GYRATOR CONSIDERATIONS

by

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INTRODUCTION

In the study of network analysis and synthesis it is customary to distinguish between reciprocal and nonreciprocal networks. The classification simply depends upon whether or not the network satisfies the Reciprocity Theorem. That is, the ratio of excitation at one port to response at another port is independent of which of the two ports is excited.

If a network contains only resistance, capacitance, self and mutual inductances it is necessarily reciprocal. Networks containing other types of elements are usually nonreciprocal.

Quite often the class of nonreciprocal networks is incorrectly identified with the class of active networks. Ordinarily active networks are nonreciprocal but, conversely nonreciprocal networks need not be active.

The distinction between active and passive nonreciprocal networks is clarified and studied by adding a fifth basic element to the list of conventional passive elements. Although several basic nonreciprocal elements have been proposed the most widely studied is the gyrator, introduced by Tellegen[1] in 1948. The inclusion of the gyrator permits analysis and synthesis of nonreciprocal networks. Its value has also been demonstrated in the synthesis of reciprocal networks.

The ideal gyrator is closely related to several other network models. The characteristics of these models will be given along with a realization of them using the gyrator.

The ideal gyrator has many physical realizations. Several of these are given in the last chapter. Because the gyrator can be physically realized, it has value, not only as an abstract mathematical model, but also as a practical network element.

PROPERTIES OF GYRATORS AND NETWORKS CONTAINING GYRATORS

The ideal gyrator is defined by the symbols in Fig. 1 and by Eqs. 1 and 2 below.

$$V_1 = -R\ I_2 \qquad (1)$$

$$V_2 = R\ I_1 \qquad (2)$$

The constant, real coefficient R has the dimension of ohms and is referred to as the gyration resistance. Notice the linear relationship between variable V_1 and variable I_2, and also between V_2 and I_1. The gyrator "gyrates" an output current into an input voltage and an output voltage into an input current. Conversely, the input variables are gyrated into the appropriate output variables. If the secondary is short-circuited, the primary terminals appear as an open circuit and vice versa.

The gyrator is a passive element. This may be demonstrated by summing the instantaneous power going into the device at the ports. Non-negative total, instantaneous power into the device indicates a passive network. Let P_1 and P_2 be the instantaneous power in at ports one and two, respectively. Then the power into the gyrator at any instant of time is $P_1 + P_2$. This may be written in the form

$$P_1 + P_2 = v_1\ i_1 + v_2\ i_2 \qquad (3)$$

where the small v's and i's represent instantaneous values of

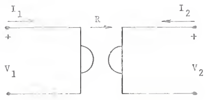

Fig. 1. Gyrator symbol.

voltage and current. If v_2 and i_2 from Eqs. 1 and 2 are substituted into Eq. 3, the total instantaneous power into the gyrator is found to be zero. That is,

$$P_1 + P_2 = v_1 i_1 + (Ri_1)(-\frac{1}{R} v_1)$$

$$P_1 + P_2 = 0 \tag{4}$$

This indicates that the ideal gyrator is not only passive but also lossless.

The open-circuit impedance, short-circuit admittance, and chain matrices for the gyrator are given below as the coefficient matrices in Eqs. 5, 6, and 7 respectively.

$$\begin{bmatrix} V_1 \\ V_2 \end{bmatrix} = \begin{bmatrix} 0 & -R \\ R & 0 \end{bmatrix} \begin{bmatrix} I_1 \\ I_2 \end{bmatrix} \tag{5}$$

$$\begin{bmatrix} I_1 \\ I_2 \end{bmatrix} = \begin{bmatrix} 0 & \frac{1}{R} \\ -\frac{1}{R} & 0 \end{bmatrix} \begin{bmatrix} V_1 \\ V_2 \end{bmatrix} \tag{6}$$

$$\begin{bmatrix} V_1 \\ I_1 \end{bmatrix} = \begin{bmatrix} 0 & R \\ \frac{1}{R} & 0 \end{bmatrix} \begin{bmatrix} V_2 \\ -I_2 \end{bmatrix} \tag{7}$$

The H parameters, quite often used in transistor networks, do not exist for the gyrator.

As defined the gyrator violates the theorem of reciprocity. The theorem of reciprocity states that the ratio of excitation at port one, to response at port two be identical to the ratio of excitation at port two to response at port one. More

Specifically, if a voltage is applied at port one ($V_1 = E$) and the current measured at short-circuited port two ($I_2 = I$), then the current measured at short-circuited port one, when the voltage is applied at port two ($V_2 = E$), will also be I. This is concisely stated in Eq. 8.

$$\left.\frac{I_1}{V_2}\right|_{V_1 = 0} = \left.\frac{I_2}{V_1}\right|_{V_2 = 0} \tag{8}$$

With reference to the open-circuit impedance matrix, this is equivalent to stating $y_{12} = y_{21}$, or by a dual development $z_{12} = z_{21}$. Equation 6 shows that this condition is not satisfied by the gyrator.

If $z_{12} \neq z_{21}$ for a two-port network, the network is said to be nonreciprocal. A special case of nonreciprocity occurs when $z_{12} = -z_{21}$. Networks possessing this property, of which the gyrator is an example, are said to be antireciprocal.

An interesting physical consequence of this property of antireciprocity may be demonstrated by first loading port two of the gyrator and examining transmission from one to two (Fig. 2), and by then loading port one and examining transmission from two to one (Fig. 3). Transmission from one to two shall be defined as V_2/V_1, when a voltage is applied at port one only ($V_1 = E$), and a load applied at port two only ($V_2 = -r\ I_2$). Transmission from two to one shall likewise be defined as V_1/V_2, when the same voltage is applied at port two ($V_2 = E$), and the same load applied at port one ($V_1 = -rI_1$). Using Eqs. 1 and 2 it may be verified that transmission from port one to two, Fig. 2, is as

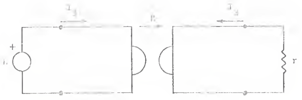

Fig. 2. Gyrator with loaded secondary.

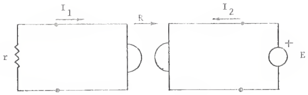

Fig. 3. Gyrator with loaded primary.

given in Eq. 9.

$$\frac{V_2}{V_1} = \frac{V_2}{E} = \frac{r}{R} \tag{9}$$

Similarly, transmission from two to one, Fig. 3, is given in Eq. 10.

$$\frac{V_1}{V_2} = \frac{V_1}{E} = -\frac{r}{R} \tag{10}$$

The negative sign, present in Eq. 10 and absent in Eq. 9, is an indication of antireciprocity. The negative sign in Eq. 10 may be viewed as 180 degrees of phase shift for transmission from port two to port one. Equation 9 indicates the absence of any phase shift for transmission in the forward direction.

Notice that the transmission in the forward and reverse direction differ only in sign and not in magnitude. This results because $|Z_{12}| = |Z_{21}|$ and $Z_{11} = Z_{22} = 0$ for the gyrator.

Transmission in the forward and reverse directions will have the same sign for reciprocal networks. Phase shift in one direction will necessarily equal the phase shift in the opposite direction. Although the signs will always be identical the magnitudes, of course, need not be.

Another very important property of the gyrator is that of impedance inverting. If the gyrator is terminated in an impedance Z_ℓ, the impedance looking into port one will be the reciprocal of Z_ℓ scaled by the factor R^2. When a capacitor is connected to the secondary terminals, Fig. 4, the impedance between the primary terminals is the same as that of an inductor. In terms of Laplace transform functions, when $Z_\ell = 1/CS$, $Z_{in} = R^2CS$ where Z_ℓ denotes the load at port two and Z_{in} denotes the input impedance at port one. The impedance R^2CS is that of an inductor of R^2C henries. In general, $Z_{in} = R^2/Z_\ell$. The poles and zeroes of Z_ℓ are respectively the zeroes and poles of Z_{in}. This same relationship exists when loading the primary and observing the impedance at the secondary.

Above, the gyrator was considered to be a one-port after loading the other port with Z_ℓ. Additional information may be gained by examining the effect of cascading a general two-port with a gyrator. It should be expected that the two-port network thus derived would be nonreciprocal due to the antireciprocal gyrator. In Fig. 5, the gyrator precedes the general two-port. The coefficient matrix in Eq. 11 is the impedance matrix for the combined two-port network of Fig. 6.

$$\begin{bmatrix} V_1 \\ \\ V_2 \end{bmatrix} = \begin{bmatrix} \dfrac{R^2}{z_{11}} & -\dfrac{Rz_{12}}{z_{11}} \\ \\ \dfrac{Rz_{12}}{z_{11}} & \dfrac{z_{11}z_{22}-z_{12}^2}{z_{11}} \end{bmatrix} \begin{bmatrix} I_1 \\ \\ I_2 \end{bmatrix} \qquad (11)$$

This matrix equation may be verified by noting that the output voltage of the gyrator is the input voltage of the general two-port and that the output current of the gyrator is the negative of the input current of the general two-port. Equation 11 indicates that the network of Fig. 5 is not just nonreciprocal but

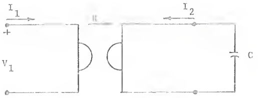

Fig. 4. Gyrator as impedance inverter.

Fig. 5. General two-port and gyrator cascaded, with gyrator on input side.

antireciprocal, if the general two-port is assumed reciprocal ($z_{12} = z_{21}$). Any reciprocal network cascaded with one gyrator will yield an antireciprocal combination. Notice also, the open-circuit input impedance, z_{11}, has been inverted and scaled by R^2 just as Z_ℓ was inverted and scaled in the previous example. If the general two-port network precedes the gyrator in cascade, as shown in Fig. 6, a similar matrix would exist. This is the coefficient matrix of Eq. 12.

$$\begin{bmatrix} V_1 \\ V_2 \end{bmatrix} = \begin{bmatrix} \dfrac{z_{11}z_{22} - z_{12}^2}{z_{22}} & -\dfrac{Rz_{12}}{z_{22}} \\ \dfrac{Rz_{12}}{z_{22}} & \dfrac{R^2}{z_{22}} \end{bmatrix} \begin{bmatrix} I_1 \\ I_2 \end{bmatrix} \qquad (12)$$

The resulting network is of course, antireciprocal as before, but it is now the open-circuit output impedance, z_{22}, which has been inverted and scaled. Later the effect of more than one gyrator in cascade will be considered.

So far only one-port and two-port networks have been investigated. Only the port voltages and currents have been considered in equation form. Quite often it becomes necessary to analyze networks having many meshes or loops and many exposed terminals. To analyze such a network one might write Kirchhoff current and voltage equations and solve for the unknown variables; or perhaps the well known node-voltage method[2] could be used. Another common method of analyzing a network with many meshes and terminals is the mesh-current method[2]. The mesh-current

Fig. 6. General two-port and gyrator cascaded, with gyrator on output side.

matrix equations can normally be written by inspection for a network of R's, L's, M's and C's. Matrix algebra may then be used to find the unknowns. Examples of this are given in most network analysis texts. If gyrators are also included, in addition to the other passive elements, the mesh-current equations take on a slightly different form[3]. An example network containing a gyrator is shown in Fig. 7. Equation 13 is the matrix of mesh-current equations for this four-mesh network.

$$\begin{bmatrix} V_1 \\ 0 \\ 0 \\ V_4 \end{bmatrix} = \begin{bmatrix} R_1 & -R_1+R & 0 & 0 \\ -R_1-R & R_1+L_1S & -L_1S-MS & 0 \\ 0 & -L_1S-MS & R_2+L_1S+L_2S+2MS+\frac{1}{CS} & R_2 \\ 0 & 0 & R_2 & R_2+L_3S \end{bmatrix} \begin{bmatrix} I_1 \\ I_2 \\ I_3 \\ I_4 \end{bmatrix} \qquad (13)$$

The self-impedance terms, along the main diagonal, in this example are not effected by the addition of the gyrator. The gyration resistance, R, occurs only in the mutual impedance terms, indicating coupling between meshes. Notice that because of R, the Z matrix is no longer symmetric about the main diagonal ($z_{12} \neq z_{21}$). The R term in z_{ij} and the corresponding R term in z_{ji} will always be of opposite sign. The methods of solving the matrix equations are not changed by the addition of the gyrator, only the Z matrix itself has been altered.

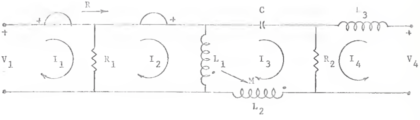

Fig. 7. Four mesh gyrator network.

THE GYRATOR AS A TOOL IN NETWORK SYNTHESIS

The synthesis of reciprocal, linear, lumped, time-invariant, passive networks is a topic of much study. It is a well known fact that all networks having the above properties may be physically realized using only resistors, capacitors, inductors and mutual inductances. If gyrators are also included in the list of available elements, all linear, lumped, time-invariant, passive, (LLTP), networks may be realized whether reciprocal or nonreciprocal. The use of gyrators in synthesis is illustrated with two examples.

Generally, an LLTP network may be represented by open-circuit impedance parameters. An example of the open-circuit impedance matrix for a two-port is given by Eq. 5. More generally for an n-port network, the n open-circuit impedance equations may be written with matrices as in Eq. 14.

$$\begin{bmatrix} V_1 \\ V_2 \\ \cdot \\ \cdot \\ \cdot \\ V_n \end{bmatrix} = \begin{bmatrix} z_{11} & z_{12} & \cdots & z_{1n} \\ z_{21} & z_{22} & \cdots & z_{2n} \\ & & \cdots & \\ & & \cdots & \\ & & \cdots & \\ z_{n1} & z_{n2} & \cdots & z_{nn} \end{bmatrix} \begin{bmatrix} I_1 \\ I_2 \\ \cdot \\ \cdot \\ \cdot \\ I_n \end{bmatrix} \qquad (14)$$

This expression is not to be confused with the mesh-current equations given in Eq. 13. The mesh-current matrix of Eq. 13

has one row and column for every mesh or loop, regardless of the number of exposed terminals. The coefficient matrix of Eq. 14 has one row and column for every port or pair of exposed terminals. The network represented by Eq. 14 may have many hidden meshes which are not made obvious by the impedance equations.

If an impedance matrix equation, such as Eq. 14, is to represent a physically realizable network (reciprocal or non-reciprocal) the coefficient matrix must satisfy certain requirements[3]. To more easily understand these requirements, the coefficient matrix, Z, of Eq. 14 is written in Eq. 15 as the sum of a hermitian matrix, Z_H, and a skew-hermitian matrix, Z_S.

$$Z = Z_H + Z_S \tag{15}$$

The elements of the hermitian matrix, Z_H, satisfy the conditions, $z_{ij} = z^*_{ji}$, $i \neq j$, and z_{ii} real for all i, with the asterisk denoting the complex conjugate. The elements of the skew-hermitian matrix, Z_S, satisfy the conditions, $z_{ij} = -z^*_{ji}$, $i \neq j$, and z_{ii} imaginary for all i. Every open-circuit impedance matrix may be written as the sum of Z_H and Z_S. This is shown by Eq. 16 and Eq. 17, which allow one to solve for Z_H and Z_S in terms of matrix Z.

$$Z_H = \frac{1}{2} [Z + Z^{*T}] \tag{16}$$

$$Z_S = \frac{1}{2} [Z - Z^{*T}] \tag{17}$$

The asterisk again denotes complex conjugate and the superscript T denotes the matrix transpose. Equations 18, 19, and 20 illustrate the splitting of a two-port Z matrix into Z_H and Z_S.

$$Z = \begin{bmatrix} z_{11} & z_{12} \\ z_{21} & z_{22} \end{bmatrix} = \begin{bmatrix} r_{11} + jx_{11} & r_{12} + jx_{12} \\ r_{21} + jx_{21} & r_{22} + jx_{22} \end{bmatrix} \tag{18}$$

$$Z_H = \begin{bmatrix} r_{11} & \frac{r_{12}+r_{21}}{2} + j\frac{x_{12}-x_{21}}{2} \\ \frac{r_{12}+r_{21}}{2} - j\frac{x_{12}-x_{21}}{2} & r_{22} \end{bmatrix} \tag{19}$$

$$Z_S = \begin{bmatrix} j\,x_{11} & \frac{r_{12}-r_{21}}{2} + j\frac{x_{12}+x_{21}}{2} \\ -\frac{r_{12}-r_{21}}{2} + j\frac{x_{12}+x_{21}}{2} & j\,x_{22} \end{bmatrix} \tag{20}$$

In general, Z_H and Z_S are both nonreciprocal. If the two-port matrix, Z, were reciprocal, both Z_H and Z_S would also be reciprocal. Furthermore, the elements of Z_H would all be real, (Z_H = ReZ), and the elements of Z_S, all imaginary, (Z_S = ImZ). The matrices Z_H and Z_S arising from the general case, Eq. 14, are similar to Eq. 19 and Eq. 20, respectively.

The conditions required of the impedance matrix for representing a physically realizable network may now be stated. The necessary and sufficient conditions for a linear, passive network, which possesses an open-circuit impedance matrix with complex number elements, to be physically realizable as a lumped passive network is that the hermitian part of the impedance matrix be positive definite or positive semidefinite.[3] Thus, all the principal minors of Z_H must be non-negative.

A method of synthesizing a nonreciprocal network from its open-circuit impedance matrix is given by Carlin.[3] The procedure begins by splitting Z, as was done in Eq. 15, and synthesizing Z_H and Z_S individually. The networks realizing Z_H and Z_S must both contain gyrators if the real and imaginary parts of the open-circuit impedance matrix are nonreciprocal. With reference to Eq. 20, Z_S may be broken down into an antireciprocal real matrix and a symmetrical imaginary matrix. The antireciprocal real matrix represents an all-gyrator network. The synthesis of the symmetrical imaginary part of Z_S is always possible and the procedure is straight forward. The network represented by Z_H, although always realizable, is not so easily synthesized because the real and imaginary parts of Z_H are not individually realizable. After the networks of Z_H and Z_S have been constructed, the two are placed in series to form the total network represented by Z. The many details of Carlin's complete procedure are omitted here because of the excessive length. It must be remembered that the real and imaginary parts of z_{ij} above are constants and not functions of frequency.

The use of gyrators in network synthesis is not limited to nonreciprocal networks. Although all reciprocal, LLTP networks may be realized without the aid of gyrators, occasionally a reduction in the number of elements results when gyrators are employed. An example of this is Hazony's modification of Darlington's driving point synthesis. Darlington's original synthesis procedure is described in detail in most network synthesis texts.[4] Here it is briefly reviewed before discussing Hazony's modification.

Darlington's procedure provides a method of realizing any realizable driving point impedance, Z(s), provided that Z(s) is not a pure reactance function. The network described by Z(s) is a two-terminal or one-port network. It is obvious that a synthesis of this kind can be performed without gyrators since reciprocity has meaning only for networks of two or more ports.

The impedance function Z(s) is assumed to be the input impedance, Z_{in}, of a two-port network terminated in a one ohm resistor. This input impedance in terms of the open circuit impedance parameters of the unknown two-port is given by Eq. 21.

$$Z_{in} = z_{11} \frac{1 + (z_{11}z_{22} - z_{12}z_{21})/z_{11}}{1 + z_{22}} \tag{21}$$

This function may be written as a ratio of polynomials in s as shown by Eq. 22.

$$Z(s) = \frac{m_1 + n_1}{m_2 + n_2} \tag{22}$$

In Eq. 32, the numerator (denominator) polynomial of $Z(s)$ has been written as the sum of an even polynomial, $m_1(m_2)$, and an odd polynomial, $n_1(n_2)$. By equating Eqs. 21 and 22, z_{11} and z_{22} can be expressed as ratios of even to odd polynomials or odd to even polynomials. The procedure of solving for the z parameters ensures realizability. Equating Eqs. 21 and 22 yields two distinct cases given by Eqs. 23 and 24.

$$z_{11} = \frac{m_1}{n_2} \tag{23a}$$

$$z_{22} = \frac{m_2}{n_2} \tag{23b}$$

$$z_{12}z_{21} = \frac{m_1 m_2 - n_1 n_2}{n_2^2} \tag{23c}$$

$$z_{11} = \frac{n_1}{m_2} \tag{24a}$$

$$z_{22} = \frac{n_2}{m_2} \tag{24b}$$

$$z_{12}z_{21} = \frac{n_1 n_2 - m_1 m_2}{m_2^2} \tag{24c}$$

If gyrators are not permitted at this point, z_{12} must equal z_{21} and Eqs. 23c and 24c yield, respectively, Eqs. 25 and 26.

$$z_{12} = \frac{\sqrt{m_1 m_2 - n_1 n_2}}{n_2} \tag{25}$$

$$z_{12} = \frac{\sqrt{n_1 n_2 - m_1 m_2}}{m_2} \tag{26}$$

Although synthesizing a network satisfying z_{11} and z_{22} for either case can easily be done, z_{12} in Eq. 25 or 26 may present a problem. The expressions for the transfer impedance, z_{12}, are often irrational functions. Originally, Darlington overcame this difficulty with the use of surplus factors. If z_{12} proves to be irrational in both cases, Eq. 25 and 26, the numerator and denominator of Z(s) are multipled by a surplus factor, in the form of a polynomial, m_o+n_o, as shown in Eq. 27.

$$Z(s) = \frac{m_1+n_1}{m_2+n_2} \cdot \frac{m_o+n_o}{m_o+n_o} \tag{27}$$

The surplus factors obviously do not alter Z(s). However, starting with Eq. 27 and deriving the new expression for z_{12} shows that a rational function is obtained provided that Eq. 28 is satisfied.

$$m_o^{\ 2} - n_o^{\ 2} = m_1 m_2 - n_1 n_2 \tag{28}$$

Equation 28 is then used to find the surplus factors. This procedure always ensures the rationality of z_{12}, and makes it possible to realize Z(s) using only reciprocal elements. Unfortunately however, the surplus factors increase the number of elements in the network.

To avoid the need of surplus factors, Bashkow[4] proposed the use of a gyrator. If gyrators are permitted, z_{12} need not be equal to z_{21} and the square roots in Eqs. 25 and 26 are no longer necessary. To show this, Eqs. 23c and 24c are rewritten in terms of m_o and n_o from Eq. 28.

$$z_{12}z_{21} = \frac{m_o^2 - n_o^2}{n_2^2} \tag{29}$$

$$z_{12}z_{21} = \frac{n_o^2 - m_o^2}{m_2^2} \tag{30}$$

Equations 29 and 30 are written in terms of the surplus factor notation only to emphasize that the factoring suggested by Eq. 28 is required. Surplus factors have not been used to modify Z(s). Factoring the expressions in Eqs. 29 and 30 yield, respectively, Eqs. 31 and 32.

$$z_{12} = \frac{m_o + n_o}{n_2} \tag{31a}$$

$$z_{21} = \frac{m_o - n_o}{n_2} \tag{31b}$$

$$z_{12} = \frac{n_o + m_o}{m_2} \tag{32a}$$

$$z_{21} = \frac{n_o - m_o}{m_2} \tag{32b}$$

Now z_{12} and z_{21} are rational functions of s, although they are not equal. To complete the synthesis, the reciprocal part of z_{12} and z_{21} should be separated from the antireciprocal part. The antireciprocal part may be realized as a gyrator; and the reciprocal part of z_{12} and z_{21} along with z_{11} and z_{22} may be realized as a lossless network. If these two networks are connected in series and the combination terminated in a one ohm resistor, the input impedance will be the desired Z_{dp}. An example of Darlington's synthesis using a gyrator is given by Hazony[4]. By relaxing the reciprocity condition and permitting gyrators, a reduction in the number of network elements was achieved for Darlington's driving point synthesis.

USING GYRATORS IN MODELING OTHER USEFUL DEVICES

The gyrator is a basic network element. Several other basic elements are closely related to the gyrator. The ideal transformer and the ideal gyrator are both passive, lossless coupling devices, although the transformer is reciprocal and the gyrator is anti-reciprocal. The isolator and circulator are passive two-port elements used in constructing nonreciprocal networks. The negative impedance converter is an active nonreciprocal two-port model. In this chapter it is shown that the gyrator may be used in modeling the ideal transformer, the isolator, the circulator, and the negative-impedance converter.

A transformer[5] consists of two coils, L_1 and L_2, magnetically coupled by a mutual inductance, M, as shown in Fig. 8. The coefficient of coupling between the coils, k, is given by Eq. 33.

$$k = \frac{|M|}{\sqrt{L_1 L_2}} \tag{33}$$

The coefficient of coupling of physical transformers is always less than unity. If k is made equal to one, as the limiting case, the transformer is said to be perfect. If, in addition, L_1 and L_2 approach infinity in such a manner that L_1/L_2 remains finite, the transformer is said to be ideal. Equations 34 and 35 describe the ideal transformer shown in Fig. 9.

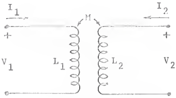

Fig. 8. A transformer.

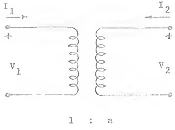

Fig. 9. An Ideal transformer.

$$V_1 = \frac{1}{a} V_2 \tag{34}$$

$$I_1 = -a\ I_2 \tag{35}$$

The turns ratio *a* is a constant real number and may be positive or negative. While the perfect transformer may be closely approximated physically, the ideal transformer must be considered a theoretical two-port model. The ideal transformer is necessarily reciprocal since it is composed of only self and mutual inductance. The chain matrix equation for the ideal transformer is given by Eq. 36.

$$\begin{bmatrix} V_1 \\ I_1 \end{bmatrix} = \begin{bmatrix} \frac{1}{a} & 0 \\ 0 & a \end{bmatrix} \begin{bmatrix} V_2 \\ -I_2 \end{bmatrix} \tag{36}$$

The Z and Y equations do not exist.

Keeping the ideal transformer in mind, note the effect of connecting two gyrators in cascade as shown in Fig. 10. It should be recalled that the overall chain matrix for cascaded networks is simply the product of the individual matrices in their respective order. The chain matrix for the gyrator was given in Eq. 7. The overall chain matrix for gyrator R and R' in cascade is the coefficient matrix of Eq. 37.

Fig. 10. Two gyrators cascaded.

$$\begin{bmatrix} V_1 \\ I_1 \end{bmatrix} = \begin{bmatrix} 0 & R \\ \frac{1}{R} & 0 \end{bmatrix} \times \begin{bmatrix} 0 & R' \\ \frac{1}{R'} & 0 \end{bmatrix} \begin{bmatrix} V_2 \\ -I_2 \end{bmatrix}$$

$$\begin{bmatrix} V_1 \\ I_1 \end{bmatrix} = \begin{bmatrix} \frac{R}{R'} & 0 \\ 0 & \frac{R'}{R} \end{bmatrix} \begin{bmatrix} V_2 \\ -I_2 \end{bmatrix} \quad (37)$$

Equations 36 and 37 are identical if the turns ratio is made equal to R'/R. Thus two gyrators in cascade are equivalent to an ideal transformer. It is easy to show that if three gyrators, or in fact any odd number, are cascaded, the resulting two-port is identical to a single gyrator. Likewise, any even number of gyrators in cascade yield an ideal transformer. A trivial solution exists if, in Eq. 34, R is equal to R'. This is equivalent to an ideal transformer of unity turns ratio or, more simply, two wires connecting port one to port two.

The ideal transformer cannot be used to realize an ideal gyrator, for nonreciprocal devices can never be realized using only reciprocal elements. Normally the ideal transformer and ideal gyrator are both considered basic elements. A truly basic network element should never be equivalent to a combination of other basic network elements. Since the ideal transformer may be constructed using gyrators, ambiguity exists among the basic and non-basic elements. More is said concerning this interesting philosophical question by Duinker.[6] Duinker develops a set of basic elements for non-linear networks.

Besides the gyrator, many other nonreciprocal models have been proposed. One of these is a passive lossy device called the isolator. The isolator is a two-port network which transmits energy in only one direction. The open-circuit impedance matrix of Eq. 38 represents a network which transmits only from input to output, that is, the input voltage is independent of the output current.

$$\begin{bmatrix} V_1 \\ V_2 \end{bmatrix} = \begin{bmatrix} z_{11} & 0 \\ z_{21} & z_{22} \end{bmatrix} \begin{bmatrix} I_1 \\ I_2 \end{bmatrix} \tag{38}$$

Likewise, Eq. 39 is the short-circuit admittance matrix for a network which transmits only from input to output.

$$\begin{bmatrix} I_1 \\ I_2 \end{bmatrix} = \begin{bmatrix} y_{11} & 0 \\ y_{21} & y_{22} \end{bmatrix} \begin{bmatrix} V_1 \\ V_2 \end{bmatrix} \tag{39}$$

If the (2,1) element in Eq. 38 or 39 were zero, instead of the (1,2) element, transmission would be in the opposite direction. The impedance matrix for the gyrator resistor network of Fig. 11 is equivalent to Eq. 38. This may easily be shown by adding the impedance matrix for the resistor to the impedance matrix for the gyrator, Eq. 40, as suggested by Fig. 11.

$$\begin{bmatrix} Z_{total} \end{bmatrix} = \begin{bmatrix} R & R \\ R & R \end{bmatrix} + \begin{bmatrix} 0 & -R \\ R & 0 \end{bmatrix}$$

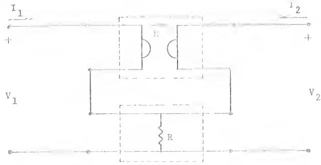

Fig. 11. Isolator formed by adding Z matrices.

$$\begin{bmatrix} V_1 \\ V_2 \end{bmatrix} = \begin{bmatrix} R & 0 \\ 2R & R \end{bmatrix} \begin{bmatrix} I_1 \\ I_2 \end{bmatrix} \tag{40}$$

Note the equivalence of the gyration resistance, R, and the dissipative resistance, R. As predicted, the input voltage, V_1, is dependent only on I_1, thus, energy is transmitted only from port one to port two. Similarly, the network of Fig. 12 has an admittance matrix of the form of Eq. 39. The total admittance matrix is found by adding the individual admittance matrices of the paralleled networks.

$$\begin{bmatrix} Y_{total} \end{bmatrix} = \begin{bmatrix} \frac{1}{R} & -\frac{1}{R} \\ -\frac{1}{R} & \frac{1}{R} \end{bmatrix} + \begin{bmatrix} 0 & \frac{1}{R} \\ -\frac{1}{R} & 0 \end{bmatrix}$$

$$\begin{bmatrix} I_1 \\ I_2 \end{bmatrix} = \begin{bmatrix} \frac{1}{R} & 0 \\ -\frac{2}{R} & \frac{1}{R} \end{bmatrix} \begin{bmatrix} V_1 \\ V_2 \end{bmatrix} \tag{41}$$

As stated for Eq. 39, the input current depends only on input voltage so transmission from port two to one is prohibited.

It is interesting to note that a procedure similar to this, using a lossy antireciprocal two port device, was published by McMillan[7] prior to the introduction of the gyrator. Tellegen makes reference to McMillan's article in his original paper[1] introducing the gyrator.

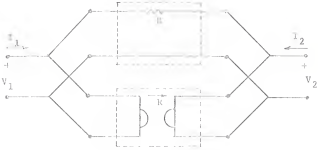

Fig. 12. Isolator formed by adding Y matrices.

The *negative-impedance converter*[8], or NIC, is another nonreciprocal model. The NIC is an active two-port network with the property that the driving point impedance at one port is the negative of the load impedance connected to the other port. This device is also termed *negative-immittance converter* to indicate that an admittance, as well as an impedance, will be converted. The NIC symbol is shown in Fig. 13, and the chain matrix equation is given by Eq. 42.

$$\begin{bmatrix} V_1 \\ I_1 \end{bmatrix} = \begin{bmatrix} -k & 0 \\ 0 & k \end{bmatrix} \begin{bmatrix} V_2 \\ -I_2 \end{bmatrix} \tag{42}$$

The symbol $\underline{k}$, in Eq. 42, is called the conversion ratio of the NIC. Although k appears in the matrix equation, its value of k has no effect on the impedance conversion. The nonreciprocal NIC may be constructed using the gyrator if an active element is included in the synthesis, as in Fig. 14. The active element is necessary because active networks can never be synthesized using only passive elements. The chain matrix for the total network of Fig. 14 is found in Eq. 43 by multiplying the chain matrices of the gyrator and the resistive T.

$$\begin{bmatrix} V_1 \\ I_1 \end{bmatrix} = \begin{bmatrix} 0 & R \\ \frac{1}{R} & 0 \end{bmatrix} \times \begin{bmatrix} 0 & R \\ -\frac{1}{R} & 0 \end{bmatrix} \begin{bmatrix} V_2 \\ -I_2 \end{bmatrix}$$

$$\begin{bmatrix} V_1 \\ I_1 \end{bmatrix} = \begin{bmatrix} -1 & 0 \\ 0 & 1 \end{bmatrix} \begin{bmatrix} V_2 \\ -I_2 \end{bmatrix} \tag{43}$$

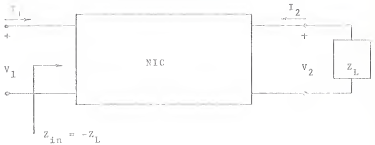

Fig. 13. Negative-impedance converter.

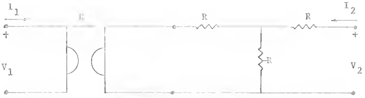

Fig. 14. Gyrator realization of negative-impedance converter.

The conversion ratio, L, for this development is independent of the gyration resistance, R, but the gyration resistance must be equal to the positive and negative impedances of the Z network.

Also, the NIC can be used with positive resistors to construct an ideal gyrator[9]. This, of course, would be an active realization of a passive device.

The circulator[10], as represented by Fig. 15, is still another nonreciprocal model. It is a passive device with three or more ports. The circulator of Fig. 15 is a three-port or three-channel circulator. A signal at port one of the circulator will be transmitted to port two. Likewise, transmission is from port two to three and from port three to one. The signals are circulated in the direction of the arrow of Fig. 15. The open-circuit impedance matrix for the three-port circulator is given by Eq. 44.

$$\begin{bmatrix} V_1 \\ V_2 \\ V_3 \end{bmatrix} = \begin{bmatrix} 0 & -R & R \\ R & 0 & R \\ -R & -R & 0 \end{bmatrix} \begin{bmatrix} I_1 \\ I_2 \\ I_3 \end{bmatrix} \qquad (44)$$

This is equivalent to the open-circuit impedance matrix of an ideal gyrator with a third pair of terminals added, as shown in Fig. 16. The equivalence can be recognized if it is noticed that primary current into the gyrator is $I_1 + I_3$, secondary current into the gyrator is $I_2 - I_3$, and that $V_3 = V_1 - V_2$. The circulator properties of this configuration may be verified by applying a voltage, E, at port one ($V_1 = E$), and a load, R,

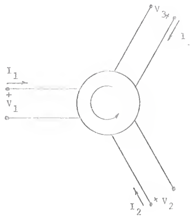

Fig. 15. Three port circulator.

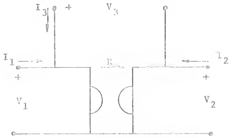

Fig. 16. Gyrator as circulator.

at port one. For this case V_1 and V_2 are equal and V_3 is zero. That is, power is transferred perfectly from port one to two. Applying the voltage at port two and loading port three with R yields similar results. Gyrators can be used to build circulators of more than three channels also, but a general method of doing so is too lengthy for this report.

A complete list of nonreciprocal models has not been given. Only those elements closely related to the gyrator have been examined, so that the gyrator itself may be better understood.

GYRATOR REALIZATIONS

The ideal gyrator was introduced as a basic nonreciprocal network element. As a mathematical model, the device has proven to be quite useful. Physical approximations to the ideal gyrator have, in recent years, enlarged its importance. Of the many proposed gyrator realizations, a few have become important commercially. Although some of the realizations that follow are passive and others are active, the port behavior of all of the realizations is, of course, passive.

Gyrator action may be achieved by mechanically coupling a <u>magneto-mechanical</u> transducer and an <u>electromechanical</u> transducer[11] as illustrated in Fig. 17. Mechanical force in a magnetomechanical transducer, such as a loudspeaker, is proportional to the current in the driving coil. Mechanical force in an electromechanical transducer, such as a capacitor microphone, is proportional to voltage. This electromechanical device approximates the ideal gyrator by gyrating currents into voltages. The electromechanical gyrator is passive but not lossless. The open-circuit input and output impedance, z_{11} and z_{22}, are not zero as would be necessary for a good approximation. A mechanical system such as this will operate only at low frequencies.

The physical phenomenon of the electromechanical gyrator is used in the piezoelectric-piezomagnetic gyrator.[11,12] Most of these realizations consist of a piezoelectric ceramic material bonded to a piezomagnetic ferrite material. Resonance in the

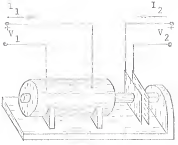

Fig. 17. Electromechanical gyrator.

piezoelectric element allows high frequency operation in a narrow bandwidth. Operation of the piezoelectric-piezomagnetic gyrator is similar to that of the electromechanical gyrator except for the narrow frequency range.

A very practical gyrator realization may be made by utilizing the Hall effect in a semiconductor material. In 1879, Hall discovered that a potential difference is developed across a current carrying metal strip when the strip is placed in a magnetic field. This effect is larger in semiconductors than in metals. The Hall effect gyrator[13] shown in Fig. 18 consists of a germanium crystal placed in a magnetic field. Four electrical contacts are made with the crystal to form the two ports of the gyrator. Nonreciprocity of the Hall effect may be explained as follows. The positive charge carriers making up I_1 are deflected to their left in a plane normal to the magnetic field. These charges cause V_2 to be positive. The positive charge carriers making up I_2 will also be deflected to their left in a plane normal to the magnetic field, this time causing V_1 to be negative. The current, magnetic field, and Hall field strength are all at right angles to one another. The current-voltage relationship in a Hall effect gyrator is analogous to the force-velocity relationship of a mechanical gyroscope.[4] The Hall effect gyrator is a passive realization. It is not loss-free but has high efficiency. Since the Hall effect doesn't depend on resonance, the gyrator thus formed is a wide band device.

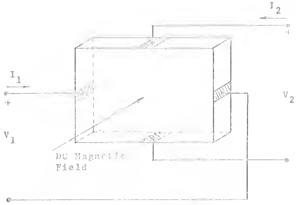

Fig. 18. Hall effect gyrator.

An efficient, wideband gyrator has been developed for use at microwave frequencies. The microwave gyrator[16] shown in Fig. 19, makes use of the Faraday rotation in a ferrite. In Fig. 19, E_1 is the direction of positive polarization of electromagnetic waves at the input guide. Positive polarization at the output guide is indicated by E_2. The guide has been twisted 90° while going from rectangular to cylindrical to rectangular. The length of the ferrite material is such that a wave passing through the material will be rotated by 90°. The curved arrow in Fig. 19 indicates the direction of rotation in the ferrite material. Notice that transmission from port one to port two, which is the same direction as the parallel d.c. magnetic field, envolves a counterclockwise rotation, while transmission from two to one envolves a clockwise rotation. As a wave enters port one, positively polarized as E_1, the wave is rotated 90° to line up with the output port, which has been twisted 90°. The wave has experienced zero phase shift while going from port one to two. A wave entering port two, positively polarized, will be rotated clockwise 90° to line up with the input port but it is now polarized negatively with respect to E_1. Thus the wave has experienced 180° of phase shift while traversing the device in the reverse direction. The microwave gyrator is one of the better approximations to an ideal gyrator. It is passive, very efficient, and operates over a wide band of frequencies in the microwave range.

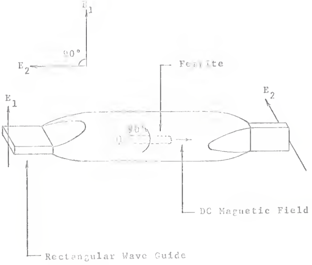

Fig. 19. Microwave gyrator.

Many active realizations of the ideal gyrator have been proposed and none of these have been constructed very successfully. It seems reasonable that vacuum tubes and transistors might be used to realize a gyrator because networks containing these active devices are almost always nonreciprocal.

If only one transistor or triode is used for a gyrator realization the input and output ports of the gyrator must have one terminal in common. Shekel[15] has named this device a 3-terminal gyrator and proposed a symbol for it which is similar to the circulator symbol of Fig. 15. A single triode in the common cathode configuration is nonreciprocal but not antireciprocal. Negative and positive resistors must be used in conjunction with the triode if the realization is to be antireciprocal and lossless.

Transistor gyrator realizations[16,17,18] are becoming important in filter design. Normally the realizations are a connection of two transistor amplifiers in parallel. Equation 45 shows how the short circuit admittance matrix of the gyrator from Eq. 6 may be broken down into two voltage-controlled current sources or idealized amplifiers.

$$\begin{bmatrix} 0 & \frac{1}{R} \\ -\frac{1}{R} & 0 \end{bmatrix} = \begin{bmatrix} 0 & \frac{1}{R} \\ 0 & 0 \end{bmatrix} + \begin{bmatrix} 0 & 0 \\ -\frac{1}{R} & 0 \end{bmatrix} \tag{45}$$

One amplifier must have 0° phase shift while the other must have 180° phase shift. The input and output impedances of the amplifiers must be very high for the main diagonal terms in the matrices of Eq. 45 to approach zero. These amplifiers each contain several transistors. These transistor gyrators have an upper frequency limit in the kilocycle range.

Operation of vacuum tube and transistor gyrators is frequency limited because of the active elements themselves and because of the associated biasing and coupling elements. The active elements must operate within the linear parts of their characteristics.

SUMMARY

The ideal gyrator was introduced as a basic nonreciprocal two-port device. It is passive, lossless and antireciprocal.

The gyrator concept is useful in network analysis in clarifying the distinction between reciprocal and nonreciprocal networks. The gyrator is also a useful tool in the synthesis of electrical networks. All linear, lumped, time-invariant, passive networks, whether reciprocal or nonreciprocal, may be realized using the gyrator and conventional passive elements.

The ideal gyrator is similar to several other network devices. The gyrator is used to model the ideal transformer, the isolator, the circulator and the negative-impedance converter.

Several of the many physical realizations for the gyrator have been given. Passive realizations may utilize the Hall effect in a semiconductor or the Faraday rotation in a ferrite. Active realizations using the vacuum tube and transistor are also important.

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GYRATOR CONSIDERATIONS

by

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AN ABSTRACT OF A MASTER'S REPORT

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