A STUDY OF THE PLANE STRESS OR STRAIN FINITE ELEMENT ANALYSIS FOR SOLUTION OF STRESS DISTRIBUTION IN PLANE ELASTIC CONTINUA

by

THOMAS C. HELBING

B.S., Kansas State University, 1965

A MASTER'S REPORT

submitted in partial fulfillment of the

requirements for the degree

MASTER OF ARCHITECTURE

College of Architecture and Design

KANSAS STATE UNIVERSITY
Manhattan, Kansas

1969

Approved by:

Eugene Thorson

Major Professor

TABLE OF CONTENTS

SYNOPSIS

The "finite element method" for solution of stress distribution in a plane elastic continuum is studied in this report. This approximate method of analysis can be used for obtaining a solution to previously intractable problems. An existing "finite element method" computer program is made operational as a requirement of the report. A simple problem will be solved using a classical "exact method" and will then be analyzed using the "finite element method" to get an idea of the correspondence of the results of the two methods. A problem for which a "classical method" does not exist will then be analyzed using the "finite element method" to illustrate the power of its application.

INTRODUCTION

Structural analysts are becoming increasingly aware of the power of numerical methods in providing reasonably accurate solutions to complex problems which heretofore relied upon approximate calculations of doubtful validity. The plane stress or strain finite element analysis is one of these numerical methods which, when coupled with a high-speed computer, can provide quick solutions that converge to "exact method" answers.

O. C. Zienkiewicz and Y. K. Cheung[1], in their book, have presented the theory behind the finite element method and a computer program which applies the plane stress or strain finite element analysis to a plane elastic continua.

The plane elastic continua is divided into a finite number of nodal points which are interconnected to form triangular elements. Force-displacement relationships are determined for these triangular elements. "Displacement method" equations[2] in matrix notation are formed with the displacements of the nodal points as unknowns. Inversion of the force-displacement matrix and multiplication by the force matrix leads to a solution for the unknown displacements. These displacements are used to calculate the stresses at the centroids of the triangular elements which are then converted to principal stresses and their angle of deviation from the original X-Y coordinate system.

The general "displacement method" equation is given as

$$\{F\}^e = [k]^e \{\delta\}^e + \{F\}_p^e \qquad (1)$$

in which $\{F\}^e$ represents the force matrix composed of forces at the nodal points, $[k]^e$ represents the force-displacement or stiffness matrix determined from the element properties, $\{\delta\}^e$ represents the nodal displacement for a particular element, and $\{F\}_p^e$ represents the nodal forces due to body forces.

The general equation used to solve for the stresses is given as

$$\{\sigma\}^e = [S]^e \{\delta\}^e \tag{2}$$

in which $\{\sigma\}^e$ represents the stress matrix composed of the stress in the X- and Y-directions and the shear stress, and $[S]^e$ represents the stress-displacement matrix determined from the material properties. A further explanation of equations (1) and (2) will be given in the section on derivations.

The finite element method computer program taken from reference 1 and included in this report is written in FORTRAN IV language and in its present form is intended for use on an IBM 360-series computer.

Even though it is limited to the solution of problems which lie in the X-Y plane, the finite element program (FINELEM) has a wide range of application. Each element can have one of up to 10 different sets of elastic properties for a given problem and any constant thickness.

The program is not limited to isotropic materials. Anisotropic materials, which are "stratified" and have rotational symmetry in the plane of the strata, can also be solved. When the direction of the strata in a transversely isotropic material is inclined to the X-axis, a transformation matrix included in the program relates the stresses back to the major X-Y coordinates.

The accuracy of the stress solutions obtained using FINELEM is dependent upon the fineness of the triangular grid. An area of a certain problem with an expected high stress or variable stress should be divided into a finer grid than an area with an expected constant stress. An infinite number of combinations of loading conditions and support conditions can be approximated using this method.

In this report, a simple problem with a solution based on a well-known "classical" method is compared with the solution using FINELEM. The correctness of the program and the accuracy of the method is studied using this simple problem. Once the program is judged to be performing correctly, a complex problem is solved to illustrate the method's usefulness and versatility.