AN EXPERIMENTAL INVESTIGATION OF WORD SOLVING PROBLEMS IN SEVENTH AND EIGHTH GRADES WITH DIFFERENT MATHEMATICAL BACKGROUNDS

by 45

GORDON ROBERT SCHUMACHER

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Approved by:

Richard E. Owens
Major Professor

TABLE OF CONTENTS

TABLE OF CONTENTS (continued)

LIST OF TABLES

LIST OF TABLES (continued)

LIST OF TABLES (continued)

LIST OF TABLES (continued)

CHAPTER I

INTRODUCTION

During recent years, many schools have changed from either a traditional, or transitional, to a "modern" mathematics program, as did the West Allis-West Milwaukee Joint School District Number One. In 1963-64, grades three and four were still using a traditional arithmetic text, Making Sure of Arithmetic, published by Silver Burdett. In the 1964-65 schoolyear, the fifth grade continued to use the traditional text while the fourth grade changed to the Greater Cleveland Mathematics Program of the Science Research Associates. The mathematical series used included texts one-twelve for intermediate grades.[1]

The Greater Cleveland Mathematics Program was completely modern and different from previously-used texts.

> This modern math program, the most widely used in the nation, is designed with the philosophy in mind that it is necessary to understand the concepts underlying mathematical operations before learning the computational skills for doing the operations.[2]

This program also includes all mathematical elements

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[1]Robert Johnson, Assistant Superintendent of Schools, West Allis-West Milwaukee Joint City School District Number One.

[2]General Catalog, Science Research Associates, Inc., 1967, p. 55.

taught in the intermediate grades and provides students with a better understanding and working knowledge of mathematical relationships.

In the 1964-65 schoolyear, the fourth grade class started with the Greater Cleveland Mathematics Program of Science Research Associates, Incorporated. This class has continued to use this series. The twelve books were designed principally for grades four, five, and six. When many of the instructors realized they would not complete the full series of twelve books during the 1966-67 schoolyear, they decided to use books eleven and twelve with the present Laidlaw texts in the seventh grade.[3]

The fifth grade class in the 1964-65 schoolyear used the traditional book, Making Sure of Arithmetic, published by Silver Burdett. This class has continued with a traditional program and is the eighth grade the researcher used in this study.

I. STATEMENT OF THE PROBLEM

In 1964-65, the fourth grade departed from the traditional mathematics program of the school system; they had studied the Greater Cleveland Mathematics Program from 1964-65

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[3] Robert Johnson, Assistant Superintendent of Schools, West Allis-West Milwaukee Joint City School District Number One.

through the 1967-68 schoolyear. This class was the seventh grade tested by the researcher in this investigation.

The eighth grade tested by the researcher has studied only the traditional mathematics program through the grades.

Therefore, this investigation was undertaken to see whether the modern mathematics program of the seventh grade was giving the students a better foundation for techniques in problem solving than the traditional mathematics program of the eighth grade. Because of the difference in the mathematical backgrounds, a difference in the approach, techniques, and understanding should be evident.

II. WHAT IS "MODERN" MATHEMATICS?

In order to be able to define the term "modern" mathematics, the following explanation is necessary. Mathematical programs have changed in both content and structure in certain areas to form a program referred to as "modern" arithmetic or "modern" mathematics. When using the term "modern", it usually refers to a modern program in arithmetic. Such a program differs from a traditional program both in content and spirit, having new emphases and points of view. Therefore, modern mathematics is not newly-discovered mathematics, but mathematics new to the curriculum of the elementary and secondary levels of our schools.[4]

The modern spirit referred to in modern mathematics is a spirit of inquiring and discovery that is instilled in the pupils. This spirit helps the students use ideas already acquired as a means of discovery of new ideas.[5]

Modern mathematics introduces some mathematical concepts earlier than was previously considered possible. This does not mean, however, that an arithmetic program that merely accelerates the content is necessarily a modern program.[6] The content must be such that the student gains insight into number relationships, principles of operations, and patterns among numbers. Another important aspect of "modern arithmetic" or "modern mathematics" is the precise use of the vocabulary. An example of this would be the use of the words numeral and number. A number is a quantitative concept, and a numeral is a name for a number.[7]

The new arithmetic or mathematics defines certain basic principles carefully and systematically. Today more

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[4]Silver Burdett Company, What is Modern Mathematics? (The Resourceful Teacher No. VII. Park Ridge, Illinois, 1966,) p. 1.

[5]Ibid.

[6]Ibid.

[7]Ibid., p. 2.

emphasis is placed on principles for two basic reasons: (1) to give the student tools for building upon new facts and concepts from what he already has acquired and (2) to give the student an appreciation, no matter how elementary, of the basic structure of mathematics.[8]

The content of the modern mathematics program may include some geometric concepts as early as grades one and two. The geometric ideas are developed further in subsequent years. The pupil in the elementary grades learns not only to identify geometric shapes but to discover many of their properties. The child usually enjoys geometry if the discovery approach is used. Another source of enjoyment for him is seeing these different geometric shapes in everyday happenings.[9]

Other characteristics of a modern mathematics program are unification and integration of mathematical ideas and procedures. In more modern mathematics, separate treatment of various branches of mathematics becomes more difficult and even inadvisable in a modern approach to the subject. Also, increased emphasis must be given to basic principles and patterns of our number system, and to the properties of

[8]Ibid., p. 3.

[9]Ibid., p. 4.

operations from which we abstract generalizations.[10]

Other characteristics of modern mathematics are more precise definitions, terminology, and notations. Many familiar definitions of traditional mathematics have been replaced by new definitions that are more precise. Considerably much more time is being taken in the classrooms to get acquainted with the "language of mathematics".[11]

Previous traditional mathematical programs emphasized the use of rules instead of the formation and foundation of mathematical concepts. In modern mathematics, if a "line segment" is meant, it is not spoken of as a "line". In discussing a length of a line, mathematicians say "the measure of the line segment is ... inches", and not "the line segment is ... inches".[12]

III. DEFINITION OF TERMS

The _seventh and eighth grades_ used in this study refer to the students enrolled in seventh and eighth grades respectively during the 1967-68 schoolyear in the West Allis-West Milwaukee Joint City School District Number One. Only those

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[10] A. B. Evenson, _Modern Mathematics: Introductory Concepts and Their Implications_ (Chicago: Scott, Foresman and Company, 1962), p. 4-5.

[11] _Ibid._, p. 5.

[12] _Ibid._

who attended the school system since the 1963-64 schoolyear, when the modern mathematics program was inaugurated, were used.

Word solving problems, often referred to as story problems, are those in the form of sentences or paragraphs, from which the student must choose only that information necessary to solve them.

In the chapter entitled RESULTS, the researcher will refer to a careless error as a careless computational mistake rather than an error due to lack of knowledge or procedure. For example, a student might have used a correct approach in solving a word problem, but during his computation he may have added or subtracted incorrectly, thus committing a careless error.

Problem solving abilities are those which rely on (1) translating to mathematical symbols, (2) complete and clear understanding of mathematical symbols, (3) skills in computational processes, and (4) knowledge of the basic principles of mathematics and of problem solving so the students may apply them correctly in varying situations.[13]

Techniques are those mathematical methods and skills used to solve a problem.

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[13]A. B. Evenson, *Modern Mathematics: Introductory Concepts and Their Implications* (Chicago: Scott, Foresman and Company, 1962), page 4, 5.

Approach consists of the initial techniques or steps used to formulate the equation to be solved.

IV. IMPORTANCE OF THE STUDY

In this age of rapid change, more attention must be given to careful problem solving techniques. Modern mathematics had been changed to cope with a better understanding of the basic principles and foundations of mathematics. Therefore, modern mathematics should help in problem solving techniques. All of the modern mathematical implications mentioned previously, should have a large impact on problem solving at all levels. Before any word problem can be solved, a student must be able to translate the words of the problem to mathematical symbols. Therefore, in order to be able to translate, a student needs a clear understanding of all mathematical symbols he needs to use. A student must also be skilled in all computational processes to be able to solve the mathematical symbols used in his mathematical statement.[14]

Since modern mathematics cannot possibly foresee every type of problem a student may encounter in life, it can only hope to teach the basic principles of mathematics and problem solving necessary to be able to apply these principles

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[14] A. B. Evenson, *Modern Mathematics: Introductory Concepts and Their Implications* (Chicago: Scott, Foresman and Company, 1962), page 7.

correctly in the various types of situations.[15]

According to the Houghton Mifflin Company, the following question appears to be unresolved: Compared to traditional mathematics, has modern mathematics improved a student's ability to solve word problems? The teacher's manual states:

> In some of the more "modern" of the experimental programs, instruction in problem solving has been de-emphasized. Whether the amount of instructional attention given in the past to problem solving was defensible is still a matter for experimentation and debate. But if the newer instructional procedures are, in fact, superior, they should result in improved performance in problem solving ability.[16]

V. ASSUMPTIONS AND LIMITATIONS OF THE STUDY

The researcher had to make several assumptions when matching the students for the testing. First, he had to assume that the result from the *Iowa Tests of Basic Skills* for each student was the best possible score the student could achieve and that no student had copied or otherwise cheated in the test.

In matching the students, the researcher had two scores from which to choose from the Arithmetic Problem Solving section of the *Iowa Tests of Basic Skills*. The grade

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[15] *Ibid.*

[16] E. F. Lindquist, and A. N. Hieronymus, *Teacher's Manual: Iowa Tests of Basic Skills* (Boston: Houghton Mifflin Company, 1964), p. 50.

equivalent score or the percentile score could have been used in the matching. However, the percentile scores had large gaps. The percentiles near the mean differed by as much as five percentile points for each tenth of a grade in grade equivalent scores.[17] Therefore, percentiles were not used as a matching tool.

Grade equivalent scores could have been used, but these results also were not accurate when differing from the mean of the group. In addition, not all scores were reported to the nearest tenth in each grade. The pupils in grade eight could have had grade equivalent scores of 8.1, 8.4, 8.5, 8.7, and 8.9, but had no scores between these listed scores. The pupils in grade seven could have had grade equivalent scores of 7.0, 7.3, 7.6, and 7.8, but had no scores between these listed scores. Therefore, it was impossible in most cases to have exactly one year's difference in grade equivalent scores. The researcher then converted all grade equivalent scores to standard scores for purposes of matching, using the formula:

$$\text{Standard Score} = \frac{\text{Student Score (G. E.)} - \text{Mean for Group (G. P.)}}{\text{Standard Deviation for this grade on this test as reported in the manual}}$$

Thus all grade equivalent scores were changed to standard

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[17] E. F. Lindquist, and A. N. Hieronymus, Manual for Administrators, Supervisors, and Counselors: Iowa Tests of Basic Skills (Boston: Houghton Mifflin Company, 1964), p. 63-64.

scores. Again, since not all scores were reported to the nearest tenth of grade equivalent, the matching was only as close as the scores permitted the researcher to determine in the matching of standard scores. The actual results may be found in the appendix.

Because the West Allis-West Milwaukee Joint School District Number One changed to the modern mathematics program so abruptly, many of the faculty may not have adjusted their forms of teaching, terminology, methods, and approaches. However, this study was begun on the basic assumption that all faculty members did make this adjustment completely and immediately when the modern program was adopted.

Problem solving skills cannot be determined only on the ability to do certain word problems, knowledge of certain concepts, and ability to translate word solving problems to mathematical symbols. Thus, it can only be assumed the questions that were used would show certain differences based on the two previous mathematical backgrounds.

As many as possible of the students must be questioned individually to determine if a difference exists; however, because of the time element, the researcher was unable to question all the students extensively and to test all the questions on a large population. If the researcher had had time, he could have checked all the questions with a larger population and tried to establish some norms for the group.

Time was also a factor in attempting to have the students solve enough problems of each type that correspond to a per cent of each type in each grade.

The types of problems were given in eight categories. The same eight categories that Houghton Mifflin Company uses in their tests of arithmetic in the Iowa Tests of Basic Skills were employed by the researcher in this study: currency, decimals, fractions, geometry, measurement, per cents, ratio and proportion, and whole numbers.[18] Since the content of most texts can be categorized into these eight types, the researcher decided to use these eight categories in this study also.[19]

The differences stated in the conclusion were found to be true for this specific group tested at West Allis-West Milwaukee Joint City School District Number One. These differences may not necessarily be prevalent in other school districts with similar backgrounds. There also may be other differences the investigator was unable to discover because of either the sampling group or the questions posed to the students by the investigator. He can only assume he has

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[18]E. F. Lindquist, and A. N. Hieronymus, Teacher's Manual: Iowa Tests of Basic Skills (Boston: Houghton Mifflin Company, 1964), p. 50

[19]Edward Drahozal, Ph. D., Test Consultant, Houghton Mifflin Company, Geneva, Illinois.

found most differences of major importance to make the study a valid and complete report.

VI. REVIEW OF RELATED RESEARCH AND LITERATURE

The view many mathematicians have regarding problem solving has been restricted to so-called word problems or story problems. Many mathematicians fail to perceive that much of mathematical learning itself is problem solving. The more students develop their mathematical learning itself as problem solving, the better able the students are to cope effectively with new problems that may arise in both mathematics and social contexts.[20]

The use of number sentences is important in finding solutions to verbal problems. To write number sentences for problems, the student must be able to translate verbal sentences and phrases into the language of mathematics, which consists of numbers and symbols.[21] Solutions to verbal problems are important, but there are other important concepts other than the solutions to the problems.

The highly symbolic language of mathematics makes it useful as an aid in solving problems. This language contains

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[20] J. Fred Weaver, "Applications and Problem Solving," _The Arithmetic Teacher_, XII (October, 1965), p. 413.

[21] National Council of Teachers of Mathematics, _Topics in Mathematics for Elementary School Teachers_ (Number Sentences Booklet Number 8. Washington, D. C., 1964), p. 30.

symbols comparable to parts of the English language, such as, nouns, verbs, phrases, and punctuation marks. These punctuation marks are used to clarify the meaning of either a mathematical phrase or sentence.[22]

In chapter three, EXPECTED OUTCOMES, the researcher stated that modern mathematics programs placed increased emphasis on the concepts, approaches to concepts, and terminology. However, the number of word problems, by actual count, decreased when the modern mathematics programs were first inaugurated.[23]

Some critics of the new mathematics programs state that many of the large experimental programs have ignored applications and relations to science instruction. Mathematics reform during the past few years has been for the greater part in the hands of pure mathematicians. Many agree that this has been important as the first step in making a mathematics program truly contemporary in concept and spirit. If this program is to be complete, then applications of this pure mathematics program are necessary as the second step.[24]

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[22]National Council of Teachers of Mathematics, Topics in Mathematics for Elementary School Teachers (Number Sentences Booklet Number 8. Washington, D. C., 1964), p. 1.

[23]Edward Drahozal, Ph. D., Test Consultant, Houghton Mifflin Company, Geneva, Illinois.

[24]Howard F. Fehr, "Sense and Nonsense in a Modern School Mathematics Program," The Education Digest, XXXI (April, 1966), pp. 16, 17.

Even though pure mathematicians were involved in setting up some programs, high school and college teachers together with psychologists and others have set up other mathematics programs, e.g., School Mathematics Study Group.[25]

Howard F. Fehr made the following statement which might truly clarify one point in the philosophy of any mathematics instructor:

> It is essential for the student of mathematics to acquire the faculty of being able, by his own wit, to learn more mathematics, to solve new problems, to adapt his past knowledge to new knowledge and new points of view, . . .[26]

While some mathematicians have stated we do not give enough emphasis to problem solving, others hold the opposite opinion. These mathematicians believe our view basically consists of the so-called social applications. Consequently, the term usefulness has been restricted.[27]

As mentioned previously, the number of word problems or story problems had decreased with the new modern mathematics programs. If story problems have decreased, then the emphasis on these problems may not necessarily be evident.

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[25]Floyd J. Coppedge, Ph. D., Assistant Professor of Education, Kansas State University, Manhattan, Kansas.

[26]Howard F. Fehr, "Sense and Nonsense in a Modern School Mathematics Program," <u>The Education Digest</u>, XXXI (April, 1966), pp. 16, 17.

[27]J. Fred Weaver, "Applications and Problem Solving," <u>The Arithmetic Teacher</u>, XII (October, 1965), p. 412.

N. Wesley Earp once stated that when special emphasis was placed on problem solving, improved achievement resulted. This was proven true when students were given a good many well-chosen problems to solve. He then concluded that achievement of an acceptable level of problem solving skills was possible if the teacher emphasized the development of these skills and applied the suggestions given for improving these skills.[28]

Problem solving skills can be improved, but not by simply assigning word problems. Solving problems is a complex process. Research has told us there seems to be no one best procedure for problem solving. Research also has told us, though, that some systematic approach is better than none at all.[29]

Many authors of textbooks state the procedure necessary in solving a problem. The last step in most cases is labeling the answer. An article by Herbert Hannon states that labeling is very important and definitely should be taken into consideration. His summary points out this important aspect:

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[28]N. Wesley Earp, "Problem Solving--Arithmetic's Persistent Dilemma," School Science and Mathematics, LXVII (February, 1967), p. 188.

[29]G. W. Brown, "Improving Instruction in Problem Solving in Ninth Grade Mathematics," School Science and Mathematics, LXIV (May, 1964), p. 344.

Three examples have been used to explore the desirability of using labels in problem solving. The point of view presented was that whenever measurement units were involved, these units might be defined as a numeral for 1. Number operations in the usual sense could then be used with the labels used as numerals, i.e., names for numbers.

First, $3a + 2a = (3 + 2)a$, where a is an arbitrary unit of measurement defined as 1.

Also, $3a \times 4a = (3 \times 4) \times (a \times a)$, where a^2 is defined as 1, and finally as "a" squared.

Finally, $3 \times 4a = (3 \times 4)a$, where "a" again is defined as 1--the arbitrary unit of measurement.

Certainly the above point of view permits the separation of the units of measurement (labels) from the number operations used in the usual sense and as such merits some consideration in problem solving--the practical aspects of mathematics.[30]

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[30]Herbert Hannon, "Label × Label = Label^2--A Point of View," School Science and Mathematics, LXVIII (June, 1968), p. 510

CHAPTER II

DESIGN OF THE STUDY

The study consisted of matching and testing pairs of students in the seventh and eighth grades of one junior high school in the West Allis-West Milwaukee Joint City School District Number One. The researcher questioned the students to determine if their abilities to solve word problems were different. The questions used were from eight areas: currency, decimals, fractions, geometry, measurement, per cents, ratio and proportion, and whole numbers.

I. AREAS OF TESTING

Based upon several "modern" textbooks in use today and upon the guide of Houghton Mifflin Company publications, the researcher found the areas of problem solving to be categorized into eight types. The eight areas, as listed above in the introductory paragraph, might be represented in both concept and problem solving techniques. Since the researcher was interested primarily in the latter, he used verbal ideas containing some or all of these techniques. The techniques of each area are as follows:

1. Currency: reading and writing amounts; counting; relating value of coins and making change.
2. Decimals: reading and writing; fraction, decimal,

per cent equivalents; fundamental operations using decimals.

3. Fractions: part of a whole or group; relative sizes; reducing; fundamental operations using fractions.
4. Geometry: recognizing geometric figures; dimensions, perimeters, areas of polygons.
5. Measurement: quantity; time; temperature; weight; areas and volume; fundamental operations with denominate numbers.
6. Per cent: meaning and use.
7. Ratio and proportion: meaning and use.
8. Whole numbers: reading and writing; rounding; measurement; averaging; fundamental operations.

These were the basic areas used to determine if a difference in problem solving was apparent.[31]

The modern mathematics series may differ slightly in techniques, vocabulary, or approach, but all series spend approximately the same amount of time on each area and cover each area at about the same time during the year. This is especially true in the lower grades.[32]

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[31]E. F. Lindquist, and A. N. Hieronymus, _Manual for Administrators, Supervisors, and Counselors: Iowa Tests of Basic Skills_ (Boston: Houghton Mifflin Company, 1964), p. 37.

[32]_Ibid._, p. 36