

ANALYTICAL AND NUMERICAL SOLUTIONS OF SOME
NON-NEWTONIAN ENTRANCE REGION FLOW PROBLEMS

by 4⁵

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CHAPTER 1

INTRODUCTION

When a viscous fluid enters a conduit (a circular pipe or a flat duct) from a very large reservoir through a well-rounded entrance, the assumed flat velocity profile at the entry will gradually change as fluid moves down the conduit because the walls of the conduit tend to retard the flow. Eventually, the velocity profile will develop to a form which remains unchanged with respect to the direction of flow. The flow in the region of changing velocity profile is so-called hydrodynamic entrance region flow.

Applying the concepts of boundary layer theory, the boundary layer form at the walls, beginning with zero thickness at the entry, and increase continuously in the down stream direction. At a certain distance from the entrance, the boundary layers become so thick that they meet each other at the center line of the conduit (excluding the cases of fluids with a yield stress). During the formation of the boundary layers in the entrance region, the flat velocity profile near the walls changes to a curved one due to the viscous drag at the fluid-solid interface, and the velocity distribution outside the boundary layer remains uniform. Since the total flow rate is constant along the conduit for steady-state flow, the fluid in the center core of the flow is accelerated in consequence of the altered velocity profile in the boundary layers. Because of increasing kinetic energy and higher viscous friction the pressure drop in the entrance region is considerably larger than that in the fully developed region where the velocity profile is invariant with respect to the direction of flow.

Originally the study of entrance region flow was of interest in connection with the correction of capillary viscometric data. It became of