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A VOLTERRA SERIES APPROACH TO CALCULATING  
THE PROBABILITY OF ERROR IN A NONLINEAR  
DIGITAL COMMUNICATION CHANNEL

by

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## CHAPTER I

### INTRODUCTION

Many, perhaps most, of the communication systems in use today contain nonlinear elements. Unfortunately most of the techniques for solving linear systems are not applicable to nonlinear systems. If a linear approximation of a nonlinear system does not yield satisfactory results, a designer is faced with three possible alternatives. The engineer can actually construct the system and then measure its performance. Aside from the financial cost, the designer can easily find himself spending more time and energy on the "nuts and bolts" aspects without ever addressing the central design problem. An attractive alternative to prototyping an unproven design is simulation. Simulation is relatively easy and straightforward to implement. Due to the physical limitations of simulation hardware, high speed communication channels are often modeled at a fraction of the desired rate. A disadvantage of simulation then becomes the very long time intervals that may be required to obtain a confident estimate of the performance of a system. The analytic approach offers the advantage that the designer can describe the system more precisely. Unfortunately, many systems are either impossible to describe analytically, or the resulting description may be so complex as to be unmanageable.

One approach which can be used for a large class of nonlinear systems involves the use of Volterra series [1,2,3]. An overdriven

transistor amplifier is one example of a physical system having sufficiently well behaved nonlinearities with memory which can be analytically described in terms of a Volterra series. This paper will consider the model of the nonlinear digital communication channel shown in Figure 1.

The basic mathematical model of the Volterra series approach to calculating probability of error is developed in Chapter II. The computer algorithms used to implement the mathematical model are described in Chapter III. Results for two channels, one having Gaussian filters and the other having casual filters, are presented in Chapter IV. The results from the channel having Gaussian filters are essentially that of Benedetto, Biglieri, and Daffara [4].

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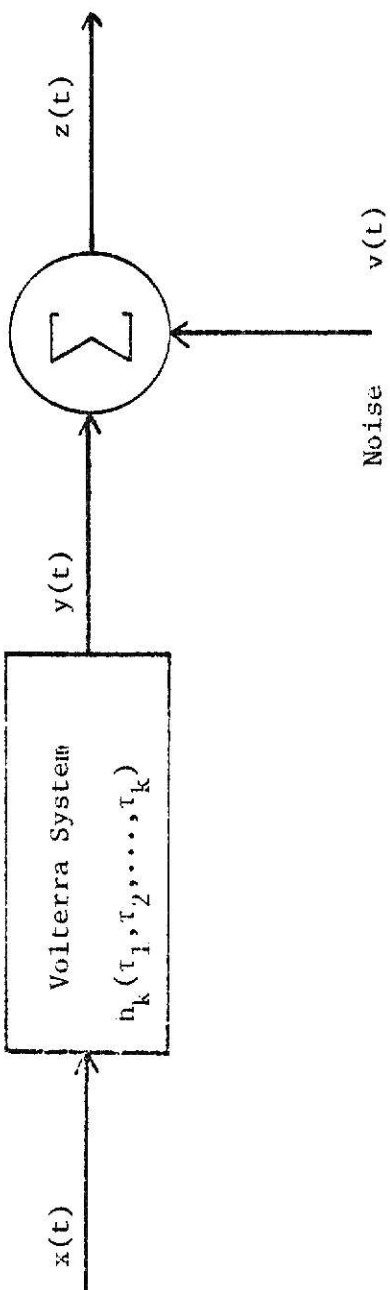


Figure 1. Communication Channel Model