

AQUIFER PARAMETER ESTIMATION BY
QUASILINEARIZATION AND INVARIANT IMBEDDING

by

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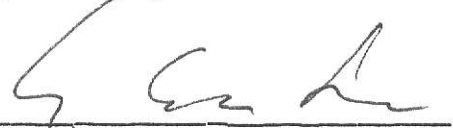
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CHAPTER 1

FUNDAMENTAL CONCEPTS

1.1 INTRODUCTION

A large class of simulation and mathematical models are often used in analyzing problems in engineering, science and industries. Most models are of parametric type in which parameters used in deriving the governing equation are not directly measurable and have to be determined from historical records. Frequently, they involve differential equations of two-point or multipoint boundary value type. In these problems, the boundary conditions are specified at two points or multipoints. To complicate the matter, the governing differential equations for a majority of such problems are nonlinear and cannot be solved analytically. The solutions must be obtained by numerical methods. Numerically, the difficulties are caused by the fact that not all the conditions are given at one point.

Methods for the numerical solution of such problems can be separated into two groups, the iterative and the non-iterative methods. Among such methods, quasilinearization and invariant imbedding, classified into the iterative and the non-iterative method, respectively, are presented. Quasilinearization is an iterative approach allied with linear approximation while invariant imbedding represents a completely different formulation of the original problem.

The purpose of this study is to use these two methods for