional development in order for teachers and students to engage in problem solving and productive classroom discourse.

It is also important to note that these commonly used curriculum products represent only a small subset of resources available to teachers. As stated in chapter 3, the seven products analyzed were selected due to their score on Edreports but teachers have Google at their fingertips and can find a very wide variety of resources online that may include some of the 11 features of CGI used to develop the coding framework. However, anecdotally, nothing as comprehensive as this Guidebook was found. There are many places to find “CGI problems” including Teachers Pay Teachers which produces over 190 results using those terms, but very few, if any, come with robust lesson support beyond the problems themselves. A teacher could of course purchase these resources for themselves and it is likely very helpful for a teacher to have a list of problems that they could use for each day of the school year. However, this is still not enough support for them to learn the best ways to use the problems to enact a meaningful lesson and a productive discussion.

5.3 Recommendations for Curriculum Developers

In the Statement of Problem section in Chapter 1, I described NCTM’s 8 Effective Teaching Practices (NCTM, 2014). I explained that adopting these practices, for many teachers, would mean a major shift in how they deliver lessons and would require a great deal of learning on their part. I offered CGI (Carpenter et al., 1999) and the 5 Practices for Orchestrating Productive Mathematics Discussions (Smith & Stein, 2018) as a solution for supporting teachers to enact these practices. I also discussed the benefits of self-directed learning for teachers and how educative curricula could play a role in that process (Land, Tyminski & Drake, 2015). Considering what was discovered regarding the lack of support through resources to implement lessons that incorporate elements of CGI and the 5 Practices, recommendations for curriculum developers, teachers, and schools adopting mathematics curricula will be detailed in this section.

Curriculum developers have an opportunity to make a substantial impact on the instructional practices used in schools. By creating products that school districts might adopt, they provide a resource with, in many cases, all the materials and directions that teachers need to provide a year-long mathematics experience for their class. If teachers followed these programs closely, the activities, problems, questions, things the teacher does and says all have the potential to impact the extent to which children understand the many discreet concepts being presented at each grade level. Curriculum developers should take care to consider the knowledge and experience of the teacher in addition to students. By embedding educative components or creating a fully educative product, curriculum developers have the potential to help teachers learn skills and develop their knowledge for teaching (Davis & Krajcik, 2005). It is important that these educative features are not only present, but explained with a clear purpose for teachers (Collopy, 2003).

Additionally, NCTM's 8 Effective Teaching Practices (NCTM, 2014) would make a strong framework for curriculum developers to consider using when developing products with educative features as they represent research-based teaching practices that are highly effective for any K-12 mathematics classroom. NCTM emphasizes that these practices are partly intended to create equitable mathematics instruction (NCTM, 2014). All students need and deserve high-quality mathematics instruction and therefore teachers must have an opportunity to learn the practices necessary to help all students master grade-level mathematics. Most teachers use a commercially available mathematics curriculum product with varying levels of modification to their lessons, or they use pieces and parts of multiple commercially available products (Kaufman et al., 2020). Thus, it would be ideal if teachers went to their curriculum resource and found ample support to implement these effective practices.

For example, one of NCTM's 8 Effective Teaching Practices is "Posing Purposeful Questions" which is an example of a practice that can be honed through experience. However, it is also a practice that curriculum developers have an opportunity to assist teachers in understanding how to enact on a day-to-day basis. Purposeful questions are those that advance student thinking about the concept being presented. The student (rather than the teacher) is put in a position to *do* the thinking – noticing patterns and developing conjectures, drawing conclusions about the similarities and differences between approaches to solving the same problem. Furthermore, purposeful questions are asked by a teacher who knows where the learning started and where it is going. The questions are the path that take students toward lesson objectives. However, it can be challenging to learn how to ask these purposeful questions (Childs & Glenn-White, 2018). A teacher must know the content of the lesson deeply and how it connects to other lessons. They must understand the mathematics itself and their students' understanding of the mathematics, including misconceptions and common errors (Ball et al., 2005). Thus, curriculum developers should consider adding in these purposeful questions in strategic places throughout their curriculums and rationales for purposeful questions so that teachers can come to understand what makes a question a great one, how students might answer it and why it is best to ask it in a particular part of the lesson.

I also recommend that curriculum developers include far more opportunities for open-ended problem solving (Small, 2009). This is because asking students to engage in open ended problem solving in the way it is presented in CGI lessons, is an opportunity for teachers to develop and hone all of NCTM's Effective Teaching Practices and many standards and lesson objectives can be taught largely through open-ended problems and problem solving. Further, instruction that focuses on open-ended problem solving can make an objective that might traditionally be very teacher-centered into one that is student-centered just by focusing on student thinking and classroom discourse. For example, the Common Core Standard 1.NBT.2 is a First-Grade standard that asks students to "understand that the two digits of a two-digit number represent amounts of tens and ones" (National Governors Association Center for Best Practices & Council of Chief State School Officers, 2010). This standard is commonly taught through direct instruction as indicated by the content analysis. Students were told to represent the numbers by drawing sticks and circles in Math Expressions (Fuson & Houghton Mifflin Harcourt Publishing Company, 2018). First, circles represented ones. Later, the teacher was directed to draw a line through groups of 10 circles to show that that now represents a ten and later, the circles in the line were removed, leaving the lines to represent 10 and additional circles to represent ones. Conversely, through the daily solving of problems in the Guidebook, students are encouraged to represent two-digit numbers in ways that they think of on their own. Teachers are encouraged to provide Unifix cubes but the way in which these cubes are used to create the two-digit numbers in the story problems must be decided by the students. The student representation of a two-digit number with the cubes or some other drawing may look similar to the sticks and circles used in Math Expressions but the experience is quite different. This is because it is the student who determined the meaning and representation of the number and they are the ones that explain that meaning rather than the teacher. I highly encourage curriculum developers to consider regular, open-ended, student-centered, problem solving as an avenue to address many, if not most, of the standards.

Another recommendation for curriculum developers is the inclusion of far more support for teachers to facilitate discussions about problem solving or any mathematics topic. Rich discussions should be a central component of all math classrooms and facilitating discussions involves a great deal of MKT that not all teachers have developed. Curriculum developers should support teachers to develop this skill by providing sample dialogue for the whole class as well as partners and small groups. Curriculum developers should include in the lessons examples of student ideas that might be shared and how the teacher can follow up on those ideas and use them to advance student thinking. Curriculum developers should also include management and engagement techniques that will help teachers to facilitate productive discussions. These might include "turn and talks" with a partner or small group or requirements to respond in writing or on a whiteboard before engaging in the discussion. Teachers can find these strategies through other materials, but it is advantageous to have them embedded into the lessons they are already using.

5.4 Recommendations for Schools

School districts spend considerable amounts of money and time researching and adopting mathematics curriculum materials. While it is important that these materials are aligned with the Common Core Standards and that teachers find them easy to use, there are other factors that school districts should greatly consider. Resource adoption committees should review the extent to which teacher learning is provided by the curriculum resource. Self-directed learning through the curriculum could be a professional development opportunity that comes at little additional cost and may not require extensive time outside the classroom (Davis & Krajcik, 2005). Even if a product is not meant to be an educative curriculum, districts should look for educative features that might provide teacher learning aligned with school goals. For example, if a district has a goal to incorporate manipulatives into instruction, they might consider looking for a curriculum product that not only includes manipulatives, but also has instruction on open-ended ways to use them, why they help students understand concepts, and the kinds of questioning and discussion opportunities that would best help students engage with others' ideas and representations using manipulatives. As previously discussed, many school districts do not have ample time for professional development and many school districts do not have the funding for extensive professional development. The curriculum product that they purchase could be the lone opportunity to provide a learning experience for teachers, so it is critical that schools consider teacher learning when adopting a new product.

5.5 Recommendations for Teachers

Teachers interested in improving their practice have access to many books designed for teacher learning and instructional improvement. Teachers must understand that the techniques they read about must be practiced and honed. Further, much of their professional learning will be in their own classroom, in practice, with students present and they should not give up quickly while trying to acquire new skills and knowledge. The Guidebook presented as part of this dissertation is very repetitive at times, prompting teachers to ask the same questions and to create very similar day-to-day experiences for students. This is because it takes practice to acquire techniques like asking the right question to the right students at the right time. Teachers must be willing to practice and reflect constantly, considering what they had hoped the outcome of their lesson would be and what they will do differently tomorrow to keep getting closer to that ideal. They must understand this takes time and effort.

Teachers must also understand that their evaluation of the resources they select for use with their students is critical. Teachers now have access to a greater wealth of resources than ever before. They are by no means limited to the curriculum product(s) adopted by their school district. It is not enough that a resource “covers the standard” or addresses the topic that they are hoping to teach. The slideshow, video or worksheet they have selected to use during a lesson are not the lesson itself. The people in the classroom create the lesson. What matters is the discussion, the questions, the curiosities and the collaboration between different minds. Student thinking must be front and center in every classroom. Teachers must be able to consider this while selecting resources and be able to evaluate whether a lesson or material that they have found or that their school has adopted will allow the students to do the thinking and will allow the teacher to facilitate rich learning that builds onto student thinking. Materials that are selected only because they are easy to use or address a certain standard, in many cases, should be left behind.

Lessons and student materials can be found online for any standard or topic. However, it is critical that teachers understand that regardless of the curriculum resource used, it is the adult who is facilitating the learning for students. Teachers' ability to pose meaningful questions, elicit student thinking, facilitate rich discussions, foster curiosity and perseverance can be taken to any classroom and applied using any curriculum resource. Teachers should seek out resources such as educative curriculum materials that will help *them* grow as mathematics educators.

5.6 Future Research

Research on Educative Curriculum Materials suggests that teachers read instructional materials in different ways (Land et al., 2015, Remillard, 2000). Therefore, it is important to understand how this CGI Guidebook is read, interpreted, and used in practice. This type of research could be conducted through user surveys, interviews, or analysis of dialogue in an online community whose members are using the CGI Guidebook and engaging in discussions about the product and the lessons. Research could also be conducted through classroom observations and discussions with teachers about their use and interpretation of the Guidebook.

A potential next study might be centered around *how* the Guidebook is read and *why* it is read the way it is. Do teachers read lessons start to finish or do they instead search for certain components and skip right to those? If so, why? Also, what is the result of the way they read on their in-class lesson implementation? As previously explained, it is important that writers of Educative Curriculum Materials make *their* purpose very clear to the reader (Land et al., 2015). This type of research could also shed light on the reader's purpose and the extent to which the teacher found what they were initially looking for. What did they already know about CGI and what were they hoping to gain for their practice? Qualitative research on how the Guidebook is used could likely answer these questions and be used to make improvements.

Additionally, the Guidebook is broken down into Modules with Chapters contained inside each module. Research on the knowledge gained and general interpretation of the chapters might inform the development of future CGI Guidebooks as the chapters are the pieces that would largely remain unchanged in Guidebooks for different grade levels. Again, it would be helpful to know what teachers were looking for when they selected this Guidebook and the extent to which the chapters met that need and helped them to alter their mathematics instructional practices.

The CGI Guidebook in this dissertation is designed for First Grade teachers which still leaves a need for CGI Guidebooks written for other grade levels. Understanding how the Guidebook is used is important for the development of these future guidebooks but also for the future development of Educative Curriculum Materials in general. There is still limited research on ECMs and their effectiveness. An ECM in the form of a CGI Guidebook is, currently, nonexistent. Therefore, it is important to ultimately come to understand its impact on teacher learning and instruction. Are teachers able to carry out the lessons as they are intended through the Guidebook? What is the effect on student learning and teachers' Mathematical Knowledge for Teaching? The purpose of this Guidebook is to change the way that teachers deliver mathematics instruction and to implement NCTM's Effective Teaching Practices so there is a need for research on the extent to which the Guidebook was able to do that.

5.7 Guidebook Next Steps

The Guidebook presented in this dissertation contains only one of six modules. The Table of Contents provides a roadmap for completing the modules and chapters making up the guidebook. As future modules are added, a gradual release will occur where the lessons will contain less pedagogical support for teachers as they make their way through the guidebook. For example, problem types that teachers have experienced in Module 1 will not necessarily be explained again in future lessons and only "see page" will be used to direct teachers back for reminders. The dialogue support will also be reduced with the assumption that teachers have read many sample class discussions and have practiced asking similar questions and facilitating similarly productive discussions in their classroom. In their place, questioning strategies will be provided in future chapters along with other support based on the "5 Practices for Orchestrating Productive Mathematics Discussions."

The Table of Contents provides a look at the chapters for teacher learning that will be added throughout the remainder of the Guidebook. These chapters are inserted intentionally during times that teachers are likely ready for new learning and in many cases, where they will have a chance to practice a new skill shortly after reading. For example, the chapter on "Common Strategies with Larger Numbers" appears in Module 3 when teachers will actually be posing problems with two-digit numbers. This is important because the strategies many students will likely employ with these numbers will look different in many ways than the strategies teachers have seen their students use up to this point. Similarly, each new topic for teacher learning is presented in a chapter embedded within the lessons throughout the Guidebook.

In addition to completing this first Guidebook, I would like to use research on the use of the Guidebook to continue developing what would complete a K-5 CGI Guidebook series. The Guidebook created as part of this dissertation is very specific to First Grade, using Grade 1 standards, problems and common strategies. All of these elements change drastically through the grade levels and each grade level requires its own Guidebook with problems and discussions unique to specific grade level expectations.

5.8 Closing Thoughts

NCTM's 8 Effective Teaching Practices were identified in this dissertation as powerful practices that can *all* be learned and implemented through Cognitively Guided Instruction. The problem, however, is that there is a lack of support for teachers to learn how to enact CGI. The purpose of this dissertation was to better understand the extent to which the principles of CGI are contained within common mathematics curriculum materials and to create a CGI Guidebook that would act as both a curriculum that provides learning for students and an educative experience for teachers. Many traditional public schools are not able to provide extensive professional development to their staff that leads to lasting change and increased student achievement. Curriculum designers could be one solution to this problem by providing teachers with products that go beyond addressing standards. Curriculum designers can and should create materials that allow teachers to build Mathematical Knowledge for Teaching – essential understanding that can be used to help students make meaningful connections and truly master grade-level mathematics. Students have the right to understand mathematics. Every teacher should see themselves as the most powerful tool for helping students succeed. Materials matter and they should be used to support adults to become the master teachers that every student deserves.

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Appendix A CGI Mini Unit

Lesson 1

Standard: 1.OA.2: Solve word problems that call for addition of three whole numbers whose sum is less than or equal to 20, (e.g. by using objects, drawings, and equations with a symbol for the unknown number to represent the problem.)

Lesson Focus & Goals for Students
In this lesson, students will solve a "Join Result Unknown" problem with only 2 addends. This will help them understand what CGI or "Number Stories" is all about. They will learn the structure of a lesson and most importantly, they will notice that the teacher allows them to solve in ways they choose and the teacher values their thinking.
Lesson Goal for Teacher
Teacher will understand the steps and structure to a CGI lesson. The teacher will be able to pose the problem, allow students to solve on their own, select 3 students to share and facilitate a discussion about the students' strategies.

Part 1: Pose the Problem

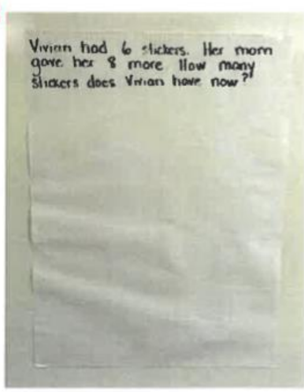
5 Min

Directions: Write the following problem on the top of a sheet of chart paper and place on board. Gather students together on the carpet. Read the problem to students. Ask two students to retell the problem to check for understanding. Then, ask the class to use their hands to show "more or less" to answer the following comprehension question: "In the end, did Vivian have more than 6 stickers or less than 6?" Ask students to talk about why. Call on students who are incorrect to share their ideas first and allow other students to correct them.

Problem: Vivian had 6 stickers. Her Mom gave her 8 more. How many stickers does she have now?	Number Sentence to Match Problem: 6 + 8 = 14

About the problem: This is a **Join Result Unknown** problem. The action of Vivian's Mom giving her 8 more supports students because they can model that action in their strategy (called "Direct Modeling"). The 6 was written first to see if any students switch the addends. The numbers may also reveal students who know doubles facts and are able to use those to solve. The numbers also push students over 10, to allow the teacher to see if students are using 10 as a friendly benchmark.

Write the problem on your chart paper and place on board before lesson begins



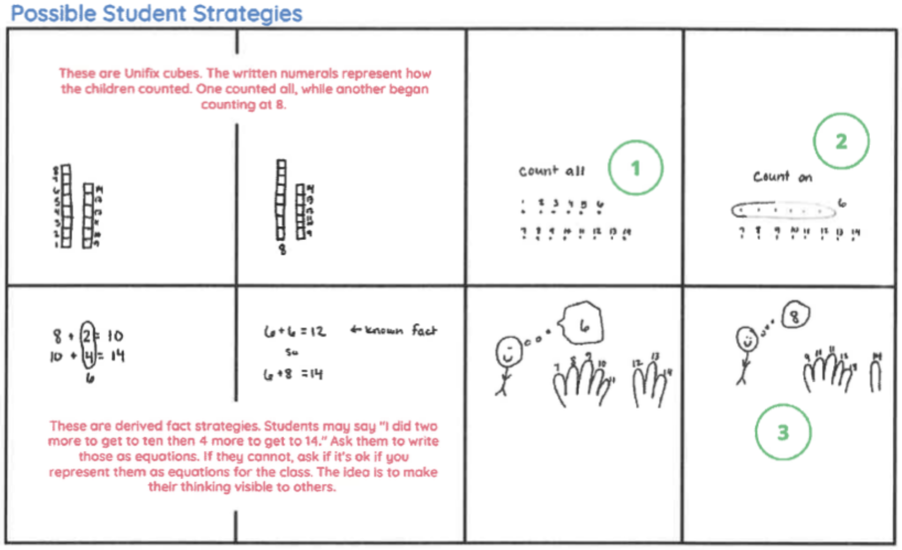
Hand signals that indicate "more" or "less" push all students to think and participate.

Part 2: Students solve independently

10 Min

Directions: Students should return to their seats and work to solve the problem independently. Ask students questions to better understand their work and thinking. Encourage them to do their best to represent or draw their strategy, even if they solved mentally. **Do not show students how to solve the problem.** If they are struggling, try asking them questions that might help them understand the actions in the problem. Suggest that they draw a picture or utilize a tool such as counters. However, you should not tell them how to do so. Move throughout the room and try to glance at what each student is doing. Look for the strategies in the next section or similar strategies. **Choose three students to share.** The guide in the following section shows three selected strategies numbered in green and should be shared in that particular order. Read the rationale for the selection and sequencing of strategies. You may choose to select and sequence the same strategies if they are present in your classroom. If not, you may choose a similar sequence of strategies. Consider ordering them beginning with the least sophisticated - pictorial, concrete, simple. However, do not share with students the reason for the order.

Possible Student Strategies



Why the strategies were selected and rationale for the particular order

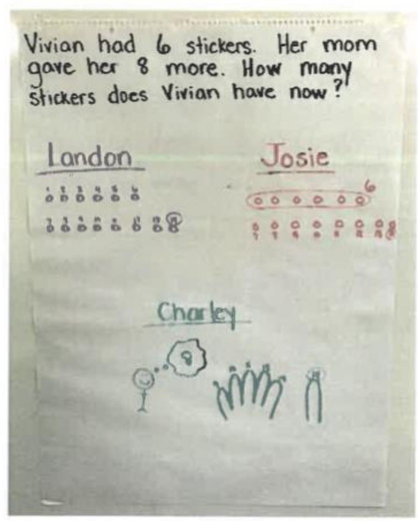
The selection and sequencing of strategies are important choices that will determine the direction of the class discussion and the extent to which it helps meet your overall goals. It is important that you select and sequence with both your lesson goal and students in mind. Today, the lesson goal is for students to see that you value their thinking and there are many "right" ways to solve CGI problems. The first strategy is a direct modeling "count all" strategy - the simplest type that many of your students likely used. Student #2 used counting ON which will be compared to student #1's strategy. **Do not emphasize that one is "better"** - only that they have similarities and differences. #3 was chosen because the child used a different tool (fingers) AND she started with the larger addend - all great discussion points. If very few children counted on, you may swap that out for a "count all" cubes strategy shown above. The derived fact equation strategies are pretty sophisticated and might be better to discuss later on. But, still make note of who is using those.

Part 3: Facilitate the discussion

15 Min

Directions: Bring students back to the carpet. The selected students will share in the order you chose and you will draw what they did on the chart paper. Your drawing might look very similar to the drawings in the "Possible Strategies" section. However, it is important that the child's thinking is represented accurately so your drawing/representation may be different. Ask each student questions as they explain their strategy. You may use the suggested questions below for ideas. Ask the class questions even if they are questions that the student has already addressed. This is an opportunity for students to learn how to listen actively and be able to explain others' ideas. **When the lesson is over, find a place on your wall to display the completed chart paper. A large space that fits the previous 3-4 problems is ideal.** These serve as a student reference for future problems.

How your chart paper may look once students have shared.



Whole Class Discussion Questions	
Questions to ask students as they share: **For Student 1:** What do the circles/dots represent? Why did you have to count all of them? Why did you write the numbers on each dot? **For Student 2:** Why did you circle the 6 dots all together? Did you have to count them again? Why not? **For Student 3:** I noticed that you started with the 8, did anyone else notice that? Did you get the same total? Why? How did that work?	**Questions to ask the rest of the class anytime during the discussion:** How did student #1 (say their name) count? How was that different than student #2? What was the same about their strategies? Where is student #3's 8? How did that work? Who can retell her strategy? Are there a lot of ways to solve this problem? Do you think there are a lot of ways to solve any problem? I think so too! I can't wait to see the many different ways you solve tomorrow.

Post Lesson Reflection for the Teacher

Directions: It is critical that you look at student work and think about the ideas that were shared by students during your lesson today. After the lesson, reflect on the following questions. If possible, discuss with a colleague who is trying the CGI problems along with you.

What surprised you about your students' work and thinking?

How were your students' strategies both different and similar to the strategies you saw in the "Possible Student Strategies" section?

What strategies were most abundant in your classroom?

What tools did students prefer to use? (Fingers, counters, cubes etc) Why do you think that is?

Look ahead at the problem in the next lesson. Based on what you saw in your classroom today, what strategies do you think students might use to solve this problem?

Lesson 2

Standard: 1.OA.2: Solve word problems that call for addition of three whole numbers whose sum is less than or equal to 20, (e.g. by using objects, drawings, and equations with a symbol for the unknown number to represent the problem.)

Lesson Focus & Goals for Students
In this lesson, students will solve a "Join Result Unknown" problem with only 2 addends, with the goal to work toward 3 in coming days. Students will remember the structure of a CGI lesson and will understand that they are expected to try their best to solve the problem using their own strategies. Students will consider the ideas they heard yesterday for support. **Note: Yesterday's chart paper should be displayed on the wall.**
Lesson Goal for Teacher
Teacher will understand the steps and structure to a CGI lesson. The teacher will be able to ask questions to both the sharing and listening students that will help to make connections between the strategies.

Part 1: Pose the Problem

5 Min

Directions: Write the following problem on the top of a sheet of chart paper and place on board **(See Lesson 1 for example)**. Gather students together on the carpet. Read the problem to students. Ask two students to retell the problem to check for understanding. Then, ask the class to use their hands to show "more or less" **(See Lesson 1)** to answer the following comprehension question: "In the end, did Fred have more or less than 7 leaves?" Ask students to talk about why. Call on students who are incorrect to share their ideas first and allow other students to correct them.

Problem: Fred had 7 leaves. He found 11 more outside at recess. How many leaves does Fred have now?	Number Sentence to Match Problem: 7 + 11 = 18

About the problem: This is another **Join Result Unknown** problem. It is ideal to have students solve the same problem type twice in a row so that students who did not have access to the problem on day one will have a chance to try that type again after the support of hearing how their peers solved the previous day.

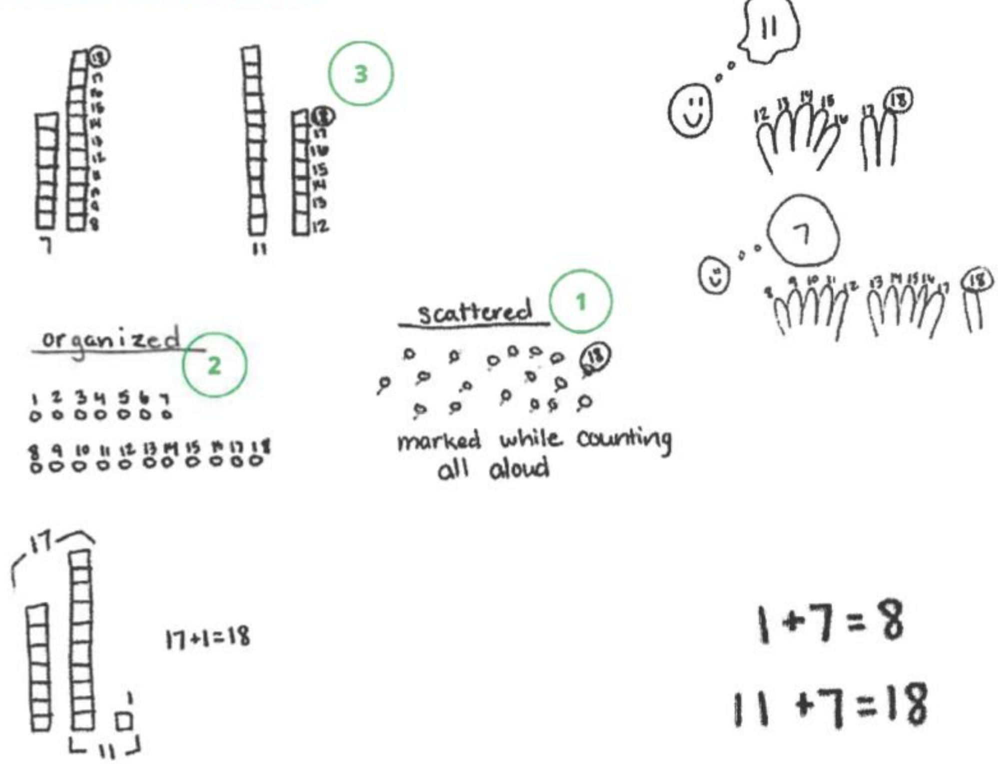
The action of finding 11 more leaves supports students because they can model that action in their strategy (called "Direct Modeling"). The numbers were increased from yesterday to slightly elevate the difficulty. Ultimately, this unit will result in students adding 3 addends within 20. These numbers will begin a conversation about adding a double digit and single digit number, which they will do when they combine 3 in a couple days. If you held a discussion about reversing the order of the number in the previous lesson, there is also an opportunity to try that out here - which is why the 7 was placed first.

Part 2: Students solve independently

10 Min

Directions: Students should return to their seats and work to solve the problem independently. Ask students questions to better understand their work and thinking. Encourage them to do their best to represent or draw their strategy, even if they solved mentally. **Do not show students how to solve the problem.** If they are struggling, try asking them questions that might help them understand the actions in the problem. Suggest that they draw a picture or utilize a tool such as counters. However, you should not tell them how to do so. Move throughout the room and try to glance at what each student is doing. Look for the strategies in the next section or similar strategies. **Choose three students to share.** The guide in the following section shows three selected strategies numbered in green and should be shared in that particular order. Read the rationale for the selection and sequencing of strategies. You may choose to select and sequence the same strategies if they are present in your classroom. If not, you may choose a similar sequence of strategies. Consider ordering them beginning with the least sophisticated - pictorial, concrete, simple. However, do not share with students the reason for the order.

Possible Student Strategies



Why the strategies were selected and rationale for the particular order

The selection and sequencing of strategies are important choices that will determine the direction of the class discussion and the extent to which it helps meet your overall goals. It is important that you select and sequence with both your lesson goal and students in mind. Today, the lesson goal is for students to be able to put together two addends and for the teacher to be able to ask questions that will draw students' attention to the mathematical connections between the different strategies. **It is fine to have the same child share two days in a row.** This may sometimes happen because you are sharing strategies, not students. It is important that it is not seen as "someone's turn." Instead, you are selecting sharing students in strategic ways that will lead to a rich discussion about their approaches to the problem. Also keep in mind that the share-out is for the students listening so think about what they need to see and hear. Today, the first student was selected to compare their strategy to student 2, assuming there are students who may be miscounting because of scattered dot pictures. This will help them see how lines or organization strategies can prevent miscounting. The third student shared to discuss both starting with the larger number and counting on. You will find that students in your class use one or both of these strategies. They are both excellent strategies to share and discuss often. There may be other counting on or reversing addend strategies in your classroom that you would like to share instead.

Part 3: Facilitate the discussion

15 Min

Directions: Bring students back to the carpet. The selected students will share in the order you chose and you will draw what they did on the chart paper. Your drawing might look very similar to the drawings in the "Possible Strategies" section. However, it is important that the child's thinking is represented accurately so your drawing/representation may be different. Ask each student questions as they explain their strategy. You may use the suggested questions below for ideas. Ask the class questions even if they are questions that the student has already addressed. This is an opportunity for students to learn how to listen actively and be able to explain others' ideas.

When the students you selected have finished sharing, your chart paper will again look like the example in the previous lesson.

Some Best Practices:

- Put the child's name above the work - it's their strategy after all!
- Use a different color for each to make them easier to differentiate, especially if you are sharing more than 3 strategies (which you certainly can!)
- When you're ready, include the equation or number sentence to match the child's work beneath their work. Ask the sharing child or the class what equation might match their strategy. Think about the order in which they thought about the numbers.

Whole Class Discussion Questions	
Questions to ask students as they share: **For Student 1:** Was is challenging to keep track of your counting? What did you do to keep track? **For Student 2:** I see that you lined your circles up. Tell us about that, why did you did that? **For Student 3:** I noticed that you started with the 11, did anyone else notice that? Did you get the same total? Why? How did that work? Also, you started counting at 11 instead of 1, tell us more about that.	**Questions to ask the rest of the class anytime during the discussion:** How did student #1 (say their name) count? How was that different than student #2? What was the same about their strategies? Cold call (call on students whether they have their hand up or not) students and ask them why student #3 made an 11 stick, what does that represent? How did he count all of the leaves? Who can restate that?

Post Lesson Reflection for the Teacher

Directions: It is critical that you look at student work and think about the ideas that were shared by students during your lesson today. After the lesson, reflect on the following questions. If possible, discuss with a colleague who is trying the CGI problems along with you.

How many of your students are using derived facts, counting on or starting with the larger number? Have you taught these strategies? Does this surprise you? How might this impact your instruction on these topics going forward?

What surprised you about your students' work and thinking today? Was it different than day 1?

How were your students' strategies both different and similar to the strategies you saw in the "Possible Student Strategies" section?

Did the tools students chose to use change at all?

How are students engaging in the discussion? What strategies could you use to increase engagement? Tomorrow, try "cold calling" and "Turn and talk to your partner."

Lesson 3

Standard: 1.OA.2: Solve word problems that call for addition of three whole numbers whose sum is less than or equal to 20, (e.g. by using objects, drawings, and equations with a symbol for the unknown number to represent the problem.)

Lesson Focus & Goals for Students
In this lesson, students will solve a "Part-Part-Whole: Part Unknown" problem with only 2 addends, with the goal of 3 tomorrow. Students will be able to use strategies that they have been using the previous two days to solve this slightly more challenging problem type. **Note: Yesterday's chart paper should be displayed on the wall.**
Lesson Goal for Teacher
Teacher will understand the steps and structure to a CGI lesson. The teacher will be able to ask questions to both the sharing and listening students that will help to make connections between the strategies. The teacher will be able to notice the various intuitive strategies students are using to solve.

Part 1: Pose the Problem

5 Min

Directions: Write the following problem on the top of a sheet of chart paper and place on board **(See Lesson 1 for example)**. Gather students together on the carpet. Read the problem to students. Ask two students to retell the problem to check for understanding. Then, ask the class to use their hands to show "more or less" **(See Lesson 1)** to answer the following comprehension question: "Did this machine have more than 9 gum balls or less than 9 gumballs?" Ask students to talk about why. Call on students who are incorrect to share their ideas first and allow other students to correct them.

Problem: A gum ball machine had 9 red gum balls and 8 green gum balls. How many gum balls were in the machine?	Number Sentence to Match Problem: $9 + 8 = 17$

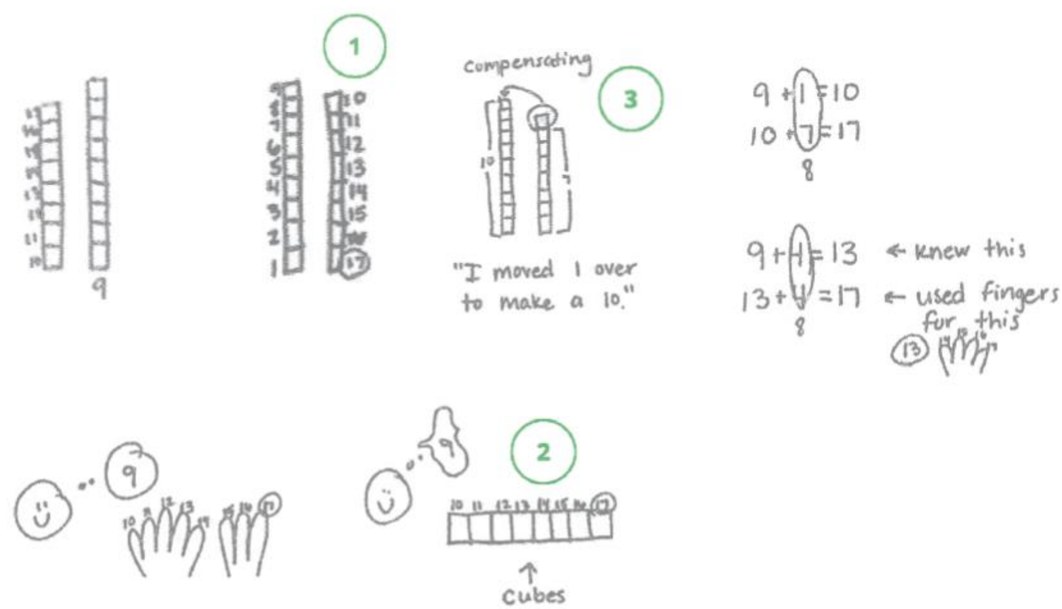
About the problem: This problem type is called "Part-Part-Whole: Whole Unknown" and it is slightly more challenging than the Join Result Unknown problems your students have solved. This is because there is no action in the problem. The gum balls are present in the machine but the action of giving or finding in the previous problem is not there to support direct modelers. However, they will likely intuitively use some of the strategies that they used in the two previous lessons. Let students grapple with this on their own. It is not necessary to discuss the different problem types and do not tell students they can use the same strategy as yesterday. In fact, you should never suggest strategies to students before they solve the problem. Follow the same routine as previous days and allow students to get to work!

Part 2: Students solve independently

10 Min

Directions: Students should return to their seats and work to solve the problem independently. Ask students questions to better understand their work and thinking. Encourage them to do their best to represent or draw their strategy, even if they solved mentally. **Do not show students how to solve the problem.** If they are struggling, try asking them questions that might help them understand the actions in the problem. Suggest that they draw a picture or utilize a tool such as counters. However, you should not tell them how to do so. Move throughout the room and try to glance at what each student is doing. Look for the strategies in the next section or similar strategies. **Choose three students to share.** The guide in the following section shows three selected strategies numbered in green and should be shared in that particular order. Read the rationale for the selection and sequencing of strategies. You may choose to select and sequence the same strategies if they are present in your classroom. If not, you may choose a similar sequence of strategies. Consider ordering them beginning with the least sophisticated - pictorial, concrete, simple. However, do not share with students the reason for the order.

Possible Student Strategies

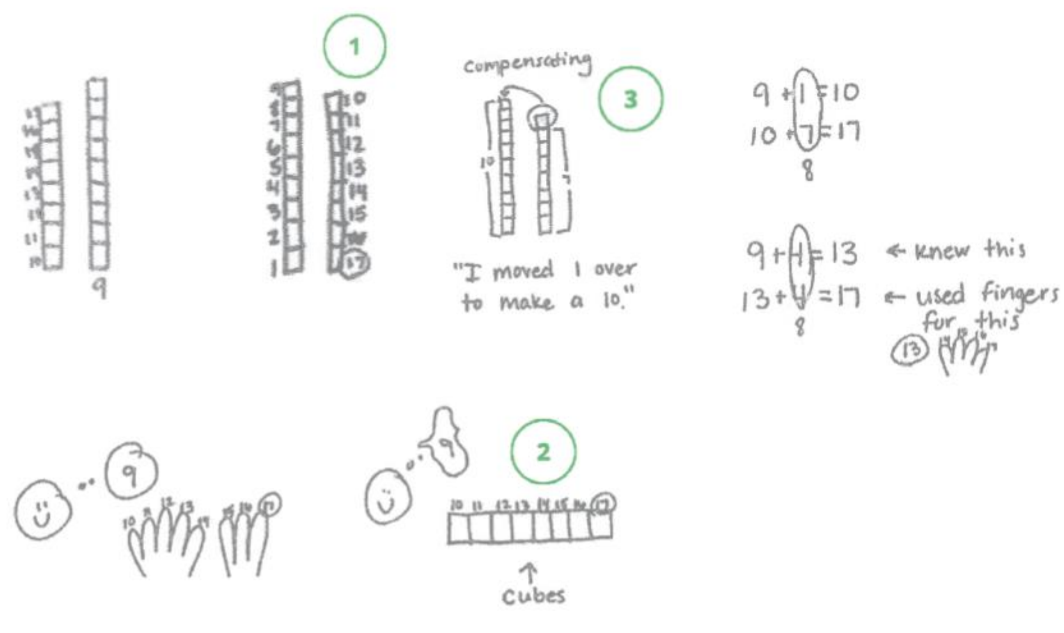


Part 2: Students solve independently

10 Min

Directions: Students should return to their seats and work to solve the problem independently. Ask students questions to better understand their work and thinking. Encourage them to do their best to represent or draw their strategy, even if they solved mentally. **Do not show students how to solve the problem.** If they are struggling, try asking them questions that might help them understand the actions in the problem. Suggest that they draw a picture or utilize a tool such as counters. However, you should not tell them how to do so. Move throughout the room and try to glance at what each student is doing. Look for the strategies in the next section or similar strategies. **Choose three students to share.** The guide in the following section shows three selected strategies numbered in green and should be shared in that particular order. Read the rationale for the selection and sequencing of strategies. You may choose to select and sequence the same strategies if they are present in your classroom. If not, you may choose a similar sequence of strategies. Consider ordering them beginning with the least sophisticated - pictorial, concrete, simple. However, do not share with students the reason for the order.

Possible Student Strategies



Whole Class Discussion Questions	
Questions to ask students as they share: **For Student 1:** Your cubes started out as 10 sticks. Can you show us how you made a 9 and an 8. How did you know you needed to do that? What do those represent? **For Student 2:** Why do you have only 8 cubes? Where is the 9? How did that work? **For Student 3:** When you moved that cube, did you know it would still be the same amount? How? Why did you move it to the 9 instead of moving one to the 8?	**Questions to ask the rest of the class anytime during the discussion:** Cold call students who were not able to solve the problem. Ask them to retell strategy #1. Why did he make a 9 stick and an 8 stick? What do those represent? What are they from the story? Are those the gum balls? What did he do next? Did he count all of them? Why? Let's talk about when student #3 moved that cube over. Who can retell why she did that? How did that work, how was it the same amount? Tell me what you think of this number sentence: 9+8=10+7. **Turn and talk to your partner.** Is that true? How does that align with her work?

Post Lesson Reflection for the Teacher

Directions: It is critical that you look at student work and think about the ideas that were shared by students during your lesson today. After the lesson, reflect on the following questions. If possible, discuss with a colleague who is trying the CGI problems along with you.

What did you notice about your students' work and thinking today? Was it different than past lessons?

How were your students' strategies both different and similar to the strategies you saw in the "Possible Student Strategies" section?

Did the tools students chose to use change at all?

Tomorrow, your students will be solving a problem with 3 addends. What kinds of strategies do you expect to see? Remember that you will not be preteaching strategies. Based on what you've seen so far, what do you think they will come up with on their own?

Lesson 4

Standard: 1.OA.2: Solve word problems that call for addition of three whole numbers whose sum is less than or equal to 20, (e.g. by using objects, drawings, and equations with a symbol for the unknown number to represent the problem.)

Lesson Focus & Goals for Students
In this lesson, students will solve a "Part-Part-Part-Whole: Part Unknown" problem with 3 addends, the ultimate goal of this mini unit. The problems they have solve and the strategies they have seen will support them to be able to solve this problem using their own ideas. Students will understand that the addends can be put together in different orders and the total remains the same. **Note: Yesterday's chart paper should be displayed on the wall.**
Lesson Goal for Teacher
The teacher will be able to select and sequence strategies in a way that highlights that the addends can be switched in various ways and that mathematicians choose to put together numbers that are friendly to them. To do this, the teacher will also write the number sentence to match each student's work.

Part 1: Pose the Problem

5 Min

Directions: Write the following problem on the top of a sheet of chart paper and place on board **(See Lesson 1 for example).** Gather students together on the carpet. Read the problem to students. Ask two students to retell the problem to check for understanding. Then, ask the class to use their hands to show "more or less" **(See Lesson 1)** to answer the following comprehension question: "Did the gym have more or less than 8 balls?" Ask students to talk about why. Call on students who are incorrect to share their ideas first and allow other students to correct them.

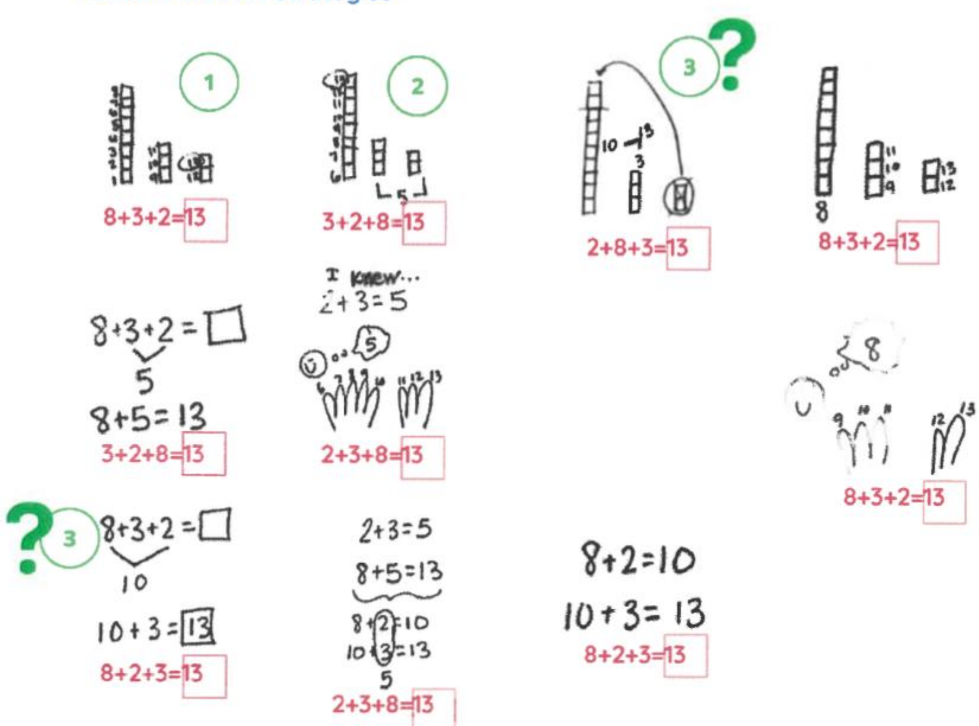
Problem: In the gym, there were 8 soccer balls, 3 basketballs and 2 footballs. How many balls were in the gym?	Number Sentence to Match Problem: 8 + 3 + 2 = 13
About the problem: This problem type is a "Part-Part-Part-Whole: Whole Unknown" problem because now there are 3 addends and still no action in the problem. Students will find the total number of balls in the gym and there two different ways the number selection might support students. Students may have worked with 5 partners and the 3 + 2 may jump out at them as a fact they know. Similarly, students may see the 8 and 2 and know that that is 10. Both of these are great starting points for solving and hopefully you will be able to share both for variety and different discussion points.	

Part 2: Students solve independently

10 Min

Directions: Students should return to their seats and work to solve the problem independently. Ask students questions to better understand their work and thinking. Encourage them to do their best to represent or draw their strategy, even if they solved mentally. **Do not show students how to solve the problem.** If they are struggling, try asking them questions that might help them understand the actions in the problem. Suggest that they draw a picture or utilize a tool such as counters. However, you should not tell them how to do so. Move throughout the room and try to glance at what each student is doing. Look for the strategies in the next section or similar strategies. **Choose three students to share.** The guide in the following section shows three selected strategies numbered in green and should be shared in that particular order. Read the rationale for the selection and sequencing of strategies. You may choose to select and sequence the same strategies if they are present in your classroom. If not, you may choose a similar sequence of strategies. Consider ordering them beginning with the least sophisticated - pictorial, concrete, simple. However, do not share with students the reason for the order. Today, the number sentence to match each student's work is beneath the strategy. Use this to support your discussion and write the number sentence beneath each student's work on your chart paper.

Possible Student Strategies



Why the strategies were selected and rationale for the particular order

Today, students were challenged with a problem with 3 addends and you can see by the number of strategies (and of course there are many more!) that there are a number of ways to take your whole class conversation. It's important to keep your lesson goal in mind. Today, the goal for students is to be able to solve this on their own as always, but also to see that the addends can be put together in different orders. So, the 3 strategies selected above present 3 different orders to add the numbers. It is important to highlight this in the wrap up so look for this no matter what strategies you choose. Strategy #1 is a very simple, concrete, count all strategy for those students who were not able to complete this problem. Strategy #2 starts the conversation about grouping these numbers strategically.Think about the two #3 strategies presented. Which one might make more sense for your class? The cubes are more concrete but some students may be ready to see and discuss the equations. Could you share them both and have 4 strategies today? Absolutely. Do what is best for your students based on their thinking that you are seeing.

Part 3: Facilitate the discussion

15 Min

Directions: Bring students back to the carpet. The selected students will share in the order you chose and you will draw what they did on the chart paper. Your drawing might look very similar to the drawings in the "Possible Strategies" section. However, it is important that the child's thinking is represented accurately so your drawing/representation may be different. Ask each student questions as they explain their strategy. You may use the suggested questions below for ideas. Ask the class questions even if they are questions that the student has already addressed. This is an opportunity for students to learn how to listen actively and be able to explain others' ideas.

When the students you selected have finished sharing, your chart paper will again look like the example in Lesson 1.

Whole Class Discussion Questions	
Questions to ask students as they share: **For Student 1:** I see you made 3 sticks or groups of cubes. How do you know you should do that? Why did you have to count ALL those cubes? **For Student 2:** How did you count the 3 cubes and the 2 cubes, did you just know that? Why did you go to those first? **For Student 3:** What numbers did you group together first? Why?	**Questions to ask the rest of the class anytime during the discussion:** Cold call students who were not able to solve the problem. Ask them why student 1 had 3 numbers and why he counted all of them. What do those sticks represent? Why do they all need to be counted? Have students retell a particular strategy - cold call students for this. Ask them about the order that the sharing student added in. Did student 3 add in the same way or a different way than student 2? Look at the number sentences to match their work. **So it looks like when adding 3 numbers, you can add in different ways and what happens? Does that happen every time?**

Post Lesson Reflection for the Teacher

Directions: It is critical that you look at student work and think about the ideas that were shared by students during your lesson today. After the lesson, reflect on the following questions. If possible, discuss with a colleague who is trying the CGI problems along with you.

What did you notice about your students' work and thinking today? Was it different than past lessons? Did they group the numbers in different ways than adding from left to right?

Did more students make 5 first or make 10? How could that information help you if you were writing your own problem for tomorrow?

How were your students' strategies both different and similar to the strategies you saw in the "Possible Student Strategies" section?

Which students are needing more support to solve these problems? How could you use cold calling to make sure they are actively listening and engaged in the lesson?

Lesson 5

Standard: 1.OA.2: Solve word problems that call for addition of three whole numbers whose sum is less than or equal to 20, (e.g. by using objects, drawings, and equations with a symbol for the unknown number to represent the problem.)

Lesson Focus & Goals for Students

In this lesson, students will solve a "Part-Part-Part-Whole: Part Unknown" problem with 3 addends, the ultimate goal of this mini unit. The problems they have solve and the strategies they have seen will support them to be able to solve this problem using their own ideas. Students will understand that the addends can be put together in different orders and the total remains the same. They will also be able to think about **strategic ways to group the addends.**

Note: Yesterday's chart paper should be displayed on the wall.

Lesson Goal for Teacher

The teacher will be able to select and sequence strategies in a way that highlights that the addends can be switched in various ways and that mathematicians choose to put together numbers that are friendly to them. To do this, the teacher will also write the number sentence to match each student's work and **will draw students' attention to these number sentences throughout the discussion.**

Part 1: Pose the Problem

5 Min

Directions: Write the following problem on the top of a sheet of chart paper and place on board **(See Lesson 1 for example)**. Gather students together on the carpet. Read the problem to students. Ask two students to retell the problem to check for understanding. Then, ask the class to use their hands to show "more or less" **(See Lesson 1)** to answer the following comprehension question: "Did Luke have more or less than 7 stickers on his page?" Ask students to talk about why. Call on students who are incorrect to share their ideas first and allow other students to correct them.

Problem: Luke had a sticker page with 4 star stickers, 7 heart stickers and 6 smiley face stickers. How many stickers were on his page?	Number Sentence to Match Problem: $4 + 7 + 6 = 17$

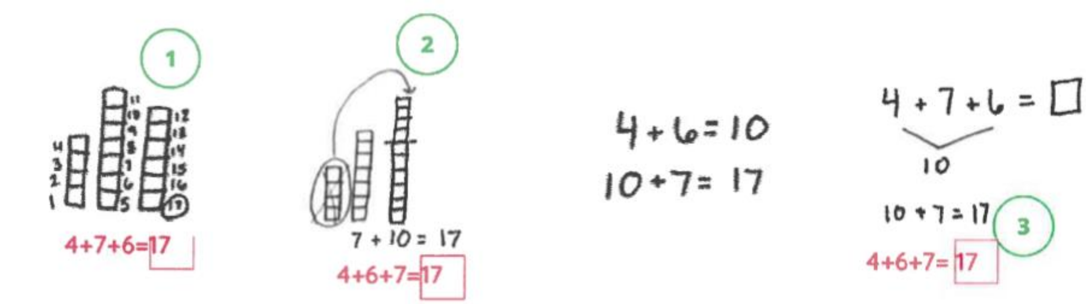
About the problem: This problem type is a "Part-Part-Part-Whole: Whole Unknown" problem because now there are 3 addends and still no action in the problem. It is important that 1st graders come to know the partners of 10 and ultimately use their deep understanding of our base 10 number system to solve a wide variety of problems which is why there is a 10 partnership in this problem. The numbers have increased slightly from the previous problem but still remain with a total within 20, the first grade standard this unit aims to address.

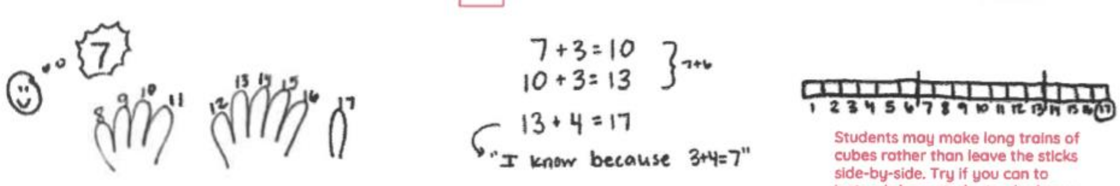
Part 2: Students solve independently

10 Min

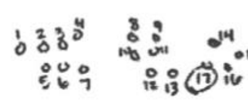
Directions: Students should return to their seats and work to solve the problem independently. Ask students questions to better understand their work and thinking. Encourage them to do their best to represent or draw their strategy, even if they solved mentally. Do not show students how to solve the problem. If they are struggling, try asking them questions that might help them understand the actions in the problem. Suggest that they draw a picture or utilize a tool such as counters. However, you should not tell them how to do so. Move throughout the room and try to glance at what each student is doing. Look for the strategies in the next section or similar strategies. Choose three students to share. The guide in the following section shows three selected strategies numbered in green and should be shared in that particular order. Read the rationale for the selection and sequencing of strategies. You may choose to select and sequence the same strategies if they are present in your classroom. If not, you may choose a similar sequence of strategies. Consider ordering them beginning with the least sophisticated - pictorial, concrete, simple. However, do not share with students the reason for the order. Today, the number sentence to match each student's work is beneath the strategy. Use this to support your discussion and write the number sentence beneath each student's work on your chart paper.

Possible Student Strategies





Students may make long trains of cubes rather than leave the sticks side-by-side. Try if you can to instead share students who leave them side-by-side because this allows them to see the tens or separate numbers. Alternatively, you can share students who make a train with alternating colors that allow them to still see tens or other important numbers that help them count.



Some students may still be making direct modeling pictures of scattered dots or circles. This is ok! However, draw their attention (during the discussion) to the advantages of cubes - the organization and the way it allows you to easily count - this will be important later when the numbers are large.

Why the strategies were selected and rationale for the particular order

As always, it's important to keep your lesson goal in mind when selecting strategies to share. Yesterday, the goal was for students to see that the order of the addends can be changed. In addition to that, it is important today that they discuss and understand strategic ways to group the addends, including finding 10 partners. Student #1 shared for the same reason as past days - to support those students who did not know how to get started on the problem. This was a direct modeling, count all strategy that they could use to access the problem. The next strategies move students toward the lesson goal of understanding the advantage of grouping by tens. #2 shows a concrete way to do this which relates nicely to #3. It will be important in the discussion to draw students' attention to the similarity between those strategies. If these strategies are not present in your classroom, find other strategies that group the 4 and 6 first.

Part 3: Facilitate the discussion

15 Min

Directions: Bring students back to the carpet. The selected students will share in the order you chose and you will draw what they did on the chart paper. Your drawing might look very similar to the drawings in the "Possible Strategies" section. However, it is important that the child's thinking is represented accurately so your drawing/representation may be different. Ask each student questions as they explain their strategy. You may use the suggested questions below for ideas. Ask the class questions even if they are questions that the student has already addressed. This is an opportunity for students to learn how to listen actively and be able to explain others' ideas.

When the students you selected have finished sharing, your chart paper will again look like the example in Lesson 1.

Whole Class Discussion Questions	
Questions to ask students as they share: **For Student 1:** I see you made 3 sticks or groups of cubes. How do you know you should do that? Why did you have to count ALL those cubes? **For Student 2**: Why did you put those two sticks together first? **For Student 3**: What numbers did you group together first? Why?	**Questions to ask the rest of the class anytime during the discussion:** Cold call students who were not able to solve the problem. Ask them why student 1 had 3 numbers and why he counted all of them. What do those sticks represent? Why do they all need to be counted? Have students retell a particular strategy - cold call students for this. Ask them about the order that the sharing student added in. Did student 3 add in the same way or a different way than student 2? What are the number sentences to match their work? Are they the same? **I noticed several students grouped the 4 and 6 together today - turn and talk to your partner about why students may have chosen to group the addends that way.**

Post Lesson Reflection for the Teacher

Directions: It is critical that you look at student work and think about the ideas that were shared by students during your lesson today. After the lesson, reflect on the following questions. If possible, discuss with a colleague who is trying the CGI problems along with you.

What did you notice about your students' work and thinking today? Was it different than past lessons? Did they group the numbers in different ways than adding from left to right?

Did any more students notice the 10 partners in the problem and try to add those first? If not, what else could you do in your classroom to support their understanding of the partners of 10?

Congratulations on completing this Mini Unit! By now, you should understand the structure of Cognitively Guided Instruction lessons and you should have learned a bit about your students' thinking and ability to solve problems on their own. Using this knowledge, what problems could you give them next? Here are a couple ideas:

Ian had 8 red markers, 1 blue marker and 9 yellow markers at his table. How many markers did he have?	Ceci had _ pink beads, _ black beads and _ gold beads. How many beads did she have?

What numbers do you think your students should work with next?

Appendix B CGI Mini Unit Feedback Form

CGI Mini Unit Feedback Survey

This survey should only be completed AFTER implementing all lessons from the mini unit with your class. Your honest feedback will be used to improve the product. Thank you for taking the time to teach all the lessons and complete this survey!

* Required

1. How would you describe your knowledge of Cognitively Guided Instruction (CGI) BEFORE you implemented this mini unit?

   Mark only one oval.

   - I had never heard of CGI.
   - I knew what CGI was but had never used CGI in my own classroom.
   - I was beginning to use Cognitively Guided Instruction in my classroom this year.
   - I had been using Cognitively Guided Instruction in my classroom for 1 year or more.

2. Choose the best definition of Cognitively Guided Instruction. *

   Mark only one oval.

   - Teaching students how to solve word problems.
   - Providing word problem experiences for students.
   - Teaching math concepts through word problems.
   - Using student thinking to drive mathematics instruction.

3. Did you add or omit lessons? Did you change the wording, numbers or problem type for any lesson? Describe these changes and what made you decide to make the changes. If you did not modify any of the lessons, you may state that instead.

4. Choose the best description of how you read the lessons. *

Mark only one oval.

- I read parts of the lesson.
- I read the entire lesson each day.
- I read the whole lesson on the first day then parts of the lessons on subsequent days.

5. If you responded that you read parts of the lesson, please briefly describe which parts you read:

6. Which features of the lessons were most clear and helpful to you when planning and implementing the lesson?

Check all that apply.

- Standard and Lesson Goal for the student
- Lesson Goal for the Teacher
- A problem provided daily
- Student Worksheet
- "About the Problem" section
- Directions for Part 1: Posing the Problem
- Directions for Part 2: Students Solve Independently
- Possible Student Strategies
- Rationale for why you might select similar strategies and sequence them in a particular order
- Part 3: Directions for Facilitating the Discussion
- Whole Class Discussion Questions
- Post Lesson Reflection for the Teacher

7. Which features of the lessons were the LEAST clear and helpful to you when planning and implementing the lesson?

Check all that apply.

Standard and Lesson Goal for the student
Lesson Goal for the Teacher
A problem provided daily
Student Worksheet
"About the Problem" section
Directions for Part 1: Posing the Problem
Directions for Part 2: Students Solve Independently
Possible Student Strategies

Rationale for why you might select similar strategies and sequence them in a particular order
Part 3: Directions for Facilitating the Discussion
Whole Class Discussion Questions
Post Lesson Reflection for the Teacher

8. As you were reading, did you notice any lesson features that were designed to help you learn about your students' thinking?

Mark only one oval.

- Yes, I noticed features that were meant for teacher learning.
- No, I did not notice features that were meant for student learning.

9. As you were reading, did you notice any lesson features that were designed to help you learn how to teach CGI lessons?

Mark only one oval.

- Yes, I noticed features that were designed to help me learn about CGI lessons.
- No, I thought of this only as a curriculum for student learning.

10. On a scale of 1-5, how comfortable would you feel making your own CGI lesson plan after the completion of this unit? *Mark only one oval.*

1 I would not
2
3
4
5 I would be able to write the problem, select and sequence strategies and lead a rich

11. Think of any part of the lesson that was unclear or confusing. Is there any other feedback that you could provide to improve this product - layout, clarity, directions, images?

Appendix C CGI Mini Unit Responses

CGI Mini Unit Feedback Survey

2 responses

Publish analytics

How would you describe your knowledge of Cognitively Guided Instruction (CGI) BEFORE you implemented this mini unit?

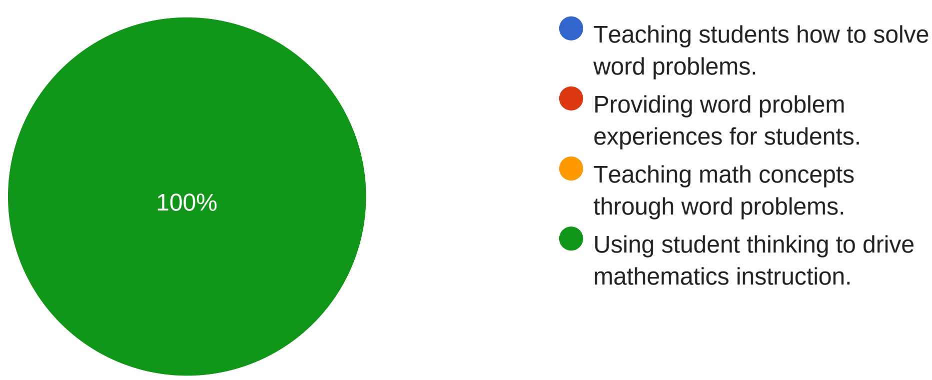
Copy

2 responses

Choose the best definition of Cognitively Guided Instruction.

Copy

2 responses



Did you add or omit lessons? Did you change the wording, numbers or problem type for any lesson? Describe these changes and what made you decide to make the changes. If you did not modify any of the lessons, you may state that instead.

2 responses

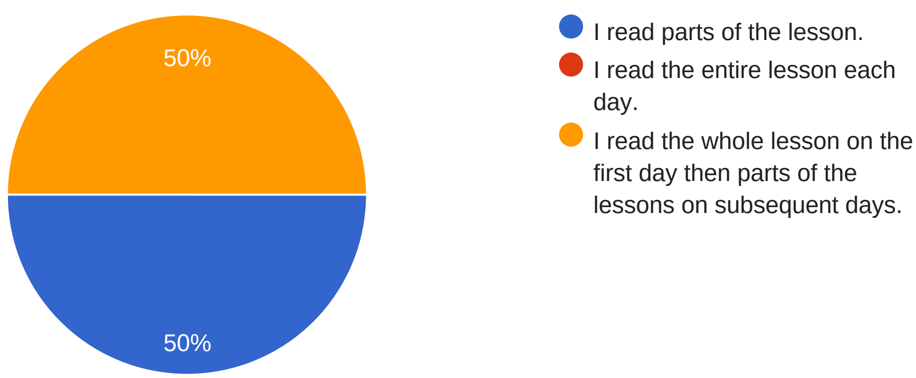
No

I only taught lessons 3-5. My class understands the addition of two addends so lesson 3 provided me with an opportunity to teach with CGI and a good review for the class. I also decided on lesson 5 not to discuss the ways the kids could solve the problem and just had them try to solve the problem on their own.

Choose the best description of how you read the lessons.

Copy

2 responses



If you responded that you read parts of the lesson, please briefly describe which parts you read:

2 responses

Pose the Problem, Students Solve Independently, Facilitate the discussion

On lesson 1, I read the whole lesson but skipped over a few of the "why the strategy works" information. For the next lessons, I read the introduction and skimmed the rest of the lessons. I made sure to read the parts that described the students solving independently. I thought the prompts and suggestions in that section were enlightening. I also found value in the pictures of possible student strategies. The class also enjoyed sharing their solutions all together and creating the chart paper with me.

Which features of the lessons were most clear and helpful to you when planning and implementing the lesson?

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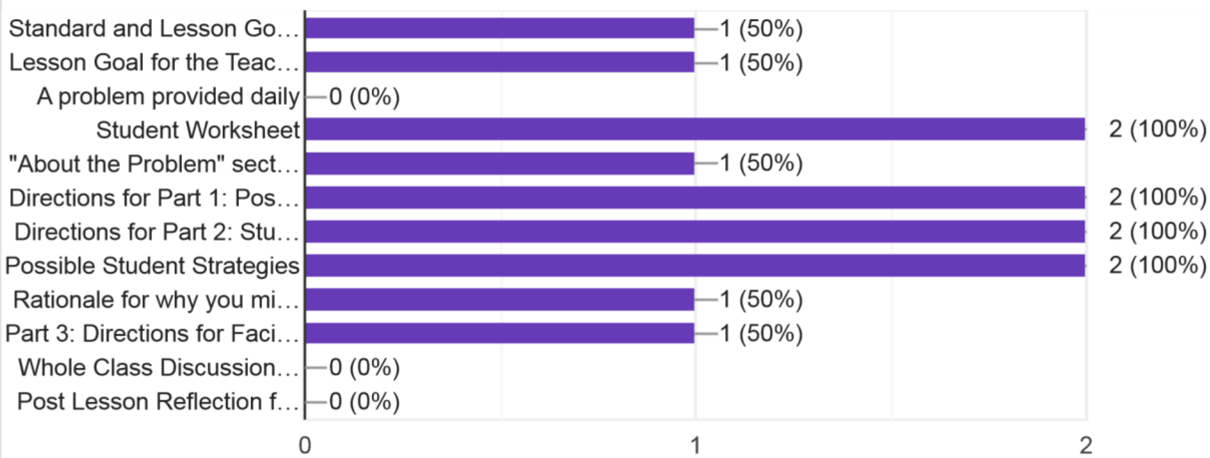
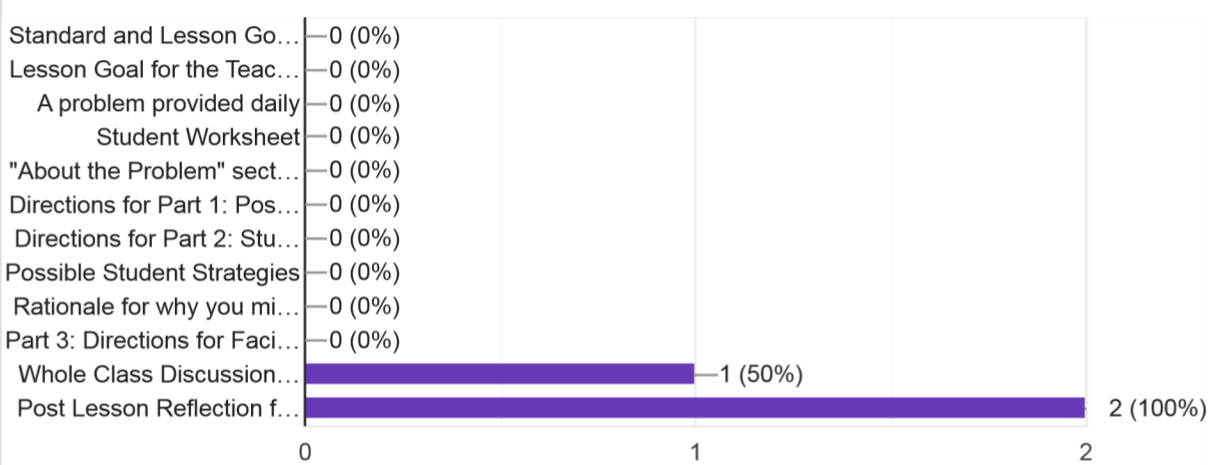
2 responses



Which features of the lessons were the LEAST clear and helpful to you when planning and implementing the lesson?

Copy

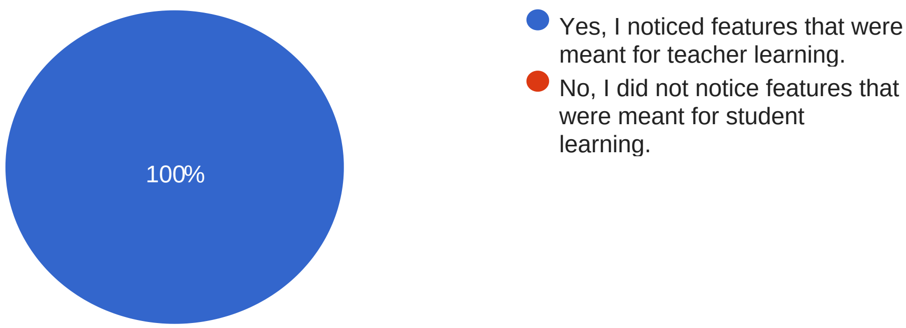
2 responses



As you were reading, did you notice any lesson features that were designed to help you learn about your students' thinking?

Copy

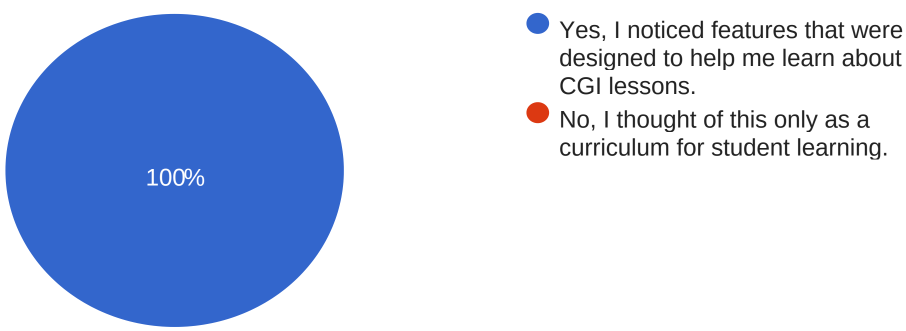
2 responses



As you were reading, did younotice any lesson features that were designed to help you learn how to teach CGI lessons?

Copy

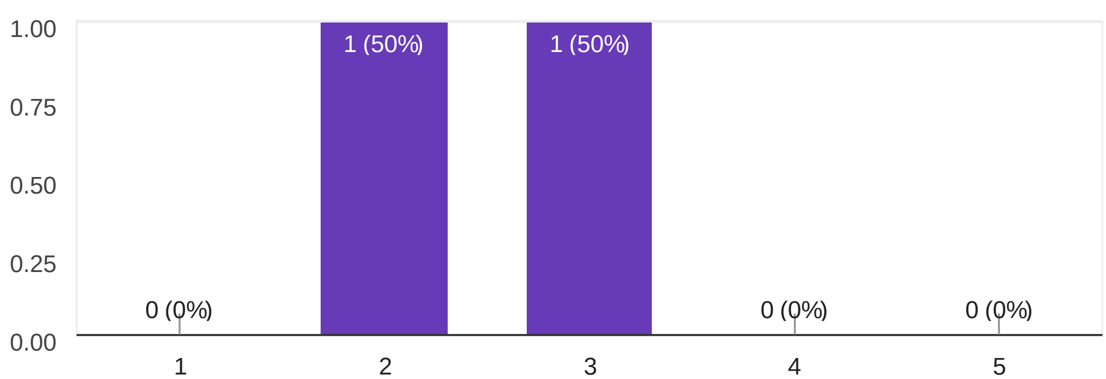
2 responses



On a scale of 1-5, how comfortable would you feel making your own CGI lesson plan after the completion of this unit?

Copy

2 responses



Think of any part of the lesson that was unclear or confusing. Is there any other feedback that you could provide to improve this product - layout, clarity, directions, images?

2 responses

None were confusing. Thanks so much!

n/a

Appendix D IRB Form

KANSAS STATE UNIVERSITY | University Research Compliance Office

TO: Michael Lawson
Elementary Education OLD

Proposal Number: IRB-11302

FROM: Lisa Rubin, Chair
Committee on Research Involving Human Subjects
DATE: 10/11/2022

RE: Proposal Entitled, "Creating a Cognitively Guided Instruction Guidebook: The Research and Development of an Educative Curriculum for Teachers."

The Committee on Research Involving Human Subjects / Institutional Review Board (IRB) for Kansas State University has reviewed the proposal identified above and has determined that it is EXEMPT from further IRB review. This exemption applies only to the proposal - as written – and currently on file with the IRB. Any change potentially affecting human subjects must be approved by the IRB prior to implementation and may disqualify the proposal from exemption. Based upon information provided to the IRB, this activity is exempt under the criteria set forth in the Federal Policy for the Protection of Human Subjects, **45 CFR §104(d), category:Exempt Category 1.**

Certain research is exempt from the requirements of HHS/OHRP regulations. A determination that research is exempt does not imply that investigators have no ethical responsibilities to subjects in such research; it means only that the regulatory requirements related to IRB review, informed consent, and assurance of compliance do not apply to the research.

Any unanticipated problems involving risk to subjects or to others must be reported immediately to the Chair of the Committee on Research Involving Human Subjects, the University Research Compliance Office, and if the subjects are KSU students, to the Director of the Student Health Center.

Electronically signed by Phill Vardiman on 10/11/2022 12:44 PM ET
On Behalf of IRB Chair

203 Fairchild Hall, Manhattan, KS 66502 | (785) 532-3224 | fax: (785) 532-3278