AN EXPERIMENTAL INVESTIGATION OF NATURAL CONVECTION IN MERCURY AT LOW GRASHOF NUMBERS

by

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INTRODUCTION

Physically, free or natural convection flow arises from a difference in density in the fluid. Visualize a heated surface exposed to colder air in a room. Since the density of air near the heated surface is less than that of mainbody of the air, buoyant forces cause an upward flow of air near the surface. If the surface were colder than the air, because of greater density near the surface, the air would flow downward. In either case heat is conducted through the air layers and is carried away by bulk motion or convection. Although both conduction and convection are involved, the process is called "natural" or "free" convection.

In recent years, there has been a growing interest in the use of low-melting metals as heat transfer media. Liquid metals have had limited applications in the field of heat transfer, but the developments in handling and metering liquid metals and their suitability for high temperature, high-heat-flux applications have led to a considerable amount of theoretical and experimental investigation in this field. Liquid metals have been used extensively to cool valves in aircraft engines. Power generation using mercury instead of water as the working fluid in boilers has been carried out commercially since 1922. The present interest in liquid metals in the field of heat transfer stems from their use in atomic reactors. The use of nuclear reactors in spacecraft or satellite systems has been suggested and the cooling of such reactors would be in general achieved using liquid metals. The natural convection effects may be quite significant and, in fact, there may be an advantage in eliminating the pumping system and using natural convection for heat transfer.

Theoretically, the natural convection phenomenon has been of interest in that it represents a system of coupled differential equations. This coupling results from the fluid flow being caused by the temperature gradient in the thin boundary layer surrounding the heated object. These equations and their solutions have been subject of numerous papers. However, experimental verifications of the analytical work have not been as numerous.

This work experimentally investigates the laminar, steady, two dimensional, free convection heat transfer from a vertical flat plate with a uniform heat flux, immersed in mercury at rest. The work is especially directed toward the condition of low Grashof numbers (Gr_x^*) where appreciable deviation from the boundary layer theory are predicted by the perturbation analysis. There have been no experimental investigations for fluids at small Grashof numbers.

REVIEW OF LITERATURE

This review is concerned with research in the field of natural convection heat transfer, the results of which have been published until early 1970. Recent literature has been especially well surveyed.

In 1881, L. Lorenz [83]* presented, in his pioneer paper, a theoretical analysis for natural convection from a vertical, isothermal, flat plate in air at rest. He assumed that the fluid flow was parallel to the plate, and that the fluid temperature at any point depended only on the distance from the plate. Although inadequate assumptions were used, his results were in agreement with the then-existing experimental results. In 1928, Nusselt and Jurges [96] improved Lorenz's work by taking into account his improper assumptions, and measured the temperature field in air at 100° C.

In 1904, L. Prandtl [103] simplified the Navier-Stokes equations by dividing the flow into two regions; a thin boundary layer along the solid surface where viscous effects were important and the velocity gradient normal to the wall was large, and a bulk flow region where viscous effects could be neglected. The first exact** solution of the boundary layer equations for the case of an isothermal vertical flat plate was developed by

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* The number corresponds to the reference in the Bibliography.

** A solution which is obtained using a similarity transformation is conventionally called "exact", even though the solutions are obtained numerically via the digital computer.

E. Pohlhausen. In 1930, he applied a similarity transformation method he had used in 1921 [101] for the boundary layer on a semi-infinite flat plate parallel to a uniform flow. He showed how the resulting partial differential equations could be transformed into ordinary differential equations with a single independent variable, i.e., a similarity variable. Pohlhausen obtained a numerical solution of these ordinary differential equations for a Prandtl number of 0.733 and compared the results with experimental values of the temperature and veloxity gradients in air which were measured by Schmidt and Beckmann [120] in 1930.

An experimental correlation derived in 1934 by H. Lorenz [84] based on measurements on a vertical hot plate in oil was given by Schlichting [118]. In 1935 and 1936, Weise [152] and Saunders [108], obtained extensive data for short vertical plates, which were later correlated by McAdams [91].

In 1939, Saunders [107] also presented approximate solutions for air and compared his results with those of Pohlhausen [120] and Squire [53]. Saunders obtained some experimental data for mercury and water and appears to have been the first to study free convection in a liquid metal. However, the plate he used was attached to a surface of fireclay containing a heater coil, and it was a portion of the wall surface. Therefore, it had no leading and trailing edges.

In 1946, H. Schuh [122] extended Pohlhausen's calculations to high Prandtl numbers of 10, 100, and 1000, and Sugawara and Michiyoshi [138] obtained numerical solutions for comparatively small Prandtl numbers of 0.03, 0.09, and 0.5.

In 1953, Ostrach [98], starting from the complete steady state equations for variable properties, determined the conditions under which E. Pohlhausen's equations adequately describe the physical process. These equations were then solved numerically for various Prandtl numbers: 0.01, 0.72, 0.733, 1, 2, 10, 100 and 1000. Ostrach compared his solutions with the experimental data in air by Schmidt and Beckmann [120], and pointed out that, in general, the agreement was good for small values of the similarity variable and less satisfactory for larger values of the similarity variable.

All the approaches mentioned so far were concerned with isothermal vertical plates or planes. The present work, however, is concerned with a uniformly heated, vertical plate. In 1956, Sparrow and Gregg [131] analyzed laminar free convection from a uniformly heated (constant heat flux), vertical, flat plate by determining a similarity transformation which would reduce the boundary-layer equations to ordinary differential equations. These equations were solved numerically for Prandtl numbers of 0.1, 1, 10 and 100, and Dotson's experimental data [23] for air were used to verify the theory. In 1964, Chang, et al. [15] extended Sparrow and Gregg's solution to small Prandtl numbers of 0.01 and 0.03 for liquid metals.

In 1955, Sparrow [129] used the approximate Von Karman-Pohlhausen integral method to solve the boundary layer equations for natural convection from a vertical plate with a nonuniform wall heat flux and wall temperature for Prandtl numbers of 0.01 to 1000. Results for the case of constant heat flux were in good agreement with those obtained using the similarity transformation [131].

In 1956, Finston [38] showed that according to the boundary layer theory the problem of free convection past a vertical plate had an exact solution for a surface temperature which is proportional to a power of the distance from the leading edge of the plate. Foote [39], in 1958, extended Finston's method and obtained the solution by asymptotic expansions, and Niuman and Pohlhausen [94] numerically evaluated Finston's equations for a Prandtl number of 0.733.

Sparrow [128], in his 1956 Ph.D. Thesis, included the temperature dependence of physical properties when he solved boundary layer equations for an isothermal, vertical, flat plate. He developed a reference temperature for perfect gases and liquid metals (mercury) so that the results from the constant property analysis could be used to approximate the values of temperature-dependent physical properties. Sparrow and Gregg [133], in 1958, extended the analysis to treat the variable fluid-property problem in free convection. Tanaev [142], in 1956, studied the effect of variable viscosity on laminar free convection of a gas. Gebhart [45], in 1962, indicated that viscosity dependence could result in an inflexion point for gases at low temperature or for higher Prandtl number fluids whose viscosities are large.

In 1959, Fujii [41] applied a modified integral method for laminar free convection from a vertical flat plate and supplemented Squire's approximation [54]. He also treated non-isothermal surface problems. In addition, he analyzed turbulent free convection from a vertical surface [40]. Bobco [6], in 1959, also used the integral technique to obtain an approximate solution in closed form to the problem of nonuniform wall heat flux.

In 1960, Yang [156] established the necessary and sufficient conditions required for the existence of similarity solutions to the problem of steady and unsteady free convection on vertical plates with various surface temperature distributions.

Then in 1961, he presented an improved integral procedure for compressible fluids in laminar natural convection near a flat vertical plate [153].

In 1962, Gebhart [42] investigated the effect of viscous dissipation in the natural convection boundary layer close to a vertical flat plate, and concluded that the effect is small for all Prandtl numbers where the only buoyant force constitutes body force. Acrivos [2] showed how an approximate but accurate expression could be obtained for the rate of heat and mass transfer in laminar boundary-layer flows by piecing together certain asymptotic solutions. Acrivos pointed out that free convection is mathematically similar to forced convection if the velocity is replaced by a characteristic velocity.

Scherberg [114], in 1962, investigated the effect of leading edge configuration, and found that it did not affect the velocity and temperature profiles at sufficient upstream distances except shifting the relative starting point of boundary layer. Scherberg [112], in 1965, extended his previous work by matching the solutions at the leading edge with the known solution immediately above this region. Arbitrary surface temperature variations along a vertical flat plate were treated in 1964 by Scherberg [113] using an integral technique.

In 1958, Schecter and Isbin [111] presented theoretical and experimental analyses of natural convection heat transfer in water at 4° C where water has its maximum density. Similarity equations for an isothermal plate were solved on an analog computer. The results indicated that fluid motion should occur in both the upward and downward directions simultaneously. These results were verified experimentally on a one-foot square, vertical, aluminum, isothermal plate immersed in a large container of water. Goren [56], in 1966, solved this same problem (without any knowledge of Schecter and Isbins' work) and concluded that the convection currents would be reduced. Vanier and Tien [147] extended Goren's work in 1967. They showed that heat transfer coefficients investigated by Goren were too low by 15 per cent and were restricted to plate temperatures of less than 8° C. They found more accurate solutions which are applicable to plate temperatures of up to 35° C.

Most of the analytical solutions of laminar free convection presented up to 1964 were based on Prandtl's boundary layer assumptions which apply for large Grashof numbers. In 1964, Yang and Jerger [158], presented for the first time a perturbation analysis which they expected to be more accurate for moderate Grashof numbers. Their analytical method was similar to that applied by Kuo [76] to forced convection flow over a horizontal plate. However, the free convection problem is more complicated due to the coupled motion and energy equations. Numerical solutions were presented for Prandtl numbers of 0.72 and 10.

In 1965, Suriano, Yang, and Donlon [139] developed a perturbation method for extremly small Grashof numbers (less than one). In 1968, Suriano and Yang [140] extended the previous work [139] for small and moderate Grashof numbers (Rayleigh number from 1 up to 300) with Prandtl numbers of 0.72 and 10. They concluded that effects of viscous dissipation were negligible, and that the effects of Grashof number (or Rayleigh number) on the free convection phenomena may be broken down into three regions with different characteristics. For Rayleigh numbers up to unity, heat is transferred by pure conduction and slow flow field varies about linearly with Grashof numbers. In the Rayleigh number region from one to about fifty, convection effects start to be increasingly more important, while conduction still persists, especially in the immediate neighborhood of the plate. For Rayleigh numbers above fifty, a boundary layer type of behavior is developed along the vertical plate. This is accompanied by sharp increase in Nusselt numbers. Suriano and Yang indicated that as Rayleigh number increases beyond 300, both the flow and energy fields would vary monotonically toward true boundary layer behaviors. The numerical results were compared with the existing experimental correlations [13, 59, 65, 91] and it was claimed that, even though there exist discrepancies between the theoretical and the experimental results, this theoretical work could be considered as reasonable, considering the order of magnitude of discrepancies between the experimental correlations.

The perturbation analysis assumed that streamlines leaving the trailing edge of the plate would essentially be parallel to the plate. Yang [154], in 1964, theoretically studied the momentum and energy fields in this laminar wake region above the plate. He carried out numerical calculations for Prandtl numbers of 0.72 and 10, and found that spreading of the boundary layer was rather gradual and that the assumption of streamlines parallel to the plate was well justified.

In 1966, Chang, Akins, and Bankoff [16] extended Yang and Jergers' [158] perturbation analysis to the case of a uniformly heated plate. They presented numerical solutions for Prandtl numbers of 0.01, 0.03 and 0.1.

In 1969, Julian and Akins [68] carried out experiments on natural convection from a uniformly heated, vertical, flat plate in water and mercury in the range of moderate Grashof number. The results were in good agreement with the similarity and integral solutions to the boundary layer equations. The dimensionless profiles also showed the trends, with position up the plate, predicted by the first order perturbation analysis [16]. The present work shows the experimental results on natural convection as an extension of Julian and Akins' work, especially at low Grashof numbers where appreciable deviation from the boundary layer solution was predicted.

In 1967, Hayday, Bowlus, and McGraw [60] numerically solved a nonsimilar free convection problem, the nonsimilarity of the flow being generated by step discontinuities in surface temperature. Results were compared with experimental correlation of Schetz and Eichhorn [117] and formed a theoretical basis for their experiments.

In 1967, O'Brien and Shine [97] investigated some effects of an electric field on heat transfer from a vertical plate in free convection. They showed that the local heat transfer from a vertical plate increased significantly in the presence of a large electric field.

Gebhart, Dring, and Polymeropoulous [50], in 1967, studied the transient natural convection from a vertical sheet following a step input (with time) of heat. Gebhart and Dring [49], in 1967, showed the rate of propagation of leading edge effects up the plate. Also in 1967, Polymeropoulous and Gebhart [102], presented the results of an experimental investigation of the behavior of artificial disturbances produced by an oscillating ribbon in the free convection boundary layer over a vertical uniform flux plate. Dring and Gebhart [24], in 1968, theoretically investigated the nature of instability and disturbance amplification for the basis of Polymeropoulous and Gebharts' experiments [102]. In the same year, Knowles and Gebhart [72], showed that thermal capacity coupling exists between the fluid and the wall which generates the flow. This coupling was shown to have a first order effect for particular Grashof-number Wave-number products. In 1969, Gebhart [47] summarized what was known at that time concerning the incipient instability and transition of laminar flow.

In 1968, Takhar [141] presented a numerical solution for the development of free convection from a semi-infinite flat plate, which was isothermal up to a certain length from the leading edge and was insulated for the rest of its length. He pointed out that at the insulated part above the isothermal part of the plate, the velocity and temperature distributions behave as if the heated part were put in as a line source of heat at the base of the insulated part.

In 1967, Tien [143] applied an integral method to study the laminar natural convection heat transfer from both an isothermal and a non-isothermal plate to a power-law fluid. Tien and Tsue [144], in 1969, presented an approximate solution for the problem of determining the laminar natural convection heat transfer between a vertical, isothermal plate and a fluid whose rheological behavior is characterized by Ellis' model. The results were obtained by integral method and compared with available experimental data.

In 1968, Sparrow and Guinle [135] investigated the deviations from classical boundary layer theory at small Prandtl numbers. They evaluated effects of transverse pressure variations, and streamwise shear stress and conduction phenomena, which are neglected in the conventional boundary layer theory. It was shown that for very low Prandtl numbers, the Grashof number must be quite large in order that the classical boundary layer results are applicable. It was also pointed that the local heat transfer exceeds that of classical boundary layer theory.

In the same year, Cygan and Richardson [22] used a transcendental approximation to the velocity and temperature profiles to obtain integral solutions of the natural convection boundary layer at small Prandtl number.

Papailiou and Lykoudis [100], in 1968, performed an experiment to show the effect of a magnetic field on a laminar natural convection of electrically conclucting fluid. The case examined in this experiment was that of a vertical hot plate of uniform temperature with mercury as conducting fluid, in the presence of a transverse magnetic field. The existence of similarity solutions theoretically investigated by Lykoudis [85] and independently by Gupta [58] in 1962 was fully established by the experiment.

Experimental results investigated in 1968 concerning turbulent natural convection from a vertical flat plate were in general agreement with each other even though there were some discrepancies in a number of the details. Cheesewright [18]