

BULK QUEUEING MODELS AND THEIR APPLICATIONS

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CHAPTER I

INTRODUCTION

Management problems commonly arise from operational systems where men, machines or materials form a queue for some type of servicing. One of the principal characteristics of such systems is the uncertainty of randomness which is generally associated with arrival flow, service times, or both. In order to design such a dynamic system, usually referred to as queueing system, for optimal performance, it is essential to consider the effect of any random elements that may be present.

Queueing theory is concerned with the development of mathematical models to predict the behavior of systems that provides service for randomly arising demands. It has been applied to various problems. Queueing theory has formed a prominent place among modern analytical techniques of operations research, since it has considerable potential as a means for decision making in industry.

1.1. Bulk Queueing Theory

In a queueing process, customers arrive at the service station. They form a queue if the service station is busy. After being served, the customers depart from the service station. Customers may arrive singly or in batch (bulk). They may also be served in bulk. In case of the bulk arrival and/or bulk service, such systems are referred to as bulk queueing processes. For example, several people may go to a restaurant as a group, and obtain service together. Another example is that an elevator serves a group of people at the same time.

It is obvious that we are considering a more general type of queueing system which can yield the single arrival and single server as special cases. Bulk queueing theory has been often applied to a variety of industrial problems. To specify a queueing process, the distribution function and arrival pattern must be known. Also the service pattern, discipline and service time distribution must be described.

This chapter includes the reviewed literature of bulk queueing theory. Chapter II discusses the basic concepts of queueing theory in general. It covers the basic structure of queueing processes, types of queueing processes and methods of solutions. Chapter III is devoted to the analysis of various bulk queueing models. Some of these models are derived mathematically for the transient and steady-state conditions. Chapter IV discusses the applications of bulk queueing.

1.2. Literature Review

Considerable research has been done on bulk queueing processes. Most of this work is theoretical. Bulk queueing literature are reviewed below.

Bailey [4] has investigated a simple queueing process in which customers arrive at random, form a single queue in order of arrival, and are served in batches. The size of each batch has a fixed maximum. In most of the results obtained the time intervals between successive service are assumed to be independent χ^2 distribution with an even number of degrees of freedom. The equilibrium distribution of queue length has been studied by the imbedded markov chain method. Expressions for the mean and variance of queue length, and the mean waiting time are given. A useful inequality for the latter is also available in the special case of