A STUDY OF THE CHIRP Z-TRANSFORM AND ITS APPLICATIONS

by

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CHAPTER I

INTRODUCTION

The z-transform is of significant importance in processing discrete data. With the development of the Fast Fourier transform (FFT), it became economically practical to numerically evaluate the z-transform of a finite number of time samples [4]. The discrete Fourier transform (DFT) is a special case of the z-transform since it evaluates the z-transform along the $j\omega$-axis of the s-plane. The FFT has been the backbone of much of the digital signal processing work to date. It is useful in its own right for the spectral information it provides. Perhaps an even more important feature of the FFT is that it can be used as a means to computing numerical convolutions and correlations.

However, the FFT allows us to compute the z-transform along only a very restricted contour. Not only must the contour be the unit circle of the z-plane, output points must be taken uniformly over the entire unit circle. The chirp z-transform (CZT) is an algorithm for computing the z-transform which was developed to overcome these restrictions of the FFT [1]. In Chapter I, the CZT is defined in relation to the general (infinite series) z-transform. Its properties are described and compared with those of the DFT. Then, the computational algorithm for evaluating the CZT is developed using the FFT as its basic component.

In order that the applications might be investigated, a program implementing the CZT was written. Chapter III demonstrates two applications of the CZT: enhancement of poles in spectral analysis, and high resolution narrow-band frequency analysis. Finally, in Chapter IV the report is summarized and a recommendation is made for future work in one possible area of application. An appendix shows a listing of a program in which the CZT appears as a sub-program.

CHAPTER II

THE CHIRP-Z TRANSFORM

2.1 The Z-Transform

The z-transform is a fundamental tool for the analysis of discrete-data systems. It is closely related to the Laplace transform, whose value in studying continuous data systems is well known. In order to see the need for defining the z-transform, consider the discrete function x*(t) which was obtained by ideally sampling the continuous waveform x(t) every T seconds (see Fig. 2.1). The sequence x*(t) can be written as

$$x^*(t) = x(t)\delta_T(t) = \sum_{n=0}^{\infty} x(nT)\delta(t - nT), \tag{2 - 1}$$

where $\delta_T(t)$ is a periodic train of unit strength impulses spaced T seconds apart, ie.,

$$\delta_T(t) = \sum_{n=0}^{\infty} \delta(t-nT). \tag{2 - 2}$$

Denoting the Laplace transform of x*(t) by $X^*(s)$, we obtain

$$X^*(s) = \int_0^{\infty} [\sum_{n=0}^{\infty} x(nT)\delta(t-nT)]e^{-st}dt \tag{2 - 3}$$

Interchanging the order of integration and summation in Eq. (2-3) and subsequently integrating, there results

$$X^*(s) = \sum_{n=0}^{\infty} x(nT)e^{-nsT}. \tag{2 - 4}$$

Let us now introduce a new complex symbol

$$z = e^{sT}. \tag{2 - 5}$$