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APPLICATION OF THE PRINCIPLES OF DECISION UNDER RISK TO A CASE STUDY

by

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CHAPTER I

INTRODUCTION

1.1 DEFINITION

Decision Making

The decision making process involves getting the facts about a problem, weighing them with specified criteria, and then deciding which of several alternatives to select.

After a decision has been made and action taken, time and hind sight may show that a better choice among the alternatives could have been found.

1.2 BACKGROUND HISTORY

Decision making theories and methods have dominated the management literature in the past decade. An investigation by Greenwood [4] mentioned that before 1950, decision making was not used in management literature and was not given much importance. Management was more inclined towards human relations, organization theory and economic analysis, than towards decision theory. Later, more emphasis was laid on business decision making. Greenwood [5] added that decision making and methods have been developed in attempts to resolve particular management problems and from the perspective of particular academic disciplines, especially psychology, sociology, mathematics, statistics, and logic. That is why the literature on decision making is scattered and as yet not properly gathered or integrated.

Between 1945 and 1948, an exhaustive survey was made on the literature of decision making by Paul Wasserman and Fred S. Silander. The findings were published in a summarized form by Cornell University in 1958 under a McKinsey Foundation grant entitled Decision Making --- An Annnotated Bibliography [15]. The findings revealed that decision making was used in small groups concerning psychological studies of individual, group and leadership factors. The ideas of management decision making was originated by psychologists, mathematicians, and statisticians; its methods being derived from the fields of mathematics and statistics.

1.3 ROUTINE DECISIONS

Decisions play an important role in our everyday lives--whether or not to study for a quiz, what to order for lunch, or what color tie (or dress) to wear--these are the sorts of decisions we must make daily. Of course not all decisions are trivial. Many involve millions of dollars or even life and death. Indeed, decision making may constitute one of the highest forms of human activity.

The decision to repair the motor of an old car may seem perfectly sound to the owner in the light of an anticipated trip and the car's generally sound condition, at least at the time the decision is made. If the car is totally wrecked in an accident a week after the repair is made, the owner may lose all the money invested in repair, but can he be blamed for a faulty decision? If an investment based on some decision appears highly speculative with very little chance of working out well, can one be credited with a "good" decision, if it brings return beyond one's expectation? In one sense the answer to this question may be 'yes', in another 'no'.

1.4 HOW TO MAKE A BUSINESS DECISION

Peter Drucker [3] said that business decisions will always have to be based on judgement. They will always remain decisions for a future which will continue to be unpredictable. They will always entail risks. But studies made until now have reached a point where every businessman by following fairly simple steps can greatly improve his performance as a decision maker. There are basically four steps involved in decision making and they may be enumerated as follows:

1. Defining the problem--what kind of problem have we to solve? What is its critical factor? When do we have to solve it? What is the cost involved in its solution?
2. Defining expectations: What do we want to gain by solving it?
3. Developing alternative solutions: Which of several plans offers the surest way to avoid unexpected outcomes.
4. Knowing what to do with the decision after it is reached, i.e. implementation of the decision.

Attention to these rules will help the businessman avoid the three most common pitfalls in the making of business decisions. These are:

1. Finding the right answer for the wrong problem-few things are as useless.
2. Making the decision at the wrong time.
3. Making decisions that do not result in action.

Paul Jedamus and Robert Frame [7] explained that if the procedure discussed above is followed step by step, the decision made will be the best, not with certainty but with higher probability (confidence).

1.5 DECISION MAKING IN OTHER FIELDS

Haywood [6] (1954, ex-colonel, U.S. Air Force), an authority on military decision, described military doctrine known as "the estimate of situation" as follows in five formal steps:

1. Determination of the mission.
2. Description of situation and courses of action.
3. Analysis of opposing courses of action.
4. Comparison of own causes of action.
5. The decision.

Klee and Garland [9] used the tree diagram for solving a solid waste disposal planning problem. They used the generalized procedure which is discussed in Chapter 2.

1.6 DIFFERENT SITUATIONS UNDER WHICH ONE HAS TO DECIDE

There are three situations under which one has to decide as explained by Archer [1]:

1. Decision under certainty
2. Decision under uncertainty
3. Decision under risk

1.6.1 Decision Under Certainty

In this situation, the payoff resulting from the selection of a particular strategy is known. It is assumed that the payoff resulting from the decision can be precisely measured; in other words, only one state of nature is assumed to exist. Prediction is involved, based on assumed outcomes. The assumption of certainty simplifies the decision but ignores variations in condition which often exist, leading to improper decisions.

Example

A man wants to invest one thousand dollars for three years. From the present trend of market interest rate, he can choose either of two alternatives, viz.,

1. Invest \$1000 at 5% compounded annually for three years, or
2. Invest \$1000 at 5½% compounded annually for two years and for the third year at 4% compounded annually.

The criterion for selection of a particular alternative is to maximize the interest earned. The solution to the above problem according to this criterion is as follows:

Alternative 1

F = Future value of the deposit after n years

$= P(1+i)^n$

where,

P = present amount

i = interest rate per period

n = number of interest period

F = a future sum of money

The future value of the deposit after three years using this relation is

$F_1 = 1000(1+5/100)^3$

$= 1000\ x(1.05)^3$

$= 1000\ x\ 1.168$

$= \$1168.$

Alternative II

The future value of the deposit after two years is

$F_2 = 1000(1+11/200)^2$

$= 1000(1.055)^2$

$= 1000\ x\ 1.113$

$= \$1113.$

For the third year he has

$P = \$1113.$

$i = 4$ percent

$n = 1$

The future value of deposit after the third year is

$F_3 = 1113(1+4/100)$

$= 1113\ x\ 1.04$

$= \$1157.52$

From these calculations the first alternative will be selected, since the future value of deposit after three years is greater than from the second alternative.

1.6.2 Decision Under Uncertainty

No information is available concerning the relative frequency with which any state of nature will occur. In this situation, the possible criteria for selecting the optimum strategy are:

a. Maximin criterion.

b. Minimax criterion.

c. Maximax criterion.

d. Hurwicz criterion.

1.6.2.a The Maximin Criterion

The payoff matrix is expressed in terms of profit. In this case the decision maker regards nature as an antagonist and expects the worst possible outcome (the smallest profit). He therefore selects the strategy that will yield the greatest minimum profit. It is one of the most conservative (pessimistic) decision rules.

Example (From Sasaki [13], Page 229)

A decision maker is considering whether to sell ice cream or hot dogs at the baseball game. The payoff (profit) matrix that represents the consequences of possible action under the given states of nature are shown in Table 1.

TABLE 1. PAYOFF (PROFIT) MATRIX

State of nature	ACTION	
	To sell ice cream a_1	To sell hot dogs a_2
Sunshine (θ_1)	\$100	\$98
Cloudiness or rain (θ_2)	\$ 20	\$97

Given these data, what action should a decision maker take using the maximin criterion for selecting a strategy?

If action a_1 is taken the worst that can happen is to earn only \$20. Similarly if the decision maker takes action a_2 the worst that can happen is to earn \$97. The maximum of the minimum profit is \$97; so the optimum decision rule is to take action a_2, that is, to sell hot dogs.

1.6.2.b The Minimax Criterion

The payoff matrix is expressed in terms of loss. In this case, the decision maker expects the worst possible outcomes (the greatest loss), and selects the strategy that will yield the smallest loss.

Example (From Sasaki [13], Page 223)

The states of nature are 'rain' and 'no rain', and the possible actions are 'stay at home', 'go without an umbrella' and 'go out with an umbrella'. The payoff matrix is given in Table 2.

TABLE 2. PAYOFF (LOSS) MATRIX

State of nature	ACTION		
	Stay at home a_1	Go without an umbrella a_2	Go with an umbrella a_3
Rain (θ_1)	4	3	2
No rain (θ_2)	4	0	6

From this data, what would be the best decision using the minimax criterion?

If action a_1 is taken, the worst that can happen (maximum loss) is 4. If action a_2 is taken, the worst is 3. Similarly for action a_3, the worst that can happen is 6. Now the minimum of the maximum loss is 3. Therefore action a_2 will be taken, that is, go without an umbrella.

1.6.2.c The Maximax Criterion

The payoff matrix is usually expressed in terms of profit. In this the payoffs are checked for each strategy and the decision maker selects the strategy with the highest possible payoff. If the criterion is maximax, then in the previous example (Section 1.6.2.a) one will select the first strategy (sell ice cream) because its highest possible payoff is greater than the highest possible payoff of the second strategy. That is, the seller receives $100 for the first strategy compared to $98 for the second strategy.

1.6.2.d The Hurwicz Criterion

The Hurwicz criterion uses the weighted average of the minimum and the maximum payoffs to select the best strategy.

Example (From Sasaki [13] , Page 234)

For three different states of nature, there are two actions (a_1 and a_2) and payoff matrix is given in Table 3.

In applying the Hurwicz criterion, weights are designed to reflect the decision maker's subjective opinion. The weight given to minimum payoff is chosen arbitrarily by decision maker, say, 3/4 and to maximum is ¼. Now the minimum for action (strategy) a_1 is \$2, while the maximum payoff is \$100. Therefore, the Hurwicz criterion would evaluate strategy 1 at the value

$$V = 3/4 \times 2 + 1/4 \times 100$$
$$= 26.5$$

Similarly, the Hurwicz criterion would evaluate strategy 2 at the value

$$V = 3/4 \times 1 + 1/4 \times 99$$
$$= 25.5$$

Hence, by this criterion, the decision maker should select strategy 1.

TABLE 3. PAYOFF (PROFIT) MATRIX

	ACTION	
State of nature	a_1	a_2
A	\$100	\$99
B	\$3	\$97
C	\$2	\$1

1.6.3 Decision Under Risk

In this case the decision maker must review the payoff matrix resulting from the various states of nature, along with their probabilities of occurrence. In order to arrive at a decision, the payoff is weighed by the associated probability. The expected value of a strategy is the sum of the payoffs, each multiplied by its respective probability of occurrence. The appropriate decision is to select the strategy with optimum expected value (largest, for maximization of the payoff unit).

Example

The payoffs mentioned below are profits. The criterion for selection of a strategy is to maximize the profit. There are three states of nature which occur with probabilities (.25,.5,.25) as shown in Table 4.

TABLE 4. PAYOFF (PROFIT) MATRIX

State of nature	Probability of occurrence	Strategies: Inventory Level				
		S_1	S_2	S_3	S_4	S_5
		200	250	300	350	400
A	0.25	100	90	70	40	0
B	0.5	100	120	120	90	50
C	0.25	50	100	140	190	160
Expected Values		87.5	107.5	105	102.5	65

The strategies represent different inventory levels. For a particular strategy, the profit is different for the several states of nature as shown above. As mentioned in the beginning, the profit is to be maximized, therefore a strategy with maximum average profit will be chosen.

The optimal strategy is therefore the stocking of 250 units, for the expected value (the average profit from such a decision in the long run) is higher (90 x 1/4 + 120 x 1/2 + 140 x 1/4 = 107.5) than for any other strategy, as summarized in the bottom line of Table 4.

CHAPTER II

THE GENERALIZED PROCEDURE FOR DECISION PROBLEM UNDER RISK

The generalized procedure for analyzing the decision problem in extensive form (as opposed to the so-called normal form where all possible strategies are listed with a table of expected losses) is as follows:

1. Chart the decision flow diagram in the form of a tree (see [11] , page 10).
2. Assign payoffs (or utilities) at the tips of the branches of the tree.
3. Assign probabilities at all chance forks.
4. Average out and fold back.

We demonstrate with a simple example the process of finding the strategy which maximizes profit.

The problem is to select a strategy (out of two available), which will maximize the profit. The entries in the payoff matrix are the gross profit. For each strategy, there are two states of nature with their probability of occurrence as shown in Table 5.

TABLE 5. PAYOFF (GROSS PROFIT) MATRIX

State of Nature	Probability of occurrence	Strategy: Inventory level		
		S_1	S_2	S_3
		STOCK 100 UNITS	STOCK 200 UNITS	STOCK 300 UNITS
A	0.25	100	80	60
B	0.75	100	200	175

States of nature A and B represent possible selling outcomes (low sales volume versus high sales volume).

Assume that the expenses incurred for maintaining any inventory level is \$0.25 per unit.

Solution

The demonstration of generalized procedure with the above simple problem is as follows:

2.1 CHART THE DECISION FLOW DIAGRAM

Since there are three strategies (inventory levels), there are three alternatives which are presented in the tree diagram 2.1 (with two states of nature A and B for each strategy).

The symbol small square (☐) represents an act, where the decision maker has to decide which strategy to select with respect to the selection criterion. The small circle (0) represents an event, where the cost is averaged out.

2.2 ASSIGN PAYOFFS AT THE TIPS OF BRANCHES OF THE TREE

The expenses incurred for maintaining inventory level is \$0.25 per unit. Therefore:

For strategy S_1, the cost of maintaining inventory level is = 100 x 0.25
= \$25.

For strategy S_2, the cost of maintaining inventory level is = 200 x .25
= \$50.

For strategy S_3, the cost of maintaining inventory level is = 300 x .25
= \$75.

Figure 2.2 is the tree diagram with the payoffs on the branches of

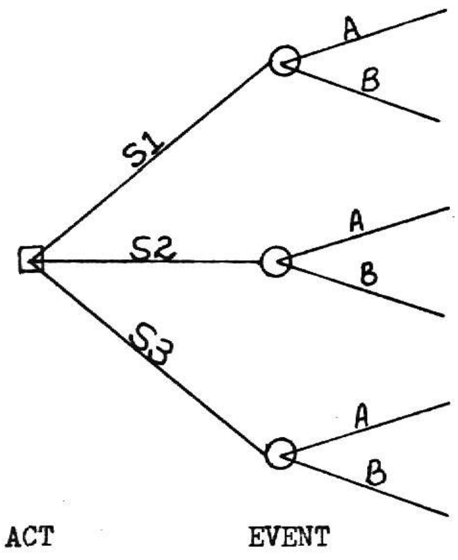

FIGURE 2.1 -Tree Diagram With Possible Acts and Events.

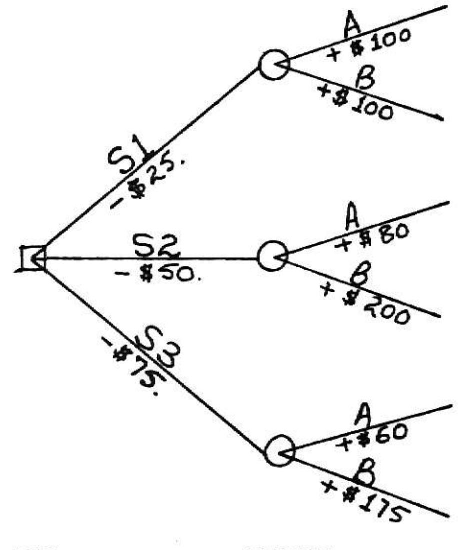

FIGURE 2.2 -Tree Diagram With Partial Cash Flows due to

Individual Acts and Events.

the tree. A negative sign with a cost shows outflow and a positive sign shows inflow.

2.3 ASSIGN PROBABILITIES AT ALL CHANCE FORKS

Since the probability at the chance (event) fork is known, the Figure 2.3 represents the third step, with total cash flow shown on the extreme right in small boxes.

2.4 AVERAGE OUT AND FOLD BACK

The starting point for carrying out this computation is the right end of the tree (see Figure 2.3). The total cash flow along the path is written in a small box. Now, proceeding from right to left, one has to check the first node to ascertain whether it is an act or event. The first node here is an event, therefore the cash flows are averaged out and written in small box (above node E1, E2, and E3), in Figure 2.4.

The numbers in small boxes at node E1, E2 and E3 in Figure 2.4 are arrived at as follows:

Node E 1

$$\text{Expected payoff} = 1/4 \times 75 + 3/4 \times 75$$
$$= 18.75 + 56.25$$
$$= \$75.$$

Node E 2

$$\text{Expected payoff} = 1/4 \times 30 + 3/4 \times 150$$
$$= 7.5 + 112.5$$
$$= \$120.$$

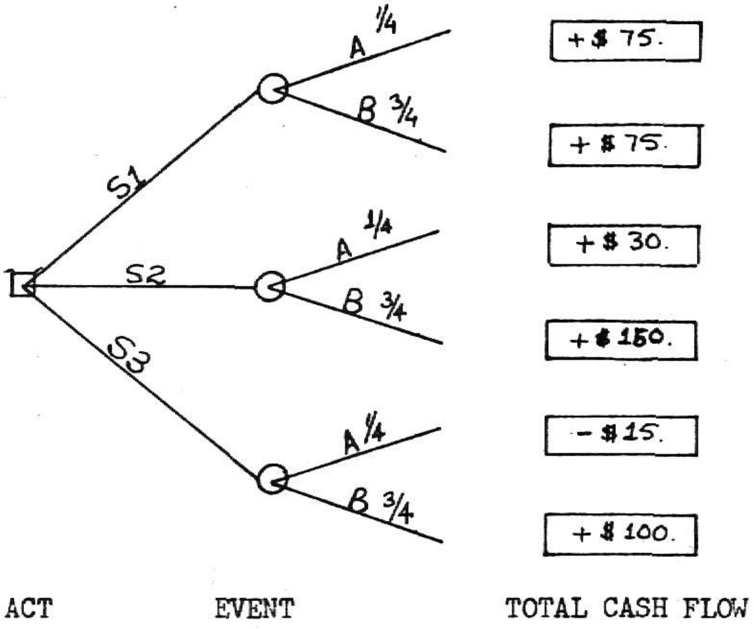

FIGURE 2.3 -Tree Diagram With Total Cash Flows due to Act-Event Sequences.

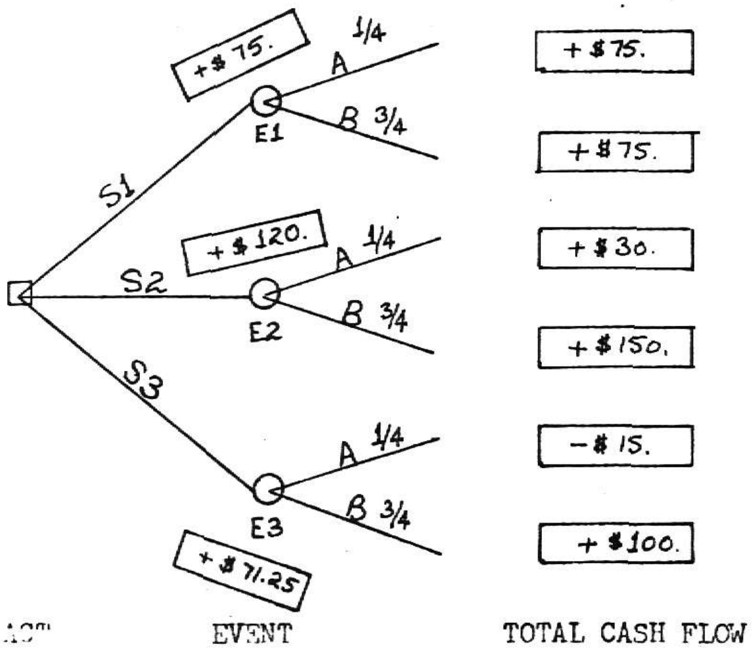

FIGURE 2.4 -Tree Diagram With Final Description.

Node E 3

$$\text{Expected payoff} = 1/4 \times (-15) + 3/4 \times 100$$
$$= -3.75 + 75$$
$$= \$71.25$$

The preceding node is an act in every case, so the branch with maximum partial cash flow will be selected. That is branch S_2.

Therefore, the optimum decision is to maintain inventory level at two hundred units.

CHAPTER III

BACK GROUND OF THE PROBLEM

The problem depicted in this report was made for class room study. An 'Applied Decision Theory' course (550 - 751) is taught in the Industrial Engineering Department at Kansas State University. The teaching schedule is shown in Table 6.

TABLE 6. TEACHING SCHEDULE

TIME SPENT	TOPIC	REFERENCE
One week	Basic ideas of Bayesian Analysis	Measuring Uncertainity an Elementry Introduction to Bayesian Statistics By Samual A. Schmitt.
Three weeks	Prior and Posterior Distribution for Binomial, Normal, Poisson and Gamma Distribution.	Instructor's notes
Seven weeks	Decision Making Under Uncertainty and Risk.	1. Elementry Decision Theory. By Chernoff and Moses. 2. Introduction to Bayesian Decision Processes By Bruce Morgan. 3. Decision Analysis By Howard Raiffa.

TABLE 6 (continued)

TIME SPENT	TOPIC	REFERENCE
One week	A Case Study Involving Decision Making Under Risk	<u>Rositex Company Case</u> Case Number ICH 12C19 EA-C 741 From Harvard Business School
Two weeks	Game Theory	<u>Two--Person Game Theory</u> By Anatol Rapoport.

For the past two years (1970, 1971) the Rositex Company case (1964) was used as a teaching example in the class; the instructor felt that it was time for new problem. The author was asked by the instructor to construct a real world problem for class room use. The main features of the problem were patterned after the Rositex Company case, which discussed a chemical producing company, producing a chemical at a time when the company was shifting to its new plant. The problem presented here concerns manufacturing flywheels under different situations. Since the author's brother in India runs a foundry, manufacturing diesel engine parts, and the author had worked there for three months, he had some knowledge about how decisions were made and how a flywheel was manufactured. The data used in this problem was received from author's brother.

CHAPTER IV

THE MANUFACTURING PROCESS

For manufacturing a flywheel, the following is the sequence of operations for arriving at a finished article:

4.1 SEQUENCE OF OPERATIONS

1. Pattern Making
2. Molding
3. Running of Cupola (melting and pouring)
4. Cleaning and Inspecting
5. Machining
    - (a) setting
    - (b) drilling
    - (c) boring
    - (d) turning and facing

4.2 FOUNDRY PROCESSES

Foundry processes consist of making molds, preparing and melting the metal, pouring the metal into molds, cleaning the castings and reclaiming the sand for reuse. The product of the foundry is a casting, in our case a flywheel.

A casting is defined as a "metal object obtained by allowing molten metal to solidify in a mold", the shape of the object being determined by the shape of the mold cavity. The following are the principle factors in getting a sound flywheel casting:

1. Pattern Making
2. Core Making
3. Molding
4. Melting and Pouring
5. Cleaning and Inspection

4.2.1 Pattern Making

The first step in making a casting of a flywheel is to prepare a model, known as a pattern, which differs in a number of respects from the resulting casting. These differences, known as pattern allowances, compensate for metal shrinkage, provide sufficient metal for machined surfaces, and facilitate molding.

For hand molding and machine molding the type of pattern used is "single", as there are different types of pattern, like single, gated, match-plate, cope and drag, etc. For hand molding the pattern is made of wood (white pine) but for machine molding, it is made of metal, usually aluminum.

The pattern to be made for a given part depends largely on the judgement and experience of the pattern maker and is goverened by the pattern cost and the number of castings to be made. If the casting is large, it is cast singly in a mold.

4.2.1.1 Pattern Allowance

Although the pattern is used to produce a casting of the desired dimensions, it is not dimensionally identical with the casting. For metallurgical and mechanical reasons, a number of allowances must be made on the pattern if the casting is to be dimensionally correct.

4.2.1.1.a Shrinkage Allowance

Shrinkage Allowance is a correction for solidification shrinkage of the metal and its contraction during cooling to room temperature. Pattern shrinkage allowance is the amount the pattern must be made larger than the casting to provide for total contraction. There are tables on "Pattern Shrinkage Allowance" available.

4.2.1.1.b Machine Finish Allowance

Machine Finish Allowance is the amount that dimensions on a casting are made oversize to provide stock for machining. This allowance largely depends upon the metal, the casting design, and method of casting and cleaning.

4.2.1.1.c Pattern Draft

Draft is the taper allowed on vertical faces of a pattern to permit its removal from the sand without tearing the mold cavity surfaces. For hand molding it is 1/16 inch per foot and for machine molding it is about one degree.

4.2.1.1.d Size Tolerance

The variation which may be permitted on a given casting dimension is called its tolerance and is equal to the difference between the minimum and the maximum limits for any specified dimension. This size tolerance allowance is given on a casting weighing one thousand pounds or more. Since the flywheel casting is about one hundred and twenty five pounds, there is no need of this allowance.

4.2.1.1.e Distortion Allowance

Distortion allowance applies only to those castings of irregular shape which are distorted in the process of cooling because of metal shrinkage. But in its present case the casting is not of irregular shape so the question of this allowance does not arise.

4.2.2 Core Making

Cores are forms, usually made of sand, which are placed into a mold cavity to form the interior surfaces of the castings. The void space between the core and mold cavity surface is what eventually becomes the casting. In making a flywheel there is no need for a core.

4.2.3 Molding

Molding consists of all operations necessary to prepare a mold for receiving molten metal. It usually implies ramming molding sand around a pattern placed in a supporting frame, withdrawing the pattern to leave the mold cavity, setting of cores in the mold cavity, and closing of the mold.

There are a number of sand molding processes; the one used for making flywheels is green-sand-molding.

Green-Sand-Molding

Green sand is a plastic mixture of sand grains, clay, water, and other materials which can be used for molding and casting processes. The sand is called "green" because of the moisture present and is thus distinguished from dry sand.

The basic steps in green sand molding are:

1. preparation of the pattern

2. making the mold
3. core setting (not applicable here)
4. closing and weighting

In the case of hand molding the floor molding process is applied, because the casting is heavy enough (the flywheel weighs one hundred and twenty pounds) to result in handling difficulty. The Figure 4.1 shows the floor molding process.

4.2.4 Melting and Pouring

The preparation of molten metal for casting is referred to simply as "melting". Melting is usually done in a specifically designated area of the foundry, and the molten metal is transferred to the molding area in ladles where the molds are poured.

Casting is essentially a remelting process, accomplished in a furnace especially designed for the quantity and type of metal to be cast. For flywheel casting the furnace used is known as cupola as shown in Figure 4.2.

Iron castings are made by remelting scrap iron along with pig iron in the cupola. The construction is very simple, consisting of a vertical stack lined with a refractory material, with provisions for introducing an air blast near the bottom.

Operation of the Cupola

The first operation in preparing a cupola is to clean out the slag and refuse on the lining and around the tuyeres from the previous run. If the lining is broken, it should be repaired so that there are no cracks on the lining. All cracks are closed with fire clay, and a layer of black molding sand is placed on the bottom. This black sand is rammed down and

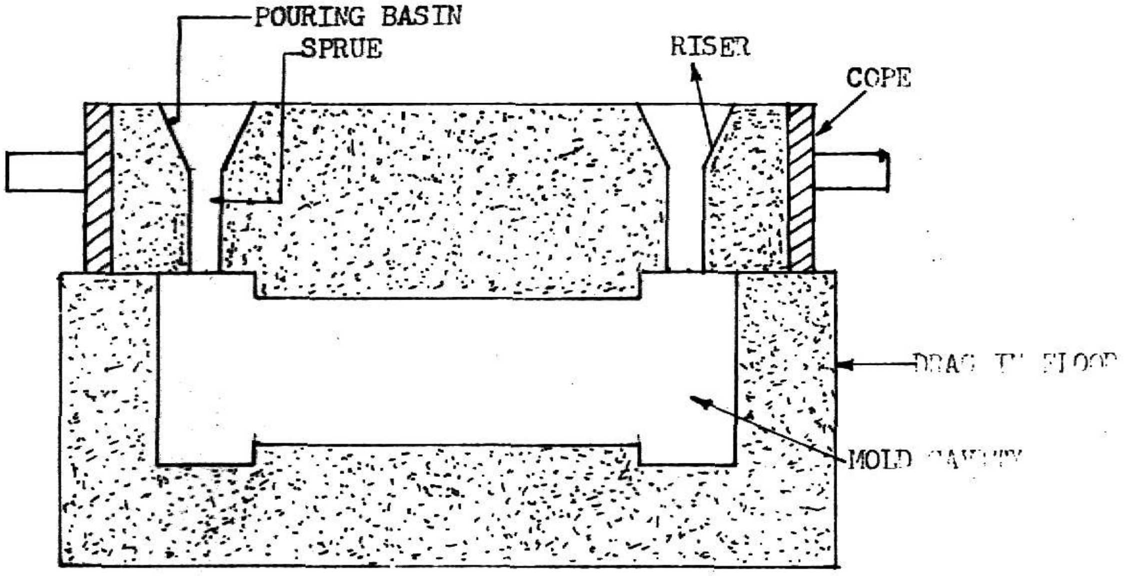

FIGURE 4.1 -Floor Molding.

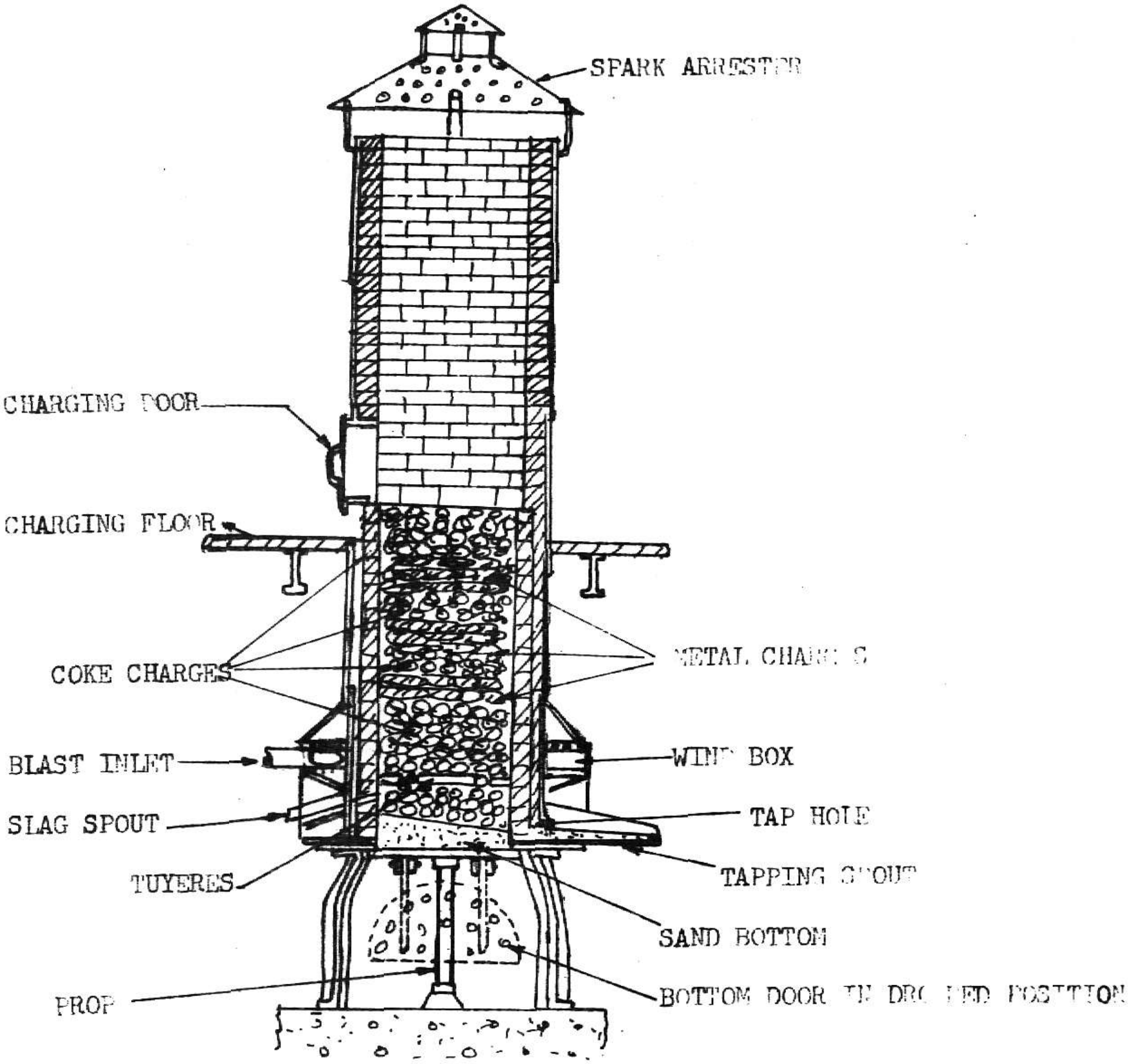

FIGURE 4.2 -Cross Section of a Cupola.

given a slope toward the spout to a depth not less than four inches at the lowest point. A small tap hole about 3/4 to 1 inch in diameter is provided.

The firing of a cupola is started 2½ to 3 hours before the first metal to be tapped. A sufficient amount of wood should be used to ignite a bed of coke. Coke is added from time to time until the bed is built up to its proper height above the tuyeres. The height of coke bed ranges from 20 to 50 inches above the top of the tuyeres. As soon as the coke bed is thoroughly ignited, the pig iron and scrap iron are introduced or charged. Alternate charges of coke and iron are made in a ratio of one part of coke to six or eight parts of iron measured by weight. In addition to charging iron and coke, a fluxing material (limestone) is used to remove impurities in the iron, protect the iron from oxidation, and render the slag more fluid for easy removal from the cupola. Slag that is formed floats on the metal accumulated on the hearth and flows continuously from the slag hole at the rear of the cupola during the heat.

After the cupola is fully charged up to the charging door, the iron should soak in heat about 3/4 of an hour or longer. Before turning on the blast, the tuyere openings must be closed. After the blast has been on a few minutes, molten metal starts accumulating in the hearth. The tap hole is then stopped up until a sufficient amount of molten metal is accumulated in the cupola to warrant pouring operations. After this the tap hole of the cupola is opened intermittently, allowing the metal to flow into a large ladle. It is then closed again with a conical clay plug called a "bot". The ladle filled with molten metal is taken to the molding