

EVALUATION OF ONE CLASSICAL AND TWO BAYESIAN ESTIMATORS  
OF SYSTEM AVAILABILITY USING MULTIPLE ATTRIBUTE  
DECISION MAKING TECHNIQUES

by

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Most importantly, I am deeply grateful to my husband, Thomas, for providing constant encouragement and unwavering support during the course of this project.

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