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PLANE STRESS FINITE ELEMENT ANALYSIS OF BEAMS WITH WEB OPENINGS

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I. Introduction

In present construction practice, holes are frequently cut in the webs of W shape beams to permit the passage of utility components or to provide access to the inside of box beams. Sometimes the holes are cut in the web without any attempt to locally reinforce the web. While at other times the beam is locally reinforced with doubler plates, angles welded to the web, or bars or flats welded to the periphery of the hole.

When an opening is cut in the web of the beam, the beam may be weakened in the vicinity of the opening to the extent that reinforcing is required. Tests have shown that stresses predicted on the basis of modified elementary beam theory may lead to an unsafe design of beams with holes.

Therefore, to get an indication of the stress distribution that actually exists in beams with holes, a more accurate and more powerful method, the finite element method, was used in this report to analyze the elastic stresses around a rectangular opening in the web of a W shape beam.

The beam [Fig. 1] is simply supported with a concentrated load applied at the center of the span. With various M/V ratios and with and without reinforcing bars, the stresses in a portion 30 inches wide, centered about the hole [Fig. 3] were determined using the finite element method as incorporated in the ICES-STRUDLE computer program.

In this report, the finite element method will be briefly reviewed and the results calculated by the finite element method will be compared with those obtained both experimentally and by the Vierendeel method.

II. Literature Review

In the 1920's Muskelishvili (1)* developed a practical method of solving the so-called plane problem of the theory of elasticity and in particular, the problem of the stress distribution in a plane or thin plate which is weakened by any type of hole. Since then the problem of stress concentration due to such openings has been much studied and many papers concerning this problem have been published (2, 3, 4, 5, 6).

In the past few years a concentrated effort has been made by steel industry and university investigators (7) to develop analytical and experimental information on steel beams with web openings.

In 1966, Bower (11), using the theory of elasticity method incorporating complex variable techniques investigated the stresses around an hole in a W shape beam. From this investigation, it was concluded that : a). the applicability of the analysis depends on the size of the web hole and on the magnitude of the moment-shear ratio at the hole ; b.) the stress distributions near the hole in uniformly loaded beams are widely different in magnitude and in appearance from the distributions occuring in beams without holes ; c.) the solution is valid for predicting stresses near the opening providing the opening depth does not exceed half the web depth.

Later Bower (12) used the Vierendeel method to calculate the elastic stresses around rectangular holes in the webs of W shape beams. It was concluded that this method provided a reasonably accurate prediction of the stresses in the vicinity of a rectangular hole except for stress concentrations near the corners.

From Bower's (12) experimental study of the stresses in W shape beams

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* Numbers in parentheses refer to corresponding item in the References.

with web-openings, it was concluded that : a.) the theoretical results based on the theory of elasticity and Vierendeel method are reasonably accurate, and b.) the elasticity analysis is complex and requires a computer solution, while the Vierendeel analysis is relatively simple to perform.