The elongation of beams in reinforced concrete special moment resisting frames

by

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Abstract

Special moment resisting frames (SMRF) are intended to protect the structure from earthquake motions through a ductile inelastic response. This thesis evaluates the performance of reinforced concrete SMRFs with an emphasis on the second level beams. Due to previous research, it is concluded that two-dimensional finite element analyses (2D-FEA) and three-dimensional finite element analyses (3D-FEA) have different results when evaluating the same structure. Due to this, the thesis used a 3D-FEA to analyze frames based on Design Example 7 in the *2006 IBC Structural/Seismic Design Manual* (Appendix A). While looking at the frame as a whole, the first of two parametric studies was performed over the columns. Using LS-DYNA the columns' forces, displacement, moment, and curvature were evaluated. From these results, it was concluded that in SMRF, columns are not acting per current design assumptions due to the elongation of beams. Using the knowledge gained in the first parametric study, a second parametric study was performed on the second level floor beams. Focusing on the beam elongation, this thesis evaluates multiple frames with different load combinations using LS-DYNA to find the displacement of the reinforcement in the beams. With the results, an equation to calculate the elongation of beams was proposed, as well as an average percentage of the elongation in reinforced concrete SMRF. The equation and average percentage of elongation aim to provide a standard design consideration for the elongation of beams.

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Chapter 1 - Introduction

With the recent rise of natural disasters, incorporating systems to withstand the load conditions being applied is very important. Special Moment Resisting Frames (SMRF) are often used to protect the structures from such motions caused by the natural disasters. Typically, SMRF are designed and analyzed using two-dimensional finite element analyses (2D-FEA) due the accessibility and ease of use. However, recent research proves that three-dimensional finite element analysis (3D-FEA) outperforms 2D-FEA for analyzing structures experiencing extreme loads. This thesis evaluates the performance of a reinforced concrete SMRF described in Design Example 7 of the *2006 IBC Structural/Seismic Design Manual* (Appendix A) using a 3D-FEA software. This thesis focuses on the elongation occurring in the second level floor beams, in order to provide an equation to calculate the elongation of beams as well as an average percentage of the elongation in reinforced concrete SMRF. The equation and average percentage of elongation aim to provide a standard design consideration into reinforced concrete SMRFs. To achieve this standard design consideration, two parametric studies were performed for this thesis.

The first parametric study was the seven-story, five-bay, SMRF described directly out of Design Example 7. Using LS DYNA, a moment vs. curvature graph was created to determine how each column of the frame acted under the same axial force. Next, the total displacement of each base column of a seven-story building was found. This was done by two different methods and then compared: the first method was using LS DYNA and the second method was hand calculations. Using LS-DYNA, the curvature of the columns were found, along with the shear deformations, and the moment diagrams of each column. From these results the total displacement was calculated using basic mechanics. Due to the results of the parametric study of the columns, the beams of the frame became the focus of the thesis.

The second parametric study, the main focus of this thesis, was the study of beam elongation in SMRFs. Using knowledge gained from the parametric study of the columns and adjustments made to the frame provided by Design Example 7, the beam elongation was analyzed for multiple frames undergoing different load combinations. Frames from five bays to one bay were analyzed for both seven stories and six stories resulting in ten different SMRFs. Each frame was analyzed under two different load combinations as well: equation 5 – dead load plus live load, and equation 7 – just dead load. Using LS-DYNA, displacements were found for the second level floor beams of all twenty (20) models to create beam deformation charts. Using the data collected, the column drifts and rotations were calculated, the beam deformation was calculated, and the neutral axis at the joints was calculated. From these results, along with simple trigonometry, an equation is proposed for determining the elongation of beams; and using algebra an average percentage of the elongation of the SMRFs was calculated.

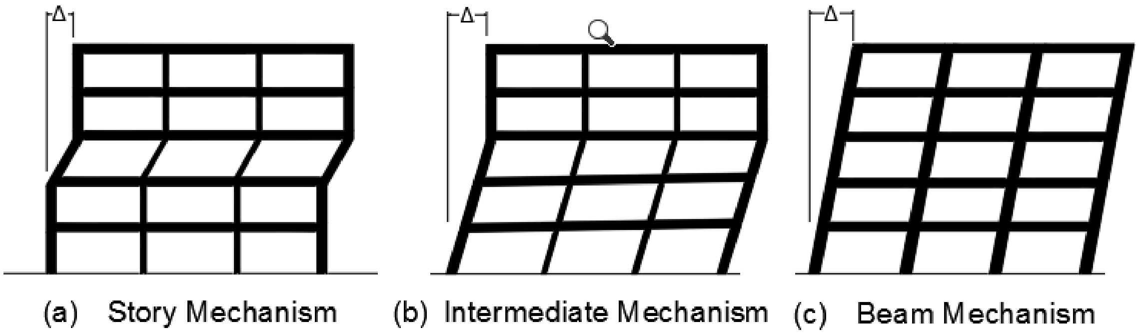
Chapter 2 - Literature Review

Concrete is used for many projects because of its ability to last for long spans of space and its flexibility of shape. While concrete is a widely used material it is also a very complex material due to its composite nature. When making concrete, water, cement, aggregate and sand are mixed together to create a pourable material that hardens and reaches its design strength in 28 days. From the time a project is poured until when the project is demolished, the concrete is exposed to elements, such as wind, seismic activity, snow, rain, ice, etc., that slowly deteriorates the structure of the concrete until a failure is in process. These loads affect the design of a structure and how the structure fails. Due to the multiple materials, concrete is never created the same every time. Because of this composite nature, concrete performs well under compression and does poorly under tension and due to load it will crack because of the low tensile strength causing the column's stiffness level to reduce. Excessive cracking could eventually lead to column failure. However, the stiffness varies from column to column and can cause an internal force that could crack members not yet cracked (Kara & Dundar, 2009). Due to concrete's composite nature, concrete structures handle all loads very differently than other materials.

2.1. Special Moment Resisting Frames

Every building experiences multiple loads, but the most extreme loading type is seismic loading. When comparing the damage from an earthquake compared to a tornado, it can be observed that the earthquake causes greater damage on structures and this is because seismic loading not only produces a force in the horizontal direction but also in the vertical direction that cycle through rapidly. However, the horizontal motions dominate over the vertical motions even during high seismic activity (Watanabe, Ohtsu & Sakamoto, 1997). Additionally, the direction of the seismic loads has a significant effect on the failure mode of the columns (Pham & Li, 2013). Due to the effect of earthquakes on concrete structures, ACI 318 adopted Special Moment-Resisting Frames (SMRF). In the 2005 ACI 318 update, all reinforced concrete moment-frame structures built in high seismic areas were to be SMRF (“IBC”, 2006). Earthquake-resistant buildings’ main purpose is to protect the structure from earthquake motions through ductile inelastic response of the seismic force resisting system. To prevent collapse, ductile concrete frames were introduced into building codes. These are designated as Special Moment-Resisting Frames (SMRF) and have special detailing requirements including location and amount of steel reinforcement. One of the main ways SMRFs protect against the sway caused by earthquakes, is by the strong column/weak beam frame. This concept is meant to produce a sway as shown in Figure 1 (b) and (c) and prevent the sway shown in Figure (a).

“This strong-column/weak-beam principle is fundamental to achieving safe behavior of frames during strong earthquake ground shaking” (Bracci, Moore, & Bugeja, 1999). SMRFs are also designed to predict the formation of plastic hinges so that the desired mechanisms can occur.



(a) Story Mechanism (b) Intermediate Mechanism (c) Beam Mechanism

Figure 1: Diagram of Mechanisms

2.2. Plastic Hinge

As loading progress and displacement increases, plastic hinges are formed at the base of columns and the ends of beams (Phillippi & Hegemier, 2014), this can be seen in Figure 2. A plastic hinge is the location of plastic bending occurring on a structure; typically, the most damage occurs at these locations. Once a plastic hinge has formed, it allows for large amount of rotation to occur under a plastic moment caused by loads acting on the structure (Kozik, 2015). Two different types of plastics hinges can form on a frame during an extreme loading: reversing and unidirectional plastic hinges, these are explained in more detail in the sections to follow.

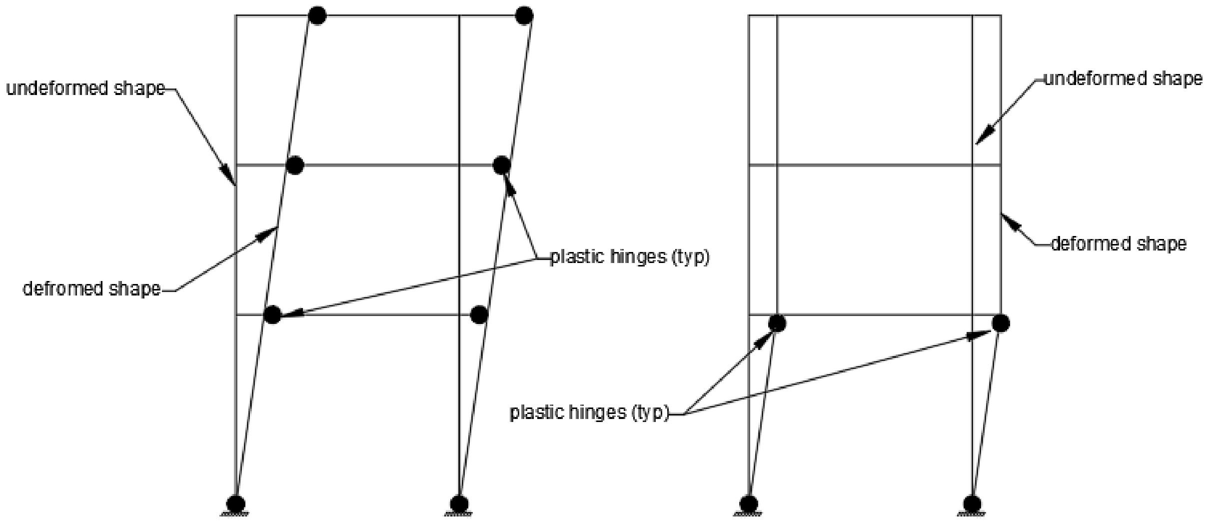
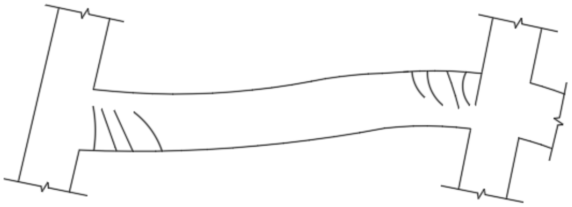


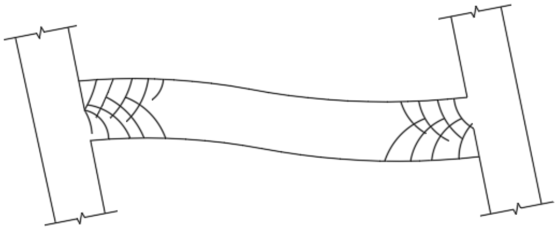
Figure 2: Diagram of plastic hinge locations

2.2.1. Reversing Plastic Hinge

When experiencing a seismic load, two plastic hinges develop on a beam, one with a negative bending moment and the other with a positive bending moment (Davidson & Fenwick, 1995). When the load direction reverses, so does the positive and negative bending moments on the plastic hinges, as seen in Figure 3. For reversing plastic hinge, the plastic hinges form at the face of the column due to the maximum moment being greater than the elastic capacity.



(a) Sway to right

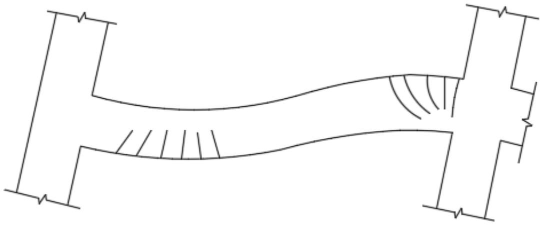


(b) Sway to left

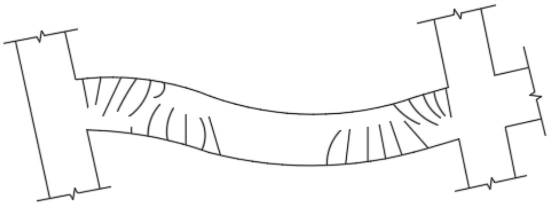
Figure 3: Diagram of reversing plastic hinge movement

2.2.2. Unidirectional Plastic Hinge

When the gravity shear is greater than the maximum lateral shear, unidirectional plastic hinges occur. Due to the gravity shears being greater, at the mid span of the beam there is a shear point force of zero. Davidson & Fenwick (1995) explain, "when the structure sways to the right, a negative moment hinge forms against the right-hand column face and the positive moment hinge develops in the span on the left-hand side of the beam center line." When the load direction reverse, so do the positive and negative sides, however there is still a point of zero moment at mid span. Figure 4 shows the deformation of the beam caused by unidirectional plastic hinges.



(a) Sway to right



(b) Sway to left

Figure 4: Diagram of unidirectional plastic hinge movement

2.3. Elongation in Plastic Hinges

Plastic hinges have been researched extensively, however, the elongation of the plastic hinges have been typically overlooked. While there is not much research over the topic, Richard C. Fenwick and others at the University of Auckland, in Auckland, New Zealand, have researched and published multiple articles pertaining to the elongation occurring in reinforced concrete SMRFs. Fenwick and his colleagues published these articles approximately between the years of 1993 and 2003. A large portion of Fenwick's work has been aimed at the research towards the elongation of plastic hinges that occur in beams. As described in the sections above, there are two types of plastic hinges that form during a serve earthquake: unidirectional and reversing. In 1993, Fenwick and B.J. Davidson's published *Elongation in Ductile Seismic-Resistant Reinforced Concrete Frames*. For this article, they performed the same tests on two beams, these beams were tested as simple cantilevers springing from an anchorage block. One beam had the loading to create an unidirectional plastic hinge, and the other the loading in which a reversing plastic hinge forms. The two beams underwent the same type of loading and the results Fenwick and Davidson came up with are to follow.

For the unidirectional plastic hinge forming beam, it failed at the first load cycle at a displacement ductility of 14. The strains in the top of the steel yielded while the strains in the bottom did not. Fenwick and Davidson were successfully able to use equation (2-1) to predict the elongation of an unidirectional plastic hinge. The equation measures the elongation at mid-depth assuming the strain is zero, in terms of rotation.

$$elongation = \theta(d - d')/2 \tag{2-1}$$

$d - d'$ is the distance of reinforcement between top and bottom centroids

For the beam with a reserving plastic hinge, the failure occurred at the first load cycle of the displacement ductility of 10 and the failure was caused by buckling in compression. With each loading cycle, the reinforcement in the compression zone elongated. This is because "the reversal of loading direction causes the reinforcement in the compression zone, which had been yielding in tension, to not fully yield back in compression" (Fenwick & Megget, 1993). With the compression zone elongation represented as *e* and the rotation of the plastic hinge as θ, Fenwick and Davidson used equation (2-2) to predict the elongation of the reversing plastic hinge.

$$elongation = e + \theta(d + d')/2 \tag{2-2}$$

Compared to the actual experiment, the predicted elongation did not match up. This was due to the increased length in the compression zone, *e*. As mentioned before, the elongation is due to effects cause by the reversing of the load, the change of loading caused the compression force to be less than the tension force which caused an inelastic rotation. The compression zone also increased in length because of the concrete cracking; when the concrete cracked, a wedging action of aggregate particles occurred, this also explained why the reinforcement could not fully yield back into compression.

In the same article, Fenwick and Davidson tested a multi-story frame; this frame was built to the New Zealand building code at the time to withstand motions for a severe earthquake. They tested two different frames, one where the beams are represented by sub-assembly elements that included elongation effects (elongating model), and one were the beams' plastic hinge zone was represented by non-elongating sub-assembly (non-elongating model). In the non-elongation model, the beams acted as stiff ties ensuring the deflection profiles of all columns stayed the same. For the elongation model, that was not the case, the beam elongation caused the columns to be pushed outwards. Ultimately, this caused the maximum plastic hinge rotation to occur at the base of the columns. At the top level of the frame, portal action associated with gravity loads created a larger axial compression in the beam which increased the flexural stiffness and decreased the elongation. The tests performed on the frame also revealed that strain hardening caused an increase in the maximum moment, which caused an increase in rotation of the plastic hinge, and this ultimately caused an increase in elongation.

Through the findings of Fenwick and Davidson in *Elongation in Ductile Seismic-Resistant Reinforced Concrete Frames,* it was concluded that "plastic hinges elongate by two to four percent of the member depth before strength degradation occurs" (1995). From this experiment, it can also be concluded that the lengthening of the reinforcement on compression zones happening from cycle to cycle, is a major contribution to the elongation in the reversing plastic hinges.

Based on the findings stated above J.G. Matthews, J.B. Mandar, and D.K. Bull, out of the University of Canterbury in New Zealand, proposed an analysis method and equation of their own to predict the total elongation of concrete elements. In *Prediction of Beam Elongation in Structural Concrete Members Using a Rainflow Method*, Matthews, Mander, and Bull state that beam elongation occurs when a "new" rotation occurs (2004). The "new" rotation is referred to as the additional rotation occurred that was not achieved during the previous load cycle. Beam elongation only occurs when the deformation and rotation exceeds the previous deformation and rotation. Due to this, the "Rainflow Counting" method was used to create an equation to predict elongation. With this method, the "new" rotation was counted and Equations (2-3) and (2-4) was created to predict the total elongation in terms of rotation.

For a plastic hinge at a given rotation:

$$\delta_i^{el} = \theta e_{cr} \tag{2-3}$$

θ is the rotation the beam undergoes

e_{cri} is the eccentricity between the centroid and the neutral axis

For the maximum beam elongation for a given frame:

$$\delta_{max}^{el} = \left(\left|\theta_p^+\right| + \left|\theta_p^-\right| + \theta_y\right) \sum_{i=1}^{n} e_{cri} \tag{2-4}$$

θ_p^+ is the maximum positive plastic rotation

θ_p^- is the maximum negative plastic rotation

θ_y is the yield drift

e_{cri} is the distance between the beam centroid and instantaneous center of rotation (neutral axis)

With equation (2-3), Matthews, Mander, and Bull presented new research to compare their predictions to previous researched beams and frames as well as their own beams for research. Testing both individual plastic hinges and a frame, the equation used for prediction verified the results of their research; equation (2-3) also verified the results from previous research done by Fenwick et al (1891), Restrepo et al (1993), and Lau (2001).

Through the resultant equation and findings of Matthew, Mander, and Bull's work, it can be concluded that rotation and the location of the beam's neutral axis in comparison to the column are important factors in determining the elongation of reinforced concrete frames.

2.4. Frame Performance

In 2017, Donald Phillippi and Gabrielle Liuzza authored *Three-Dimensional Non-Linear Analyses of Special Reinforced Concrete Moment Frames*, the focus of this article was to determine the column-shear force distribution at the bottom story of the frame. A seven story, five bay special moment resisting frame was used to conduct the research. An elastic two-dimensional finite element analysis (2D-FEA) was done on a readily available software, that is often used for design and analysis. This model was compared to a model created for a three-dimensional finite element analysis (3D-FEA). With the 3D non-linear dynamic general-purpose program, the 3D-FEA model controls included a "hour-glassing, length of simulated time, method of vertical loading, rate of horizontal displacement, size of time steps, integration method, and type of integration set-up for the elements" (Phillippi & Liuzza, 2017). The results of comparison between 2D-FEA and 3D-FEA are shown in Table 1.

The results for this experiment show that what is expected to happen, does not. The shear-force distribution force at the first level is significantly different that the prediction provided by the 2D-FEA. For the last column of the frame (column 6) 3D-FEA predicted approximately double the prediction of the 2D-FEA. This is caused by the increased axial load acting on the compression columns as well as beam elongation creating drifts in the frame. However, the results between the 3D-FEA and 2D-FEA of the shear-force for the columns at level two are comparable. The two models have a comparable drift but looking at the 3D-FEA, the last column's drift is about one and a half times larger than the first column's drift. This can be explained by the frame becoming stiffer due to an increased axial over turning loads and the beams elongating.

Table 1: Results from 2D-FEA versus 3D-FEA (Phillippi & Liuzza, 2017)

Method	Parameter	First Floor Columns					
		1	2	3	4	5	6
2D-FEA	Axial (kN)	381	-1300	-1287	-1285	-1255	-1665
	Shear (kN)	294	419	427	427	437	323
	Top M (kN-m)	-137	-472	-461	-461	-474	-230
	Base M (kN-m)	1119	1391	1361	1360	1390	1148
	Joint Δ (mm)	8.59	8.60	8.60	8.59	8.57	8.53
	Joint θ (rad.)	0.00278	0.00238	0.00239	0.00238	0.00237	0.00263
3D-FEA	Axial (kN)	265	-1237	-1282	-1275	-1224	-1812
	Shear (kN)	65	281	357	447	550	647
	Top M (kN-m)	186	-19	-131	-240	-342	-356
	Base M (kN-m)	379	956	1061	1257	1476	1801
	Joint Δ (mm)	5.84	7.25	8.57	10.01	11.71	14.50
	Joint θ (rad.)	0.00178	0.00180	0.00193	0.00217	0.00243	0.00358

Method	Parameter	Second Floor Beams									
		1L	1R	2L	2R	3L	3R	4L	4R	5L	5R
2D-FEA	Axial (kN)	0	0	-8	-8	-22	-22	-35	-35	-40	-40
	Shear (kN)	96	287	80	278	86	278	86	277	93	284
	M (kN-m)	589	-787	534	-768	534	-767	532	-765	549	-807
	Elongation (mm)	0.01		0.00		-0.01		-0.02		-0.04	
3D-FEA	Axial (kN)	-81	-94	-197	-235	-334	-331	-371	-380	-337	-355
	Shear (kN)	59	217	115	262	127	267	163	331	196	349
	M (kN-m)	323	-615	461	-735	534	-820	594	-864	649	-1102
	Elongation (mm)	1.41		1.32		1.44		1.70		2.79	

Method	Parameter	Second Floor Columns					
		1	2	3	4	5	6
2D-FEA	Axial (kN)	286	-1099	-1095	-1093	-1071	-1380
	Shear (kN)	219	445	441	439	442	283
	Base M (kN-m)	463	879	870	867	870	587
3D-FEA	Axial (kN)	222	-1139	-1078	-1078	-1082	-1464
	Shear (kN)	131	427	430	451	488	288
	Base M (kN-m)	426	898	912	899	878	390

Chapter 3 - Research Background

3.1. 2006 IBC Example

This thesis evaluates the performance of a reinforced concrete SMRF described in Design Example 7 of the *2006 IBC Structural/Seismic Design Manual* (Appendix A). This frame consists of four seven-story, five-bay SMRF on each perimeter wall as can be seen in Figure 5.

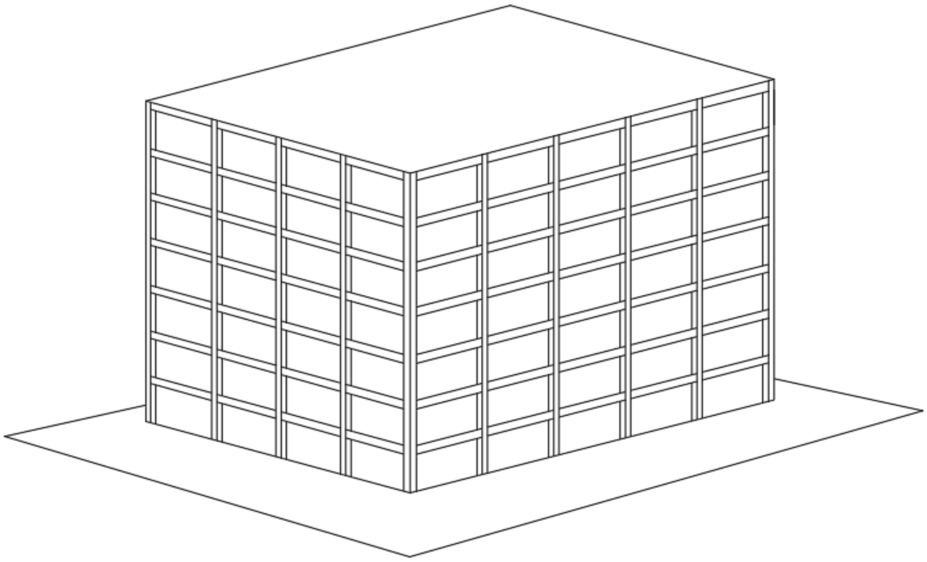


Figure 5: Sketch of the seven story concrete SMRF building

When sizing the members, a two-dimensional analysis was used for the analysis. Using the portal method, the exterior columns' nodal forces were half the amount as the interior column's nodal forces. Using the 2D frame program forces were found and members were designed and detailed. Appendix A shows the Design Example produced by IBC.

For the beam, design members are chosen based on a cracked structure for the worst case scenario. The design "considers the frame beam to a combined section including consideration of the adjacent slab for both compression and tension stresses" (ACI 318 Section 318 Section 1836.2.1, 2014). An important design for the beams is that the shear design allows for the formation of plastic hinges at a certain distance from the column face. SMRFs require a strong column – weak beam design to ensure the plastics hinges occur in the beams of the frame. This is achieved by designing the columns to have nominal bending strengths 120 percent stronger than beams (ACI 318 Section 18.7.3.2, 2014). Typically for column design, the governing shear strength process prohibits the column shear from exceeding the moment strength of the beams framing into the joints, this forced plastic hinges to form on the beams rather than the columns. From the Design Example 7, Figure 6 shows the detailing of the beams and columns in the SMRF.

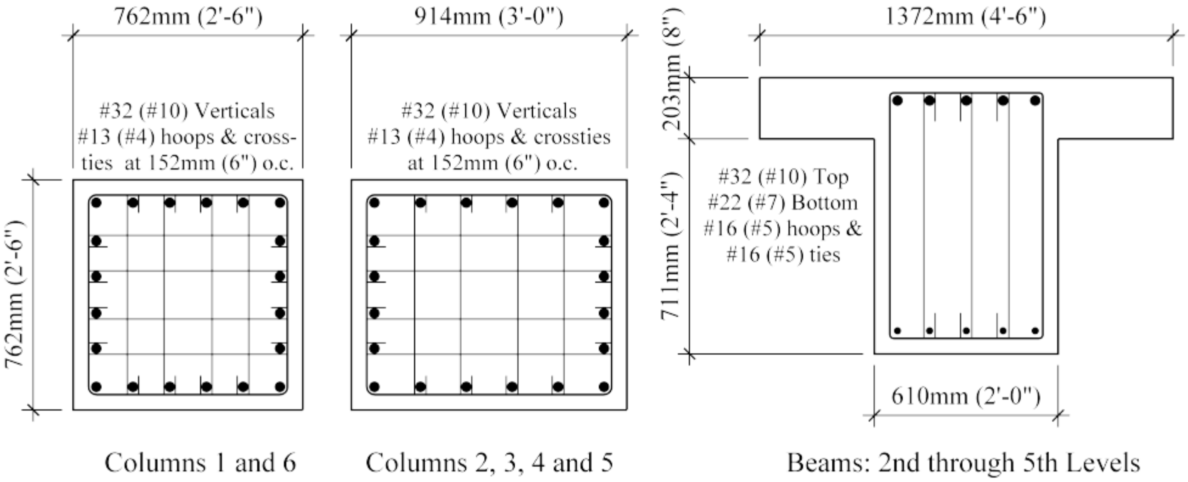


Figure 6: Diagrams of the reinforcement of the building components

3.2. Finite Element Analyses

Currently, the structural analysis and design of a concrete building is determined using computer programs such as RISA 3D and SAP2000. While these programs work, they are not 100% accurate due to a lack of research done on 3D models (Xiao, Kunnath, Li, Zhao & Lew, 2015). Another computer program, LS-DYNA, is starting to become more popular for performing research due to its dynamic nature. To follow is a comparison of each program based on their benefits and drawbacks.

3.2.1. RISA

RISA is one of the most used computer program for structural analysis and design in industry. With multiple products and the ability to use all types of building material, this program is used for everyday design. RISA has the ability to create a three-dimensional model as shown in Figure 7, and from this model, an analysis can be made for the structure.

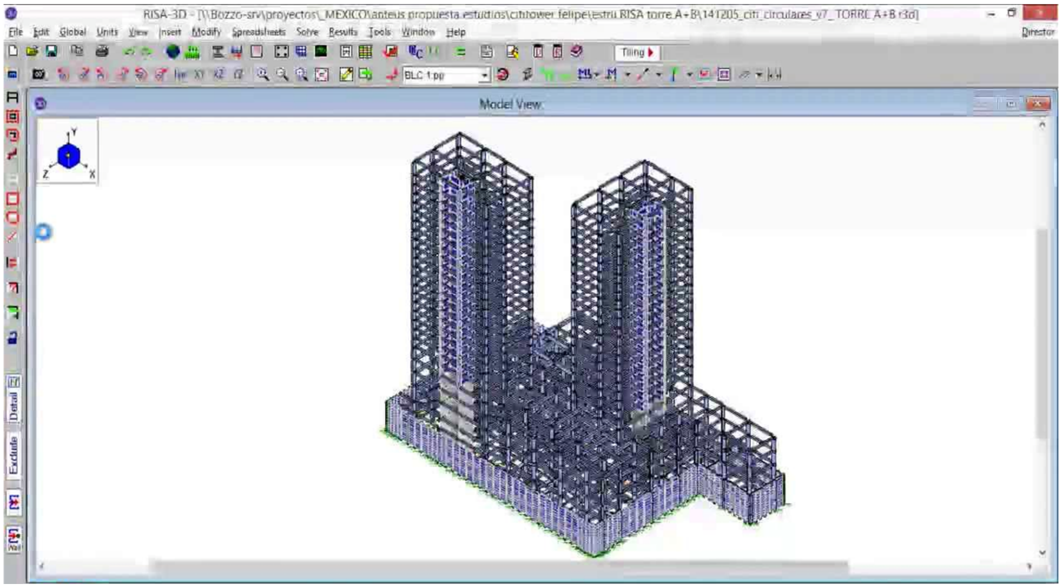


Figure 7: Diagram of a RISA 3D model

Benefits

RISA was the first program to introduce a graphical interface that was adept for creating larger and smaller structures. For this reason, many companies in industry started utilizing RISA and as its popularity grew, so did its power (McNaghten, 2016). Along with the ability to customize the shape properties of the components of the structures, RISA stands out among other programs when researching concrete (Goldberg, 2007). Another benefit of RISA is that it is very user friendly through program tutorials and layout; this helps with speeding along the process of analysis and design.

Drawbacks

For the research of concrete, RISA lacks the three-dimensional aspects of the behavior of a structure. RISA claims to be three dimensional, however this simply means that the structure can be modeled in three-dimensions and that the elements have three dimensional properties. However, the program does not take the three-dimensional movement of the model into effect, nor can the user control much of the model.

3.2.2. SAP2000

As a finite-element analysis (FEA) program, SAP2000 is often used in industry as well as in research. With the ability of multiple views, as seen in Figure 8, SAP2000 is useful for designing a structure.

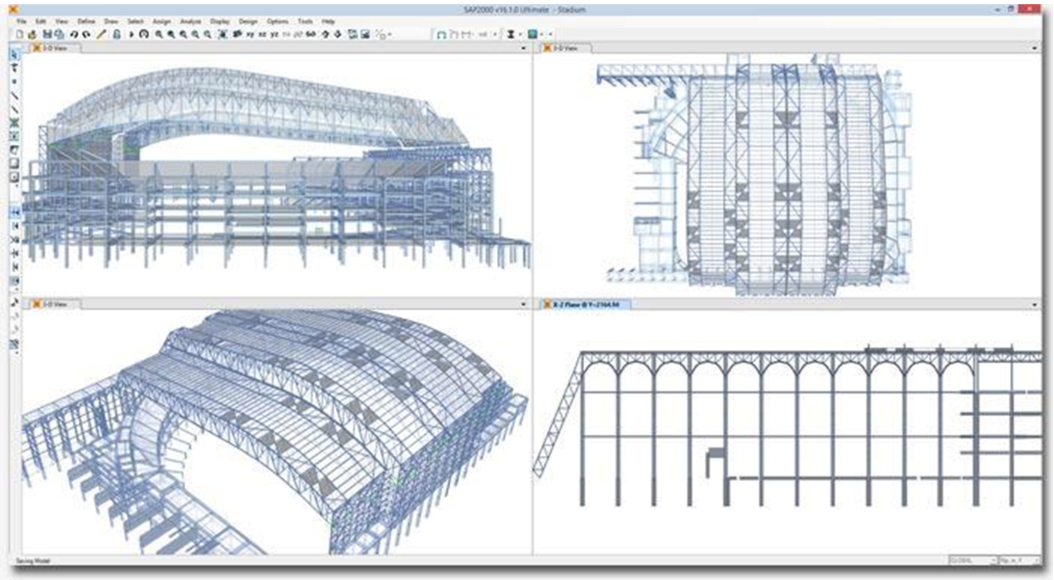


Figure 8: Diagram of a SAP2000 model

Benefits

This software is completely integrated with the ability to analyze simple and complex structures; it also allows the user to, "account for the non-linear mechanical behavior of the material" (Pasticier, Amadio & Fragiacomo, 2008). SAP2000 has many uses; one of the main uses is the analysis of a structure, this is because along with being able to view the structure in 3D, SAP2000 can analyze the structure as a hole or an individual element of the structure ("CSI Introduces SAP2000", 2009).

Drawbacks

While being able to view and analyze the structure in 3D is very beneficial in research, SAP2000 does not properly function once an element of the concrete structure becomes plastic. The program does not allow automatic control of the deflection of the structure when more than one plastic hinge reached plastic state (Pasticier, Amadio & Fragiacomo, 2008). This can cause a halt in researching because of the limitations of the software.

3.2.3. LS-DYNA

As a newer program, LS-DYNA is a non-linear dynamic program with the ability to analyze large deformation behavior in structures over time (Kenshiro, 2012). The most common use for this program is car crash data, but as the program becomes more popular, structural dynamics was included into the software. This can be seen in Figure 9.

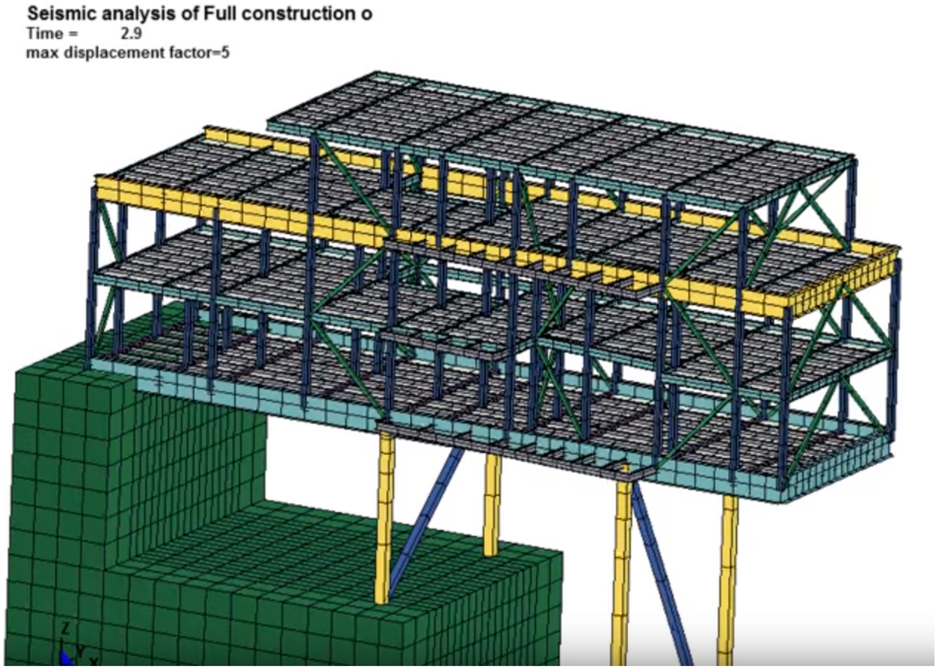


Figure 9: Diagram of LS-DYNA model

Benefits

LS-DYNA solves for the equation of motion for the problem; according to Xu and Zhang (2015), “in LS-DYNA, the study area is spatially discretized by 2D or 3D finite elements. At each time step, the equations of motion for the considered dynamic system are solved at the integration points of each element. The strain increments are determined based on the calculated nodal displacements”. Considering three-dimensional factors applied to a structure, this program can calculate displacement, velocity, and acceleration in terms of time. These values are very important when determining the loads applied to a structure and how these loads affect the structure. With the ability to use data given by LS-DYNA on other analyzing platforms, many components of concrete can be analyzed.

Drawbacks

While not widely used in industry, LS-DYNA is starting to see an increase in use for researching structures. However, because it is new to the analysis of structures, there is not a lot to compare research to.

3.2.4. Summary of Computer Programs

Both RISA and SAP2000 claim to be able to calculate all that is needed for analysis and design of a structure, however, these programs do not fully analyze every component of concrete due to the lack of control in design. Additionally, Shirmohammadi (2015) explains, “Exploration of existing model performances for predicting the behavior of several tested specimens shows a need for improvement of existing algorithms”.

Based on the computer programs described, two models were created for research purposes. The models Phillippi & Liuzza created in their articles *Column Seismic Shear Load Distribution in a Seven-Story Multi-Bay Concrete Frame* and *Three-Dimensional Non-Linear Analyses of Special Reinforced Concrete Moment Frames*, are described below.

3.2.5. Two-Dimensional Finite Element Analysis

Using the SMRF as described in Design Guide 7, a 2D-FEA, SAP2000 model was created. With the help of Design Example 7, the frame in Figure 10 was constructed of already made elements in the program using code suggested settings. The materials properties included in the frame are: modulus of elasticity, shear modulus, Poisson’s ratio, and weight density.

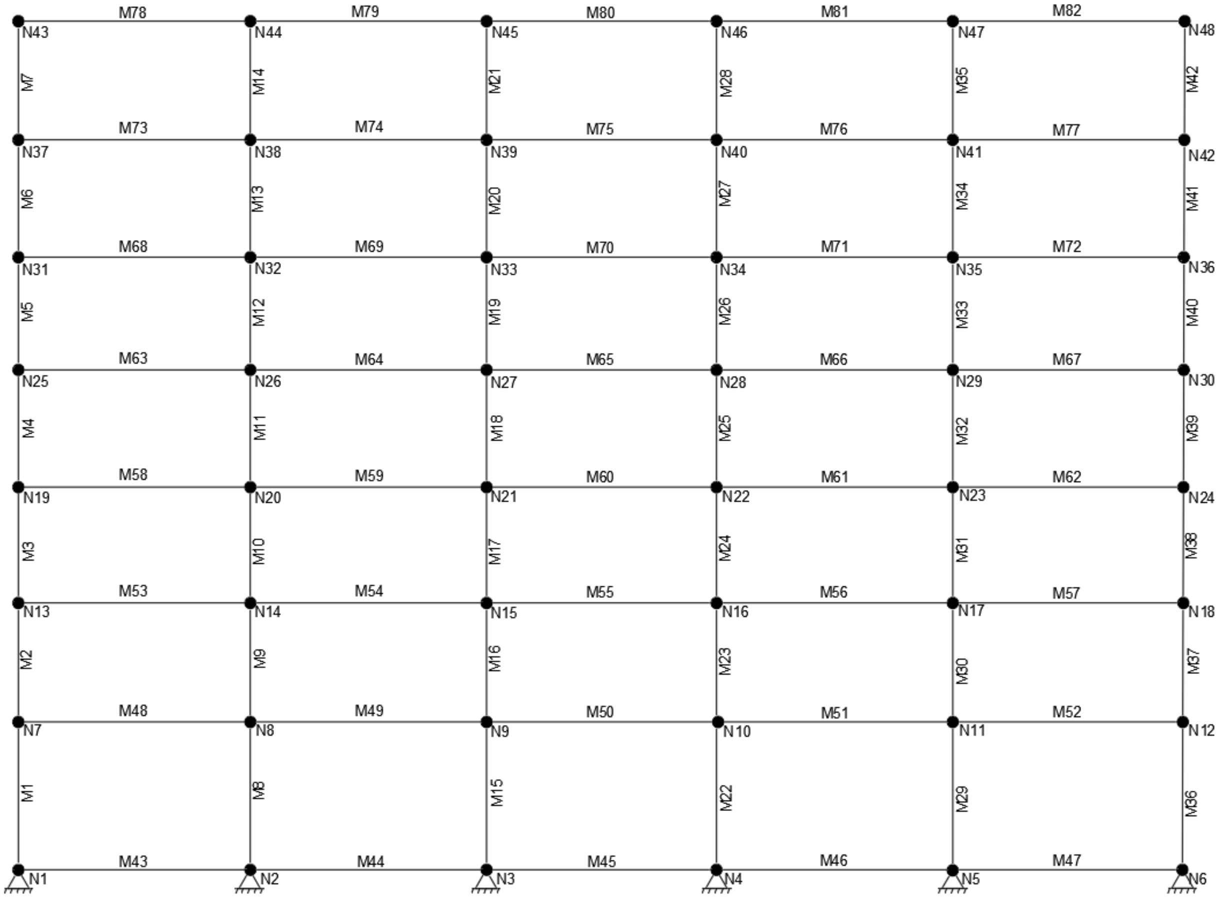


Figure 10: Diagram of the computer model used in the 2006 IBC Design Example 7

3.2.6. Three-Dimensional Finite Element Analysis

Using LS-DYNA, concrete elements were modeled as continuous surface cap solid elements as the steel reinforcement was represented as two-dimensional beam elements modeled with piecewise linear plasticity. Through the 3D-FEA, the model was controlled by the horizontal and vertical displacement to characterize quasi-static loading. Figure 11 shows the model used in LS-DYNA.

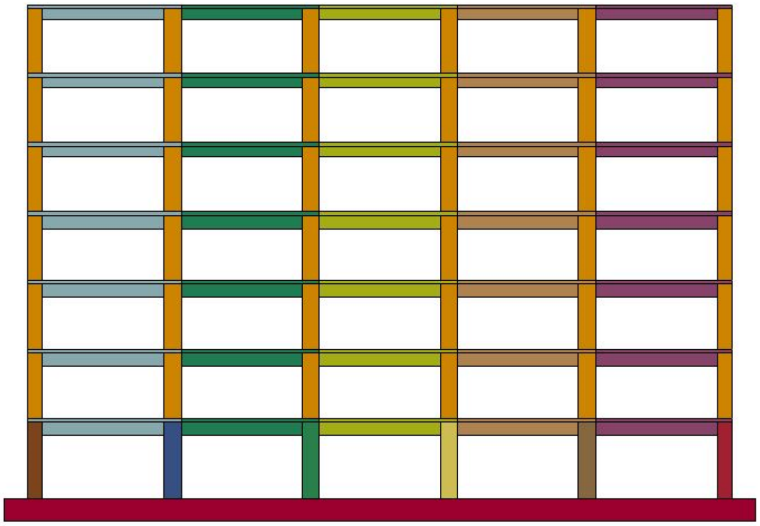


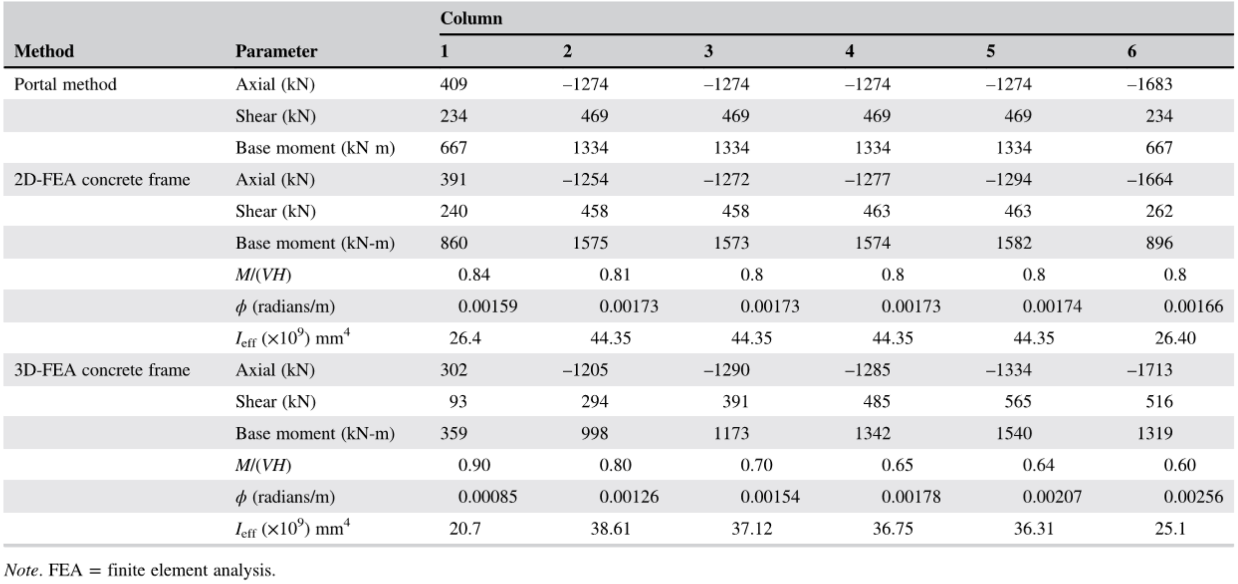
Figure 11: Image of the LS-DYNA model

As mentioned above, the model is displacement controlled; this was achieved by using the loads and forces described in Design Example 7 from the *2006 IBC Structural/Seismic Design Manual.* Inputting the prescribed code forces into LS-DYNA, the displacement of the columns was determined at the fifth level. With this data, the LS-DYNA model's columns were set to the displacement found at the prescribed code force and then the displacements were continued at the resulting rate until failure.

3.2.7. Finite Element Analysis Summary

Different FEA give different results, this is shown in Table 2. The Portal Method is the long, tedious hand calculation that is often taught in school. It states that the shear and moment for the exterior columns is half that of the interior columns. The 2D-FEA is used for everyday analysis and design because its ease of use. 3D-FEA are often only used for research and preformation based designs.

Table 2: 2006 IBC Frame Outcomes (Phillippi & Liuzza, 2017)



		Column					
Method	**Parameter**	**1**	**2**	**3**	**4**	**5**	**6**
Portal method	Axial (kN)	409	–1274	–1274	–1274	–1274	–1683
	Shear (kN)	234	469	469	469	469	234
	Base moment (kN m)	667	1334	1334	1334	1334	667
2D-FEA concrete frame	Axial (kN)	391	–1254	–1272	–1277	–1294	–1664
	Shear (kN)	240	458	458	463	463	262
	Base moment (kN-m)	860	1575	1573	1574	1582	896
	$M/(VH)$	0.84	0.81	0.8	0.8	0.8	0.8
	ϕ (radians/m)	0.00159	0.00173	0.00173	0.00173	0.00174	0.00166
	I_{eff} ($\times 10^9$) mm^4	26.4	44.35	44.35	44.35	44.35	26.40
3D-FEA concrete frame	Axial (kN)	302	–1205	–1290	–1285	–1334	–1713
	Shear (kN)	93	294	391	485	565	516
	Base moment (kN-m)	359	998	1173	1342	1540	1319
	$M/(VH)$	0.90	0.80	0.70	0.65	0.64	0.60
	ϕ (radians/m)	0.00085	0.00126	0.00154	0.00178	0.00207	0.00256
	I_{eff} ($\times 10^9$) mm^4	20.7	38.61	37.12	36.75	36.31	25.1

Note. FEA = finite element analysis.

For research purposes, LS-DYNA seems to be the best option due to the dynamic background of the software as well has user control over the model. Due to concrete being a complex material, it is very hard to fully understand; many components and elements go into analyzing and designing a concrete structure. Additionally, the complexity of the material interferes with computer programs fully grasping every component of concrete. However, because of LS-DYNA's dynamic analysis, it is able to analyze these components other programs seem to miss. As a program, LS-DYNA solves for an equation of motion meaning that at each moment, every element is analyzed. Due of this LS-DYNA has the ability to have an incremental displacement control, unlike RISA and SAP2000 which are typically load based. Due to this LS-DYNA can analyze structures beyond the maximum load for the structure. Becasue to the abilities of control, LS-DYNA was used for the analysis of this research.

3.3. Governing Regulations

For the safety, health, and welfare of the occupant, governing regulations are set in place for all aspects of buildings. For the analysis of a building, ASCE publishes a standard for minimum design loads. For structural concrete, ACI publishes the building code requirements used in the United States of America for design.

3.3.1. ASCE 7-10

The *Minimum Design Loads for Buildings and Other Structures* (ASCE 7-10) is a standard with a detailed guide to calculate the minimum design loads such as: dead load, live load, soil load, flood load, snow load, rain load, ice load, earthquake loads, and more including combinations of different loads. These loads are implemented into the design of the structure.

3.3.2. ACI 314-18

The Building Code Requirement for Structural Concrete (ACI 318-14) is a code standard for the design of reinforced concrete buildings. These regulations on design help with the durability of a structure. The main purpose of the ACI 318-14 is to provide a detailed guide of calculating the reinforcement needed in the element being designed.

Chapter 4 - Parametric Study: Columns

As mentioned in Chapter 3 Section 1, this thesis evaluates the performance of a reinforced concrete special moment resisting frame (SMRF) described in Design Example 7 of the *2006 IBC Structural/Seismic Design Manual* (Appendix A). However, the frame was modified to have equal column sizes as seen in the detailing of Figure 12.

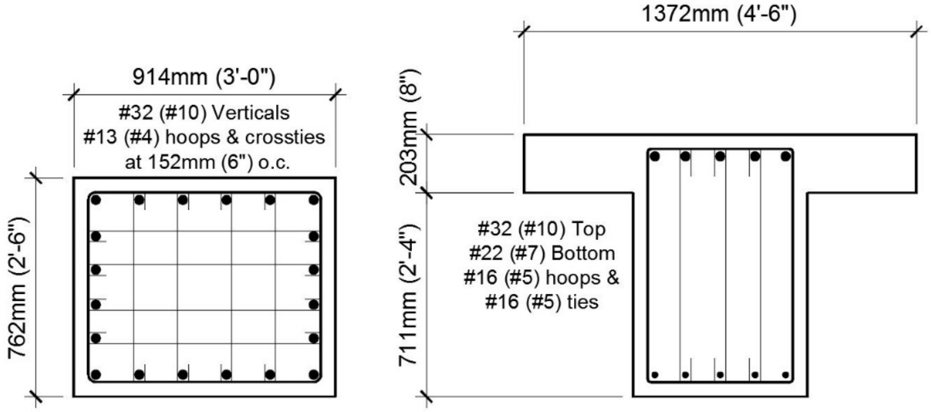


Figure 12: Diagram of reinforcement for the modified frame

Using LS-DYNA, forces were simulated to act laterally on the frame and the concrete columns, in a moment frame, were investigated. The effect of the load is illustrated in Figure 13. Since, LS-DYNA solves for an equation of motion, data produced from the program and structural mechanics can be used to create basic concrete analysis. Through the research, four things were looked into and are discussed as follows:

1. Forces
2. Force vs. Displacement
3. Moment vs. Curvature

4. Total Displacement

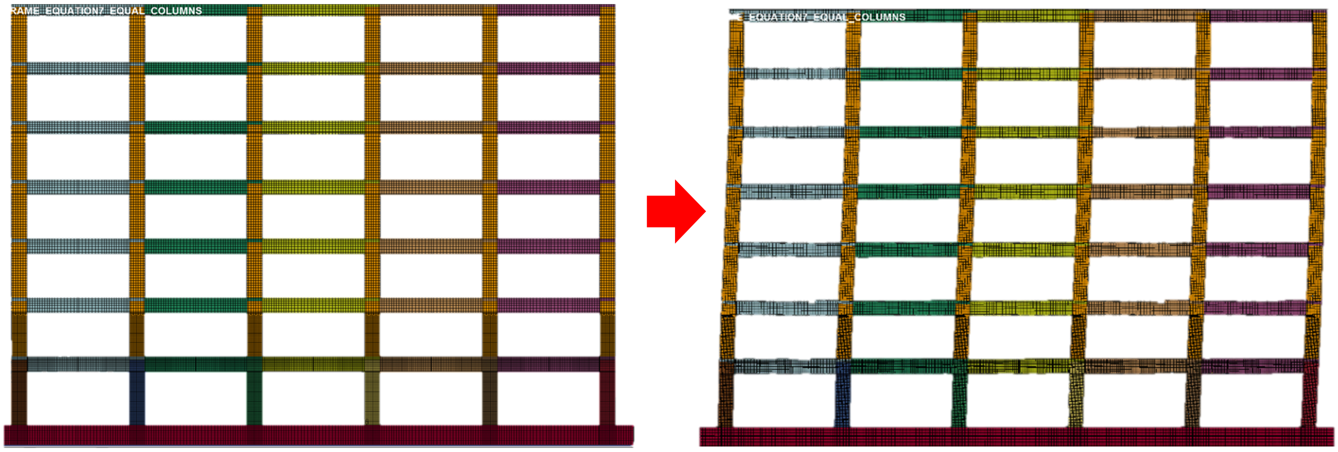


Figure 13: Diagram of LS-DYNA model showing displacement

4.1. LS-DNYA Model

The SMRF are typically designed with finite element analyses using two-dimensional beam elements for the concrete and the steel reinforcement. In the thesis presented LS-DYNA, a three-dimensional FEA that includes eight node solid elements for the concrete material and two-dimensional beam elements for the steel reinforcement, was used for analysis. Nodes were selected on the members according to Figure 14. Nodes u_1, u_3, u_5, and u_7 represent x-displacement and the z-displacement is represented by nodes u_2, u_4, u_6, and u_8. Figure 15 shows the SMRF's column call outs for reference.

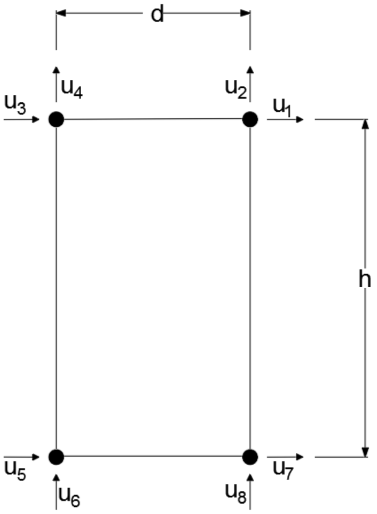


Figure 14: Diagram of node selection

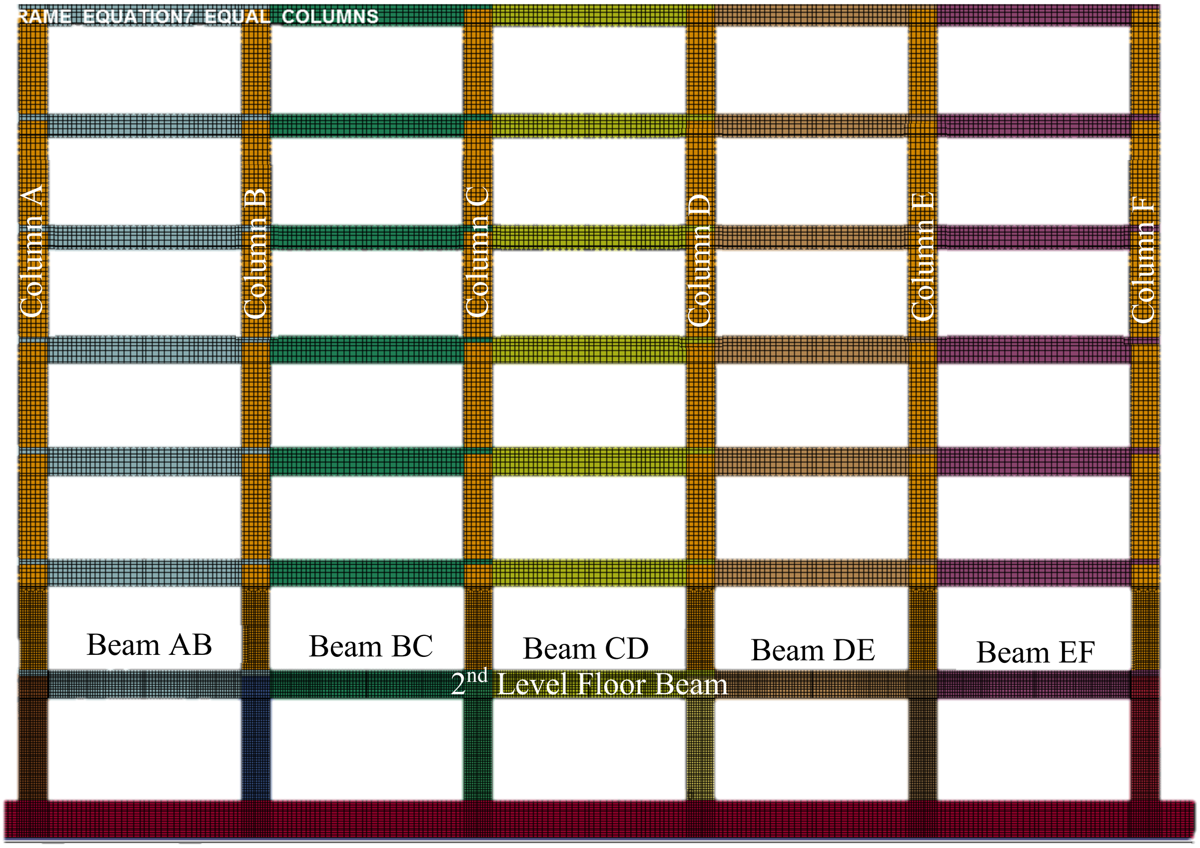


Figure 15: Diagram of SMRF references

4.2. Forces

Using the LS-DYNA model described in Section 3.2.6., the forces versus time graph was plotted directly from the program, the results are shown in Figure 16. As a note, time refers to time steps, not the actual time.

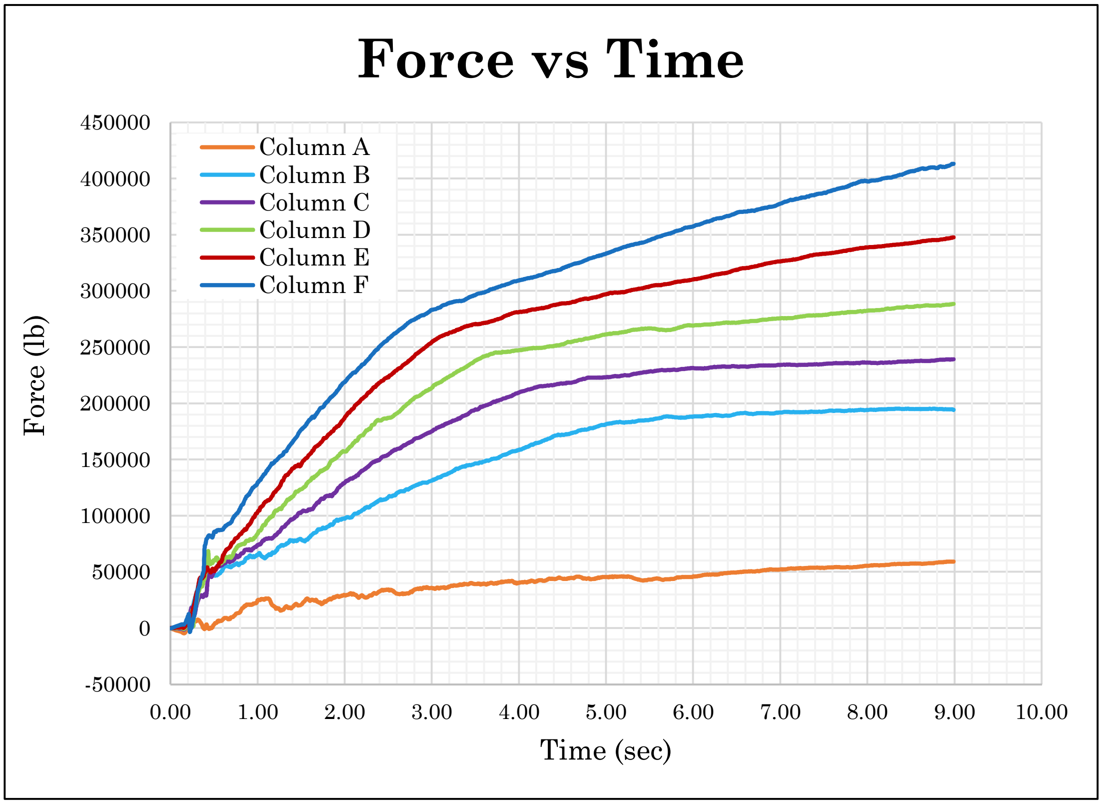


Figure 16: Force vs Time for each column

It can be seen that each column experiences a different load at any given time. Column A is wayward of the load, and it experiences the lowest amount of load. Currently, it is practiced that Column F (leeward of the load) should have the same amount of force as the Column A does, and it should be one-half than that of the inner columns.

4.3. Force vs. Displacement

From the force data used above, and the x-displacement taken from LS-DYNA, a Force vs. Displacement graph was made. This graph provides detail on how each column's force is depended on the displacement. The results are in Figure 17.

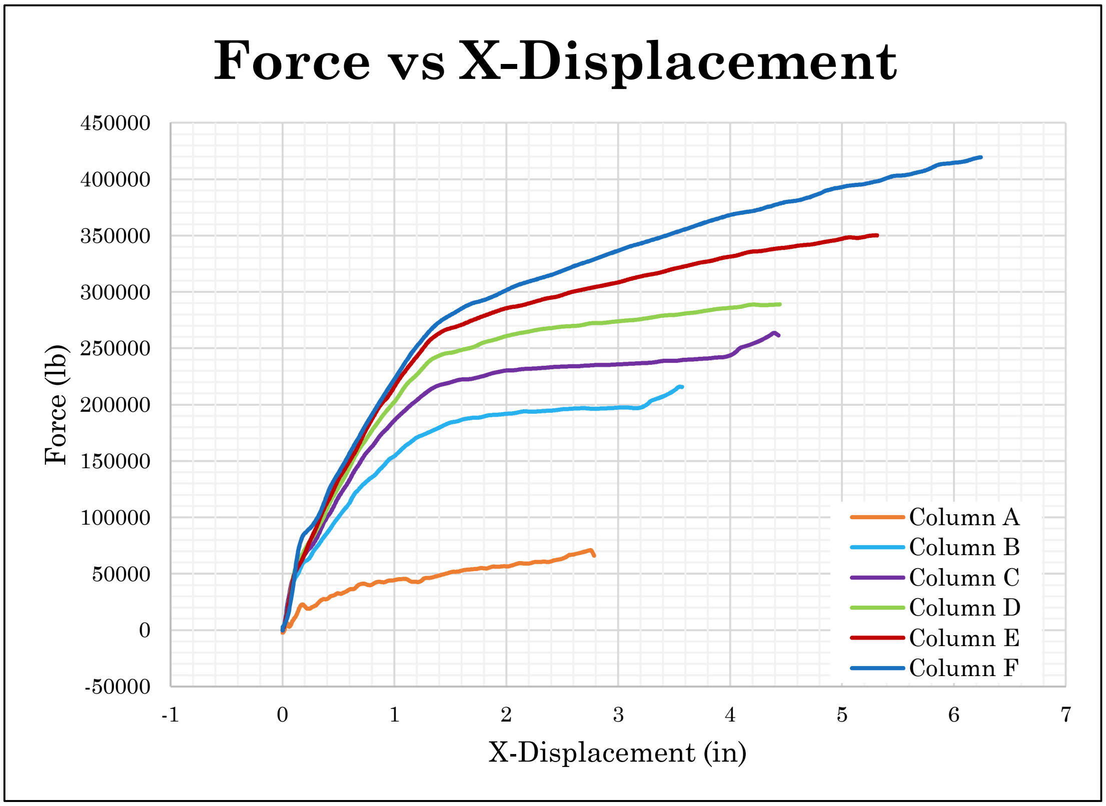


Figure 17: Force vs. X-Displacement for each column

This graph shows that every column experiences a different force due to displacement. This is because of the stiffness of the columns and the elongation of the beams.

4.4. Moment vs. Curvature

For concrete, Moment vs. Curvature graphs are used to determine many factors. These factors include determining when the concrete: begins cracking, becomes elastic, reaches ultimate design, and has a total failure, it also shows when the reinforcing bars yield. Figure 18 shows these points in a graph.

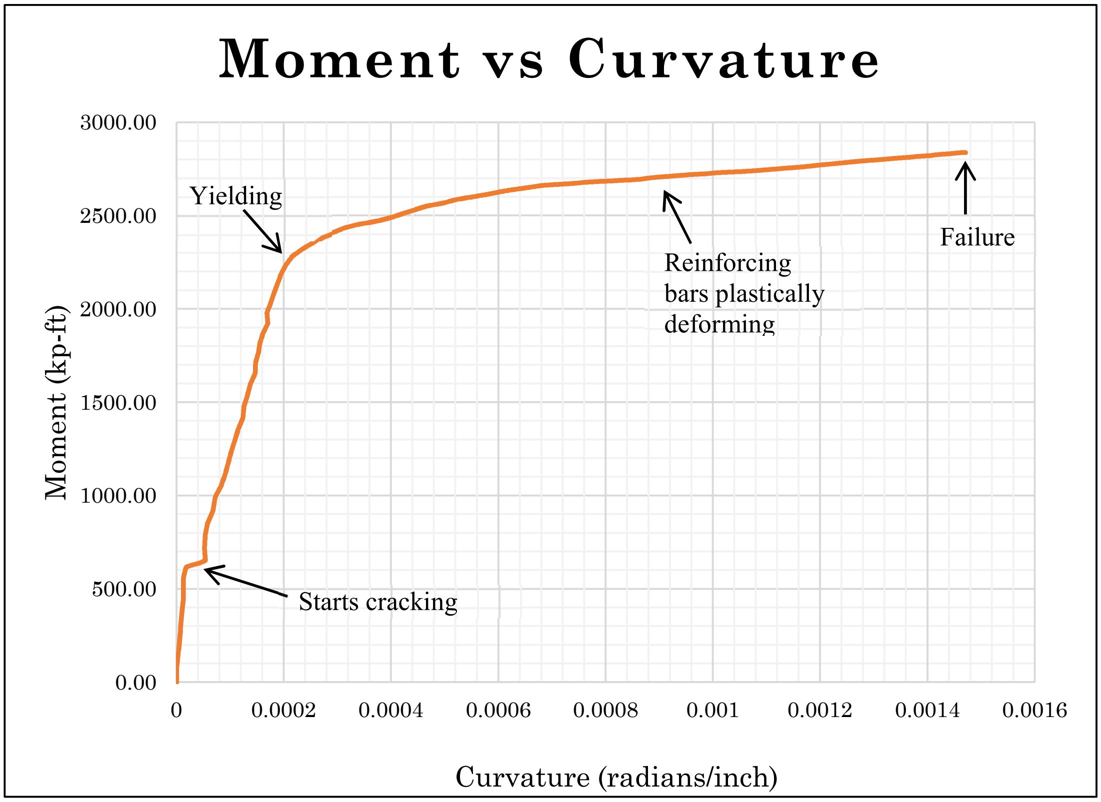


Figure 18: Moment vs Curvature curve

For this research, data was taken at the steel reinforcement (rebar); it was taken at the rebar because of the complex nature of the concrete causing errors in the data. Because the rebar is embedded into the concrete, the results are similar as if the data was taken at the concrete. First the z-displacement was found of the columns, and then the y-moment of each column was found. Using LS-DYNA, a z-displacement versus y-moment plot was created and then put into excel. Using equation (4-1) the curvature was found, and then plotted versus the y-moment.

$$\phi = \frac{\varepsilon_1 - \varepsilon_2}{d} \tag{4-1}$$

$$\varepsilon = \frac{\Delta L}{L} \tag{4-2}$$

$$\phi = \frac{(u_4 - u_6) - (u_2 - u_8)}{hd} \tag{4-3}$$

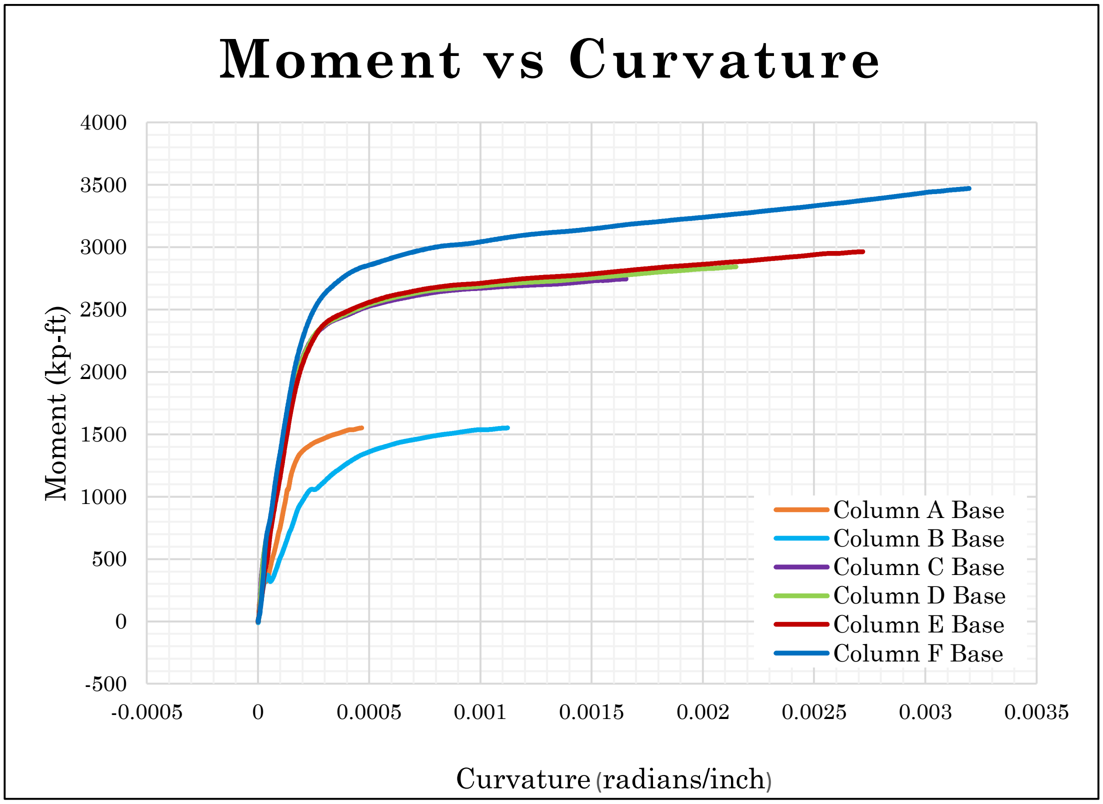


Figure 19: Moment vs Curvature for each column

Figure 19 show the results and that each of the columns undergo cracking, yielding and failure all at different times under the same axial force. As the moment and curvature of a concrete frame start to change, a shear deformation is created due to the cracking of the concrete.

4.5. Total Displacement

The displacement is how much the column moves during and after loading; it is very important for structure not to have too much deflection. Based off the shear deformation from LS-DYNA and basic mechanics, the total displacement can be determined. The total displacement was determined using two different methods:

1. Using LS DYNA and,
2. Using hand calculations.

Using LS-DYNA, the curvature of the columns was found using the same method described in section 4.4, these results are available in Appendix B. Shear deformation was found using the x-displacement and z-displacements of the frame in LS-DYNA. The data from the displacements was then applied to equation (4-4); as a reminder, u_1, u_3, u_5, and u_7 represent x-displacement and the z-displacement is represented by u_2, u_4, u_6, and u_8.

$$\gamma = \frac{u_1 + u_3 - u_5 - u_7}{2h} + \frac{u_2 - u_4 - u_6 + u_8}{2d} \tag{4-4}$$

The first part of the shear deformation equation describes the horizontal deformation while the second part of the equation describes the vertical deformation occurring. The shear deformation of the columns can be seen in Appendix B. With these results, a Curvature vs. Height graph was created to represent the displacement model; these graphs were used for the

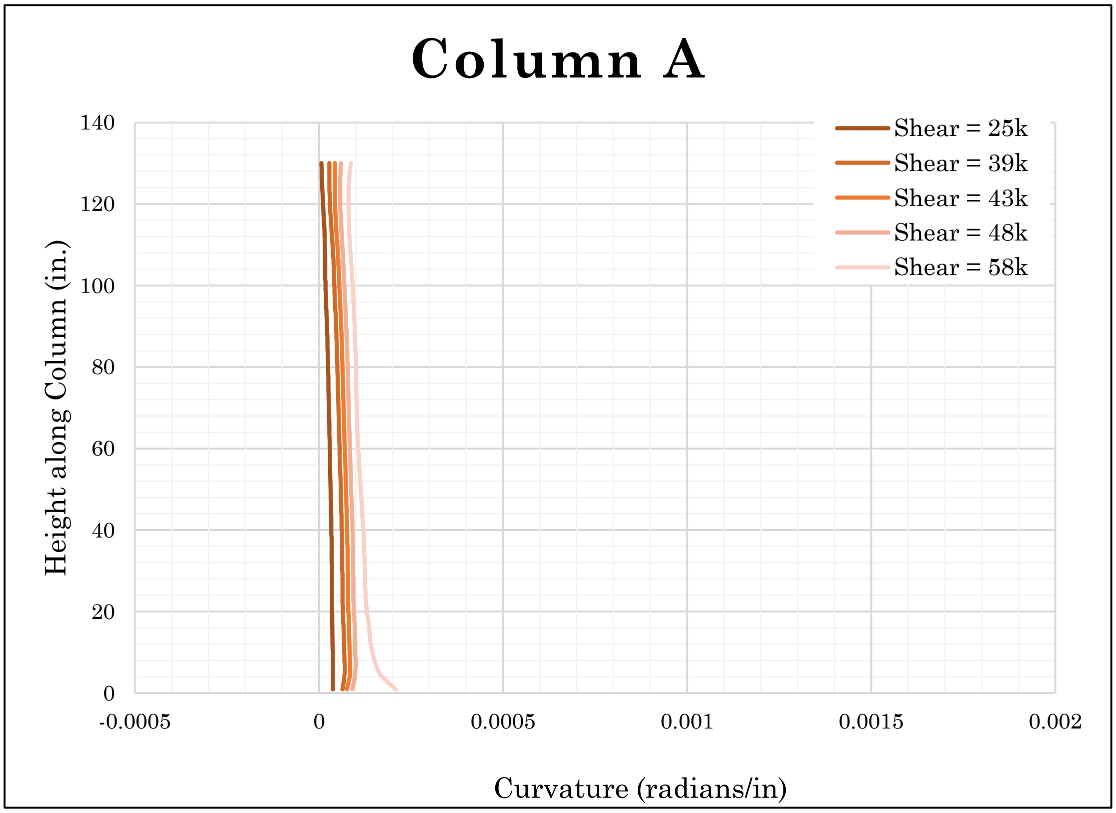


Figure 20: Curvature vs. Height used for calculating Total Displacement

hand calculations for total displacement. Figure 20 shows the Curvature vs. Height graph for Column A, the other columns' data is in Appendix B.

With these graphs, to the find the total displacement by mechanics the equations as follows.

For the flexure portion:

$$\Delta_f = (\phi_1 - \phi_y) l_p \left(h - \frac{l_p}{2}\right) + \frac{\phi_y h^2}{3} + \frac{\phi_2 h^2}{6} \qquad (4\text{-}5)$$

The height is based on the Priestley model (Mandar, Priestley, & Park, 1988) and the curvatures are from LS-DYNA.

$$h = H + 0.15 f_y d_b \qquad (4\text{-}6)$$

ϕ_1 – curvature at base

ϕ_y – yielding curvature

ϕ_2 – curvature at maximum height

$$l_p = 0.08 h_{clr} + 0.15 f_y d_b \qquad (4\text{-}7)$$

For the shear portion:

$$\Delta_s = \gamma_{avg} h \qquad (4\text{-}8)$$

γ_{avg} – based on the average shear angle over the height of the column

Total Displacement:

$$\Delta_T = \Delta_{f+} \Delta_s \qquad (4\text{-}9)$$

The value of the total displacement is then compared to LS-DNYA's total displacement, Figure 21 shows and example calculation. All graphs and calculations for this section are in Appendix B.

$$\Delta_{f,A} = (\phi_1 - \phi_y)l_p\left(h - \frac{l_p}{2}\right) + \frac{\phi_y h^2}{3} + \frac{\phi_2 h^2}{6} = 1.698\ in$$

$$\Delta_{s,A} = \gamma_{avg}h = 0.183\ in$$

$$\Delta_{total,A} = \Delta_{f,A} + \Delta_{s,A} = 1.880\ in$$

$$value\ from\ LS - DYNA = 1.886\ in$$

Figure 21: Equations used for calculating Total Displacement Compare to LS-DYNA Value

It is seen that the hand calculated displacement and the computer generated displacement are very similar, meaning that the equation produced through this research is very close to what software calculated. These graphs show that each of the six columns react differently under the varying axial forces; the leeward column has the largest shear deformation meaning that it displaces the most. The movement of the columns cause cracking in the concrete. As the cracking occurs, steel reaches its yielding strain and goes beyond then becoming plastic, and once the material is plastic it cannot go back to its original form causing a permanent elongation in the beams.

Due to these findings, it is apparent that beam elongation has a role into why concrete columns act differently than what is currently thought. The next chapter of the thesis, will look into the beam elongation and how to incorporate it into standard design.

Chapter 5 - Parametric Study: Beams

5.1. Design

In order to produce sufficient data for comparing, multiple frames were created based off the frame described in the *2006 IBC* Design Example 7. Ten different frames underwent two different load cases for the purposes of gathering data to compare. Using the same call out for reference as in Figure 14, five frames were seven stories tall, while the other five were six stories tall. Of the five frames at each height, the frames varied from having five bays to four bays to three bays to 2 bays and down to one bay. The different load cases acting on the frames refer to the gravity loads being applied; classified as equation 5, the dead and live loads were applied to the beam, while the loading case called equation 7 had dead load only. The forces applied laterally were based off the displacement determined by 2D-FEA done in the *2006 IBC* Design Example 7, the frames where put into LS-DYNA with displacement controlled lateral loads. The frames compared are described in Table 3:

Table 3: Frame Call Outs

Frame Label	Number of Stories	Number of Bays	Gravity Equation
7.5.7	7	5	7
7.4.7	7	4	7
7.3.7	7	3	7
7.2.7	7	2	7
7.1.7	7	1	7
7.5.5	7	5	5
7.4.5	7	4	5
7.3.5	7	3	5
7.2.5	7	2	5
7.1.5	7	1	5
6.5.7	6	5	7
6.4.7	6	4	7
6.3.7	6	3	7
6.2.7	6	2	7
6.1.7	6	1	7
6.5.5	6	5	5
6.4.5	6	4	5
6.3.5	6	3	5
6.2.5	6	2	5
6.1.5	6	1	5

5.2. Experiment

5.2.1. Columns

An essential part of the elongation of reinforced concrete beams is the drift and rotation of the columns the beams are attached to. For this thesis, LS-DYNA was used to determine the drift of the columns through the time versus x-displacement data. Using the same reference nodes u_1, u_3, u_5, and u_7 as shown in Figure 13, node values where taken at the exterior reinforcement along the fourth tie from the bottom and from the first tie beyond the second level floor beams. At time equal to zero seconds, the nodes were thirty inches in width and one hundred fifty-one inches in height apart. In order to find the drift of the columns Equation (5-1) was used; the equation finds the average drift of each column and is then used to find the rotation of the column, Equation (5-2).

$$\Delta = \frac{u_1 + u_3}{2} - \frac{u_5 + u_7}{2} \tag{5-1}$$

$$\theta = \tan^{-1}\left({}^{\Delta}/_{h}\right) \tag{5-2}$$

The rotation of the columns was compared to the time in order to describe the relationship of the rotations, through this it was concluded that rotation has a linear relationship with time that can be seen since in Appendix C. The frames used for this research were then created to represent the rotation occurring the columns, Figure 22 shows this representation for Frame 7.5.7 at the maximum time before failure. The other frames' column rotation representation are in Appendix C.

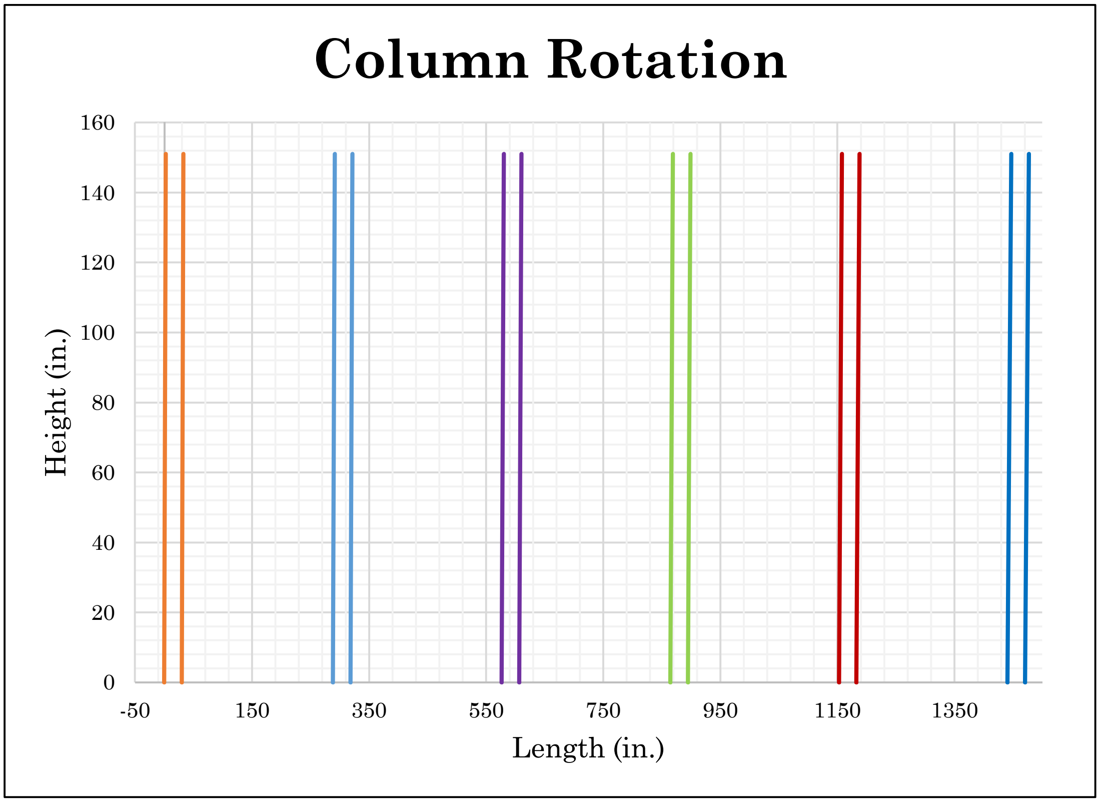


Figure 22: Rotation of Columns

5.2.2. Beams

The focus of this thesis is the elongation of beams, in order to achieve this the time versus x-displacement and the time versus z-displacement data was taken for the second level floor beams of the frame. As with the columns nodes u_1, u_2, u_3, u_4, u_5, u_6, u_7, and u_8, as shown in Figure 13, were taken at the steel reinforcement throughout the beams. Shown in Figure 14 are the locations of the nodes on the beam. With the results of the LS-DYNA displacements of the beams the deformation and neutral axis where determined.

The frames where loaded until failure, and from the displacement values in both the x and z direction, the beam deformation was graphed for each beam. The points plotted show how the reinforcement moved vertically and horizontally at the same time. To find the location at the deformed beam, the reinforcement's displacement at a certain time in both directions was added to the original location of the reinforcement in the beam. Figure 23 shows an example of the beam deformation of Beam AB for Frame 7.5.7, the rest of the beam deformations can be seen in Appendix C.

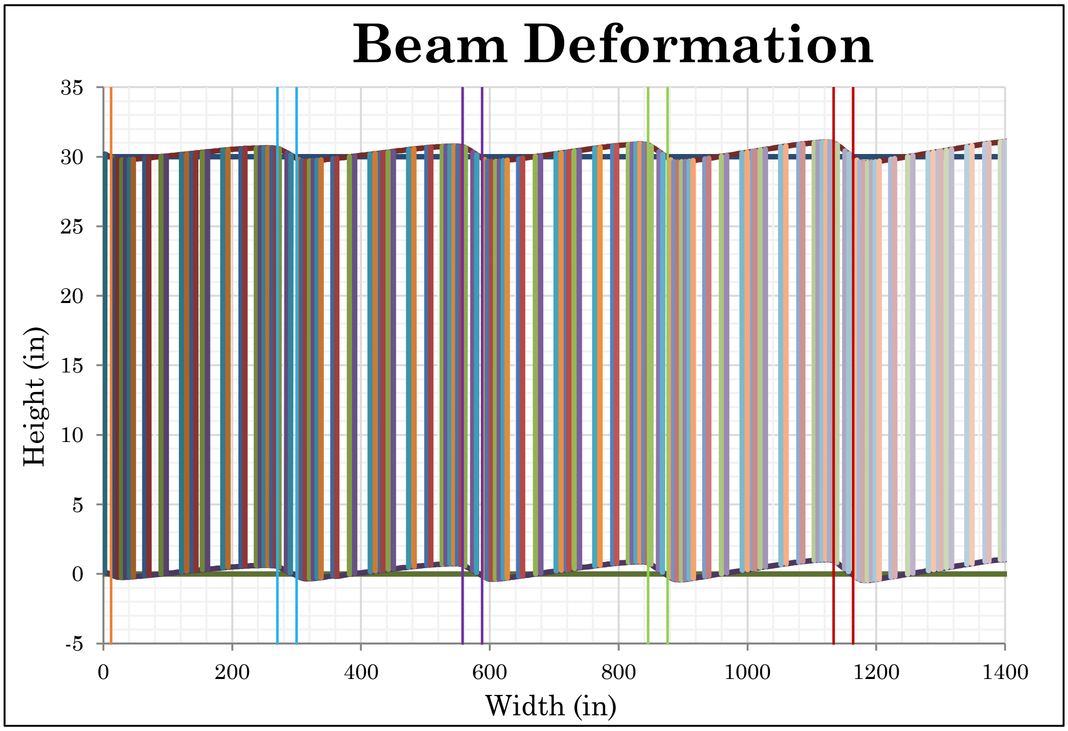


Figure 23: Beam Deformation

Due to the importance of the of neutral axis, as discovered by previous researchers, the neutral axis was determined by finding the strain of the reinforcement along the beams. The strain equation stated in Equation (4-2) is used to find the top and bottom strains of the reinforcements. Equation (5-3) and Equation (5-4) are used for finding the top and bottom strains.

$$\varepsilon_{TOP} = \frac{u_1 - u_3}{d} \tag{5-3}$$

$$\varepsilon_{BOTTOM} = \frac{u_7 - u_5}{d} \tag{5-4}$$

From the strain, the standard design process of determining the neutral axis value, c, from the compression side of the beam is described in Figure 24 and Equations (5-5), (5-6), and (5-7).

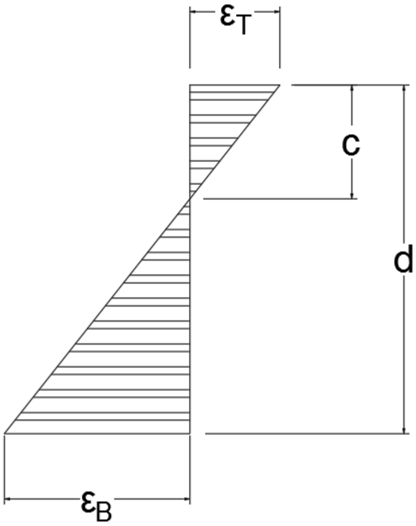


Figure 24: Diagram of the strain of concrete and reinforcement

$$\frac{\varepsilon_{TOP}}{c} = \frac{\varepsilon_{TOP} + \varepsilon_{BOTTOM}}{d} \tag{5-5}$$

$$c = \varepsilon_{TOP} \times d/(\varepsilon_{TOP} + \varepsilon_{BOTTOM}) \tag{5-6}$$

$$d - c = \varepsilon_{BOTTOM} \times d/(\varepsilon_{TOP} + \varepsilon_{BOTTOM}) \tag{5-7}$$

It is important to note that for concrete the compression strain is taken at the concrete face while the tension strain is taken at the reinforcement. The data taken from LS-DYNA take both the strains from the reinforcement in the beam. In order to follow the standard design process for the neutral axis, three inches was added to the value of c.

5.2.3. Frame

The column drift, beam deformation, and neutral axis were all graphed as a length versus height graph in order to show the deformation of the frame. Figure 25 is an example of a frame deformation, the other frame deformations are shown in Appendix C.

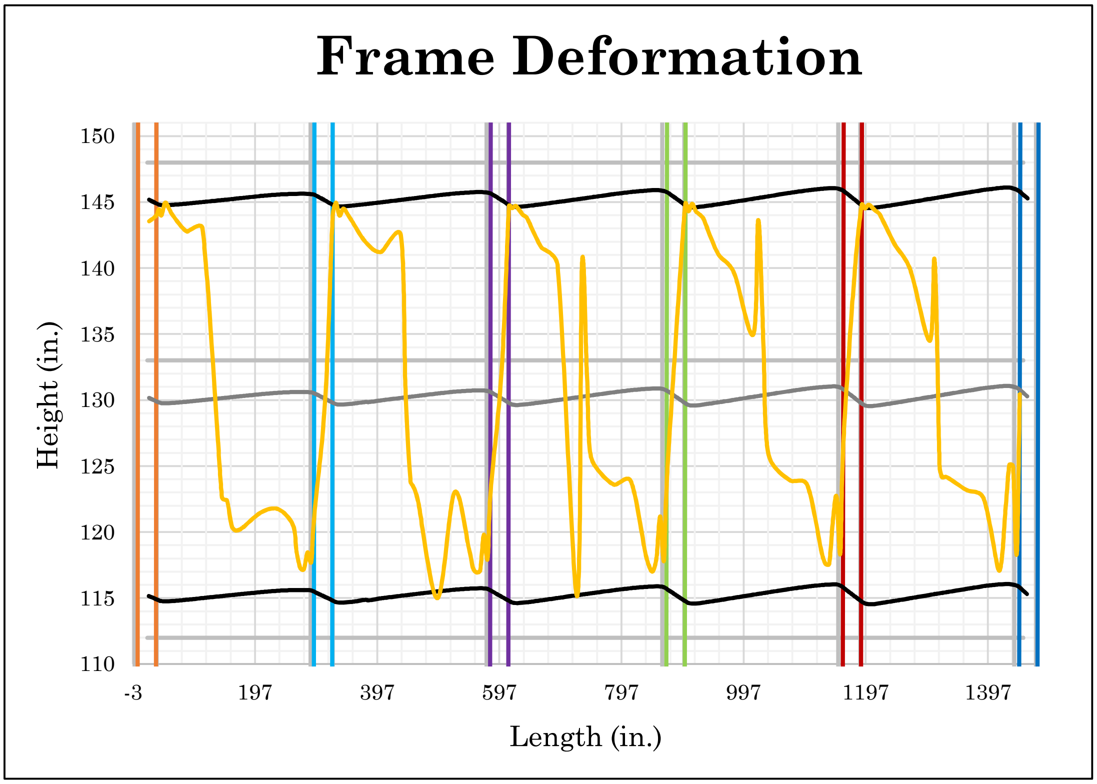


Figure 25: Frame Deformation

5.3. Results

5.3.1. Elongation Calculations

The purpose of this thesis was to propose a standard design consideration for the elongation of the beams in reinforced concrete SMRFs. With the data and graphs discussed in the previous section of this chapter along with simple math, an equation for elongation was found that can be incorporated into standard design practice. Using the frame deformation graphs discussed in Section 5.2.3 the joints of the beams and columns were focused on and analyzed to determine how much the beams are elongating. Using the deformation of the neutral axis and the location of the original centroid of the concrete beam, the elongation of the beam can be calculated. Using simple trigonometry, Equation (5-8) was used to determine the elongation at the location where the beam and column meet.

$$x = e_{cri} \tan \theta \tag{5-8}$$

x - elongation

e_{cri} - distance between deformed neutral axis and the original centroid.

θ - rotation

It should be noted that while the neutral axis location is usually taken from the compression face of the beams, for calculation purposes the neutral axis location is taken from the top face of the beam. Meaning, for the right side of the beam the neutral axis is measured from the compression side, while for the left side of the beam the neutral axis is measured from the tension side.

For example, the elongation calculation of Beams AB and BC at Column B of Frame 7.5.7 are described as follows. Figure 26 shows the deformation of the frame due to loading at the maximum time of 9.60 seconds, the following figure, Figure 27, shows the calculation of the elongation of the left end of Beam AB and the right end of Beam BC.

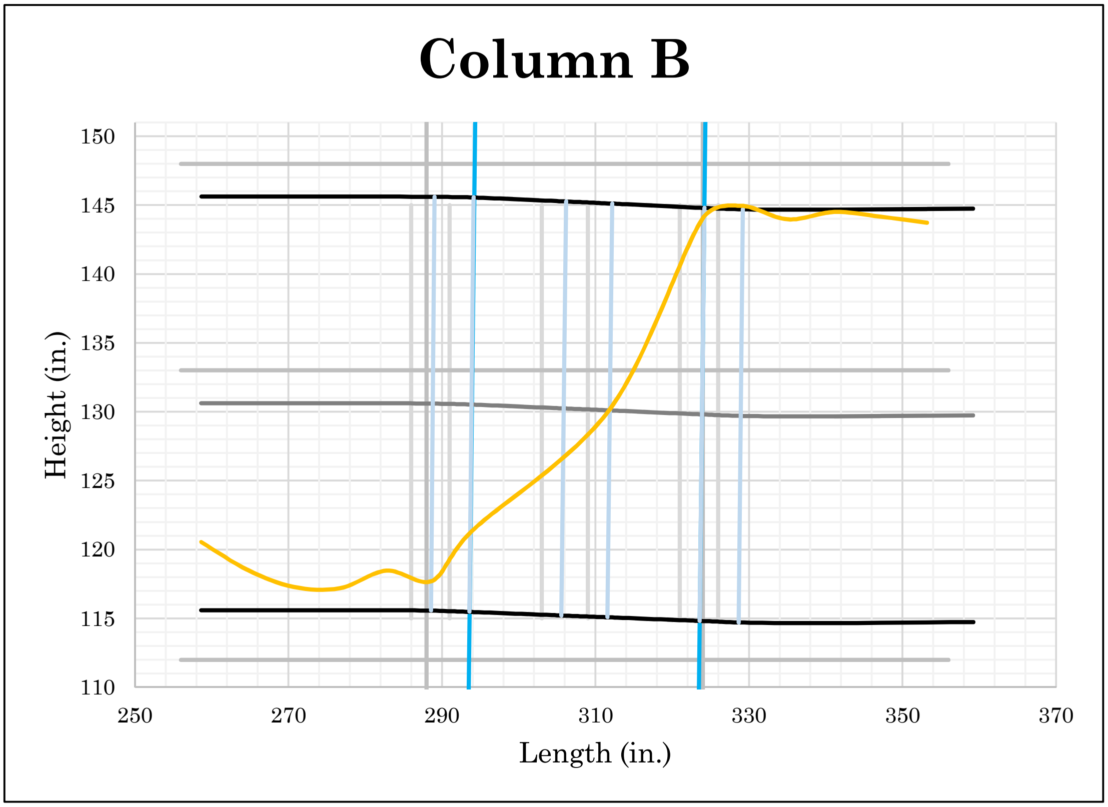


Figure 26: Deformation at the joint of Column B and the second level floor beams

7.5.7

Left Side of Column B / Right Side of Column B

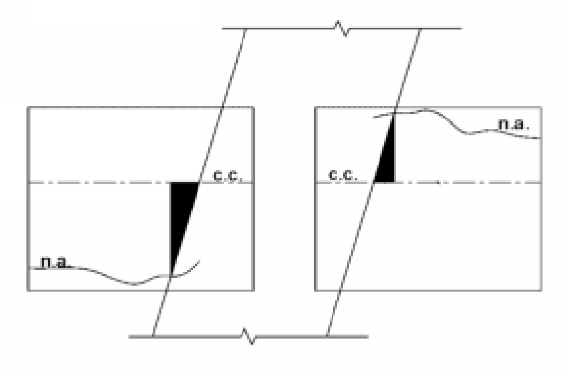
Column

Left Side of Column B			Right Side of Column B		
time, t =	7.0000	sec	time, t =	7.0000	sec
drift, Δ =	2.2309	in.	drift, Δ =	2.2309	in.
rotation, Θ =	0.8464	°	rotation, Θ =	0.8464	°

Beam

Left Side of Column B			Right Side of Column B		
time, t =	7.0000	sec	time, t =	7.0000	sec
neutral axis, n.a. =	27.1213	in.	neutral axis, n.a. =	4.6524	in.
centroid, c.c. =	15.0000	in.	centroid, c.c. =	15.0000	in.

Calculations

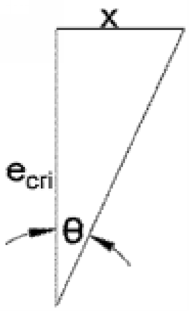


all measurements are taken from the top face of the beam

q = rotation

e_{cri} = distance between the n.a. and c.c.

x = elongation



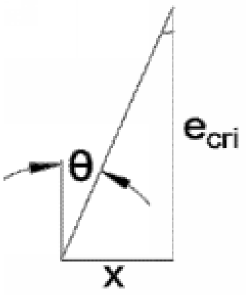
$$\tan\theta = \frac{opposite}{adjacent}$$

$$\tan\theta = \frac{x}{e_{cri}}$$

$$x = e_{cri}\tan\theta$$

e_{cri} = 12.1213 in.

x = 0.1791 in.



$$\tan\theta = \frac{opposite}{adjacent}$$

$$\tan\theta = \frac{x}{e_{cri}}$$

$$x = e_{cri}\tan\theta$$ due to similar triangles

e_{cri} = 10.3476 in.

x = 0.1529 in.

Figure 27: Calculation of beam elongation at Column B in Frame 7.5.7

From the calculations shown above, the total beam elongation, as well as frame elongations where found and compared. In order to compare all of the frames, the data for determining the elongation has a control point of seven seconds. Using Equation (5-8) the beam elongations where found for the beam and column joints, individual beams, and for the total elongation of the beams and frame. For the individual beams and total beams (as called in the result tables), the joint elongations were added as required. The percentage of elongation was also calculated for the individual beams and the total beam elongation. Using the total beam elongations and the column drifts, the total frame elongation was found as well as the percent of elongation. Tables 4 – 8 summarizes the elongation calculations just described for the all of the frames at seven seconds.

As noticed in the results, commonly the leeward beams have a shorter elongation than the beam adjacent. This is caused the fact there is not another beam next to it, without this, the column does not get pulled by another causing a greater elongation in the beams.

Table 4: Elongation of the five bay frames

Frame	**7.5.7**									
Location	**Column A**	**Column B**		**Column C**		**Column D**		**Column E**		**Column F**
	Right	**Left**	**Right**	**Left**	**Right**	**Left**	**Right**	**Left**	**Right**	**Left**
Time	7.0000	7.0000	7.0000	7.0000	7.0000	7.0000	7.0000	7.0000	7.0000	7.0000
Drift (in.)	1.8080	2.2309	2.2309	2.7131	2.7131	3.2517	3.2517	3.8632	3.8632	4.6139
Rotation (°)	0.6860	0.8464	0.8464	1.0293	1.0293	1.2336	1.2336	1.4655	1.4655	1.7502
Neutral Axis	4.5976	27.1213	4.6524	26.2572	3.5563	25.9270	3.2499	25.6786	3.2470	21.9144
Top of Conc.	36.0000	36.0000	36.0000	36.0000	36.0000	36.0000	36.0000	36.0000	36.0000	36.0000
Centroid	15.0000	15.0000	15.0000	15.0000	15.0000	15.0000	15.0000	15.0000	15.0000	15.0000
Elongation	0.1246	0.1791	0.1529	0.2023	0.2056	0.2353	0.2530	0.2732	0.3007	0.2113
	0.3036		0.3551		0.4409		0.5262		0.5120	
	0.12%		0.14%		0.17%		0.20%		0.20%	
Total Beam									**2.1379 in.**	**0.17%**
Total									**20.6187 in.**	**1.40%**
Frame	**7.5.5**									
Location	**Column A**	**Column B**		**Column C**		**Column D**		**Column E**		**Column F**
	Right	**Left**	**Right**	**Left**	**Right**	**Left**	**Right**	**Left**	**Right**	**Left**
Time	7.0000	7.0000	7.0000	7.0000	7.0000	7.0000	7.0000	7.0000	7.0000	7.0000
Drift	2.5016	3.1145	3.1145	3.8350	3.8350	4.6078	4.6078	5.4534	5.4534	6.4697
Rotation	0.9491	1.1816	1.1816	1.4548	1.4548	1.7479	1.7479	2.0683	2.0683	2.4534
Neutral Axis	4.8927	26.7116	6.3413	25.7774	3.8741	24.7809	3.0260	23.8071	3.1625	21.1375
Top of Conc.	36.0000	36.0000	36.0000	36.0000	36.0000	36.0000	36.0000	36.0000	36.0000	36.0000
Centroid	15.0000	15.0000	15.0000	15.0000	15.0000	15.0000	15.0000	15.0000	15.0000	15.0000
Elongation	0.1674	0.2416	0.1786	0.2737	0.2826	0.2985	0.3654	0.3181	0.4275	0.2630
	0.4090		0.4523		0.5810		0.6835		0.6905	
	0.16%		0.18%		0.23%		0.26%		0.27%	
Total Beam									**2.8163 in.**	**0.22%**
Total									**28.7984 in.**	**1.96%**
Frame	**6.5.7**									
Location	**Column A**	**Column B**		**Column C**		**Column D**		**Column E**		**Column F**
	Right	**Left**	**Right**	**Left**	**Right**	**Left**	**Right**	**Left**	**Right**	**Left**
Time	7.0000	7.0000	7.0000	7.0000	7.0000	7.0000	7.0000	7.0000	7.0000	7.0000
Drift	2.5873	3.1586	3.1586	3.8234	3.8234	4.5361	4.5361	5.3068	5.3068	6.2189
Rotation	0.9816	1.1983	1.1983	1.4504	1.4504	1.7207	1.7207	2.0128	2.0128	2.3584
Neutral Axis	3.0777	26.0119	3.8594	25.1537	3.1657	23.5751	3.0964	23.3637	3.1523	18.7857
Top of Conc.	36.0000	36.0000	36.0000	36.0000	36.0000	36.0000	36.0000	36.0000	36.0000	36.0000
Centroid	15.0000	15.0000	15.0000	15.0000	15.0000	15.0000	15.0000	15.0000	15.0000	15.0000
Elongation	0.2043	0.2303	0.2330	0.2571	0.2996	0.2576	0.3576	0.2939	0.4164	0.1559
	0.4346		0.4901		0.5572		0.6515		0.5723	
	0.17%		0.19%		0.22%		0.25%		0.22%	
Total Beam									**2.7058 in.**	**0.21%**
Total									**28.3369 in.**	**1.93%**
Frame	**6.5.5**									
Location	**Column A**	**Column B**		**Column C**		**Column D**		**Column E**		**Column F**
	Right	**Left**	**Right**	**Left**	**Right**	**Left**	**Right**	**Left**	**Right**	**Left**
Time	7.0000	7.0000	7.0000	7.0000	7.0000	7.0000	7.0000	7.0000	7.0000	7.0000
Drift	2.5016	3.0214	3.0214	3.8350	3.8350	4.6078	4.6078	5.4534	5.4534	6.4697
Rotation	0.9491	1.1463	1.1463	1.4548	1.4548	1.7479	1.7479	2.0683	2.0683	2.4534
Neutral Axis	4.5843	25.9907	4.1084	24.5828	3.2522	22.5980	3.6158	22.0485	3.4336	18.7914
Top of Conc.	36.0000	36.0000	36.0000	36.0000	36.0000	36.0000	36.0000	36.0000	36.0000	36.0000
Centroid	15.0000	15.0000	15.0000	15.0000	15.0000	15.0000	15.0000	15.0000	15.0000	15.0000
Elongation	0.1726	0.2199	0.2179	0.2434	0.2984	0.2319	0.3474	0.2546	0.4177	0.1624
	0.3925		0.4613		0.5302		0.6020		0.5802	
	0.15%		0.18%		0.21%		0.23%		0.22%	
Total Beam									**2.5661 in.**	**0.20%**
Total									**28.4550 in.**	**1.94%**

Table 5: Elongation of the four bay frames

Frame	**7.4.7**							
Location	**Column A**	**Column B**		**Column C**		**Column D**		**Column E**
	Right	**Left**	**Right**	**Left**	**Right**	**Left**	**Right**	**Left**
Time	7.0000	7.0000	7.0000	7.0000	7.0000	7.0000	7.0000	7.0000
Drift	1.9719	2.4352	2.4352	2.9567	2.9567	3.5388	3.5388	4.3296
Rotation	0.7482	0.9239	0.9239	1.1217	1.1217	1.3425	1.3425	1.6424
Neutral Axis	4.6774	26.7809	4.4574	25.8275	3.5733	25.2582	3.2996	21.4512
Top of Conc.	36.0000	36.0000	36.0000	36.0000	36.0000	36.0000	36.0000	36.0000
Centroid	15.0000	15.0000	15.0000	15.0000	15.0000	15.0000	15.0000	15.0000
Elongation	0.1348	0.1900	0.1700	0.2120	0.2237	0.2404	0.2742	0.1850
	0.3248		0.3820		0.4641		0.4592	
	0.13%		0.15%		0.18%		0.18%	
Total Beam							**1.6301 in.**	**0.16%**
Total							**16.8622 in.**	**1.43%**
Frame	**7.4.5**							
Location	**Column A**	**Column B**		**Column C**		**Column D**		**Column E**
	Right	**Left**	**Right**	**Left**	**Right**	**Left**	**Right**	**Left**
Time	7.0000	7.0000	7.0000	7.0000	7.0000	7.0000	7.0000	7.0000
Drift	1.9253	2.4177	2.4177	2.9958	2.9958	3.6582	3.6582	4.5024
Rotation	0.7305	0.9173	0.9173	1.1366	1.1366	1.3878	1.3878	1.7079
Neutral Axis	4.8346	26.4464	5.9833	25.3069	4.1259	24.2720	3.5917	21.7310
Top of Conc.	36.0000	36.0000	36.0000	36.0000	36.0000	36.0000	36.0000	36.0000
Centroid	15.0000	15.0000	15.0000	15.0000	15.0000	15.0000	15.0000	15.0000
Elongation	0.1296	0.1833	0.1444	0.2045	0.2157	0.2246	0.2764	0.2007
	0.3129		0.3489		0.4404		0.4771	
	0.12%		0.14%		0.17%		0.18%	
Total Beam							**1.5792 in.**	**0.15%**
Total							**17.0787 in.**	**1.44%**
Frame	**6.4.7**							
Location	**Column A**	**Column B**		**Column C**		**Column D**		**Column E**
	Right	**Left**	**Right**	**Left**	**Right**	**Left**	**Right**	**Left**
Time	7.0000	7.0000	7.0000	7.0000	7.0000	7.0000	7.0000	7.0000
Drift	2.8104	3.4313	3.4313	4.1465	4.1465	4.9176	4.9176	5.9061
Rotation	1.0663	1.3018	1.3018	1.5730	1.5730	1.8653	1.8653	2.2399
Neutral Axis	3.1690	25.7142	3.6799	24.1424	3.1012	23.4868	3.0610	19.2151
Top of Conc.	36.0000	36.0000	36.0000	36.0000	36.0000	36.0000	36.0000	36.0000
Centroid	15.0000	15.0000	15.0000	15.0000	15.0000	15.0000	15.0000	15.0000
Elongation	0.2202	0.2435	0.2572	0.2511	0.3267	0.2764	0.3888	0.1649
	0.4637		0.5083		0.6031		0.5537	
	0.18%		0.20%		0.23%		0.21%	
Total Beam							**2.1288 in.**	**0.21%**
Total							**23.3407 in.**	**1.97%**
Frame	**6.4.5**							
Location	**Column A**	**Column B**		**Column C**		**Column D**		**Column E**
	Right	**Left**	**Right**	**Left**	**Right**	**Left**	**Right**	**Left**
Time	7.0000	7.0000	7.0000	7.0000	7.0000	7.0000	7.0000	7.0000
Drift	2.7541	3.4228	3.4228	4.2059	4.2059	5.0614	5.0614	6.0929
Rotation	1.0449	1.2985	1.2985	1.5955	1.5955	1.9198	1.9198	2.3107
Neutral Axis	4.5240	25.5876	3.6834	23.4972	3.2943	22.0068	3.2743	32.9340
Top of Conc.	36.0000	36.0000	36.0000	36.0000	36.0000	36.0000	36.0000	36.0000
Centroid	15.0000	15.0000	15.0000	15.0000	15.0000	15.0000	15.0000	15.0000
Elongation	0.1911	0.2400	0.2565	0.2367	0.3260	0.2349	0.3930	0.7236
	0.4311		0.4932		0.5609		1.1167	
	0.17%		0.19%		0.22%		0.43%	
Total Beam							**2.6019 in.**	**0.25%**
Total							**24.1389 in.**	**2.04%**

Table 6: Elongation of the three bay frames

Frame	**7.3.7**					
Location	**Column A**	**Column B**		**Column C**		**Column D**
	Right	**Left**	**Right**	**Left**	**Right**	**Left**
Time	7.0000	7.0000	7.0000	7.0000	7.0000	7.0000
Drift	2.1891	2.6913	2.6913	3.2637	3.2637	4.0290
Rotation	0.8306	1.0211	1.0211	1.2382	1.2382	1.5284
Neutral Axis	3.5800	26.4647	4.5276	25.5756	3.7322	22.3009
Top of Conc.	36.0000	36.0000	36.0000	36.0000	36.0000	36.0000
Centroid	15.0000	15.0000	15.0000	15.0000	15.0000	15.0000
Elongation	0.1656	0.2043	0.1867	0.2286	0.2435	0.1948
	0.3699		0.4152		0.4383	
	0.14%		0.16%		0.17%	
Total Beam	**1.2235 in.**					**0.16%**
Total	**13.3966 in.**					**1.50%**
Frame	**7.3.5**					
Location	**Column A**	**Column B**		**Column C**		**Column D**
	Right	**Left**	**Right**	**Left**	**Right**	**Left**
Time	7.0000	7.0000	7.0000	7.0000	7.0000	7.0000
Drift	2.1573	2.6975	2.6975	3.3360	3.3360	4.1490
Rotation	0.8185	1.0234	1.0234	1.2656	1.2656	1.5739
Neutral Axis	4.7660	26.0353	5.6360	25.1522	4.3635	22.1019
Top of Conc.	36.0000	36.0000	36.0000	36.0000	36.0000	36.0000
Centroid	15.0000	15.0000	15.0000	15.0000	15.0000	15.0000
Elongation	0.1462	0.1971	0.1673	0.2243	0.2350	0.1951
	0.3433		0.3916		0.4301	
	0.13%		0.15%		0.17%	
Total Beam	**1.1650 in.**					**0.15%**
Total	**13.5049 in.**					**1.51%**
Frame	**6.3.7**					
Location	**Column A**	**Column B**		**Column C**		**Column D**
	Right	**Left**	**Right**	**Left**	**Right**	**Left**
Time	7.0000	7.0000	7.0000	7.0000	7.0000	7.0000
Drift	3.0949	3.7762	3.7762	4.5438	4.5438	5.5251
Rotation	1.1742	1.4325	1.4325	1.7236	1.7236	2.0955
Neutral Axis	3.0838	25.0179	3.4478	23.6544	3.0915	19.1760
Top of Conc.	36.0000	36.0000	36.0000	36.0000	36.0000	36.0000
Centroid	15.0000	15.0000	15.0000	15.0000	15.0000	15.0000
Elongation	0.2442	0.2505	0.2889	0.2604	0.3583	0.1528
	0.4948		0.5493		0.5111	
	0.19%		0.21%		0.20%	
Total Beam	**1.5552 in.**					**0.20%**
Total	**18.4952 in.**					**2.07%**
Frame	**6.3.5**					
Location	**Column A**	**Column B**		**Column C**		**Column D**
	Right	**Left**	**Right**	**Left**	**Right**	**Left**
Time	7.0000	7.0000	7.0000	7.0000	7.0000	7.0000
Drift	3.0417	3.7712	3.7712	4.6177	4.6177	5.6423
Rotation	1.1540	1.4307	1.4307	1.7516	1.7516	2.1399
Neutral Axis	4.2637	24.4067	3.4857	22.0350	3.0408	19.0062
Top of Conc.	36.0000	36.0000	36.0000	36.0000	36.0000	36.0000
Centroid	15.0000	15.0000	15.0000	15.0000	15.0000	15.0000
Elongation	0.2163	0.2349	0.2876	0.2151	0.3657	0.1497
	0.4512		0.5027		0.5154	
	0.17%		0.19%		0.20%	
Total Beam	**1.4693 in.**					**0.19%**
Total	**18.5423 in.**					**2.07%**

Table 7: Elongation of the two bay frames

Frame	**7.2.7**			
Location	**Column A**	**Column B**		**Column C**
	Right	**Left**	**Right**	**Left**
Time	7.0000	7.0000	7.0000	7.0000
Drift	2.4145	2.9531	2.9531	3.6836
Rotation	0.9161	1.1204	1.1204	1.3974
Neutral Axis	3.3355	26.5995	4.4100	23.5644
Top of Conc.	36.0000	36.0000	36.0000	36.0000
Centroid	15.0000	15.0000	15.0000	15.0000
Elongation	0.1865	0.2269	0.2071	0.2089
	0.4134		0.4160	
	0.16%		0.16%	
Total Beam	**0.8294 in.**			**0.16%**
Total	**9.8806 in.**			**1.63%**
Frame	**7.2.5**			
Location	**Column A**	**Column B**		**Column C**
	Right	**Left**	**Right**	**Left**
Time	7.0000	7.0000	7.0000	7.0000
Drift	2.3825	2.9738	2.9738	3.7446
Rotation	0.9039	1.1282	1.1282	1.4206
Neutral Axis	4.6900	25.4003	5.3865	23.5285
Top of Conc.	36.0000	36.0000	36.0000	36.0000
Centroid	15.0000	15.0000	15.0000	15.0000
Elongation	0.1627	0.2048	0.1893	0.2115
	0.3675		0.4008	
	0.14%		0.16%	
Total Beam	**0.7683 in.**			**0.15%**
Total	**9.8692 in.**			**1.63%**
Frame	**6.2.7**			
Location	**Column A**	**Column B**		**Column C**
	Right	**Left**	**Right**	**Left**
Time	7.0000	7.0000	7.0000	7.0000
Drift	3.3952	4.1328	4.1328	5.0969
Rotation	1.2881	1.5678	1.5678	1.9333
Neutral Axis	3.0427	24.0376	3.3749	20.3694
Top of Conc.	36.0000	36.0000	36.0000	36.0000
Centroid	15.0000	15.0000	15.0000	15.0000
Elongation	0.2689	0.2474	0.3182	0.1812
	0.5162		0.4994	
	0.20%		0.19%	
Total Beam	**1.0156 in.**			**0.20%**
Total	**13.6405 in.**			**2.25%**
Frame	**6.2.5**			
Location	**Column A**	**Column B**		**Column C**
	Right	**Left**	**Right**	**Left**
Time	7.0000	7.0000	7.0000	7.0000
Drift	3.3647	4.1602	4.1602	5.1618
Rotation	1.2765	1.5782	1.5782	1.9578
Neutral Axis	4.1077	23.4825	3.9212	19.9877
Top of Conc.	36.0000	36.0000	36.0000	36.0000
Centroid	15.0000	15.0000	15.0000	15.0000
Elongation	0.2427	0.2337	0.3052	0.1705
	0.4764		0.4757	
	0.18%		0.18%	
Total Beam	**0.9521 in.**			**0.18%**
Total	**13.6389 in.**			**2.25%**

Table 8: Elongation of the one bay frames

Frame	**7.1.7**	
Location	**Column A**	**Column B**
	Right	**Left**
Time	7.0000	7.0000
Drift	2.5001	3.1589
Rotation	0.9486	1.1984
Neutral Axis	4.0249	24.3315
Top of Conc.	36.0000	36.0000
Centroid	15.0000	15.0000
Elongation	0.1817	0.1952
	0.3769	
	0.15%	
Total Beam	**0.3769 in.**	**0.15%**
Total	**6.0359 in.**	**1.90%**
Frame	**7.1.5**	
Location	**Column A**	**Column B**
	Right	**Left**
Time	7.0000	7.0000
Drift	2.4797	3.1630
Rotation	0.9408	1.2000
Neutral Axis	4.6234	24.5643
Top of Conc.	36.0000	36.0000
Centroid	15.0000	15.0000
Elongation	0.1704	0.2003
	0.3708	
	0.14%	
Total Beam	**0.3708 in.**	**0.14%**
Total	**6.0135 in.**	**1.89%**
Frame	**6.1.7**	
Location	**Column A**	**Column B**
	Right	**Left**
Time	7.0000	7.0000
Drift	3.6061	4.5220
Rotation	1.3681	1.7153
Neutral Axis	4.2430	21.1722
Top of Conc.	36.0000	36.0000
Centroid	15.0000	15.0000
Elongation	0.2569	0.1848
	0.4417	
	0.17%	
Total Beam	**0.4417 in.**	**0.17%**
Total	**8.5698 in.**	**2.69%**
Frame	**6.1.5**	
Location	**Column A**	**Column B**
	Right	**Left**
Time	7.0000	7.0000
Drift	3.5996	4.5296
Rotation	1.3656	1.7182
Neutral Axis	4.2133	21.0661
Top of Conc.	36.0000	36.0000
Centroid	15.0000	15.0000
Elongation	0.2571	0.1820
	0.4391	
	0.17%	
Total Beam	**0.4391 in.**	**0.17%**
Total	**8.5683 in.**	**2.69%**

Table 9 compares the average elongation of the individual beams, the beams as one in a frame, as well as the frame elongation by bay, story, equation, and the story and equation combination.

Table 9: Elongation averages at seven seconds

Time @ 7.00 sec	**Individual Beam Elongation**	**Percentage**	**Total Beam Elongation**	**Percentage**	**Frame Drift and Elongation**	**Percentage**
5 Bay	0.4870 in.	0.20%	2.5565 in.	0.20%	26.5522 in.	1.81%
4 Bay	0.4962 in.	0.19%	1.9850 in.	0.19%	20.3551 in.	1.72%
3 Bay	0.4511 in.	0.17%	1.3533 in.	0.17%	15.9847 in.	1.79%
2 Bay	0.4457 in.	0.17%	0.8914 in.	0.17%	11.7573 in.	1.94%
1 Bay	0.4071 in.	0.16%	0.4071 in.	0.16%	7.2969 in.	2.29%
7 Story	0.4299 in.	0.17%	1.2897 in.	0.16%	14.2059 in.	1.68%
6 Story	0.5292 in.	0.21%	1.5876 in.	0.20%	18.5726 in.	2.24%
Equation 7	0.4682 in.	0.18%	1.4045 in.	0.18%	15.9177 in.	1.92%
Equation 5	0.4909 in.	0.19%	1.4728 in.	0.18%	16.8608 in.	1.99%
7 St. Eq. 7	0.4132 in.	0.16%	1.2396 in.	0.16%	13.3588 in.	1.61%
7 St. Eq. 5	0.4466 in.	0.17%	1.3399 in.	0.16%	15.0529 in.	1.75%
6 St. Eq. 7	0.5231 in.	0.20%	1.5694 in.	0.20%	18.4766 in.	2.24%
6 St. Eq. 5	0.5352 in.	0.21%	1.6057 in.	0.20%	18.6687 in.	2.24%
Total	0.4597 in.	0.19%	1.4387 in.	0.18%	16.3893 in.	1.91%

The results from the elongation calculations provide information that individual beams are elongating on average 0.4597 inches with a total of 1.4387 inches for the beams of the entire frame. When the column drift is included, the average elongation of the entire frame is 16.3893 inches. On average, the frame elongates 1.91% at seven seconds when comparing all of the frames. From the table, it can be concluded that the frame size has the largest effect for the elongation of a frame. The six story frame elongates almost a quarter more than the seven story, and according to the results, the less bays there are, the more the SMRF elongates. However, the results of this section are at controlled time, in order to incorporated the elongation of beams into a standard design consideration the elongation was looked at over time. The results are to follow.

5.3.2. Time Comparison

Due to the varying time durations during an extreme loading case there is no definite time to compare the beam elongations with, so instead comparing what is happening to the structure leading up to failure is more proactive. Each frame was evaluated until failure; this section will go over the time leading up to that failure for each frame. The frames were analyzed at one second, two seconds, three seconds, four seconds, five seconds, six seconds, and seven seconds; as mentioned before, in this thesis the time in seconds refers to time steps created by LS-DYNA.

To understand the elongation through the time, first the rotation at each time needs to be analyzed. As stated in section 5.2.1 the rotation and time have a linear relationship, Tables 10, 11, 12, 13, and 14 show the drift and rotation at each time.

Table 10: Rotation data for five bay frames

Frame	7.5.7												
Location	**Column A**		**Column B**		**Column C**		**Column D**		**Column E**		**Column F**		**Story-Height Ratio**
Time	**Drift**	**Rotation**	**Drift**	**Rotation**	**Drift**	**Rotation**	**Drift**	**Rotation**	**Drift**	**Rotation**	**Drift**	**Rotation**	
1.00 sec	0.1911	0.0725	0.2289	0.0868	0.2727	0.1035	0.3247	0.1232	0.3745	0.1421	0.4757	0.1805	2.4891
2.00 sec	0.4288	0.1627	0.5112	0.1940	0.6099	0.2314	0.7216	0.2738	0.8404	0.3189	1.0410	0.3950	2.4273
3.00 sec	0.6748	0.2560	0.8097	0.3072	0.9684	0.3674	1.1479	0.4355	1.3454	0.5105	1.6506	0.6263	2.4462
4.00 sec	0.9266	0.3516	1.1225	0.4259	1.3539	0.5137	1.6141	0.6124	1.9109	0.7250	2.3281	0.8833	2.5125
5.00 sec	1.1982	0.4546	1.4657	0.5561	1.7787	0.6749	2.1298	0.8081	2.5310	0.9603	3.0623	1.1618	2.5558
6.00 sec	1.4932	0.5666	1.8400	0.6981	2.2368	0.8487	2.6798	1.0167	3.1849	1.2083	3.8276	1.4521	2.5634
7.00 sec	1.8080	0.6860	2.2309	0.8464	2.7131	1.0293	3.2517	1.2336	3.8632	1.4655	4.6139	1.7502	2.5520
Max.	2.5232	0.9573	3.0945	1.1740	3.7681	1.4295	4.5260	1.7168	5.3408	2.0257	6.3202	2.3967	2.5048

Frame	7.5.5												
Location	**Column A**		**Column B**		**Column C**		**Column D**		**Column E**		**Column F**		**Story-Height Ratio**
Time	**Drift**	**Rotation**	**Drift**	**Rotation**	**Drift**	**Rotation**	**Drift**	**Rotation**	**Drift**	**Rotation**	**Drift**	**Rotation**	
1.00 sec	0.2447	0.0928	0.3009	0.1142	0.3667	0.1392	0.4383	0.1663	0.5197	0.1972	0.6357	0.2412	2.5981
2.00 sec	0.5906	0.2241	0.7241	0.2748	0.8848	0.3357	1.0640	0.4037	1.2697	0.4818	1.5514	0.5886	2.6269
3.00 sec	0.8777	0.3330	1.0815	0.4103	1.3293	0.5044	1.6062	0.6094	1.9254	0.7305	2.3466	0.8903	2.6735
4.00 sec	1.2523	0.4752	1.5621	0.5927	1.9300	0.7323	2.3392	0.8875	2.8078	1.0653	3.3981	1.2892	2.7135
5.00 sec	1.6464	0.6247	2.0638	0.7830	2.5481	0.9668	3.0861	1.1708	3.6910	1.4002	4.4322	1.6813	2.6920
6.00 sec	2.0686	0.7848	2.5857	0.9810	3.1882	1.2095	3.8487	1.4601	4.5768	1.7361	5.4589	2.0704	2.6390
7.00 sec	2.5016	0.9491	3.1145	1.1816	3.8350	1.4548	4.6078	1.7479	5.4534	2.0683	6.4697	2.4534	2.5862
Max.	2.7713	1.0514	3.4437	1.3065	4.2379	1.6076	5.0808	1.9271	5.9989	2.2750	7.1114	2.6964	2.5661

Frame	6.5.7												
Location	**Column A**		**Column B**		**Column C**		**Column D**		**Column E**		**Column F**		**Story-Height Ratio**
Time	**Drift**	**Rotation**	**Drift**	**Rotation**	**Drift**	**Rotation**	**Drift**	**Rotation**	**Drift**	**Rotation**	**Drift**	**Rotation**	
1.00 sec	0.2613	0.0991	0.3183	0.1208	0.3725	0.1413	0.4326	0.1642	0.5022	0.1906	0.6211	0.2357	2.3772
2.00 sec	0.5931	0.2251	0.7137	0.2708	0.8445	0.3204	0.9873	0.3746	1.1494	0.4361	1.3994	0.5310	2.3593
3.00 sec	0.9432	0.3579	1.1419	0.4333	1.3656	0.5182	1.6087	0.6104	1.8862	0.7157	2.2736	0.8626	2.4106
4.00 sec	1.3218	0.5016	1.6154	0.6129	1.9477	0.7390	2.3071	0.8753	2.7155	1.0302	3.2387	1.2287	2.4501
5.00 sec	1.7269	0.6552	2.1202	0.8044	2.5617	0.9719	3.0378	1.1525	3.5769	1.3570	4.2310	1.6050	2.4500
6.00 sec	2.1515	0.8163	2.6356	1.0000	3.1860	1.2087	3.7831	1.4352	4.4443	1.6859	5.2271	1.9826	2.4295
7.00 sec	2.5873	0.9816	3.1586	1.1983	3.8234	1.4504	4.5361	1.7207	5.3068	2.0128	6.2189	2.3584	2.4036
Max.	3.3526	1.2719	4.0814	1.5483	4.9477	1.8767	5.8491	2.2183	6.8117	2.5829	7.9453	3.0120	2.3699

Frame	6.5.5												
Location	**Column A**		**Column B**		**Column C**		**Column D**		**Column E**		**Column F**		**Story-Height Ratio**
Time	**Drift**	**Rotation**	**Drift**	**Rotation**	**Drift**	**Rotation**	**Drift**	**Rotation**	**Drift**	**Rotation**	**Drift**	**Rotation**	
1.00 sec	0.2447	0.0928	0.2937	0.1114	0.3667	0.1392	0.4383	0.1663	0.5197	0.1972	0.6357	0.2412	2.5981
2.00 sec	0.5906	0.2241	0.7057	0.2678	0.8848	0.3357	1.0640	0.4037	1.2697	0.4818	1.5514	0.5886	2.6269
3.00 sec	0.8777	0.3330	1.0539	0.3999	1.3293	0.5044	1.6062	0.6094	1.9254	0.7305	2.3466	0.8903	2.6735
4.00 sec	1.2523	0.4752	1.5225	0.5777	1.9300	0.7323	2.3392	0.8875	2.8078	1.0653	3.3981	1.2892	2.7135
5.00 sec	1.6464	0.6247	2.0092	0.7623	2.5481	0.9668	3.0861	1.1708	3.6910	1.4002	4.4322	1.6813	2.6920
6.00 sec	2.0686	0.7848	2.5127	0.9533	3.1882	1.2095	3.8487	1.4601	4.5768	1.7361	5.4589	2.0704	2.6390
7.00 sec	2.5016	0.9491	3.0214	1.1463	3.8350	1.4548	4.6078	1.7479	5.4534	2.0683	6.4697	2.4534	2.5862
Max.	2.7713	1.0514	3.3377	1.2663	4.2379	1.6076	5.0808	1.9271	5.9989	2.2750	7.1114	2.6964	2.5661

Table 11: Rotation data for four bay frames

Frame	7.4.7										
Location	Column A		Column B		Column C		Column D		Column E		Story-Height Ratio
Time	Drift	Rotation	Drift	Rotation	Drift	Rotation	Drift	Rotation	Drift	Rotation	
1.00 sec	0.2080	0.0789	0.2501	0.0949	0.2955	0.1121	0.3461	0.1313	0.4234	0.1607	2.0353
2.00 sec	0.4637	0.1760	0.5560	0.2110	0.6618	0.2511	0.7790	0.2956	0.9613	0.3647	2.0730
3.00 sec	0.7260	0.2755	0.8768	0.3327	1.0482	0.3977	1.2408	0.4708	1.5345	0.5822	2.1134
4.00 sec	1.0000	0.3794	1.2190	0.4625	1.4707	0.5580	1.7547	0.6658	2.1706	0.8236	2.1707
5.00 sec	1.2990	0.4929	1.5982	0.6064	1.9384	0.7355	2.3230	0.8814	2.8639	1.0865	2.2047
6.00 sec	1.6255	0.6168	2.0086	0.7621	2.4401	0.9258	2.9222	1.1087	3.5893	1.3617	2.2080
7.00 sec	1.9719	0.7482	2.4352	0.9239	2.9567	1.1217	3.5388	1.3425	4.3296	1.6424	2.1957
Max.	3.8599	1.4643	4.7230	1.7915	5.7543	2.1824	6.8398	2.5935	8.2820	3.1394	2.1456

Frame	7.4.5										
Location	Column A		Column B		Column C		Column D		Column E		Story-Height Ratio
Time	Drift	Rotation	Drift	Rotation	Drift	Rotation	Drift	Rotation	Drift	Rotation	
1.00 sec	0.1972	0.0748	0.2432	0.0923	0.2970	0.1127	0.3574	0.1356	0.4383	0.1663	2.2227
2.00 sec	0.4453	0.1690	0.5469	0.2075	0.6676	0.2533	0.8045	0.3052	0.9950	0.3775	2.2345
3.00 sec	0.6996	0.2654	0.8577	0.3254	1.0517	0.3991	1.2786	0.4851	1.5910	0.6037	2.2742
4.00 sec	0.9660	0.3665	1.1958	0.4537	1.4803	0.5617	1.8115	0.6873	2.2526	0.8547	2.3319
5.00 sec	1.2604	0.4782	1.5774	0.5985	1.9572	0.7426	2.3985	0.9100	2.9741	1.1284	2.3596
6.00 sec	1.5810	0.5999	1.9880	0.7543	2.4654	0.9354	3.0185	1.1452	3.7300	1.4150	2.3592
7.00 sec	1.9253	0.7305	2.4177	0.9173	2.9958	1.1366	3.6582	1.3878	4.5024	1.7079	2.3386
Max.	3.6141	1.3711	4.4969	1.7058	5.5356	2.0995	6.6419	2.5186	7.9935	3.0302	2.2117

Frame	6.4.7										
Location	Column A		Column B		Column C		Column D		Column E		Story-Height Ratio
Time	Drift	Rotation	Drift	Rotation	Drift	Rotation	Drift	Rotation	Drift	Rotation	
1.00 sec	0.2816	0.1069	0.3499	0.1328	0.4090	0.1552	0.4724	0.1792	0.5789	0.2197	2.0555
2.00 sec	0.6362	0.2414	0.7726	0.2932	0.9140	0.3468	1.0693	0.4057	1.3157	0.4992	2.0681
3.00 sec	1.0094	0.3830	1.2294	0.4665	1.4713	0.5583	1.7397	0.6601	2.1364	0.8106	2.1166
4.00 sec	1.4182	0.5381	1.7404	0.6604	2.0977	0.7959	2.4930	0.9459	3.0439	1.1548	2.1464
5.00 sec	1.8659	0.7080	2.2945	0.8706	2.7678	1.0501	3.2902	1.2482	3.9935	1.5149	2.1402
6.00 sec	2.3343	0.8857	2.8606	1.0853	3.4512	1.3093	4.1005	1.5555	4.9515	1.8781	2.1212
7.00 sec	2.8104	1.0663	3.4313	1.3018	4.1465	1.5730	4.9176	1.8653	5.9061	2.2399	2.1015
Max.	4.0478	1.5355	4.9300	1.8700	5.9686	2.2635	7.0268	2.6643	8.3368	3.1601	2.0596

Frame	6.4.5										
Location	Column A		Column B		Column C		Column D		Column E		Story-Height Ratio
Time	Drift	Rotation	Drift	Rotation	Drift	Rotation	Drift	Rotation	Drift	Rotation	
1.00 sec	0.2697	0.1023	0.3303	0.1253	0.4026	0.1528	0.4821	0.1829	0.5896	0.2237	2.1861
2.00 sec	0.6138	0.2329	0.7497	0.2844	0.9152	0.3473	1.1028	0.4184	1.3584	0.5154	2.2130
3.00 sec	0.9762	0.3704	1.2045	0.4570	1.4838	0.5630	1.8013	0.6835	2.2183	0.8416	2.2723
4.00 sec	1.3762	0.5222	1.7174	0.6516	2.1207	0.8046	2.5777	0.9780	3.1593	1.1986	2.2958
5.00 sec	1.8118	0.6875	2.2660	0.8598	2.7955	1.0606	3.3926	1.2871	4.1333	1.5680	2.2813
6.00 sec	2.2760	0.8635	2.8370	1.0764	3.4933	1.3253	4.2251	1.6028	5.1178	1.9412	2.2486
7.00 sec	2.7541	1.0449	3.4228	1.2985	4.2059	1.5955	5.0614	1.9198	6.0929	2.3107	2.2123
Max.	3.8149	1.4472	4.7240	1.7919	5.7922	2.1967	6.9050	2.6182	8.5595	3.2444	2.2437

Table 12: Rotation data for three bay frames

Frame	7.3.7								
Location	Column A		Column B		Column C		Column D		Story-Height Ratio
Time	Drift	Rotation	Drift	Rotation	Drift	Rotation	Drift	Rotation	
1.00 sec	0.2326	0.0883	0.2774	0.1052	0.3252	0.1234	0.4005	0.1520	1.7219
2.00 sec	0.5100	0.1935	0.6085	0.2309	0.7213	0.2737	0.8978	0.3406	1.7604
3.00 sec	0.7913	0.3003	0.9524	0.3614	1.1391	0.4322	1.4223	0.5397	1.7974
4.00 sec	1.0884	0.4130	1.3249	0.5027	1.6015	0.6077	2.0005	0.7590	1.8380
5.00 sec	1.4230	0.5399	1.7471	0.6629	2.1214	0.8049	2.6405	1.0018	1.8556
6.00 sec	1.7914	0.6797	2.2060	0.8370	2.6789	1.0164	3.3206	1.2598	1.8537
7.00 sec	2.1891	0.8306	2.6913	1.0211	3.2637	1.2382	4.0290	1.5284	1.8405
Max.	4.1659	1.5803	5.0962	1.9330	6.1859	2.3459	7.5253	2.8531	1.8064
Frame	7.3.5								
Location	Column A		Column B		Column C		Column D		Story-Height Ratio
Time	Drift	Rotation	Drift	Rotation	Drift	Rotation	Drift	Rotation	
1.00 sec	0.2217	0.0841	0.2705	0.1027	0.3290	0.1248	0.4066	0.1543	1.8338
2.00 sec	0.4951	0.1878	0.6041	0.2292	0.7373	0.2798	0.9194	0.3489	1.8571
3.00 sec	0.7720	0.2929	0.9458	0.3589	1.1639	0.4416	1.4623	0.5548	1.8940
4.00 sec	1.0661	0.4045	1.3208	0.5012	1.6376	0.6213	2.0574	0.7806	1.9298
5.00 sec	1.3991	0.5309	1.7493	0.6637	2.1701	0.8234	2.7176	1.0311	1.9424
6.00 sec	1.7654	0.6698	2.2121	0.8393	2.7409	1.0399	3.4209	1.2978	1.9378
7.00 sec	2.1573	0.8185	2.6975	1.0234	3.3360	1.2656	4.1490	1.5739	1.9233
Max.	3.6670	1.3911	4.5570	1.7286	5.5841	2.1179	7.1853	2.7243	1.9594
Frame	6.3.7								
Location	Column A		Column B		Column C		Column D		Story-Height Ratio
Time	Drift	Rotation	Drift	Rotation	Drift	Rotation	Drift	Rotation	
1.00 sec	0.2789	0.1058	0.3292	0.1249	0.3916	0.1486	0.4796	0.1820	1.7197
2.00 sec	0.6960	0.2641	0.8273	0.3139	0.9893	0.3754	1.2240	0.4644	1.7587
3.00 sec	1.1023	0.4183	1.3285	0.5041	1.6003	0.6072	1.9815	0.7518	1.7976
4.00 sec	1.5539	0.5896	1.8943	0.7187	2.2887	0.8684	2.8231	1.0711	1.8168
5.00 sec	2.0485	0.7772	2.5045	0.9502	3.0210	1.1461	3.7085	1.4069	1.8103
6.00 sec	2.5710	0.9755	3.1391	1.1909	3.7791	1.4336	4.6162	1.7510	1.7955
7.00 sec	3.0949	1.1742	3.7762	1.4325	4.5438	1.7236	5.5251	2.0955	1.7852
Max.	4.2584	1.6154	5.1907	1.9688	6.2516	2.3708	7.5237	2.8525	1.7668
Frame	6.3.5								
Location	Column A		Column B		Column C		Column D		Story-Height Ratio
Time	Drift	Rotation	Drift	Rotation	Drift	Rotation	Drift	Rotation	
1.00 sec	0.2938	0.1115	0.3586	0.1361	0.4371	0.1659	0.5411	0.2053	1.8417
2.00 sec	0.6689	0.2538	0.8167	0.3099	1.0004	0.3796	1.2477	0.4734	1.8652
3.00 sec	1.0681	0.4053	1.3200	0.5009	1.6282	0.6178	2.0305	0.7704	1.9010
4.00 sec	1.5163	0.5753	1.8903	0.7172	2.3335	0.8854	2.8958	1.0986	1.9098
5.00 sec	2.0064	0.7613	2.5012	0.9490	3.0815	1.1691	3.8030	1.4427	1.8955
6.00 sec	2.5186	0.9556	3.1299	1.1874	3.8447	1.4585	4.7200	1.7904	1.8741
7.00 sec	3.0417	1.1540	3.7712	1.4307	4.6177	1.7516	5.6423	2.1399	1.8550
Max.	3.8792	1.4716	4.7875	1.8160	5.8346	2.2128	7.7308	2.9308	1.9929

Table 13: Rotation data for two bay frames

Frame	7.2.7						
Location	Column A		Column B		Column C		Story-Height Ratio
Time	Drift	Rotation	Drift	Rotation	Drift	Rotation	
1.00 sec	0.2486	0.0943	0.3014	0.1144	0.3697	0.1403	1.4871
2.00 sec	0.5493	0.2084	0.6592	0.2501	0.8230	0.3123	1.4983
3.00 sec	0.8528	0.3236	1.0294	0.3906	1.2947	0.4913	1.5182
4.00 sec	1.1762	0.4463	1.4345	0.5443	1.8094	0.6865	1.5384
5.00 sec	1.5461	0.5867	1.8958	0.7193	2.3850	0.9049	1.5425
6.00 sec	1.9612	0.7441	2.4054	0.9126	3.0131	1.1431	1.5363
7.00 sec	2.4145	0.9161	2.9531	1.1204	3.6836	1.3974	1.5256
Max.	4.6668	1.7702	5.6934	2.1593	7.0196	2.6616	1.5042
Frame	**7.2.5**						
Location	**Column A**		**Column B**		**Column C**		**Story-Height Ratio**
Time	**Drift**	**Rotation**	**Drift**	**Rotation**	**Drift**	**Rotation**	
1.00 sec	0.2440	0.0926	0.2969	0.1126	0.3691	0.1401	1.5126
2.00 sec	0.5422	0.2057	0.6617	0.2511	0.8323	0.3158	1.5350
3.00 sec	0.8416	0.3193	1.0337	0.3922	1.3145	0.4988	1.5619
4.00 sec	1.1571	0.4390	1.4390	0.5460	1.8327	0.6954	1.5839
5.00 sec	1.5224	0.5776	1.9060	0.7232	2.4164	0.9168	1.5872
6.00 sec	1.9381	0.7354	2.4250	0.9201	3.0630	1.1621	1.5804
7.00 sec	2.3825	0.9039	2.9738	1.1282	3.7446	1.4206	1.5717
Max.	4.6367	1.7588	5.7452	2.1789	7.3752	2.7962	1.5906
Frame	**6.2.7**						
Location	**Column A**		**Column B**		**Column C**		**Story-Height Ratio**
Time	**Drift**	**Rotation**	**Drift**	**Rotation**	**Drift**	**Rotation**	
1.00 sec	0.3452	0.1310	0.4059	0.1540	0.5039	0.1912	1.4597
2.00 sec	0.7550	0.2865	0.8963	0.3401	1.1251	0.4269	1.4901
3.00 sec	1.1937	0.4529	1.4395	0.5462	1.8104	0.6869	1.5166
4.00 sec	1.6880	0.6405	2.0556	0.7799	2.5729	0.9762	1.5243
5.00 sec	2.2367	0.8486	2.7278	1.0349	3.3948	1.2879	1.5178
6.00 sec	2.8140	1.0676	3.4277	1.3004	4.2451	1.6103	1.5086
7.00 sec	3.3952	1.2881	4.1328	1.5678	5.0969	1.9333	1.5012
Max.	5.2403	1.9876	6.3700	2.4156	7.7836	2.9508	1.4853
Frame	**6.2.5**						
Location	**Column A**		**Column B**		**Column C**		**Story-Height Ratio**
Time	**Drift**	**Rotation**	**Drift**	**Rotation**	**Drift**	**Rotation**	
1.00 sec	0.3268	0.1240	0.3967	0.1505	0.4956	0.1880	1.5164
2.00 sec	0.7358	0.2792	0.8978	0.3407	1.1336	0.4301	1.5407
3.00 sec	1.1680	0.4432	1.4444	0.5480	1.8279	0.6935	1.5650
4.00 sec	1.6586	0.6293	2.0650	0.7835	2.6004	0.9866	1.5679
5.00 sec	2.2072	0.8374	2.7441	1.0411	3.4374	1.3041	1.5574
6.00 sec	2.7829	1.0558	3.4509	1.3092	4.3009	1.6315	1.5455

Table 14: Rotation data for one bay frames

Frame	7.1.7				
Location	Column A		Column B		Story-Height Ratio
Time	Drift	Rotation	Drift	Rotation	
1.00 sec	0.2584	0.0980	0.3266	0.1239	1.2641
2.00 sec	0.5679	0.2155	0.7187	0.2727	1.2656
3.00 sec	0.8740	0.3316	1.1092	0.4209	1.2690
4.00 sec	1.1999	0.4553	1.5327	0.5816	1.2773
5.00 sec	1.5678	0.5949	2.0045	0.7606	1.2786
6.00 sec	2.0033	0.7601	2.5470	0.9663	1.2714
7.00 sec	2.5001	0.9486	3.1589	1.1984	1.2635
Max.	5.5115	2.0904	6.8872	2.6115	1.2496
Frame	**7.1.5**				
Location	**Column A**		**Column B**		**Story-Height Ratio**
Time	**Drift**	**Rotation**	**Drift**	**Rotation**	
1.00 sec	0.2538	0.0963	0.3168	0.1202	1.2486
2.00 sec	0.5676	0.2154	0.7159	0.2717	1.2613
3.00 sec	0.8734	0.3314	1.1111	0.4216	1.2721
4.00 sec	1.1928	0.4526	1.5312	0.5810	1.2837
5.00 sec	1.5552	0.5901	2.0005	0.7590	1.2864
6.00 sec	1.9900	0.7551	2.5493	0.9672	1.2810
7.00 sec	2.4797	0.9408	3.1630	1.2000	1.2755
Max.	4.7426	1.7990	5.9813	2.2684	1.2612
Frame	**6.1.7**				
Location	**Column A**		**Column B**		**Story-Height Ratio**
Time	**Drift**	**Rotation**	**Drift**	**Rotation**	
1.00 sec	0.3369	0.1278	0.4403	0.1671	1.3071
2.00 sec	0.7634	0.2896	0.9795	0.3716	1.2831
3.00 sec	1.2160	0.4614	1.5578	0.5911	1.2811
4.00 sec	1.7304	0.6566	2.2094	0.8383	1.2768
5.00 sec	2.3207	0.8805	2.9411	1.1158	1.2673
6.00 sec	2.9580	1.1222	3.7247	1.4130	1.2592
7.00 sec	3.6061	1.3681	4.5220	1.7153	1.2540
Max.	6.2825	2.3825	8.0924	3.0676	1.2881
Frame	**6.1.5**				
Location	**Column A**		**Column B**		**Story-Height Ratio**
Time	**Drift**	**Rotation**	**Drift**	**Rotation**	
1.00 sec	0.3513	0.1333	0.4367	0.1657	1.2431
2.00 sec	0.7833	0.2972	0.9868	0.3744	1.2599
3.00 sec	1.2303	0.4668	1.5670	0.5946	1.2737
4.00 sec	1.7369	0.6590	2.2145	0.8402	1.2750
5.00 sec	2.3291	0.8837	2.9554	1.1213	1.2689
6.00 sec	2.9604	1.1232	3.7398	1.4187	1.2633

As mentioned before, the neutral axis is very important for the calculation of the elongation of beams. For this thesis the neutral axis distance was measured from the top face of the concrete beam, usually the neutral axis is measured from the compression face of the beam. With the deformation of the beam, at the right side of the beam the compression face is the at the top; while for the left side of the beam, compression face is at the bottom of the beam. Tables 12, 16, 17, 18, and 19 show the data for the neutral axis throughout time. It is important to point out that as the time increases so does the distance of the neutral axis from the compression face for the left sides of all of the beams.

Figure 28 shows the relationship for the neutral axis on Frame 7.5.7. Taking the neutral axis measurements from the top face of the beam, the right end of the beam has a neutral axis location of three to four inches from the compression side. For the left end of the beams, the neutral axis location ranges from 17 to 26 inches from the top of the beam face. The neutral axis location increases, from the compression face (bottom), as the beam is moving away from the location of the applied load.

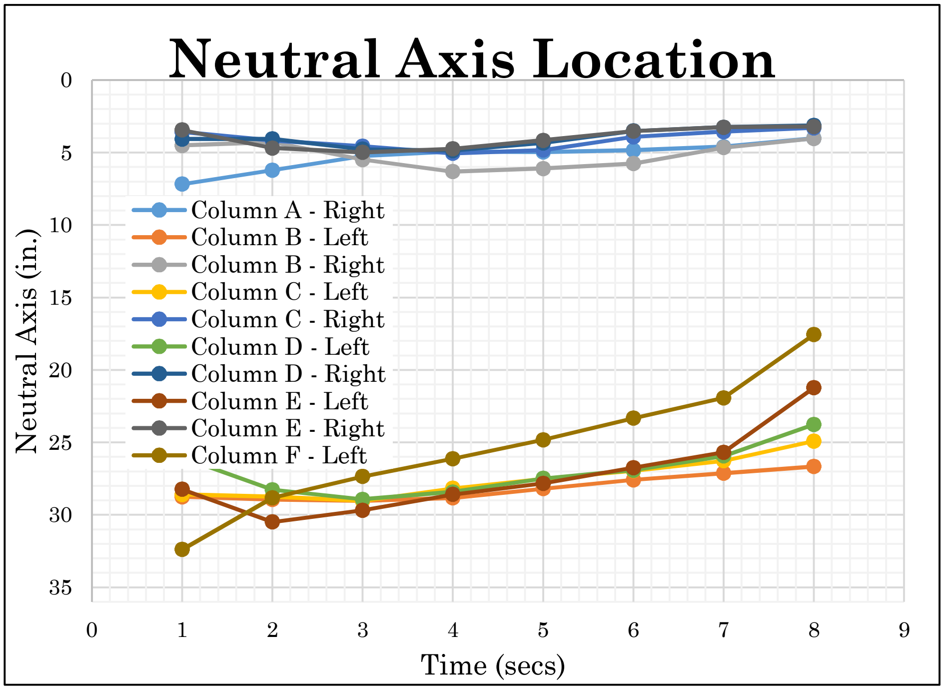


Figure 28: Neutral axis location for Frame 7.5.7

Table 15: Neutral axis data for five bay frames

Frame	7.5.7									
Location	Column A	Column B		Column C		Column D		Column E		Column F
	Right	Left	Right	Left	Right	Left	Right	Left	Right	Left
Time	Neutral Axis (in.)									
1.00 sec	7.1735	28.7510	4.5108	28.5954	3.5769	26.0233	4.0656	28.2312	3.4369	32.3793
2.00 sec	6.2078	28.9397	4.2721	28.7480	4.1630	28.2591	4.0569	30.4863	4.6834	28.8140
3.00 sec	5.2213	29.0290	5.4760	29.0278	4.5640	28.9190	4.7602	29.6711	4.9835	27.3424
4.00 sec	4.9190	28.8287	6.3061	28.1607	5.0429	28.4172	4.8621	28.5871	4.7391	26.1206
5.00 sec	4.9570	28.1789	6.0972	27.4937	4.8218	27.4760	4.3338	27.8139	4.1538	24.8182
6.00 sec	4.8331	27.5731	5.7529	26.9440	3.9101	26.9167	3.4972	26.7367	3.5155	23.3268
7.00 sec	4.5976	27.1213	4.6524	26.2572	3.5563	25.9270	3.2499	25.6786	3.2470	21.9144
Max.	4.0048	26.6639	4.0194	24.8983	3.2864	23.7584	3.1269	21.2069	3.1984	17.5472
Frame	7.5.5									
Location	Column A	Column B		Column C		Column D		Column E		Column F
	Right	Left	Right	Left	Right	Left	Right	Left	Right	Left
Time	Neutral Axis (in.)									
1.00 sec	4.8383	28.5679	4.6278	28.0802	4.9917	27.6668	4.9955	27.6731	4.1248	27.4313
2.00 sec	3.2335	29.1337	3.2241	28.6467	3.3985	28.2647	3.1210	28.2179	3.5928	26.9057
3.00 sec	3.7448	29.0081	4.8687	28.3795	4.8702	27.9006	4.9985	27.6491	4.9528	26.1936
4.00 sec	4.3830	28.5422	6.1801	27.7194	5.7536	27.1147	5.2813	26.7974	5.0125	25.1482
5.00 sec	4.7817	27.8978	6.5480	27.0943	5.6938	26.4238	4.7820	25.9124	4.3336	23.8178
6.00 sec	4.8995	27.2749	6.6280	26.4680	5.0528	25.6626	3.8312	24.8612	3.3914	22.3580
7.00 sec	4.8927	26.7116	6.3413	25.7774	3.8741	24.7809	3.0260	23.8071	3.1625	21.1375
Max.	4.8927	26.7116	6.3413	25.7774	3.8741	24.7809	3.0260	23.8071	3.1625	21.1375
Frame	6.5.7									
Location	Column A	Column B		Column C		Column D		Column E		Column F
	Right	Left	Right	Left	Right	Left	Right	Left	Right	Left
Time	Neutral Axis (in.)									
1.00 sec	4.9624	29.7050	4.3413	28.4618	3.7919	28.9351	3.8155	29.4106	3.9078	29.3873
2.00 sec	3.9930	29.5091	5.1387	29.1780	4.8884	29.5120	4.9779	30.5575	4.7753	27.0455
3.00 sec	3.5440	29.0587	5.4442	28.6036	4.9907	28.5653	4.7854	29.6235	4.5457	25.7119
4.00 sec	3.4057	28.1501	5.3846	27.6111	4.3207	27.3252	3.9525	28.5464	3.8345	24.3761
5.00 sec	3.3287	27.2064	4.7115	26.7763	3.5981	26.1397	3.3406	27.3267	3.3281	22.9042
6.00 sec	3.2190	26.5345	4.0663	25.9343	3.3162	24.8395	3.0839	25.6423	3.0533	20.9987
7.00 sec	3.0777	26.0119	3.8594	25.1537	3.1657	23.5751	3.0964	23.3637	3.1523	18.7857
Max.	3.1478	25.0128	3.5897	23.6124	3.0533	21.6736	3.2799	18.7249	3.3614	14.7197
Frame	6.5.5									
Location	Column A	Column B		Column C		Column D		Column E		Column F
	Right	Left	Right	Left	Right	Left	Right	Left	Right	Left
Time	Neutral Axis (in.)									
1.00 sec	5.0930	28.8730	3.5733	28.5529	3.9683	28.3045	4.0072	28.3683	3.7820	27.6857
2.00 sec	3.5172	29.0377	5.0860	28.3286	4.5187	28.0127	4.5386	28.1172	3.1949	26.6708
3.00 sec	4.5385	28.4745	6.2855	27.7339	5.2750	27.3338	5.0836	27.3573	6.1140	25.6212
4.00 sec	4.9655	27.6857	6.6575	26.8712	5.0929	26.2553	4.2319	26.2218	3.8451	24.2212
5.00 sec	4.9737	27.0324	6.3866	26.0848	3.9993	25.1793	3.1584	25.1785	3.1909	22.7318
6.00 sec	4.8672	26.3197	5.2119	25.2483	3.0952	23.9407	3.3757	23.8611	3.2099	20.8554
7.00 sec	4.5843	25.9907	4.1084	24.5828	3.2522	22.5980	3.6158	22.0485	3.4336	18.7914
Max.	4.3509	25.7273	3.8077	24.2056	3.3574	21.8140	3.6716	20.8302	3.4233	16.7412

Table 16: Neutral axis data for four bay frames

Frame	**7.4.7**							
Location	**Column A**	**Column B**		**Column C**		**Column D**		**Column E**
	Right	**Left**	**Right**	**Left**	**Right**	**Left**	**Right**	**Left**
Time	**Neutral Axis (in.)** (from compression side of the beam)							
1.00 sec	7.2043	28.5503	3.1570	28.3976	3.1692	28.7442	3.2239	27.4286
2.00 sec	6.2957	29.0650	4.5043	28.9921	4.6361	29.7854	4.7739	28.0435
3.00 sec	5.5171	29.1200	5.4570	28.9637	5.1671	29.3835	5.1196	26.9490
4.00 sec	5.2484	28.7604	5.8676	28.2319	5.2375	28.3330	4.9267	26.0070
5.00 sec	5.1581	28.0470	5.7265	27.3691	4.8340	27.4507	4.3659	24.7466
6.00 sec	4.9263	27.3518	5.2415	26.6035	4.1087	26.4249	3.6420	23.1701
7.00 sec	4.6774	26.7809	4.4574	25.8275	3.5733	25.2582	3.2996	21.4512
Max.	3.7372	24.1121	3.4561	22.7227	3.0207	18.3529	3.3560	13.7825
Frame	**7.4.5**							
Location	**Column A**	**Column B**		**Column C**		**Column D**		**Column E**
	Right	**Left**	**Right**	**Left**	**Right**	**Left**	**Right**	**Left**
Time	**Neutral Axis (in.)** (from compression side of the beam)							
1.00 sec	4.7679	28.5396	4.7281	28.2091	4.7861	27.9914	4.6088	27.6519
2.00 sec	3.2484	29.1432	3.0429	28.7494	3.0464	28.5159	3.5728	27.6387
3.00 sec	3.8339	28.9704	5.3337	28.3595	5.3642	27.9901	5.2157	26.8884
4.00 sec	4.5083	28.3984	6.4944	27.5610	5.9644	27.1122	5.4072	25.8560
5.00 sec	4.8140	27.7207	6.5789	26.8824	5.7082	26.2553	5.0514	24.6635
6.00 sec	4.8942	27.0696	6.4846	26.1621	5.1440	25.3272	4.3470	23.2831
7.00 sec	4.8346	26.4464	5.9833	25.3069	4.1259	24.2720	3.5917	21.7310
Max.	3.7741	24.5241	3.0675	21.9145	3.6260	18.3193	3.6457	14.9736
Frame	**6.4.7**							
Location	**Column A**	**Column B**		**Column C**		**Column D**		**Column E**
	Right	**Left**	**Right**	**Left**	**Right**	**Left**	**Right**	**Left**
Time	**Neutral Axis (in.)** (from compression side of the beam)							
1.00 sec	5.1656	28.4079	4.0483	29.4587	4.5237	30.0288	4.3442	27.7357
2.00 sec	3.9394	29.0496	4.9055	29.8494	5.1759	30.6490	4.9809	27.3862
3.00 sec	3.3176	28.8010	5.2481	28.9237	5.1529	29.4706	4.8411	26.1605
4.00 sec	3.1775	27.9214	5.1461	27.6435	4.5435	28.2446	4.0693	24.6716
5.00 sec	3.0772	26.9958	4.5352	26.3953	3.7556	26.9762	3.4539	23.1407
6.00 sec	3.0457	26.3008	3.9259	25.1658	3.3186	25.4444	3.1420	21.3545
7.00 sec	3.1690	25.7142	3.6799	24.1424	3.1012	23.4868	3.0610	19.2151
Max.	3.4977	23.9064	3.3497	21.6215	3.1880	18.0249	3.3814	13.6433
Frame	**6.4.5**							
Location	**Column A**	**Column B**		**Column C**		**Column D**		**Column E**
	Right	**Left**	**Right**	**Left**	**Right**	**Left**	**Right**	**Left**
Time	**Neutral Axis (in.)** (from compression side of the beam)							
1.00 sec	3.5698	29.0071	3.6339	28.7581	3.8863	28.7387	3.3567	32.8726
2.00 sec	3.6341	29.0998	4.7504	28.6702	4.5934	28.4596	4.5608	32.9014
3.00 sec	4.4285	28.4916	6.2325	27.8411	5.7318	27.3788	4.9801	32.9203
4.00 sec	4.9174	27.6783	6.2739	26.8959	5.3037	26.2304	4.4861	32.9294
5.00 sec	4.9592	26.8861	5.9129	25.8708	4.2181	24.9683	3.6414	32.9338
6.00 sec	4.7966	26.1292	4.6968	24.6943	3.1943	23.5874	3.0106	32.9351
7.00 sec	4.5240	25.5876	3.6834	23.4972	3.2943	22.0068	3.2743	32.9340
Max.	3.8105	24.3738	3.0347	21.4737	3.6510	18.2134	3.5683	32.9760

Table 17: Neutral axis data for three bay frames

Frame	7.3.7					
Location	Column A	Column B		Column C		Column D
	Right	Left	Right	Left	Right	Left
Time	Neutral Axis (in.)					
1.00 sec	3.6294	27.6524	4.4815	28.8162	3.5717	27.6043
2.00 sec	3.7779	28.6029	4.1009	29.9347	4.9377	27.9528
3.00 sec	3.8149	28.9787	5.6250	29.8837	5.2534	27.3338
4.00 sec	3.7424	28.8153	6.1203	28.5569	5.2339	26.3199
5.00 sec	3.5477	28.0678	5.6247	27.6554	4.8577	25.1757
6.00 sec	3.1050	27.2403	5.1881	26.7193	4.3179	23.8573
7.00 sec	3.5800	26.4647	4.5276	25.5756	3.7322	22.3009
Max.	8.7592	23.5980	3.2371	20.2414	3.1532	10.6510
Frame	**7.3.5**					
Location	**Column A**	**Column B**		**Column C**		**Column D**
	Right	**Left**	**Right**	**Left**	**Right**	**Left**
Time	**Neutral Axis (in.)**					
1.00 sec	4.4701	28.6200	4.5346	28.1757	4.6181	27.8791
2.00 sec	3.0500	29.2031	3.2361	28.8238	3.6014	27.9390
3.00 sec	4.0308	28.9184	5.6010	28.4277	5.5419	27.2096
4.00 sec	4.6286	28.2248	6.5186	27.6580	5.9153	26.2153
5.00 sec	4.8206	27.5035	6.4837	26.9605	5.6229	25.0932
6.00 sec	4.8641	26.7957	6.2398	26.1404	4.9970	23.7273
7.00 sec	4.7660	26.0353	5.6360	25.1522	4.3635	22.1019
Max.	3.8431	23.9516	3.1034	20.9804	3.3503	30.8928
Frame	**6.3.7**					
Location	**Column A**	**Column B**		**Column C**		**Column D**
	Right	**Left**	**Right**	**Left**	**Right**	**Left**
Time	**Neutral Axis (in.)**					
1.00 sec	4.8799	28.9067	3.9429	28.7627	3.9822	27.9096
2.00 sec	4.0365	29.4139	5.2503	29.9217	4.9550	27.5116
3.00 sec	3.7197	28.9297	5.5539	28.9761	5.0497	26.3370
4.00 sec	3.6553	27.8473	5.2626	27.7807	4.5547	24.7672
5.00 sec	3.5173	26.7579	4.5434	26.6137	3.8307	22.9365
6.00 sec	3.2939	25.7636	3.8132	25.2109	3.3555	21.0677
7.00 sec	3.0838	25.0179	3.4478	23.6544	3.0915	19.1760
Max.	3.2743	23.2136	3.1080	20.4037	3.2525	14.8991
Frame	**6.3.5**					
Location	**Column A**	**Column B**		**Column C**		**Column D**
	Right	**Left**	**Right**	**Left**	**Right**	**Left**
Time	**Neutral Axis (in.)**					
1.00 sec	3.6255	29.0280	3.4247	28.7606	3.3397	28.0098
2.00 sec	3.7255	29.0319	4.9571	28.6662	5.0104	27.5800
3.00 sec	4.5371	28.2867	6.2479	27.6107	5.7339	26.3313
4.00 sec	4.8830	27.3448	6.0730	26.4299	5.3156	24.8094
5.00 sec	4.8622	26.3474	5.5297	25.1074	4.5033	23.0235
6.00 sec	4.6411	25.2913	4.4898	23.5968	3.6380	21.0016
7.00 sec	4.2637	24.4067	3.4857	22.0350	3.0408	19.0062
Max.	3.6584	23.2865	3.2771	19.9934	3.3668	26.8335

Table 18: Neutral axis data for two bay frames

Frame	**7.2.7**			
Location	**Column A**	**Column B**		**Column C**
	Right	**Left**	**Right**	**Left**
Time	**Neutral Axis (in.)** (from compression side of the beam)			
1.00 sec	5.4110	28.9459	4.0045	27.9218
2.00 sec	4.5159	29.7052	5.0389	28.2686
3.00 sec	3.8808	29.6669	5.3659	27.7845
4.00 sec	3.6448	29.0344	5.5174	26.8474
5.00 sec	3.6111	28.1810	5.3058	25.8930
6.00 sec	3.5234	27.4416	4.9277	24.8359
7.00 sec	3.3355	26.5995	4.4100	23.5644
Max.	3.4575	22.3058	3.1028	15.6863
Frame	**7.2.5**			
Location	**Column A**	**Column B**		**Column C**
	Right	**Left**	**Right**	**Left**
Time	**Neutral Axis (in.)** (from compression side of the beam)			
1.00 sec	4.4113	28.7824	4.1168	28.0367
2.00 sec	3.0885	29.2785	3.7267	28.2417
3.00 sec	4.1643	28.8443	5.8858	27.6927
4.00 sec	4.6524	28.0130	6.5892	26.8256
5.00 sec	4.7969	27.1758	6.3960	25.8929
6.00 sec	4.7964	26.3417	5.9504	24.8051
7.00 sec	4.6900	25.4003	5.3865	23.5285
Max.	3.2345	20.7552	3.2564	19.2643
Frame	**6.2.7**			
Location	**Column A**	**Column B**		**Column C**
	Right	**Left**	**Right**	**Left**
Time	**Neutral Axis (in.)** (from compression side of the beam)			
1.00 sec	4.9736	29.3530	4.4331	28.2368
2.00 sec	4.0052	29.9654	5.1971	27.9925
3.00 sec	3.7261	29.2999	5.3196	26.9631
4.00 sec	3.6804	28.1262	5.0001	25.6889
5.00 sec	3.5403	26.9379	4.4935	24.1960
6.00 sec	3.2893	25.4878	3.8699	22.4040
7.00 sec	3.0427	24.0376	3.3749	20.3694
Max.	3.4377	20.5626	3.2389	14.1394
Frame	**6.2.5**			
Location	**Column A**	**Column B**		**Column C**
	Right	**Left**	**Right**	**Left**
Time	**Neutral Axis (in.)** (from compression side of the beam)			
1.00 sec	3.7289	29.2869	3.2643	28.2392
2.00 sec	3.8408	29.1077	5.1718	27.9589
3.00 sec	4.5586	28.1728	6.3028	26.8408
4.00 sec	4.7112	27.1001	6.0846	25.5354
5.00 sec	4.6548	26.0321	5.4387	23.9555

Table 19: Neutral axis data for one bay frames

Frame	**7.1.7**	
Location	**Column A**	**Column B**
	Right	**Left**
Time	**Neutral Axis (in.)**	
1.00 sec	6.6482	28.1707
2.00 sec	5.5483	28.5685
3.00 sec	4.8012	28.1566
4.00 sec	4.5594	27.3065
5.00 sec	4.4022	26.3293
6.00 sec	4.2454	25.3824
7.00 sec	4.0249	24.3315
Max.	3.1613	15.7295
Frame	**7.1.5**	
Location	**Column A**	**Column B**
	Right	**Left**
Time	**Neutral Axis (in.)**	
1.00 sec	4.2962	28.4121
2.00 sec	3.2704	28.7530
3.00 sec	4.2388	28.2874
4.00 sec	4.6103	27.4539
5.00 sec	4.7374	26.5807
6.00 sec	4.7139	25.6801
7.00 sec	4.6234	24.5643
Max.	3.9581	20.0684
Frame	**6.1.7**	
Location	**Column A**	**Column B**
	Right	**Left**
Time	**Neutral Axis (in.)**	
1.00 sec	7.6424	28.5951
2.00 sec	6.0829	28.3502
3.00 sec	5.3511	27.4195
4.00 sec	5.0430	26.1761
5.00 sec	4.7787	24.8332
6.00 sec	4.5121	23.1629
7.00 sec	4.2430	21.1722
Max.	3.2586	21.6530
Frame	**6.1.5**	
Location	**Column A**	**Column B**
	Right	**Left**
Time	**Neutral Axis (in.)**	
1.00 sec	3.5665	28.6495
2.00 sec	3.9020	28.3962
3.00 sec	4.6808	27.3156
4.00 sec	4.8711	26.1420
5.00 sec	4.7513	24.7573

With the rotation, neutral axis location, and Equation (5-8), the beam and frame elongations were found at one second, two seconds, three seconds, four seconds, five seconds, six seconds, and seven seconds. Table 20 compares the average total elongation of all the frames at the stated times.

Table 20: Average elongations at the time steps

Time	Individual Beam Elongation	Percentage	Total Beam Elongation	Percentage	Frame Drift and Elongation	Percentage
1.00 sec	0.0568 in.	0.02%	0.1778 in.	0.02%	1.6379 in.	0.19%
2.00 sec	0.1290 in.	0.05%	0.4093 in.	0.05%	3.7582 in.	0.44%
3.00 sec	0.1951 in.	0.08%	0.6105 in.	0.08%	5.9002 in.	0.68%
4.00 sec	0.2627 in.	0.11%	0.8289 in.	0.10%	8.3358 in.	0.96%
5.00 sec	0.3331 in.	0.14%	1.0523 in.	0.13%	10.9443 in.	1.27%
6.00 sec	0.4011 in.	0.16%	1.2621 in.	0.16%	13.6542 in.	1.59%
7.00 sec	0.4597 in.	0.19%	1.4387 in.	0.18%	16.3893 in.	1.91%

The results show how the elongation is changing over time, the greatest time of elongation change is from one to two seconds with an average of increased elongation by about one and half times the original elongation. As the time increases, the average elongation decreases by 20 to 30 percent as the frames reach closer to failure. This is due to the fact that the frames are reaching the plastic state in which no more deformation occurs and it becomes permanent.

5.3.3. Failure Comparison

The final time that was analyzed was the failure time of the frames. For this thesis, failure is defined as the time in which LS-DYNA has given insufficient values. The insufficient values are at time steps in which the frames are starting to collapse and when LS-DYNA deems the data invalid. All of the frames were analyzed using LS-DYNA until they fell apart. The data used for calculations was set at the time step before blow out and failure began, Table 21 shows these times. Generally, the frames with a higher number of bays fail before those with a lower number of bays, this can be correlated to the findings in Chapter 4 about the column stiffness and how the column forces are affected by the other columns.

Table 21: Maximum times for the frames

Frame	Max Time	Frame	Max Time	Frame	Max Time	Frame	Max Time
7.5.7	9.60	7.5.5	7.30	6.5.7	8.80	6.5.5	7.60
7.4.7	13.50	7.4.5	12.80	6.4.7	9.90	6.4.5	9.30
7.3.7	13.20	7.3.5	11.20	6.3.7	9.40	6.3.5	8.40
7.2.7	13.40	7.2.5	13.00	6.2.7	10.80	6.2.5	8.50
7.1.7	13.00	7.1.5	12.60	6.1.7	11.90	6.1.5	10.50

Based off the maximum times of each frame the elongations of individual beams, total beams in a frame, and of the frame were calculated using Equation (5-8), the results are shown in Tables 22 – 26. Every frame undergoes a different failure rate causing the comparison of the elongation at the failure times to apply best to design. The average of the total elongations is summarized in Table 27.

Table 22: Elongation data of failure comparison of the five bay frames

Frame	**7.5.7**									
Location	**Column A**	**Column B**		**Column C**		**Column D**		**Column E**		**Column F**
	Right	**Left**	**Right**	**Left**	**Right**	**Left**	**Right**	**Left**	**Right**	**Left**
Time	Max.	Max.	Max.	Max.	Max.	Max.	Max.	Max.	Max.	Max.
Drift (in.)	2.5232	3.0945	3.0945	3.7681	3.7681	4.5260	4.5260	5.3408	5.3408	6.3202
Rotation (°)	0.9573	1.1740	1.1740	1.4295	1.4295	1.7168	1.7168	2.0257	2.0257	2.3967
Neutral Axis	4.0048	26.6639	4.0194	24.8983	3.2864	23.7584	3.1269	21.2069	3.1984	17.5472
Top of Conc.	36.0000	36.0000	36.0000	36.0000	36.0000	36.0000	36.0000	36.0000	36.0000	36.0000
Centroid	15.0000	15.0000	15.0000	15.0000	15.0000	15.0000	15.0000	15.0000	15.0000	15.0000
Elongation	0.1837	0.2390	0.2250	0.2470	0.2923	0.2625	0.3559	0.2195	0.4174	0.1066
	0.4228		0.4720		0.5548		0.5754		0.5240	
	0.16%		0.18%		0.22%		0.22%		0.20%	
Total Beam									**2.5491 in.**	**0.20%**
Total									**28.1218 in.**	**1.91%**
Frame	**7.5.5**									
Location	**Column A**	**Column B**		**Column C**		**Column D**		**Column E**		**Column F**
	Right	**Left**	**Right**	**Left**	**Right**	**Left**	**Right**	**Left**	**Right**	**Left**
Time	Max.	Max.	Max.	Max.	Max.	Max.	Max.	Max.	Max.	Max.
Drift	2.7713	3.4437	3.4437	4.2379	4.2379	5.0808	5.0808	5.9989	5.9989	7.1114
Rotation	1.0514	1.3065	1.3065	1.6076	1.6076	1.9271	1.9271	2.2750	2.2750	2.6964
Neutral Axis	4.8927	26.7116	6.3413	25.7774	3.8741	24.7809	3.0260	23.8071	3.1625	21.1375
Top of Conc.	36.0000	36.0000	36.0000	36.0000	36.0000	36.0000	36.0000	36.0000	36.0000	36.0000
Centroid	15.0000	15.0000	15.0000	15.0000	15.0000	15.0000	15.0000	15.0000	15.0000	15.0000
Elongation	0.1855	0.2671	0.1975	0.3025	0.3123	0.3291	0.4029	0.3499	0.4703	0.2890
	0.4526		0.4999		0.6414		0.7528		0.7593	
	0.18%		0.19%		0.25%		0.29%		0.29%	
Total Beam									**3.1060 in.**	**0.24%**
Total									**31.7501 in.**	**2.16%**
Frame	**6.5.7**									
Location	**Column A**	**Column B**		**Column C**		**Column D**		**Column E**		**Column F**
	Right	**Left**	**Right**	**Left**	**Right**	**Left**	**Right**	**Left**	**Right**	**Left**
Time	Max.	Max.	Max.	Max.	Max.	Max.	Max.	Max.	Max.	Max.
Drift	3.3526	4.0814	4.0814	4.9477	4.9477	5.8491	5.8491	6.8117	6.8117	7.9453
Rotation	1.2719	1.5483	1.5483	1.8767	1.8767	2.2183	2.2183	2.5829	2.5829	3.0120
Neutral Axis	3.1478	25.0128	3.5897	23.6124	3.0533	21.6736	3.2799	18.7249	3.3614	14.7197
Top of Conc.	36.0000	36.0000	36.0000	36.0000	36.0000	36.0000	36.0000	36.0000	36.0000	36.0000
Centroid	15.0000	15.0000	15.0000	15.0000	15.0000	15.0000	15.0000	15.0000	15.0000	15.0000
Elongation	0.2631	0.2706	0.3084	0.2822	0.3914	0.2585	0.4540	0.1680	0.5250	0.0147
	0.5338		0.5906		0.6500		0.6220		0.5398	
	0.21%		0.23%		0.25%		0.24%		0.21%	
Total Beam									**2.9361 in.**	**0.23%**
Total									**35.9239 in.**	**2.44%**
Frame	**6.5.5**									
Location	**Column A**	**Column B**		**Column C**		**Column D**		**Column E**		**Column F**
	Right	**Left**	**Right**	**Left**	**Right**	**Left**	**Right**	**Left**	**Right**	**Left**
Time	Max.	Max.	Max.	Max.	Max.	Max.	Max.	Max.	Max.	Max.
Drift	2.7713	3.3377	3.3377	4.2379	4.2379	5.0808	5.0808	5.9989	5.9989	7.1114
Rotation	1.0514	1.2663	1.2663	1.6076	1.6076	1.9271	1.9271	2.2750	2.2750	2.6964
Neutral Axis	4.3509	25.7273	3.8077	24.2056	3.3574	21.8140	3.6716	20.8302	3.4233	16.7412
Top of Conc.	36.0000	36.0000	36.0000	36.0000	36.0000	36.0000	36.0000	36.0000	36.0000	36.0000
Centroid	15.0000	15.0000	15.0000	15.0000	15.0000	15.0000	15.0000	15.0000	15.0000	15.0000
Elongation	0.1954	0.2371	0.2474	0.2584	0.3268	0.2293	0.3812	0.2316	0.4599	0.0820
	0.4326		0.5058		0.5560		0.6128		0.5419	
	0.17%		0.20%		0.22%		0.24%		0.21%	
Total Beam									**2.6491 in.**	**0.21%**
Total									**31.1871 in.**	**2.12%**

Table 23: Elongation data of failure comparison of the four bay frames

Frame	**7.4.7**							
Location	**Column A**	**Column B**		**Column C**		**Column D**		**Column E**
	Right	**Left**	**Right**	**Left**	**Right**	**Left**	**Right**	**Left**
Time	Max.	Max.	Max.	Max.	Max.	Max.	Max.	Max.
Drift	3.8599	4.7230	4.7230	5.7543	5.7543	6.8398	6.8398	8.2820
Rotation	1.4643	1.7915	1.7915	2.1824	2.1824	2.5935	2.5935	3.1394
Neutral Axis	3.7372	24.1121	3.4561	22.7227	3.0207	18.3529	3.3560	13.7825
Top of Conc.	36.0000	36.0000	36.0000	36.0000	36.0000	36.0000	36.0000	36.0000
Centroid	15.0000	15.0000	15.0000	15.0000	15.0000	15.0000	15.0000	15.0000
Elongation	0.2879	0.2850	0.3611	0.2943	0.4565	0.1519	0.5274	0.0668
	0.5729		0.6554		0.6084		0.5942	
	0.22%		0.25%		0.24%		0.23%	
Total Beam	**2.4309 in.**							**0.24%**
Total	**31.8900 in.**							**2.70%**
Frame	**7.4.5**							
Location	**Column A**	**Column B**		**Column C**		**Column D**		**Column E**
	Right	**Left**	**Right**	**Left**	**Right**	**Left**	**Right**	**Left**
Time	Max.	Max.	Max.	Max.	Max.	Max.	Max.	Max.
Drift	3.6141	4.4969	4.4969	5.5356	5.5356	6.6419	6.6419	7.9935
Rotation	1.3711	1.7058	1.7058	2.0995	2.0995	2.5186	2.5186	3.0302
Neutral Axis	3.7741	24.5241	3.0675	21.9145	3.6260	18.3193	3.6457	14.9736
Top of Conc.	36.0000	36.0000	36.0000	36.0000	36.0000	36.0000	36.0000	36.0000
Centroid	15.0000	15.0000	15.0000	15.0000	15.0000	15.0000	15.0000	15.0000
Elongation	0.2687	0.2836	0.3554	0.2535	0.4170	0.1460	0.4994	0.0014
	0.5523		0.6088		0.5630		0.5008	
	0.21%		0.24%		0.22%		0.19%	
Total Beam	**2.2250 in.**							**0.22%**
Total	**30.5070 in.**							**2.58%**
Frame	**6.4.7**							
Location	**Column A**	**Column B**		**Column C**		**Column D**		**Column E**
	Right	**Left**	**Right**	**Left**	**Right**	**Left**	**Right**	**Left**
Time	Max.	Max.	Max.	Max.	Max.	Max.	Max.	Max.
Drift	4.0478	4.9300	4.9300	5.9686	5.9686	7.0268	7.0268	8.3368
Rotation	1.5355	1.8700	1.8700	2.2635	2.2635	2.6643	2.6643	3.1601
Neutral Axis	3.4977	23.9064	3.3497	21.6215	3.1880	18.0249	3.3814	13.6433
Top of Conc.	36.0000	36.0000	36.0000	36.0000	36.0000	36.0000	36.0000	36.0000
Centroid	15.0000	15.0000	15.0000	15.0000	15.0000	15.0000	15.0000	15.0000
Elongation	0.3083	0.2908	0.3804	0.2617	0.4669	0.1408	0.5407	0.0749
	0.5991		0.6421		0.6077		0.6156	
	0.23%		0.25%		0.24%		0.24%	
Total Beam	**2.4644 in.**							**0.24%**
Total	**32.7744 in.**							**2.77%**
Frame	**6.4.5**							
Location	**Column A**	**Column B**		**Column C**		**Column D**		**Column E**
	Right	**Left**	**Right**	**Left**	**Right**	**Left**	**Right**	**Left**
Time	Max.	Max.	Max.	Max.	Max.	Max.	Max.	Max.
Drift	3.8149	4.7240	4.7240	5.7922	5.7922	6.9050	6.9050	8.5595
Rotation	1.4472	1.7919	1.7919	2.1967	2.1967	2.6182	2.6182	3.2444
Neutral Axis	3.8105	24.3738	3.0347	21.4737	3.6510	18.2134	3.5683	32.9760
Top of Conc.	36.0000	36.0000	36.0000	36.0000	36.0000	36.0000	36.0000	36.0000
Centroid	15.0000	15.0000	15.0000	15.0000	15.0000	15.0000	15.0000	15.0000
Elongation	0.2827	0.2933	0.3743	0.2483	0.4353	0.1469	0.5228	1.0190
	0.5760		0.6227		0.5823		1.5417	
	0.22%		0.24%		0.23%		0.60%	
Total Beam	**3.3226 in.**							**0.32%**
Total	**33.1182 in.**							**2.80%**

Table 24: Elongation data of failure comparison of the three bay frames

Frame	7.3.7					
Location	Column A	Column B		Column C		Column D
	Right	Left	Right	Left	Right	Left
Time	Max.	Max.	Max.	Max.	Max.	Max.
Drift	4.1659	5.0962	5.0962	6.1859	6.1859	7.5253
Rotation	1.5803	1.9330	1.9330	2.3459	2.3459	2.8531
Neutral Axis	8.7592	23.5980	3.2371	20.2414	3.1532	10.6510
Top of Conc.	36.0000	36.0000	36.0000	36.0000	36.0000	36.0000
Centroid	15.0000	15.0000	15.0000	15.0000	15.0000	15.0000
Elongation	0.1722	0.2902	0.3970	0.2147	0.4853	0.2167
	0.4624		0.6117		0.7021	
	0.18%		0.24%		0.27%	
Total Beam					1.7761 in.	0.23%
Total					24.7495 in.	2.77%
Frame	7.3.5					
Location	Column A	Column B		Column C		Column D
	Right	Left	Right	Left	Right	Left
Time	Max.	Max.	Max.	Max.	Max.	Max.
Drift	3.6670	4.5570	4.5570	5.5841	5.5841	7.1853
Rotation	1.3911	1.7286	1.7286	2.1179	2.1179	2.7243
Neutral Axis	3.8431	23.9516	3.1034	20.9804	3.3503	30.8928
Top of Conc.	36.0000	36.0000	36.0000	36.0000	36.0000	36.0000
Centroid	15.0000	15.0000	15.0000	15.0000	15.0000	15.0000
Elongation	0.2709	0.2702	0.3590	0.2212	0.4308	0.7562
	0.5411		0.5802		1.1871	
	0.21%		0.22%		0.46%	
Total Beam					2.3083 in.	0.30%
Total					23.3017 in.	2.61%
Frame	6.3.7					
Location	Column A	Column B		Column C		Column D
	Right	Left	Right	Left	Right	Left
Time	Max.	Max.	Max.	Max.	Max.	Max.
Drift	4.2584	5.1907	5.1907	6.2516	6.2516	7.5237
Rotation	1.6154	1.9688	1.9688	2.3708	2.3708	2.8525
Neutral Axis	3.2743	23.2136	3.1080	20.4037	3.2525	14.8991
Top of Conc.	36.0000	36.0000	36.0000	36.0000	36.0000	36.0000
Centroid	15.0000	15.0000	15.0000	15.0000	15.0000	15.0000
Elongation	0.3307	0.2823	0.4088	0.2237	0.4864	0.0050
	0.6130		0.6325		0.4914	
	0.24%		0.25%		0.19%	
Total Beam					1.7369 in.	0.22%
Total					24.9614 in.	2.79%
Frame	6.3.5					
Location	Column A	Column B		Column C		Column D
	Right	Left	Right	Left	Right	Left
Time	Max.	Max.	Max.	Max.	Max.	Max.
Drift	3.8792	4.7875	4.7875	5.8346	5.8346	7.7308
Rotation	1.4716	1.8160	1.8160	2.2128	2.2128	2.9308
Neutral Axis	3.6584	23.2865	3.2771	19.9934	3.3668	26.8335
Top of Conc.	36.0000	36.0000	36.0000	36.0000	36.0000	36.0000
Centroid	15.0000	15.0000	15.0000	15.0000	15.0000	15.0000
Elongation	0.2914	0.2627	0.3717	0.1929	0.4495	0.6058
	0.5541		0.5646		1.0554	
	0.21%		0.22%		0.41%	
Total Beam					2.1741 in.	0.28%
Total					24.4063 in.	2.73%

Table 25: Elongation data of failure comparison of the two bay frames

Frame	**7.2.7**			
Location	**Column A**	**Column B**		**Column C**
	Right	**Left**	**Right**	**Left**
Time	Max.	Max.	Max.	Max.
Drift	4.6668	5.6934	5.6934	7.0196
Rotation	1.7702	2.1593	2.1593	2.6616
Neutral Axis	3.4575	22.3058	3.1028	15.6863
Top of Conc.	36.0000	36.0000	36.0000	36.0000
Centroid	15.0000	15.0000	15.0000	15.0000
Elongation	0.3567	0.2755	0.4486	0.0319
	0.6322		0.4805	
	0.25%		0.19%	
Total Beam	**1.1127 in.**			**0.22%**
Total	**18.4924 in.**			**3.05%**
Frame	**7.2.5**			
Location	**Column A**	**Column B**		**Column C**
	Right	**Left**	**Right**	**Left**
Time	Max.	Max.	Max.	Max.
Drift	4.6367	5.7452	5.7452	7.3752
Rotation	1.7588	2.1789	2.1789	2.7962
Neutral Axis	3.2345	20.7552	3.2564	19.2643
Top of Conc.	36.0000	36.0000	36.0000	36.0000
Centroid	15.0000	15.0000	15.0000	15.0000
Elongation	0.3613	0.2190	0.4468	0.2083
	0.5802		0.6551	
	0.22%		0.25%	
Total Beam	**1.2353 in.**			**0.24%**
Total	**18.9923 in.**			**3.13%**
Frame	**6.2.7**			
Location	**Column A**	**Column B**		**Column C**
	Right	**Left**	**Right**	**Left**
Time	Max.	Max.	Max.	Max.
Drift	5.2403	6.3700	6.3700	7.7836
Rotation	1.9876	2.4156	2.4156	2.9508
Neutral Axis	3.4377	20.5626	3.2389	14.1394
Top of Conc.	36.0000	36.0000	36.0000	36.0000
Centroid	15.0000	15.0000	15.0000	15.0000
Elongation	0.4013	0.2347	0.4961	0.0444
	0.6359		0.5405	
	0.25%		0.21%	
Total Beam	**1.1764 in.**			**0.23%**
Total	**20.5703 in.**			**3.39%**
Frame	**6.2.5**			
Location	**Column A**	**Column B**		**Column C**
	Right	**Left**	**Right**	**Left**
Time	Max.	Max.	Max.	Max.
Drift	4.4842	5.5200	5.5200	6.7003
Rotation	1.7010	2.0936	2.0936	2.5407
Neutral Axis	3.2814	21.3864	3.1296	32.8542
Top of Conc.	36.0000	36.0000	36.0000	36.0000
Centroid	15.0000	15.0000	15.0000	15.0000
Elongation	0.3480	0.2335	0.4339	0.7922
	0.5815		1.2262	
	0.23%		0.48%	
Total Beam	**1.8077 in.**			**0.35%**
Total	**18.5122 in.**			**3.05%**

Table 26: Elongation data of failure comparison of the one bay frames

Frame	**7.1.7**	
Location	**Column A**	**Column B**
	Right	**Left**
Time	Max.	Max.
Drift	5.5115	6.8872
Rotation	2.0904	2.6115
Neutral Axis	3.1613	15.7295
Top of Conc.	36.0000	36.0000
Centroid	15.0000	15.0000
Elongation	0.4321	0.0333
	0.4654	
	0.18%	
Total Beam	**0.4654 in.**	**0.18%**
Total	**12.8641 in.**	**4.05%**
Frame	**7.1.5**	
Location	**Column A**	**Column B**
	Right	**Left**
Time	Max.	Max.
Drift	4.7426	5.9813
Rotation	1.7990	2.2684
Neutral Axis	3.9581	20.0684
Top of Conc.	36.0000	36.0000
Centroid	15.0000	15.0000
Elongation	0.3468	0.2008
	0.5476	
	0.21%	
Total Beam	**0.5476 in.**	**0.21%**
Total	**11.2715 in.**	**3.54%**
Frame	**6.1.7**	
Location	**Column A**	**Column B**
	Right	**Left**
Time	Max.	Max.
Drift	6.2825	8.0924
Rotation	2.3825	3.0676
Neutral Axis	3.2586	21.6530
Top of Conc.	36.0000	36.0000
Centroid	15.0000	15.0000
Elongation	0.4885	0.3565
	0.8451	
	0.33%	
Total Beam	**0.8451 in.**	**0.33%**
Total	**15.2199 in.**	**4.79%**
Frame	**6.1.5**	
Location	**Column A**	**Column B**
	Right	**Left**
Time	Max.	Max.
Drift	5.5192	6.8927
Rotation	2.0933	2.6136
Neutral Axis	3.1246	15.5006
Top of Conc.	36.0000	36.0000
Centroid	15.0000	15.0000
Elongation	0.4341	0.0228
	0.4569	
	0.18%	
Total Beam	**0.4569 in.**	**0.18%**
Total	**12.8689 in.**	**4.05%**

Table 27: Elongation averages at maximum time

Time @ Max.	Individual Beam Elongation	Percentage	Total Beam Elongation	Percentage	Frame Drift and Elongation	Percentage
5 Bay	0.5353 in.	0.22%	2.8101 in.	0.22%	31.7457 in.	2.16%
4 Bay	0.6527 in.	0.25%	2.6107 in.	0.25%	32.0724 in.	2.71%
3 Bay	0.6663 in.	0.26%	1.9989 in.	0.26%	24.3547 in.	2.72%
2 Bay	0.6665 in.	0.26%	1.3330 in.	0.26%	19.1418 in.	3.16%
1 Bay	0.5787 in.	0.22%	0.5787 in.	0.24%	13.0561 in.	4.11%
7 Story	0.5919 in.	0.23%	1.7756 in.	0.23%	23.1940 in.	2.90%
6 Story	0.6523 in.	0.25%	1.9569 in.	0.26%	24.9543 in.	3.17%
Equation 7	0.5831 in.	0.23%	1.7493 in.	0.23%	24.5568 in.	3.15%
Equation 5	0.6611 in.	0.26%	1.9833 in.	0.25%	23.5915 in.	2.92%
7 St. Eq. 7	0.5556 in.	0.22%	1.6668 in.	0.21%	23.2236 in.	2.94%
7 St. Eq. 5	0.6281 in.	0.24%	1.8844 in.	0.24%	23.1645 in.	2.86%
6 St. Eq. 7	0.6106 in.	0.24%	1.8318 in.	0.25%	25.8900 in.	3.35%
6 St. Eq. 5	0.6940 in.	0.27%	2.0821 in.	0.27%	24.0185 in.	2.99%
Total	0.6363 in.	0.24%	1.8663 in.	0.24%	24.0742 in.	2.97%

Looking at the averages of the results, as seen in Table 27, at the maximum time as well as with comparing multiple factors, the individual beams elongate no more than 0.7 inches and no less than 0.5 inches, with an average of 0.6363 inches. The percentage of elongation for the individual beams is around 0.24%, once again while comparing multiple factors such as time, bay size, number of stories, and equations. With these results, it can be concluded that no matter what the time step is, once the beams reach that plastic yield and achieve permanent deformation, they will have elongated about 0.24%.

While looking at the total beam elongation, the percentage of elongation averages are the same as for the individual beams, however size seems to affect how much elongation will really occur. When comparing the number of bays in the frame, the results range from an average elongation of 0.5787 inches for the frames with one bay, to an average elongation of 2.8101 inches for the frames with five bays. Due to the fact that there are more beams to consider in the five bay frames than the one bay frames, the results make sense; and when looking the percentage of the total beam elongation of the one bay frames to the five bay frames it is comparable.

When comparing all of the factors of frame elongation, the frames elongated on average 24.0742 inches, which is 2.97% from the original size. When comparing the bay sizes, the percentage of elongation increases as the number of bays decreases, as discussed before this most likely has to do with the column stiffness and the column forces affected by the other columns stated in Chapter 4. Comparing the elongation averages at the controlled times and at the maximum times, the beam elongations are affected the most by the size of the frames. The smaller the frames, the larger the elongation percentage; this might have to do with the weight of the frame, the less weight there is to resist the load, the more the load can affect the frame and cause a greater elongation. The results from the comparison of the factors at the maximum time of each frame as discussed can be applied to standard design practices to consider the elongation of the beams and frames.

Chapter 6 - Conclusion

The focus of this thesis was the study of beam elongation in SMFRs and how to incorporate these findings into standard design practice. By evaluating columns, beams, and joints in 20 different configurations of SMRFs based on the Design Example 7 of the *2006 IBC Structural/Seismic Design Manual,* an equation was developed to predict beam elongation. Equations (5-8) incorporates the rotation of the column, along with the centroid and the changing location of the neutral axis of the beam to calculate the elongation occurring where the beam and the column meet.

$$x = e_{cri} \tan\theta \tag{5-8}$$

For standard design consideration of this equation, two factors must be addressed: rotation and the location of the neutral axis. The rotation of the columns can be found by using the process of finding the story drift in accordance to Section 12.8.6 of the ASCE 7-10. The other factor needed in order to predict the elongation is the difference of the centroid and neutral axis location at the face of the rotated column. The centroid is found by standard equations while the location of the neutral axis is more difficult to determine. Through the results of this research the neutral axis location was compared to the time steps in order to evaluate the relationship. However, to incorporate the elongation of beams into design practice, a way to predict the neutral axis location at the necessary location and time is needed. Once the two factors are determined at the same time step, Equation (5-8) can be applied to find the elongation of the beam at one end of a joint.

Previous research indicated that 2D-FEA and 3D-FEA have different results. As mentioned in Phillippi & Liuzza's (2017) work, the 3D-FEA has a shear force distribution for the last column about twice as large as that of the 2D-FEA. Looking at the moment, the results of the 3D-FEA at the end column is about 1.6 times as large as well. Through this thesis, it can be concluded that the reason behind this is because the 2D-FEA does not include the changing section properties into the analysis. These changing properties, specifically the changing of the neutral axis location, are factors that are altering the results between the 2D-FEA and the 3D-FEA dramatically. In order to incorporate these findings into 2D-FEA programs, Equation (5-8) can be added to each end of the beams to account for the elongation, and the prediction of the location of the neutral axis should be included.

This thesis also provided an average percent of elongation for the SMRFs. By using the beam elongation and the drifts of the columns, the average elongation of the frames were found and compared by many different factors and control points. The results go into multiple comparisons of the average frame elongation but using the results from the failure comparison can be applied to the standard design practice the most. On average, the SMRFs were elongating by about three percent at the time step right before failure. Knowing this happens, provisions can be made to the detailing requirements, such as adding more reinforcement, to prevent such from happening.

With the results of this thesis, SMRFs were proven not to perform as the way current analysis and design predict. With the columns experiencing different loads than expected, the columns' reactions to the loading differ from what is thought. This, with the combination of beam elongation, need to be considered into the analysis and design.

6.1. Recommendations

Future research is needed in order to fully incorporate the beam elongation into codes and design practices. Additional frames could be studied to compare the average elongation in order to create detailing to accommodate the elongation. Physical research should also be performed to conform the results provided in this these. Most importantly, a way to predict the location of the neutral axis at any given location along the beam and time needs to be determined. With the results from this thesis and further research, a way to design for the prevention of elongation can be discovered and make SMRFs an even better lateral system than they already are.

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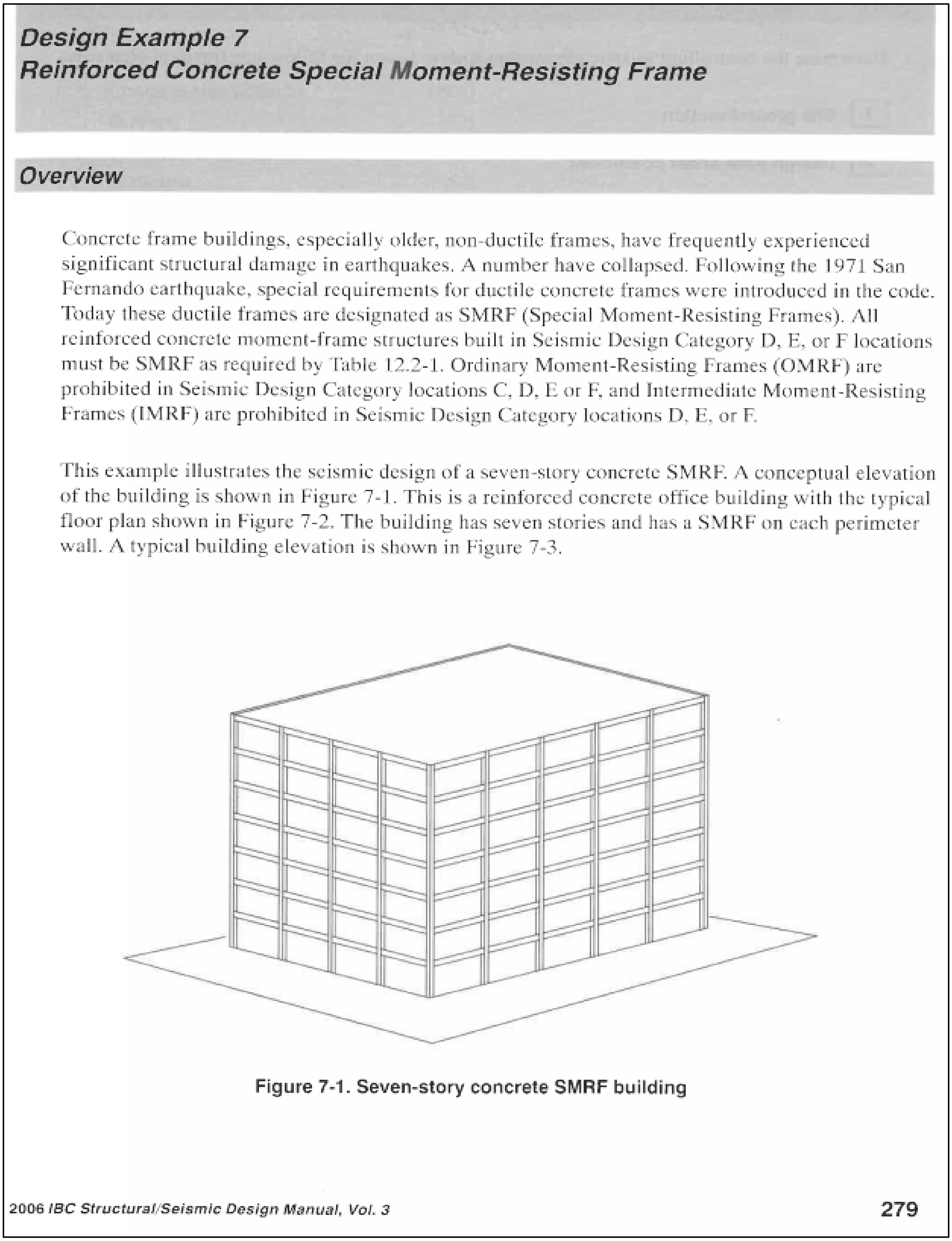
Appendix A - 2006 IBC Structural/Seismic Design Manual



Design Example 7
Reinforced Concrete Special Moment-Resisting Frame

Overview

Concrete frame buildings, especially older, non-ductile frames, have frequently experienced significant structural damage in earthquakes. A number have collapsed. Following the 1971 San Fernando earthquake, special requirements for ductile concrete frames were introduced in the code. Today these ductile frames are designated as SMRF (Special Moment-Resisting Frames). All reinforced concrete moment-frame structures built in Seismic Design Category D, E, or F locations must be SMRF as required by Table 12.2-1. Ordinary Moment-Resisting Frames (OMRF) are prohibited in Seismic Design Category locations C, D, E or F, and Intermediate Moment-Resisting Frames (IMRF) are prohibited in Seismic Design Category locations D, E, or F.

This example illustrates the seismic design of a seven-story concrete SMRF. A conceptual elevation of the building is shown in Figure 7-1. This is a reinforced concrete office building with the typical floor plan shown in Figure 7-2. The building has seven stories and has a SMRF on each perimeter wall. A typical building elevation is shown in Figure 7-3.

Figure 7-1. Seven-story concrete SMRF building

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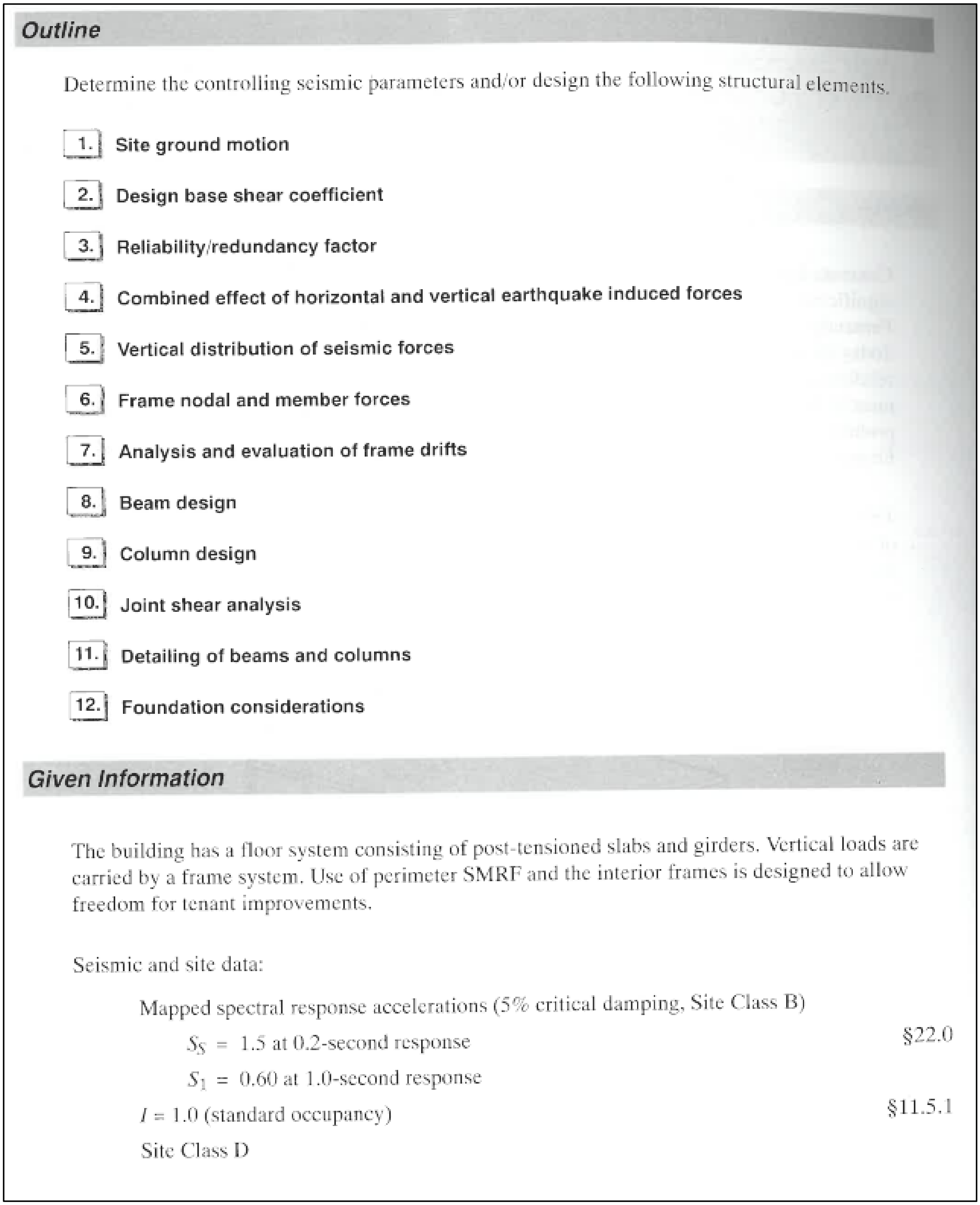
Figure 29: Design Example 7 from *2006 IBC Structural/Seismic Design Manual*



Outline

Determine the controlling seismic parameters and/or design the following structural elements.

1. Site ground motion
2. Design base shear coefficient
3. Reliability/redundancy factor
4. Combined effect of horizontal and vertical earthquake induced forces
5. Vertical distribution of seismic forces
6. Frame nodal and member forces
7. Analysis and evaluation of frame drifts
8. Beam design
9. Column design
10. Joint shear analysis
11. Detailing of beams and columns
12. Foundation considerations

Given Information

The building has a floor system consisting of post-tensioned slabs and girders. Vertical loads are carried by a frame system. Use of perimeter SMRF and the interior frames is designed to allow freedom for tenant improvements.

Seismic and site data:

Mapped spectral response accelerations (5% critical damping, Site Class B)

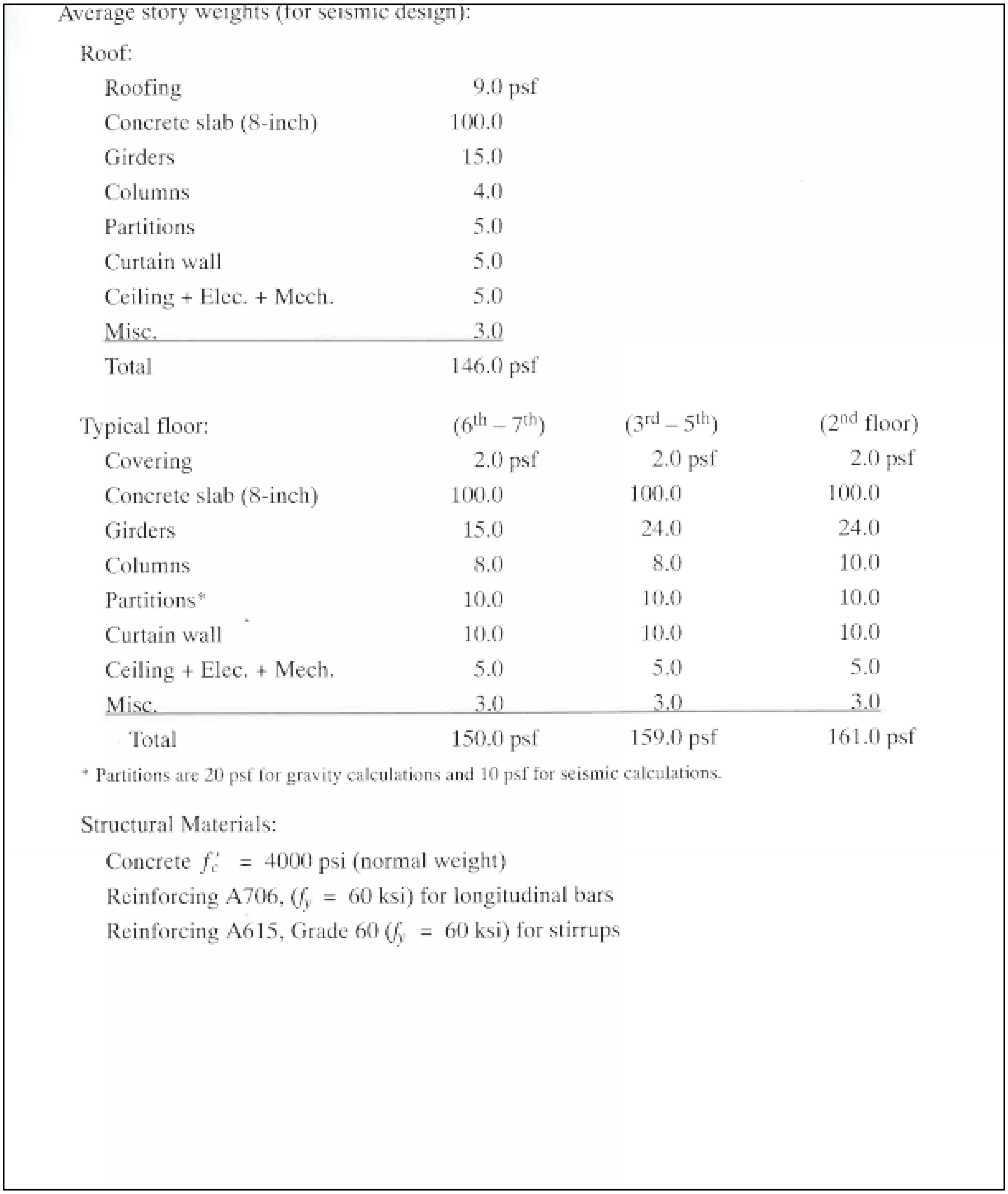
S_S = 1.5 at 0.2-second response §22.0

S_1 = 0.60 at 1.0-second response

I = 1.0 (standard occupancy) §11.5.1

Site Class D

Figure 30: Design Example 7 from *2006 IBC Structural/Seismic Design Manual*



Average story weights (for seismic design):

Roof:

Roofing	9.0 psf
Concrete slab (8-inch)	100.0
Girders	15.0
Columns	4.0
Partitions	5.0
Curtain wall	5.0
Ceiling + Elec. + Mech.	5.0
Misc.	3.0
Total	146.0 psf

Typical floor:	($6^{th} – 7^{th}$)	($3^{rd} – 5^{th}$)	(2^{nd} floor)
Covering	2.0 psf	2.0 psf	2.0 psf
Concrete slab (8-inch)	100.0	100.0	100.0
Girders	15.0	24.0	24.0
Columns	8.0	8.0	10.0
Partitions*	10.0	10.0	10.0
Curtain wall	10.0	10.0	10.0
Ceiling + Elec. + Mech.	5.0	5.0	5.0
Misc.	3.0	3.0	3.0
Total	150.0 psf	159.0 psf	161.0 psf

* Partitions are 20 psf for gravity calculations and 10 psf for seismic calculations.

Structural Materials:

Concrete f'_c = 4000 psi (normal weight)
Reinforcing A706, (f_y = 60 ksi) for longitudinal bars
Reinforcing A615, Grade 60 (f_y = 60 ksi) for stirrups

Figure 31: Design Example 7 from *2006 IBC Structural/Seismic Design Manual*

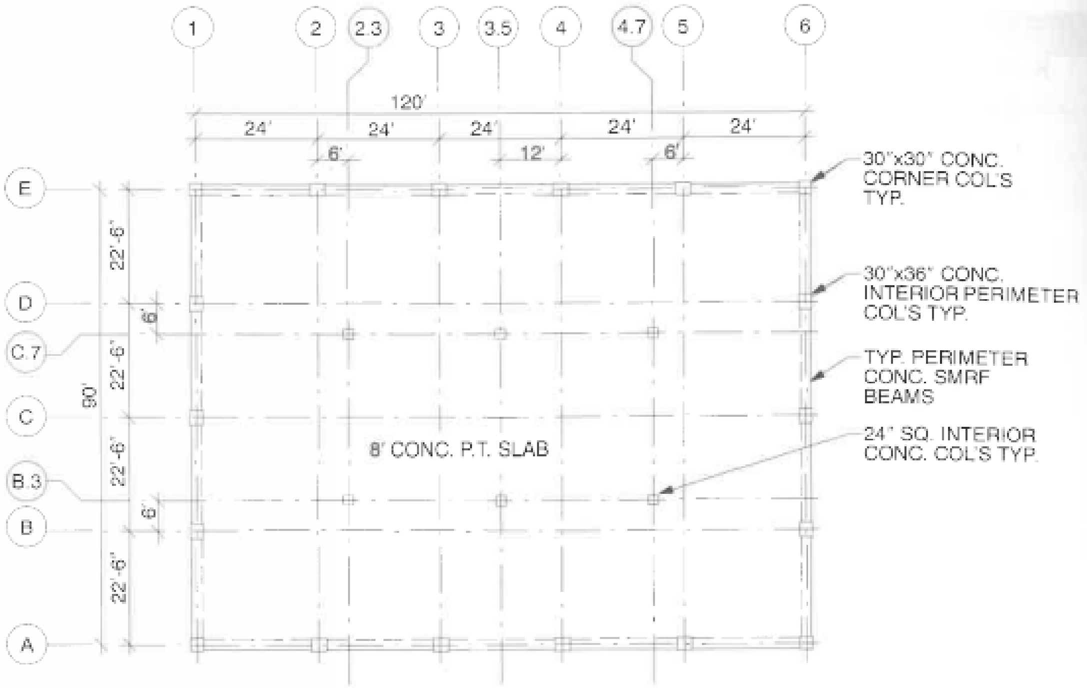


Figure 7-2. Typical floor plan

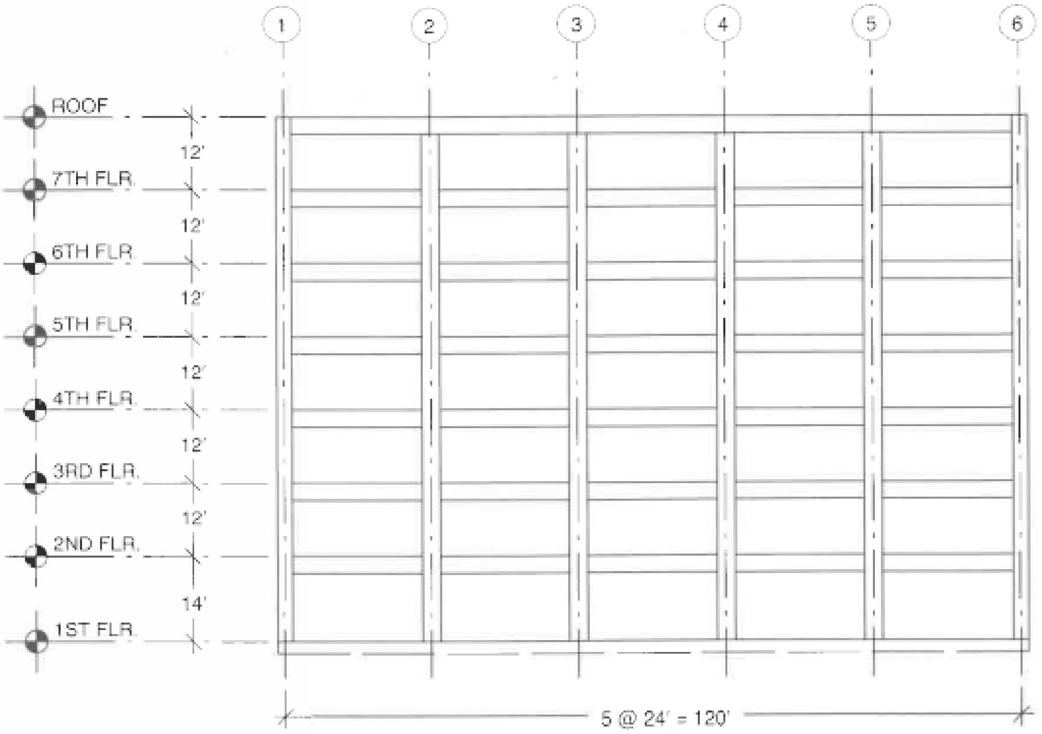
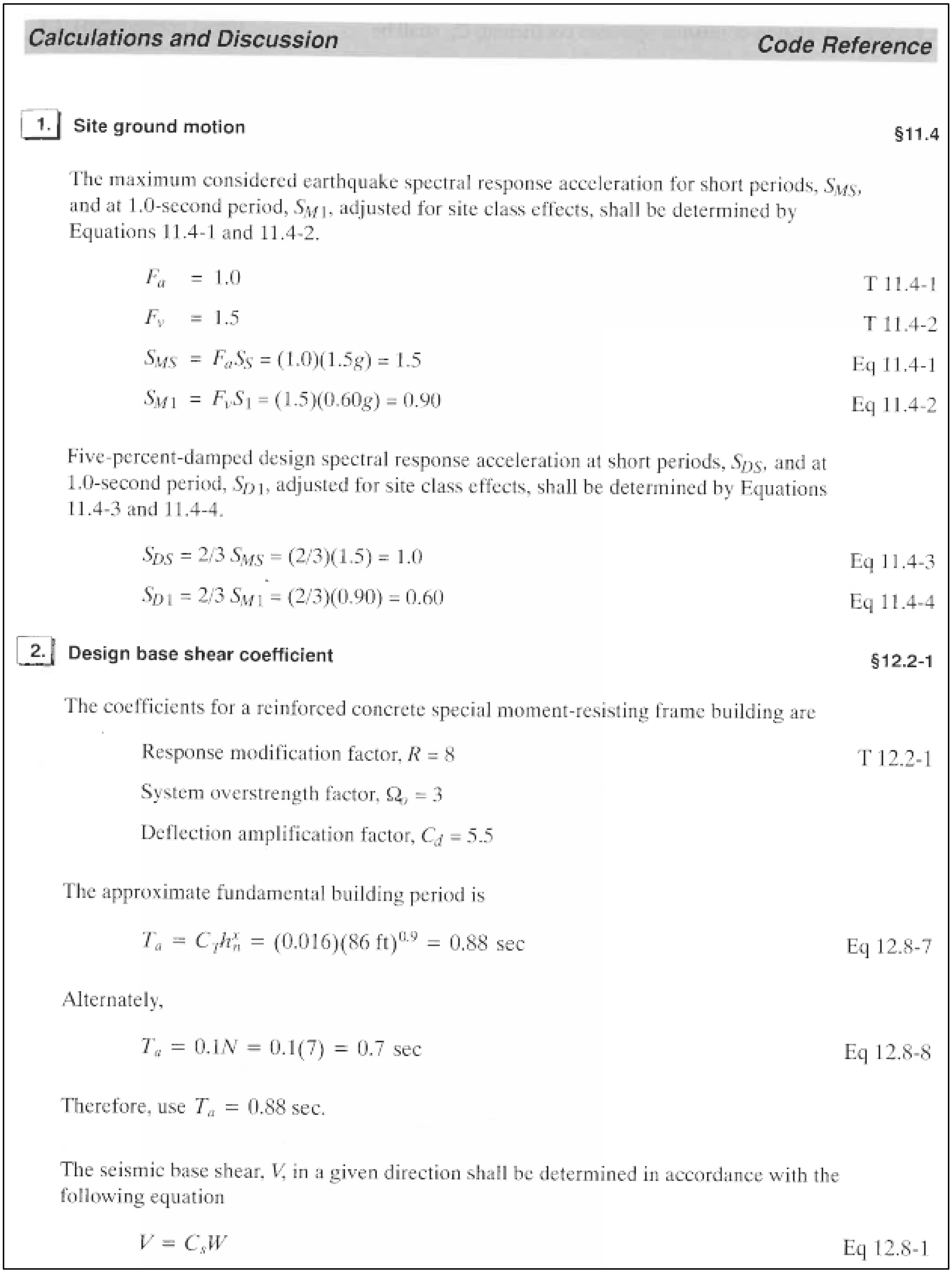


Figure 7-3. Typical frame elevation, line A

Figure 32: Design Example 7 from *2006 IBC Structural/Seismic Design Manual*



Calculations and Discussion **Code Reference**

1. Site ground motion §11.4

The maximum considered earthquake spectral response acceleration for short periods, S_{MS}, and at 1.0-second period, S_{M1}, adjusted for site class effects, shall be determined by Equations 11.4-1 and 11.4-2.

$F_a = 1.0$ T 11.4-1

$F_v = 1.5$ T 11.4-2

$S_{MS} = F_a S_S = (1.0)(1.5g) = 1.5$ Eq 11.4-1

$S_{M1} = F_v S_1 = (1.5)(0.60g) = 0.90$ Eq 11.4-2

Five-percent-damped design spectral response acceleration at short periods, S_{DS}, and at 1.0-second period, S_{D1}, adjusted for site class effects, shall be determined by Equations 11.4-3 and 11.4-4.

$S_{DS} = 2/3\ S_{MS} = (2/3)(1.5) = 1.0$ Eq 11.4-3

$S_{D1} = 2/3\ S_{M1} = (2/3)(0.90) = 0.60$ Eq 11.4-4

2. Design base shear coefficient §12.2-1

The coefficients for a reinforced concrete special moment-resisting frame building are

Response modification factor, $R = 8$ T 12.2-1

System overstrength factor, $\Omega_o = 3$

Deflection amplification factor, $C_d = 5.5$

The approximate fundamental building period is

$T_a = C_t h_n^x = (0.016)(86\text{ ft})^{0.9} = 0.88\text{ sec}$ Eq 12.8-7

Alternately,

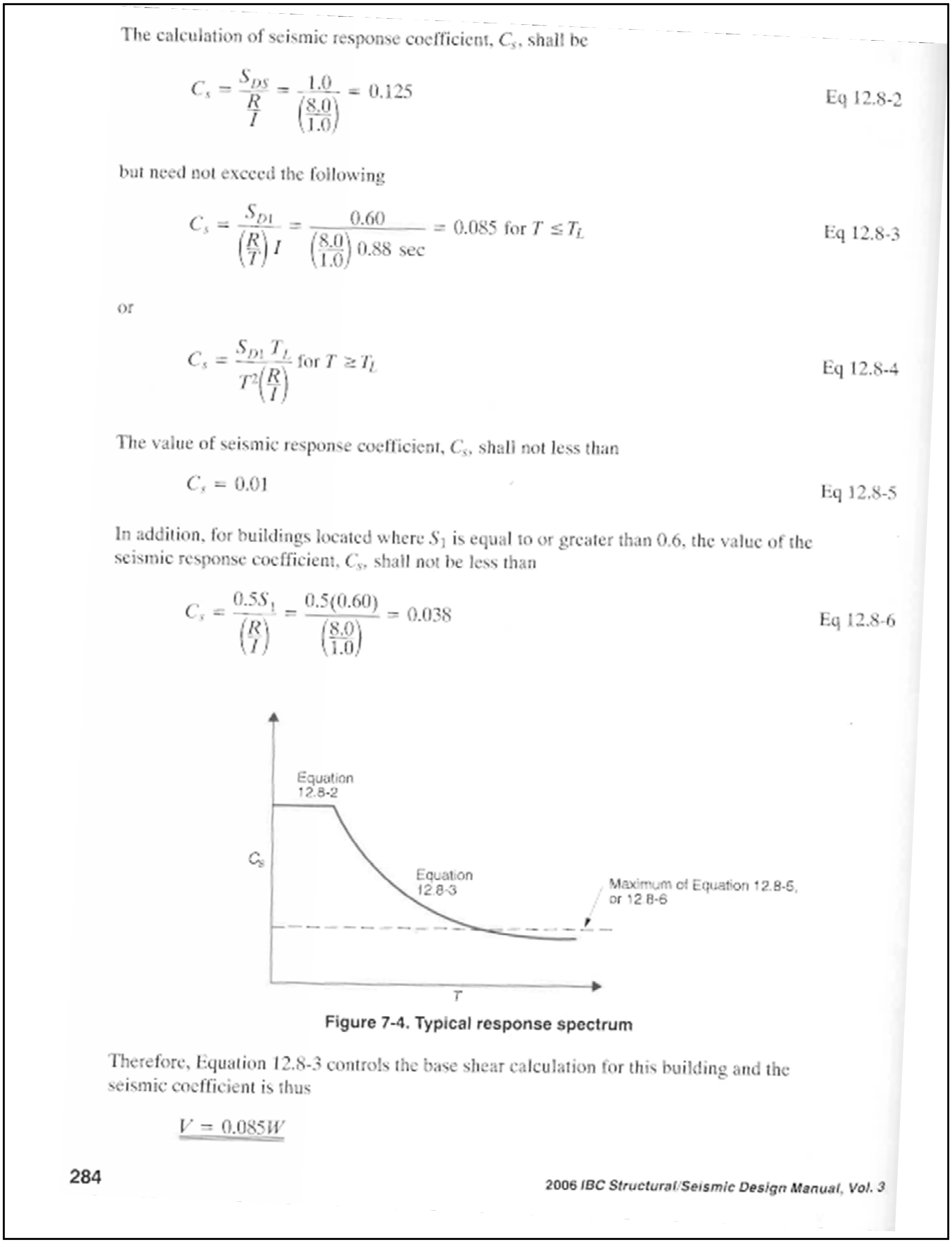
$T_a = 0.1N = 0.1(7) = 0.7\text{ sec}$ Eq 12.8-8

Therefore, use $T_a = 0.88$ sec.

The seismic base shear, V, in a given direction shall be determined in accordance with the following equation

$V = C_s W$ Eq 12.8-1

Figure 33: Design Example 7 from *2006 IBC Structural/Seismic Design Manual*



The calculation of seismic response coefficient, C_s, shall be

$$C_s = \frac{S_{DS}}{\frac{R}{I}} = \frac{1.0}{\left(\frac{8.0}{1.0}\right)} = 0.125 \qquad \text{Eq 12.8-2}$$

but need not exceed the following

$$C_s = \frac{S_{D1}}{\left(\frac{R}{T}\right)I} = \frac{0.60}{\left(\frac{8.0}{1.0}\right)0.88 \text{ sec}} = 0.085 \text{ for } T \le T_L \qquad \text{Eq 12.8-3}$$

or

$$C_s = \frac{S_{D1}\,T_L}{T^2\left(\frac{R}{I}\right)} \text{ for } T \ge T_L \qquad \text{Eq 12.8-4}$$

The value of seismic response coefficient, C_s, shall not less than

$$C_s = 0.01 \qquad \text{Eq 12.8-5}$$

In addition, for buildings located where S_1 is equal to or greater than 0.6, the value of the seismic response coefficient, C_s, shall not be less than

$$C_s = \frac{0.5S_1}{\left(\frac{R}{I}\right)} = \frac{0.5(0.60)}{\left(\frac{8.0}{1.0}\right)} = 0.038 \qquad \text{Eq 12.8-6}$$

Figure 7-4. Typical response spectrum

Therefore, Equation 12.8-3 controls the base shear calculation for this building and the seismic coefficient is thus

$$\underline{V = 0.085W}$$

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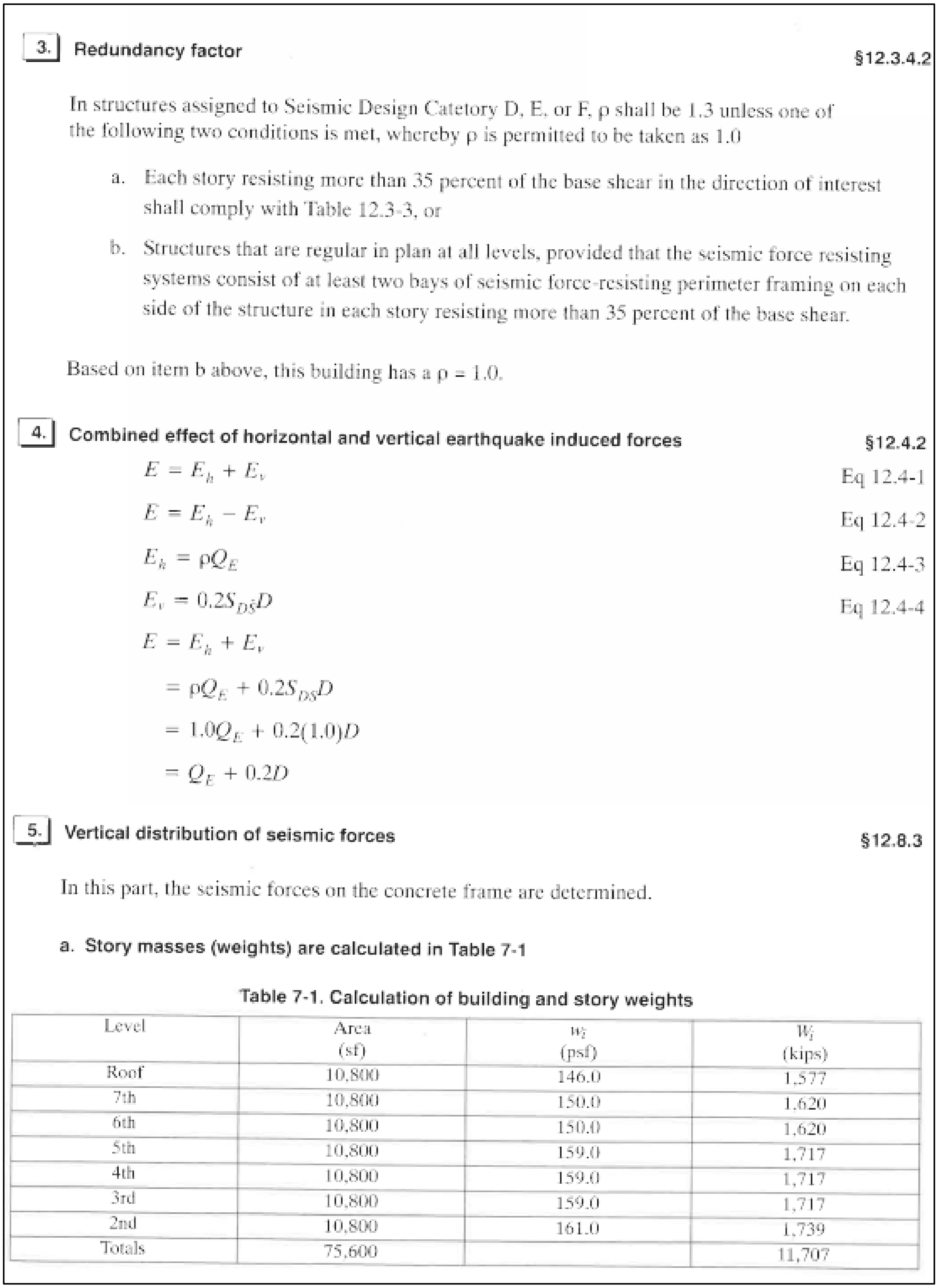
Figure 34: Design Example 7 from *2006 IBC Structural/Seismic Design Manual*



3. Redundancy factor §12.3.4.2

In structures assigned to Seismic Design Catetory D, E, or F, ρ shall be 1.3 unless one of the following two conditions is met, whereby ρ is permitted to be taken as 1.0

a. Each story resisting more than 35 percent of the base shear in the direction of interest shall comply with Table 12.3-3, or

b. Structures that are regular in plan at all levels, provided that the seismic force resisting systems consist of at least two bays of seismic force-resisting perimeter framing on each side of the structure in each story resisting more than 35 percent of the base shear.

Based on item b above, this building has a $\rho = 1.0$.

4. Combined effect of horizontal and vertical earthquake induced forces §12.4.2

$$E = E_h + E_v \qquad \text{Eq 12.4-1}$$

$$E = E_h - E_v \qquad \text{Eq 12.4-2}$$

$$E_h = \rho Q_E \qquad \text{Eq 12.4-3}$$

$$E_v = 0.2S_{DS}D \qquad \text{Eq 12.4-4}$$

$$\begin{aligned} E &= E_h + E_v \\ &= \rho Q_E + 0.2S_{DS}D \\ &= 1.0Q_E + 0.2(1.0)D \\ &= Q_E + 0.2D \end{aligned}$$

5. Vertical distribution of seismic forces §12.8.3

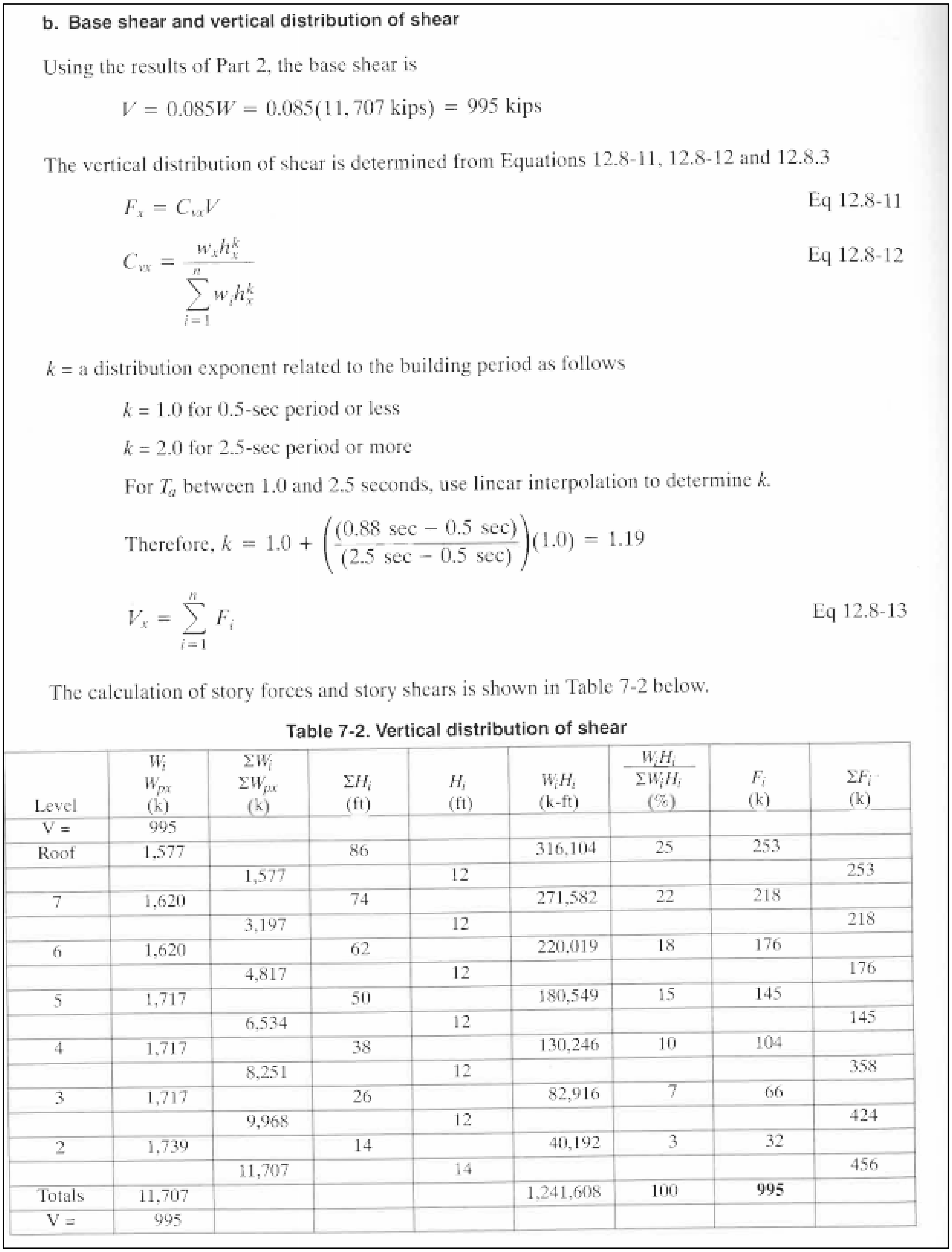
In this part, the seismic forces on the concrete frame are determined.

a. Story masses (weights) are calculated in Table 7-1

Table 7-1. Calculation of building and story weights

Level	Area (sf)	w_i (psf)	W_i (kips)
Roof	10,800	146.0	1,577
7th	10,800	150.0	1,620
6th	10,800	150.0	1,620
5th	10,800	159.0	1,717
4th	10,800	159.0	1,717
3rd	10,800	159.0	1,717
2nd	10,800	161.0	1,739
Totals	75,600		11,707

Figure 35: Design Example 7 from *2006 IBC Structural/Seismic Design Manual*



b. Base shear and vertical distribution of shear

Using the results of Part 2, the base shear is

$$V = 0.085W = 0.085(11,707 \text{ kips}) = 995 \text{ kips}$$

The vertical distribution of shear is determined from Equations 12.8-11, 12.8-12 and 12.8.3

$$F_x = C_{vx}V \qquad \text{Eq 12.8-11}$$

$$C_{vx} = \frac{w_x h_x^k}{\sum_{i=1}^{n} w_i h_x^k} \qquad \text{Eq 12.8-12}$$

k = a distribution exponent related to the building period as follows

k = 1.0 for 0.5-sec period or less

k = 2.0 for 2.5-sec period or more

For T_a between 1.0 and 2.5 seconds, use linear interpolation to determine k.

$$\text{Therefore, } k = 1.0 + \left(\frac{(0.88 \text{ sec} - 0.5 \text{ sec})}{(2.5 \text{ sec} - 0.5 \text{ sec})}\right)(1.0) = 1.19$$

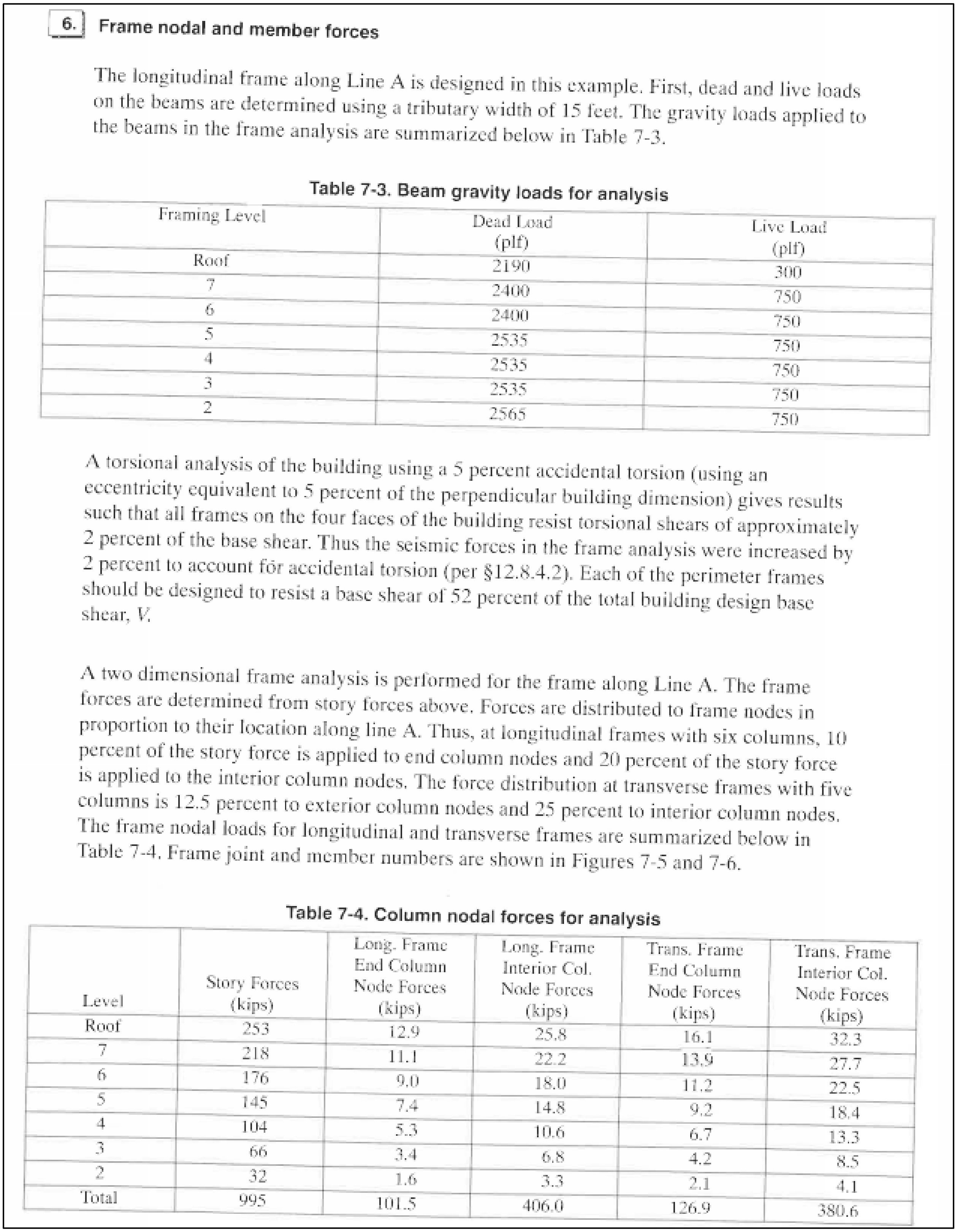
$$V_x = \sum_{i=1}^{n} F_i \qquad \text{Eq 12.8-13}$$

The calculation of story forces and story shears is shown in Table 7-2 below.

Table 7-2. Vertical distribution of shear

Level	W_i W_{px} (k)	ΣW_i ΣW_{px} (k)	ΣH_i (ft)	H_i (ft)	W_iH_i (k-ft)	$\frac{W_iH_i}{\Sigma W_iH_i}$ (%)	F_i (k)	ΣF_i (k)
V =	995							
Roof	1,577		86		316,104	25	253	
		1,577		12				253
7	1,620		74		271,582	22	218	
		3,197		12				218
6	1,620		62		220,019	18	176	
		4,817		12				176
5	1,717		50		180,549	15	145	
		6,534		12				145
4	1,717		38		130,246	10	104	
		8,251		12				358
3	1,717		26		82,916	7	66	
		9,968		12				424
2	1,739		14		40,192	3	32	
		11,707		14				456
Totals	11,707				1,241,608	100	**995**	
V =	995							

Figure 36: Design Example 7 from *2006 IBC Structural/Seismic Design Manual*



6. Frame nodal and member forces

The longitudinal frame along Line A is designed in this example. First, dead and live loads on the beams are determined using a tributary width of 15 feet. The gravity loads applied to the beams in the frame analysis are summarized below in Table 7-3.

Table 7-3. Beam gravity loads for analysis

Framing Level	Dead Load (plf)	Live Load (plf)
Roof	2190	300
7	2400	750
6	2400	750
5	2535	750
4	2535	750
3	2535	750
2	2565	750

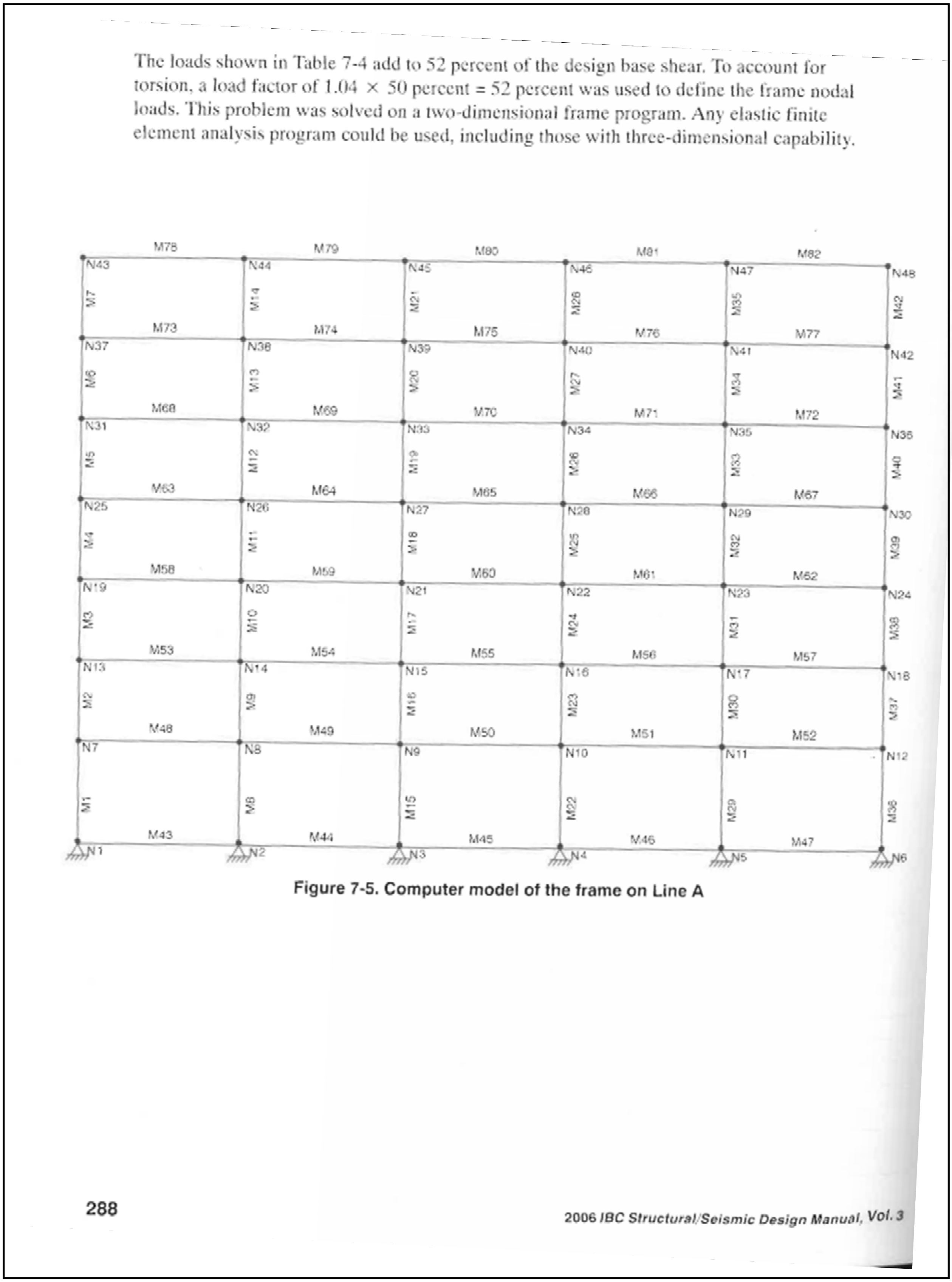
A torsional analysis of the building using a 5 percent accidental torsion (using an eccentricity equivalent to 5 percent of the perpendicular building dimension) gives results such that all frames on the four faces of the building resist torsional shears of approximately 2 percent of the base shear. Thus the seismic forces in the frame analysis were increased by 2 percent to account for accidental torsion (per §12.8.4.2). Each of the perimeter frames should be designed to resist a base shear of 52 percent of the total building design base shear, *V*.

A two dimensional frame analysis is performed for the frame along Line A. The frame forces are determined from story forces above. Forces are distributed to frame nodes in proportion to their location along line A. Thus, at longitudinal frames with six columns, 10 percent of the story force is applied to end column nodes and 20 percent of the story force is applied to the interior column nodes. The force distribution at transverse frames with five columns is 12.5 percent to exterior column nodes and 25 percent to interior column nodes. The frame nodal loads for longitudinal and transverse frames are summarized below in Table 7-4. Frame joint and member numbers are shown in Figures 7-5 and 7-6.

Table 7-4. Column nodal forces for analysis

Level	Story Forces (kips)	Long. Frame End Column Node Forces (kips)	Long. Frame Interior Col. Node Forces (kips)	Trans. Frame End Column Node Forces (kips)	Trans. Frame Interior Col. Node Forces (kips)
Roof	253	12.9	25.8	16.1	32.3
7	218	11.1	22.2	13.9	27.7
6	176	9.0	18.0	11.2	22.5
5	145	7.4	14.8	9.2	18.4
4	104	5.3	10.6	6.7	13.3
3	66	3.4	6.8	4.2	8.5
2	32	1.6	3.3	2.1	4.1
Total	995	101.5	406.0	126.9	380.6

Figure 37: Design Example 7 from *2006 IBC Structural/Seismic Design Manual*



The loads shown in Table 7-4 add to 52 percent of the design base shear. To account for torsion, a load factor of 1.04 × 50 percent = 52 percent was used to define the frame nodal loads. This problem was solved on a two-dimensional frame program. Any elastic finite element analysis program could be used, including those with three-dimensional capability.

Figure 7-5. Computer model of the frame on Line A

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Figure 38: Design Example 7 from *2006 IBC Structural/Seismic Design Manual*

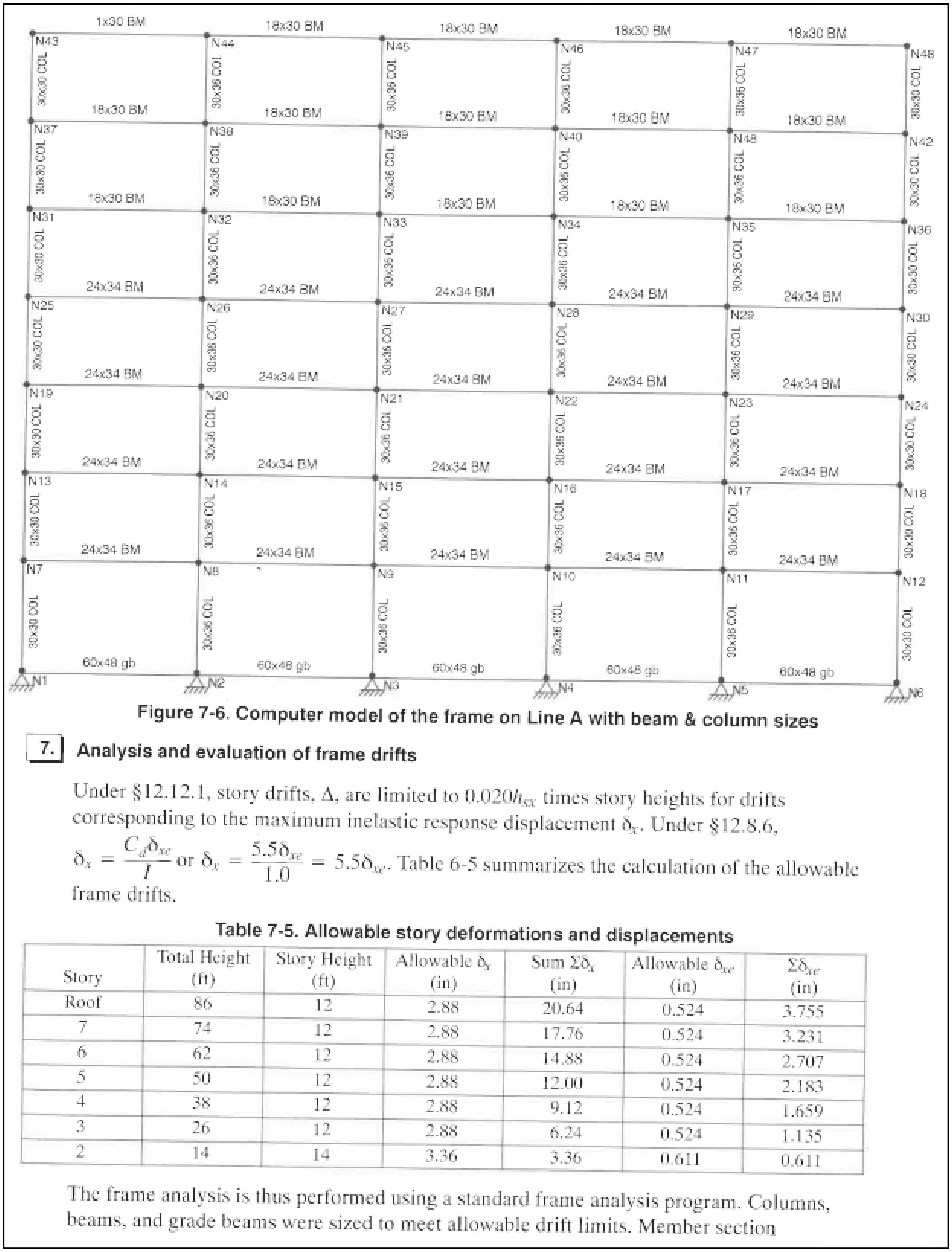


Figure 7-6. Computer model of the frame on Line A with beam & column sizes

7. Analysis and evaluation of frame drifts

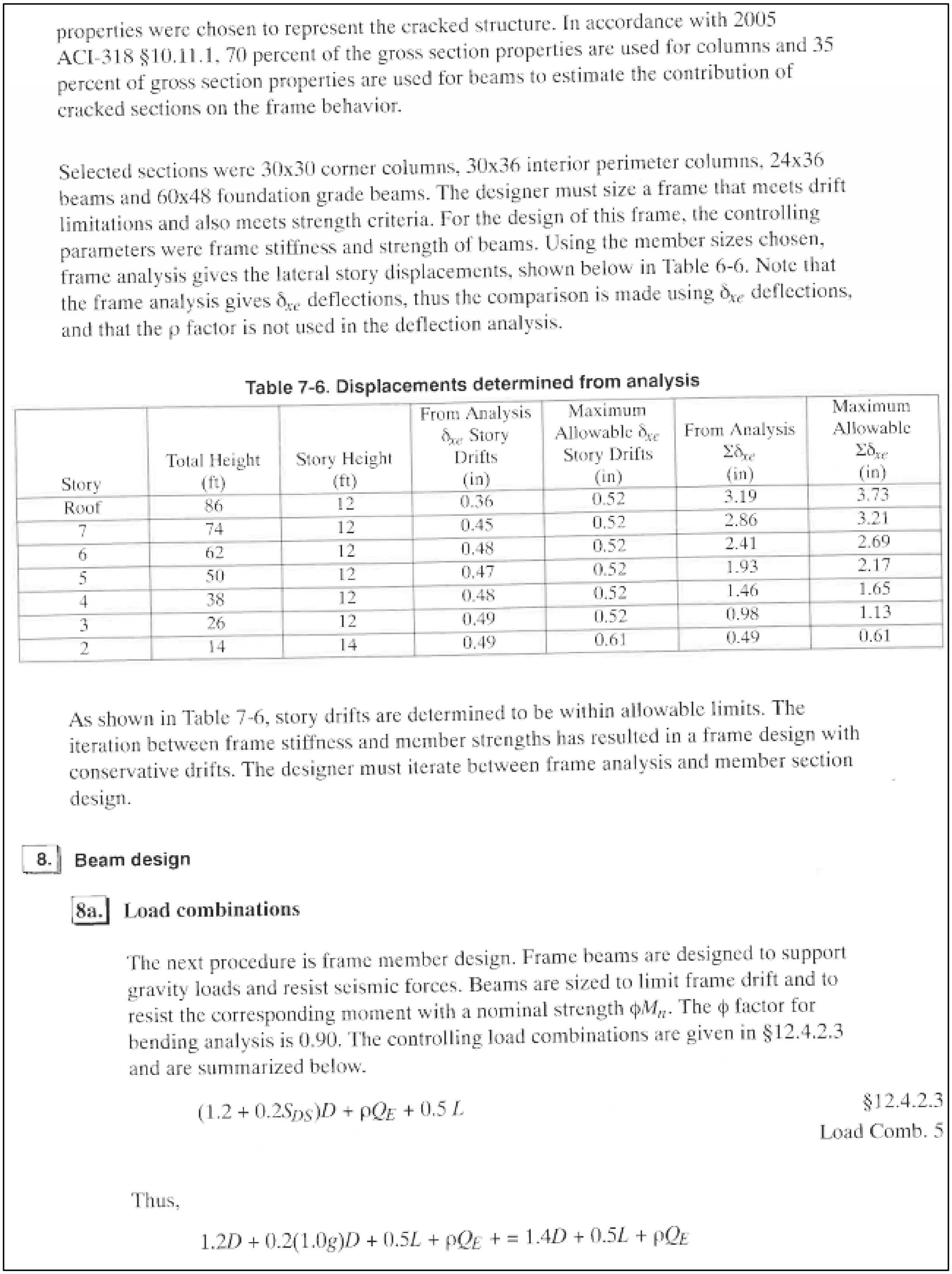
Under §12.12.1, story drifts, Δ, are limited to $0.020h_{sx}$ times story heights for drifts corresponding to the maximum inelastic response displacement δ_x. Under §12.8.6, $\delta_x = \frac{C_d\delta_{xe}}{I}$ or $\delta_x = \frac{5.5\delta_{xe}}{1.0} = 5.5\delta_{xe}$. Table 6-5 summarizes the calculation of the allowable frame drifts.

Table 7-5. Allowable story deformations and displacements

Story	Total Height (ft)	Story Height (ft)	Allowable δ_x (in)	Sum $\Sigma\delta_x$ (in)	Allowable δ_{xe} (in)	$\Sigma\delta_{xe}$ (in)
Roof	86	12	2.88	20.64	0.524	3.755
7	74	12	2.88	17.76	0.524	3.231
6	62	12	2.88	14.88	0.524	2.707
5	50	12	2.88	12.00	0.524	2.183
4	38	12	2.88	9.12	0.524	1.659
3	26	12	2.88	6.24	0.524	1.135
2	14	14	3.36	3.36	0.611	0.611

The frame analysis is thus performed using a standard frame analysis program. Columns, beams, and grade beams were sized to meet allowable drift limits. Member section

Figure 39: Design Example 7 from *2006 IBC Structural/Seismic Design Manual*



properties were chosen to represent the cracked structure. In accordance with 2005 ACI-318 §10.11.1, 70 percent of the gross section properties are used for columns and 35 percent of gross section properties are used for beams to estimate the contribution of cracked sections on the frame behavior.

Selected sections were 30x30 corner columns, 30x36 interior perimeter columns, 24x36 beams and 60x48 foundation grade beams. The designer must size a frame that meets drift limitations and also meets strength criteria. For the design of this frame, the controlling parameters were frame stiffness and strength of beams. Using the member sizes chosen, frame analysis gives the lateral story displacements, shown below in Table 6-6. Note that the frame analysis gives δ_{xe} deflections, thus the comparison is made using δ_{xe} deflections, and that the ρ factor is not used in the deflection analysis.

Table 7-6. Displacements determined from analysis

Story	Total Height (ft)	Story Height (ft)	From Analysis δ_{xe} Story Drifts (in)	Maximum Allowable δ_{xe} Story Drifts (in)	From Analysis $\Sigma\delta_{xe}$ (in)	Maximum Allowable $\Sigma\delta_{xe}$ (in)
Roof	86	12	0.36	0.52	3.19	3.73
7	74	12	0.45	0.52	2.86	3.21
6	62	12	0.48	0.52	2.41	2.69
5	50	12	0.47	0.52	1.93	2.17
4	38	12	0.48	0.52	1.46	1.65
3	26	12	0.49	0.52	0.98	1.13
2	14	14	0.49	0.61	0.49	0.61

As shown in Table 7-6, story drifts are determined to be within allowable limits. The iteration between frame stiffness and member strengths has resulted in a frame design with conservative drifts. The designer must iterate between frame analysis and member section design.

8. Beam design

8a. Load combinations

The next procedure is frame member design. Frame beams are designed to support gravity loads and resist seismic forces. Beams are sized to limit frame drift and to resist the corresponding moment with a nominal strength ϕM_n. The ϕ factor for bending analysis is 0.90. The controlling load combinations are given in §12.4.2.3 and are summarized below.

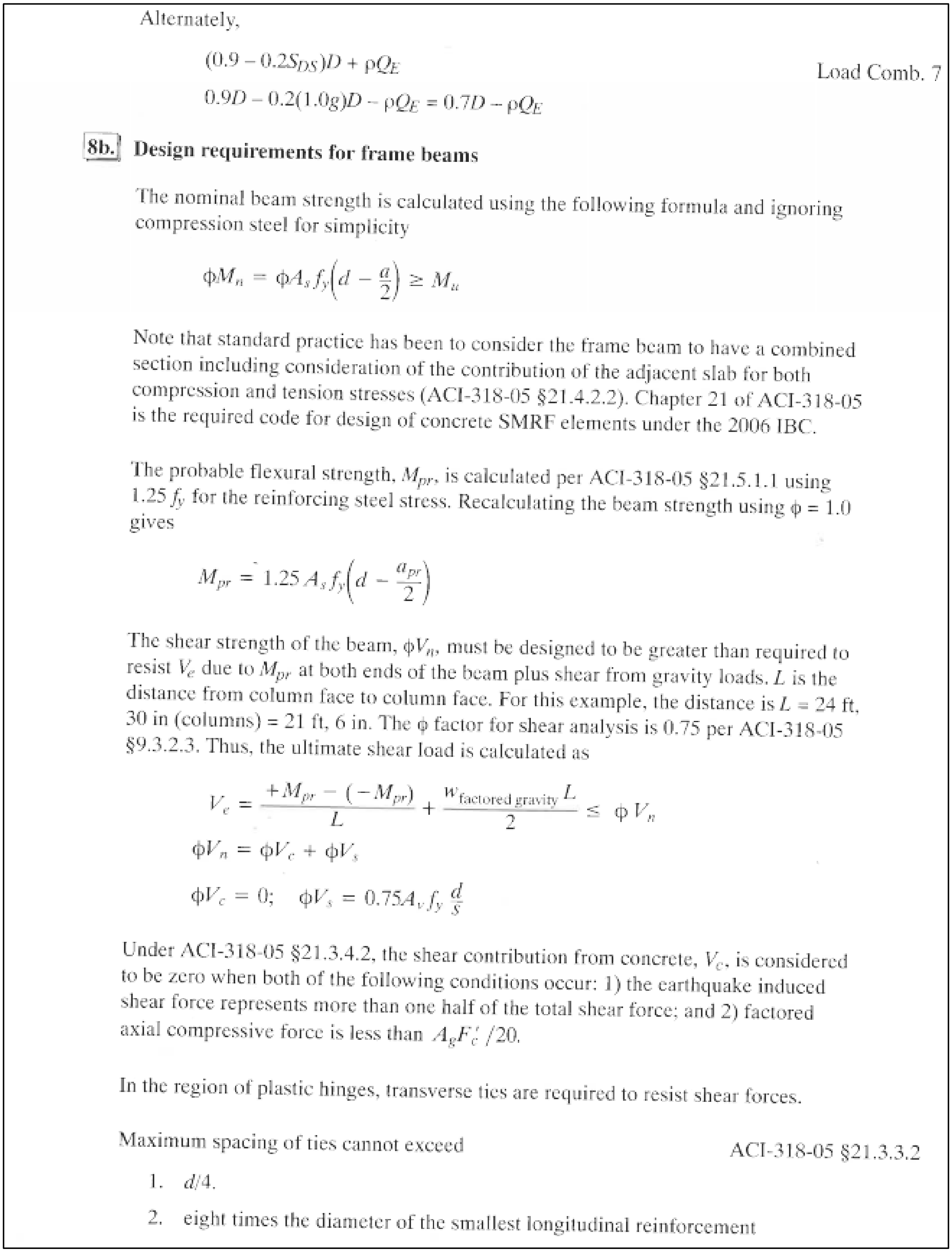
$$(1.2 + 0.2S_{DS})D + \rho Q_E + 0.5\,L$$

§12.4.2.3
Load Comb. 5

Thus,

$$1.2D + 0.2(1.0g)D + 0.5L + \rho Q_E + = 1.4D + 0.5L + \rho Q_E$$

Figure 40: Design Example 7 from *2006 IBC Structural/Seismic Design Manual*



Alternately,

$$(0.9 - 0.2S_{DS})D + \rho Q_E$$

Load Comb. 7

$$0.9D - 0.2(1.0g)D - \rho Q_E = 0.7D - \rho Q_E$$

8b. Design requirements for frame beams

The nominal beam strength is calculated using the following formula and ignoring compression steel for simplicity

$$\phi M_n = \phi A_s f_y\left(d - \frac{a}{2}\right) \geq M_u$$

Note that standard practice has been to consider the frame beam to have a combined section including consideration of the contribution of the adjacent slab for both compression and tension stresses (ACI-318-05 §21.4.2.2). Chapter 21 of ACI-318-05 is the required code for design of concrete SMRF elements under the 2006 IBC.

The probable flexural strength, M_{pr}, is calculated per ACI-318-05 §21.5.1.1 using 1.25 f_y for the reinforcing steel stress. Recalculating the beam strength using $\phi = 1.0$ gives

$$M_{pr} = 1.25 A_s f_y\left(d - \frac{a_{pr}}{2}\right)$$

The shear strength of the beam, ϕV_n, must be designed to be greater than required to resist V_e due to M_{pr} at both ends of the beam plus shear from gravity loads. L is the distance from column face to column face. For this example, the distance is L = 24 ft, 30 in (columns) = 21 ft, 6 in. The ϕ factor for shear analysis is 0.75 per ACI-318-05 §9.3.2.3. Thus, the ultimate shear load is calculated as

$$V_e = \frac{+M_{pr} - (-M_{pr})}{L} + \frac{w_{\text{factored gravity}}\, L}{2} \leq \phi V_n$$

$$\phi V_n = \phi V_c + \phi V_s$$

$$\phi V_c = 0; \quad \phi V_s = 0.75 A_v f_y \frac{d}{s}$$

Under ACI-318-05 §21.3.4.2, the shear contribution from concrete, V_c, is considered to be zero when both of the following conditions occur: 1) the earthquake induced shear force represents more than one half of the total shear force; and 2) factored axial compressive force is less than $A_g F'_c/20$.

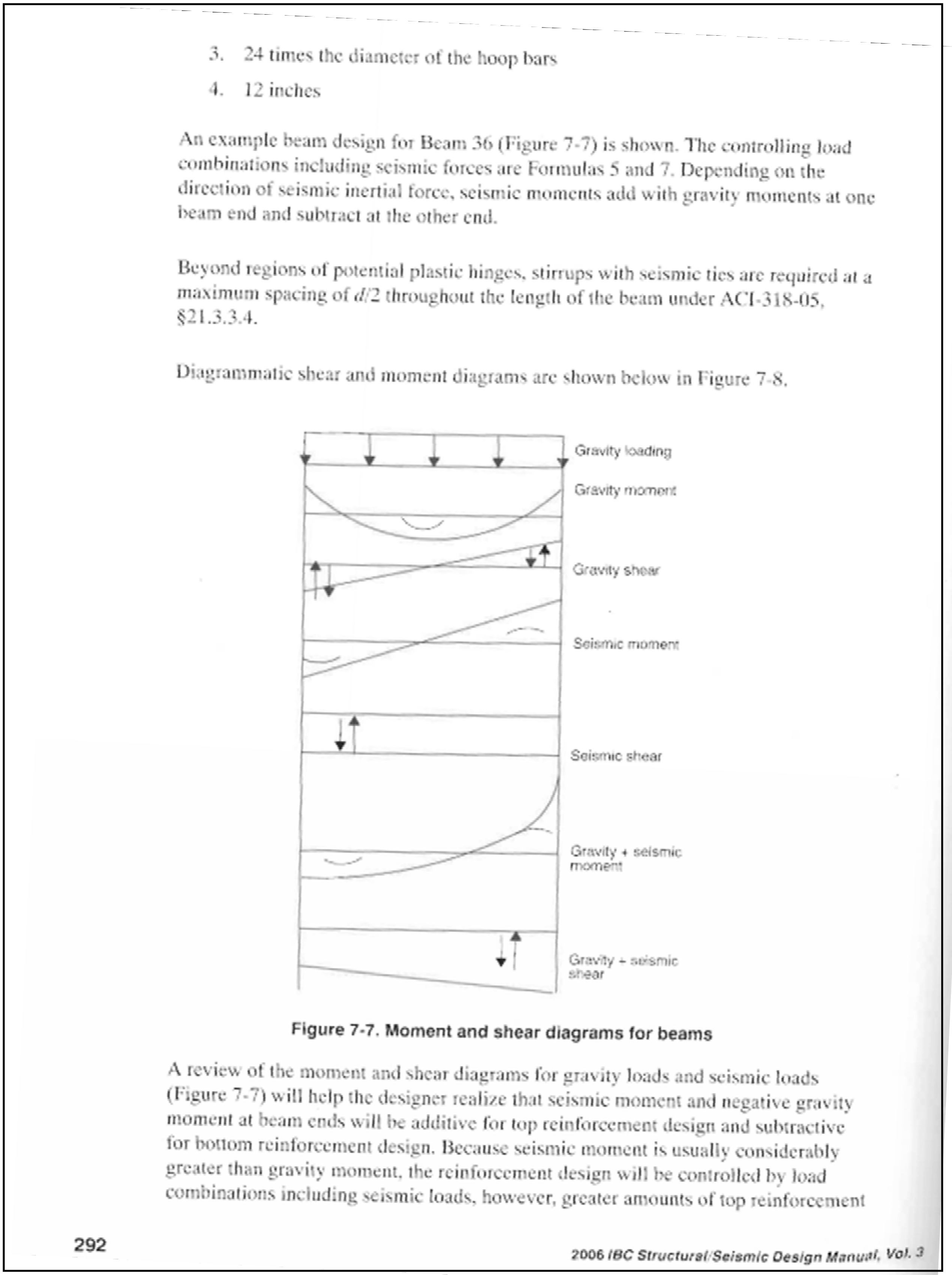
In the region of plastic hinges, transverse ties are required to resist shear forces.

Maximum spacing of ties cannot exceed

ACI-318-05 §21.3.3.2

1. $d/4$.
2. eight times the diameter of the smallest longitudinal reinforcement

Figure 41: Design Example 7 from *2006 IBC Structural/Seismic Design Manual*



3. 24 times the diameter of the hoop bars
4. 12 inches

An example beam design for Beam 36 (Figure 7-7) is shown. The controlling load combinations including seismic forces are Formulas 5 and 7. Depending on the direction of seismic inertial force, seismic moments add with gravity moments at one beam end and subtract at the other end.

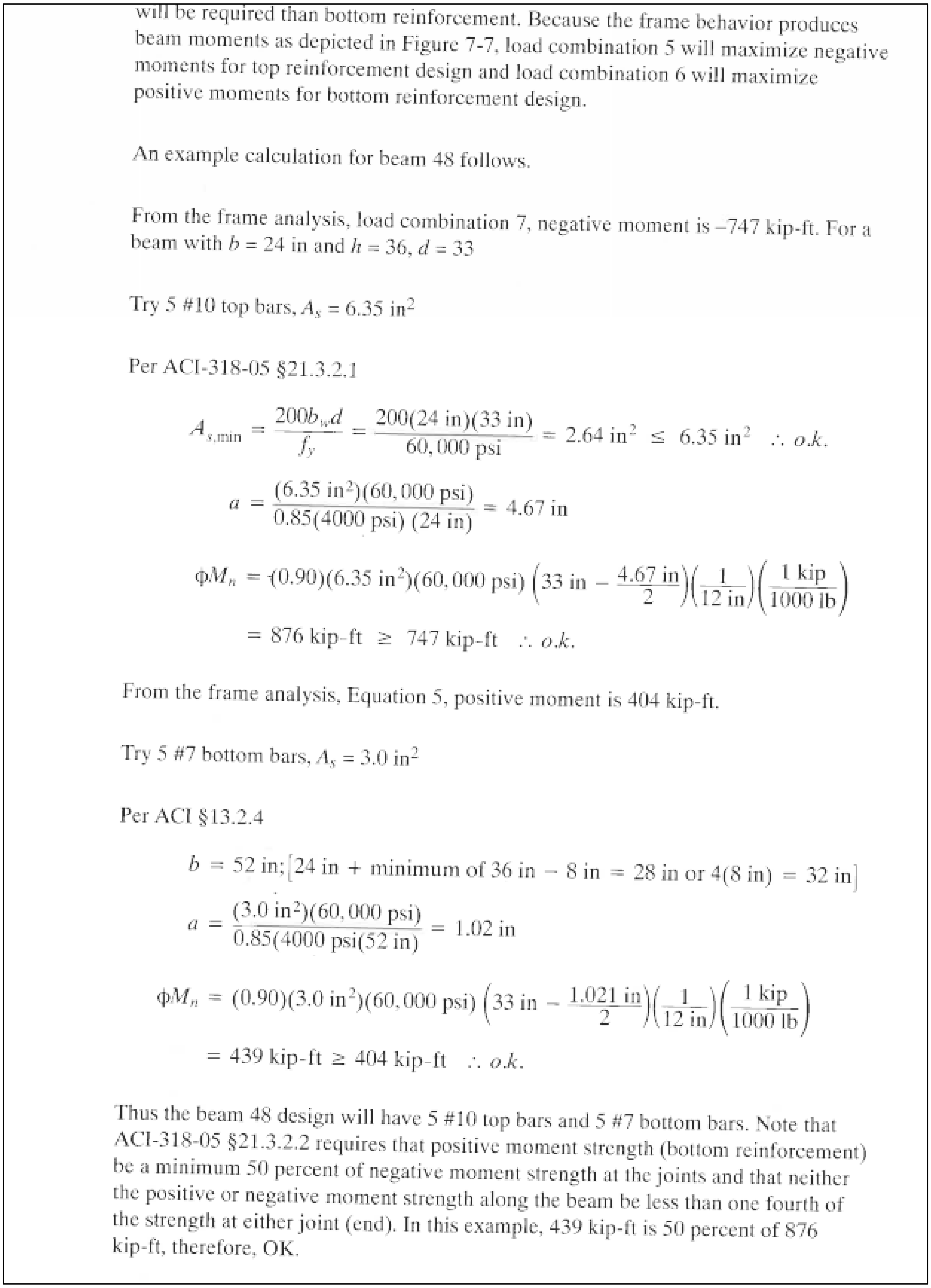
Beyond regions of potential plastic hinges, stirrups with seismic ties are required at a maximum spacing of *d*/2 throughout the length of the beam under ACI-318-05, §21.3.3.4.

Diagrammatic shear and moment diagrams are shown below in Figure 7-8.

Figure 7-7. Moment and shear diagrams for beams

A review of the moment and shear diagrams for gravity loads and seismic loads (Figure 7-7) will help the designer realize that seismic moment and negative gravity moment at beam ends will be additive for top reinforcement design and subtractive for bottom reinforcement design. Because seismic moment is usually considerably greater than gravity moment, the reinforcement design will be controlled by load combinations including seismic loads, however, greater amounts of top reinforcement

Figure 42: Design Example 7 from *2006 IBC Structural/Seismic Design Manual*



will be required than bottom reinforcement. Because the frame behavior produces beam moments as depicted in Figure 7-7, load combination 5 will maximize negative moments for top reinforcement design and load combination 6 will maximize positive moments for bottom reinforcement design.

An example calculation for beam 48 follows.

From the frame analysis, load combination 7, negative moment is –747 kip-ft. For a beam with $b = 24$ in and $h = 36$, $d = 33$

Try 5 #10 top bars, $A_s = 6.35\ \text{in}^2$

Per ACI-318-05 §21.3.2.1

$$A_{s,\min} = \frac{200 b_w d}{f_y} = \frac{200(24\ \text{in})(33\ \text{in})}{60{,}000\ \text{psi}} = 2.64\ \text{in}^2 \leq 6.35\ \text{in}^2 \quad \therefore\ o.k.$$

$$a = \frac{(6.35\ \text{in}^2)(60{,}000\ \text{psi})}{0.85(4000\ \text{psi})\ (24\ \text{in})} = 4.67\ \text{in}$$

$$\phi M_n = (0.90)(6.35\ \text{in}^2)(60{,}000\ \text{psi})\left(33\ \text{in} - \frac{4.67\ \text{in}}{2}\right)\left(\frac{1}{12\ \text{in}}\right)\left(\frac{1\ \text{kip}}{1000\ \text{lb}}\right)$$

$$= 876\ \text{kip-ft} \geq 747\ \text{kip-ft} \quad \therefore\ o.k.$$

From the frame analysis, Equation 5, positive moment is 404 kip-ft.

Try 5 #7 bottom bars, $A_s = 3.0\ \text{in}^2$

Per ACI §13.2.4

$$b = 52\ \text{in};\left[24\ \text{in} + \text{minimum of } 36\ \text{in} - 8\ \text{in} = 28\ \text{in or } 4(8\ \text{in}) = 32\ \text{in}\right]$$

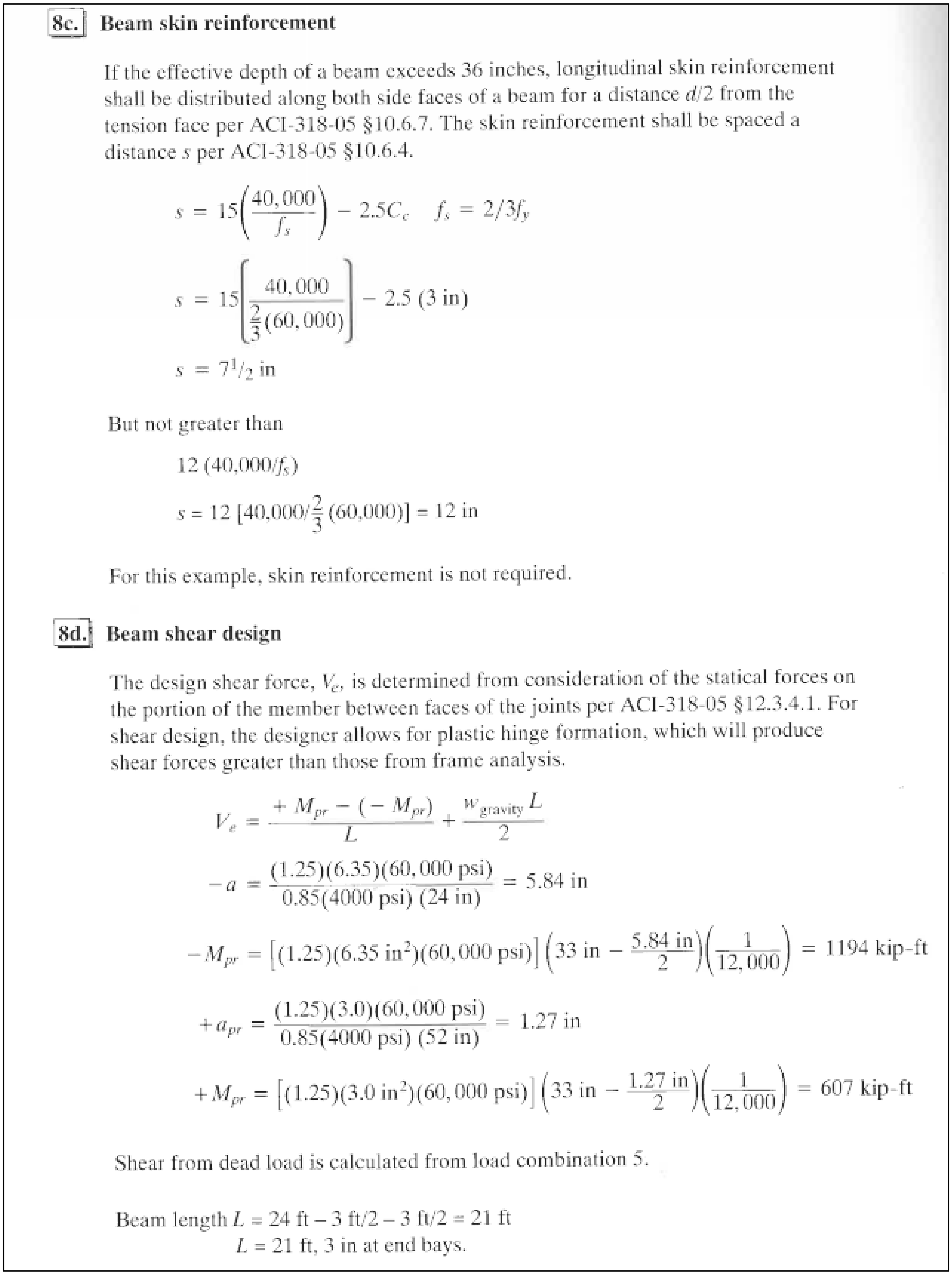
$$a = \frac{(3.0\ \text{in}^2)(60{,}000\ \text{psi})}{0.85(4000\ \text{psi}(52\ \text{in})} = 1.02\ \text{in}$$

$$\phi M_n = (0.90)(3.0\ \text{in}^2)(60{,}000\ \text{psi})\left(33\ \text{in} - \frac{1.021\ \text{in}}{2}\right)\left(\frac{1}{12\ \text{in}}\right)\left(\frac{1\ \text{kip}}{1000\ \text{lb}}\right)$$

$$= 439\ \text{kip-ft} \geq 404\ \text{kip-ft} \quad \therefore\ o.k.$$

Thus the beam 48 design will have 5 #10 top bars and 5 #7 bottom bars. Note that ACI-318-05 §21.3.2.2 requires that positive moment strength (bottom reinforcement) be a minimum 50 percent of negative moment strength at the joints and that neither the positive or negative moment strength along the beam be less than one fourth of the strength at either joint (end). In this example, 439 kip-ft is 50 percent of 876 kip-ft, therefore, OK.

Figure 43: Design Example 7 from *2006 IBC Structural/Seismic Design Manual*



8c. Beam skin reinforcement

If the effective depth of a beam exceeds 36 inches, longitudinal skin reinforcement shall be distributed along both side faces of a beam for a distance $d/2$ from the tension face per ACI-318-05 §10.6.7. The skin reinforcement shall be spaced a distance s per ACI-318-05 §10.6.4.

$$s = 15\left(\frac{40{,}000}{f_s}\right) - 2.5C_c \quad f_s = 2/3f_y$$

$$s = 15\left[\frac{40{,}000}{\frac{2}{3}(60{,}000)}\right] - 2.5\ (3\ \text{in})$$

$$s = 7\,{}^{1}/_{2}\ \text{in}$$

But not greater than

$$12\ (40{,}000/f_s)$$

$$s = 12\ [40{,}000/\tfrac{2}{3}\ (60{,}000)] = 12\ \text{in}$$

For this example, skin reinforcement is not required.

8d. Beam shear design

The design shear force, V_e, is determined from consideration of the statical forces on the portion of the member between faces of the joints per ACI-318-05 §12.3.4.1. For shear design, the designer allows for plastic hinge formation, which will produce shear forces greater than those from frame analysis.

$$V_e = \frac{+M_{pr} - (-M_{pr})}{L} + \frac{w_{\text{gravity}}L}{2}$$

$$-a = \frac{(1.25)(6.35)(60{,}000\ \text{psi})}{0.85(4000\ \text{psi})\ (24\ \text{in})} = 5.84\ \text{in}$$

$$-M_{pr} = \left[(1.25)(6.35\ \text{in}^2)(60{,}000\ \text{psi})\right]\left(33\ \text{in} - \frac{5.84\ \text{in}}{2}\right)\left(\frac{1}{12{,}000}\right) = 1194\ \text{kip-ft}$$

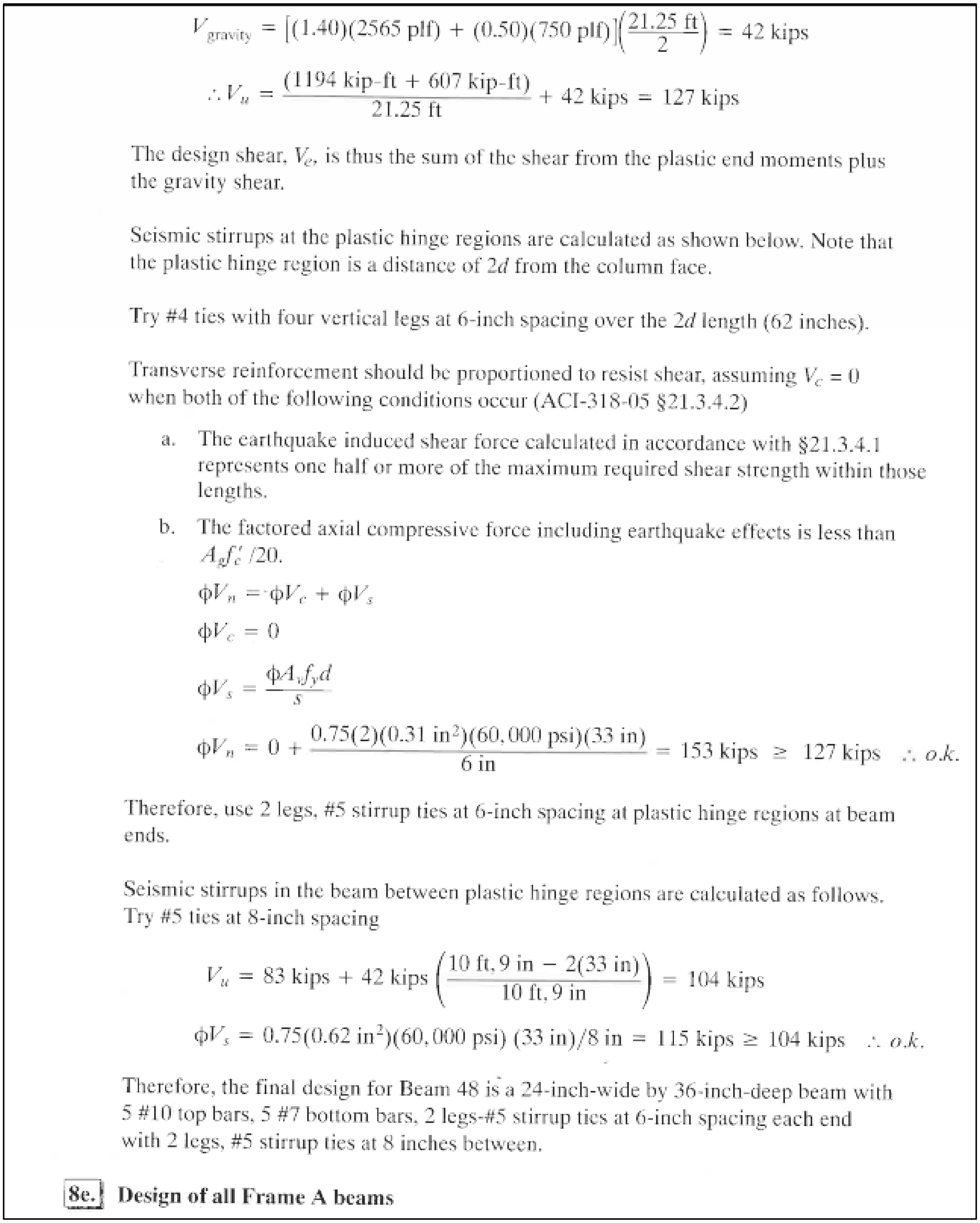
$$+a_{pr} = \frac{(1.25)(3.0)(60{,}000\ \text{psi})}{0.85(4000\ \text{psi})\ (52\ \text{in})} = 1.27\ \text{in}$$

$$+M_{pr} = \left[(1.25)(3.0\ \text{in}^2)(60{,}000\ \text{psi})\right]\left(33\ \text{in} - \frac{1.27\ \text{in}}{2}\right)\left(\frac{1}{12{,}000}\right) = 607\ \text{kip-ft}$$

Shear from dead load is calculated from load combination 5.

Beam length L = 24 ft – 3 ft/2 – 3 ft/2 = 21 ft
L = 21 ft, 3 in at end bays.

Figure 44: Design Example 7 from *2006 IBC Structural/Seismic Design Manual*



$$V_{\text{gravity}} = \left[(1.40)(2565\text{ plf}) + (0.50)(750\text{ plf})\right]\left(\frac{21.25\text{ ft}}{2}\right) = 42\text{ kips}$$

$$\therefore V_u = \frac{(1194\text{ kip-ft} + 607\text{ kip-ft})}{21.25\text{ ft}} + 42\text{ kips} = 127\text{ kips}$$

The design shear, V_e, is thus the sum of the shear from the plastic end moments plus the gravity shear.

Seismic stirrups at the plastic hinge regions are calculated as shown below. Note that the plastic hinge region is a distance of $2d$ from the column face.

Try #4 ties with four vertical legs at 6-inch spacing over the $2d$ length (62 inches).

Transverse reinforcement should be proportioned to resist shear, assuming $V_c = 0$ when both of the following conditions occur (ACI-318-05 §21.3.4.2)

a. The earthquake induced shear force calculated in accordance with §21.3.4.1 represents one half or more of the maximum required shear strength within those lengths.

b. The factored axial compressive force including earthquake effects is less than $A_g f'_c/20$.

$$\phi V_n = \phi V_c + \phi V_s$$

$$\phi V_c = 0$$

$$\phi V_s = \frac{\phi A_v f_y d}{s}$$

$$\phi V_n = 0 + \frac{0.75(2)(0.31\text{ in}^2)(60,000\text{ psi})(33\text{ in})}{6\text{ in}} = 153\text{ kips} \geq 127\text{ kips} \quad \therefore o.k.$$

Therefore, use 2 legs, #5 stirrup ties at 6-inch spacing at plastic hinge regions at beam ends.

Seismic stirrups in the beam between plastic hinge regions are calculated as follows. Try #5 ties at 8-inch spacing

$$V_u = 83\text{ kips} + 42\text{ kips}\left(\frac{10\text{ ft}, 9\text{ in} - 2(33\text{ in})}{10\text{ ft}, 9\text{ in}}\right) = 104\text{ kips}$$

$$\phi V_s = 0.75(0.62\text{ in}^2)(60,000\text{ psi})\ (33\text{ in})/8\text{ in} = 115\text{ kips} \geq 104\text{ kips} \quad \therefore o.k.$$

Therefore, the final design for Beam 48 is a 24-inch-wide by 36-inch-deep beam with 5 #10 top bars, 5 #7 bottom bars, 2 legs-#5 stirrup ties at 6-inch spacing each end with 2 legs, #5 stirrup ties at 8 inches between.

8e. Design of all Frame A beams

Figure 45: Design Example 7 from *2006 IBC Structural/Seismic Design Manual*

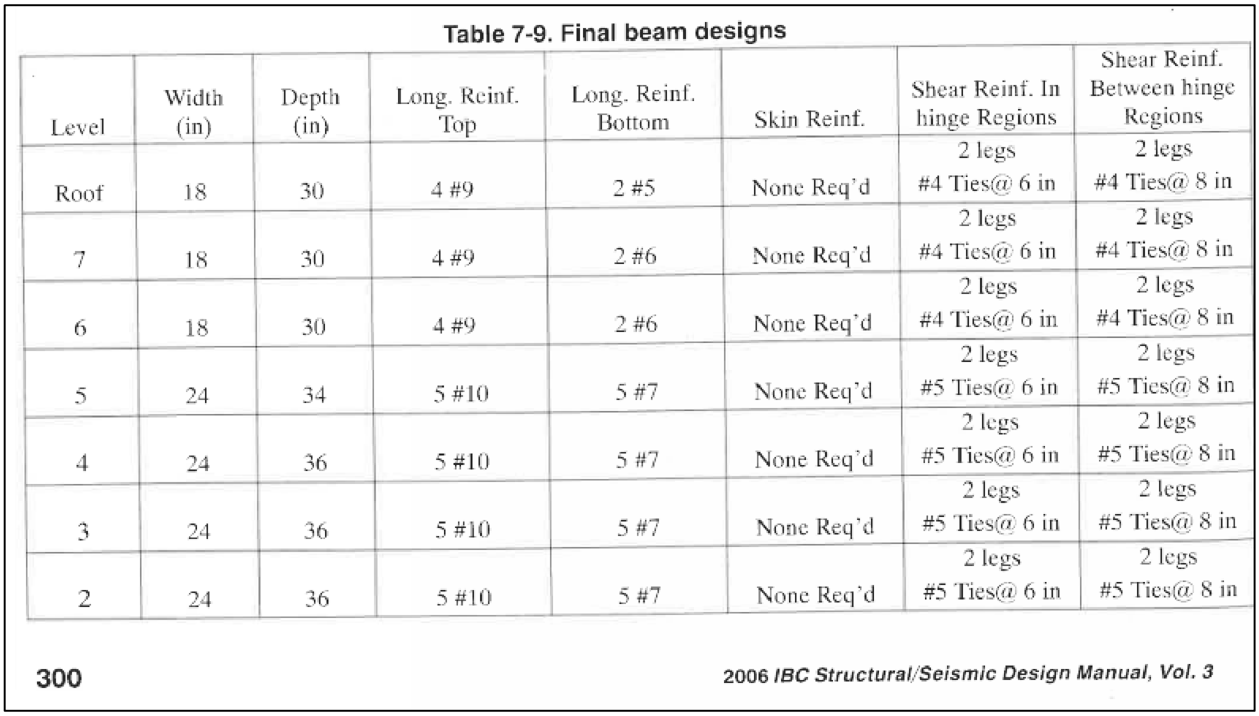


Table 7-9. Final beam designs

Level	Width (in)	Depth (in)	Long. Reinf. Top	Long. Reinf. Bottom	Skin Reinf.	Shear Reinf. In hinge Regions	Shear Reinf. Between hinge Regions
Roof	18	30	4 #9	2 #5	None Req'd	2 legs #4 Ties@ 6 in	2 legs #4 Ties@ 8 in
7	18	30	4 #9	2 #6	None Req'd	2 legs #4 Ties@ 6 in	2 legs #4 Ties@ 8 in
6	18	30	4 #9	2 #6	None Req'd	2 legs #4 Ties@ 6 in	2 legs #4 Ties@ 8 in
5	24	34	5 #10	5 #7	None Req'd	2 legs #5 Ties@ 6 in	2 legs #5 Ties@ 8 in
4	24	36	5 #10	5 #7	None Req'd	2 legs #5 Ties@ 6 in	2 legs #5 Ties@ 8 in
3	24	36	5 #10	5 #7	None Req'd	2 legs #5 Ties@ 6 in	2 legs #5 Ties@ 8 in
2	24	36	5 #10	5 #7	None Req'd	2 legs #5 Ties@ 6 in	2 legs #5 Ties@ 8 in

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Figure 46: Design Example 7 from *2006 IBC Structural/Seismic Design Manual*

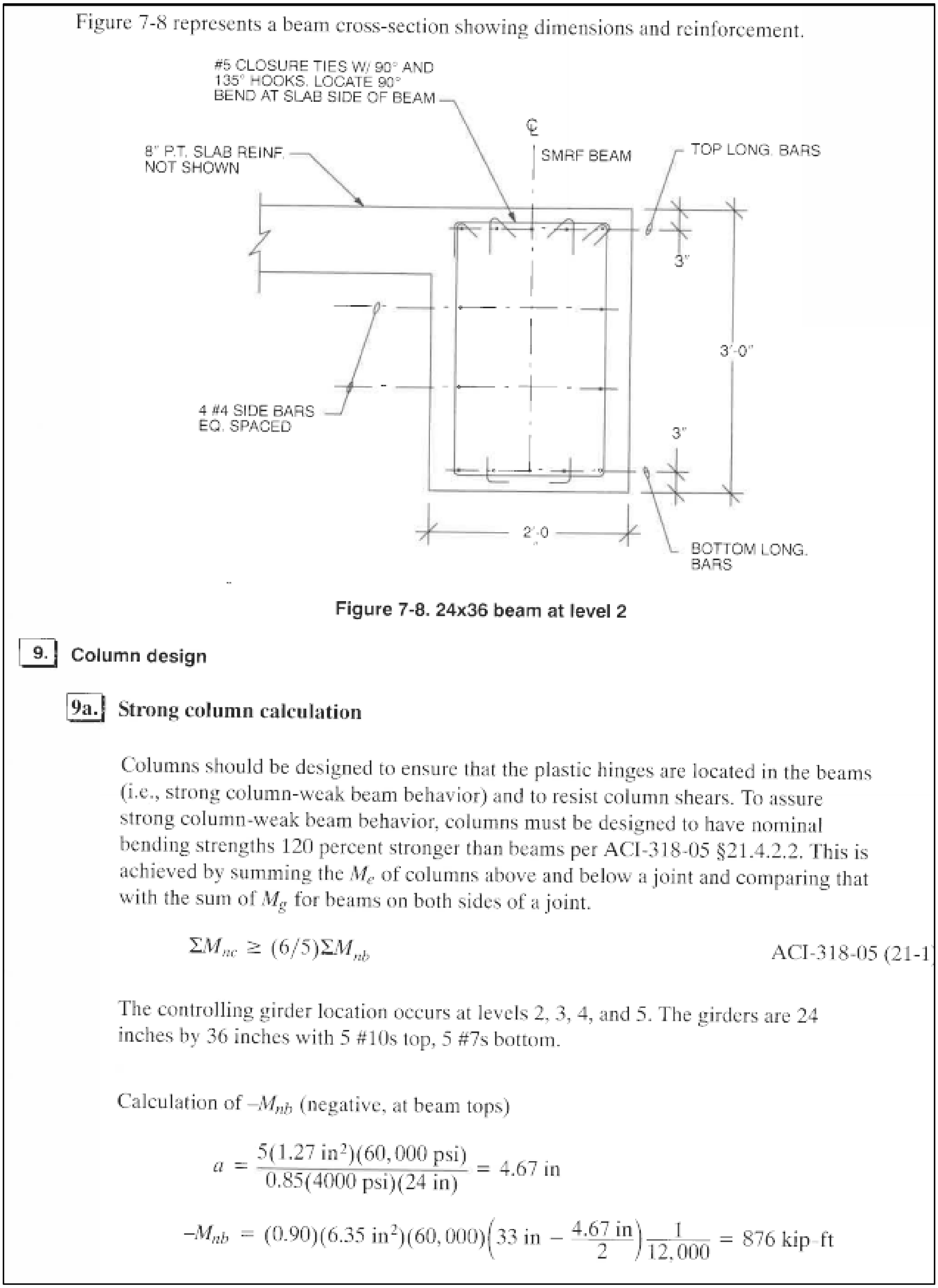


Figure 7-8 represents a beam cross-section showing dimensions and reinforcement.

Figure 7-8. 24x36 beam at level 2

9. Column design

9a. Strong column calculation

Columns should be designed to ensure that the plastic hinges are located in the beams (i.e., strong column-weak beam behavior) and to resist column shears. To assure strong column-weak beam behavior, columns must be designed to have nominal bending strengths 120 percent stronger than beams per ACI-318-05 §21.4.2.2. This is achieved by summing the M_e of columns above and below a joint and comparing that with the sum of M_g for beams on both sides of a joint.

$$\Sigma M_{nc} \geq (6/5)\Sigma M_{nb} \qquad \text{ACI-318-05 (21-1}$$

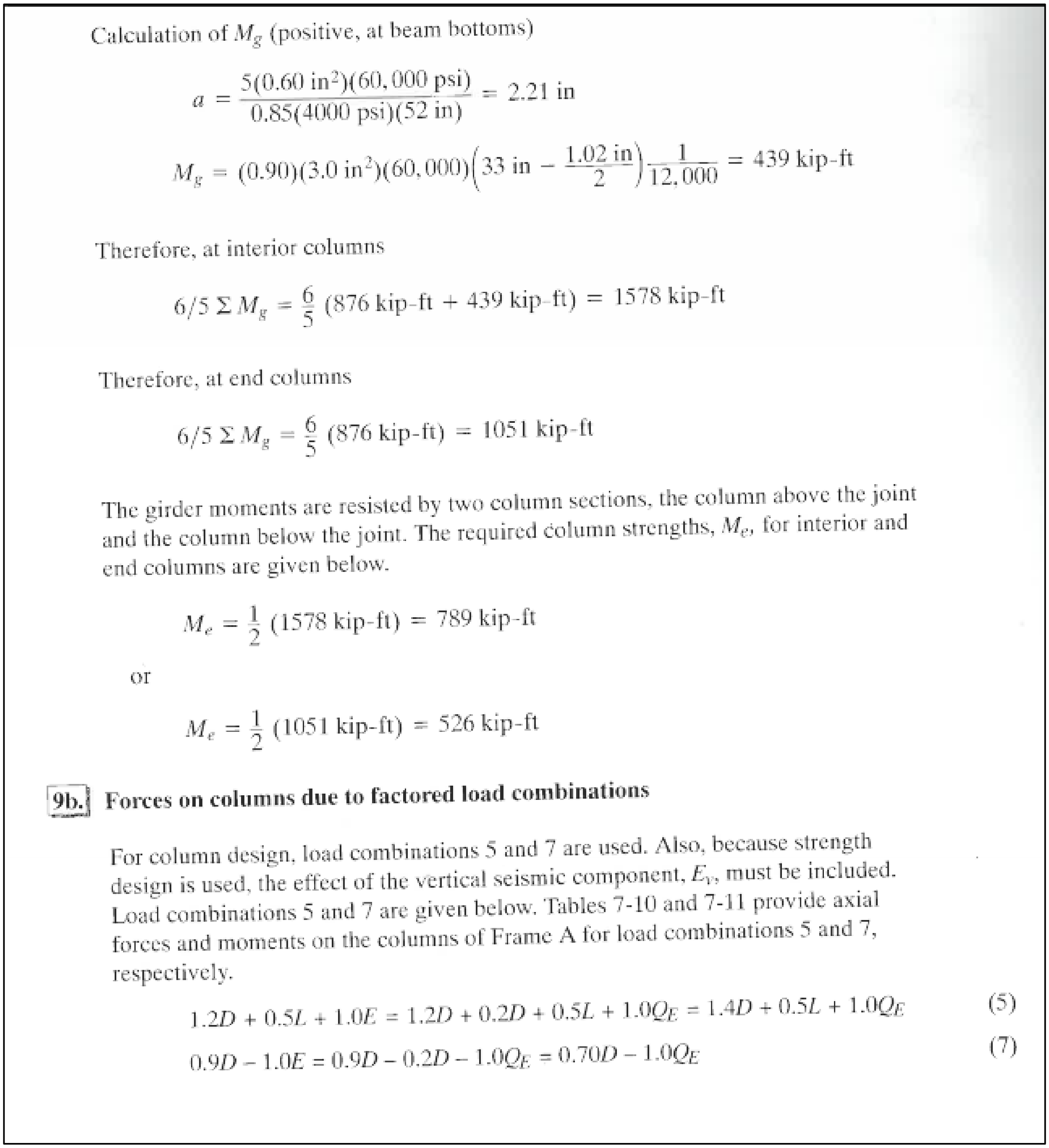
The controlling girder location occurs at levels 2, 3, 4, and 5. The girders are 24 inches by 36 inches with 5 #10s top, 5 #7s bottom.

Calculation of $-M_{nb}$ (negative, at beam tops)

$$a = \frac{5(1.27\ \text{in}^2)(60,000\ \text{psi})}{0.85(4000\ \text{psi})(24\ \text{in})} = 4.67\ \text{in}$$

$$-M_{nb} = (0.90)(6.35\ \text{in}^2)(60,000)\left(33\ \text{in} - \frac{4.67\ \text{in}}{2}\right)\frac{1}{12,000} = 876\ \text{kip-ft}$$

Figure 47: Design Example 7 from *2006 IBC Structural/Seismic Design Manual*



Calculation of M_g (positive, at beam bottoms)

$$a = \frac{5(0.60\text{ in}^2)(60,000\text{ psi})}{0.85(4000\text{ psi})(52\text{ in})} = 2.21\text{ in}$$

$$M_g = (0.90)(3.0\text{ in}^2)(60,000)\left(33\text{ in} - \frac{1.02\text{ in}}{2}\right)\frac{1}{12,000} = 439\text{ kip-ft}$$

Therefore, at interior columns

$$6/5\,\Sigma M_g = \frac{6}{5}(876\text{ kip-ft} + 439\text{ kip-ft}) = 1578\text{ kip-ft}$$

Therefore, at end columns

$$6/5\,\Sigma M_g = \frac{6}{5}(876\text{ kip-ft}) = 1051\text{ kip-ft}$$

The girder moments are resisted by two column sections, the column above the joint and the column below the joint. The required column strengths, M_e, for interior and end columns are given below.

$$M_e = \frac{1}{2}(1578\text{ kip-ft}) = 789\text{ kip-ft}$$

or

$$M_e = \frac{1}{2}(1051\text{ kip-ft}) = 526\text{ kip-ft}$$

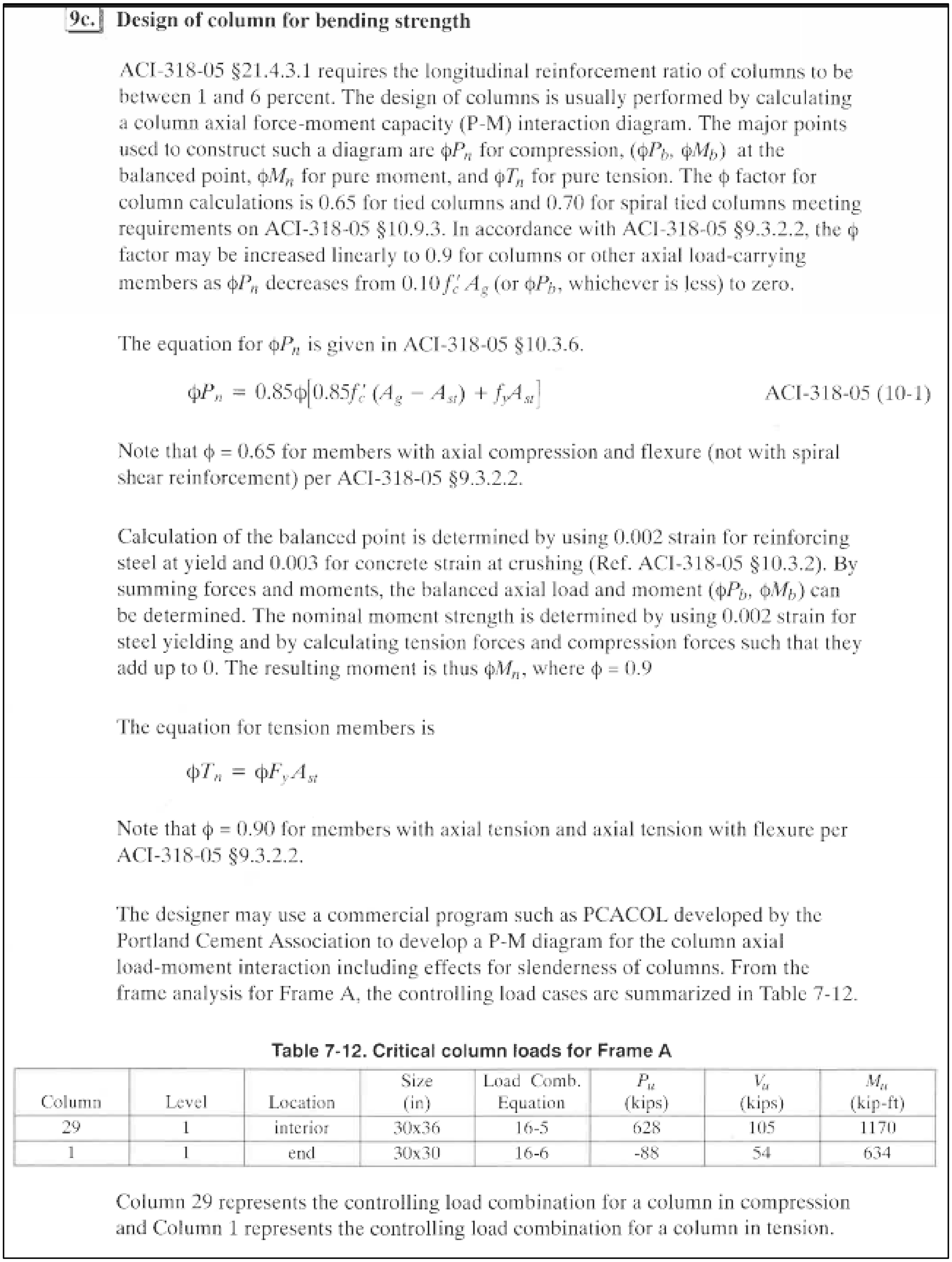
9b. Forces on columns due to factored load combinations

For column design, load combinations 5 and 7 are used. Also, because strength design is used, the effect of the vertical seismic component, E_v, must be included. Load combinations 5 and 7 are given below. Tables 7-10 and 7-11 provide axial forces and moments on the columns of Frame A for load combinations 5 and 7, respectively.

$$1.2D + 0.5L + 1.0E = 1.2D + 0.2D + 0.5L + 1.0Q_E = 1.4D + 0.5L + 1.0Q_E \quad (5)$$

$$0.9D - 1.0E = 0.9D - 0.2D - 1.0Q_E = 0.70D - 1.0Q_E \quad (7)$$

Figure 48: Design Example 7 from *2006 IBC Structural/Seismic Design Manual*



9c. Design of column for bending strength

ACI-318-05 §21.4.3.1 requires the longitudinal reinforcement ratio of columns to be between 1 and 6 percent. The design of columns is usually performed by calculating a column axial force-moment capacity (P-M) interaction diagram. The major points used to construct such a diagram are ϕP_n for compression, $(\phi P_b, \phi M_b)$ at the balanced point, ϕM_n for pure moment, and ϕT_n for pure tension. The ϕ factor for column calculations is 0.65 for tied columns and 0.70 for spiral tied columns meeting requirements on ACI-318-05 §10.9.3. In accordance with ACI-318-05 §9.3.2.2, the ϕ factor may be increased linearly to 0.9 for columns or other axial load-carrying members as ϕP_n decreases from $0.10 f'_c A_g$ (or ϕP_b, whichever is less) to zero.

The equation for ϕP_n is given in ACI-318-05 §10.3.6.

$$\phi P_n = 0.85\phi\left[0.85 f'_c (A_g - A_{st}) + f_y A_{st}\right] \qquad \text{ACI-318-05 (10-1)}$$

Note that $\phi = 0.65$ for members with axial compression and flexure (not with spiral shear reinforcement) per ACI-318-05 §9.3.2.2.

Calculation of the balanced point is determined by using 0.002 strain for reinforcing steel at yield and 0.003 for concrete strain at crushing (Ref. ACI-318-05 §10.3.2). By summing forces and moments, the balanced axial load and moment $(\phi P_b, \phi M_b)$ can be determined. The nominal moment strength is determined by using 0.002 strain for steel yielding and by calculating tension forces and compression forces such that they add up to 0. The resulting moment is thus ϕM_n, where $\phi = 0.9$

The equation for tension members is

$$\phi T_n = \phi F_y A_{st}$$

Note that $\phi = 0.90$ for members with axial tension and axial tension with flexure per ACI-318-05 §9.3.2.2.

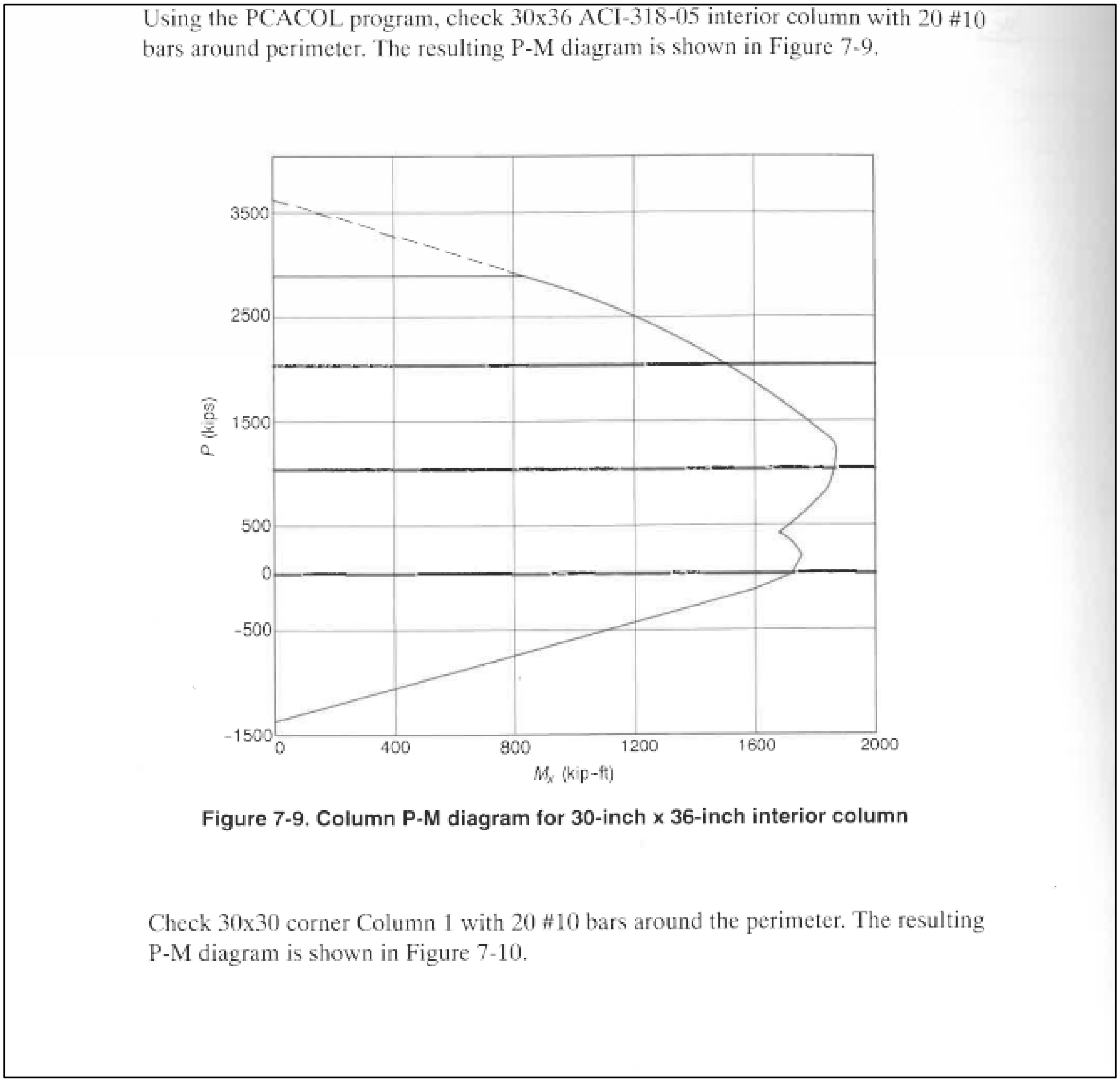
The designer may use a commercial program such as PCACOL developed by the Portland Cement Association to develop a P-M diagram for the column axial load-moment interaction including effects for slenderness of columns. From the frame analysis for Frame A, the controlling load cases are summarized in Table 7-12.

Table 7-12. Critical column loads for Frame A

Column	Level	Location	Size (in)	Load Comb. Equation	P_u (kips)	V_u (kips)	M_u (kip-ft)
29	1	interior	30x36	16-5	628	105	1170
1	1	end	30x30	16-6	-88	54	634

Column 29 represents the controlling load combination for a column in compression and Column 1 represents the controlling load combination for a column in tension.

Figure 49: Design Example 7 from *2006 IBC Structural/Seismic Design Manual*



Using the PCACOL program, check 30x36 ACI-318-05 interior column with 20 #10 bars around perimeter. The resulting P-M diagram is shown in Figure 7-9.

Figure 7-9. Column P-M diagram for 30-inch x 36-inch interior column

Check 30x30 corner Column 1 with 20 #10 bars around the perimeter. The resulting P-M diagram is shown in Figure 7-10.

Figure 50: Design Example 7 from *2006 IBC Structural/Seismic Design Manual*

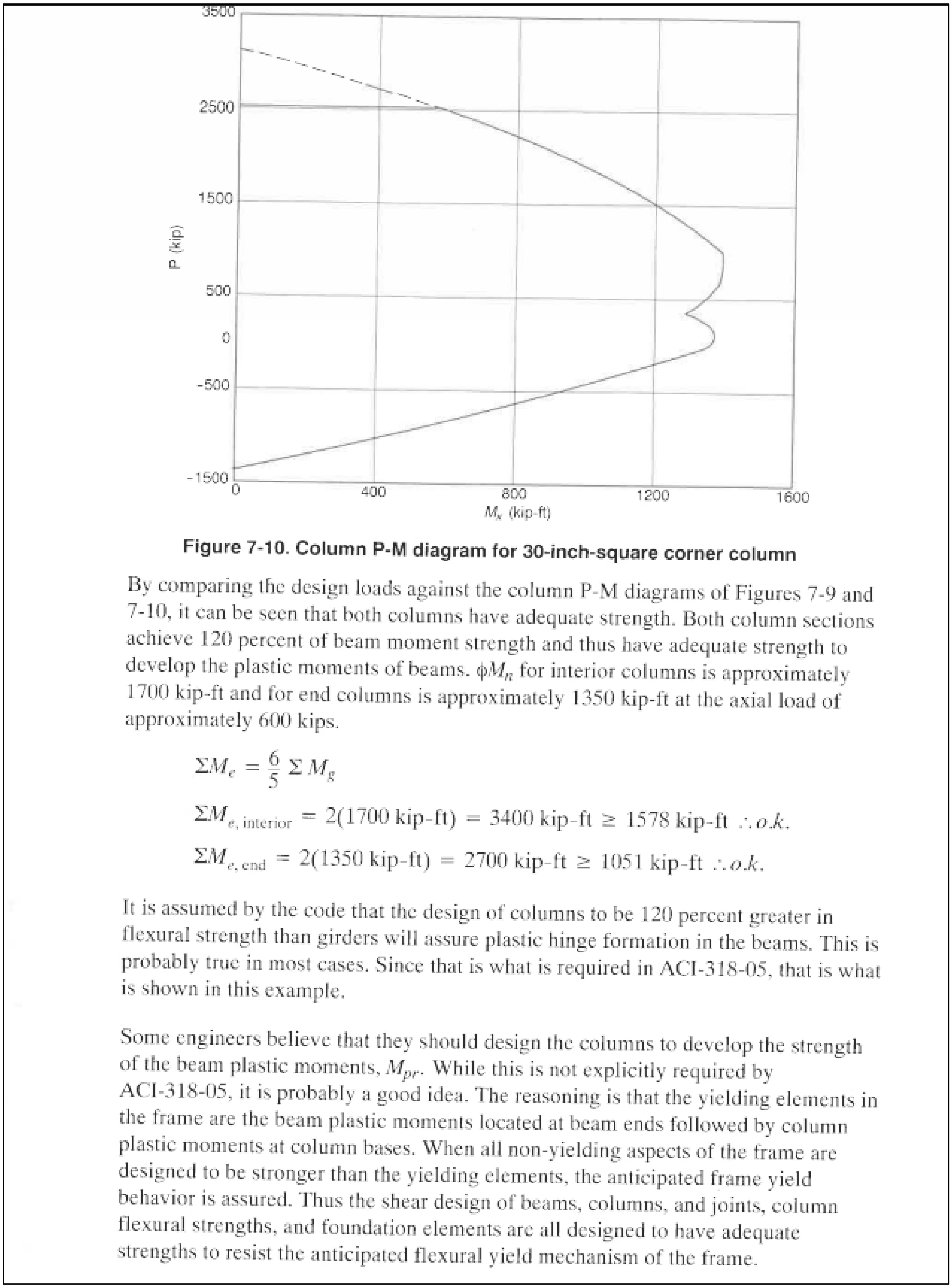


Figure 7-10. Column P-M diagram for 30-inch-square corner column

By comparing the design loads against the column P-M diagrams of Figures 7-9 and 7-10, it can be seen that both columns have adequate strength. Both column sections achieve 120 percent of beam moment strength and thus have adequate strength to develop the plastic moments of beams. ϕM_n for interior columns is approximately 1700 kip-ft and for end columns is approximately 1350 kip-ft at the axial load of approximately 600 kips.

$$\Sigma M_e = \frac{6}{5}\,\Sigma M_g$$

$$\Sigma M_{e,\,\text{interior}} = 2(1700 \text{ kip-ft}) = 3400 \text{ kip-ft} \geq 1578 \text{ kip-ft} \;\therefore o.k.$$

$$\Sigma M_{e,\,\text{end}} = 2(1350 \text{ kip-ft}) = 2700 \text{ kip-ft} \geq 1051 \text{ kip-ft} \;\therefore o.k.$$

It is assumed by the code that the design of columns to be 120 percent greater in flexural strength than girders will assure plastic hinge formation in the beams. This is probably true in most cases. Since that is what is required in ACI-318-05, that is what is shown in this example.

Some engineers believe that they should design the columns to develop the strength of the beam plastic moments, M_{pr}. While this is not explicitly required by ACI-318-05, it is probably a good idea. The reasoning is that the yielding elements in the frame are the beam plastic moments located at beam ends followed by column plastic moments at column bases. When all non-yielding aspects of the frame are designed to be stronger than the yielding elements, the anticipated frame yield behavior is assured. Thus the shear design of beams, columns, and joints, column flexural strengths, and foundation elements are all designed to have adequate strengths to resist the anticipated flexural yield mechanism of the frame.

Figure 51: Design Example 7 from *2006 IBC Structural/Seismic Design Manual*

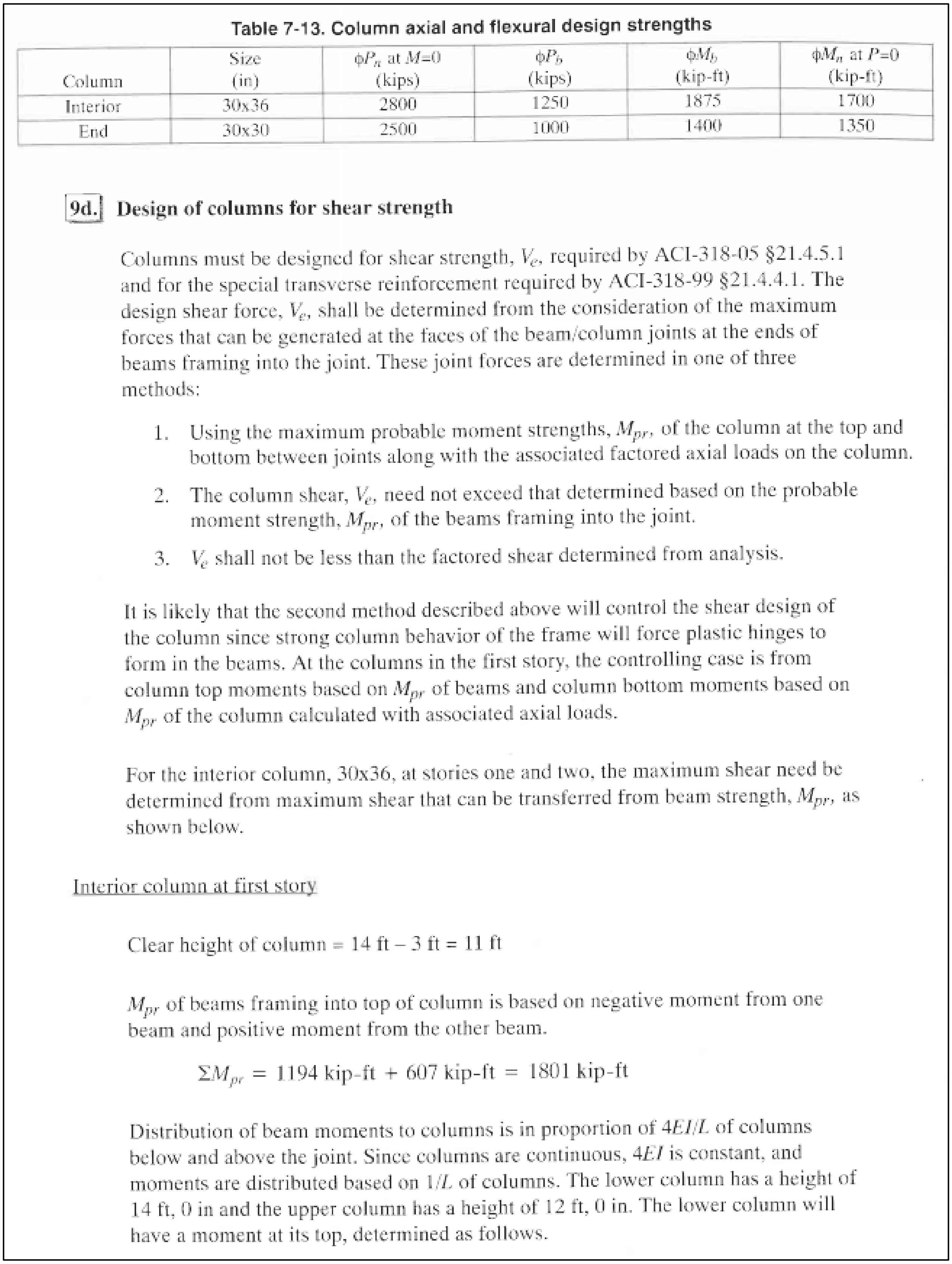


Table 7-13. Column axial and flexural design strengths

Column	Size (in)	ϕP_n at M=0 (kips)	ϕP_b (kips)	ϕM_b (kip-ft)	ϕM_n at P=0 (kip-ft)
Interior	30x36	2800	1250	1875	1700
End	30x30	2500	1000	1400	1350

9d. Design of columns for shear strength

Columns must be designed for shear strength, V_e, required by ACI-318-05 §21.4.5.1 and for the special transverse reinforcement required by ACI-318-99 §21.4.4.1. The design shear force, V_e, shall be determined from the consideration of the maximum forces that can be generated at the faces of the beam/column joints at the ends of beams framing into the joint. These joint forces are determined in one of three methods:

1. Using the maximum probable moment strengths, M_{pr}, of the column at the top and bottom between joints along with the associated factored axial loads on the column.
2. The column shear, V_e, need not exceed that determined based on the probable moment strength, M_{pr}, of the beams framing into the joint.
3. V_e shall not be less than the factored shear determined from analysis.

It is likely that the second method described above will control the shear design of the column since strong column behavior of the frame will force plastic hinges to form in the beams. At the columns in the first story, the controlling case is from column top moments based on M_{pr} of beams and column bottom moments based on M_{pr} of the column calculated with associated axial loads.

For the interior column, 30x36, at stories one and two, the maximum shear need be determined from maximum shear that can be transferred from beam strength, M_{pr}, as shown below.

Interior column at first story

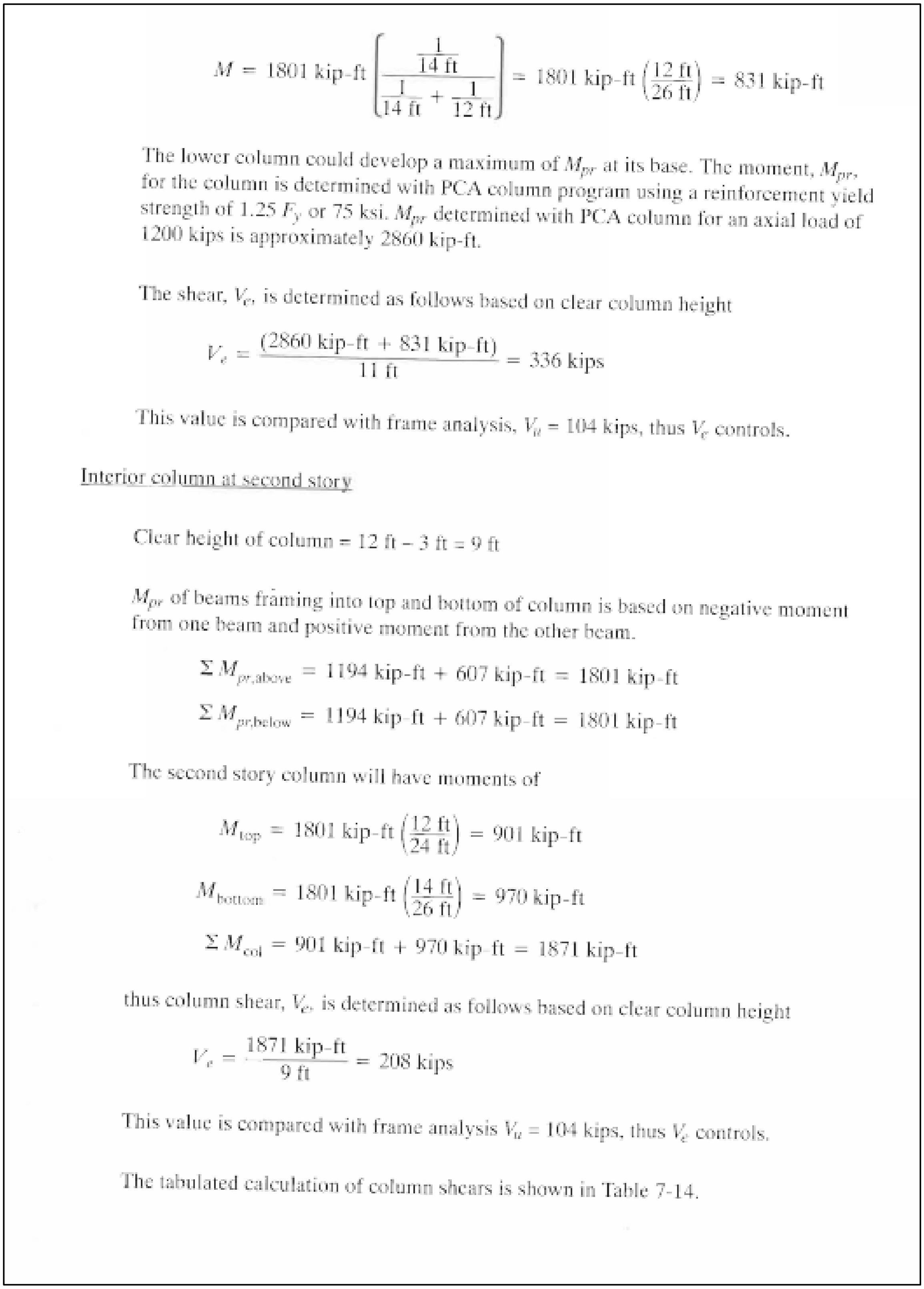
Clear height of column = 14 ft – 3 ft = 11 ft

M_{pr} of beams framing into top of column is based on negative moment from one beam and positive moment from the other beam.

$$\Sigma M_{pr} = 1194 \text{ kip-ft} + 607 \text{ kip-ft} = 1801 \text{ kip-ft}$$

Distribution of beam moments to columns is in proportion of $4EI/L$ of columns below and above the joint. Since columns are continuous, $4EI$ is constant, and moments are distributed based on $1/L$ of columns. The lower column has a height of 14 ft, 0 in and the upper column has a height of 12 ft, 0 in. The lower column will have a moment at its top, determined as follows.

Figure 52: Design Example 7 from *2006 IBC Structural/Seismic Design Manual*



$$M = 1801\text{ kip-ft}\left[\frac{\frac{1}{14\text{ ft}}}{\frac{1}{14\text{ ft}}+\frac{1}{12\text{ ft}}}\right] = 1801\text{ kip-ft}\left(\frac{12\text{ ft}}{26\text{ ft}}\right) = 831\text{ kip-ft}$$

The lower column could develop a maximum of M_{pr} at its base. The moment, M_{pr}, for the column is determined with PCA column program using a reinforcement yield strength of 1.25 F_y or 75 ksi. M_{pr} determined with PCA column for an axial load of 1200 kips is approximately 2860 kip-ft.

The shear, V_e, is determined as follows based on clear column height

$$V_e = \frac{(2860\text{ kip-ft} + 831\text{ kip-ft})}{11\text{ ft}} = 336\text{ kips}$$

This value is compared with frame analysis, V_u = 104 kips, thus V_e controls.

Interior column at second story

Clear height of column = 12 ft – 3 ft = 9 ft

M_{pr} of beams framing into top and bottom of column is based on negative moment from one beam and positive moment from the other beam.

$$\Sigma M_{pr,\text{above}} = 1194\text{ kip-ft} + 607\text{ kip-ft} = 1801\text{ kip-ft}$$

$$\Sigma M_{pr,\text{below}} = 1194\text{ kip-ft} + 607\text{ kip-ft} = 1801\text{ kip-ft}$$

The second story column will have moments of

$$M_{\text{top}} = 1801\text{ kip-ft}\left(\frac{12\text{ ft}}{24\text{ ft}}\right) = 901\text{ kip-ft}$$

$$M_{\text{bottom}} = 1801\text{ kip-ft}\left(\frac{14\text{ ft}}{26\text{ ft}}\right) = 970\text{ kip-ft}$$

$$\Sigma M_{\text{col}} = 901\text{ kip-ft} + 970\text{ kip-ft} = 1871\text{ kip-ft}$$

thus column shear, V_e, is determined as follows based on clear column height

$$V_e = \frac{1871\text{ kip-ft}}{9\text{ ft}} = 208\text{ kips}$$

This value is compared with frame analysis V_u = 104 kips, thus V_e controls.

The tabulated calculation of column shears is shown in Table 7-14.

Figure 53: Design Example 7 from *2006 IBC Structural/Seismic Design Manual*

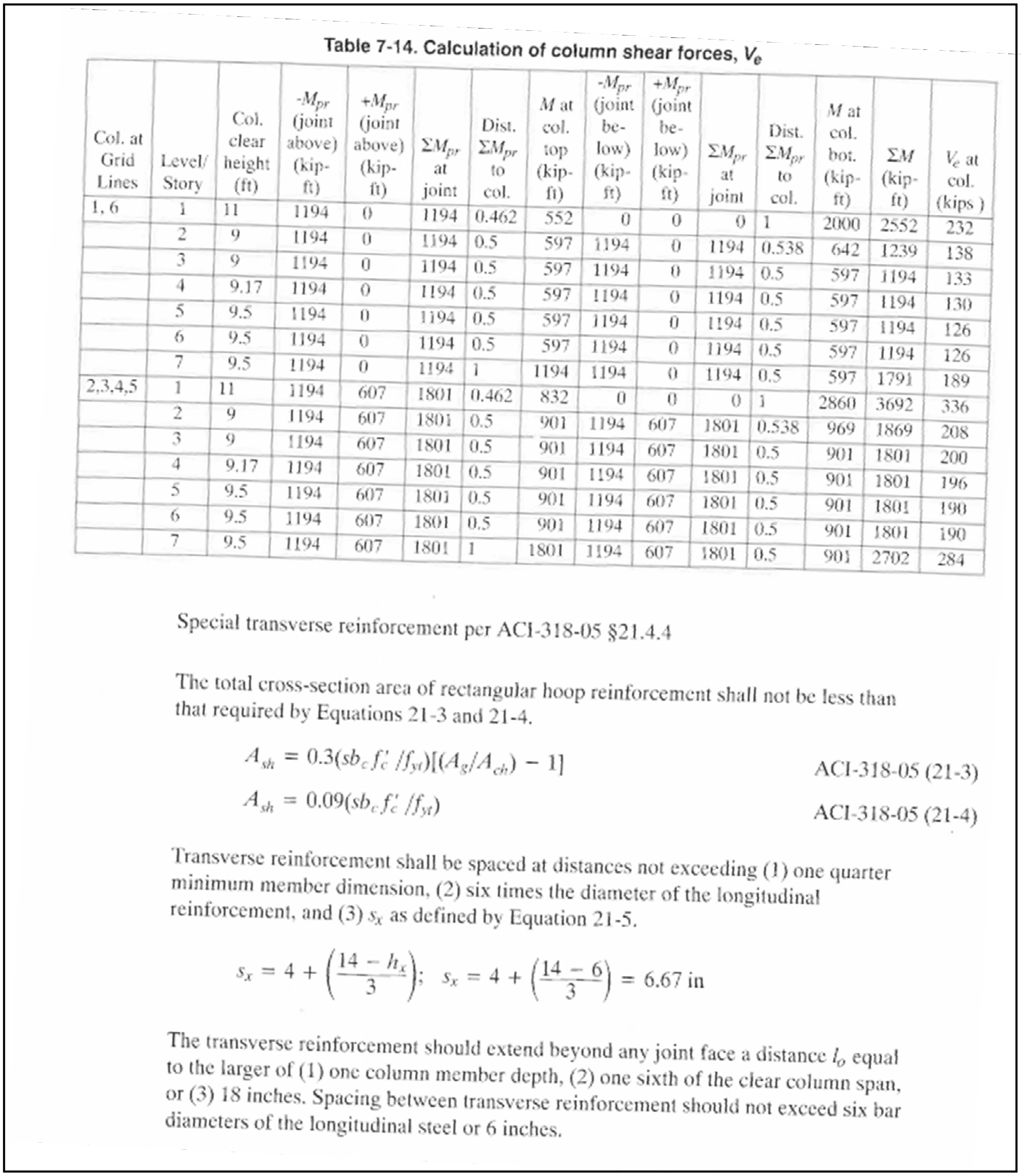


Table 7-14. Calculation of column shear forces, V_e

Col. at Grid Lines	Level/ Story	Col. clear height (ft)	$-M_{pr}$ (joint above) (kip-ft)	$+M_{pr}$ (joint above) (kip-ft)	ΣM_{pr} at joint	Dist. ΣM_{pr} to col.	M at col. top (kip-ft)	$-M_{pr}$ (joint below) (kip-ft)	$+M_{pr}$ (joint below) (kip-ft)	ΣM_{pr} at joint	Dist. ΣM_{pr} to col.	M at col. bot. (kip-ft)	ΣM (kip-ft)	V_e at col. (kips)
1, 6	1	11	1194	0	1194	0.462	552	0	0	0	1	2000	2552	232
	2	9	1194	0	1194	0.5	597	1194	0	1194	0.538	642	1239	138
	3	9	1194	0	1194	0.5	597	1194	0	1194	0.5	597	1194	133
	4	9.17	1194	0	1194	0.5	597	1194	0	1194	0.5	597	1194	130
	5	9.5	1194	0	1194	0.5	597	1194	0	1194	0.5	597	1194	126
	6	9.5	1194	0	1194	0.5	597	1194	0	1194	0.5	597	1194	126
	7	9.5	1194	0	1194	1	1194	1194	0	1194	0.5	597	1791	189
2,3,4,5	1	11	1194	607	1801	0.462	832	0	0	0	1	2860	3692	336
	2	9	1194	607	1801	0.5	901	1194	607	1801	0.538	969	1869	208
	3	9	1194	607	1801	0.5	901	1194	607	1801	0.5	901	1801	200
	4	9.17	1194	607	1801	0.5	901	1194	607	1801	0.5	901	1801	196
	5	9.5	1194	607	1801	0.5	901	1194	607	1801	0.5	901	1801	190
	6	9.5	1194	607	1801	0.5	901	1194	607	1801	0.5	901	1801	190
	7	9.5	1194	607	1801	1	1801	1194	607	1801	0.5	901	2702	284

Special transverse reinforcement per ACI-318-05 §21.4.4

The total cross-section area of rectangular hoop reinforcement shall not be less than that required by Equations 21-3 and 21-4.

$$A_{sh} = 0.3(sb_c f_c' / f_{yt})[(A_g/A_{ch}) - 1] \qquad \text{ACI-318-05 (21-3)}$$

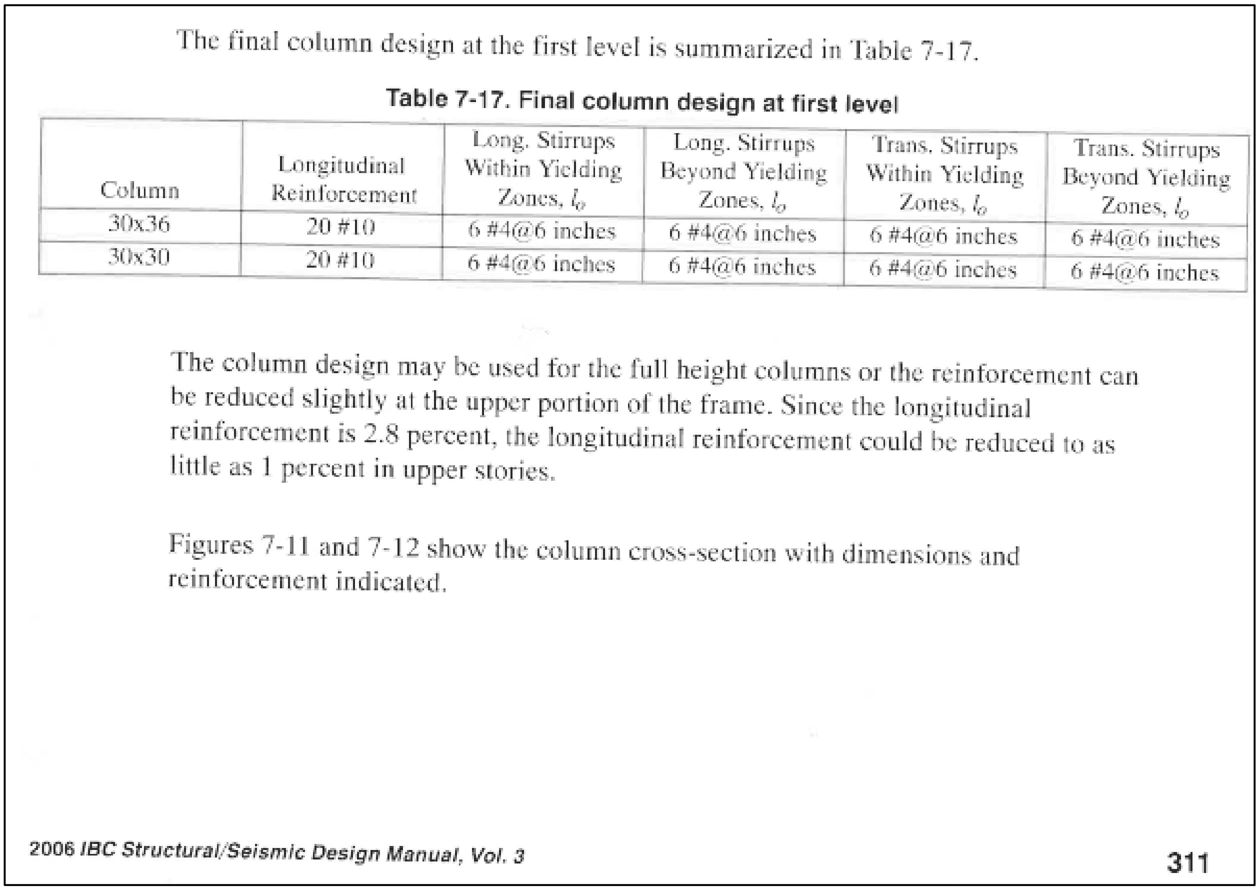
$$A_{sh} = 0.09(sb_c f_c' / f_{yt}) \qquad \text{ACI-318-05 (21-4)}$$

Transverse reinforcement shall be spaced at distances not exceeding (1) one quarter minimum member dimension, (2) six times the diameter of the longitudinal reinforcement, and (3) s_x as defined by Equation 21-5.

$$s_x = 4 + \left(\frac{14 - h_x}{3}\right); \quad s_x = 4 + \left(\frac{14 - 6}{3}\right) = 6.67 \text{ in}$$

The transverse reinforcement should extend beyond any joint face a distance l_o equal to the larger of (1) one column member depth, (2) one sixth of the clear column span, or (3) 18 inches. Spacing between transverse reinforcement should not exceed six bar diameters of the longitudinal steel or 6 inches.

Figure 54: Design Example 7 from *2006 IBC Structural/Seismic Design Manual*



The final column design at the first level is summarized in Table 7-17.

Table 7-17. Final column design at first level

Column	Longitudinal Reinforcement	Long. Stirrups Within Yielding Zones, l_o	Long. Stirrups Beyond Yielding Zones, l_o	Trans. Stirrups Within Yielding Zones, l_o	Trans. Stirrups Beyond Yielding Zones, l_o
30x36	20 #10	6 #4@6 inches	6 #4@6 inches	6 #4@6 inches	6 #4@6 inches
30x30	20 #10	6 #4@6 inches	6 #4@6 inches	6 #4@6 inches	6 #4@6 inches

The column design may be used for the full height columns or the reinforcement can be reduced slightly at the upper portion of the frame. Since the longitudinal reinforcement is 2.8 percent, the longitudinal reinforcement could be reduced to as little as 1 percent in upper stories.

Figures 7-11 and 7-12 show the column cross-section with dimensions and reinforcement indicated.

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Figure 55: Design Example 7 from *2006 IBC Structural/Seismic Design Manual*

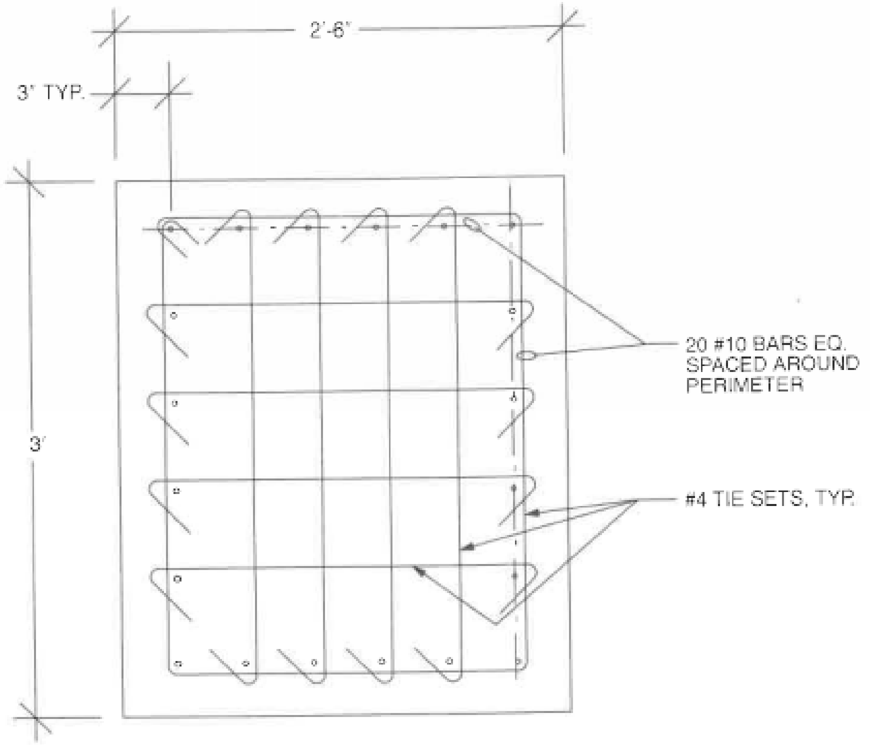


Figure 7-11. 30x36 column

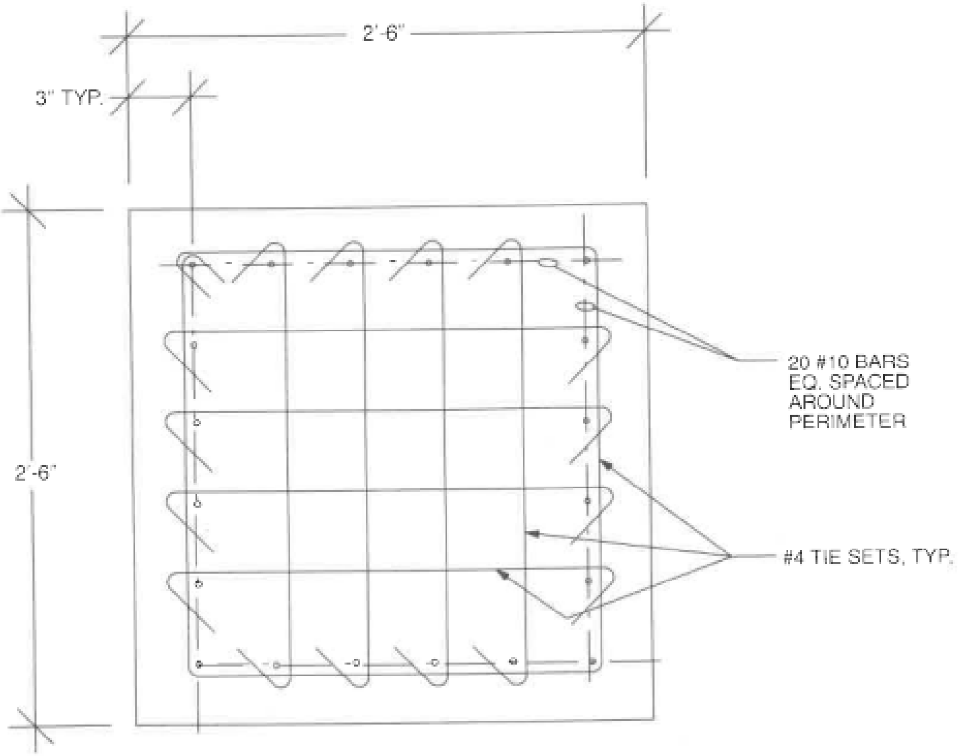
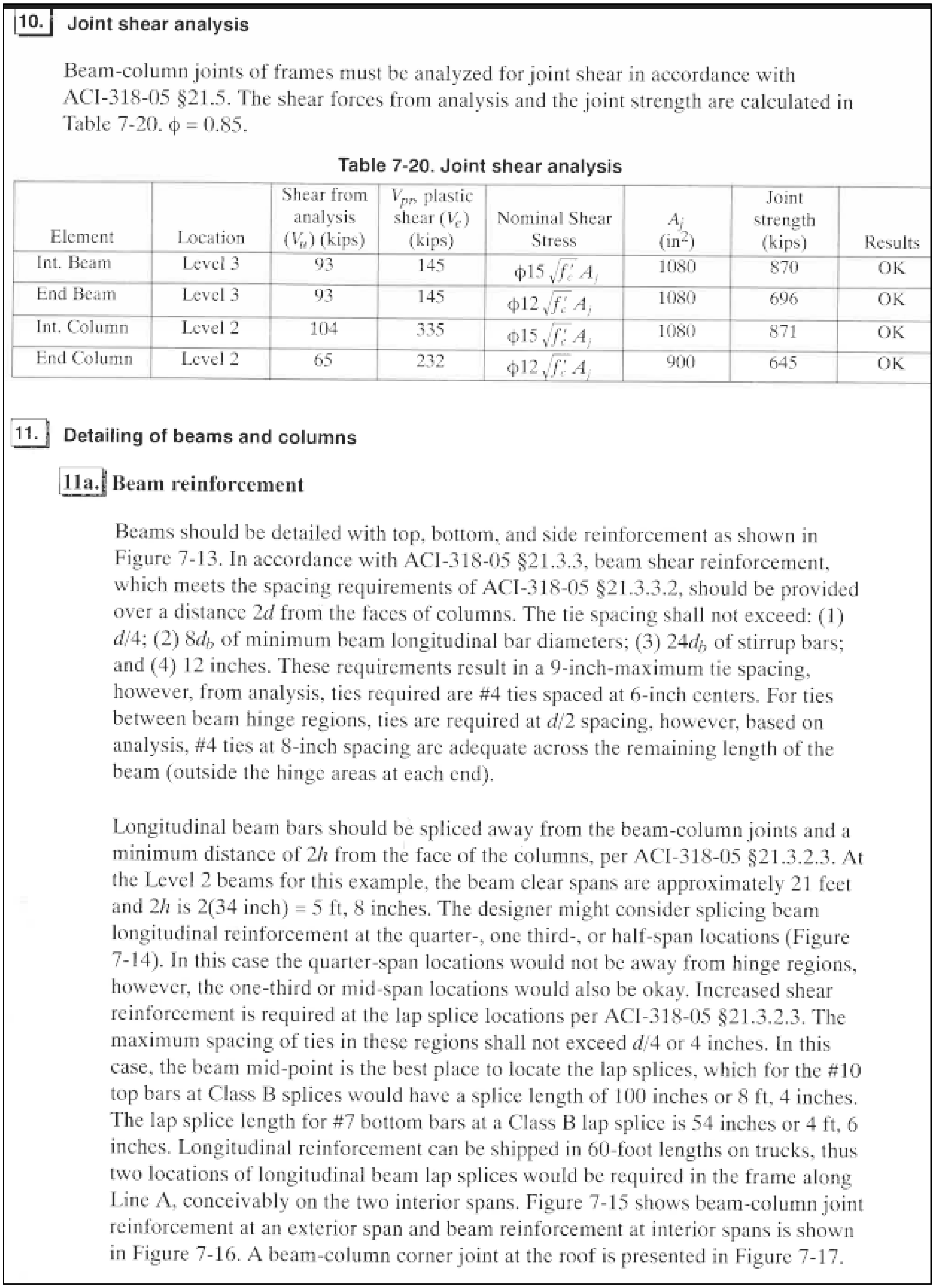


Figure 7-12. 30x30 column

Figure 56: Design Example 7 from *2006 IBC Structural/Seismic Design Manual*



10. **Joint shear analysis**

Beam-column joints of frames must be analyzed for joint shear in accordance with ACI-318-05 §21.5. The shear forces from analysis and the joint strength are calculated in Table 7-20. $\phi = 0.85$.

Table 7-20. Joint shear analysis

Element	Location	Shear from analysis (V_u) (kips)	V_{pr}, plastic shear (V_e) (kips)	Nominal Shear Stress	A_j (in^2)	Joint strength (kips)	Results
Int. Beam	Level 3	93	145	$\phi 15\sqrt{f'_c}\,A_j$	1080	870	OK
End Beam	Level 3	93	145	$\phi 12\sqrt{f'_c}\,A_j$	1080	696	OK
Int. Column	Level 2	104	335	$\phi 15\sqrt{f'_c}\,A_j$	1080	871	OK
End Column	Level 2	65	232	$\phi 12\sqrt{f'_c}\,A_j$	900	645	OK

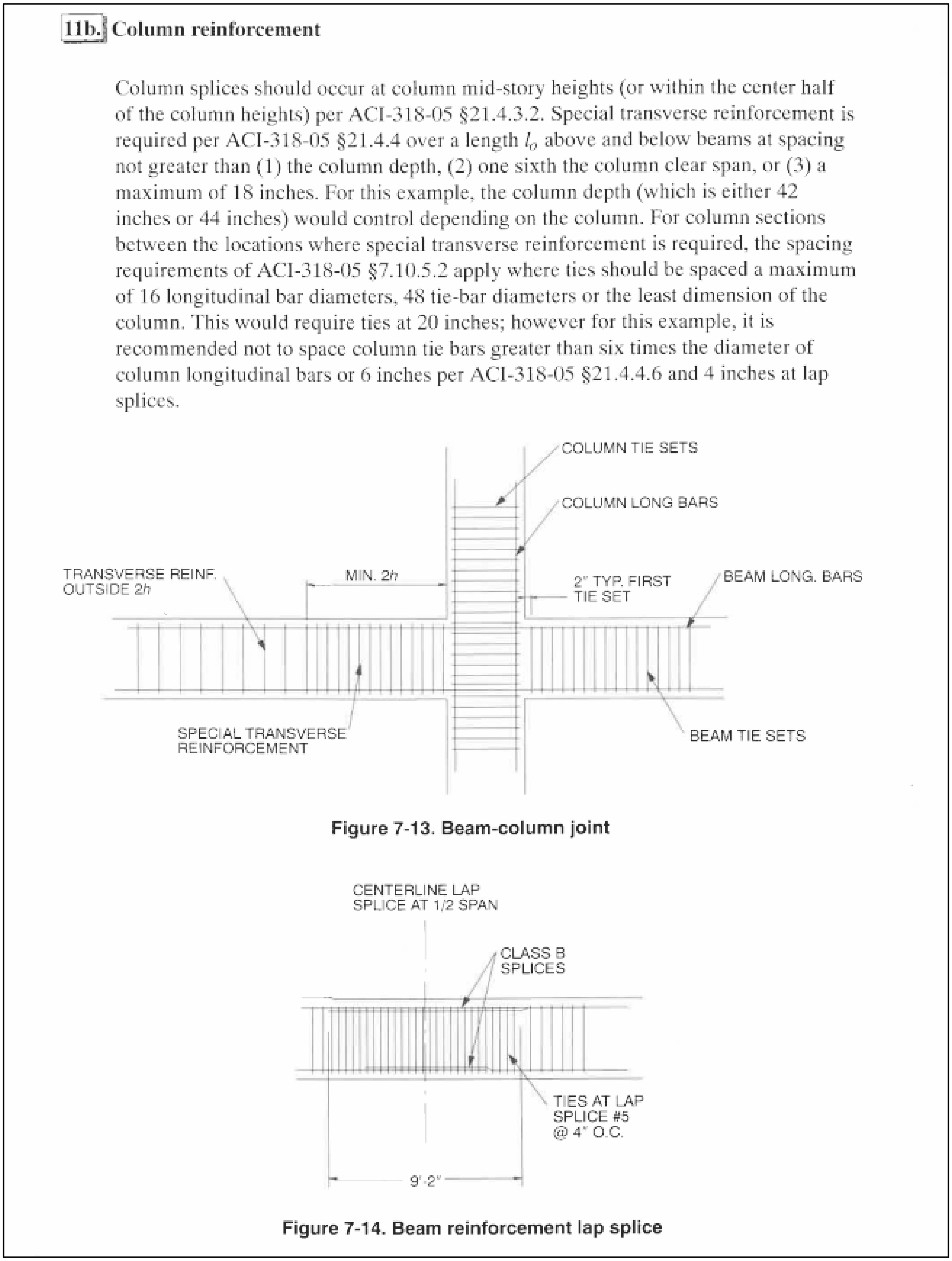
11. **Detailing of beams and columns**

11a. **Beam reinforcement**

Beams should be detailed with top, bottom, and side reinforcement as shown in Figure 7-13. In accordance with ACI-318-05 §21.3.3, beam shear reinforcement, which meets the spacing requirements of ACI-318-05 §21.3.3.2, should be provided over a distance $2d$ from the faces of columns. The tie spacing shall not exceed: (1) $d/4$; (2) $8d_b$ of minimum beam longitudinal bar diameters; (3) $24d_b$ of stirrup bars; and (4) 12 inches. These requirements result in a 9-inch-maximum tie spacing, however, from analysis, ties required are #4 ties spaced at 6-inch centers. For ties between beam hinge regions, ties are required at $d/2$ spacing, however, based on analysis, #4 ties at 8-inch spacing are adequate across the remaining length of the beam (outside the hinge areas at each end).

Longitudinal beam bars should be spliced away from the beam-column joints and a minimum distance of $2h$ from the face of the columns, per ACI-318-05 §21.3.2.3. At the Level 2 beams for this example, the beam clear spans are approximately 21 feet and $2h$ is 2(34 inch) = 5 ft, 8 inches. The designer might consider splicing beam longitudinal reinforcement at the quarter-, one third-, or half-span locations (Figure 7-14). In this case the quarter-span locations would not be away from hinge regions, however, the one-third or mid-span locations would also be okay. Increased shear reinforcement is required at the lap splice locations per ACI-318-05 §21.3.2.3. The maximum spacing of ties in these regions shall not exceed $d/4$ or 4 inches. In this case, the beam mid-point is the best place to locate the lap splices, which for the #10 top bars at Class B splices would have a splice length of 100 inches or 8 ft, 4 inches. The lap splice length for #7 bottom bars at a Class B lap splice is 54 inches or 4 ft, 6 inches. Longitudinal reinforcement can be shipped in 60-foot lengths on trucks, thus two locations of longitudinal beam lap splices would be required in the frame along Line A, conceivably on the two interior spans. Figure 7-15 shows beam-column joint reinforcement at an exterior span and beam reinforcement at interior spans is shown in Figure 7-16. A beam-column corner joint at the roof is presented in Figure 7-17.

Figure 57: Design Example 7 from *2006 IBC Structural/Seismic Design Manual*



11b. **Column reinforcement**

Column splices should occur at column mid-story heights (or within the center half of the column heights) per ACI-318-05 §21.4.3.2. Special transverse reinforcement is required per ACI-318-05 §21.4.4 over a length l_o above and below beams at spacing not greater than (1) the column depth, (2) one sixth the column clear span, or (3) a maximum of 18 inches. For this example, the column depth (which is either 42 inches or 44 inches) would control depending on the column. For column sections between the locations where special transverse reinforcement is required, the spacing requirements of ACI-318-05 §7.10.5.2 apply where ties should be spaced a maximum of 16 longitudinal bar diameters, 48 tie-bar diameters or the least dimension of the column. This would require ties at 20 inches; however for this example, it is recommended not to space column tie bars greater than six times the diameter of column longitudinal bars or 6 inches per ACI-318-05 §21.4.4.6 and 4 inches at lap splices.

Figure 7-13. Beam-column joint

Figure 7-14. Beam reinforcement lap splice

Figure 58: Design Example 7 from *2006 IBC Structural/Seismic Design Manual*

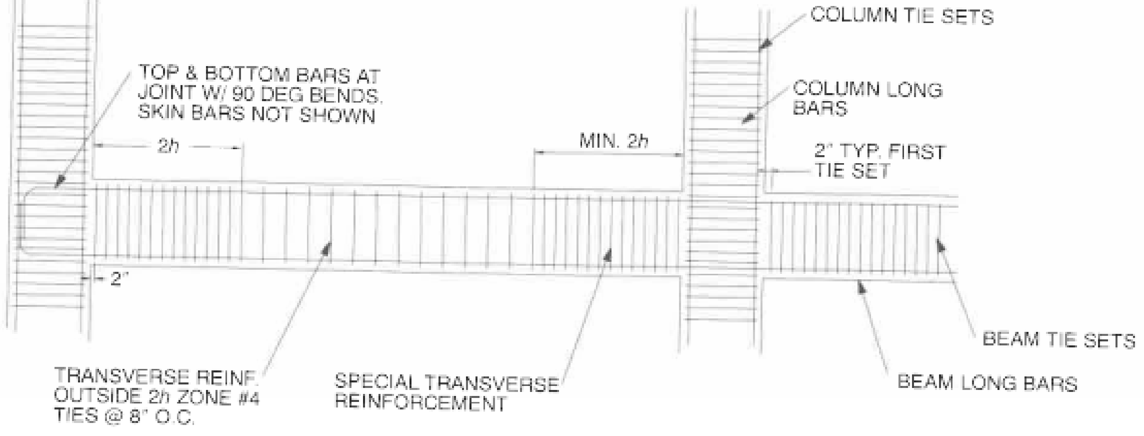


Figure 7-15. Beam-column joint reinforcement at exterior span

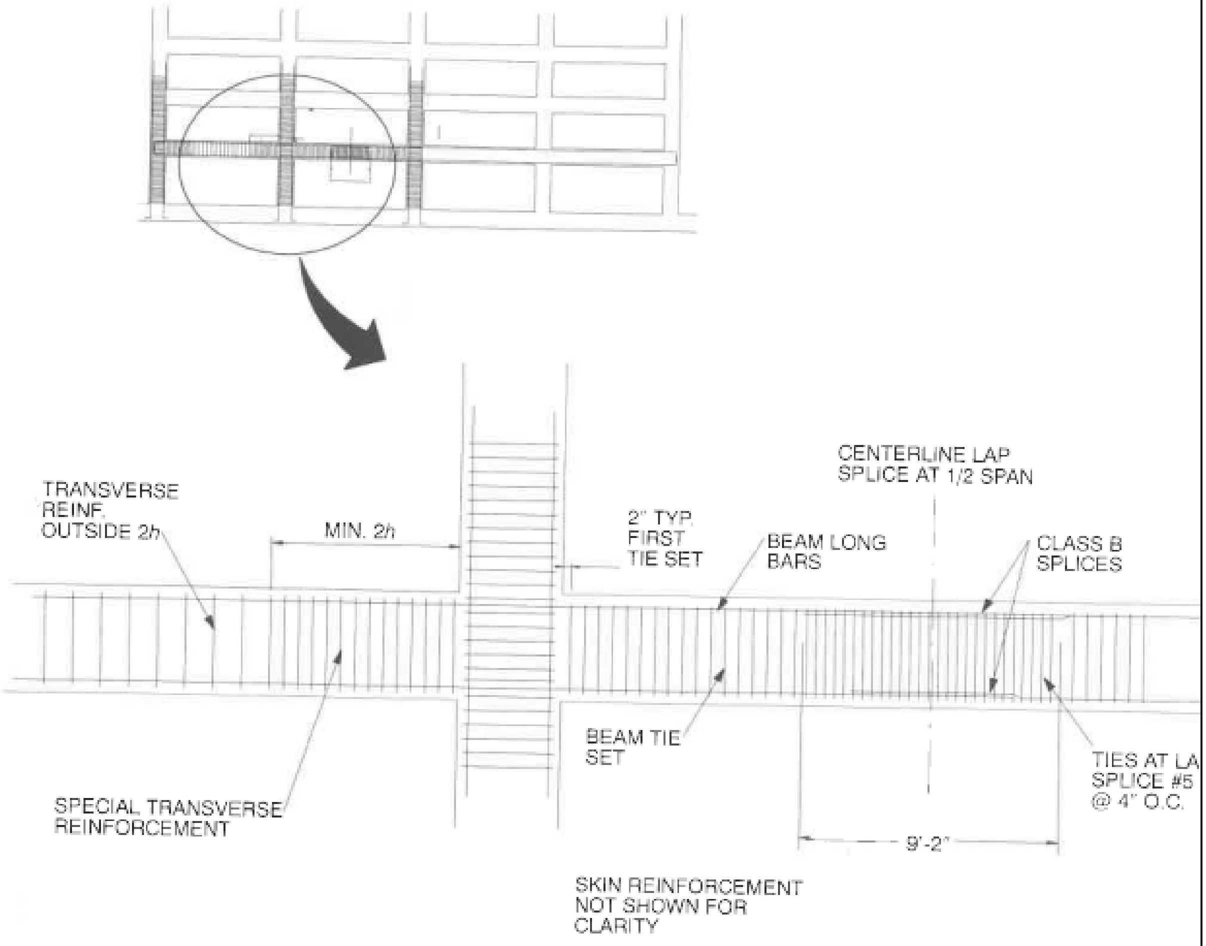


Figure 7-16. Beam reinforcement at interior spans

Figure 59: Design Example 7 from *2006 IBC Structural/Seismic Design Manual*

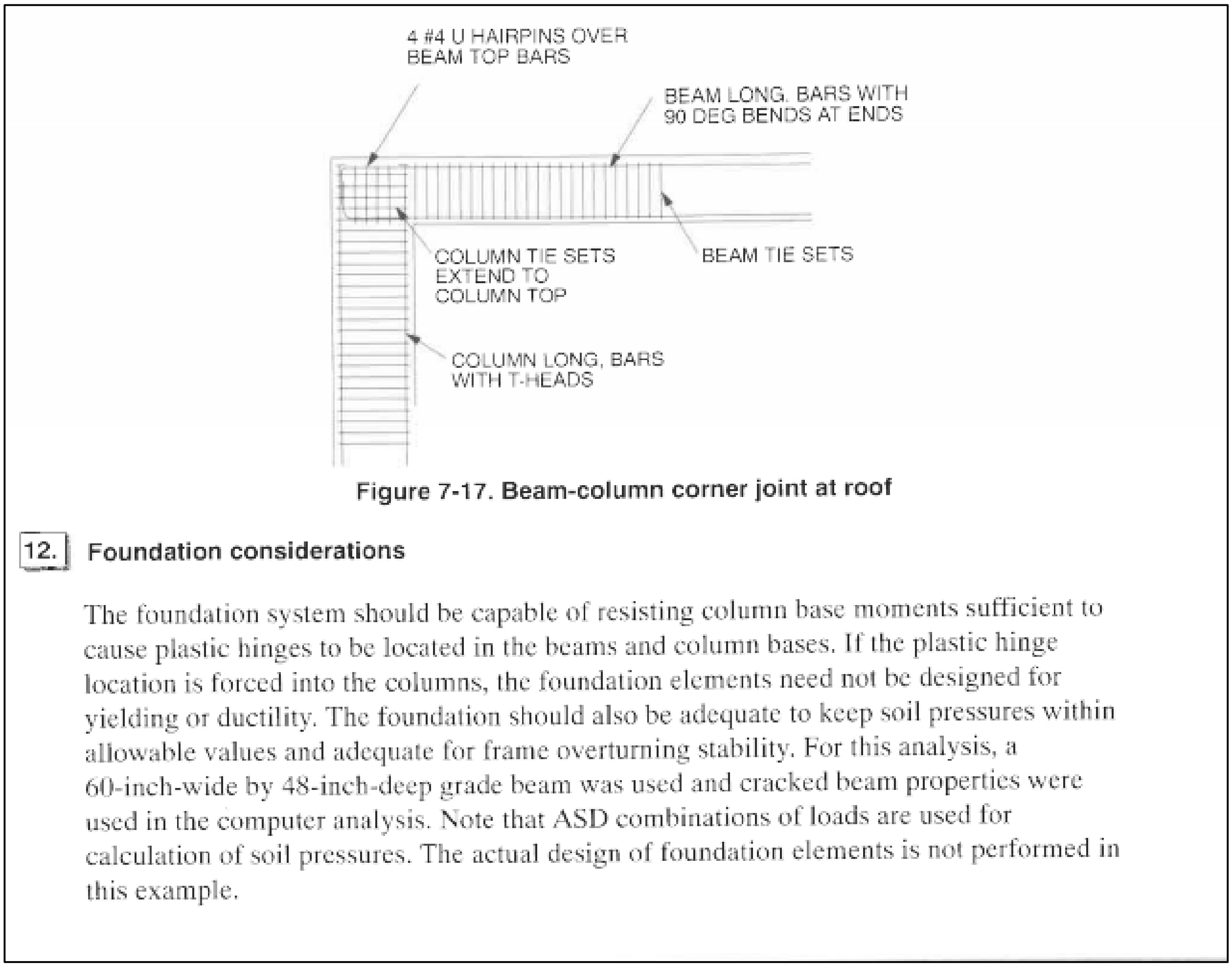


Figure 7-17. Beam-column corner joint at roof

12. **Foundation considerations**

The foundation system should be capable of resisting column base moments sufficient to cause plastic hinges to be located in the beams and column bases. If the plastic hinge location is forced into the columns, the foundation elements need not be designed for yielding or ductility. The foundation should also be adequate to keep soil pressures within allowable values and adequate for frame overturning stability. For this analysis, a 60-inch-wide by 48-inch-deep grade beam was used and cracked beam properties were used in the computer analysis. Note that ASD combinations of loads are used for calculation of soil pressures. The actual design of foundation elements is not performed in this example.

Figure 60: Design Example 7 from *2006 IBC Structural/Seismic Design Manual*

Appendix B - Total Displacement of Columns

Total Displacement According to LS-DYNA

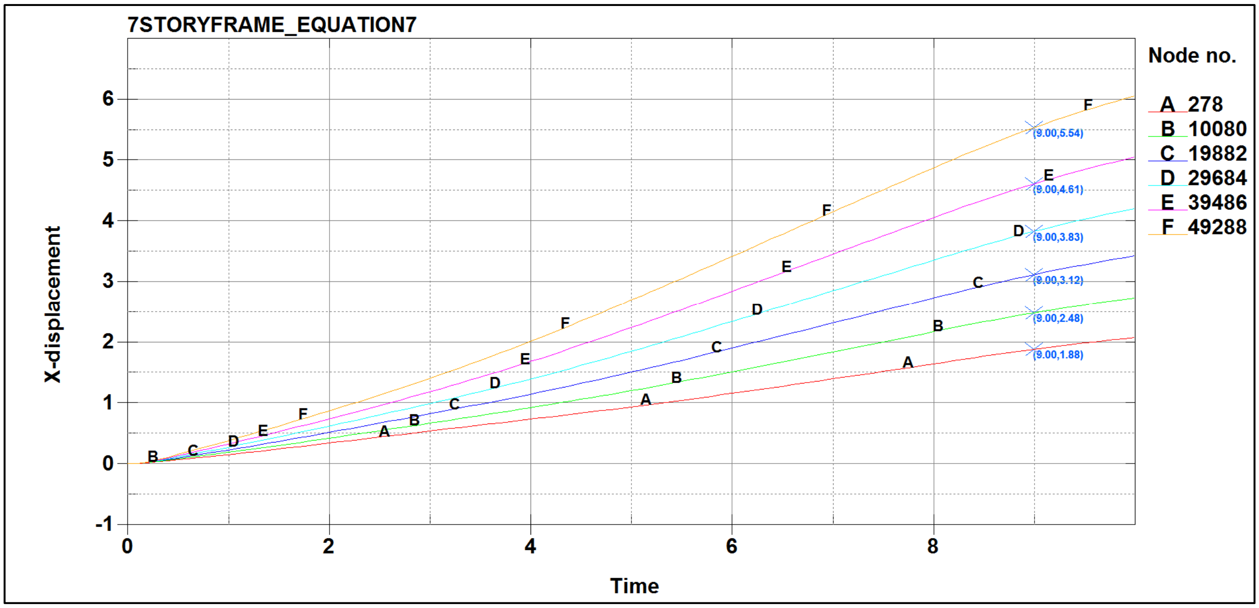


Figure 61: LS-DYNA Displacement

Table 28: X-Displacements for Frame 7.5.7

Column	Time (sec)	X-Displacement (in.)
A	9.00	1.88
B	9.00	2.48
C	9.00	3.12
D	9.00	3.83
E	9.00	4.61
F	9.00	5.54

Total Displacement by Hand Calculations

Constants: $f_y = 68$ ksi $d_b = 1.27$ in. $h_{clr} = 132$ in.

Column A

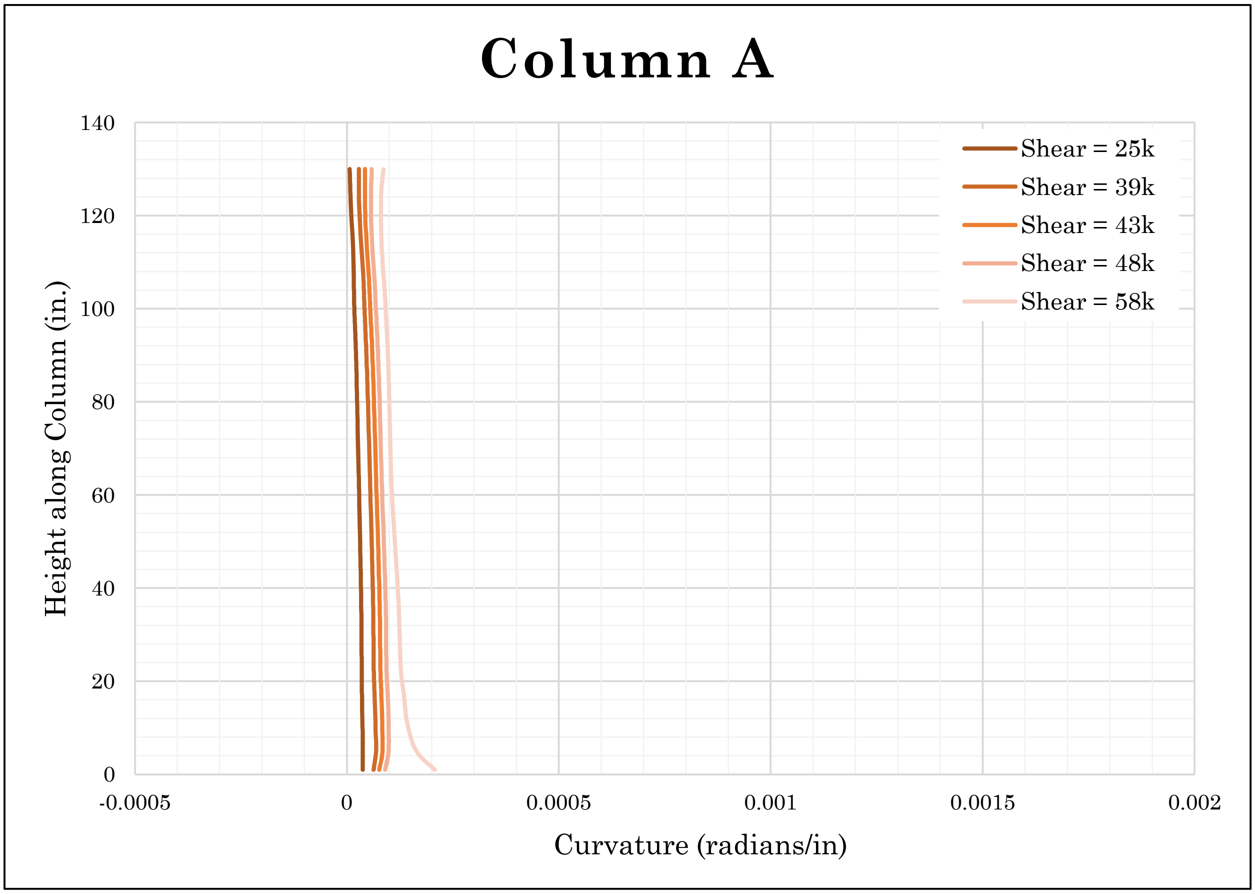


Figure 63: Curvature vs. Height Graph

Column A								
Flexural				Shear			Total Displacement	
$h =$	145.0	in	$h =$	145.0	in	$\Delta_f =$	1.698	in
$\varphi_1 =$	0.0003		$\gamma_{avg} =$	0.00126		$\Delta_s =$	0.183	in
$\varphi_y =$	0.000118							
$\varphi_2 =$	0.000086		$\Delta_s =$	0.183	in	$\mathbf{\Delta_{total} =}$	**1.880**	**in**
$l_p =$	23.5	in						
						LS-DYNA =	**1.88**	**in**
$\Delta_f =$	1.698	in						

Figure 62: Displacement Calculations

Column B

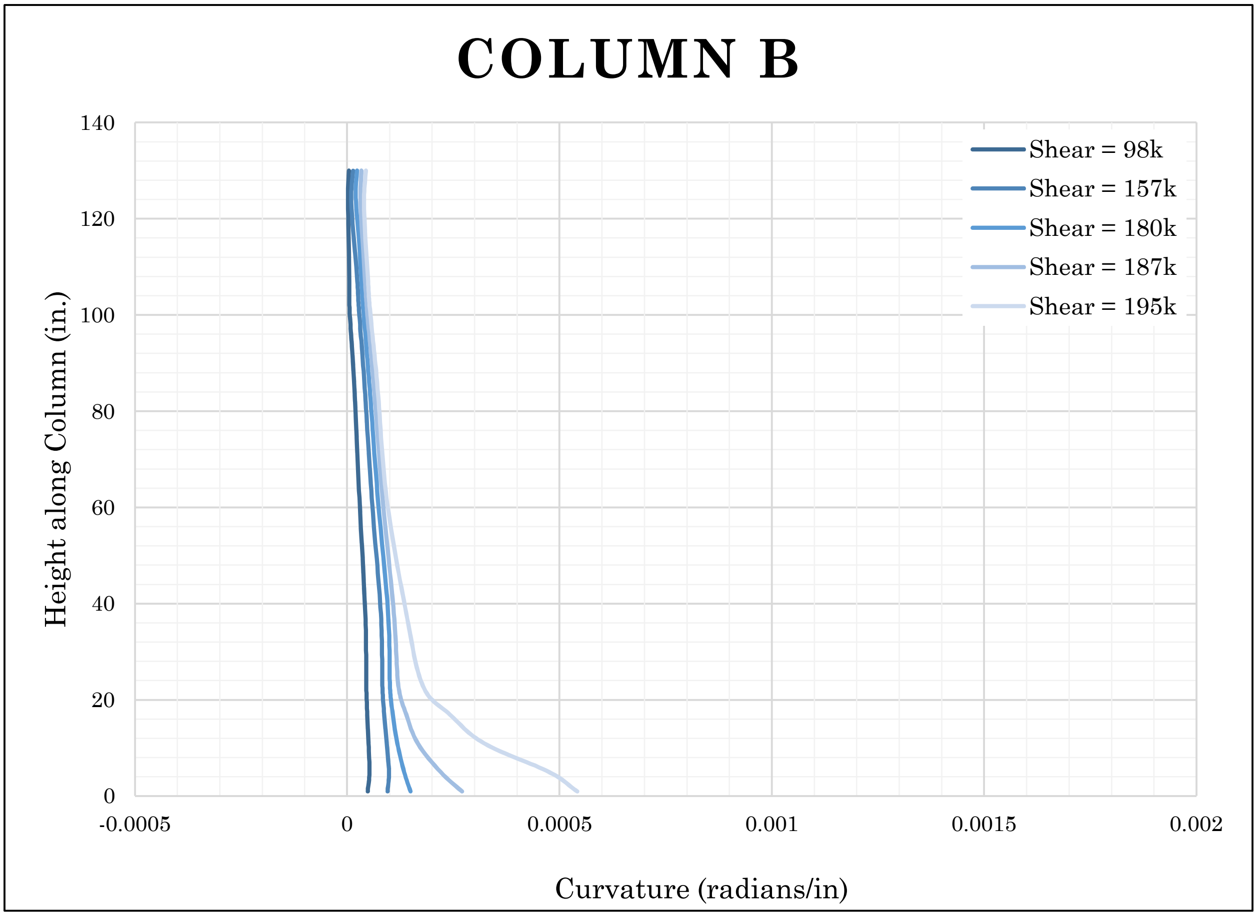
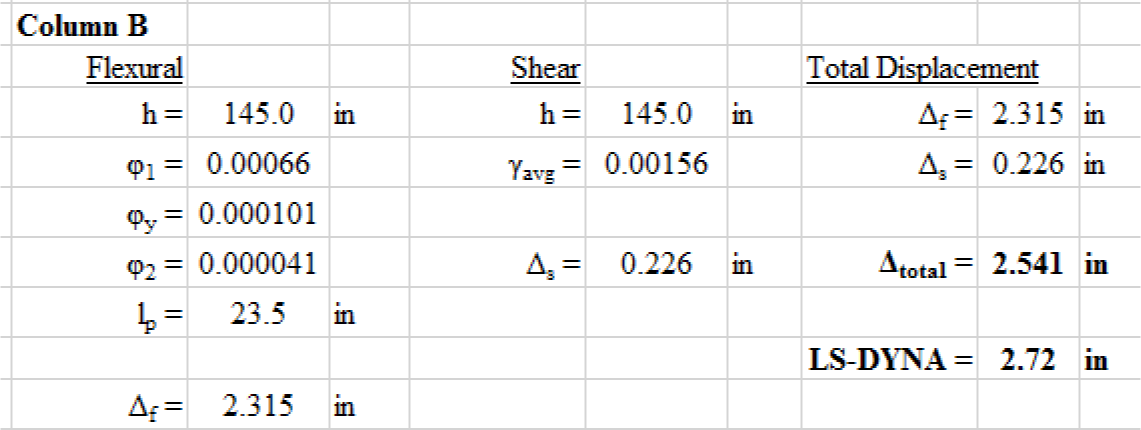


Figure 64: Curvature vs. Height Graph



Column B								
Flexural			Shear			Total Displacement		
h =	145.0	in	h =	145.0	in	Δ_f =	2.315	in
φ_1 =	0.00066		γ_{avg} =	0.00156		Δ_s =	0.226	in
φ_y =	0.000101							
φ_2 =	0.000041		Δ_s =	0.226	in	**Δ_{total} =**	**2.541**	**in**
l_p =	23.5	in						
						LS-DYNA =	**2.72**	**in**
Δ_f =	2.315	in						

Figure 65: Displacement Calculations

Column C

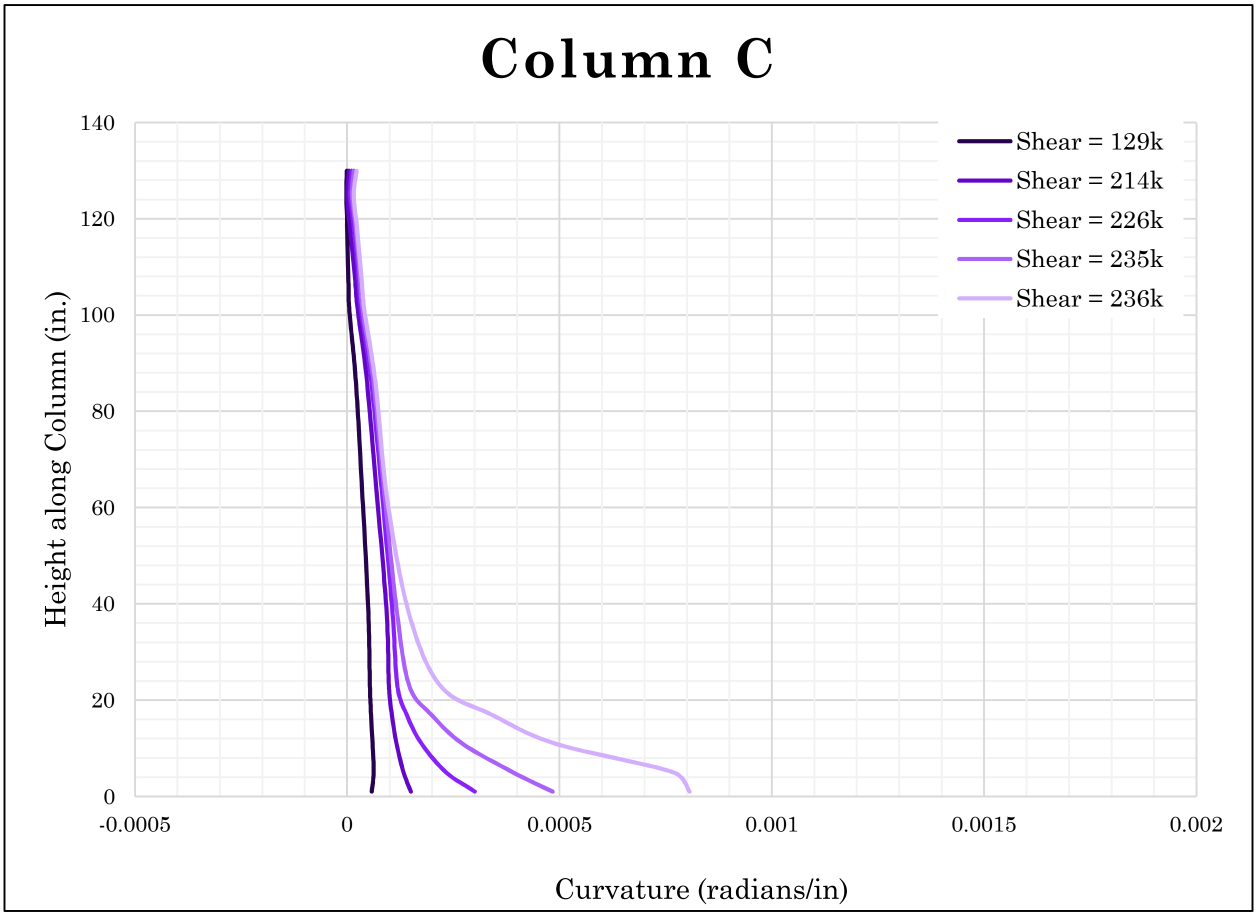
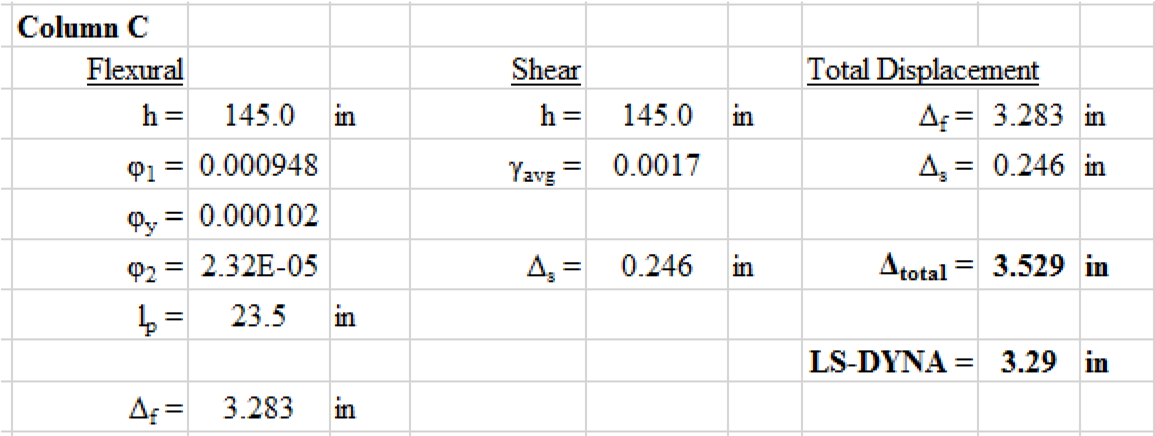


Figure 66: Curvature vs. Height Graph



Column C								
Flexural			Shear			Total Displacement		
h =	145.0	in	h =	145.0	in	Δ_f =	3.283	in
φ_1 =	0.000948		γ_{avg} =	0.0017		Δ_s =	0.246	in
φ_y =	0.000102							
φ_2 =	2.32E-05		Δ_s =	0.246	in	**Δ_{total} =**	**3.529**	**in**
l_p =	23.5	in						
						LS-DYNA =	**3.29**	**in**
Δ_f =	3.283	in						

Figure 67: Displacement Calculations

Column D

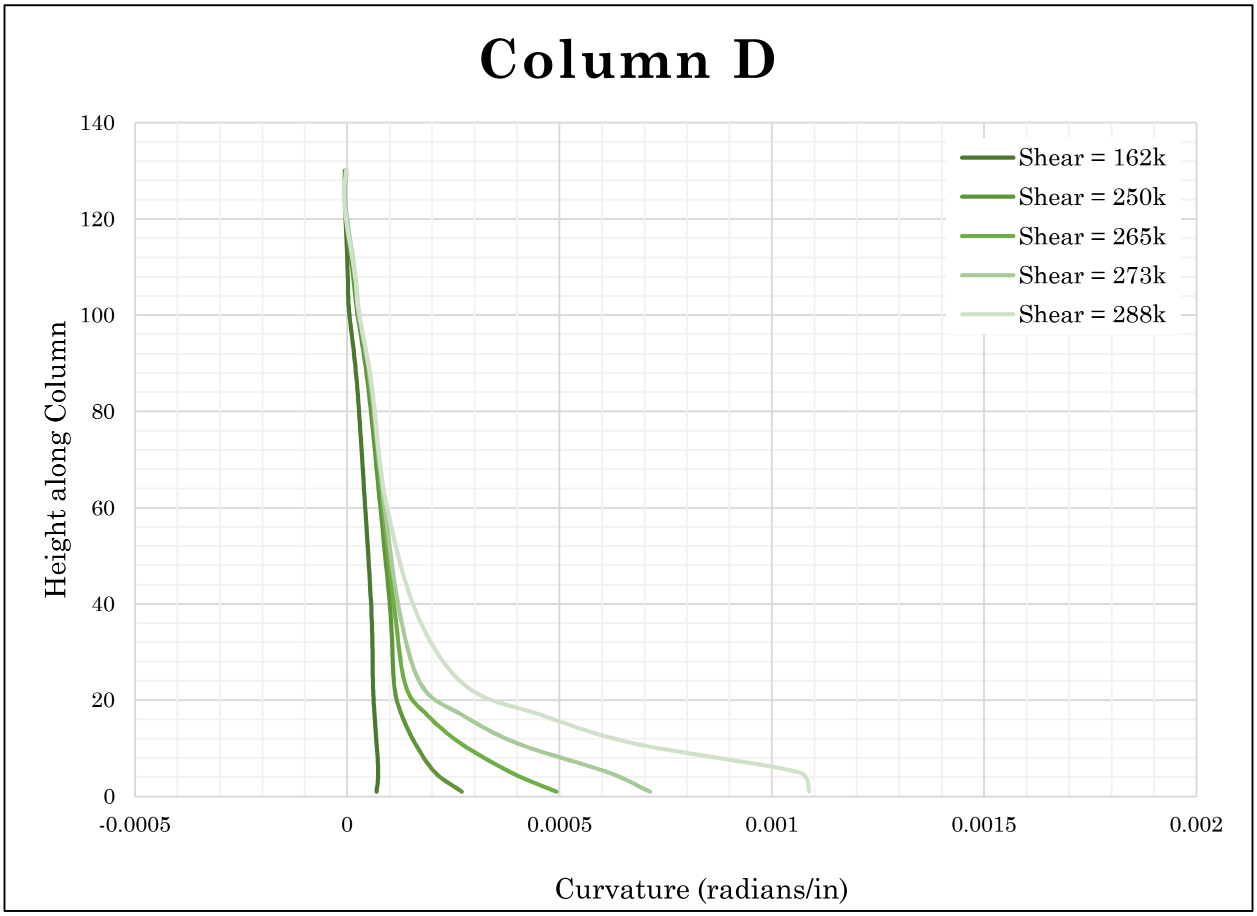
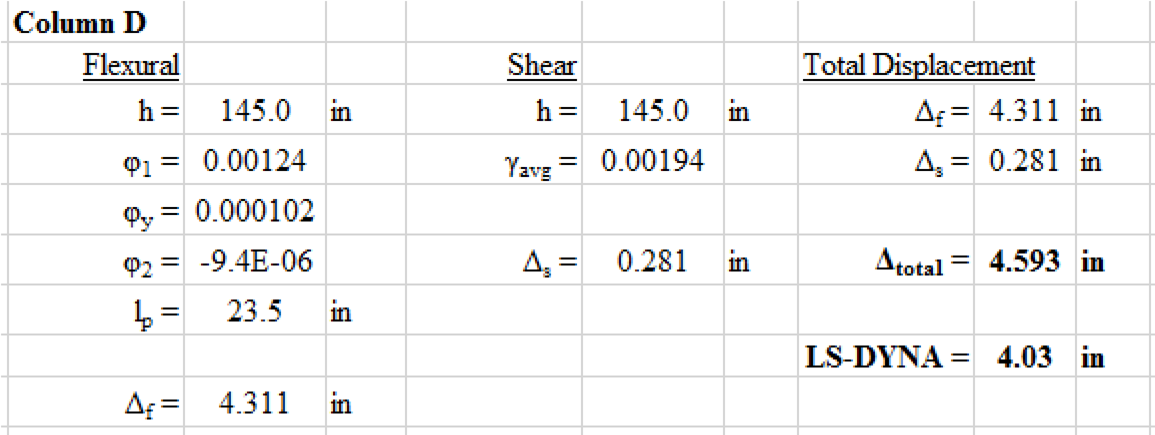


Figure 68: Curvature vs. Height Graph



Column D								
Flexural				Shear			Total Displacement	
h =	145.0	in	h =	145.0	in	Δ_f =	4.311	in
φ_1 =	0.00124		γ_{avg} =	0.00194		Δ_s =	0.281	in
φ_y =	0.000102							
φ_2 =	-9.4E-06		Δ_s =	0.281	in	**Δ_{total} =**	**4.593**	**in**
l_p =	23.5	in						
						LS-DYNA =	**4.03**	**in**
Δ_f =	4.311	in						

Figure 69: Displacement Calculations

Column E

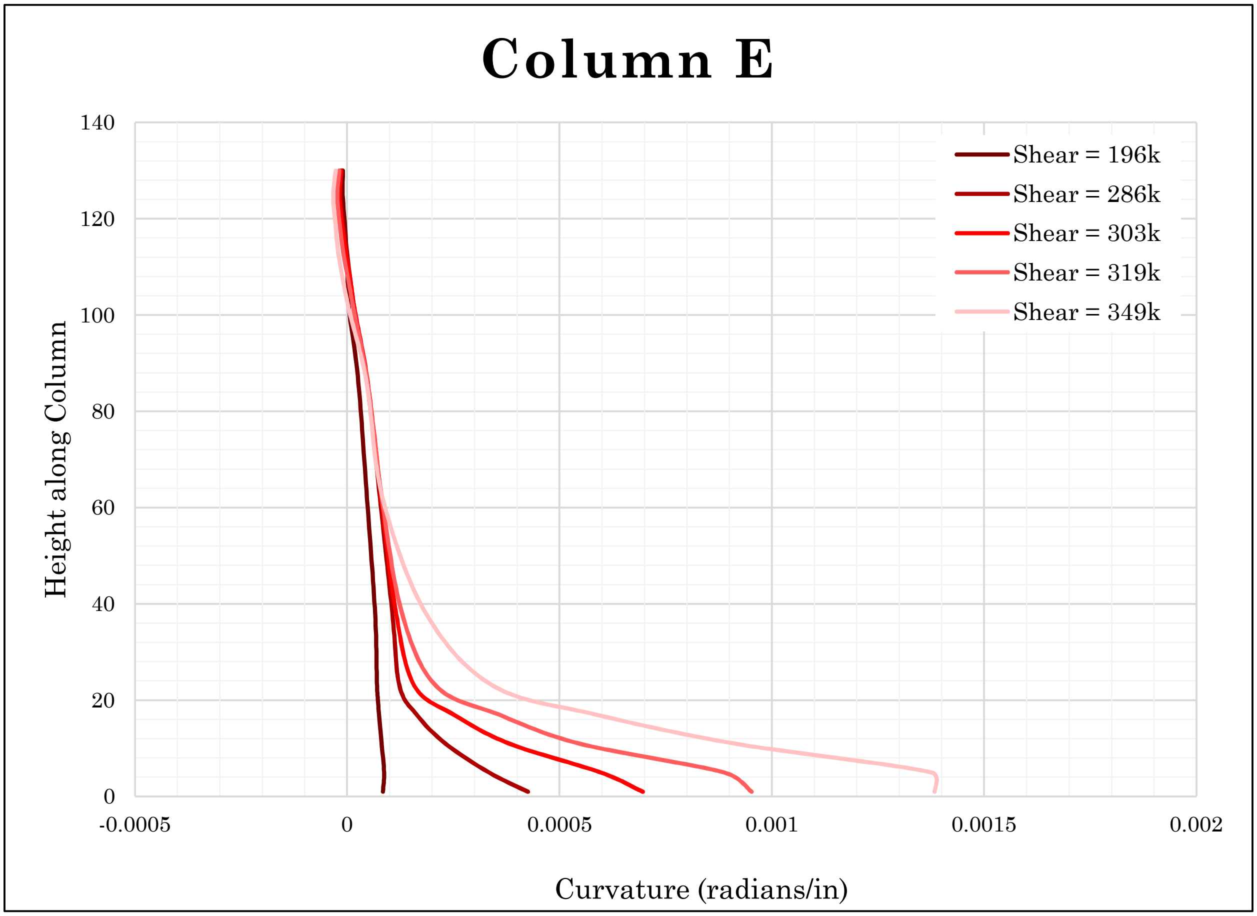
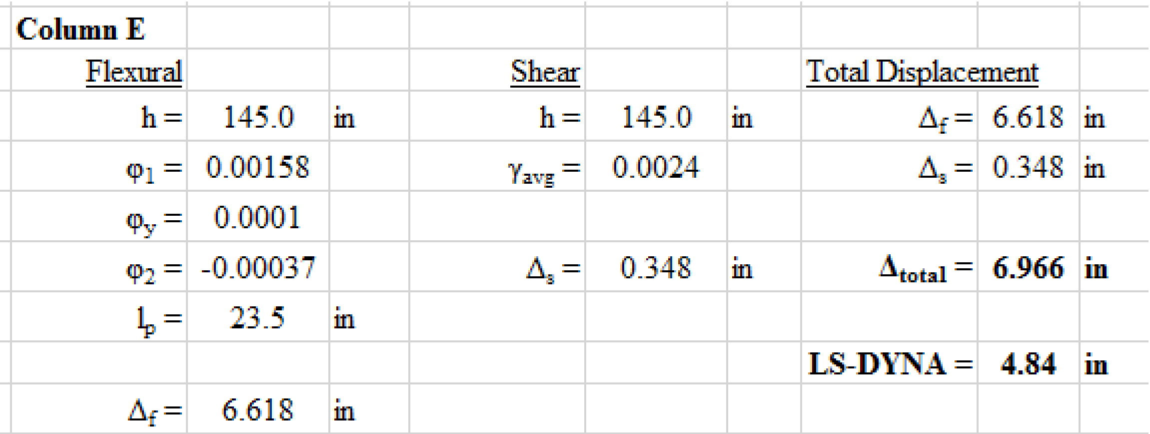


Figure 70: Curvature vs. Height Graph



Column E								
Flexural			Shear			Total Displacement		
h =	145.0	in	h =	145.0	in	Δ_f =	6.618	in
φ_1 =	0.00158		γ_{avg} =	0.0024		Δ_s =	0.348	in
φ_y =	0.0001							
φ_2 =	-0.00037		Δ_s =	0.348	in	**Δ_{total} =**	**6.966**	**in**
l_p =	23.5	in						
						LS-DYNA =	**4.84**	**in**
Δ_f =	6.618	in						

Figure 71: Displacement Calculations

Column F

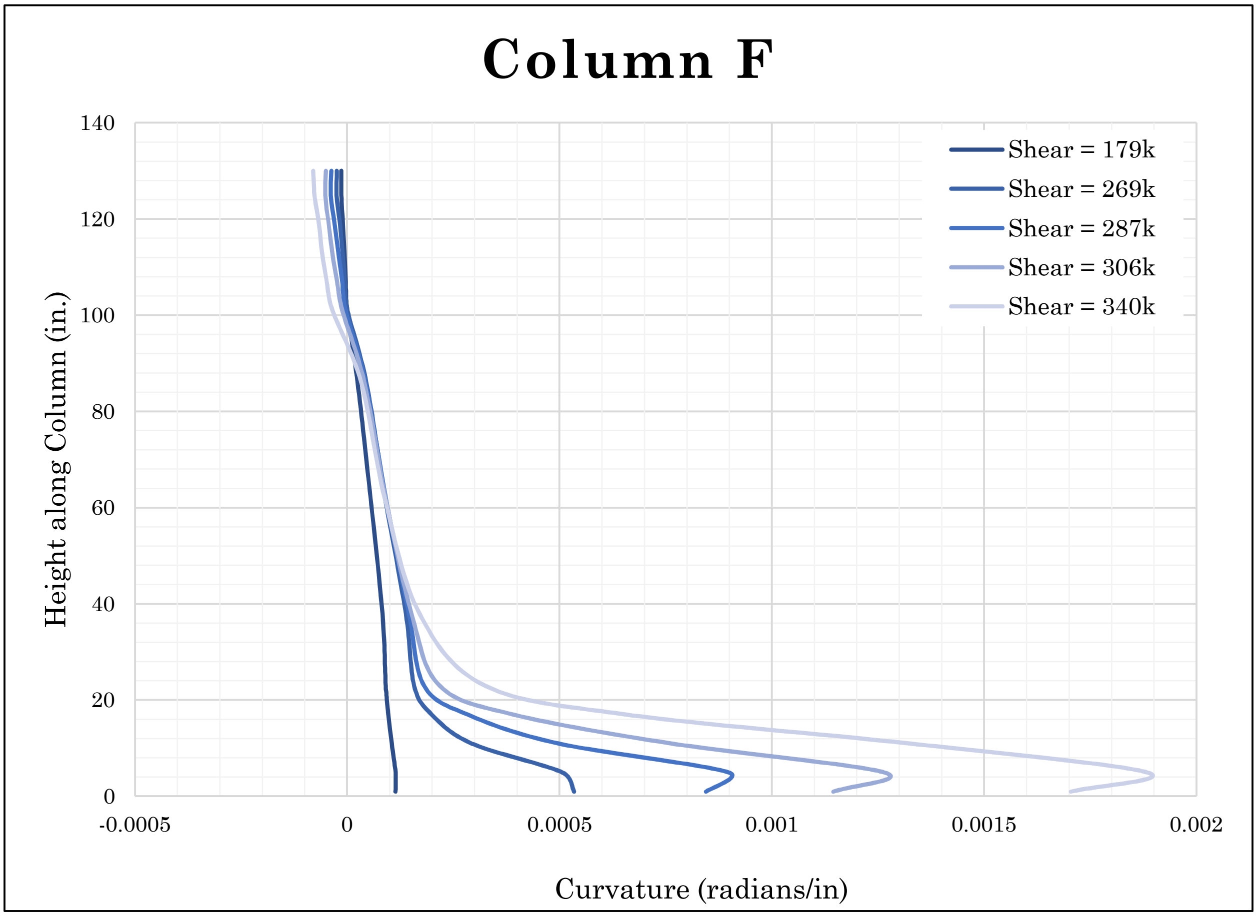
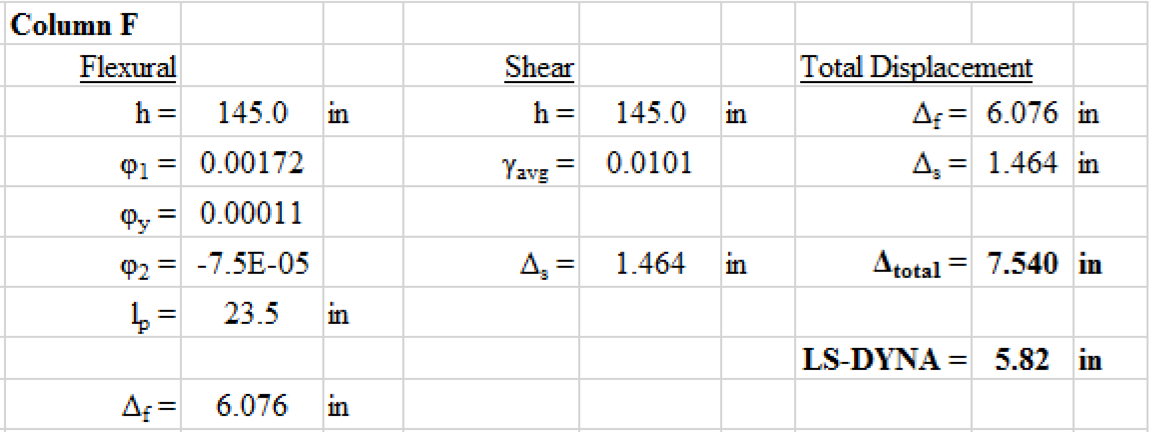


Figure 72: Curvature vs. Height Graph



Column F								
Flexural			Shear			Total Displacement		
$h =$	145.0	in	$h =$	145.0	in	$\Delta_f =$	6.076	in
$\varphi_1 =$	0.00172		$\gamma_{avg} =$	0.0101		$\Delta_s =$	1.464	in
$\varphi_y =$	0.00011							
$\varphi_2 =$	-7.5E-05		$\Delta_s =$	1.464	in	$\mathbf{\Delta_{total} =}$	**7.540**	**in**
$l_p =$	23.5	in						
						LS-DYNA =	**5.82**	**in**
$\Delta_f =$	6.076	in						

Figure 73: Displacement Calculations

Appendix C - Frame Deformations

Frame 7.5.7

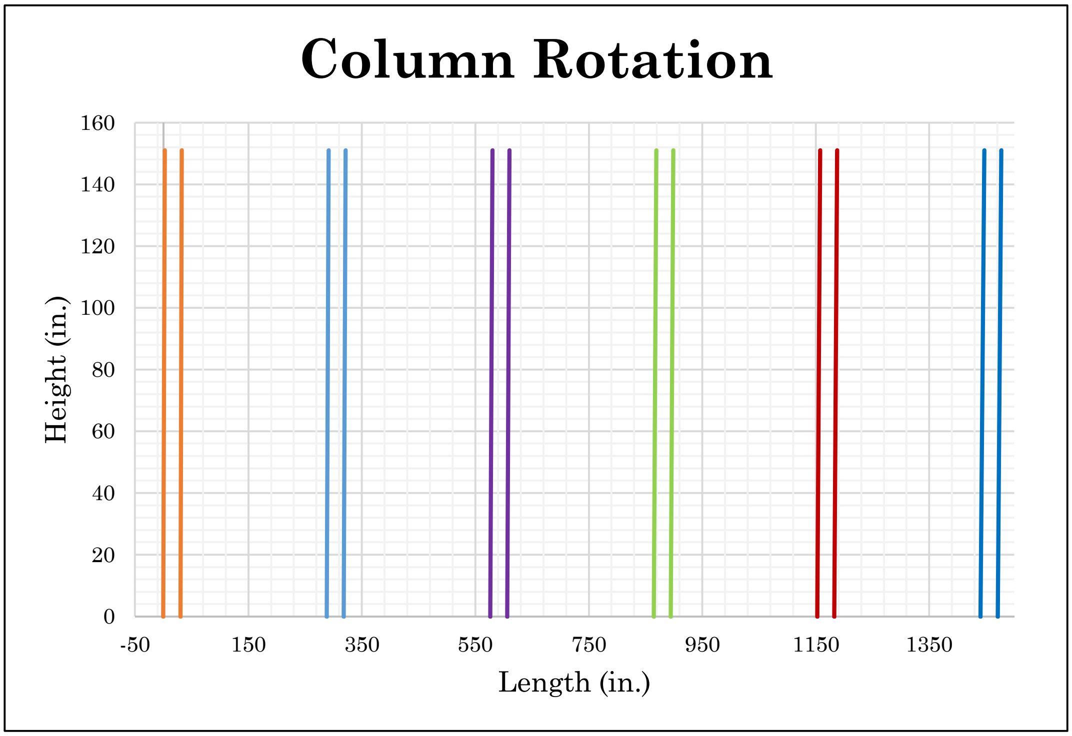


Figure 74: Column Rotation

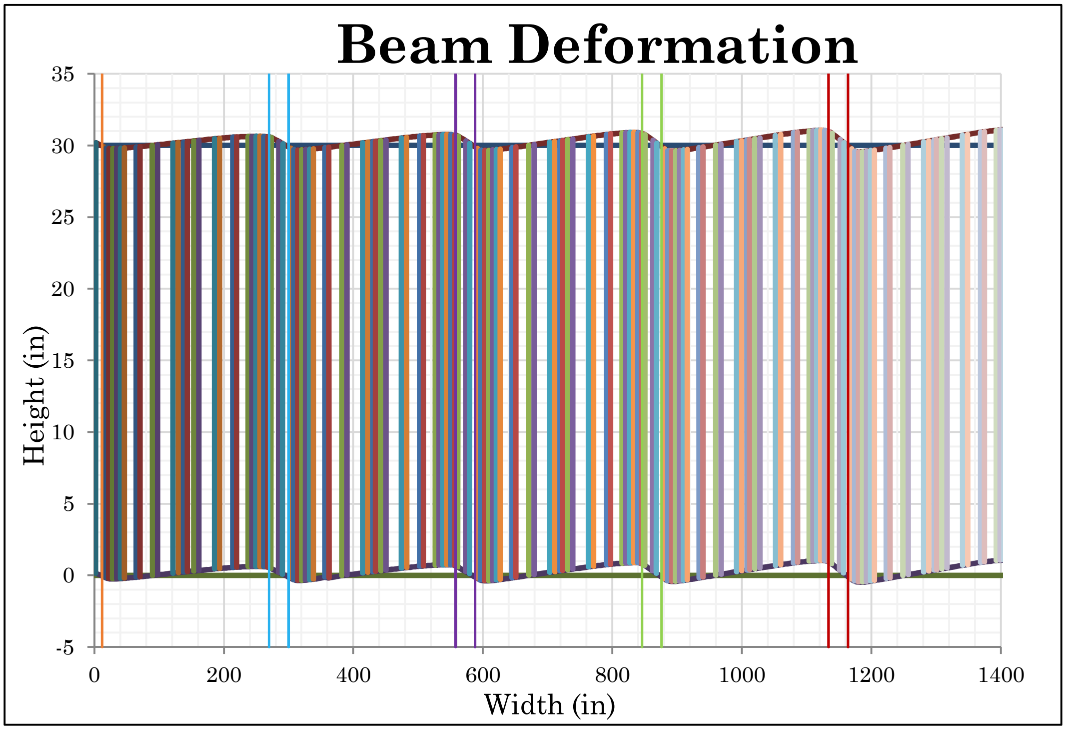


Figure 75: Beam Deformation

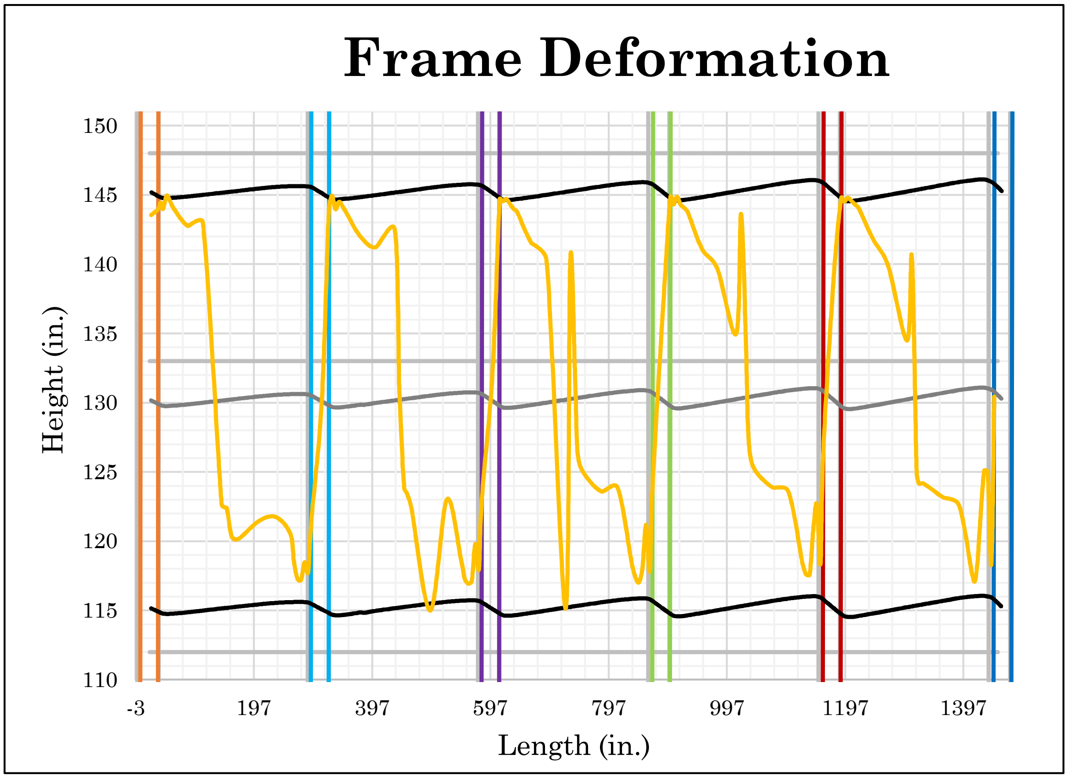


Figure 76: Frame Deformation

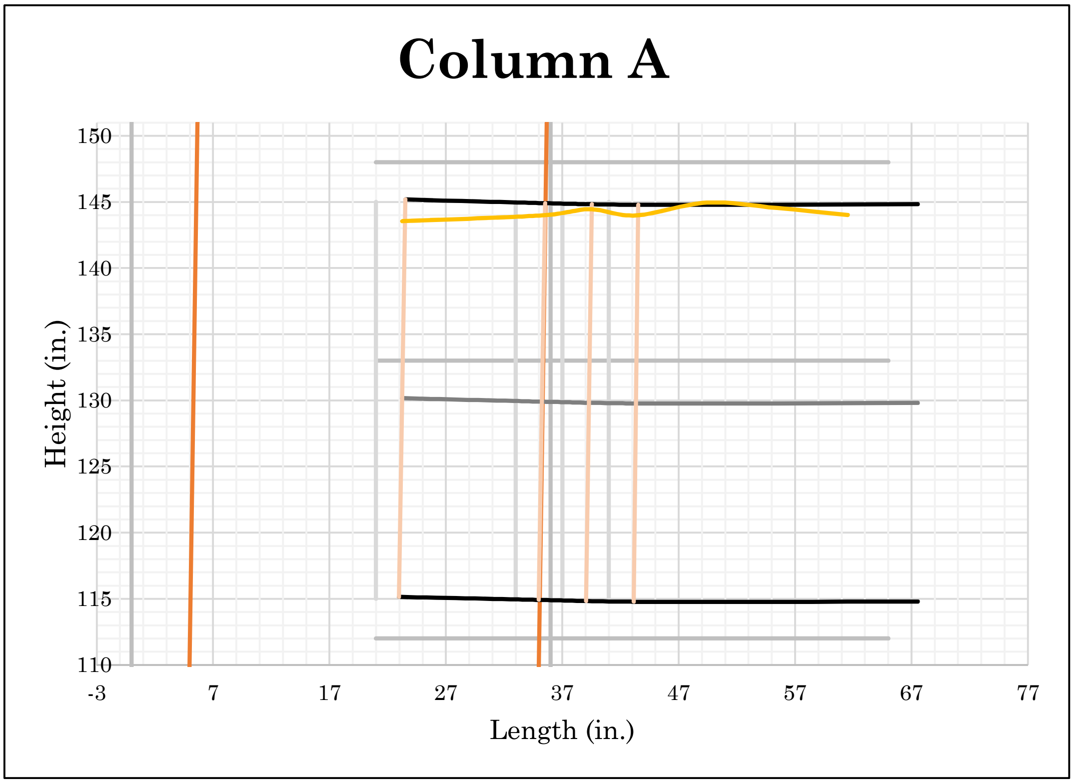


Figure 77: Joint at Column A

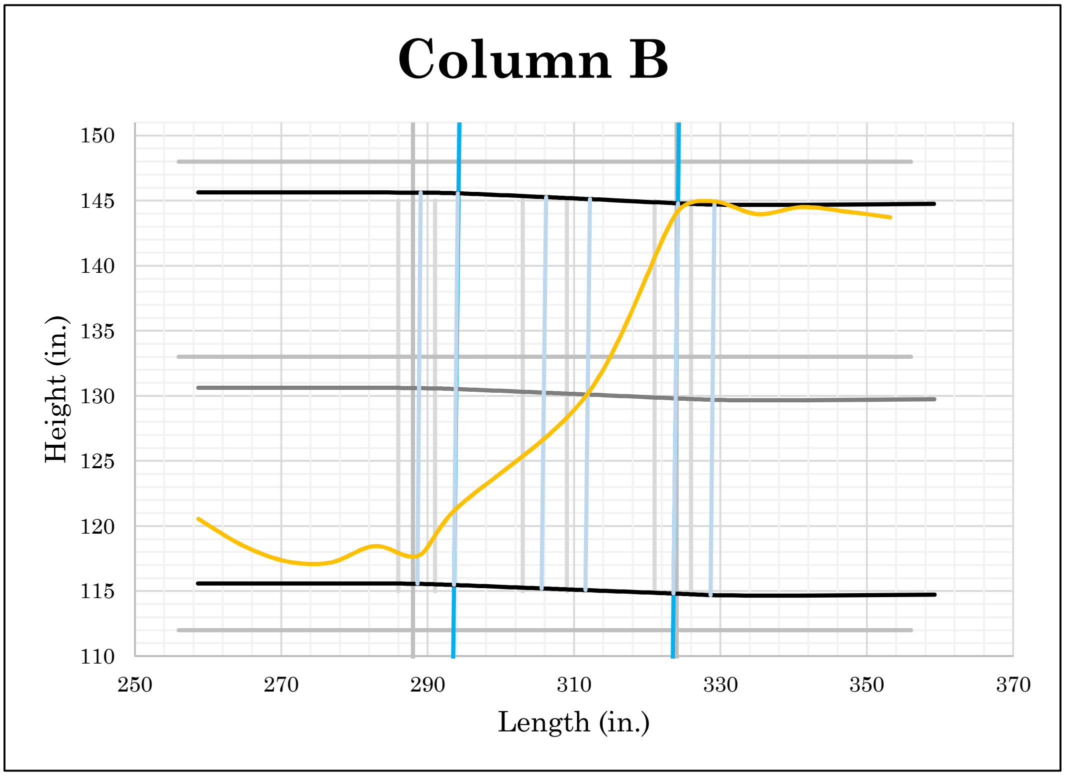


Figure 78: Joint at Column B

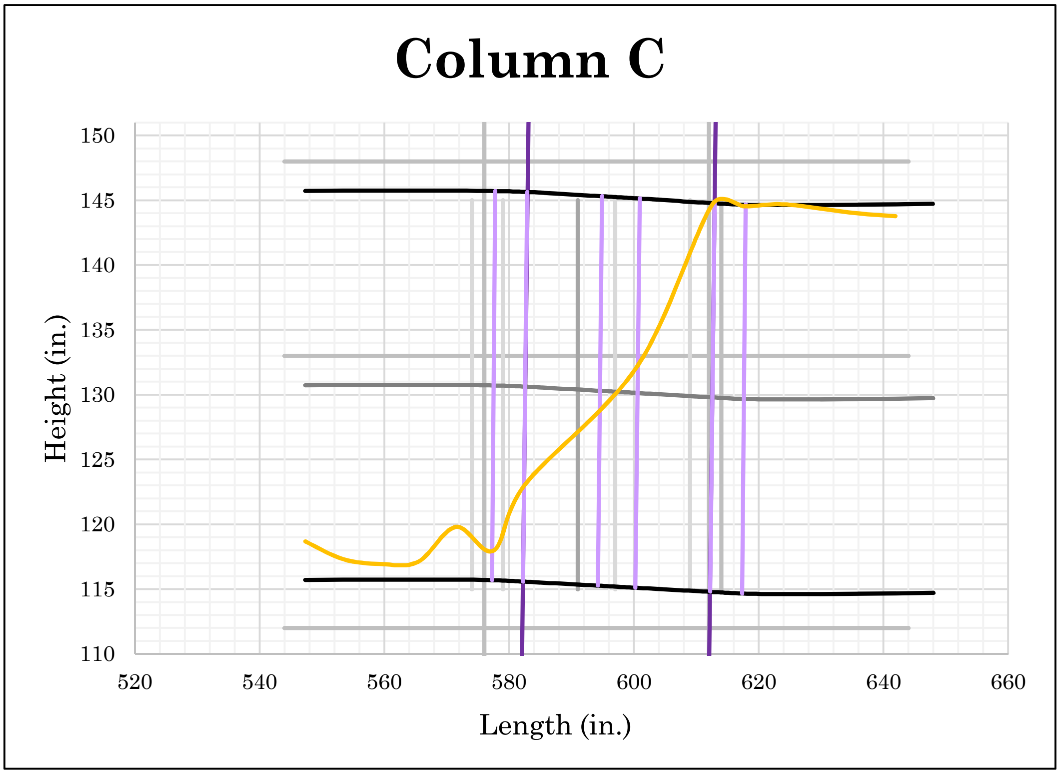


Figure 79: Joint at Column C

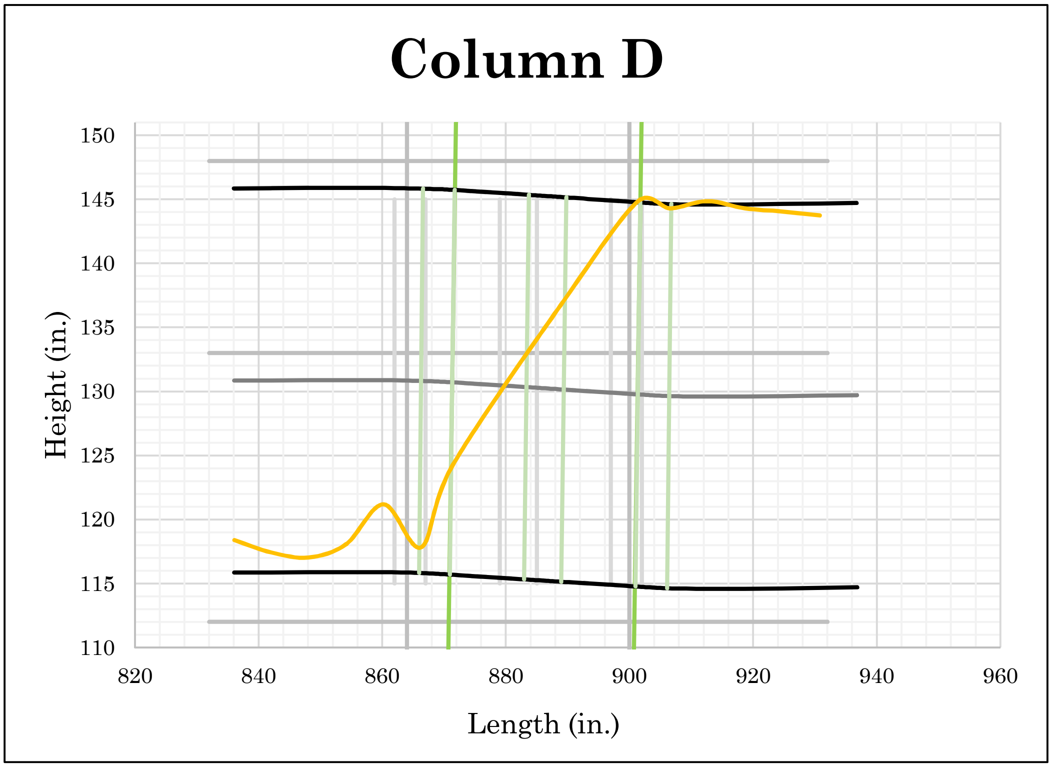


Figure 80: Joint at Column D

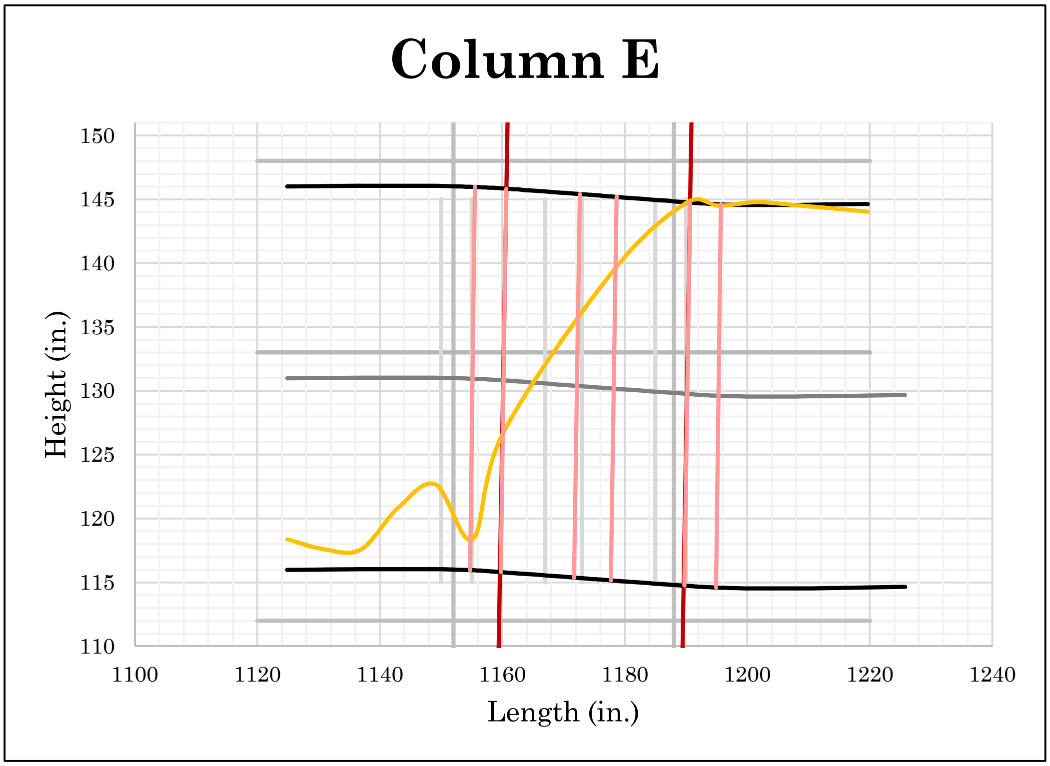


Figure 81: Joint at Column E

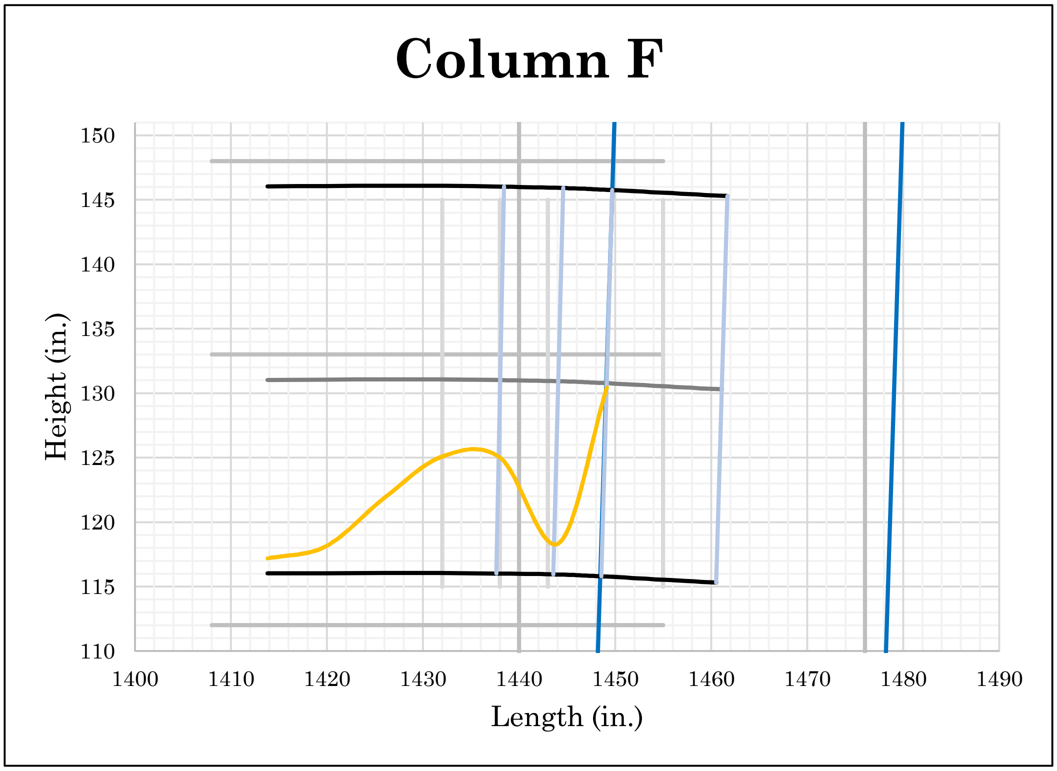


Figure 82: Joint at Column F

Frame 7.5.5

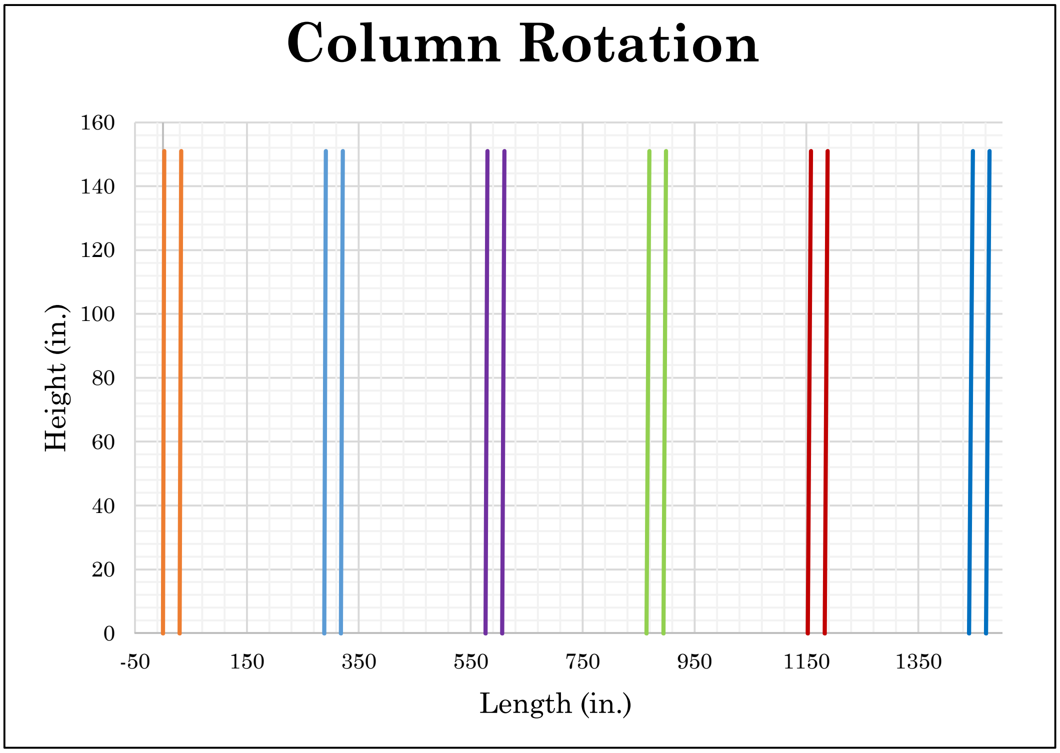


Figure 83: Column Rotation

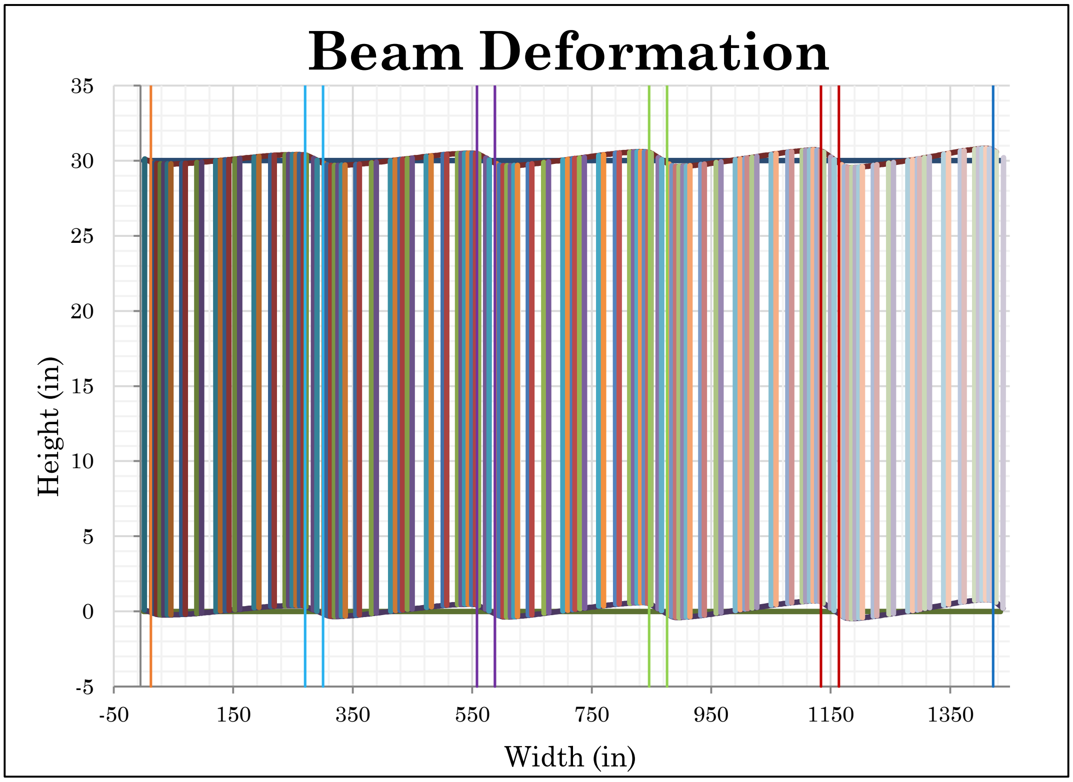


Figure 84: Beam Deformation

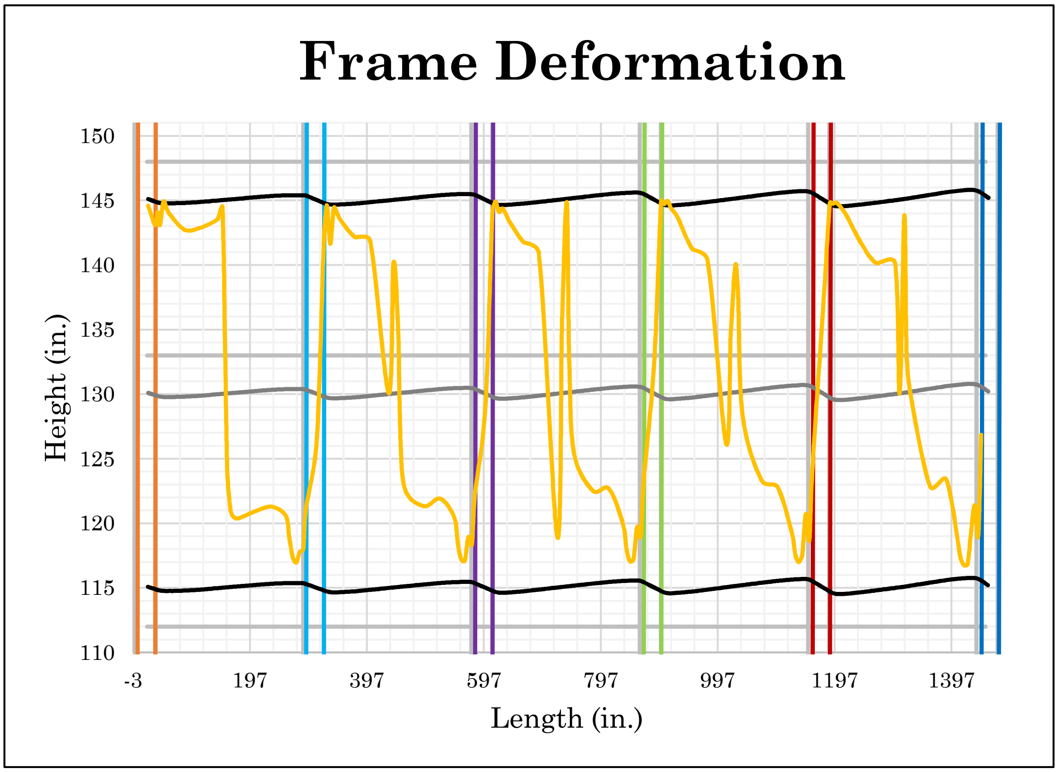


Figure 85: Frame Deformation

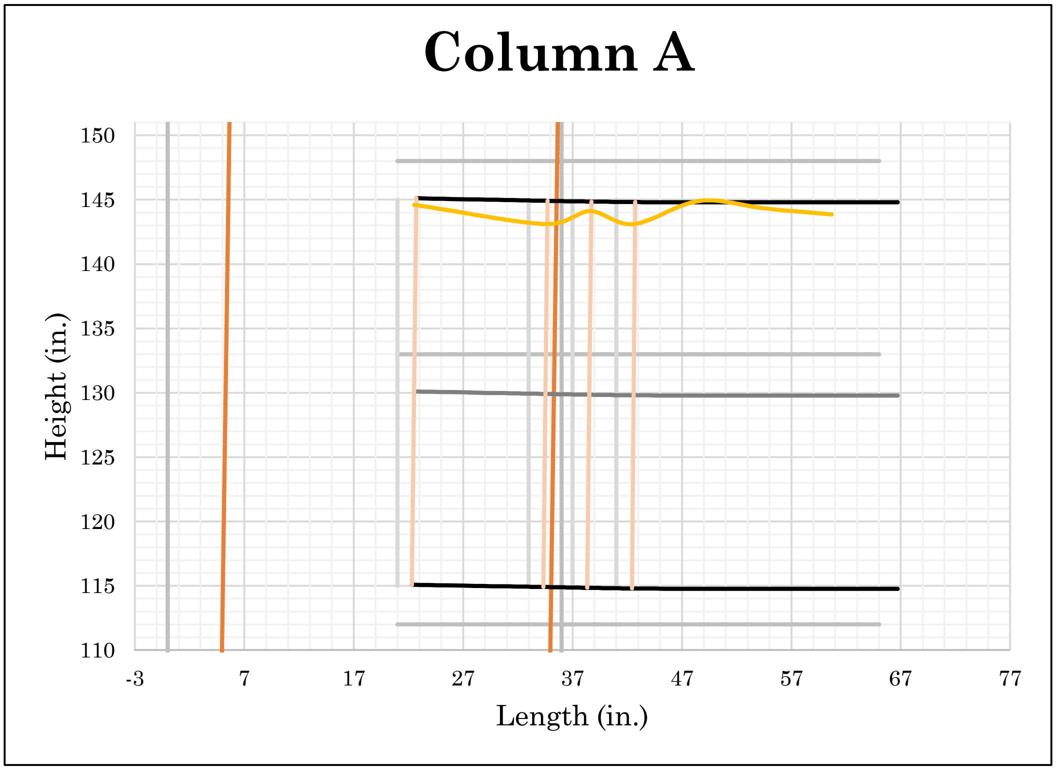


Figure 86: Joint at Column A

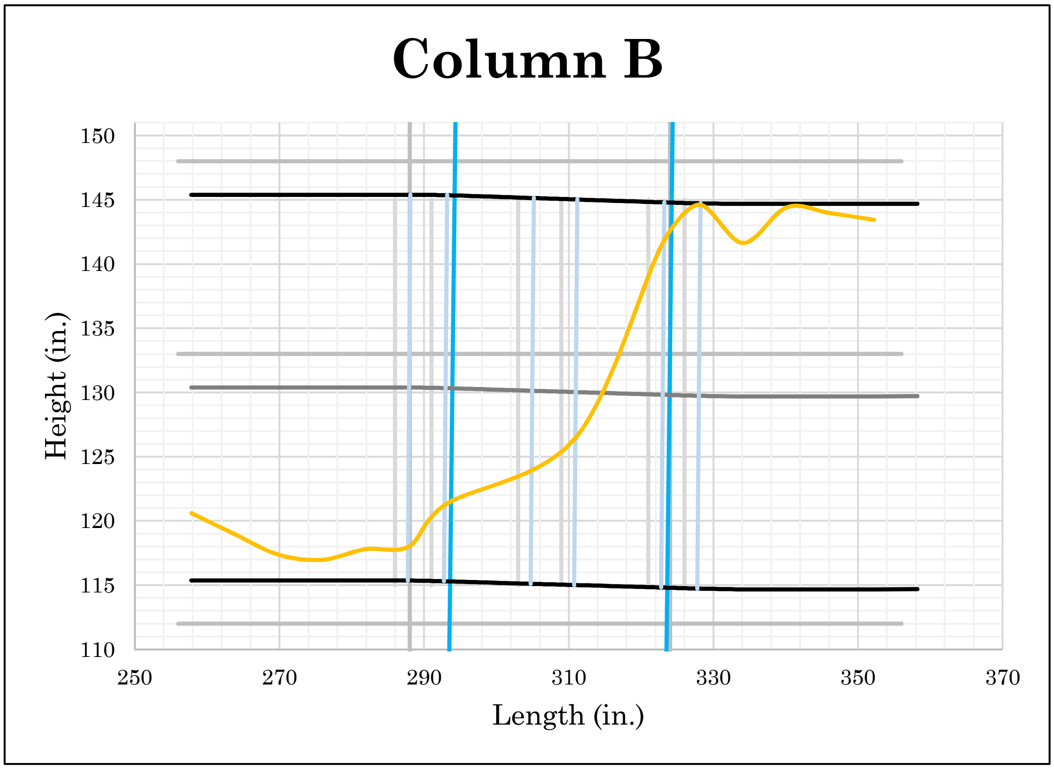


Figure 87: Joint at Column B

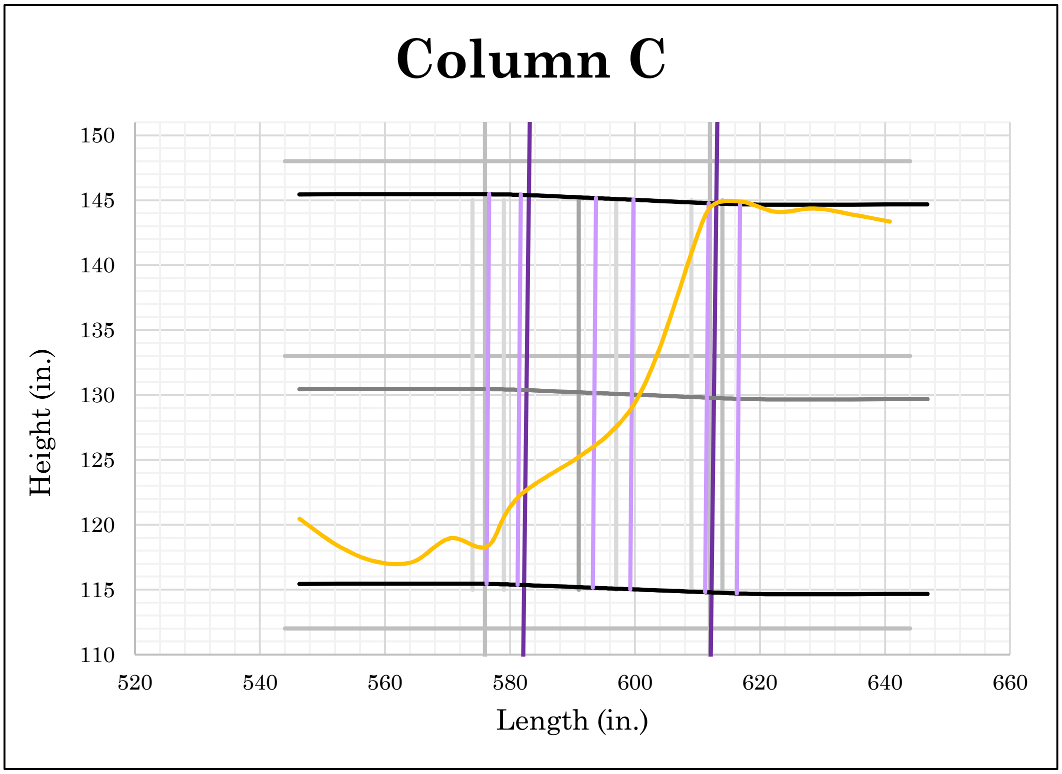


Figure 88: Joint at Column C

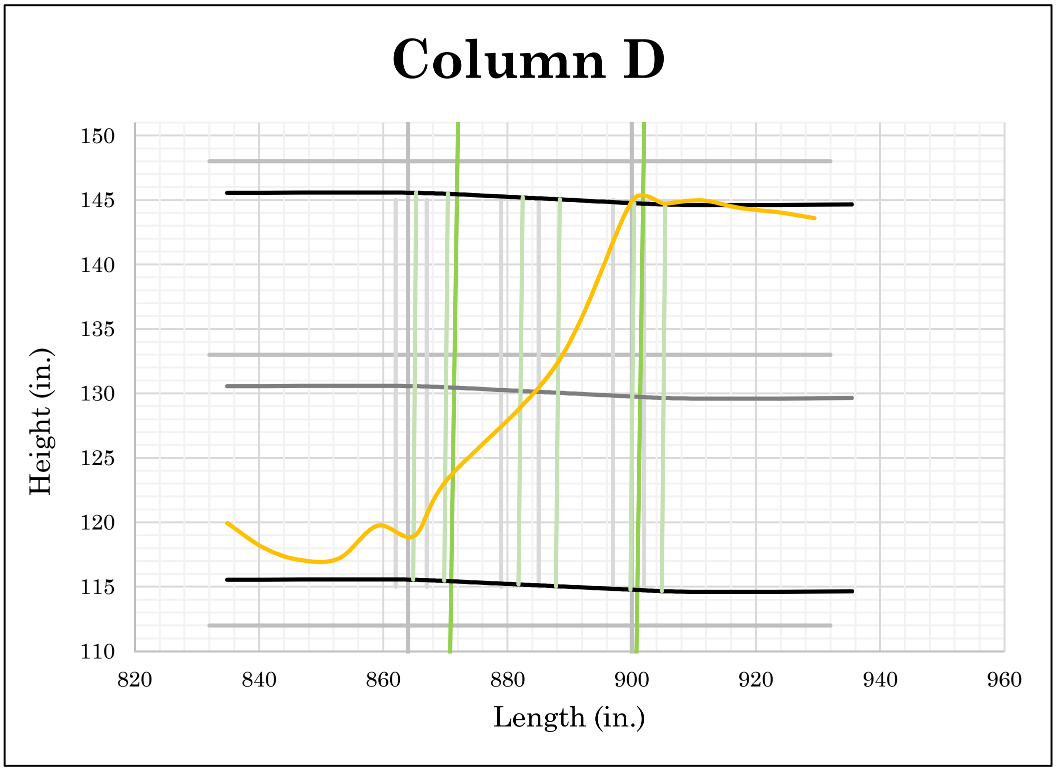


Figure 89: Joint at Column D

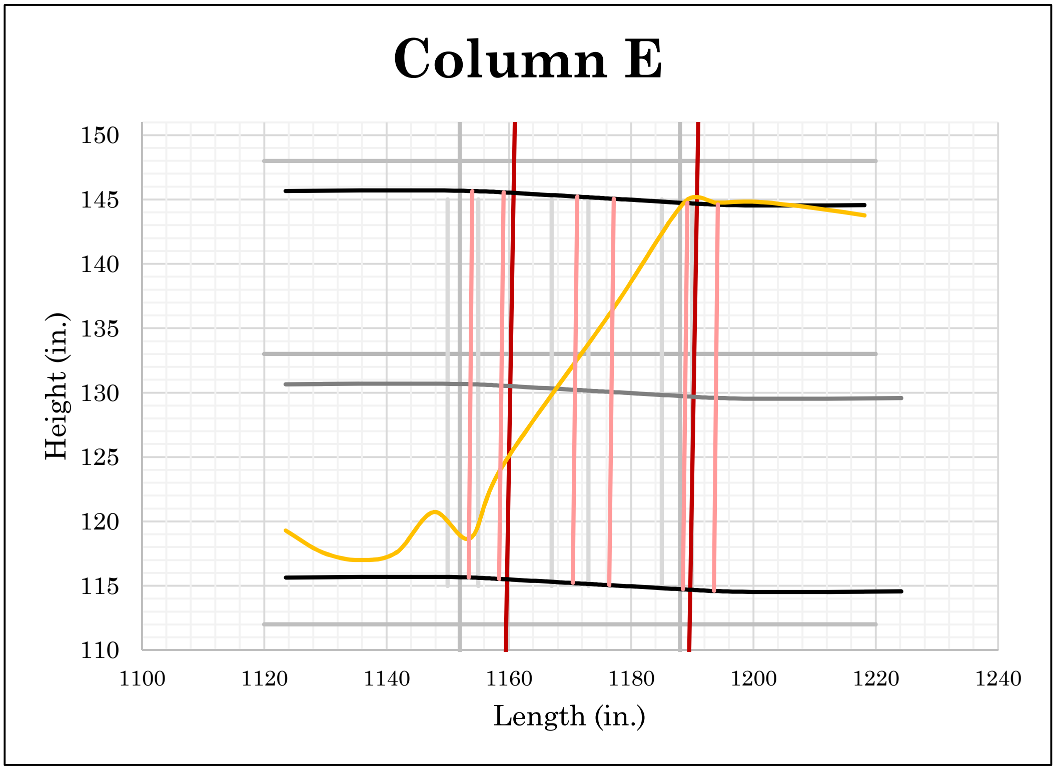


Figure 90: Joint at Column E

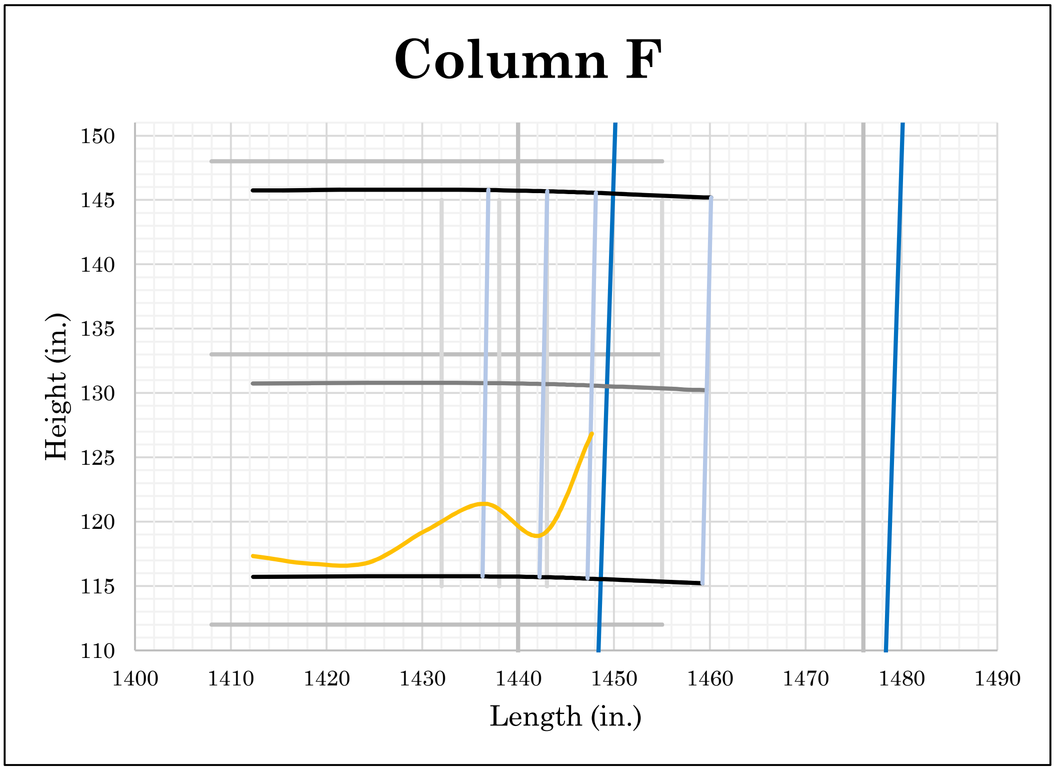


Figure 91: Joint at Column F

Frame 6.5.7

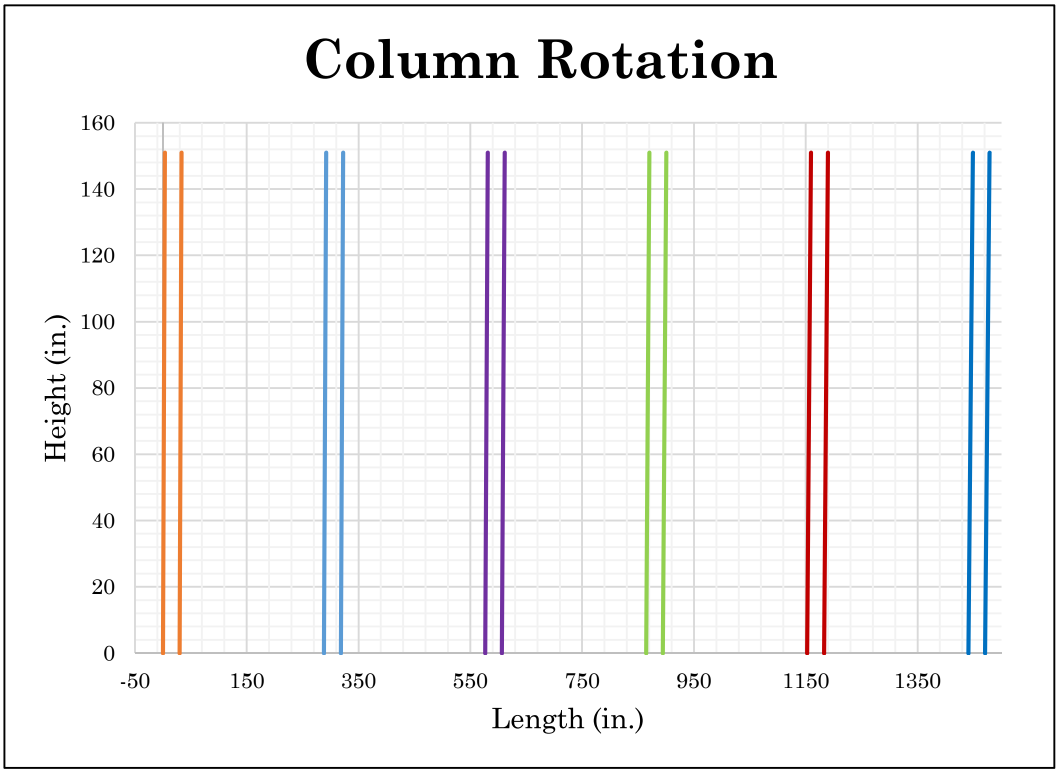


Figure 92: Column Rotation

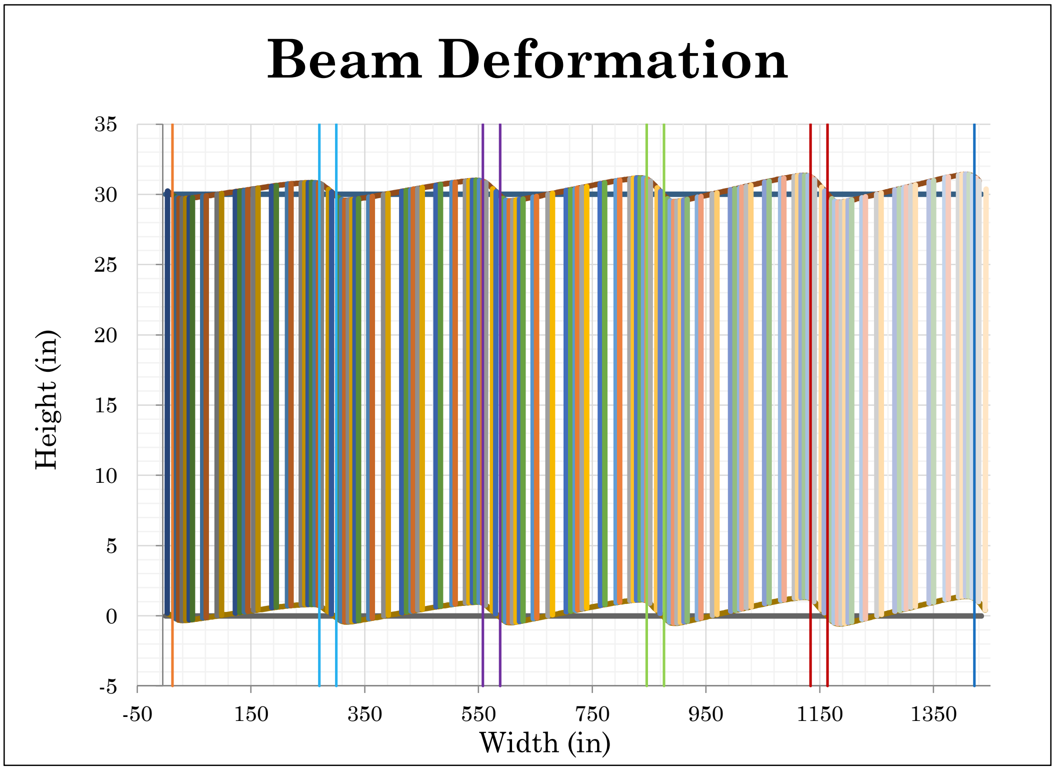


Figure 93: Beam Deformation

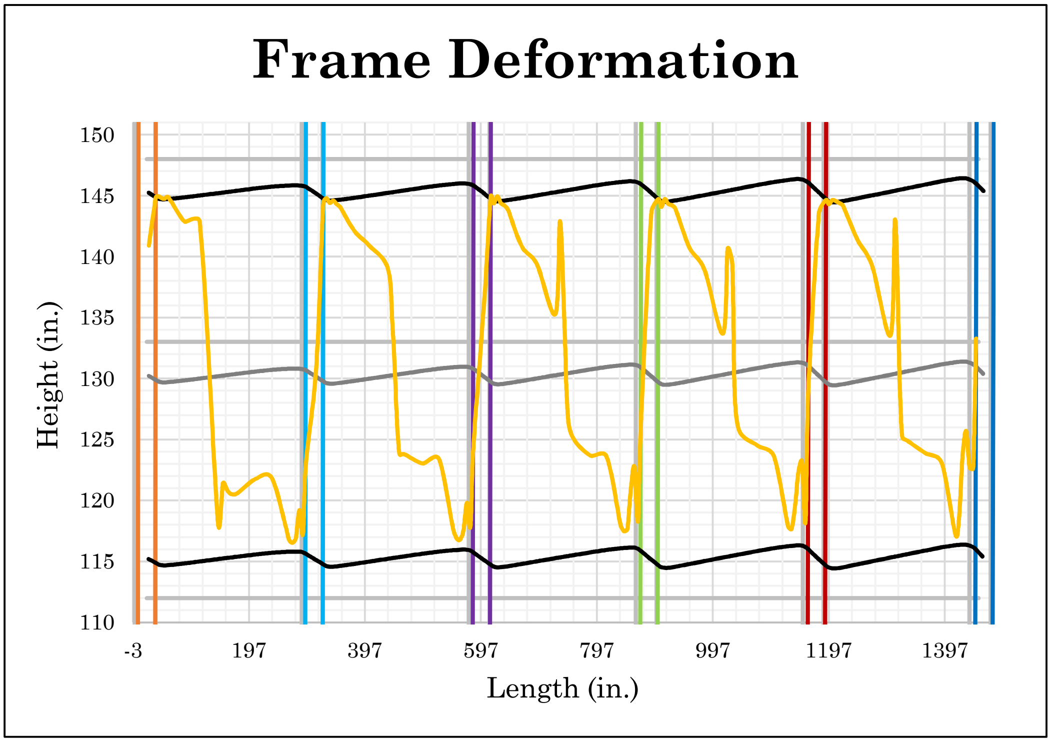


Figure 94: Frame Deformation

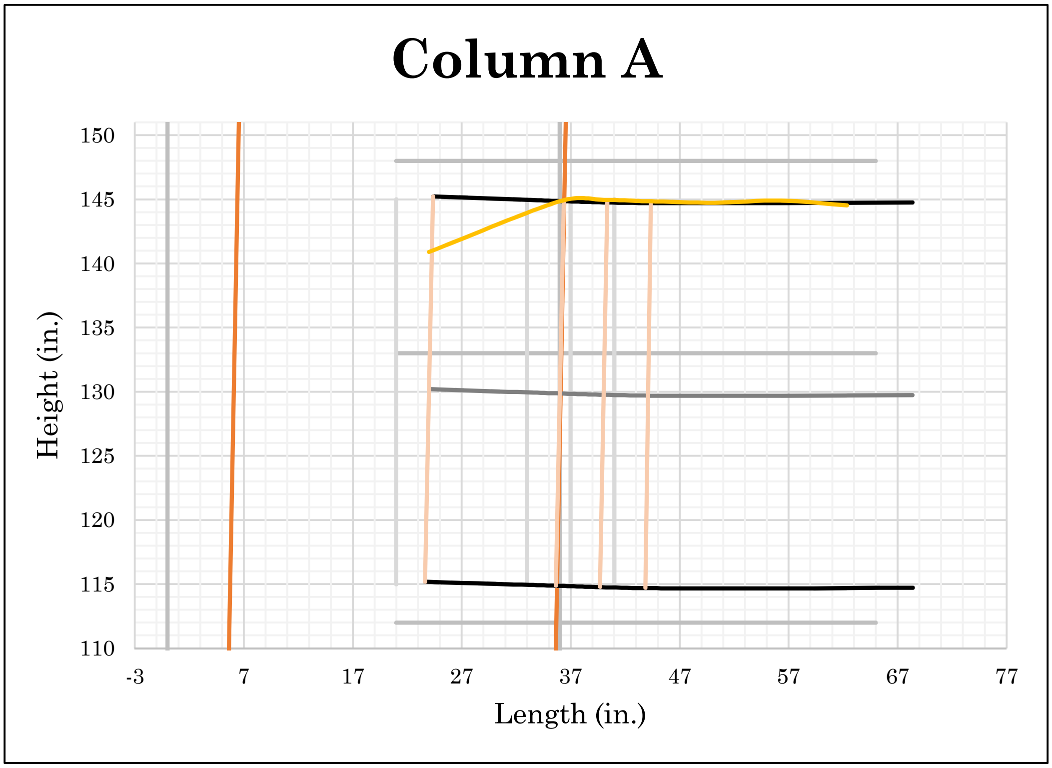


Figure 95: Joint at Column A

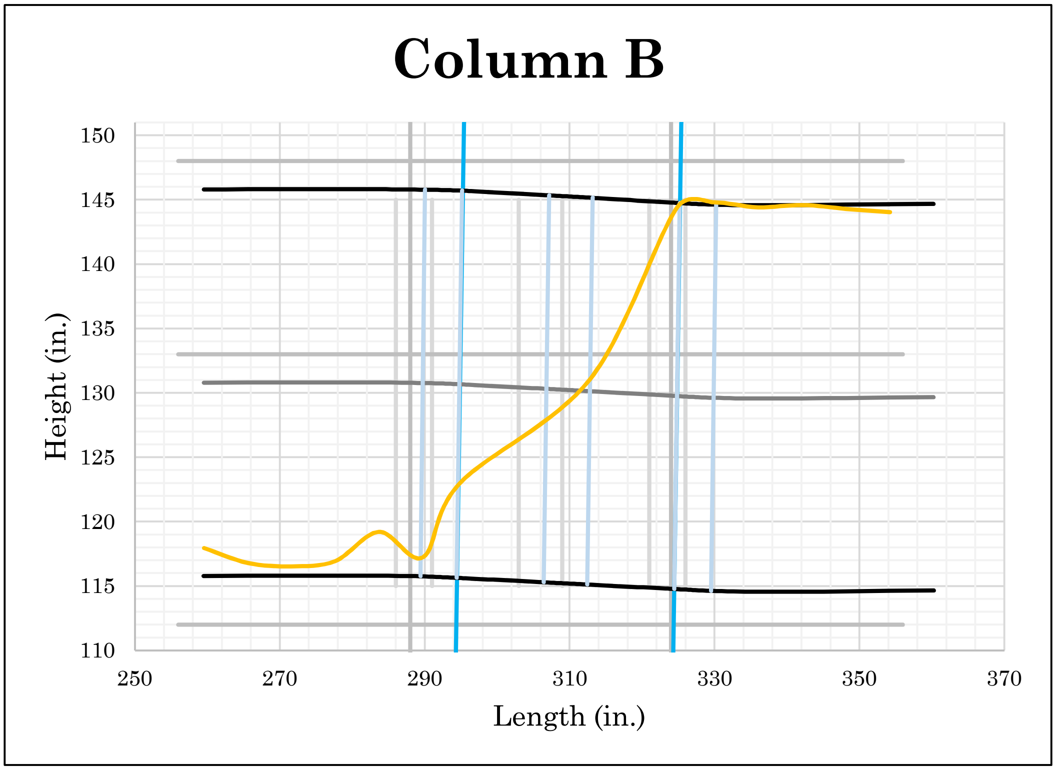


Figure 96: Joint at Column B

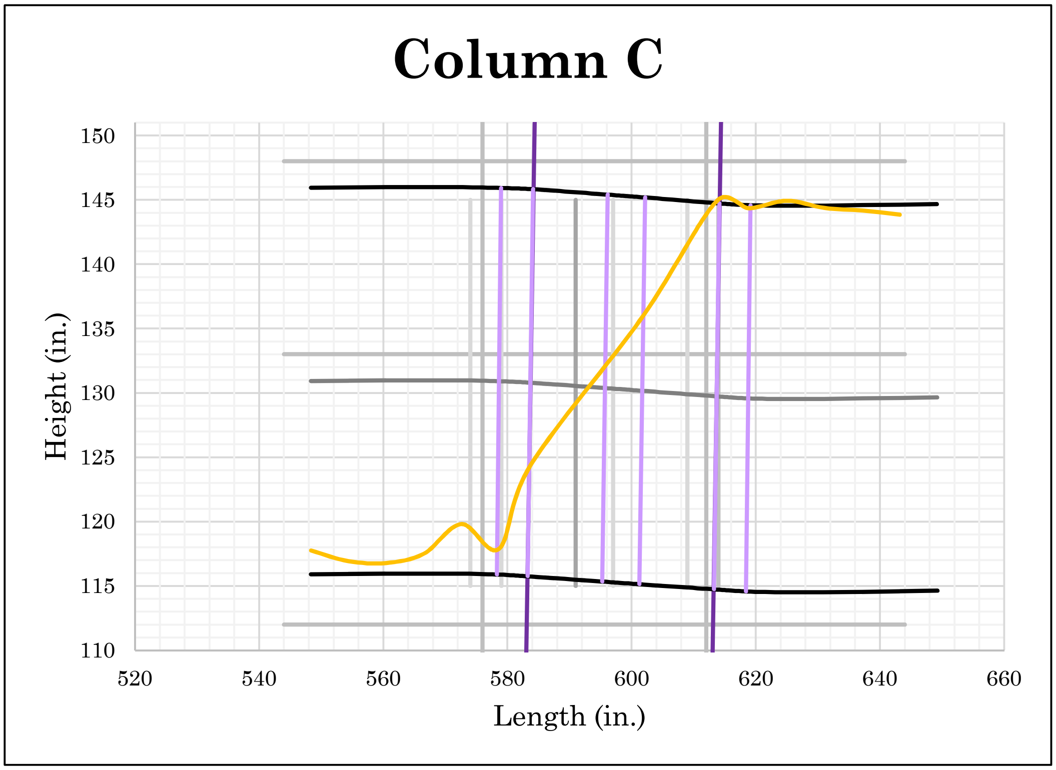


Figure 97: Joint at Column C

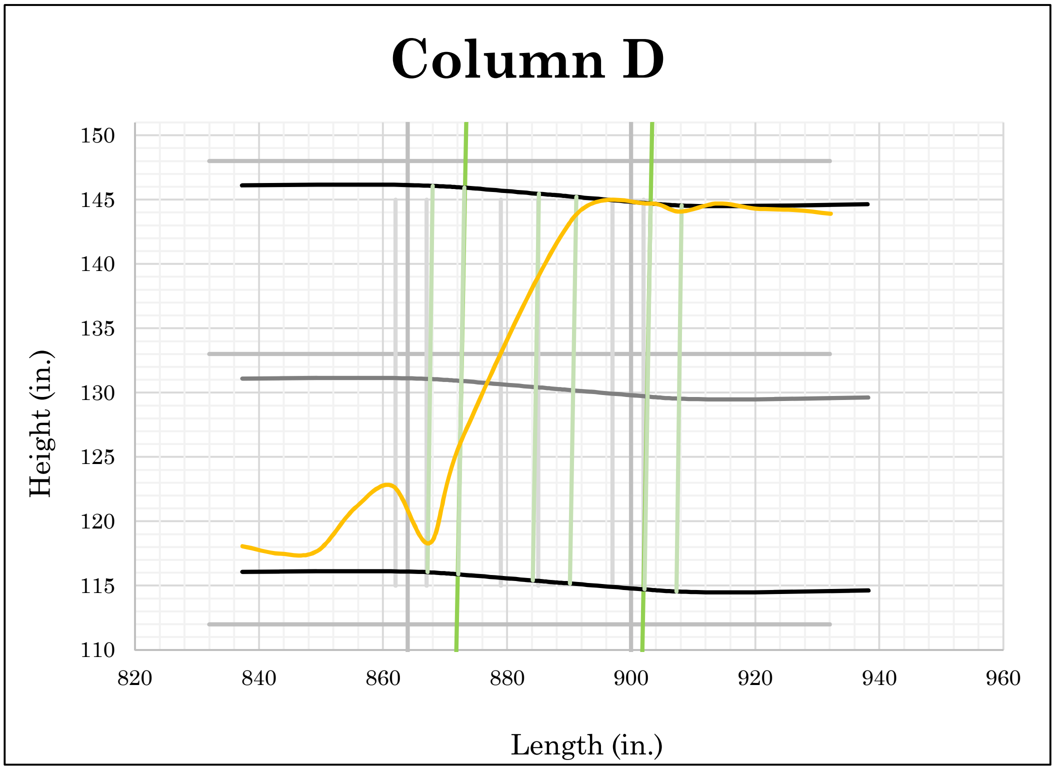


Figure 98: Joint at Column D

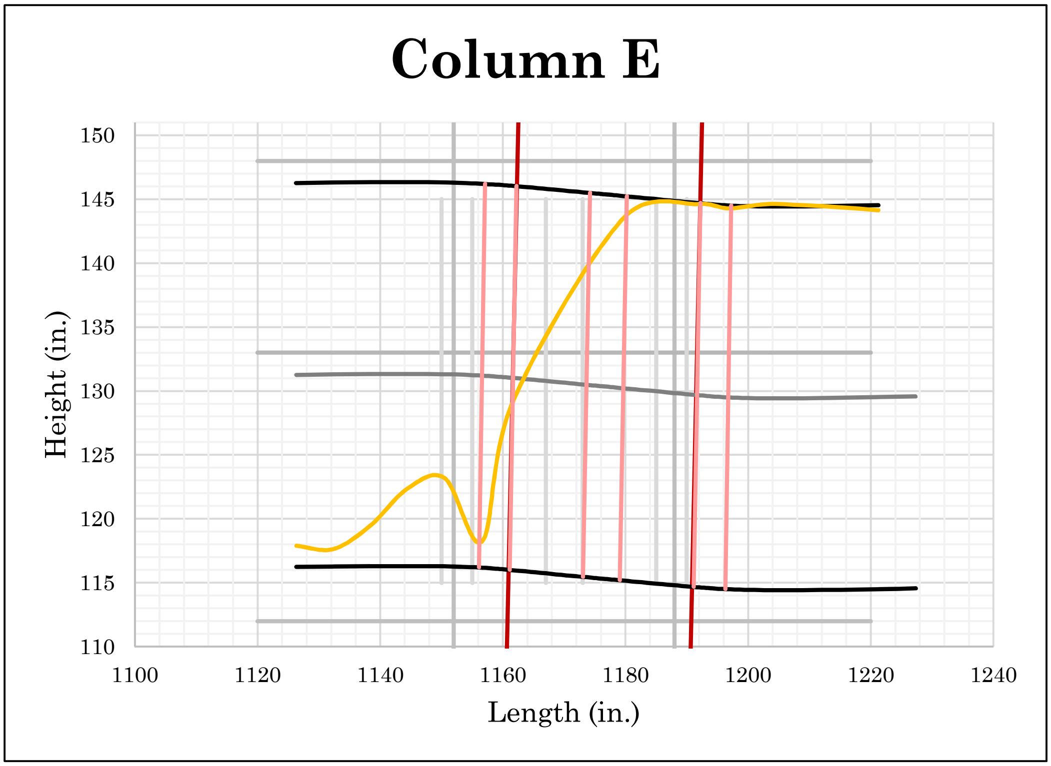


Figure 99: Joint at Column E

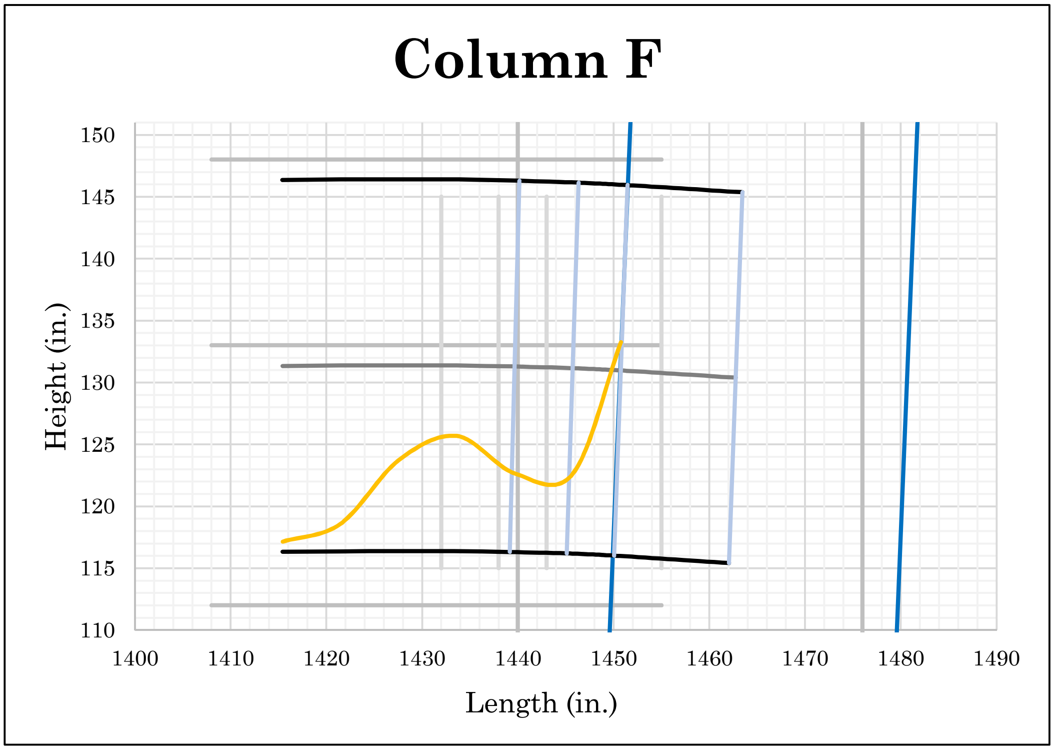


Figure 100: Joint at Column F

Frame 6.5.5

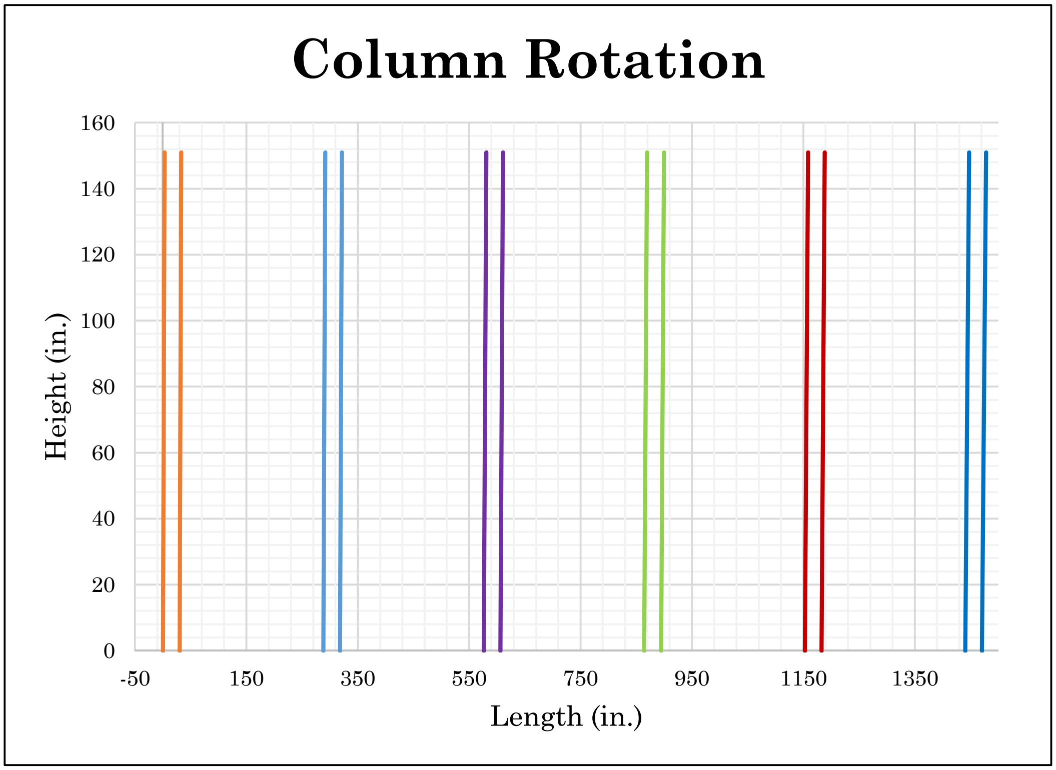


Figure 101: Column Rotation

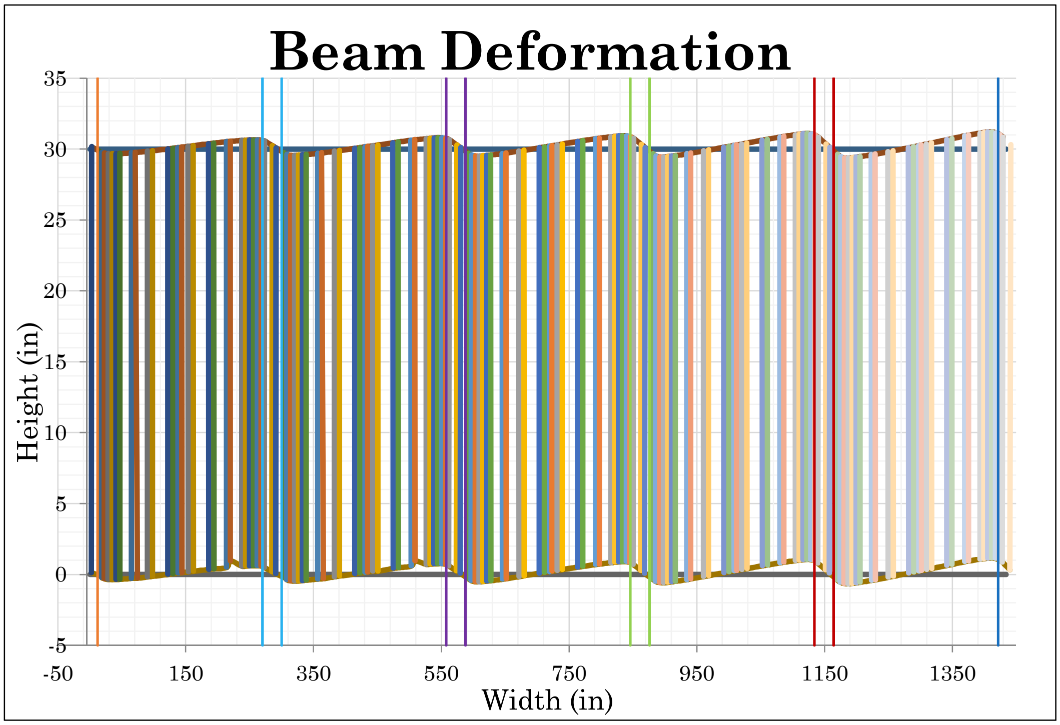


Figure 102: Beam Deformation

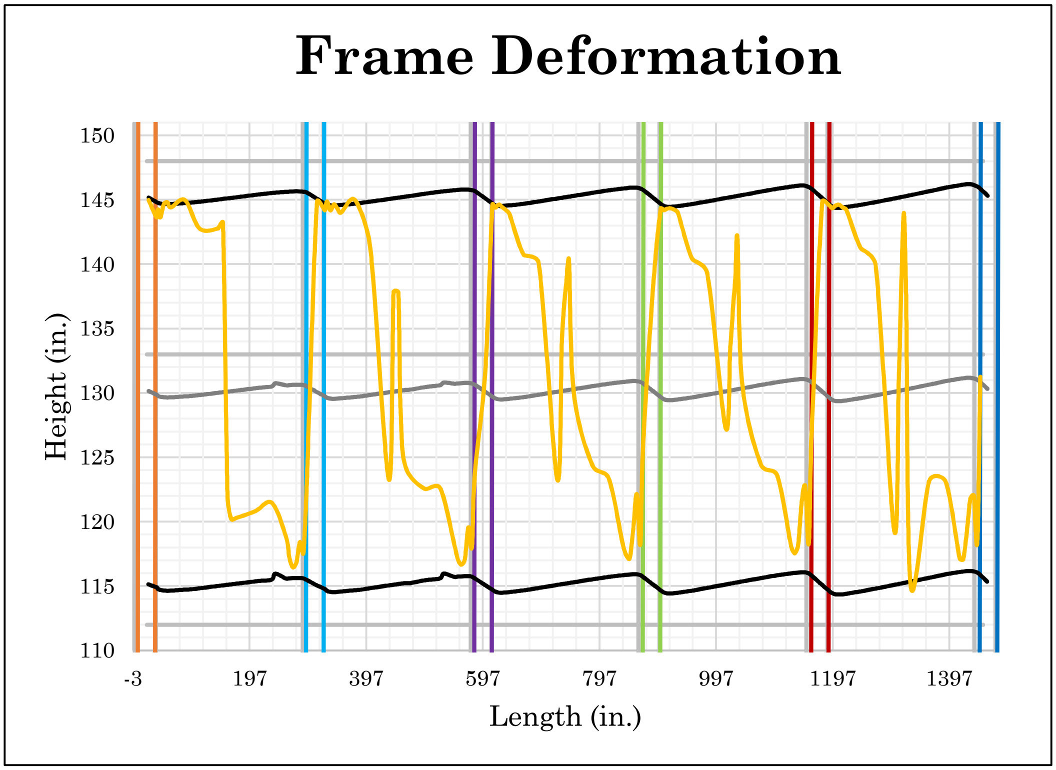


Figure 103: Frame Deformation

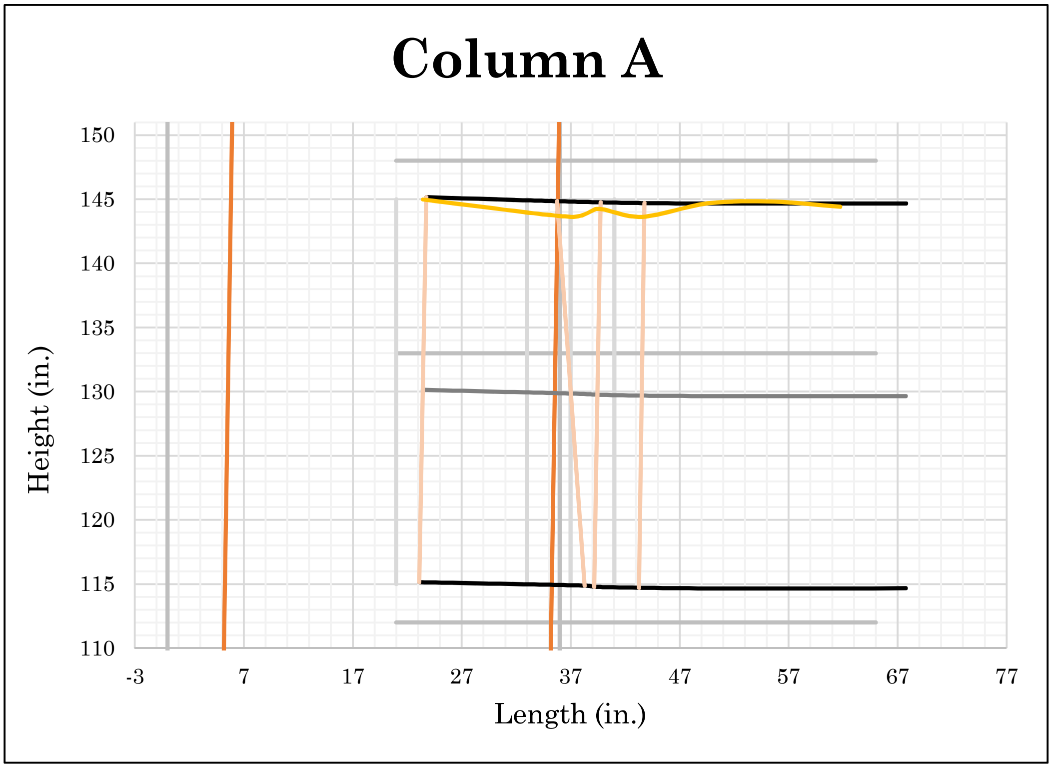


Figure 104: Joint at Column A

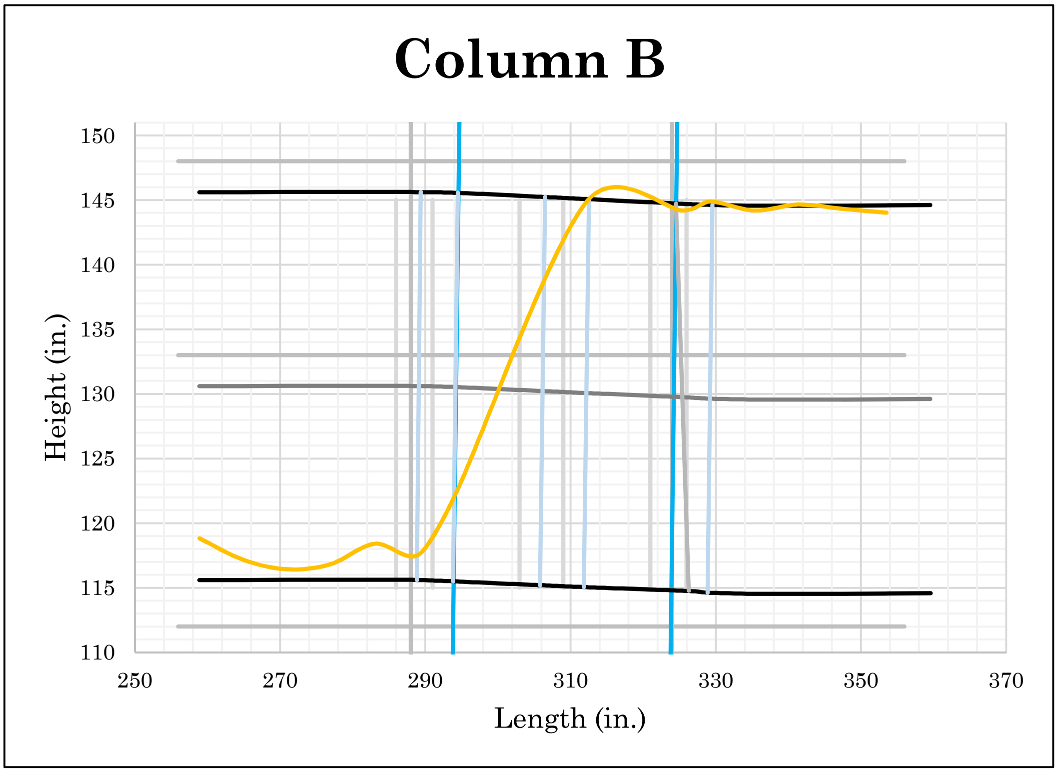


Figure 105: Joint at Column B

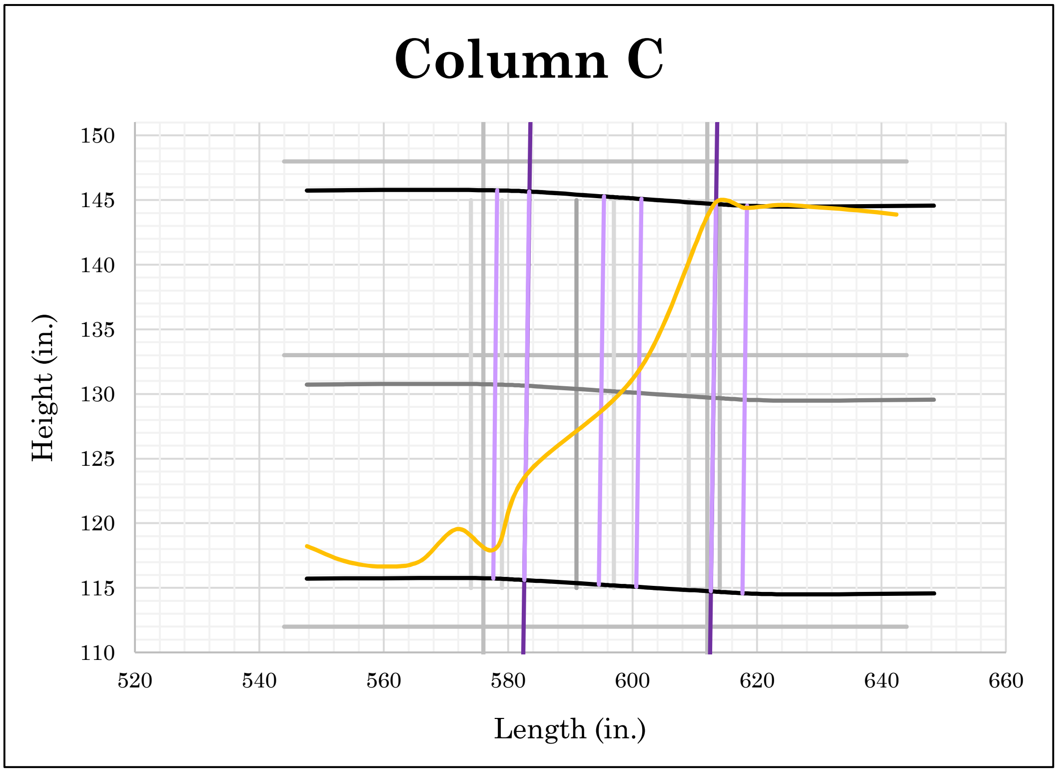


Figure 106: Joint at Column C

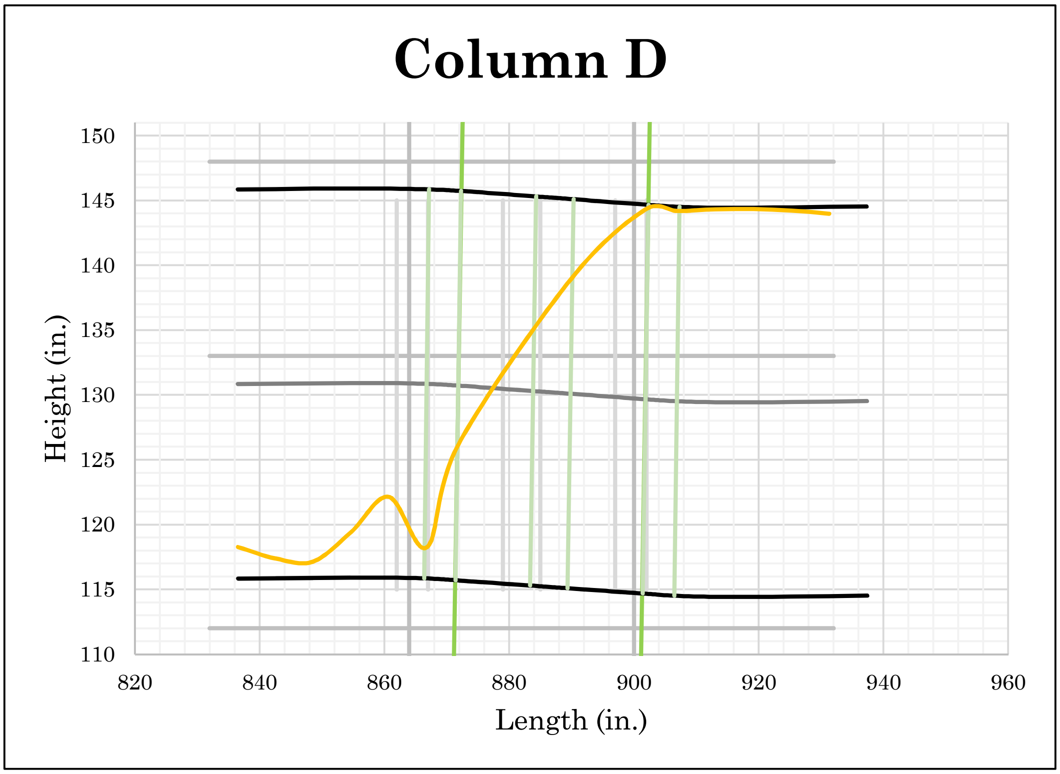


Figure 107: Joint at Column D

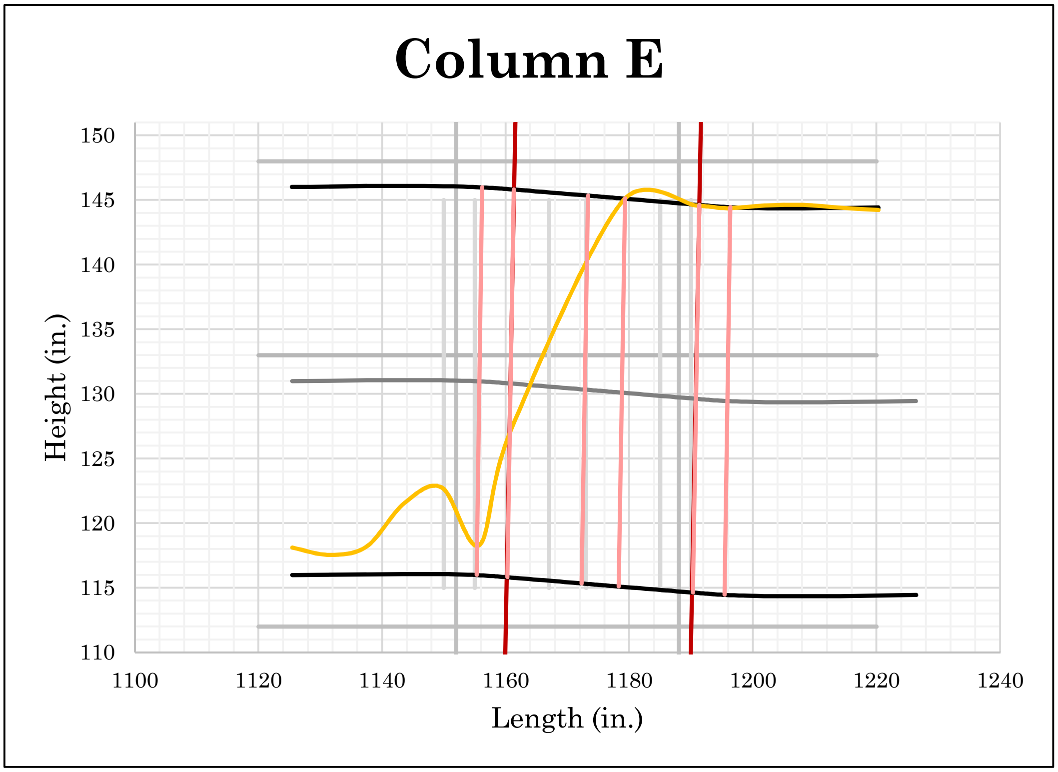


Figure 108: Joint at Column E

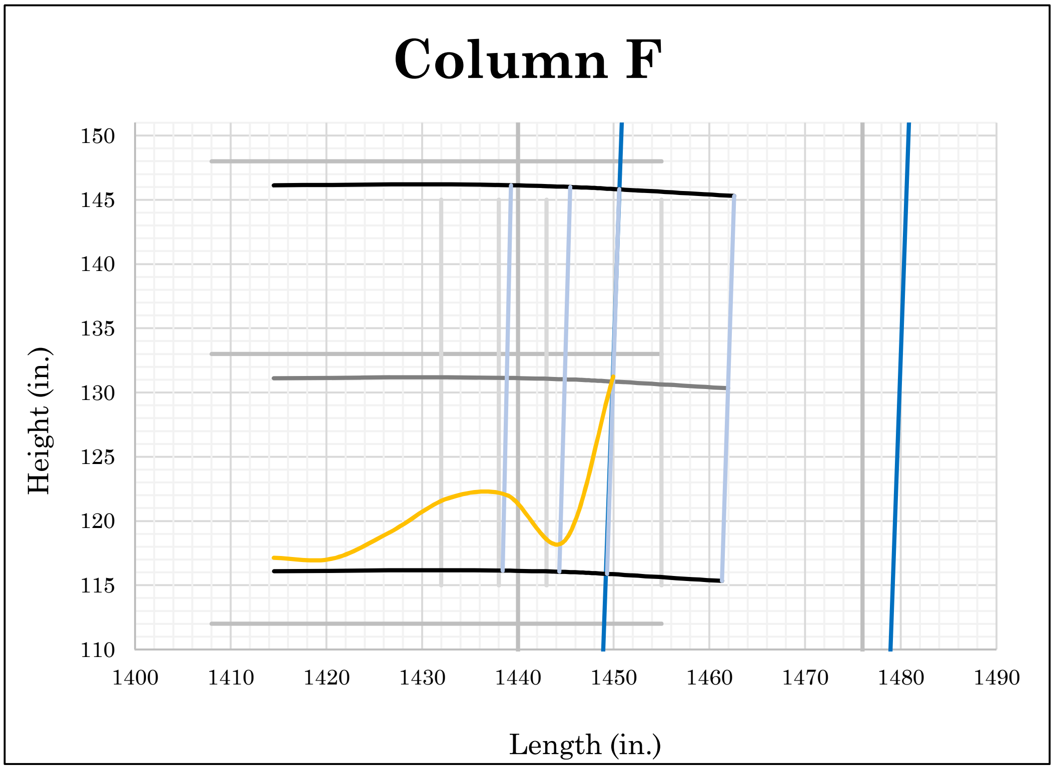


Figure 109: Joint at Column F

Frame 7.4.7

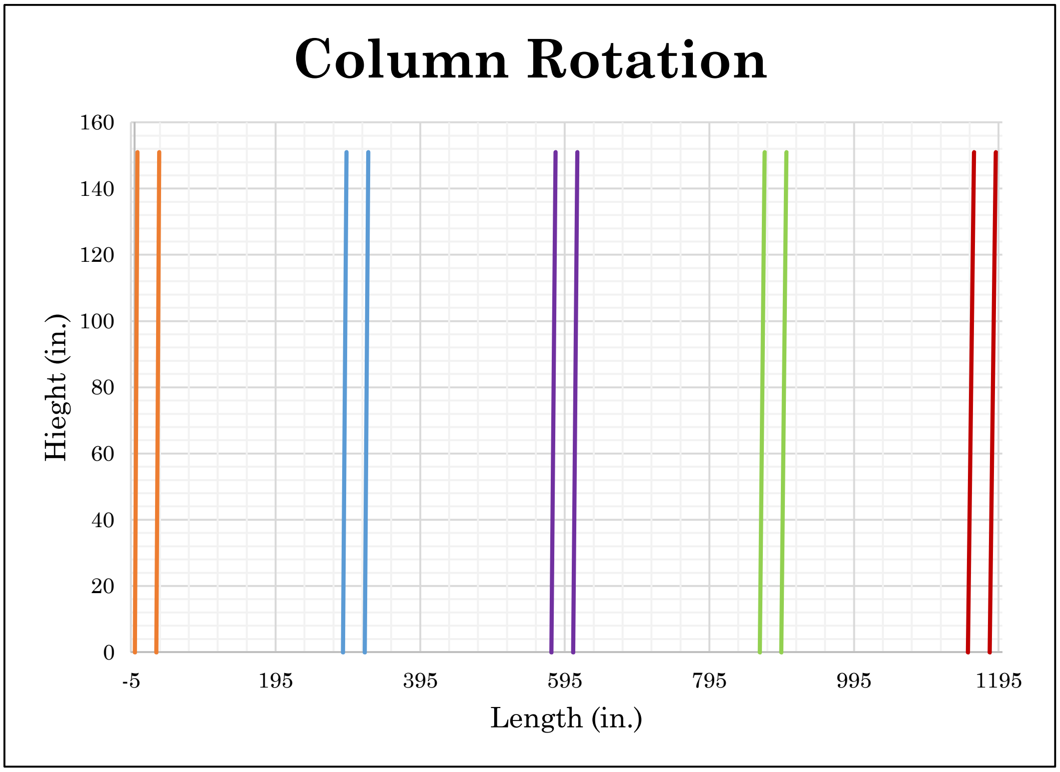


Figure 110: Column Rotation

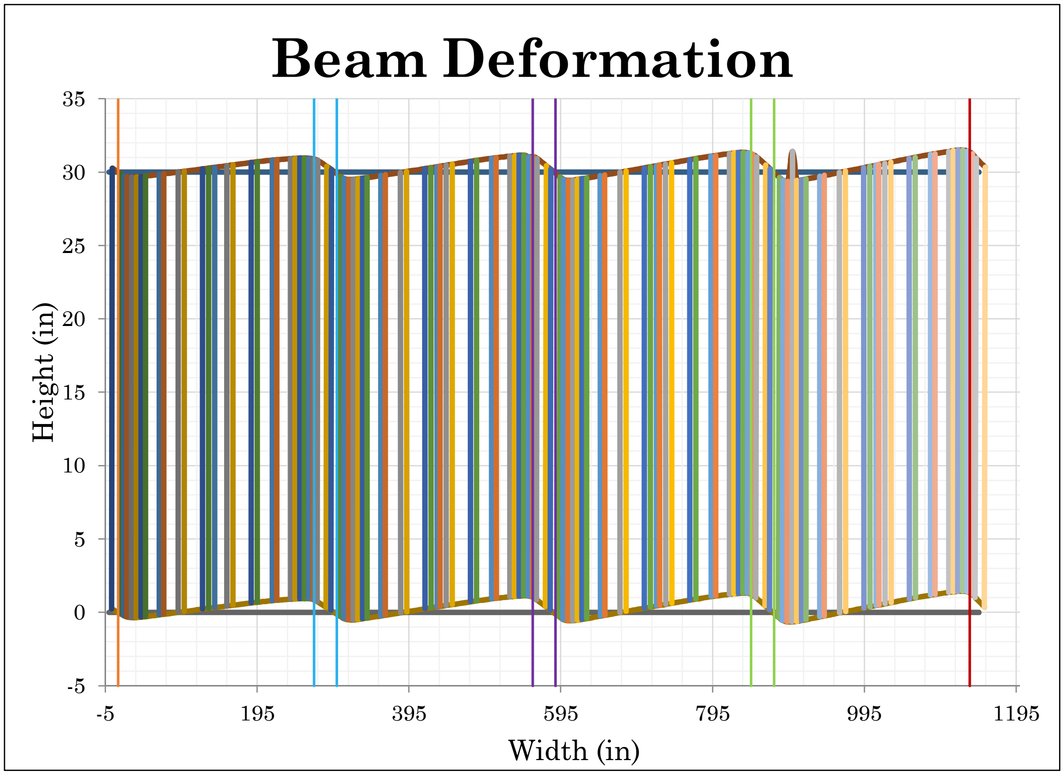


Figure 111: Beam Deformation

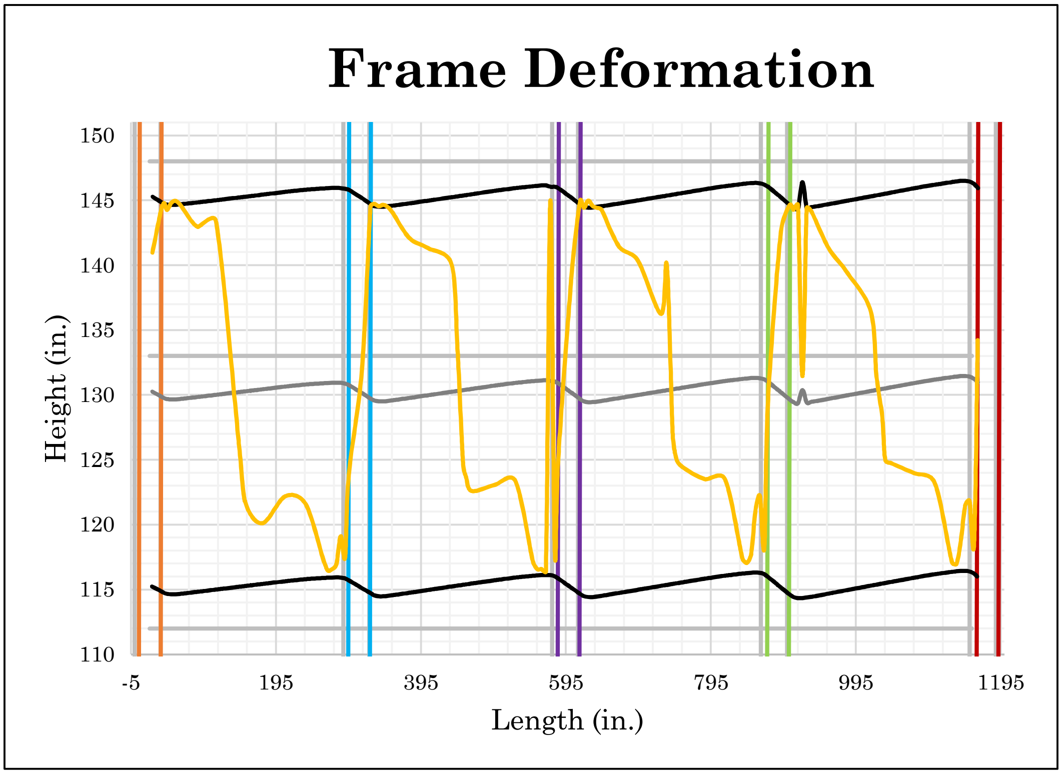


Figure 113: Frame Deformation

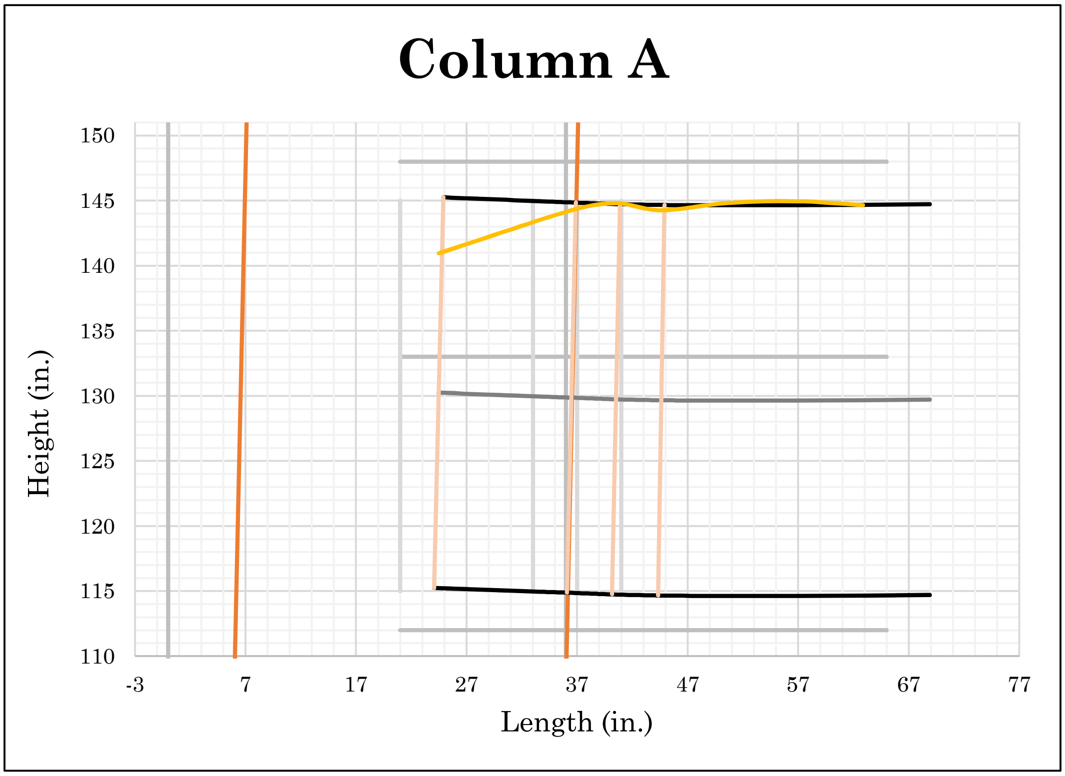


Figure 112: Joint at Column A

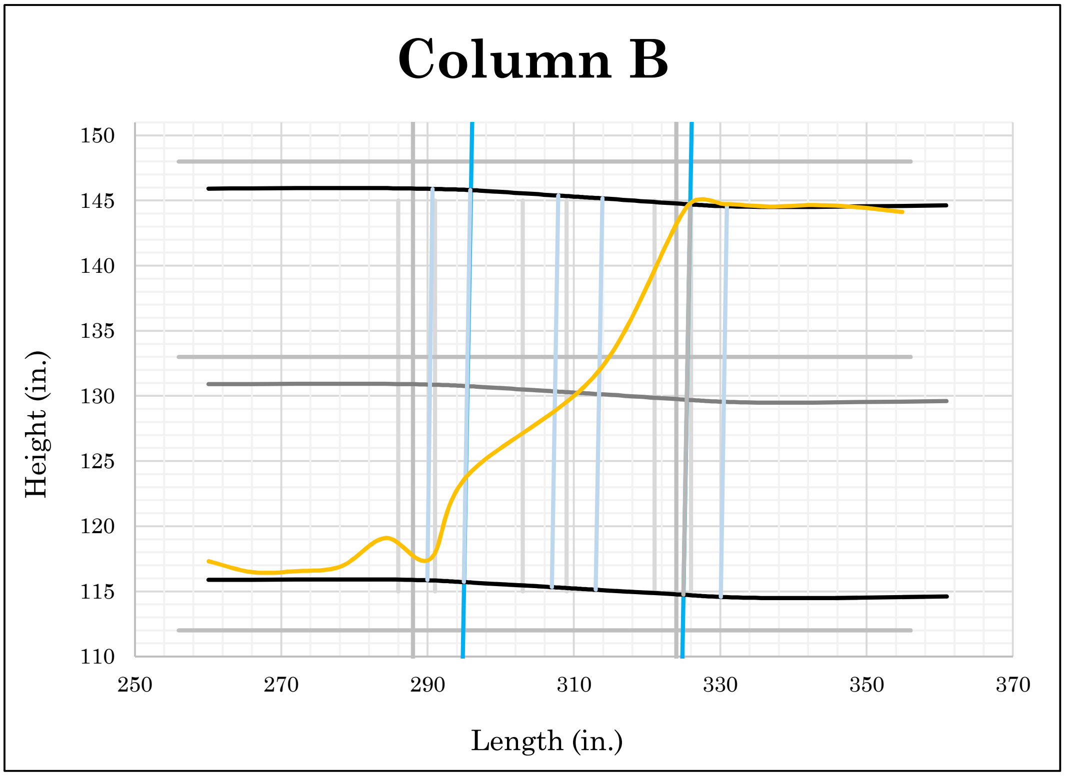


Figure 114: Joint at Column B

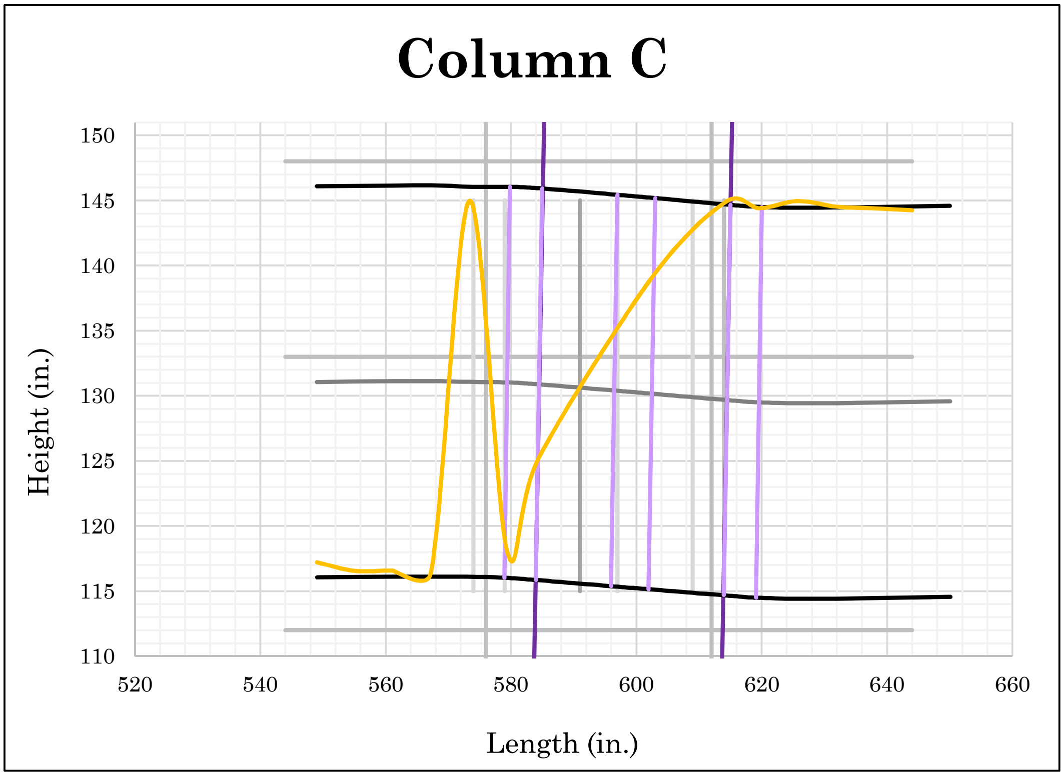


Figure 115: Joint at Column C

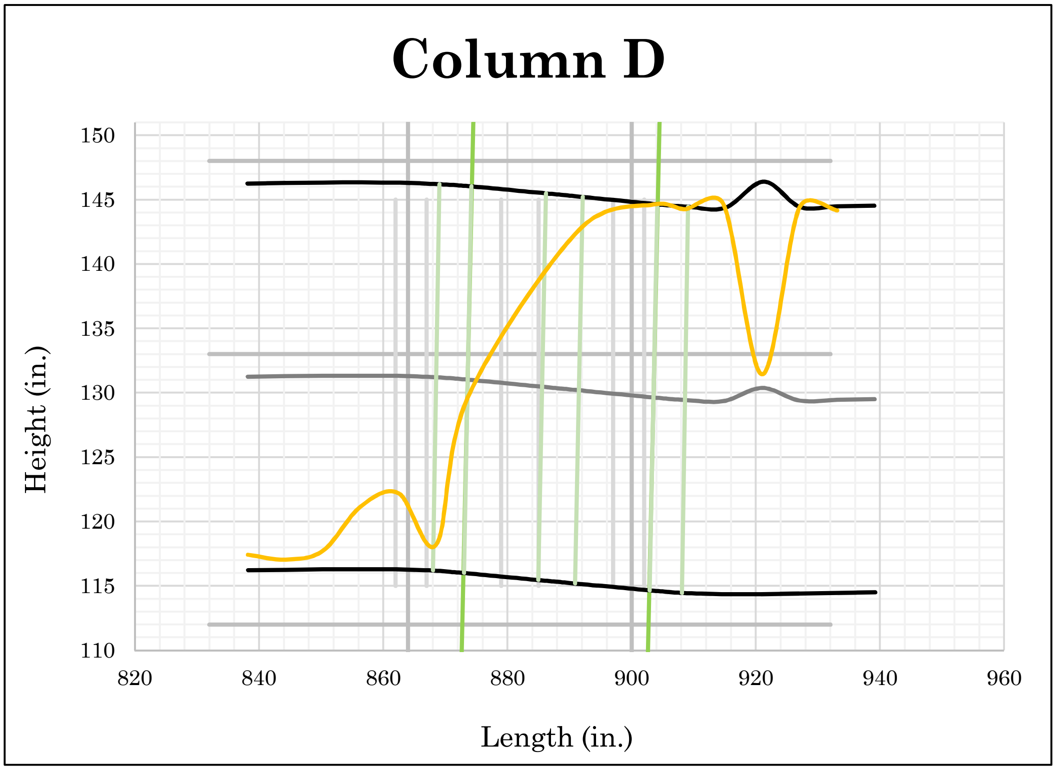


Figure 116: Joint at Column D

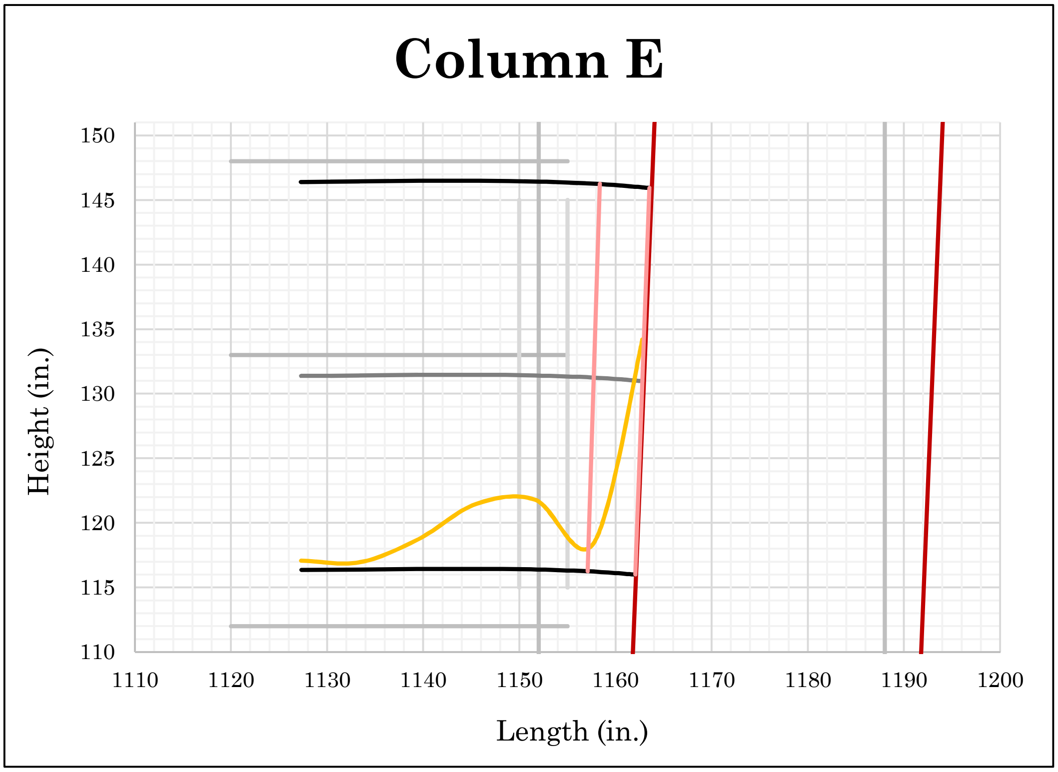


Figure 117: Joint at Column E

Frame 7.4.5

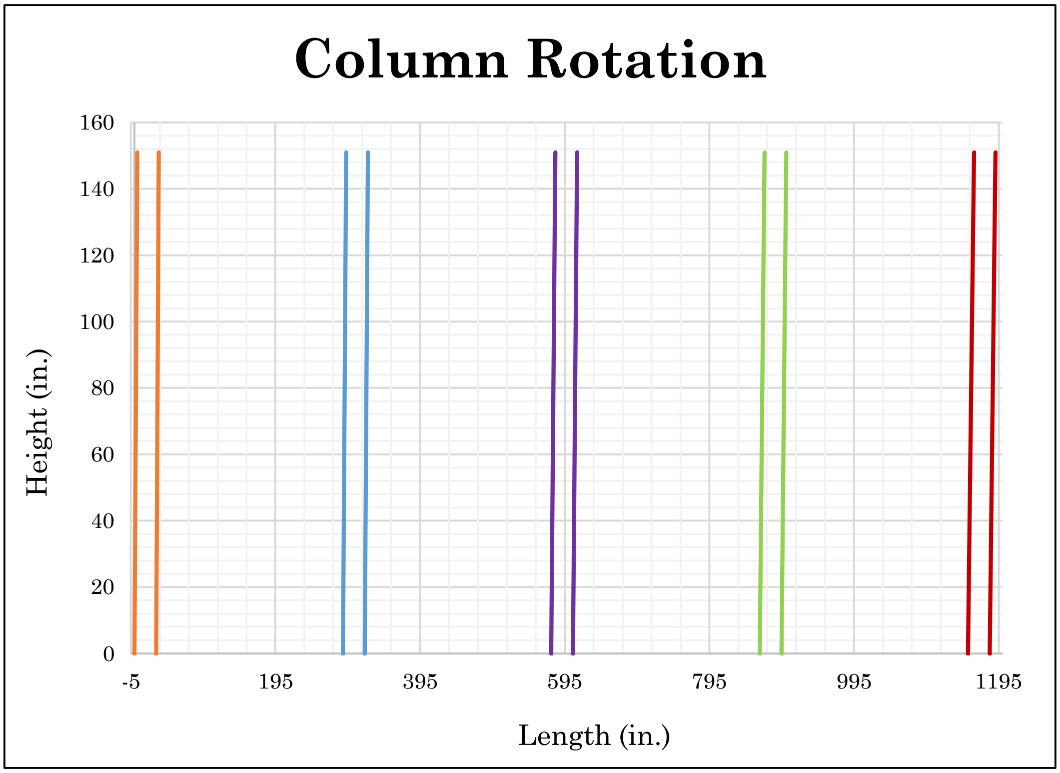


Figure 118: Column Rotation

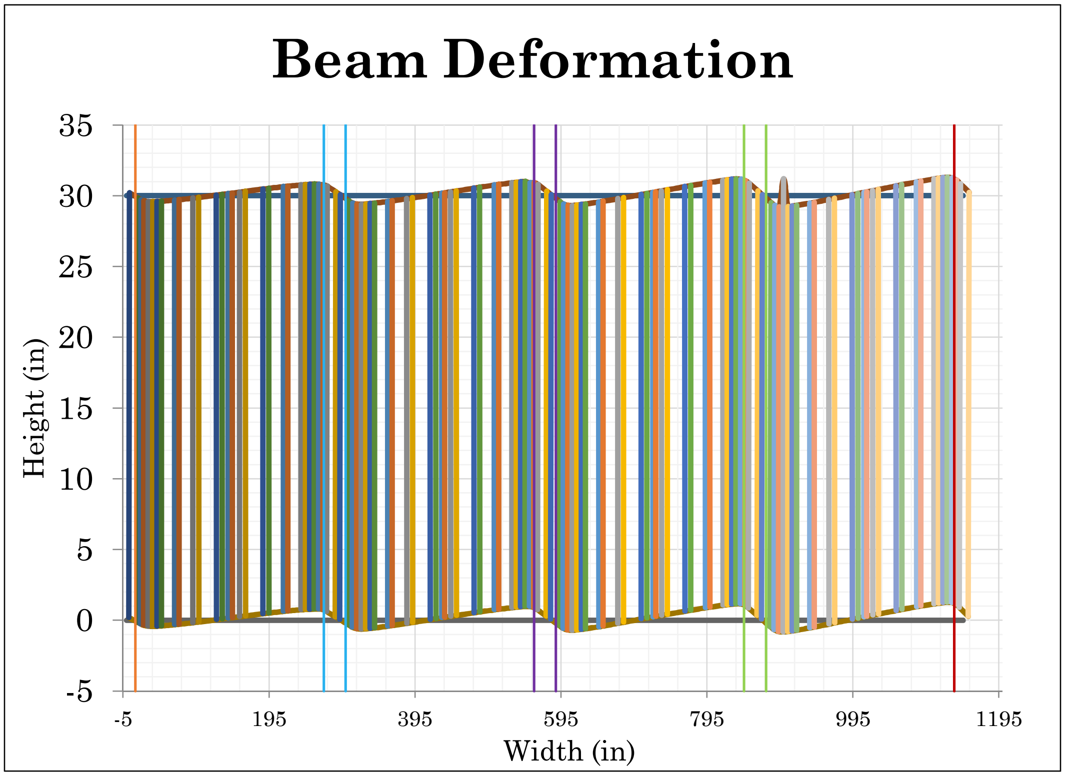


Figure 119: Beam Deformation

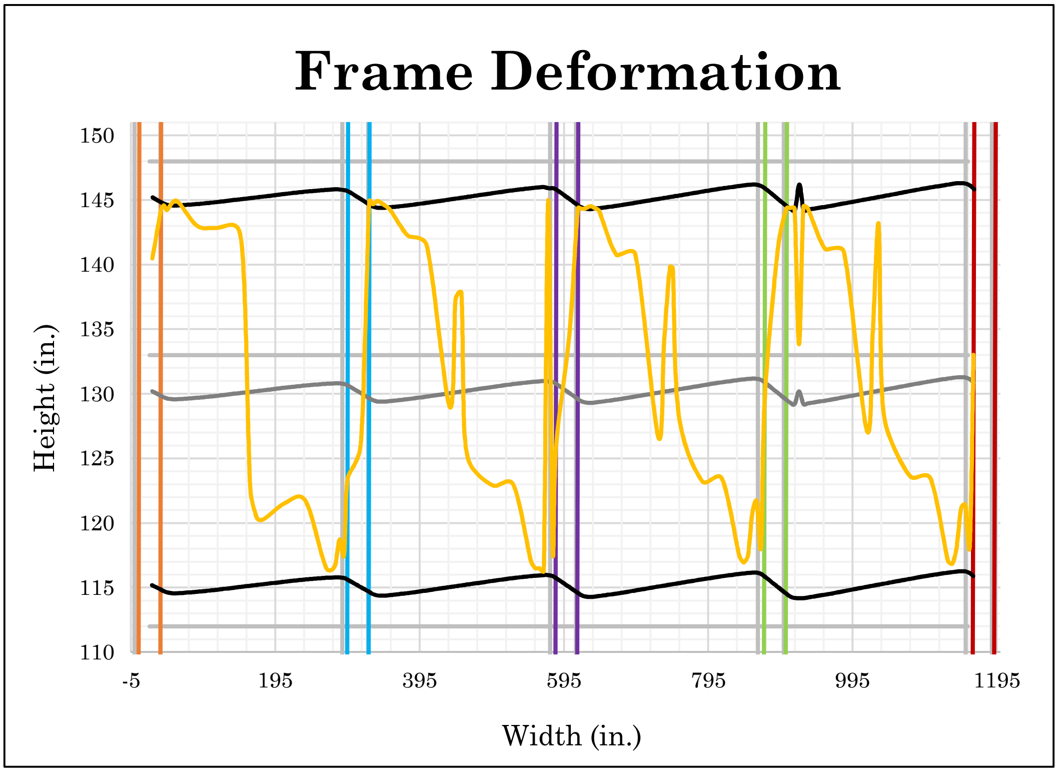


Figure 120: Frame Deformation

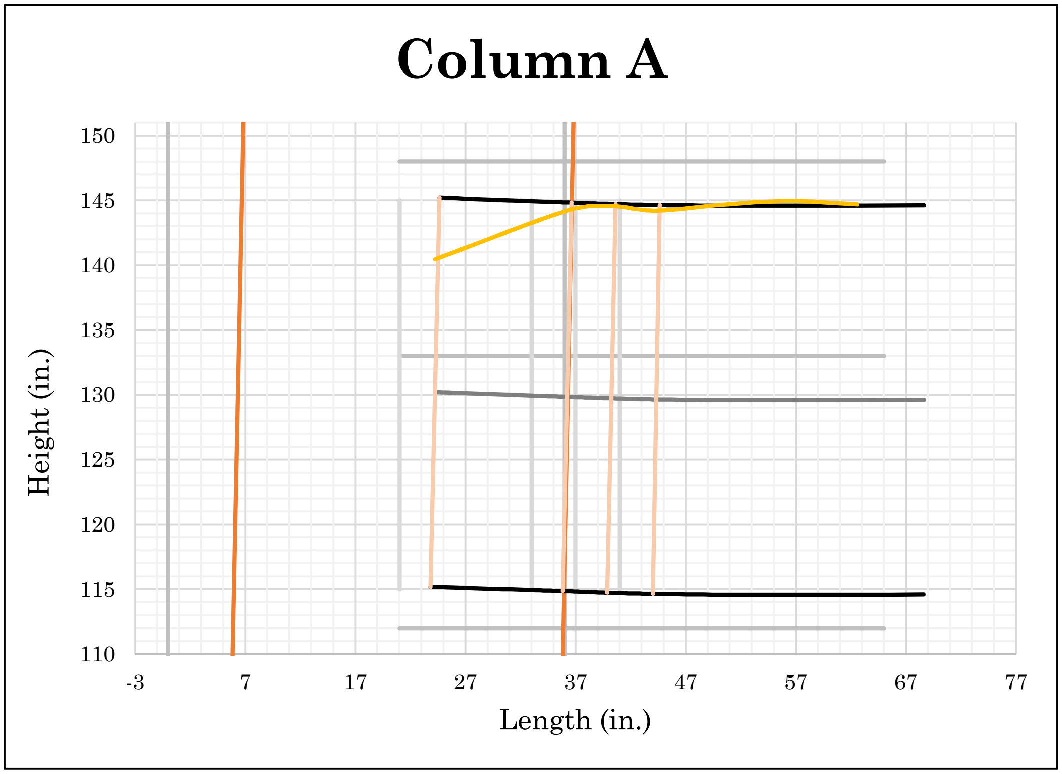


Figure 121: Joint at Column A

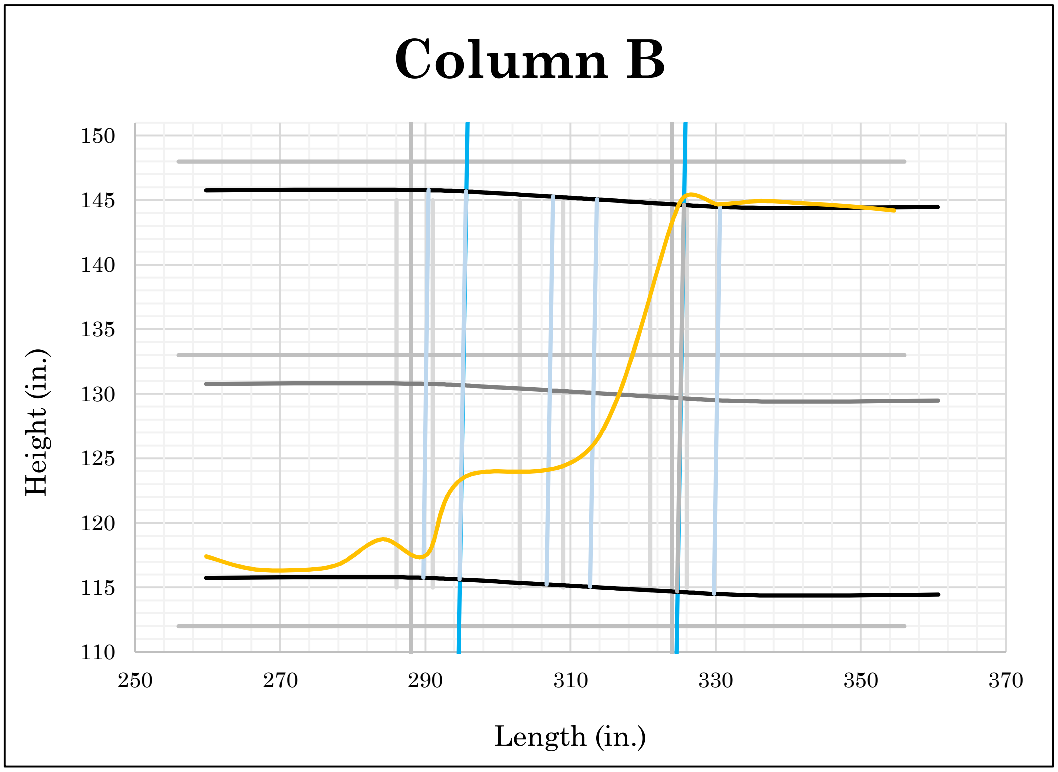


Figure 122: Joint at Column B

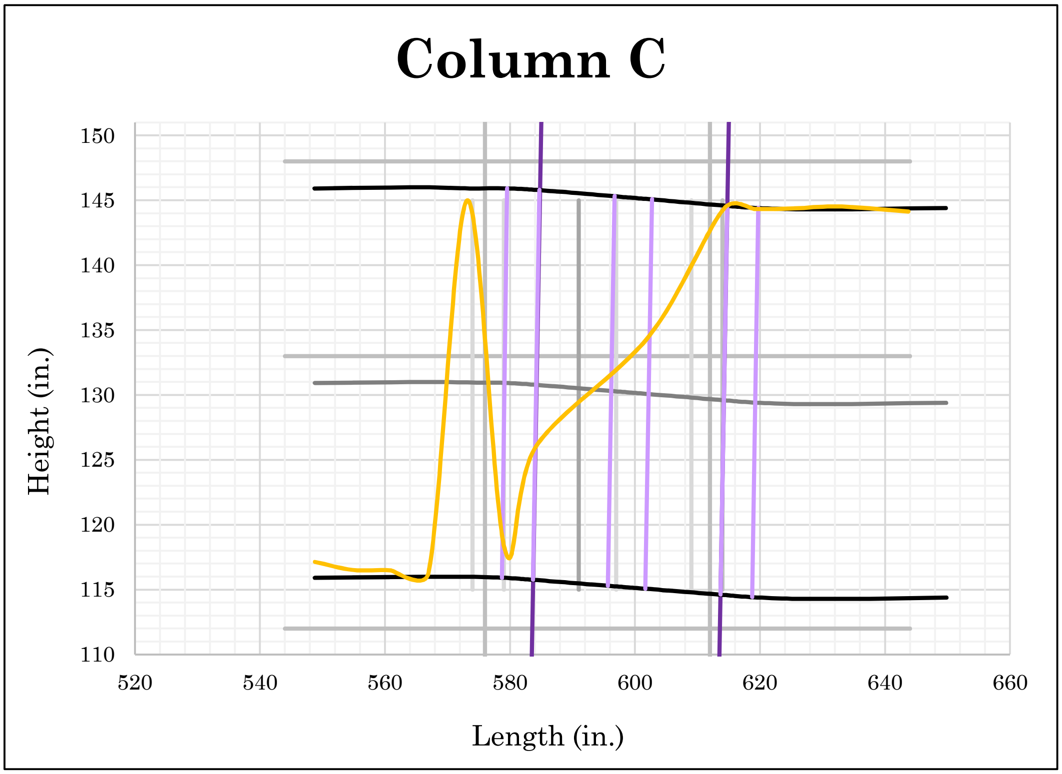


Figure 123: Joint at Column C

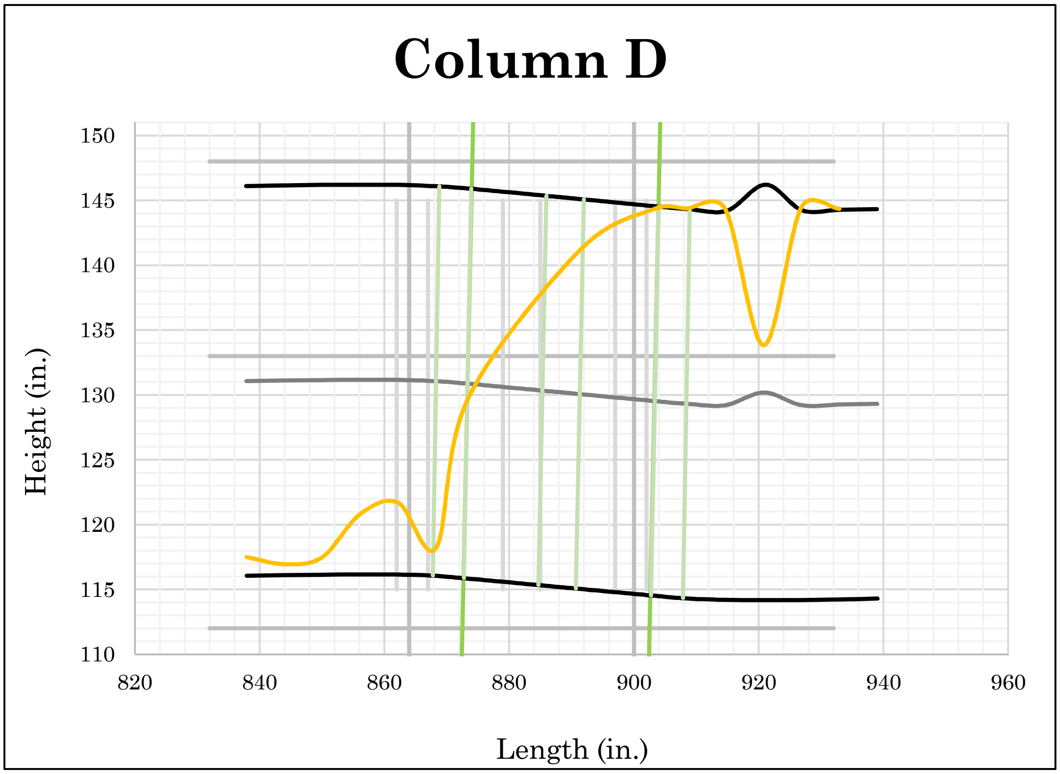


Figure 124: Joint at Column D

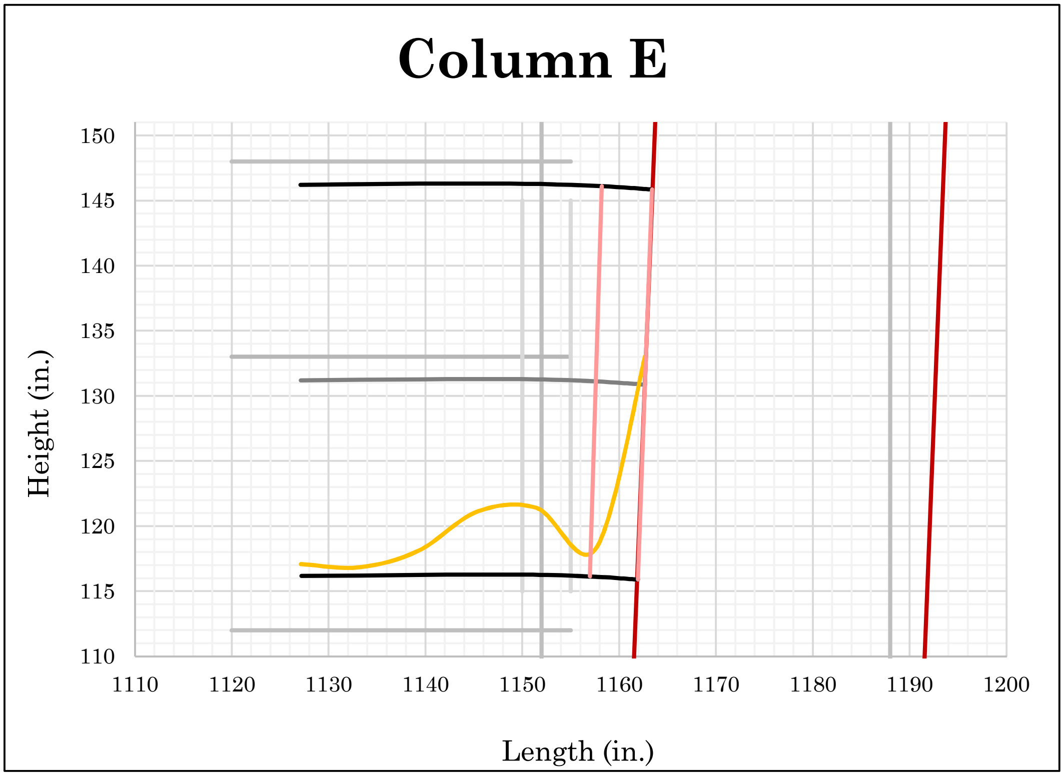


Figure 125: Joint at Column E

Frame 6.4.7

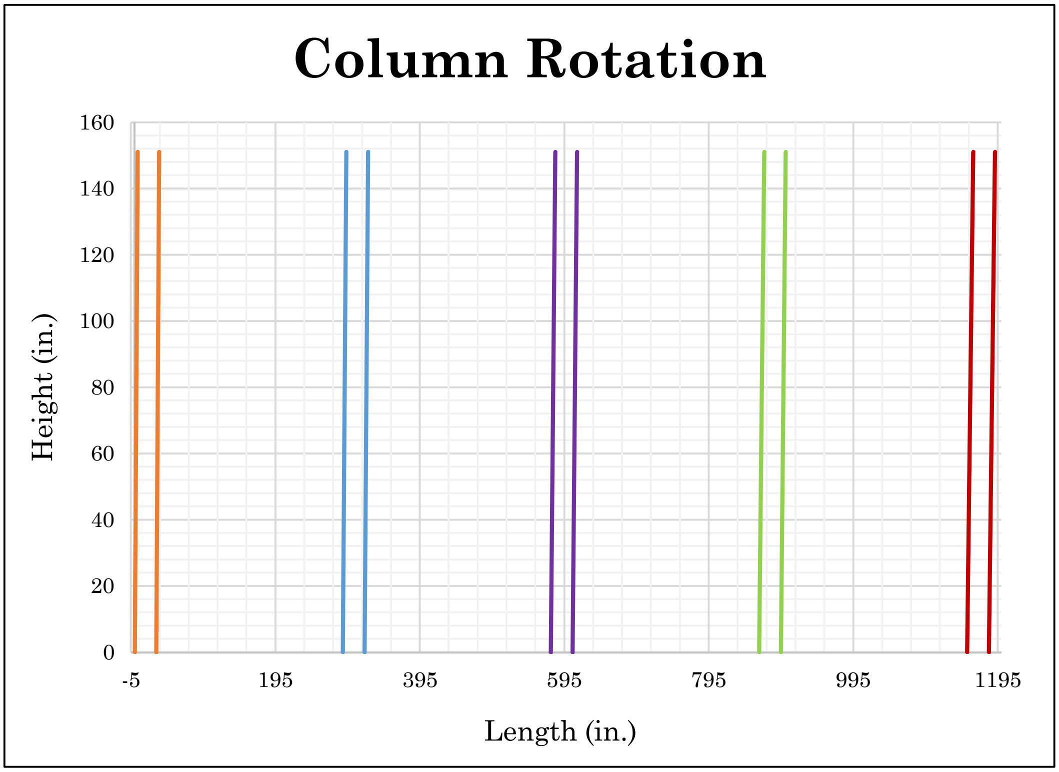


Figure 126: Column Rotation

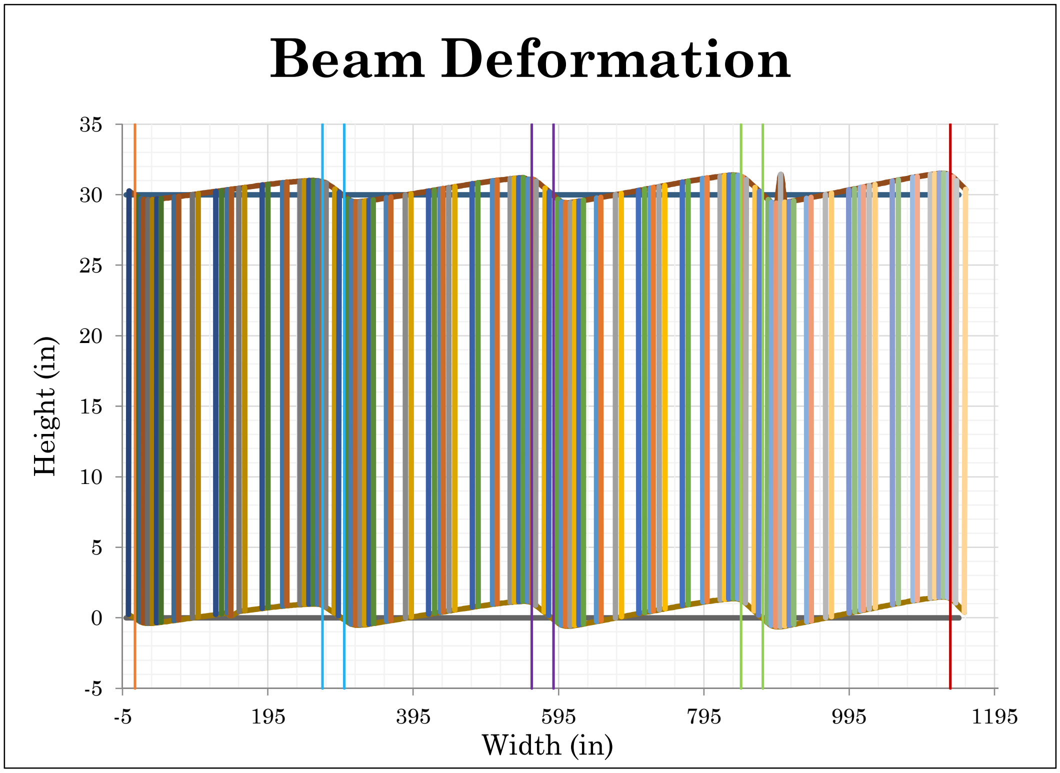


Figure 127: Beam Deformation

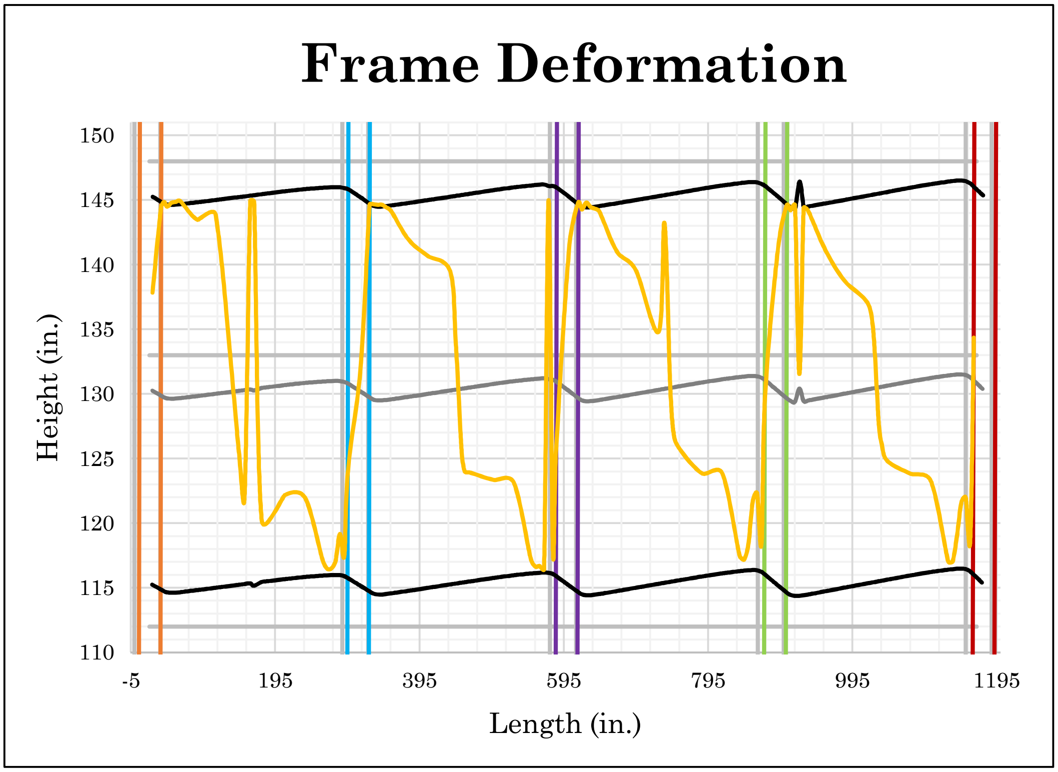


Figure 128: Frame Deformation

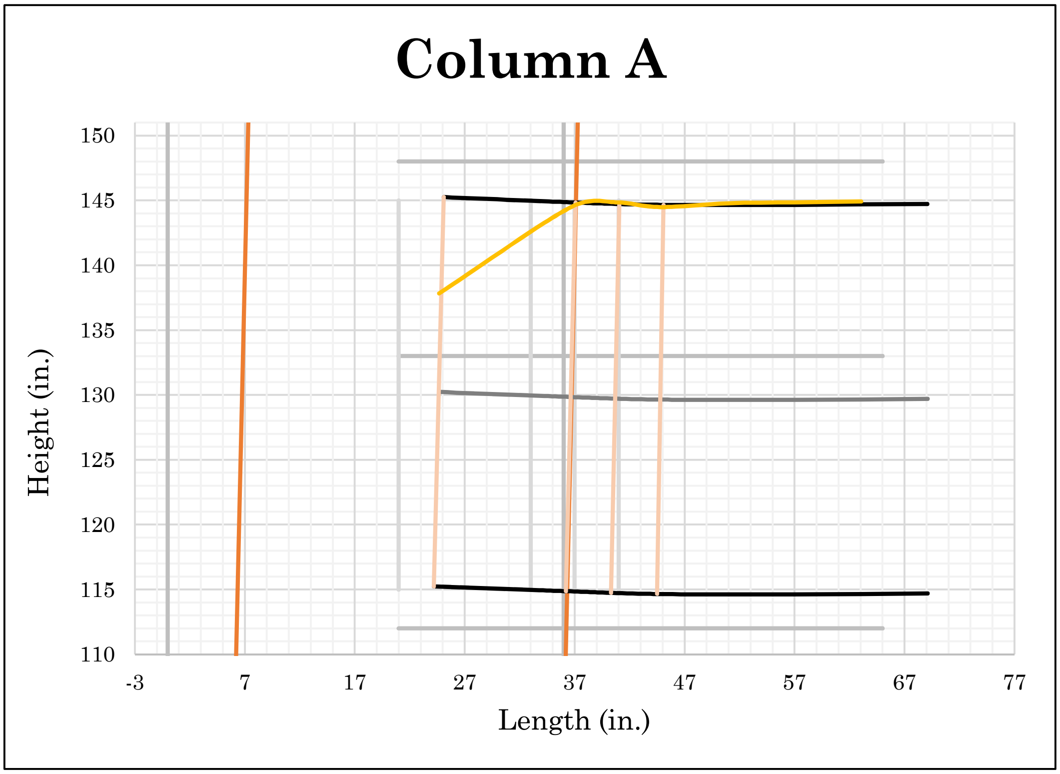


Figure 129: Joint at Column A

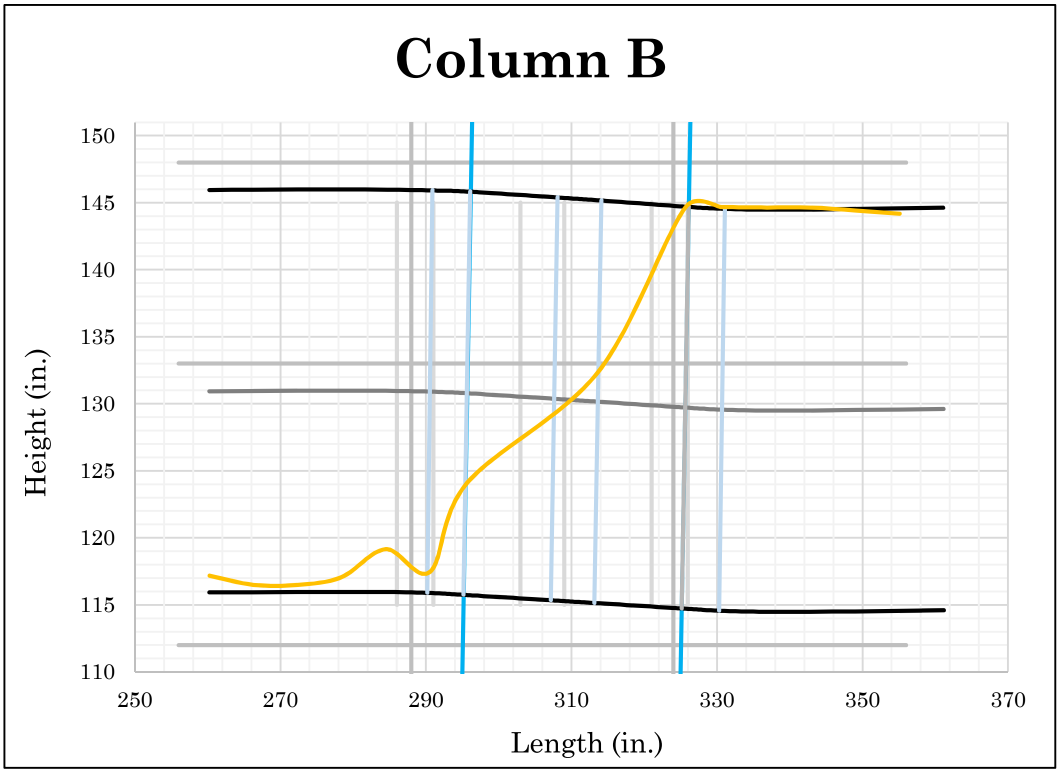


Figure 130: Joint at Column B

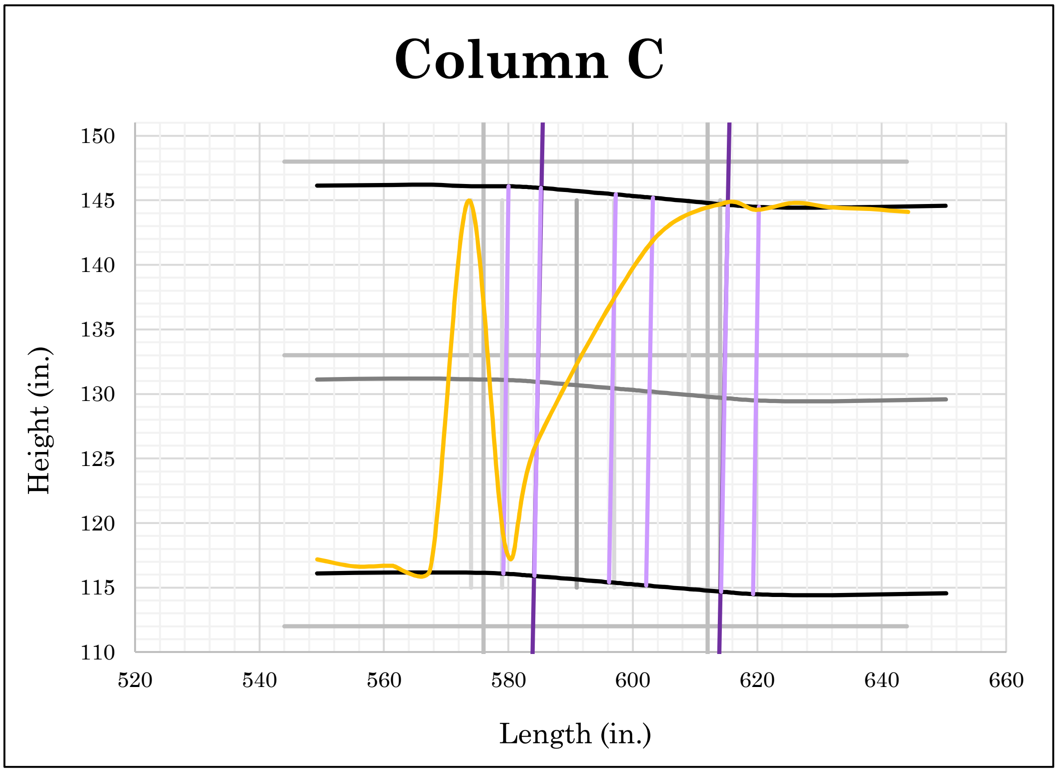


Figure 131: Joint at Column C

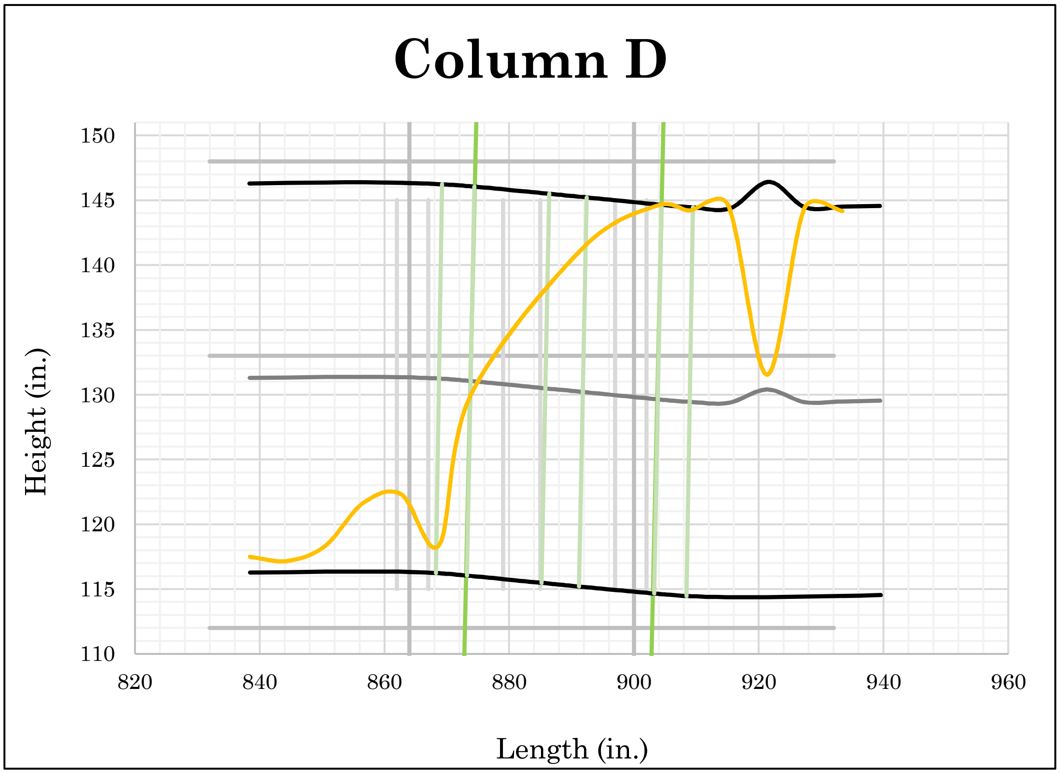


Figure 132: Joint at Column D

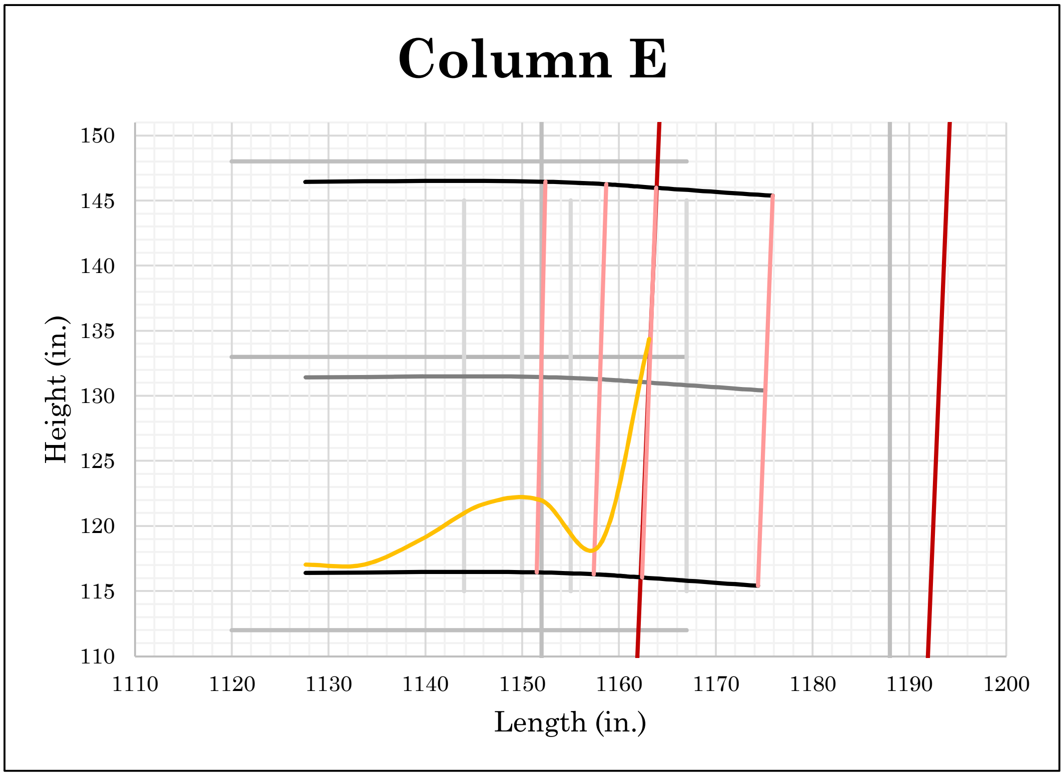


Figure 133: Joint at Column E

Frame 6.4.5

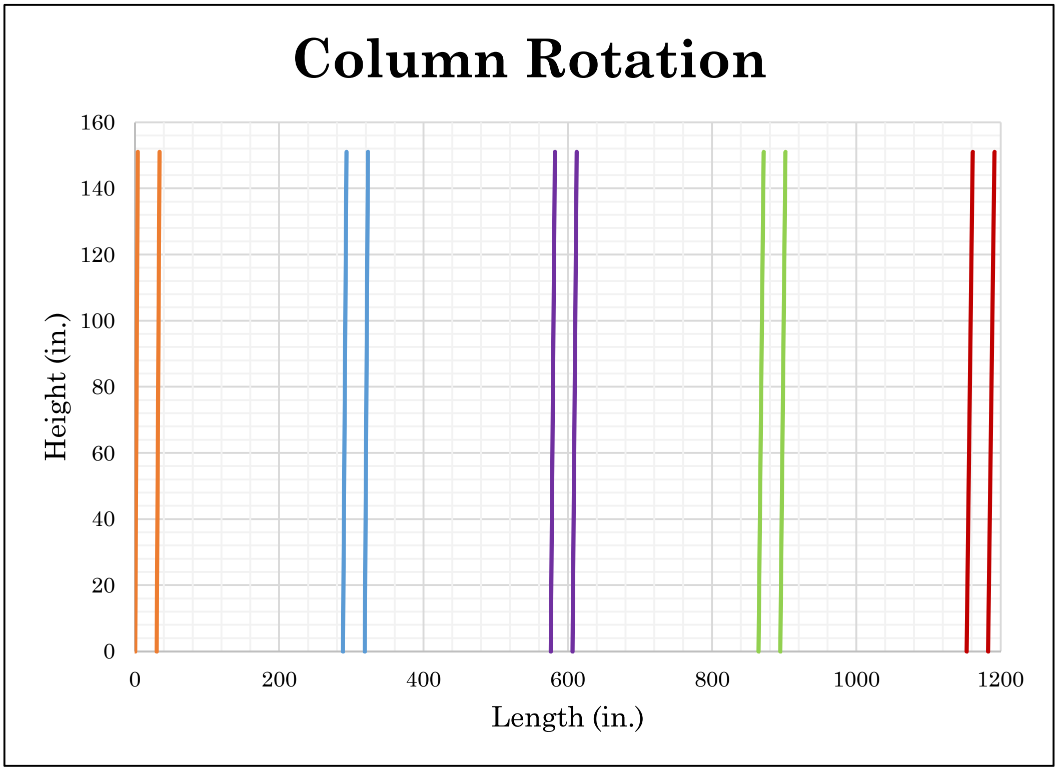


Figure 134: Column Rotation

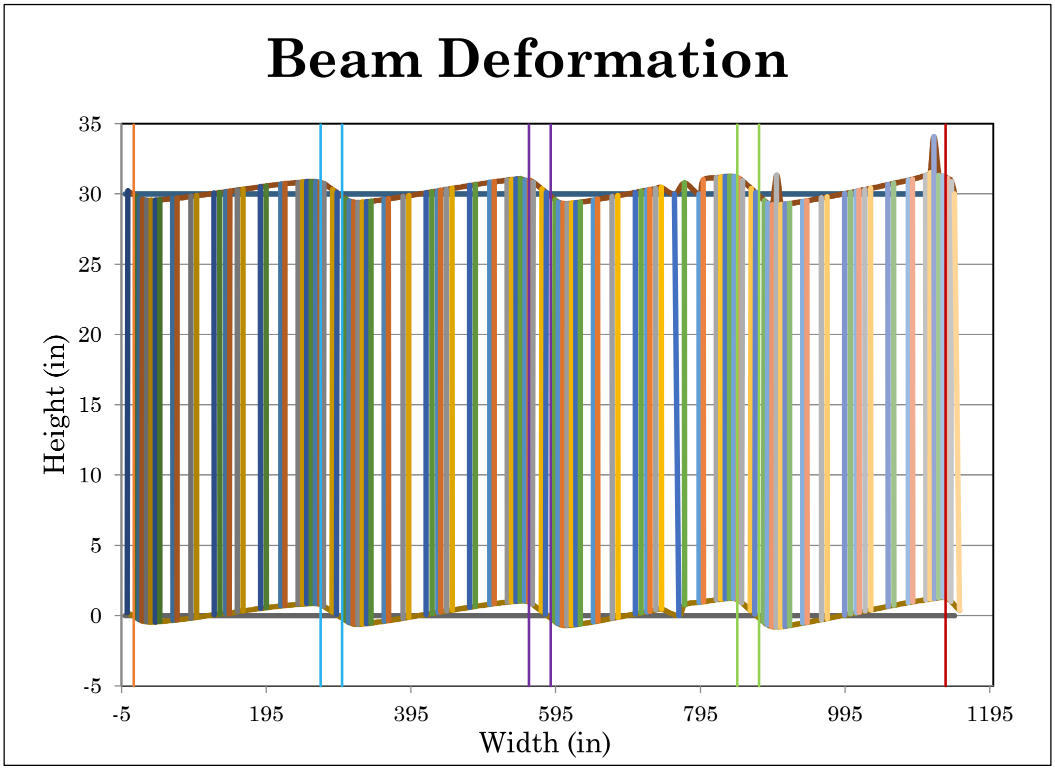


Figure 135: Beam Deformation

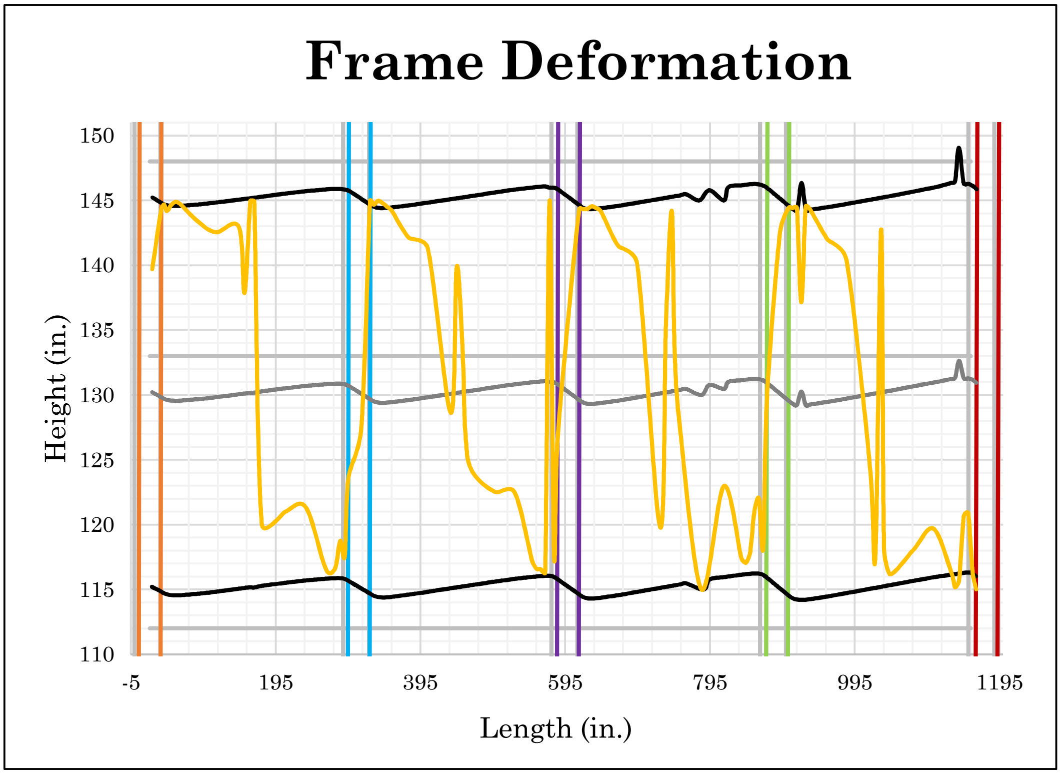


Figure 136: Frame Deformation

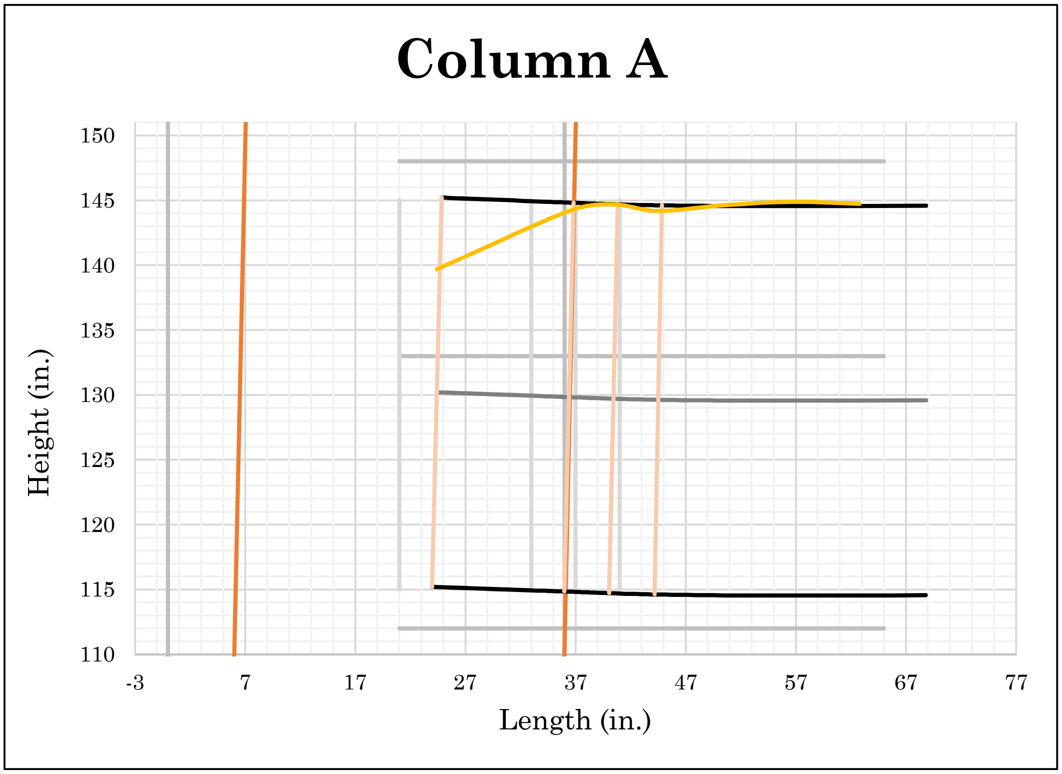


Figure 137: Joint at Column A

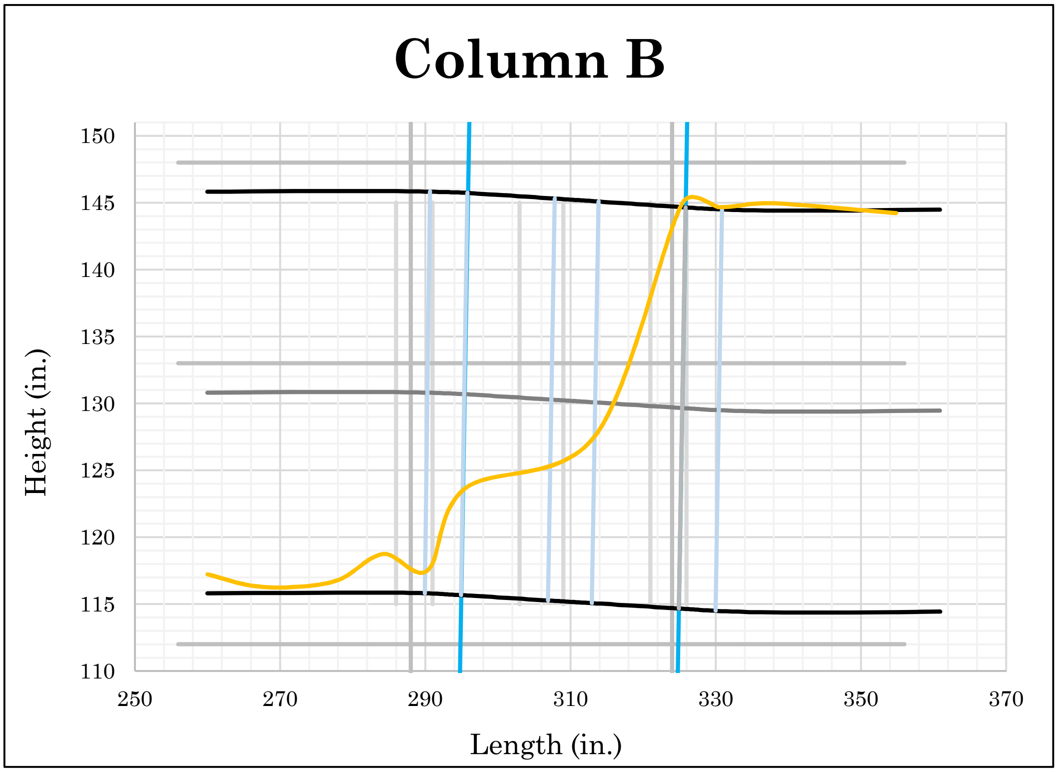


Figure 138: Joint at Column B

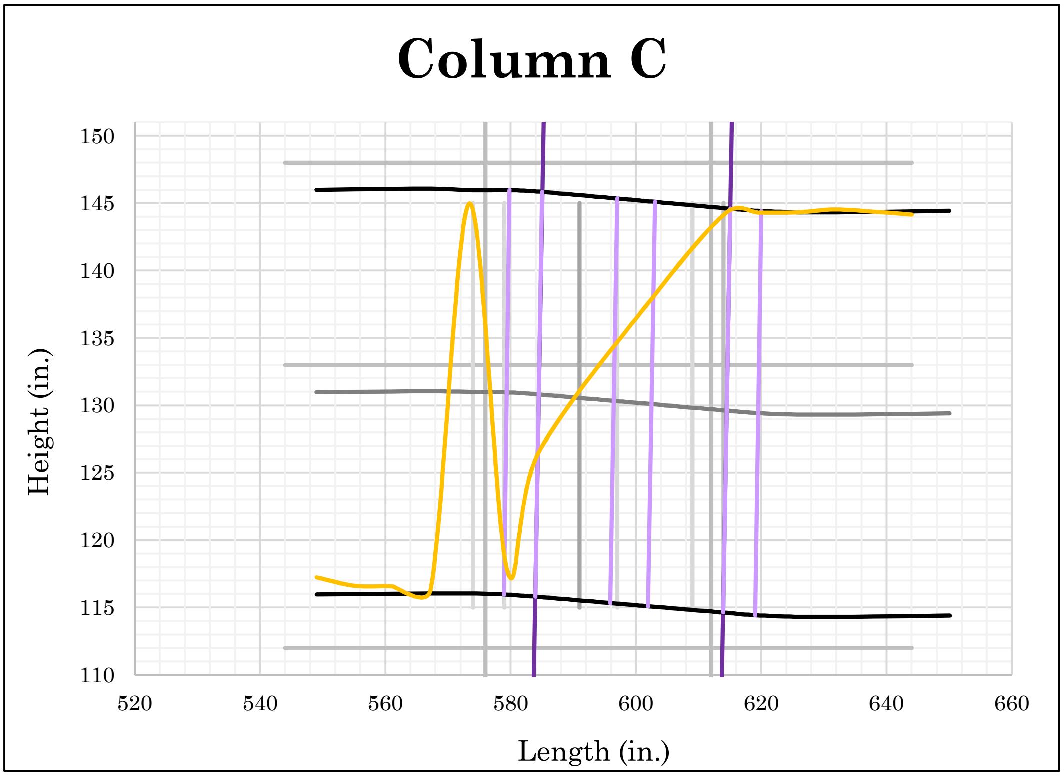


Figure 139: Joint at Column C

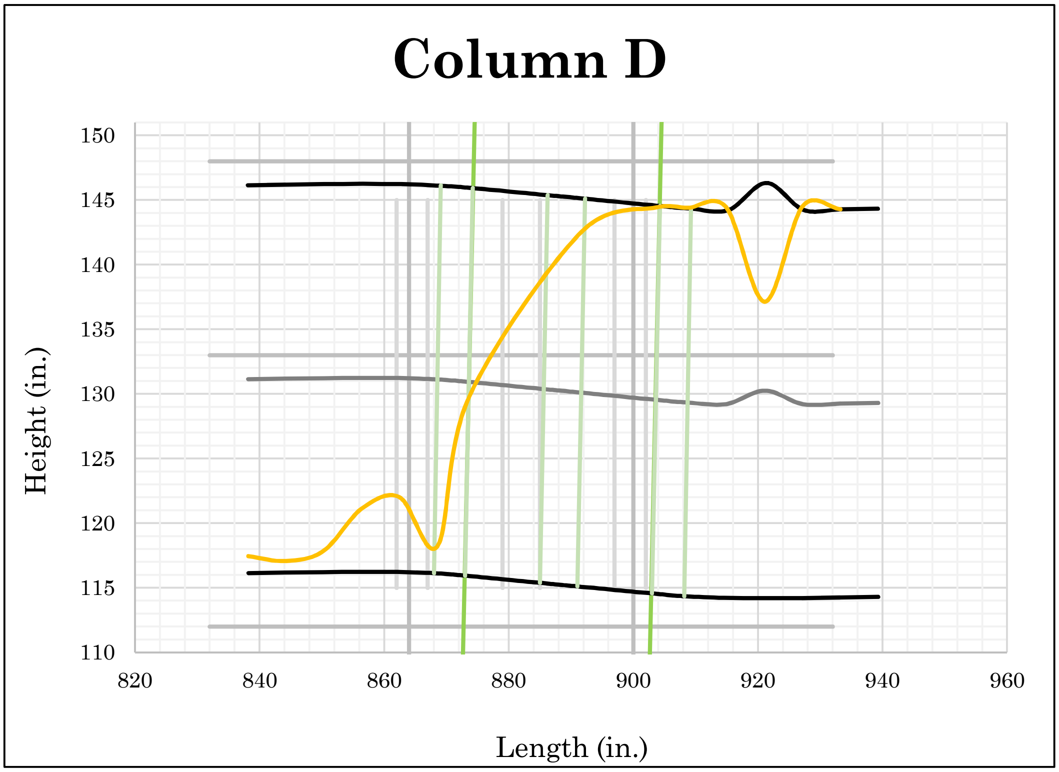


Figure 140: Joint at Column D

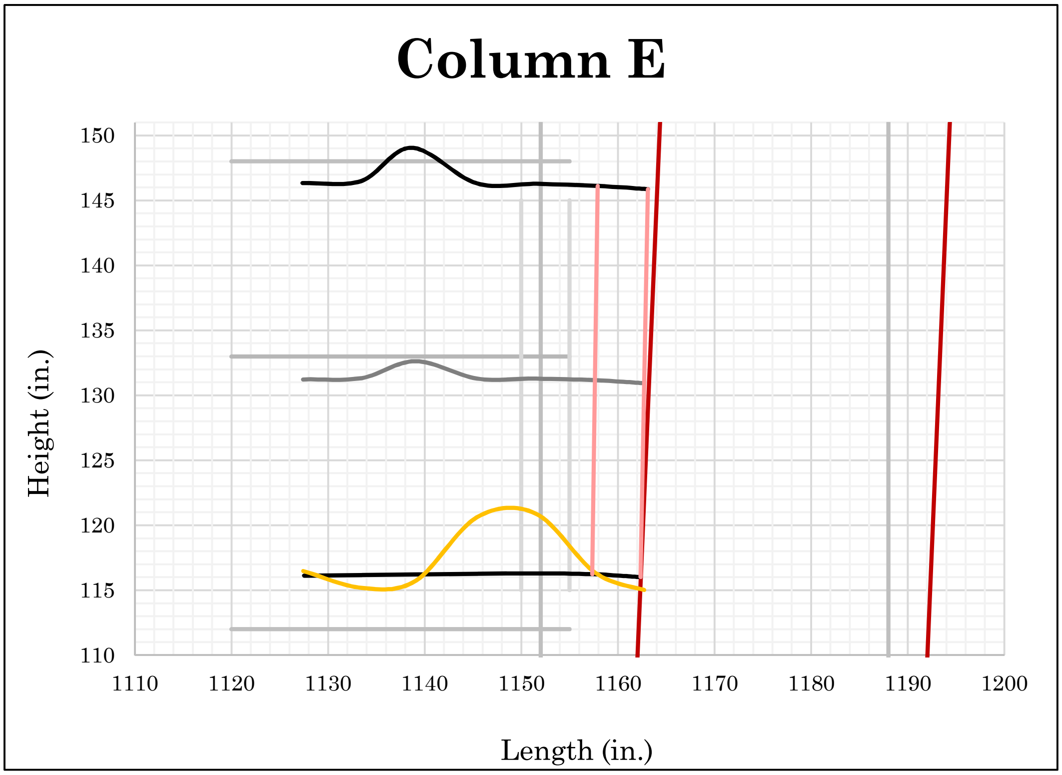


Figure 141: Joint at Column E

Frame 7.3.7

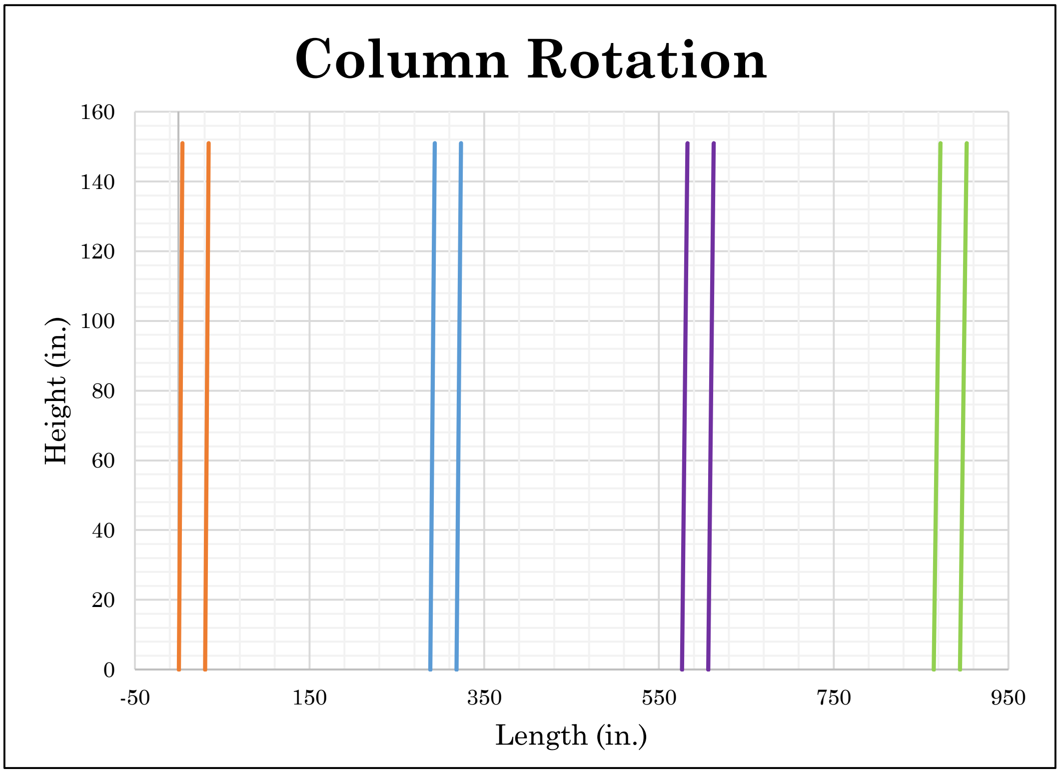


Figure 142: Column Rotation

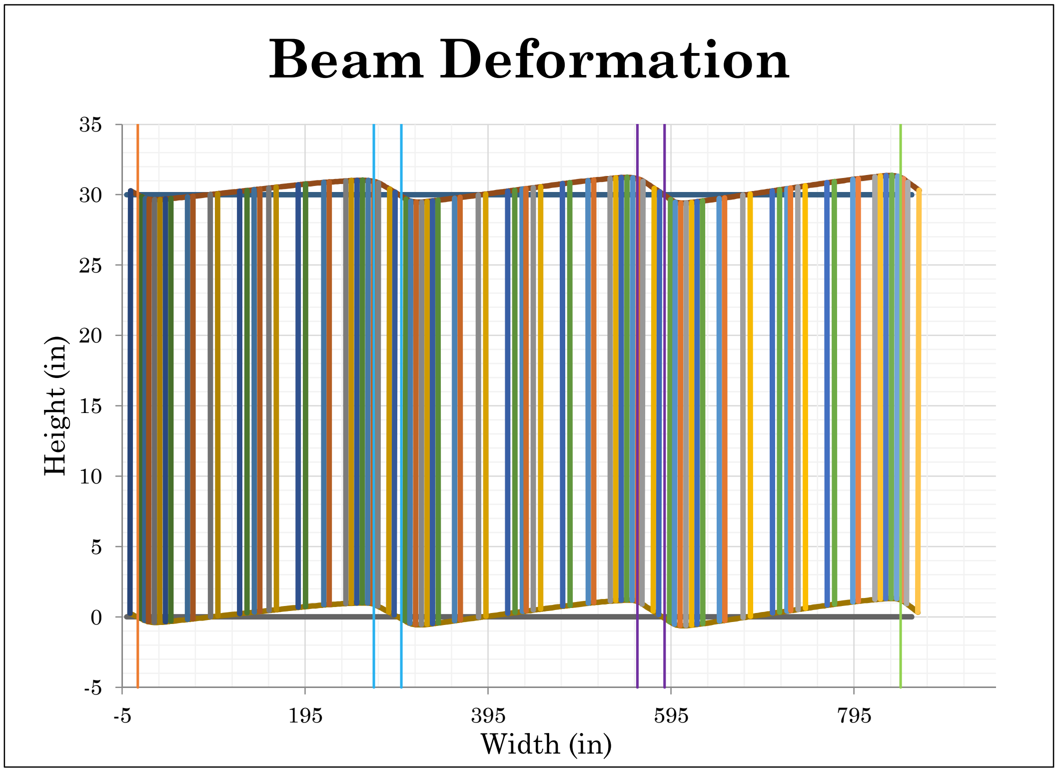


Figure 143: Beam Deformation

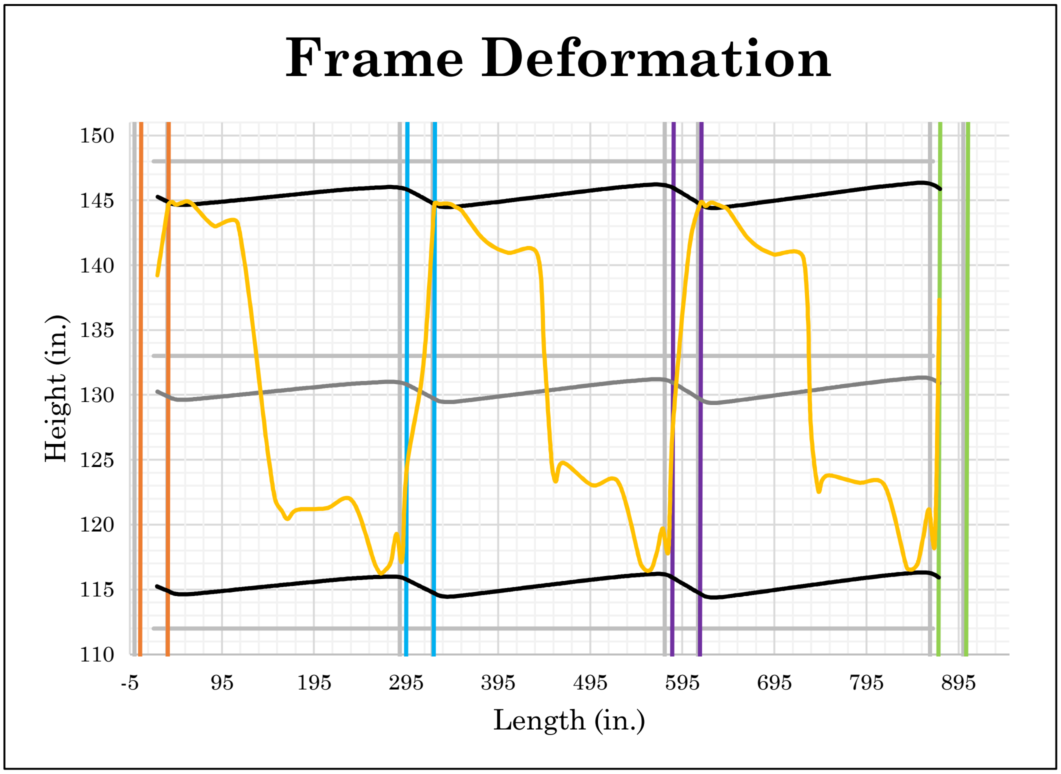


Figure 144: Frame Deformation

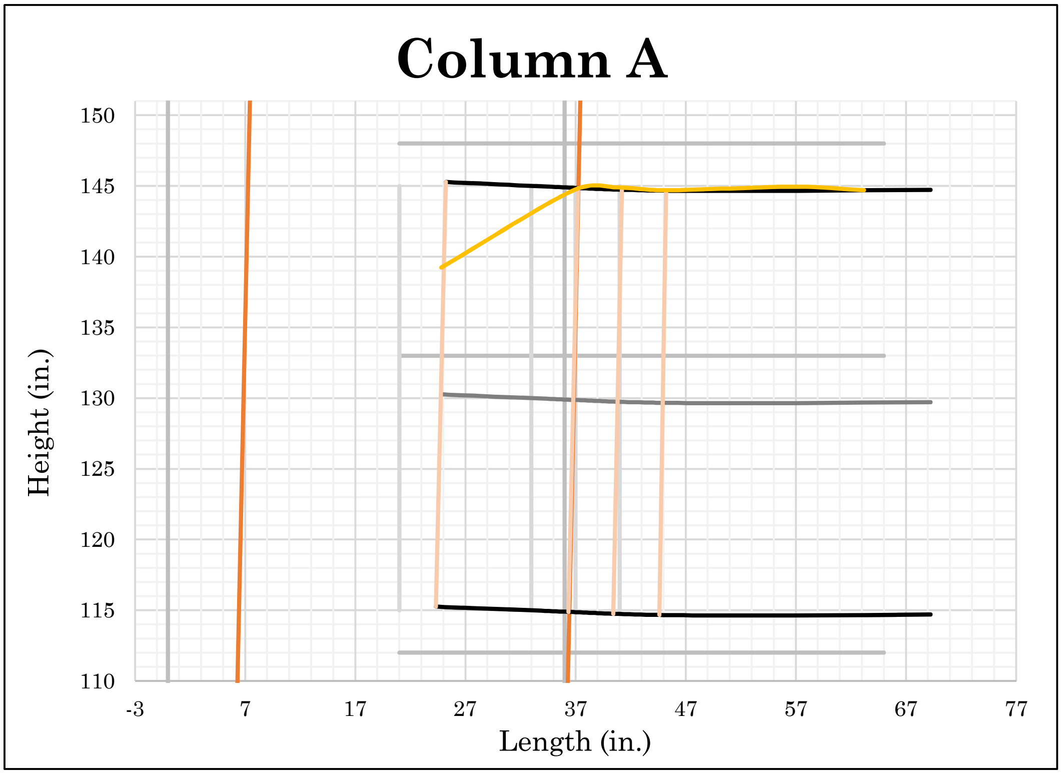


Figure 145: Joint at Column A

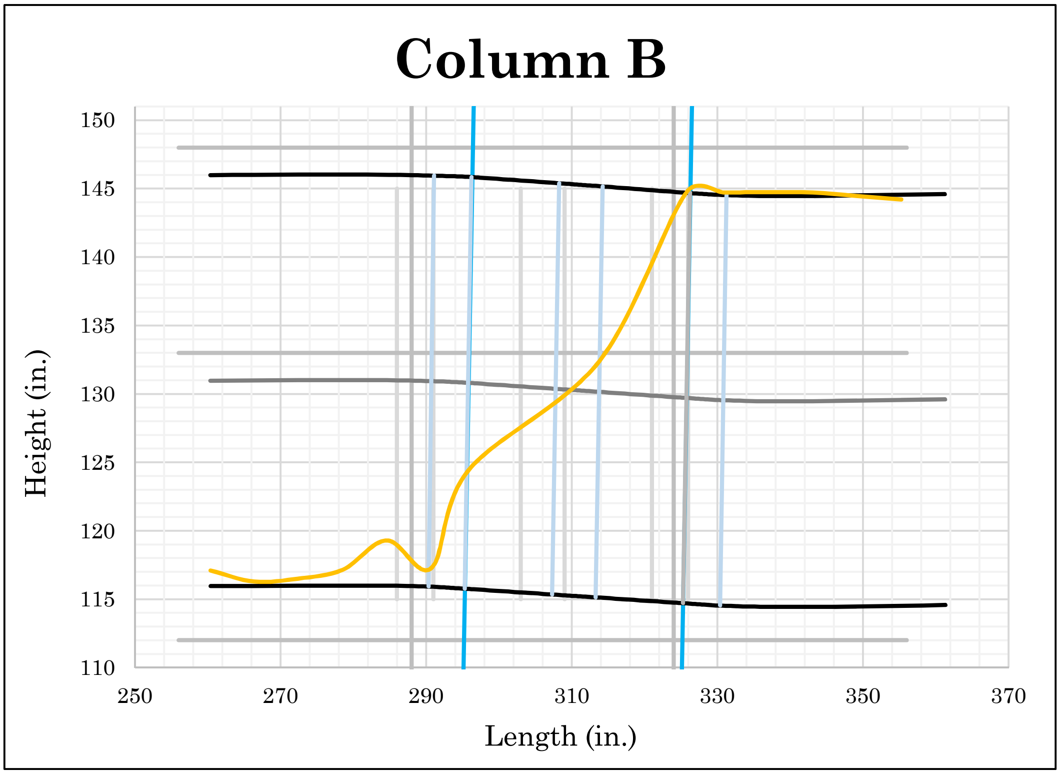


Figure 146: Joint at Column B

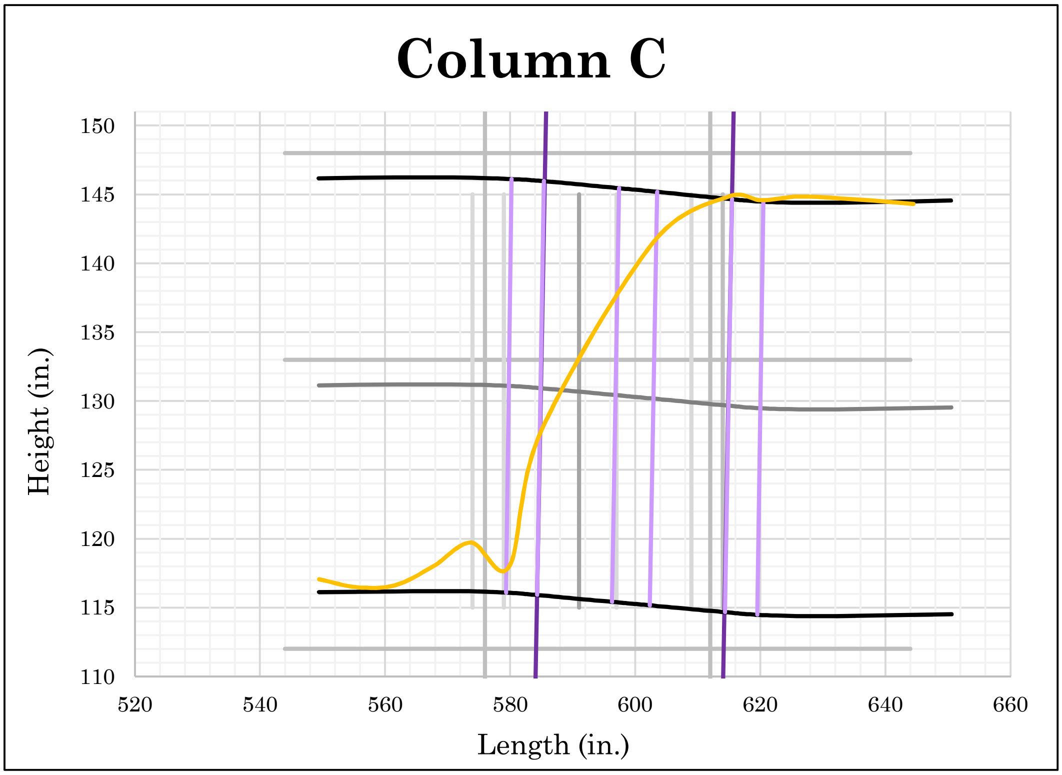


Figure 147: Joint at Column C

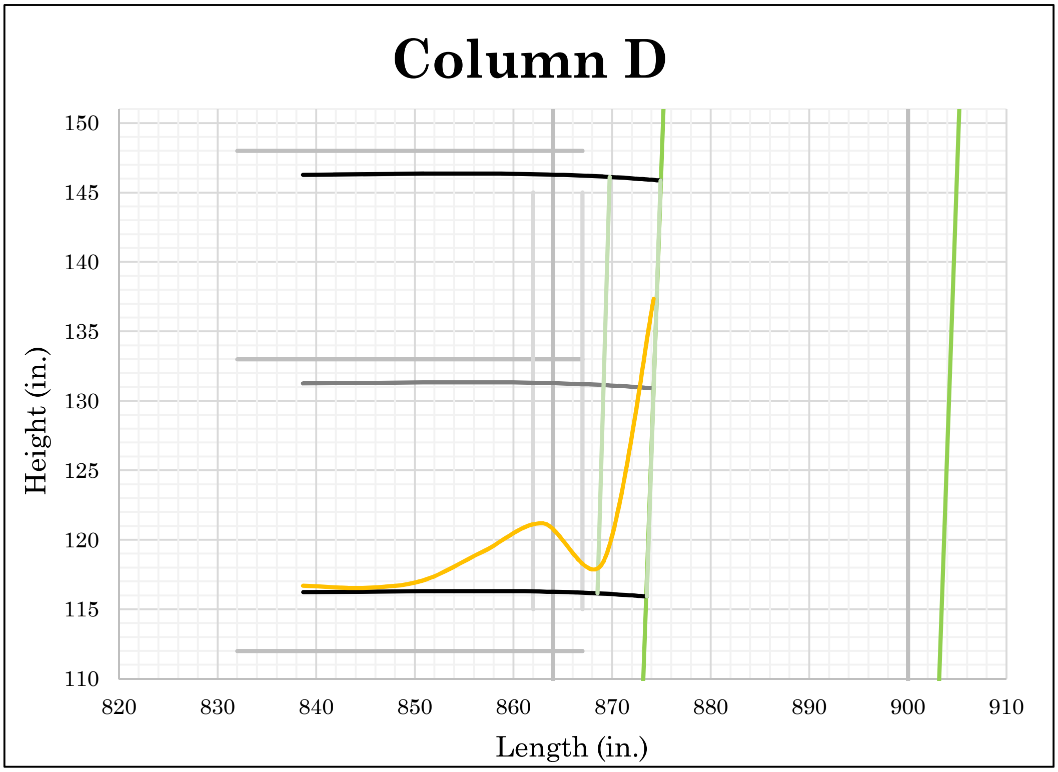


Figure 148: Joint at Column D

Frame 7.3.5

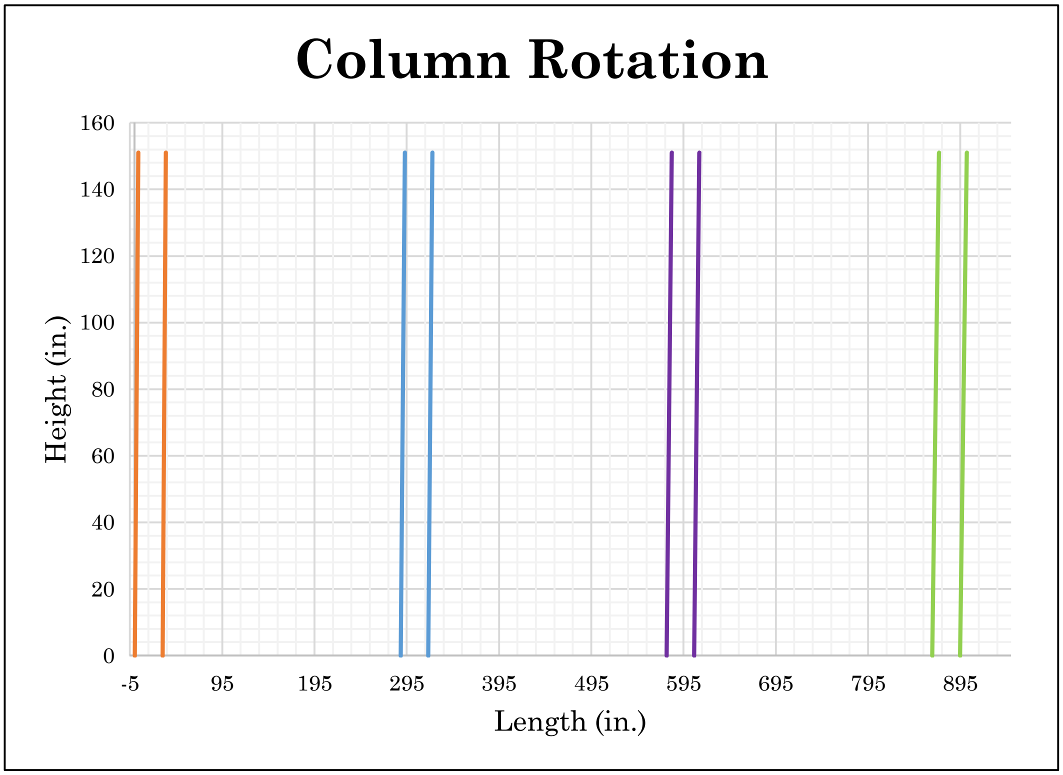


Figure 149: Column Rotation

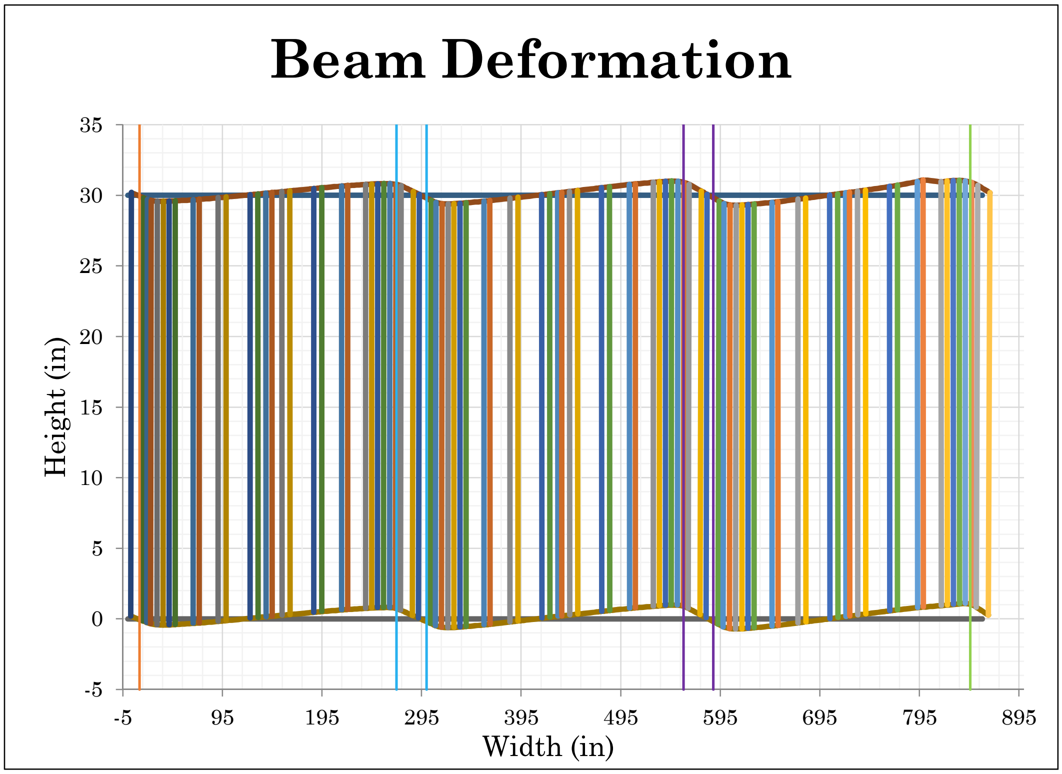


Figure 150: Beam Deformation

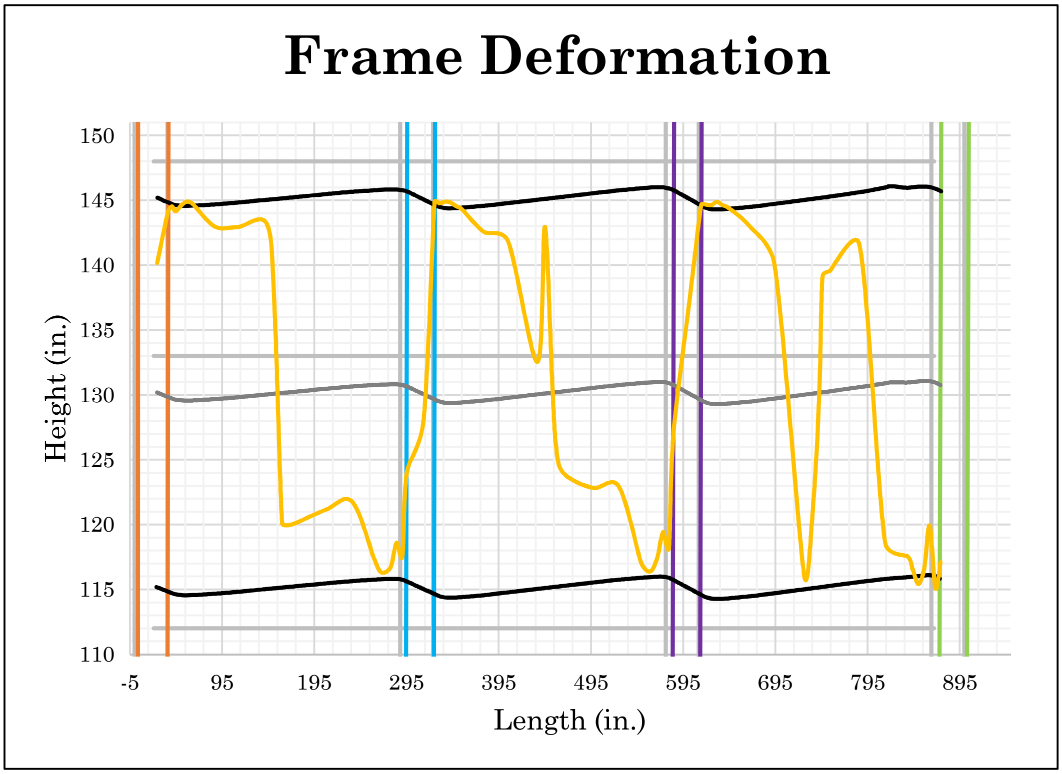


Figure 151: Frame Deformation

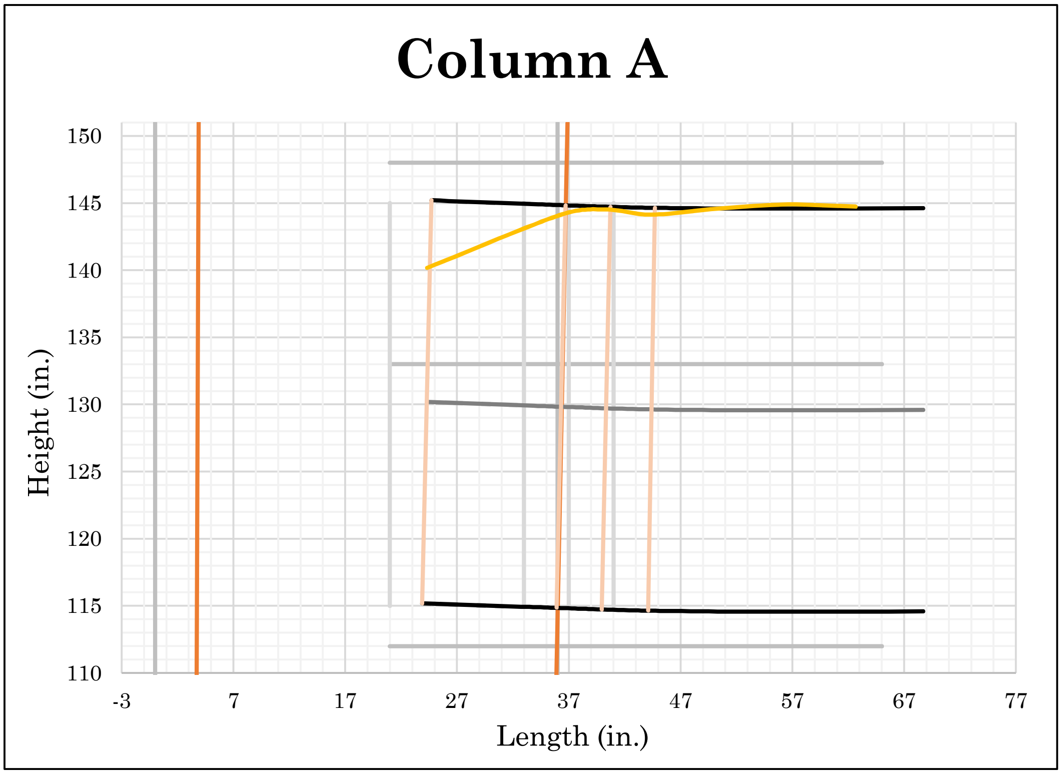


Figure 152: Joint at Column A

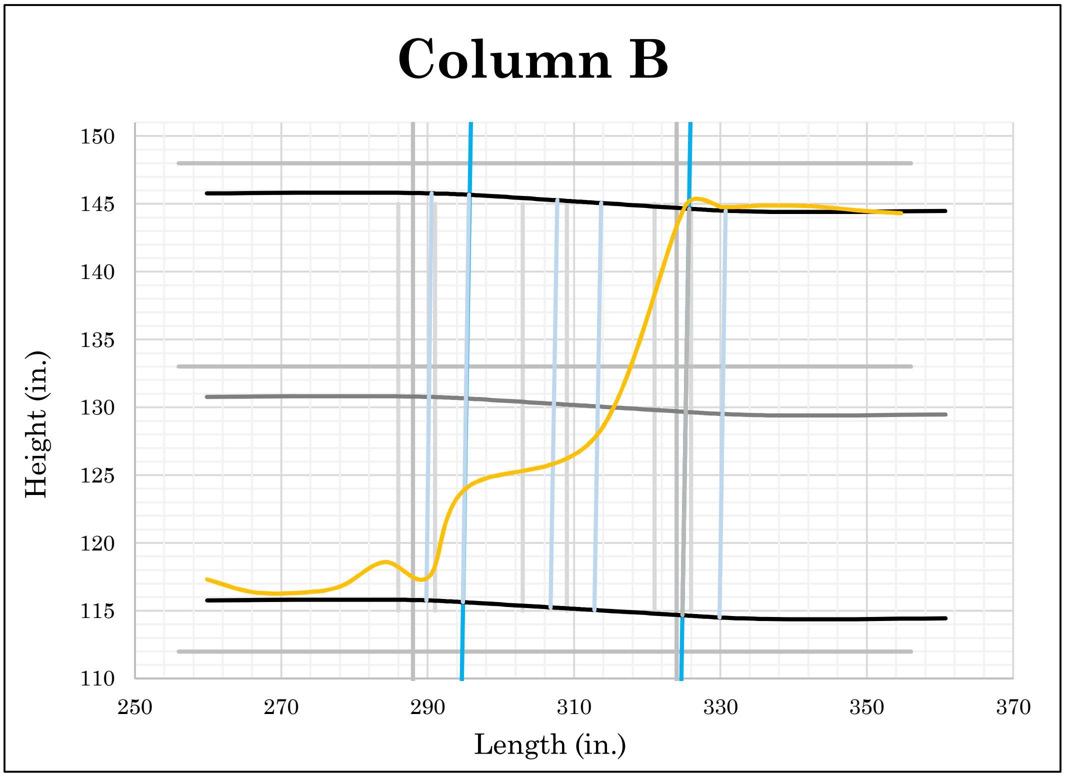


Figure 153: Joint at Column B

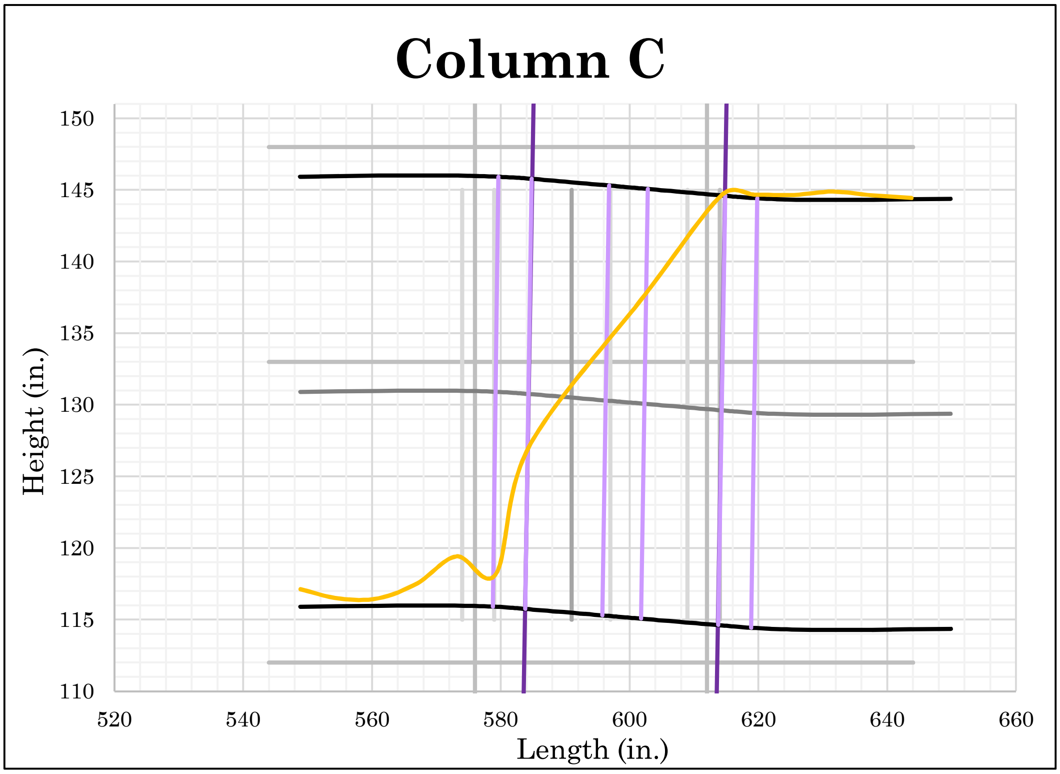


Figure 154: Joint at Column C

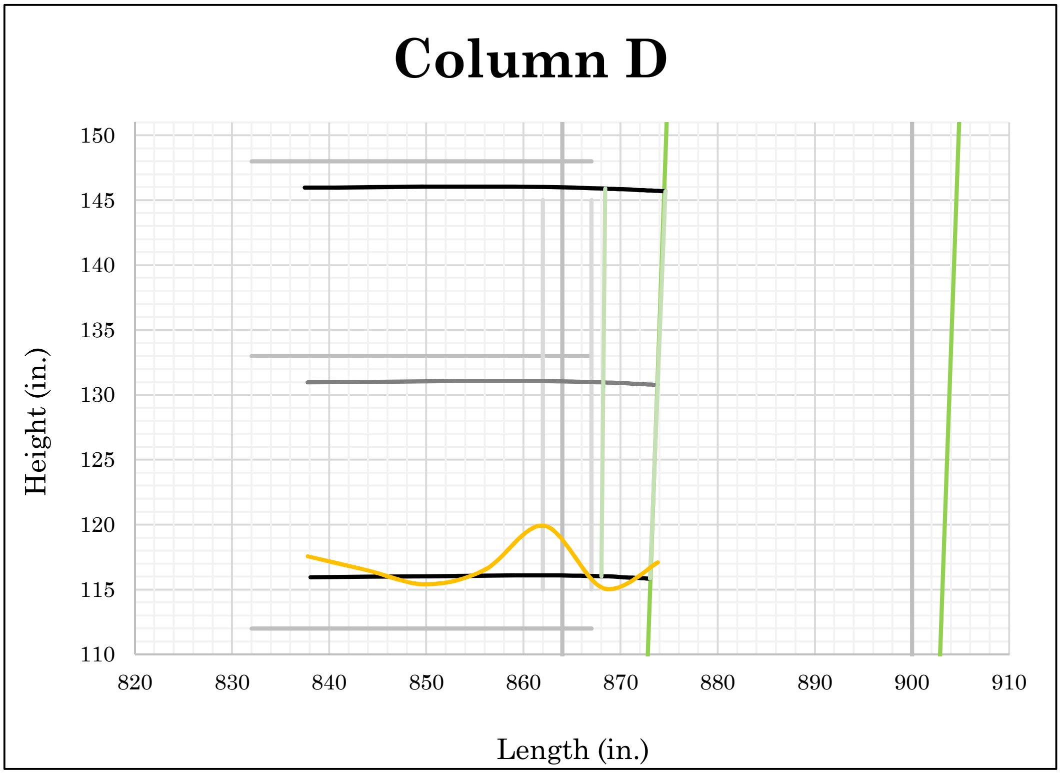


Figure 155: Joint at Column D

Frame 6.3.7

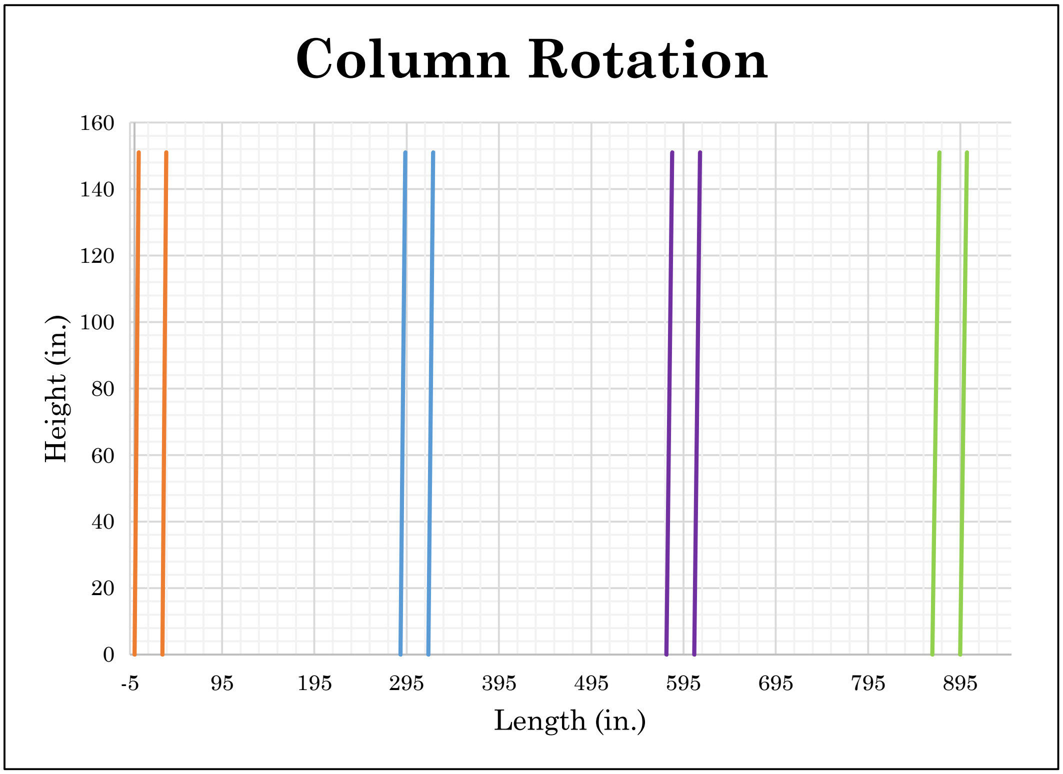


Figure 156: Column Rotation

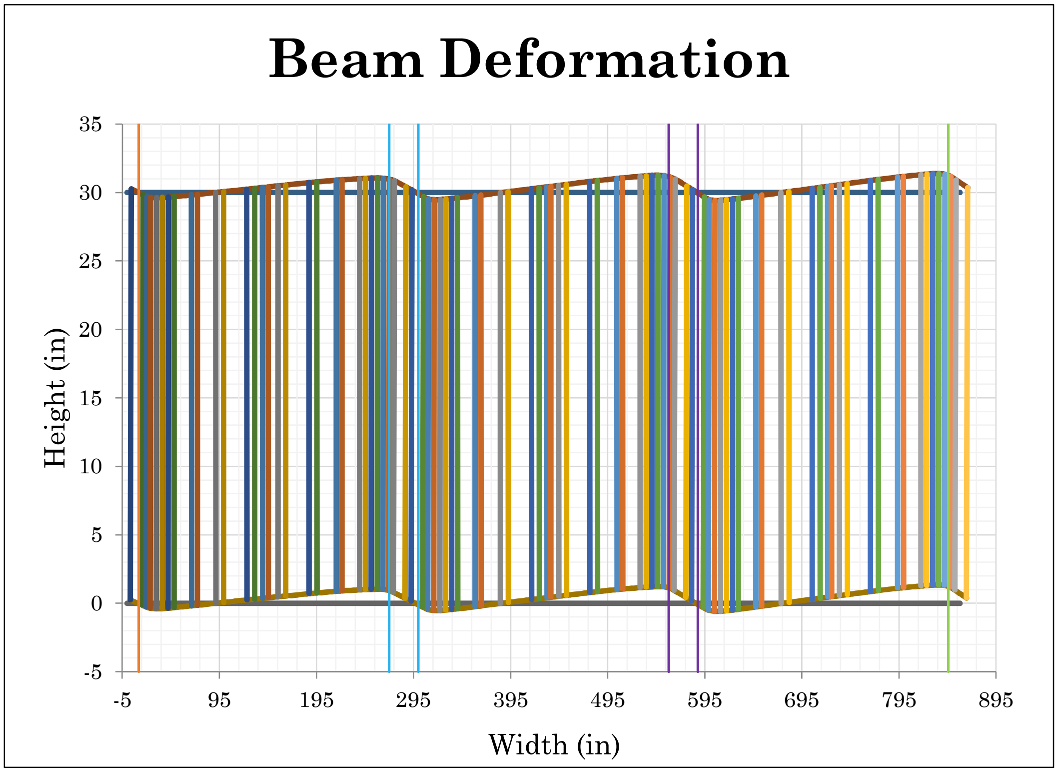


Figure 157: Beam Deformation

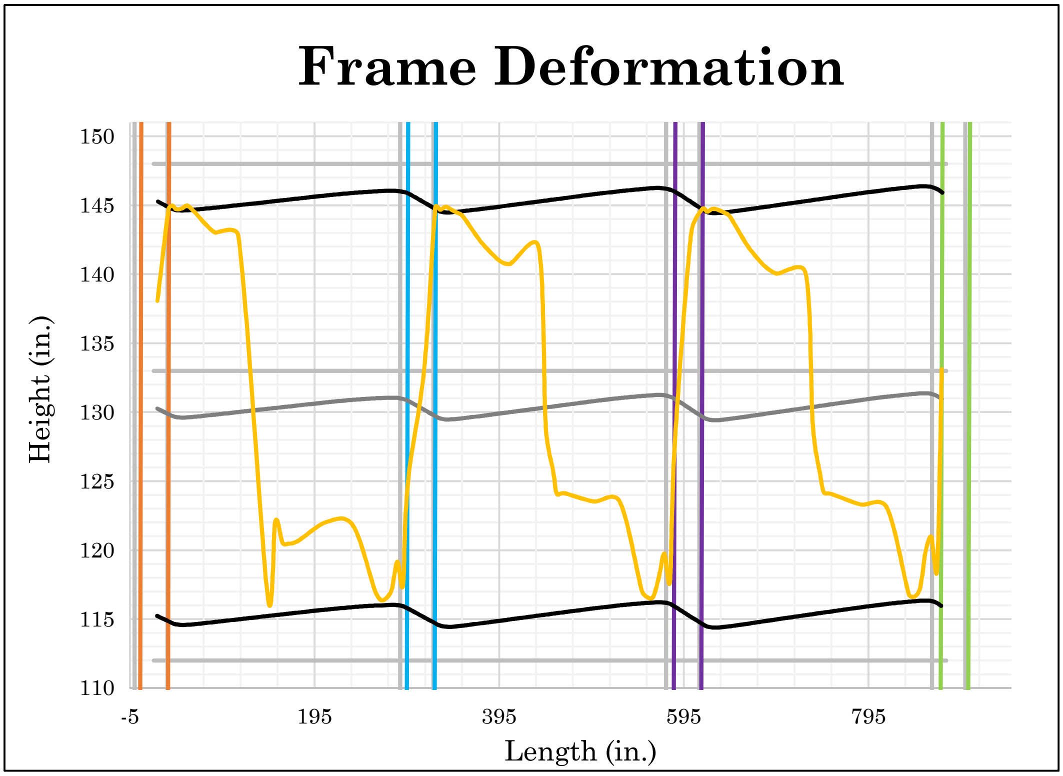


Figure 158: Frame Deformation

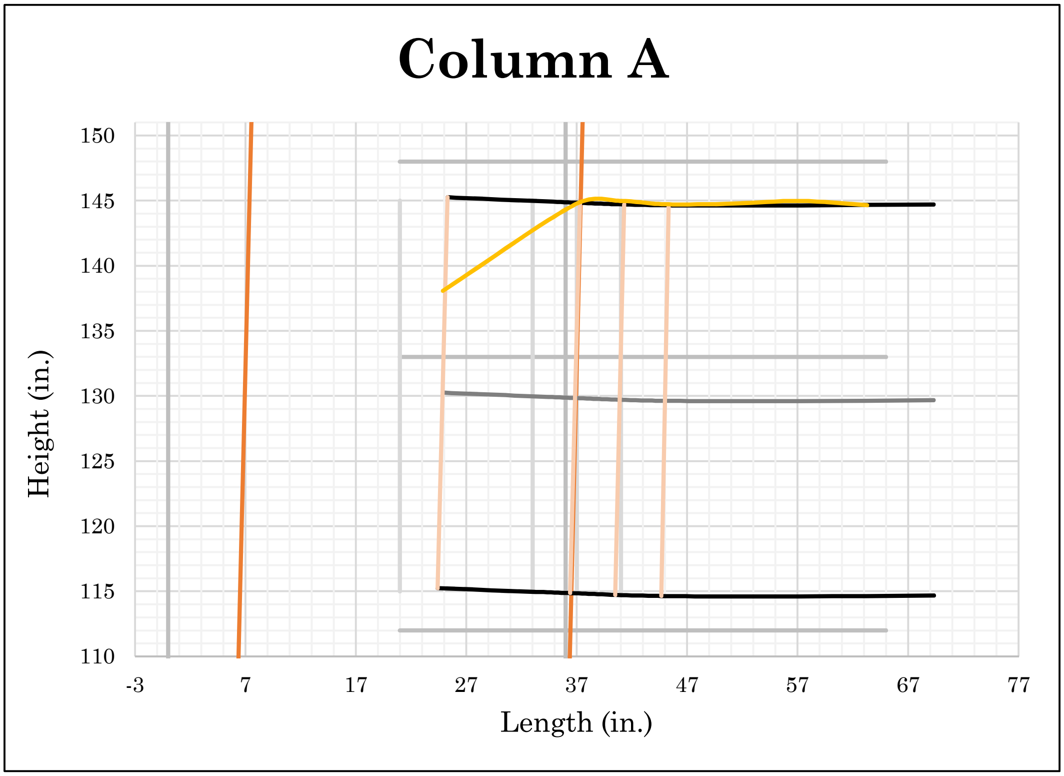


Figure 159: Joint at Column A

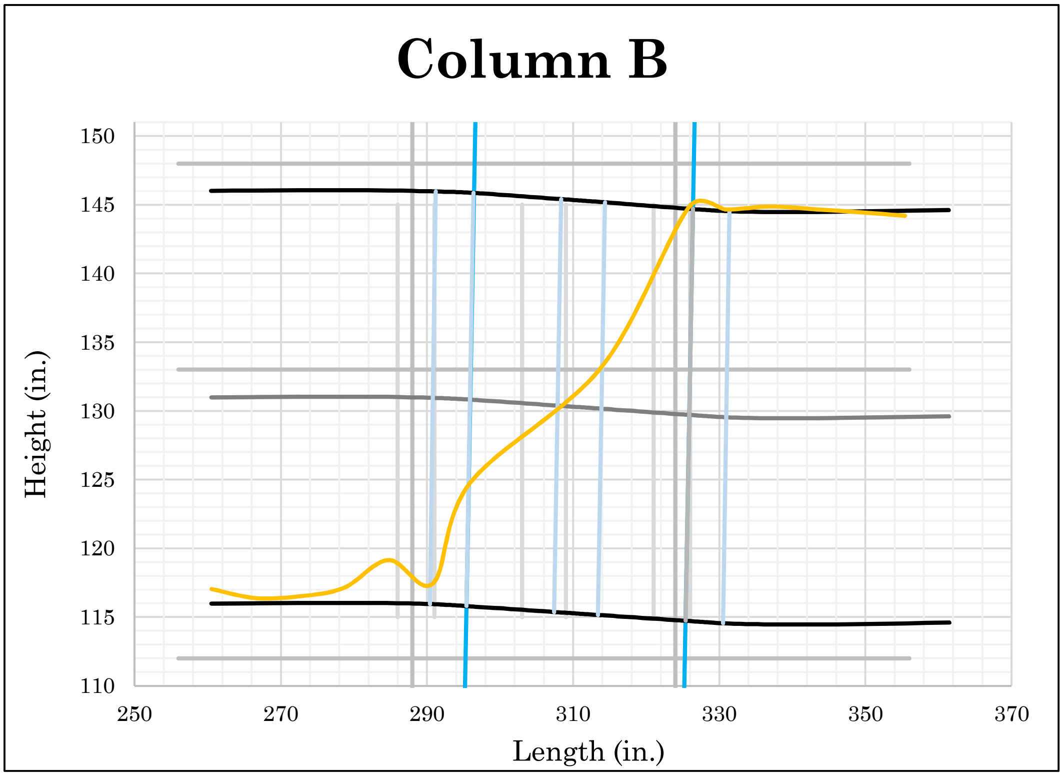


Figure 160: Joint at Column B

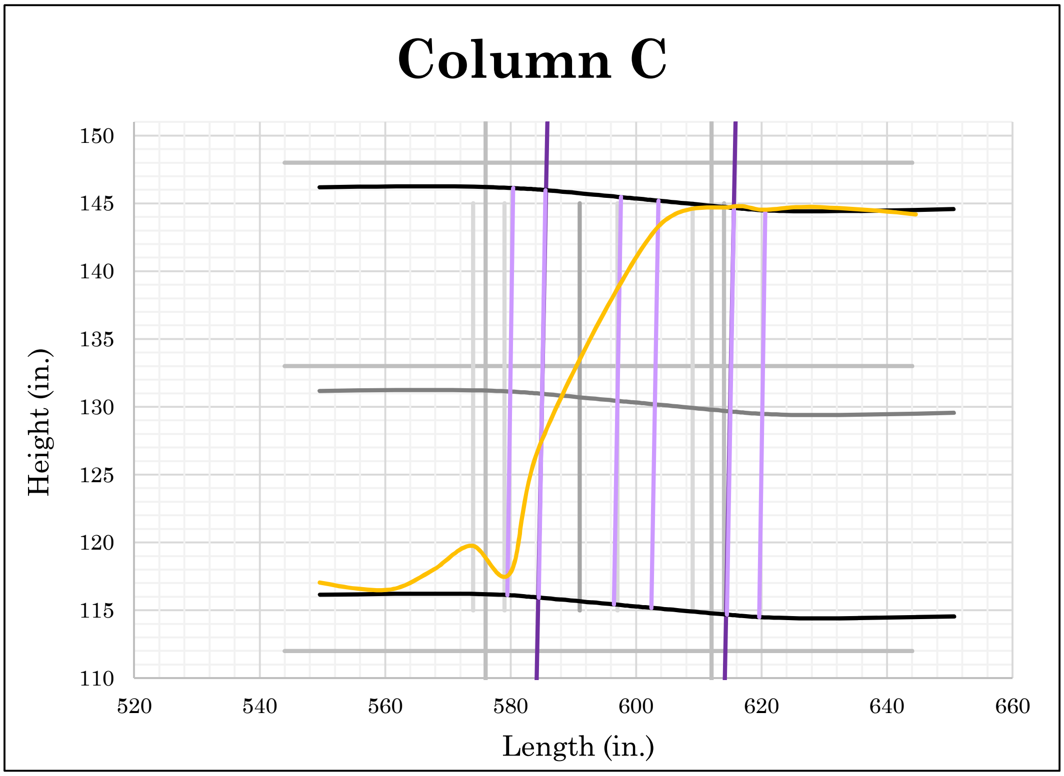


Figure 161: Joint at Column C

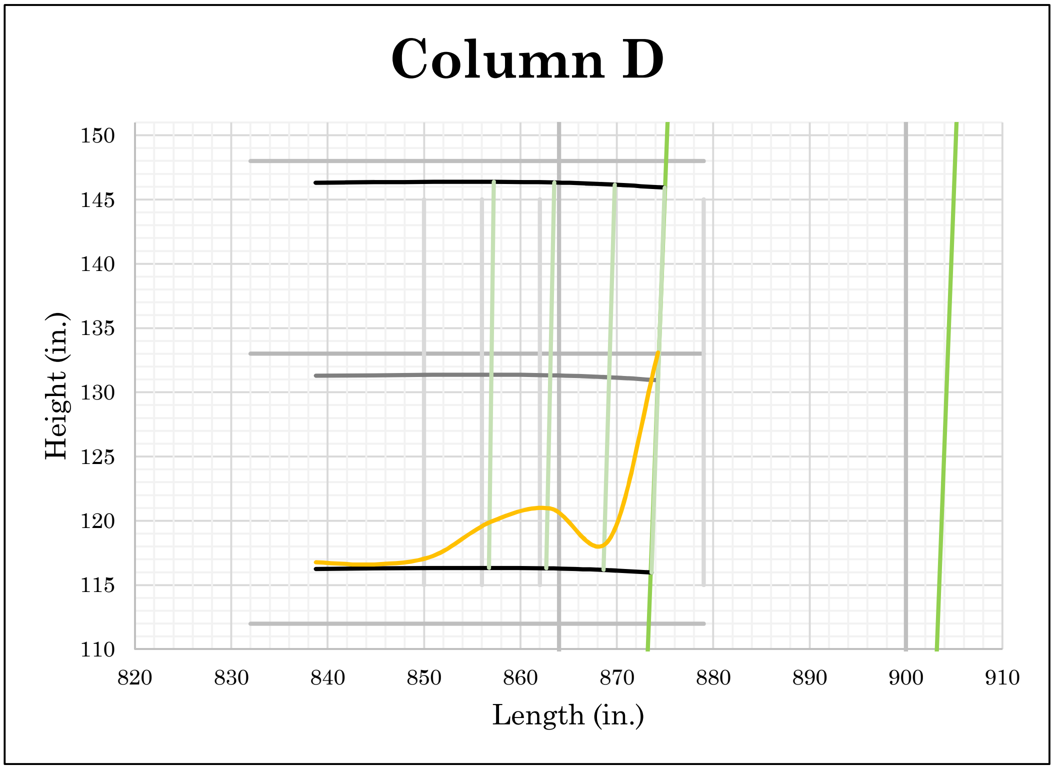


Figure 162: Joint at Column D

Frame 6.3.5

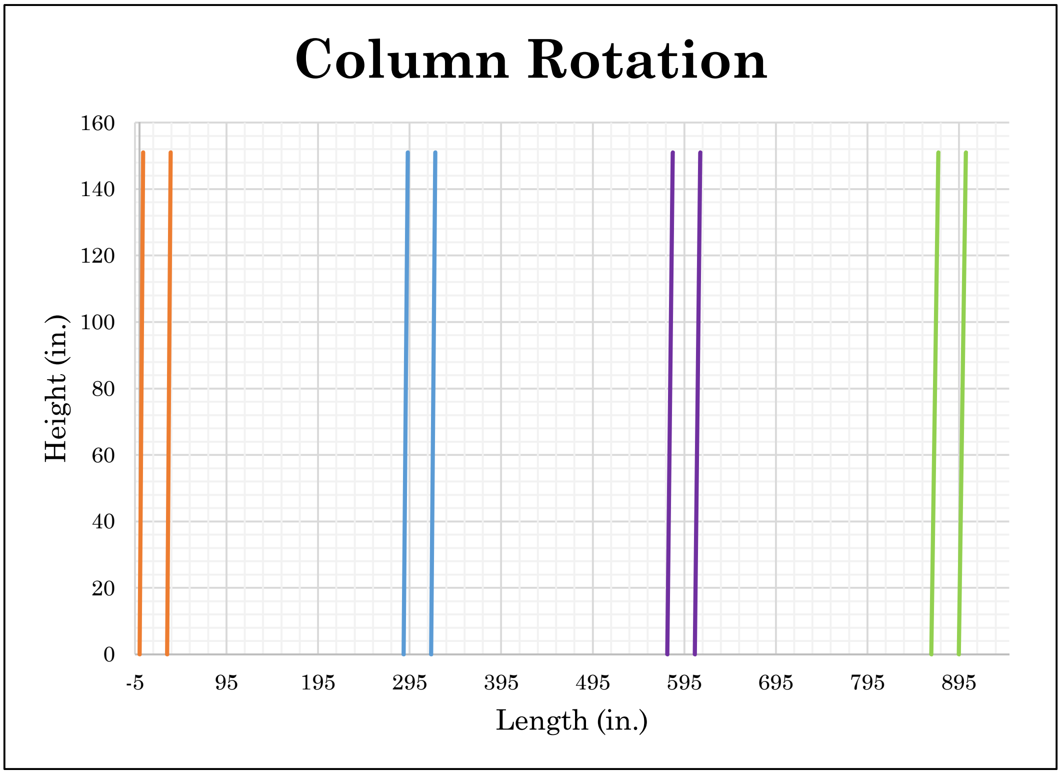


Figure 163: Column Rotation

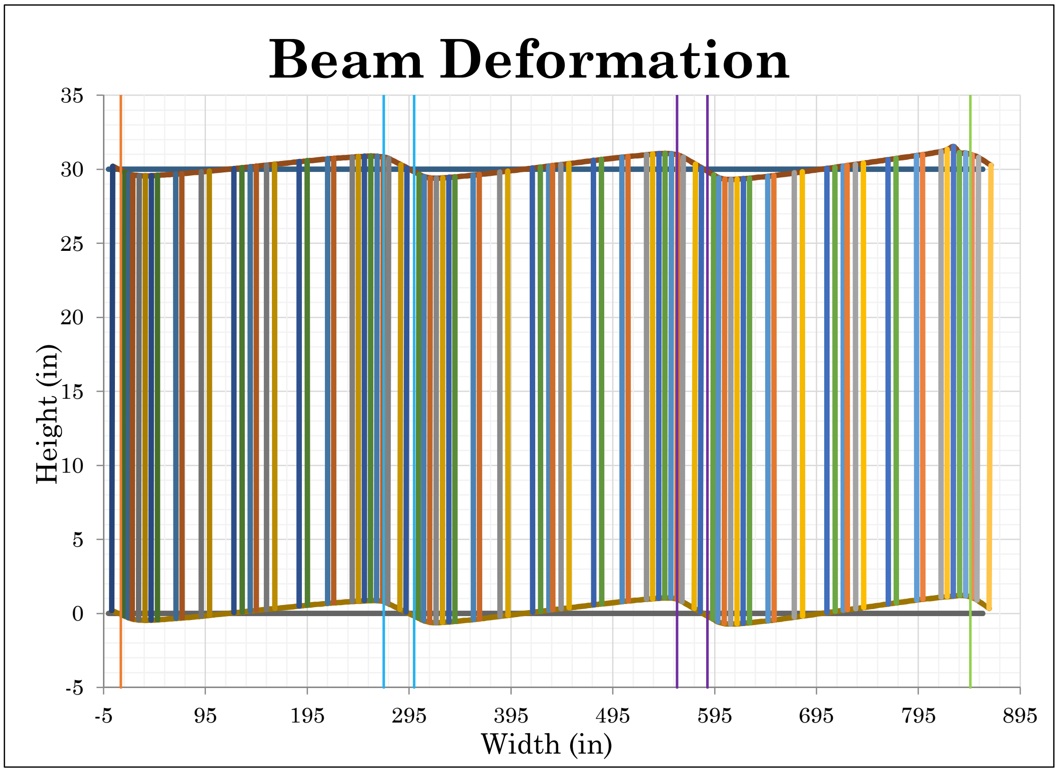


Figure 164: Beam Deformation

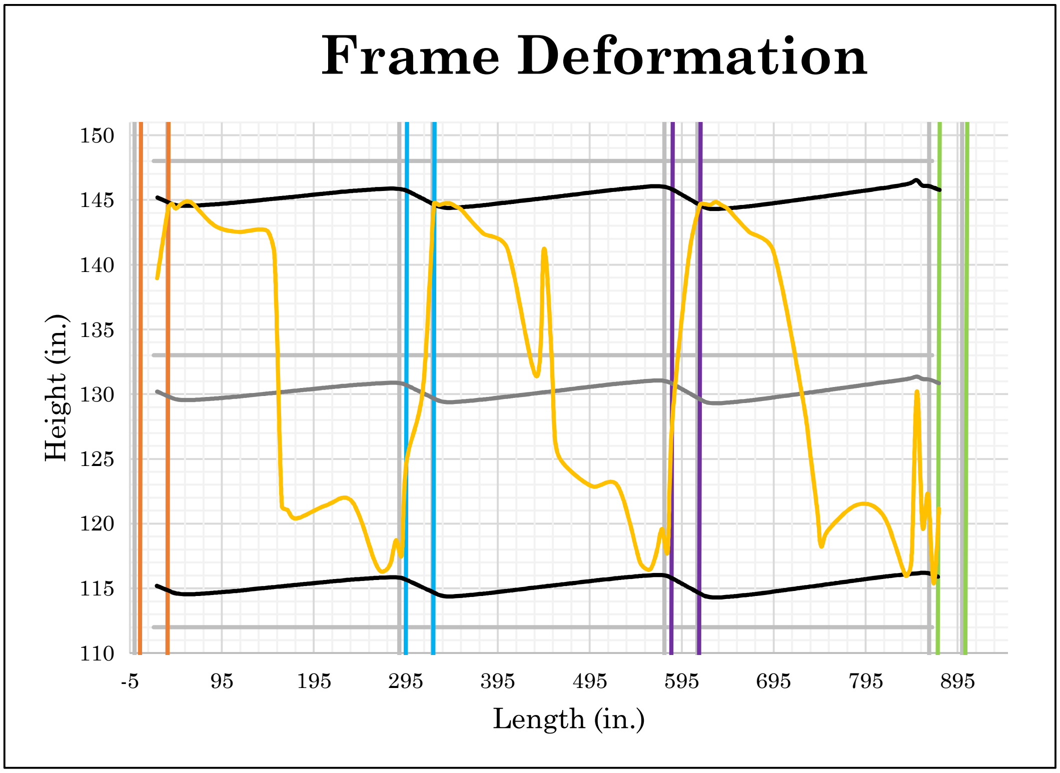


Figure 165: Frame Deformation

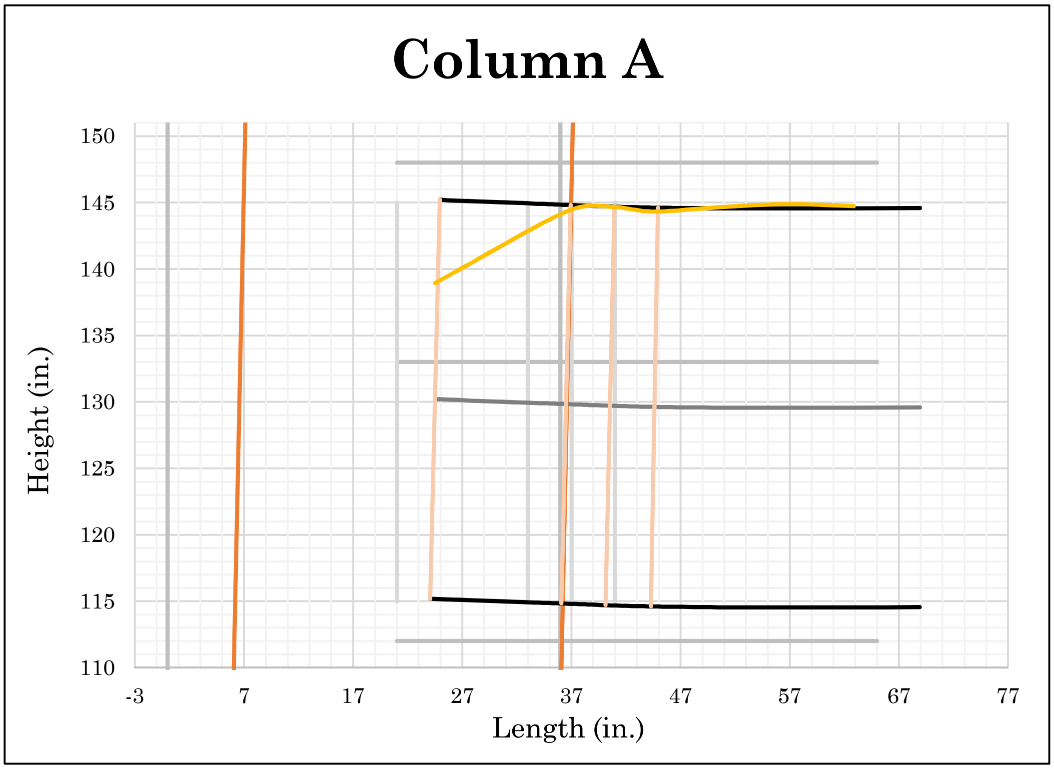


Figure 166: Joint at Column A

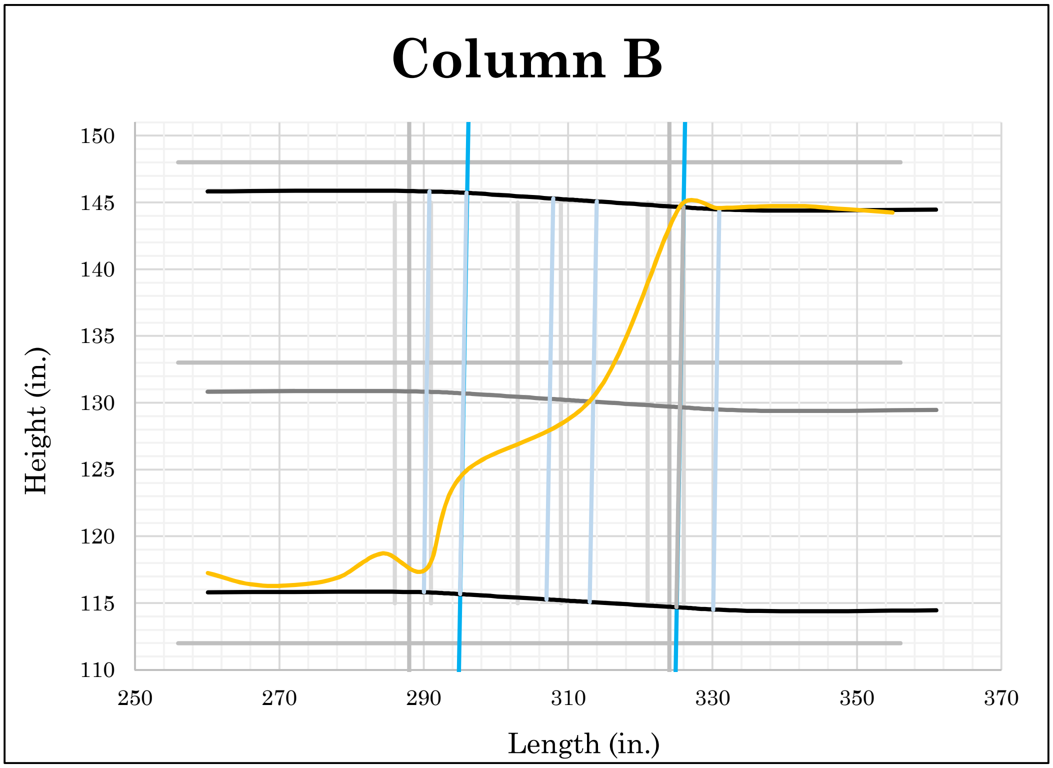


Figure 167: Joint at Column B

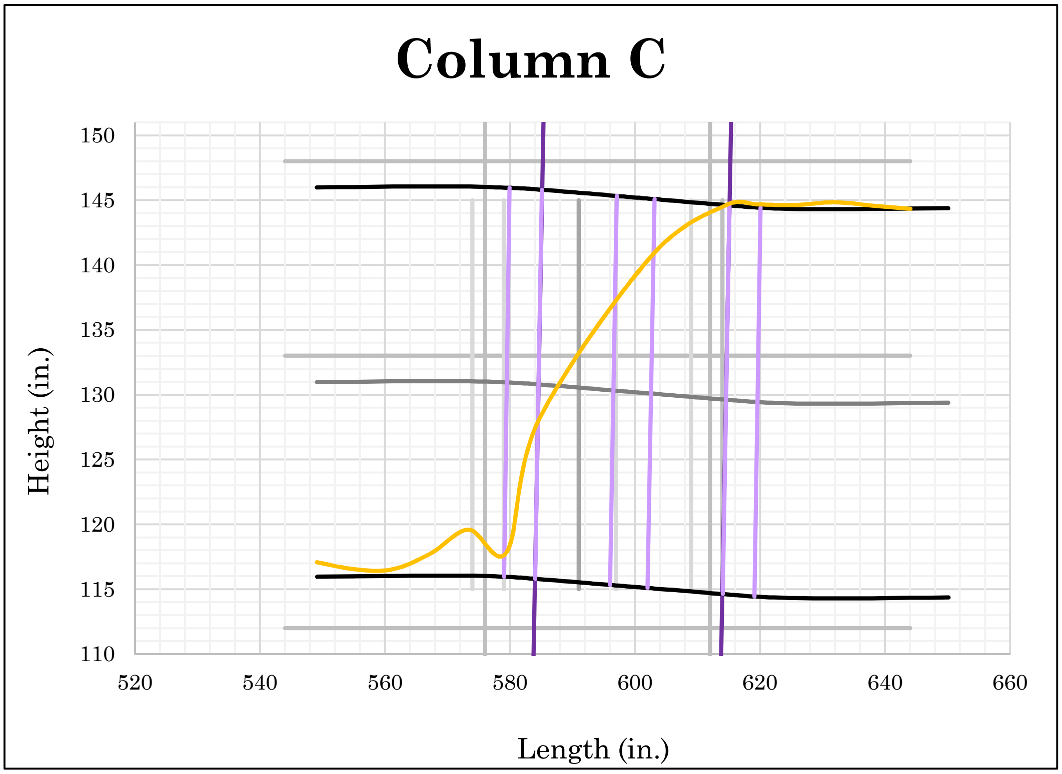


Figure 168: Joint at Column C

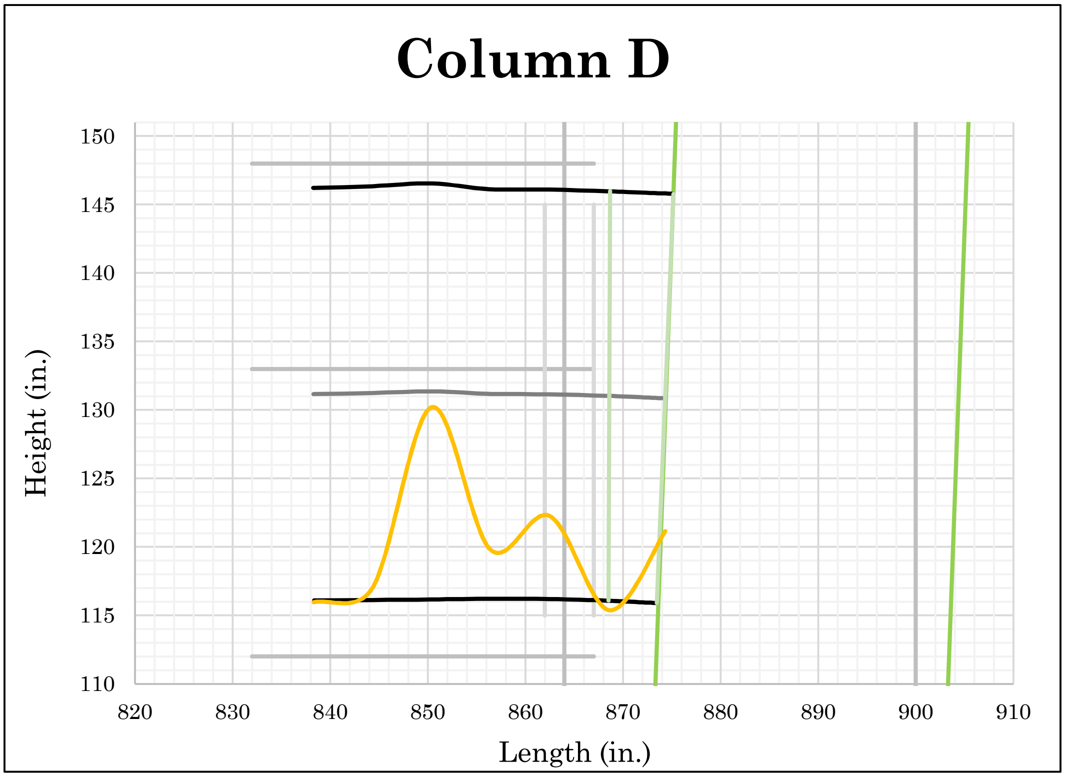


Figure 169: Joint at Column D

Frame 7.2.7

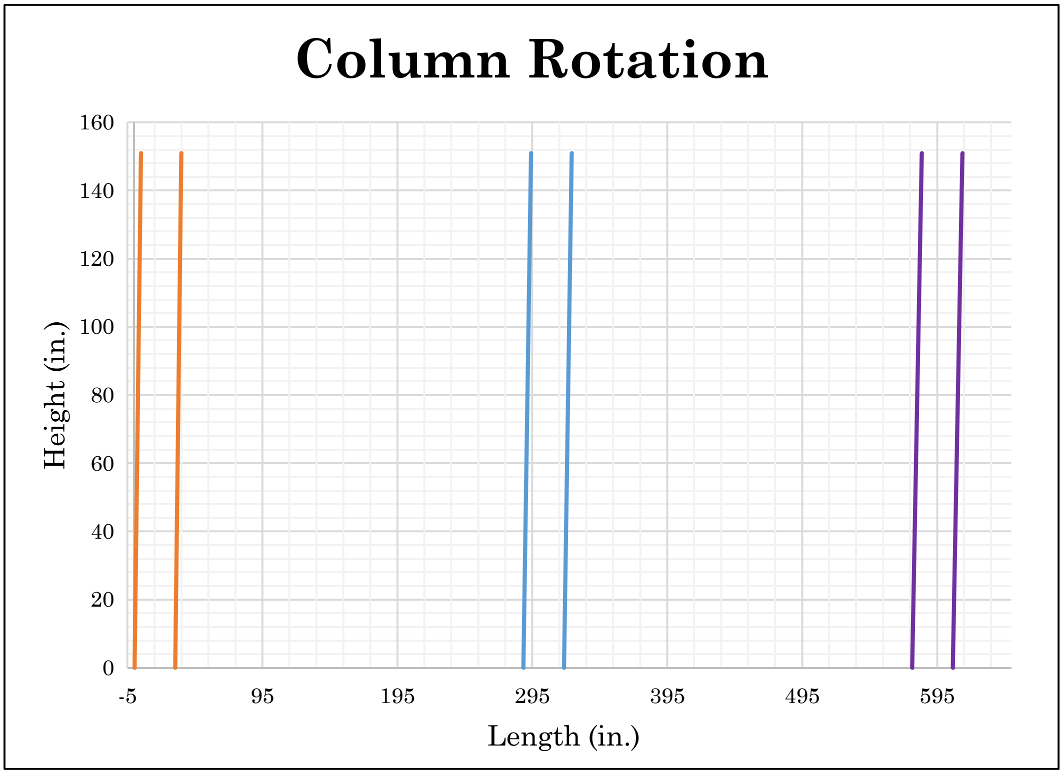


Figure 170: Column Rotation

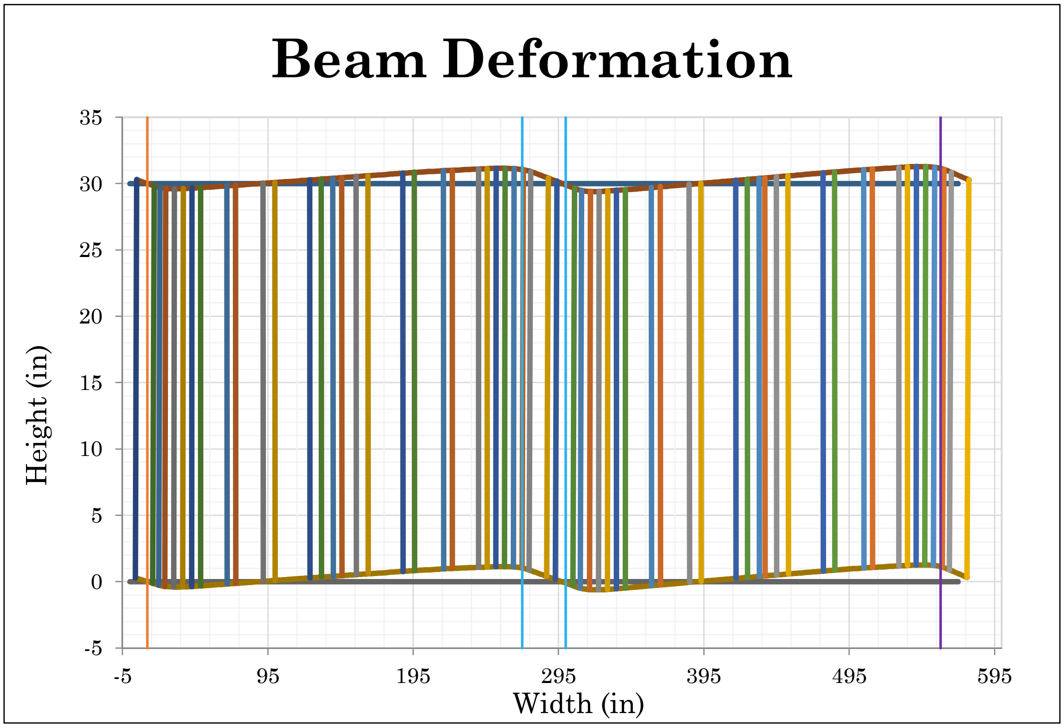


Figure 171: Beam Deformation

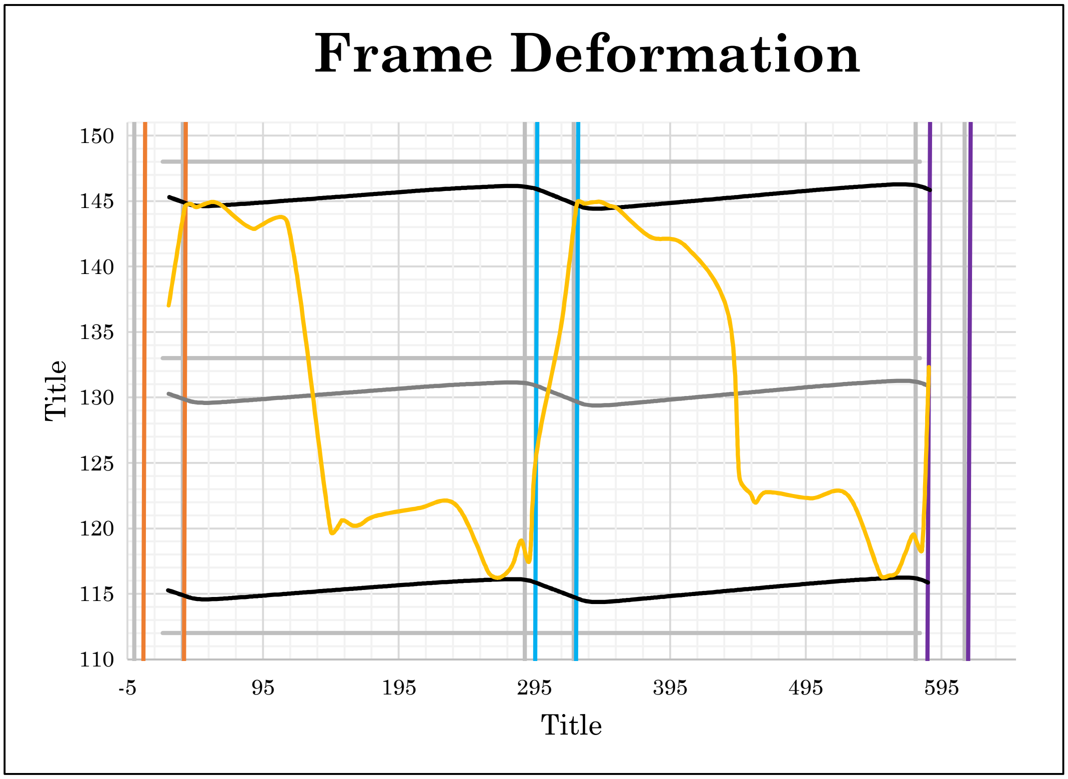


Figure 172: Frame Deformations

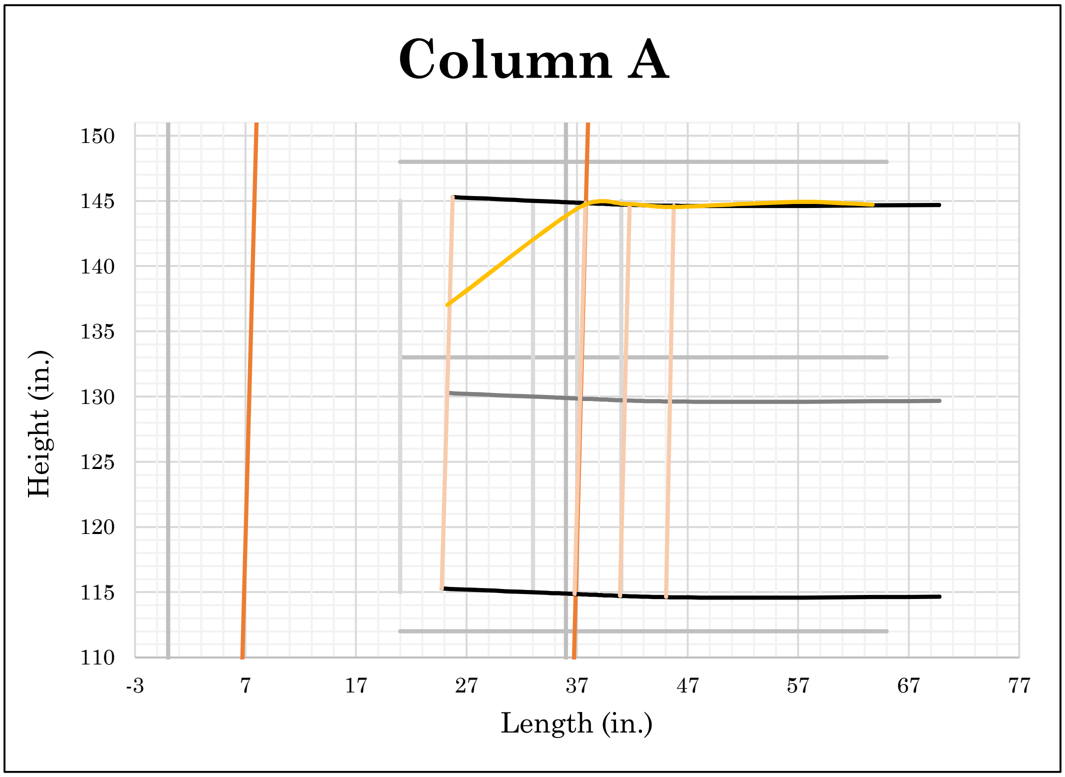


Figure 173: Joint at Column A

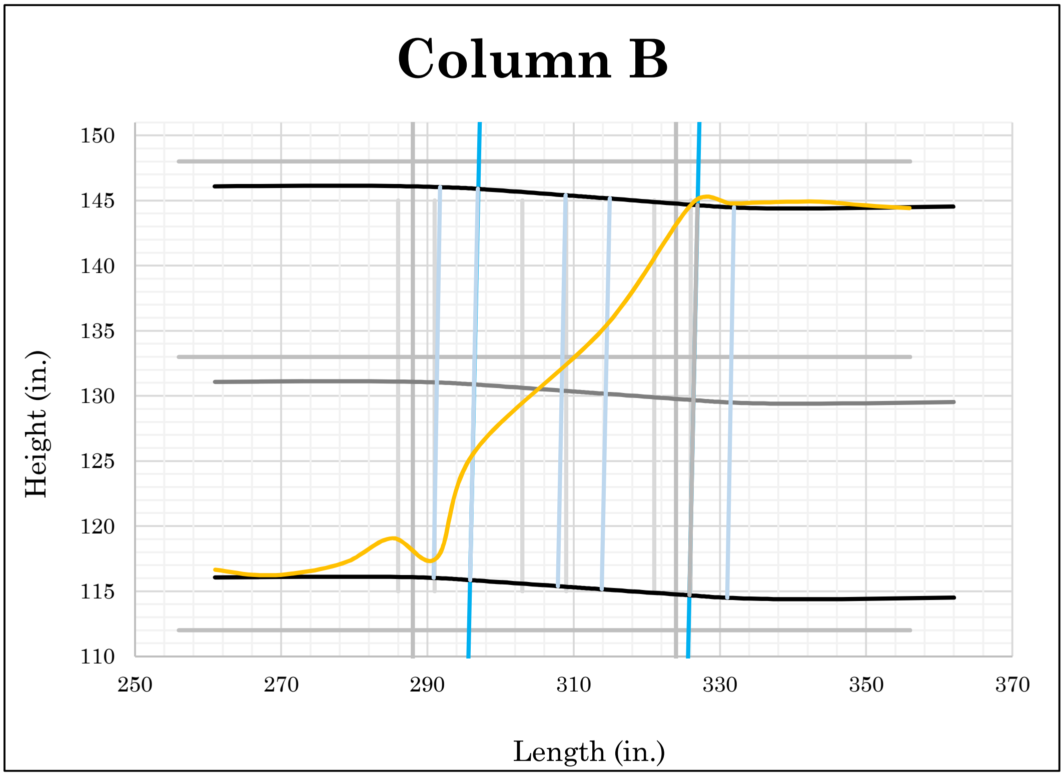


Figure 174: Joint at Column B

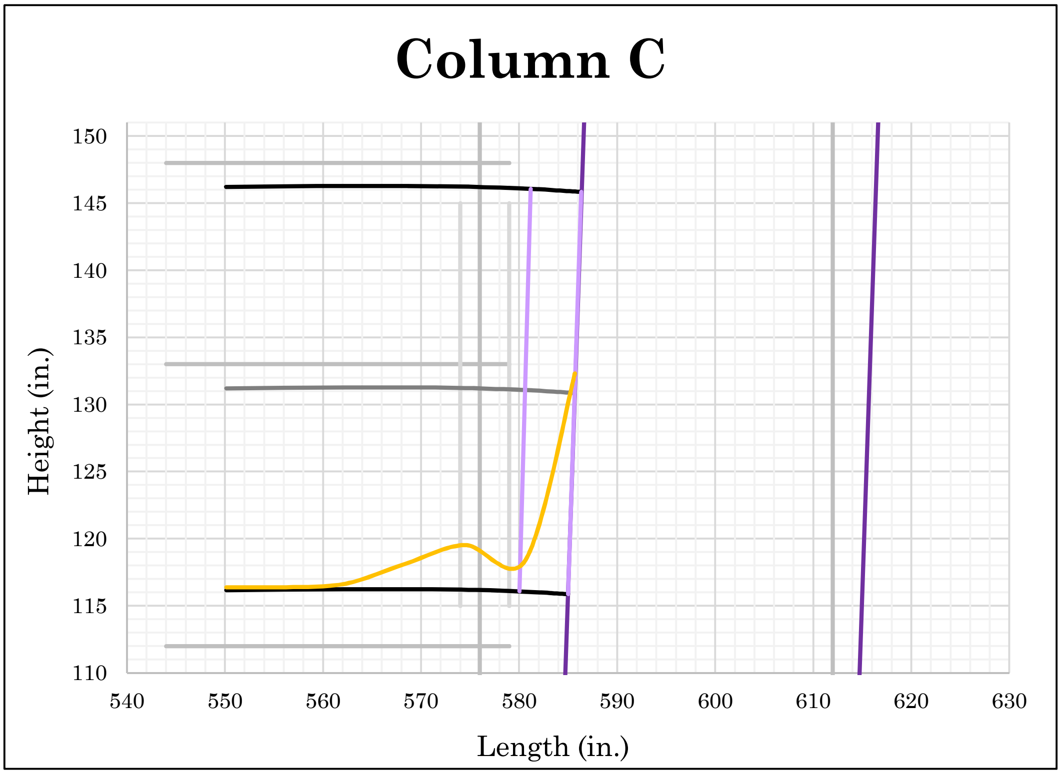


Figure 175: Joint at Column C

Frame 7.2.5

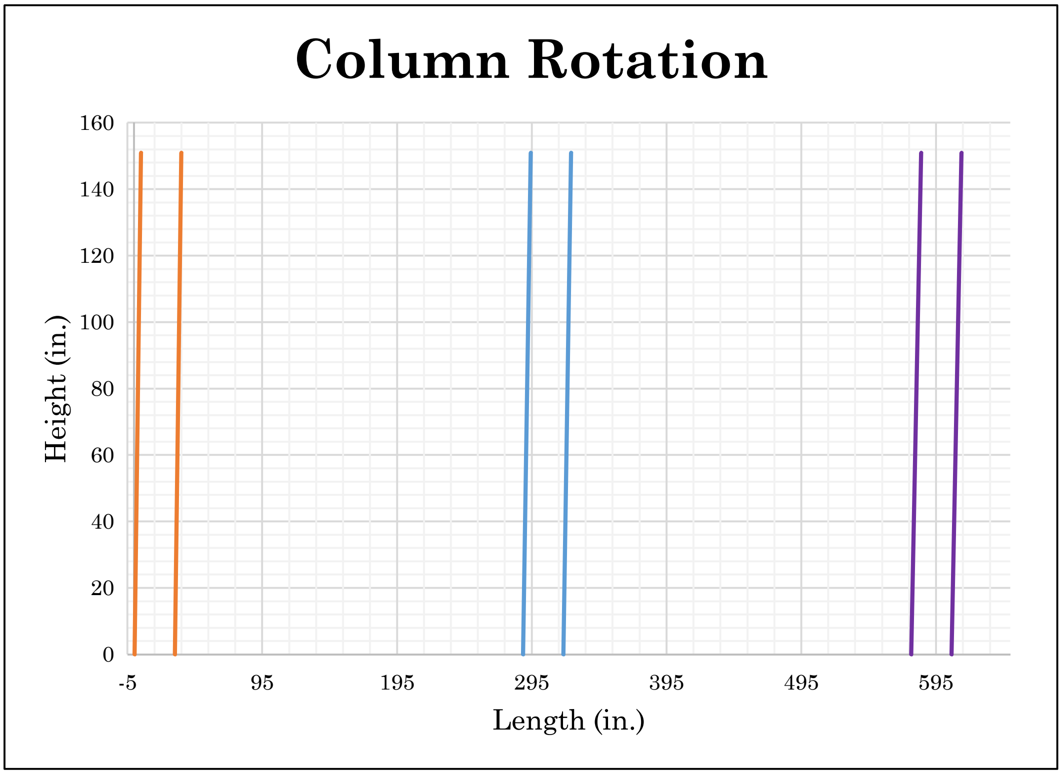


Figure 176: Column Rotation

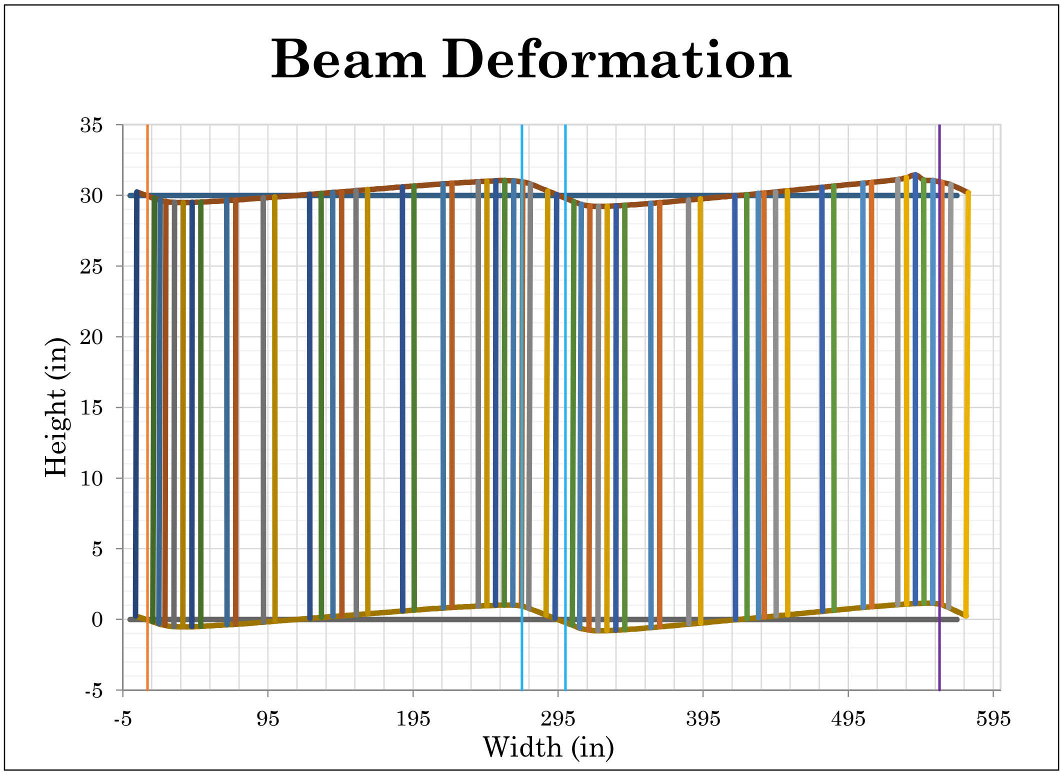


Figure 177: Beam Deformation

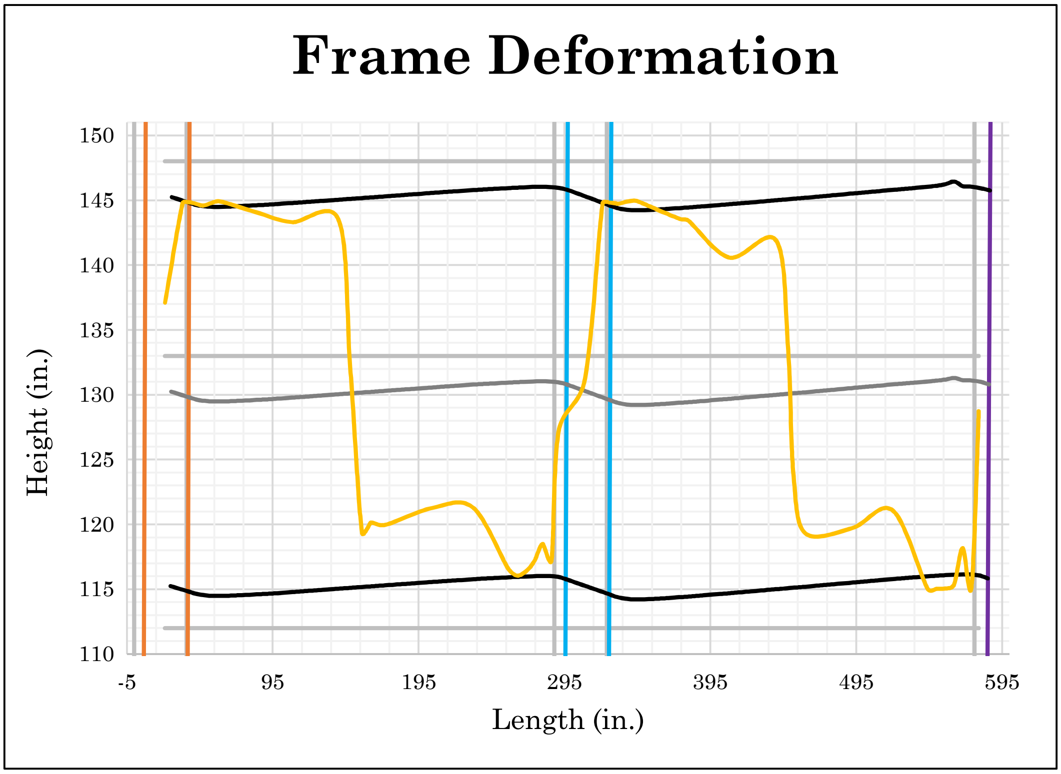


Figure 179: Frame Deformation

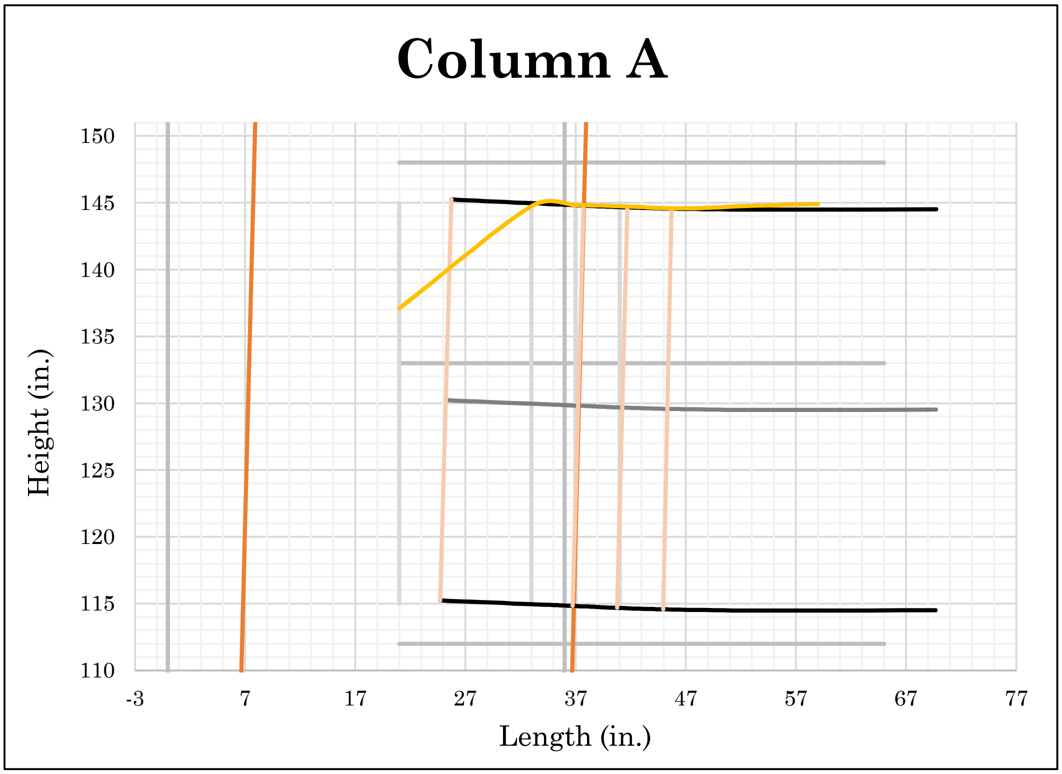


Figure 180: Joint at Column A

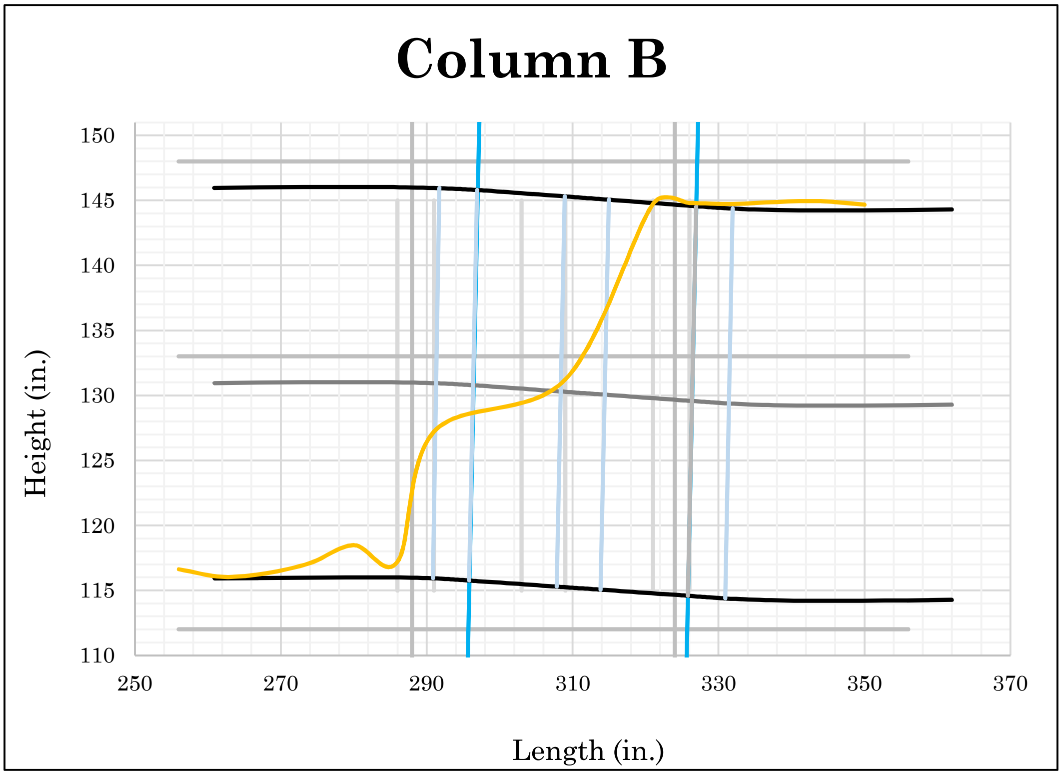


Figure 181: Joint at Column B

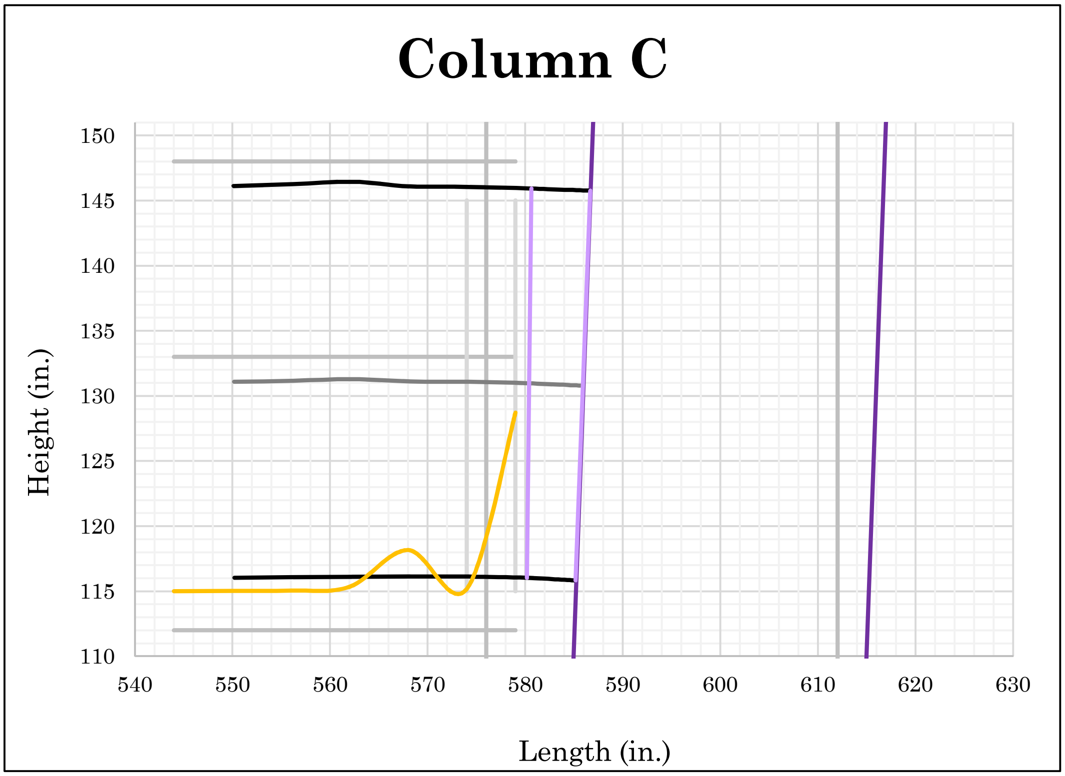


Figure 182: Joint at Column C

Frame 6.2.7

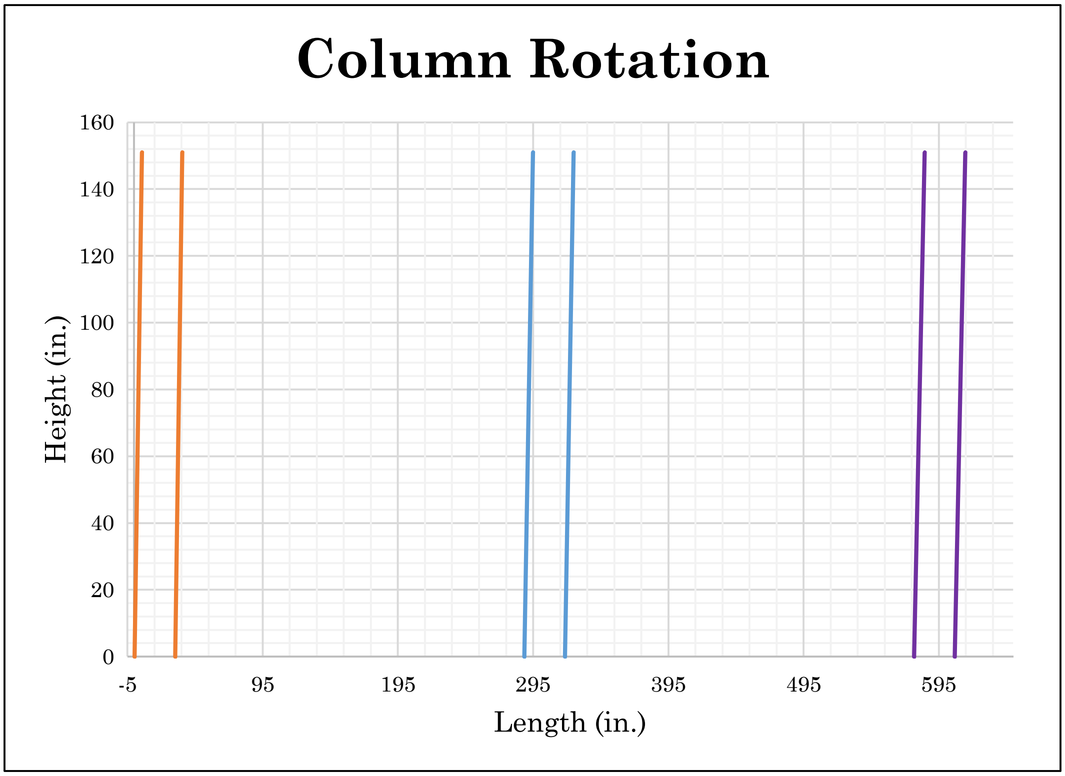


Figure 183: Column Rotation

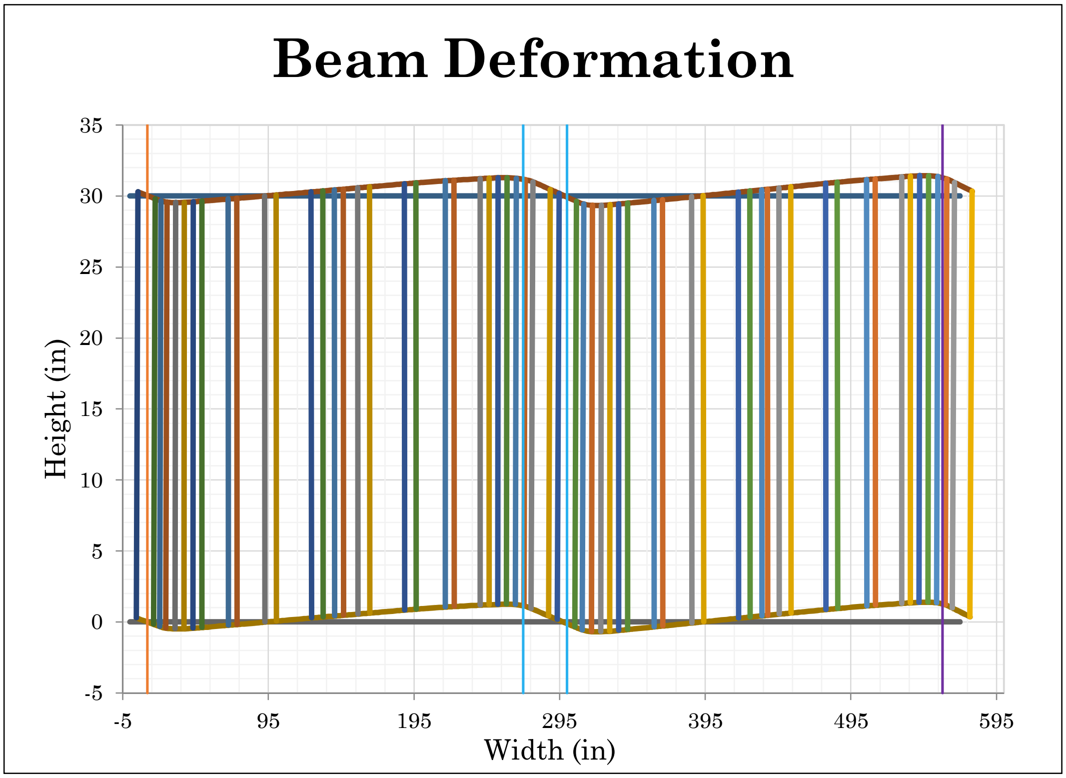


Figure 184: Beam Deformation

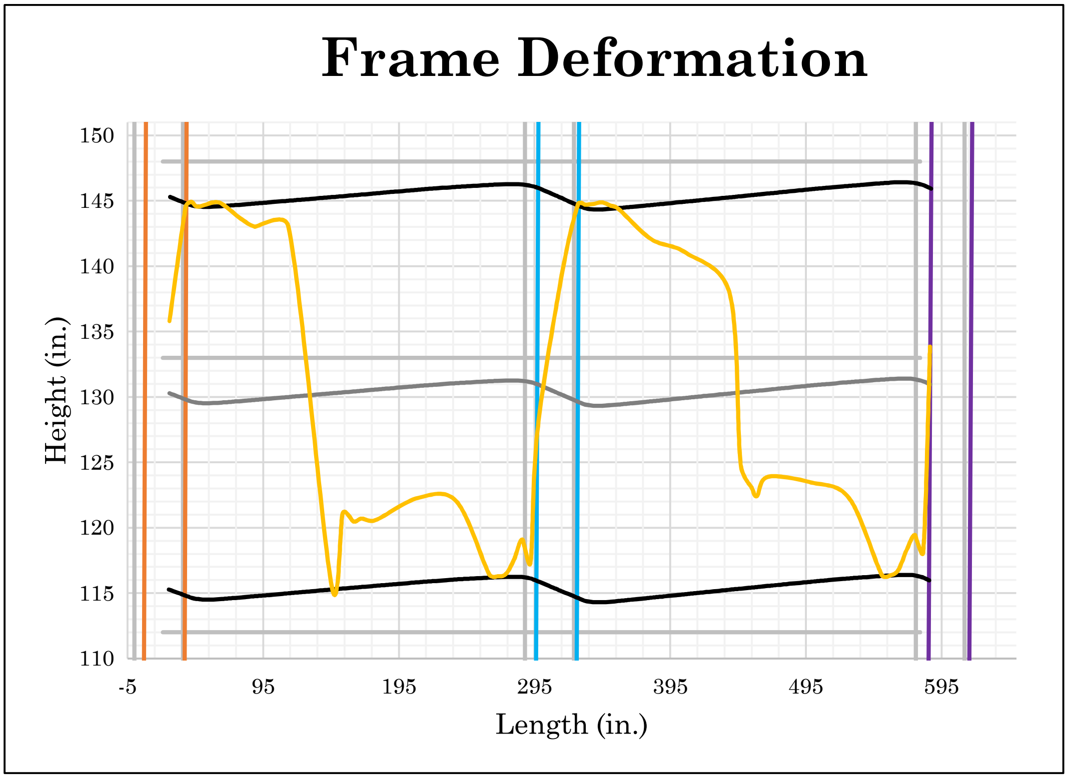


Figure 185: Frame Deformation

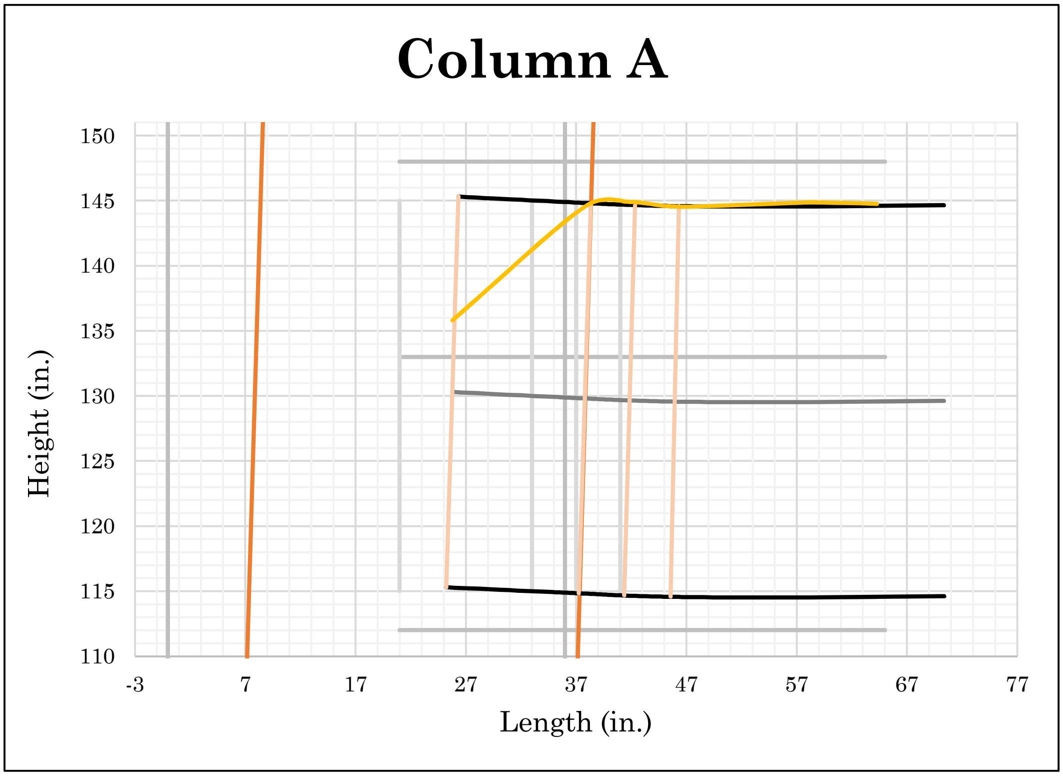


Figure 186: Joint at Column A

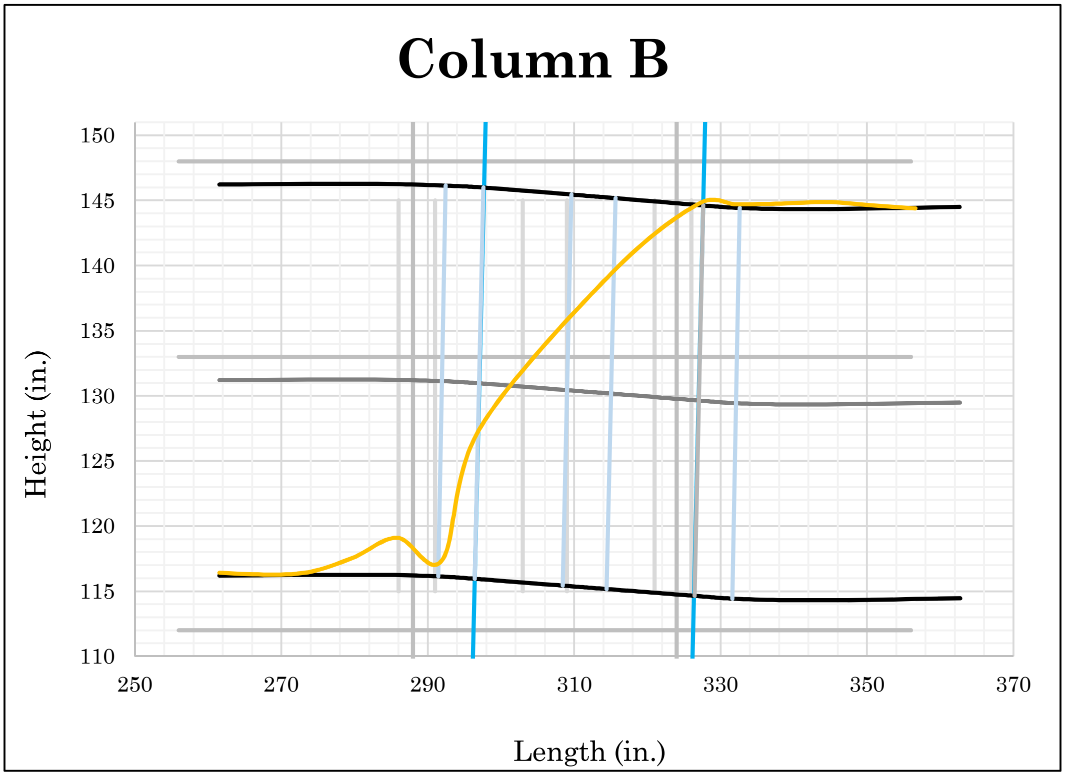


Figure 187: Joint at Column B

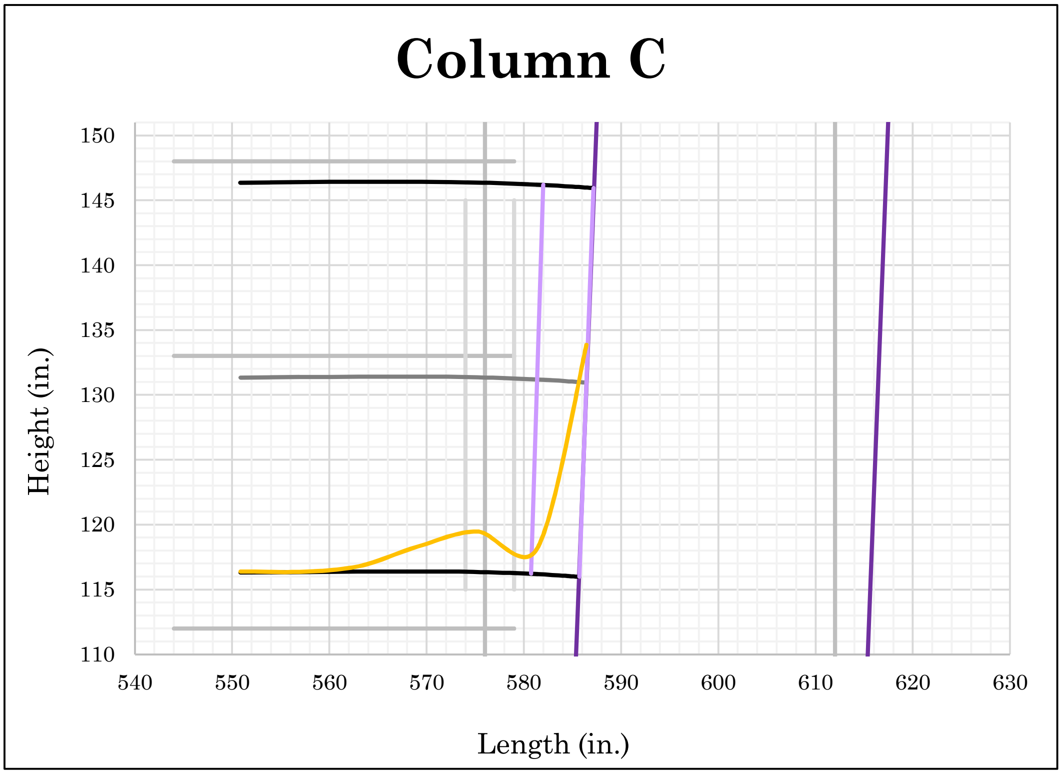


Figure 188: Joint at Column C

Frame 6.2.5

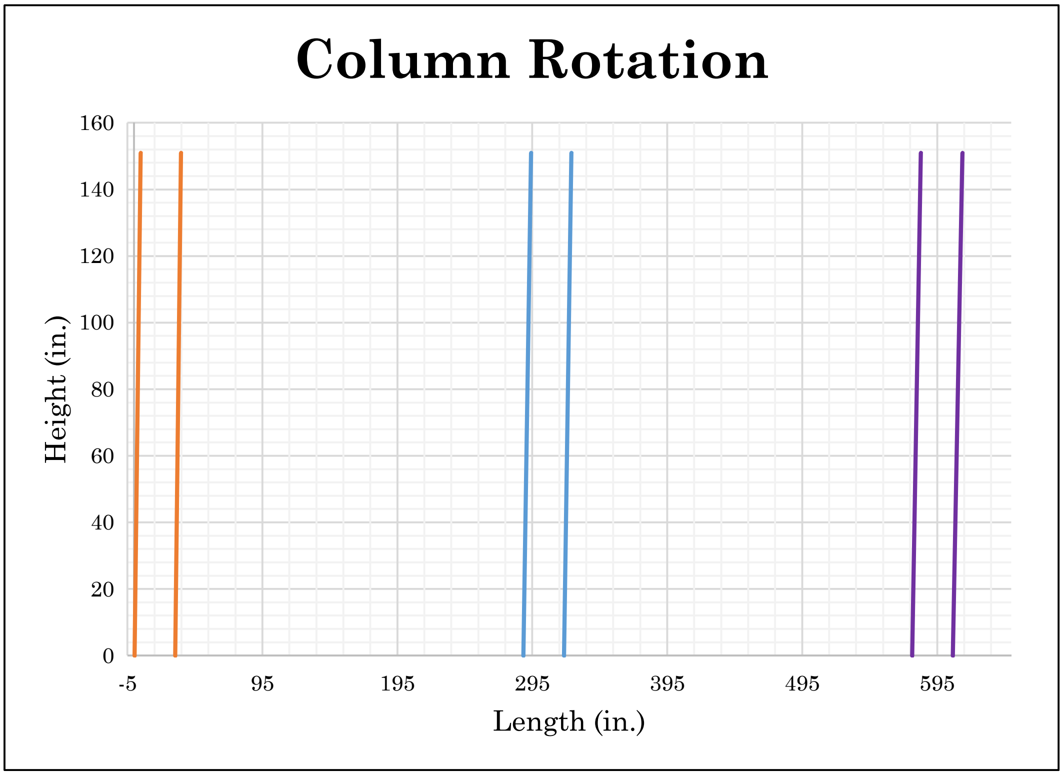


Figure 189: Column Rotation

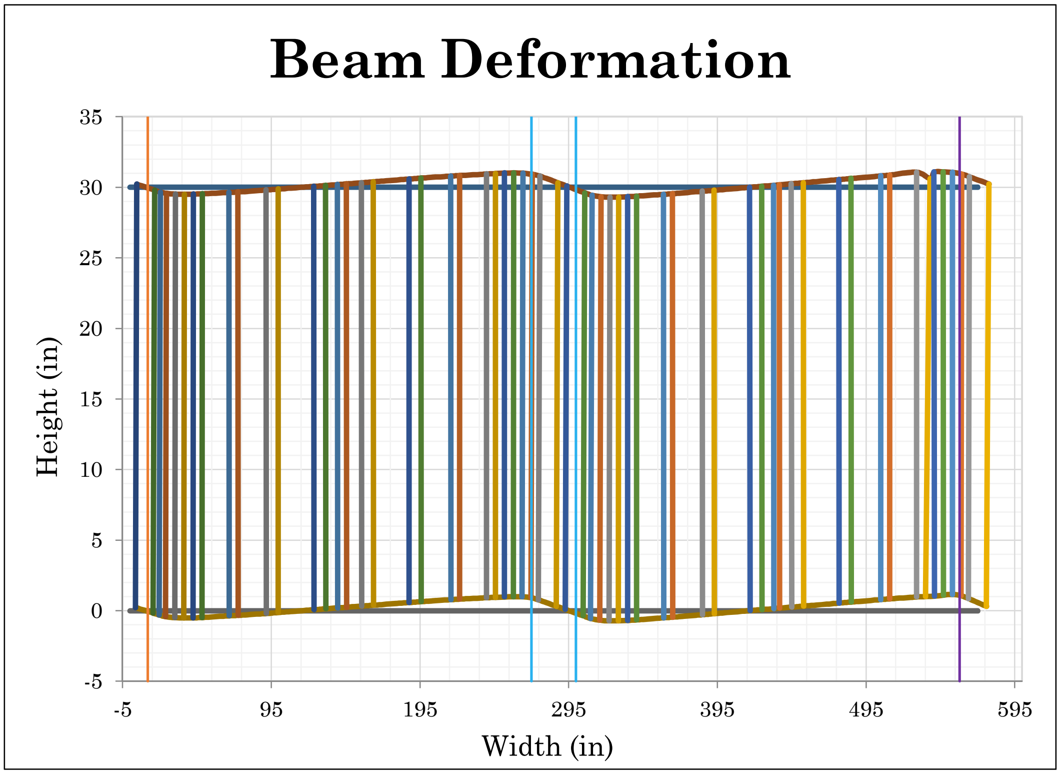


Figure 190: Beam Deformation

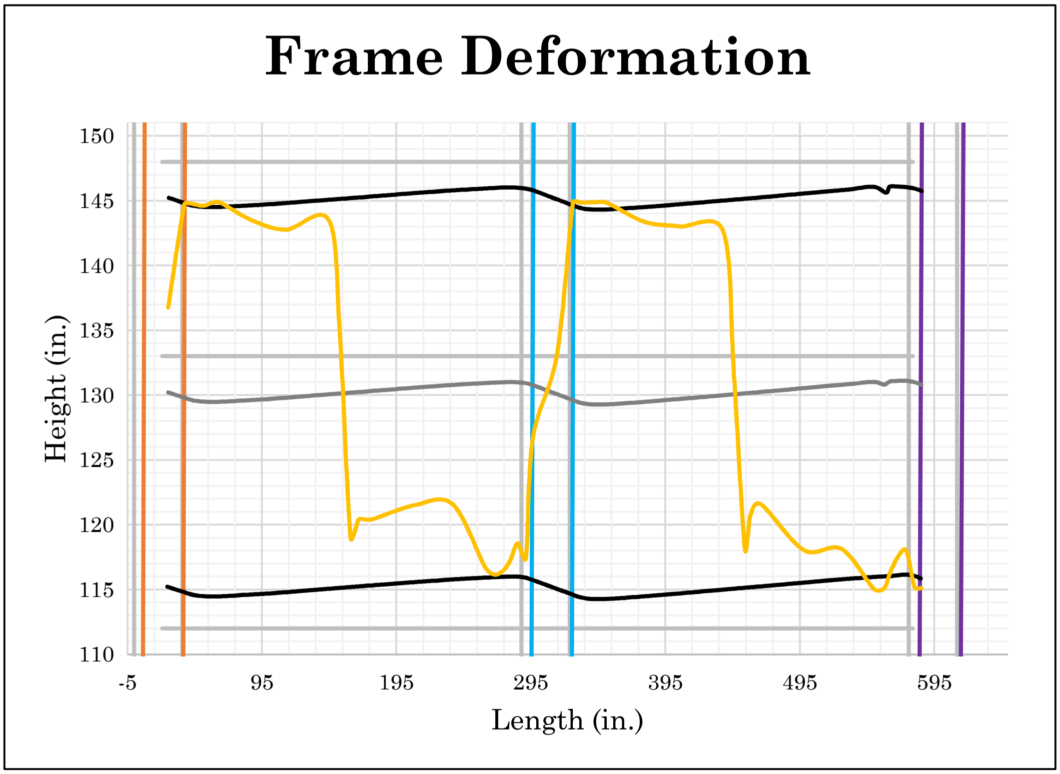


Figure 191: Frame Deformation

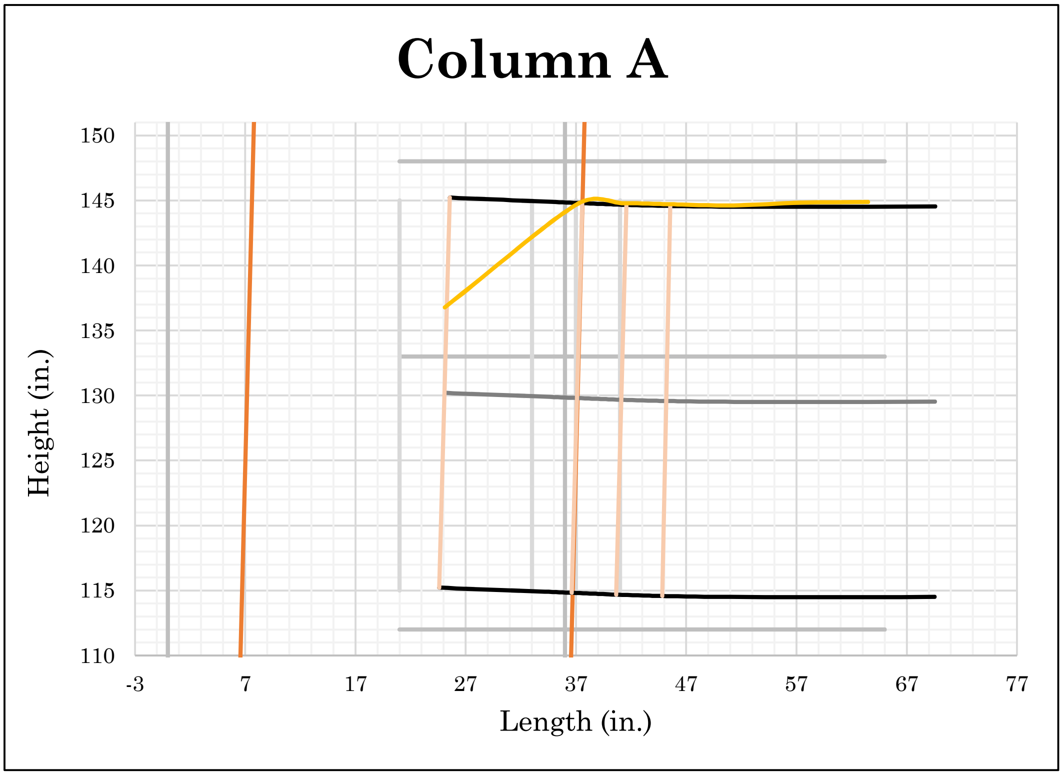


Figure 192: Joint at Column A

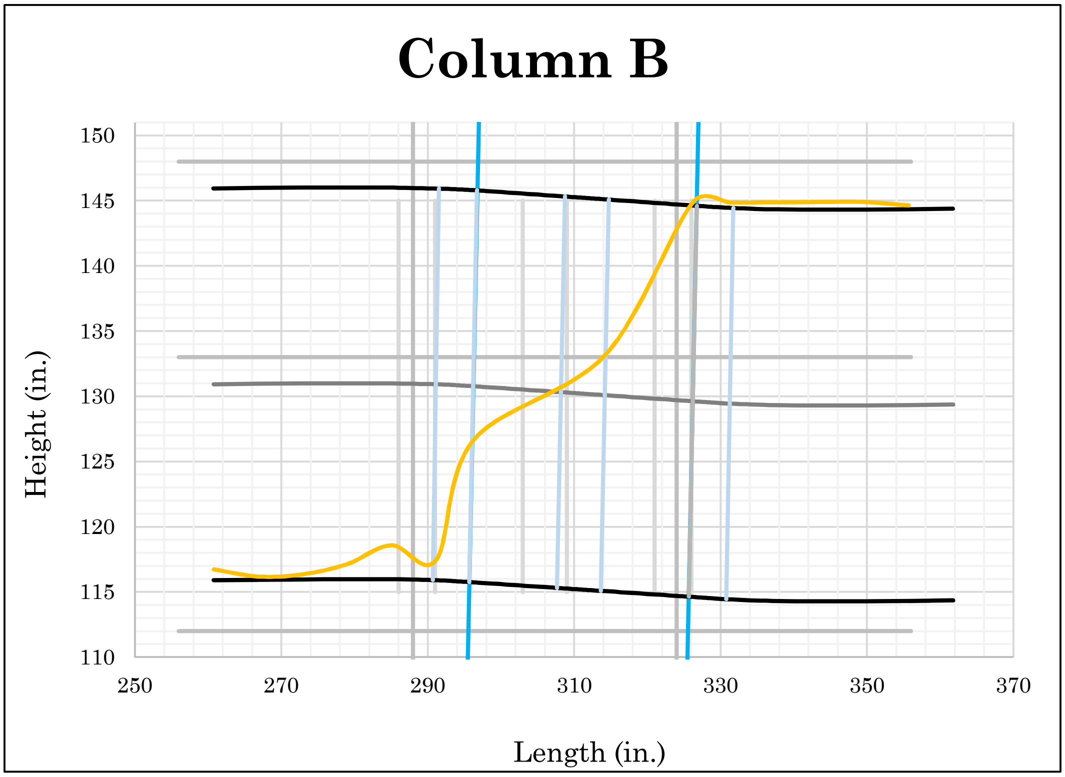


Figure 193: Joint at Column B

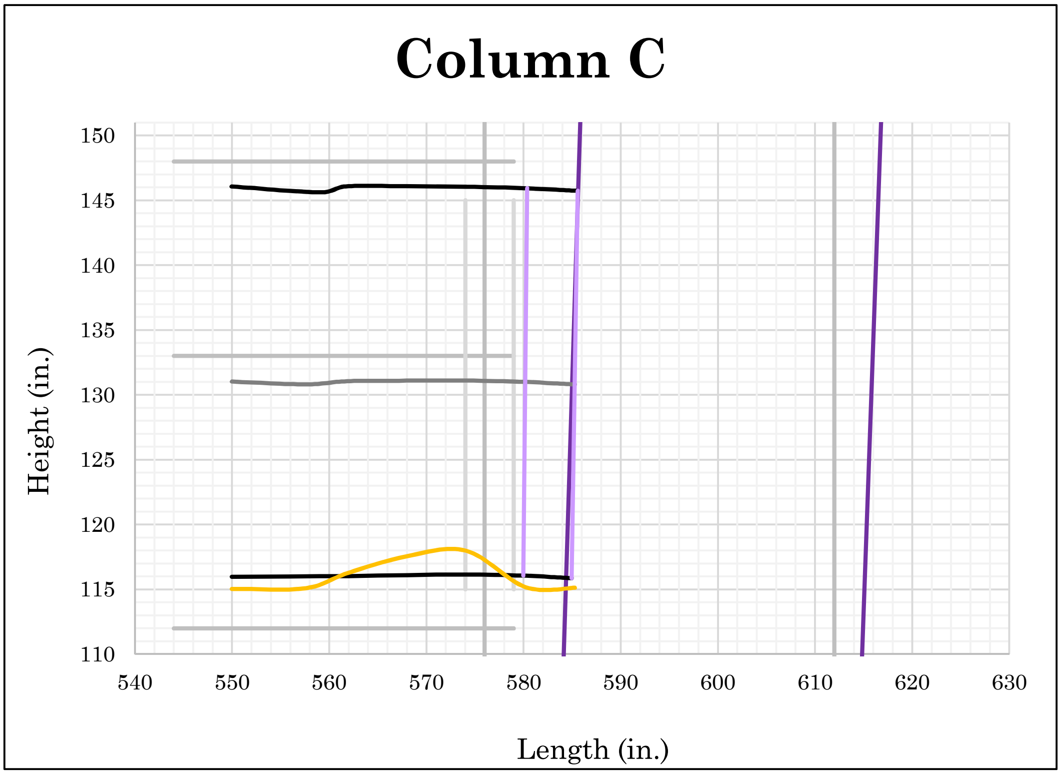


Figure 194: Joint at Column C

Frame 7.1.7

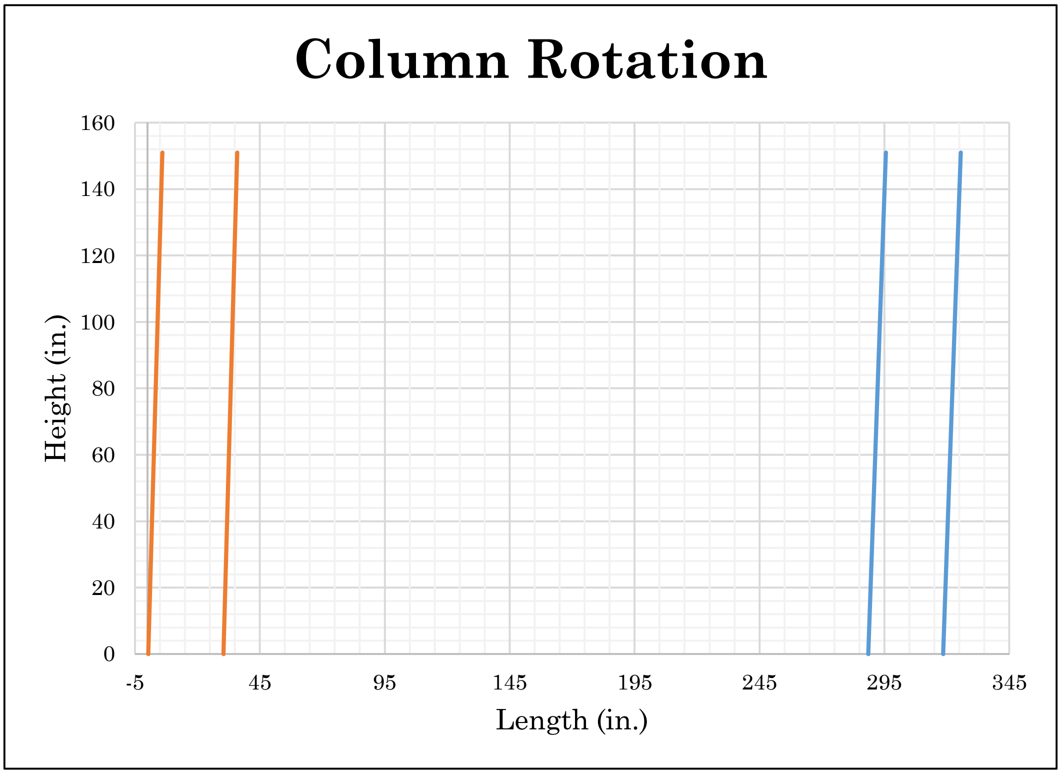


Figure 195: Column Rotation

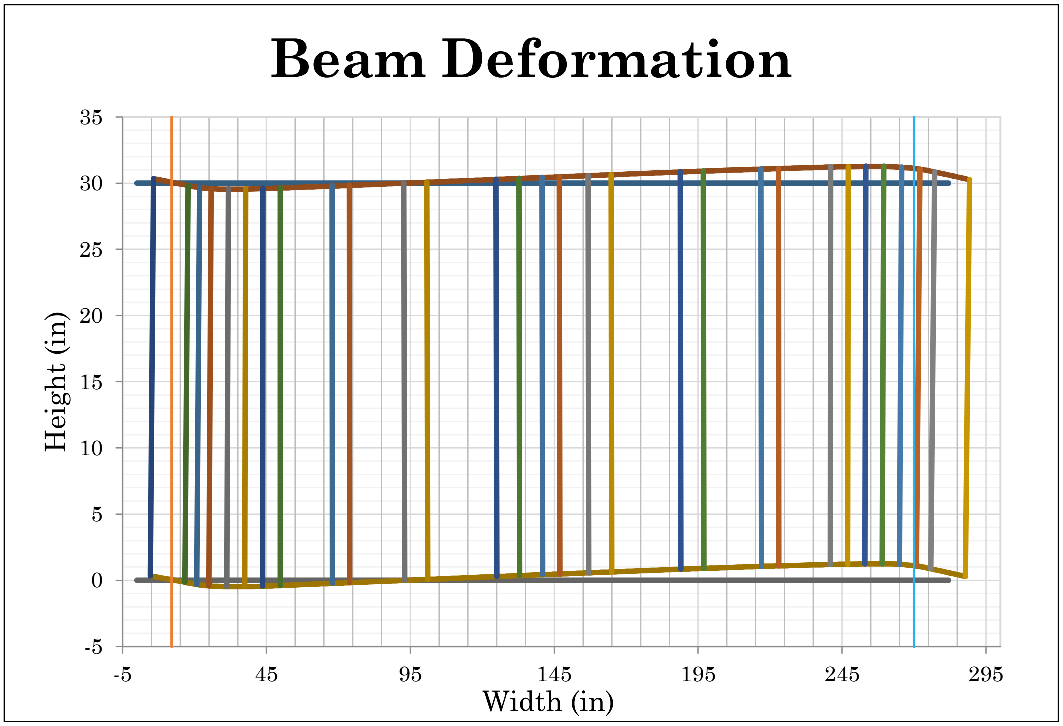


Figure 196: Beam Deformation

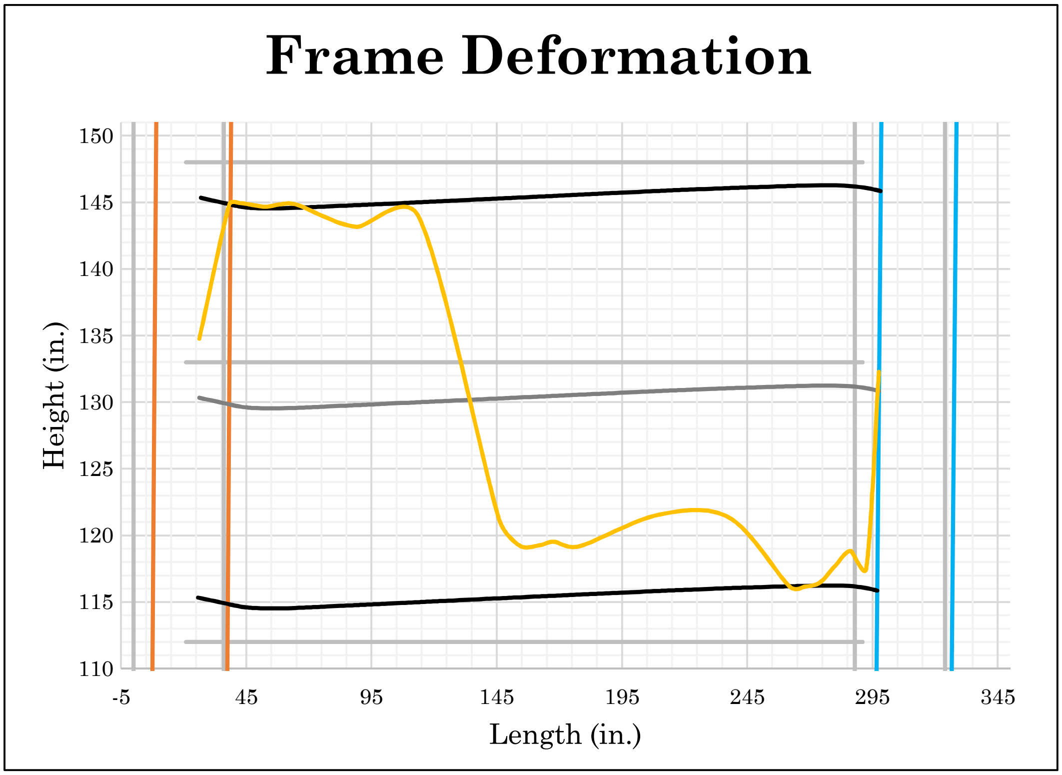


Figure 197: Frame Deformation

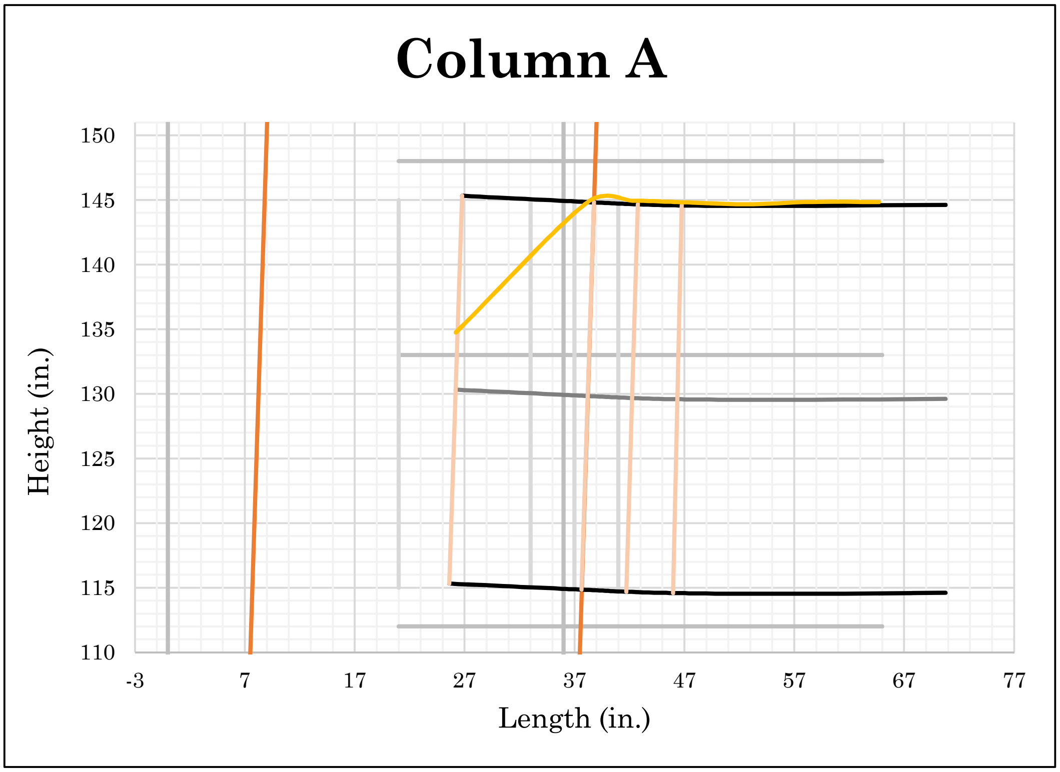


Figure 198: Joint at Column A

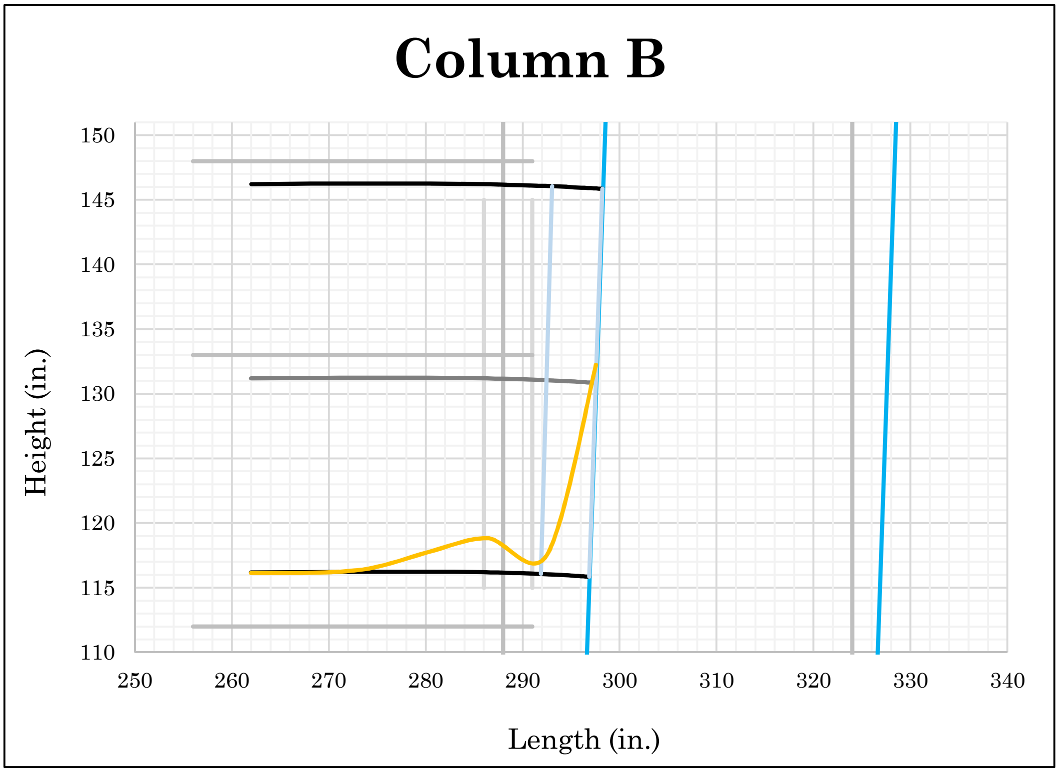


Figure 199: Joint at Column B

Frame 7.1.5

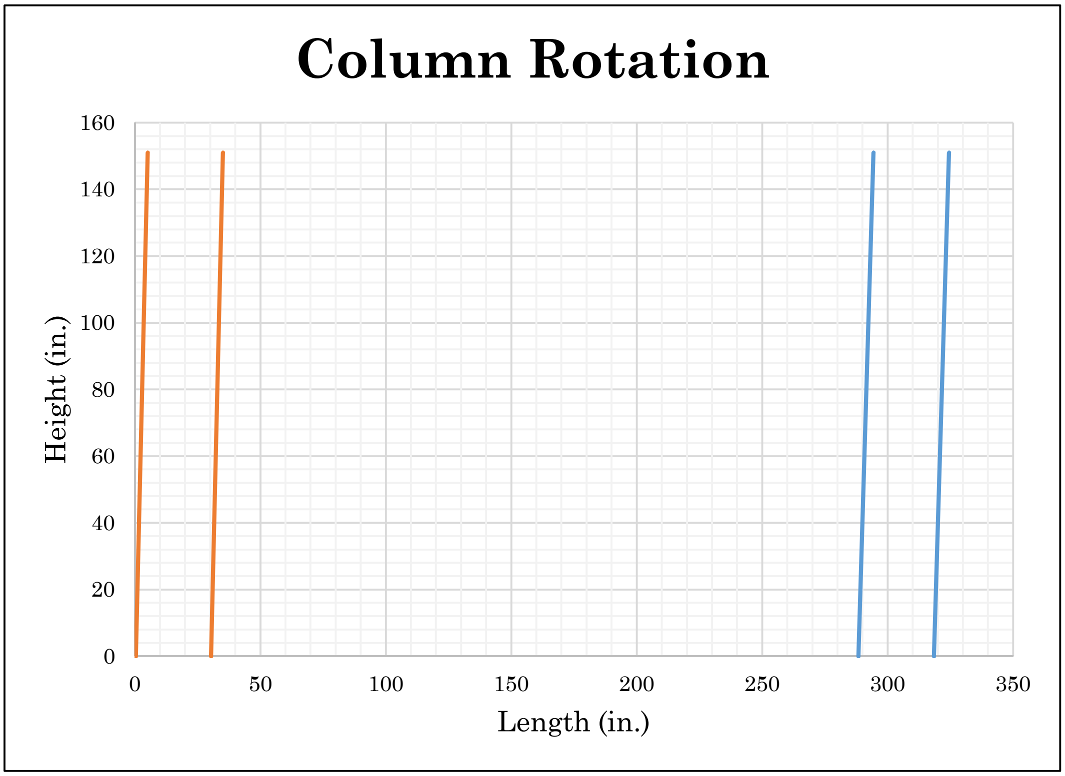


Figure 200: Column Rotation

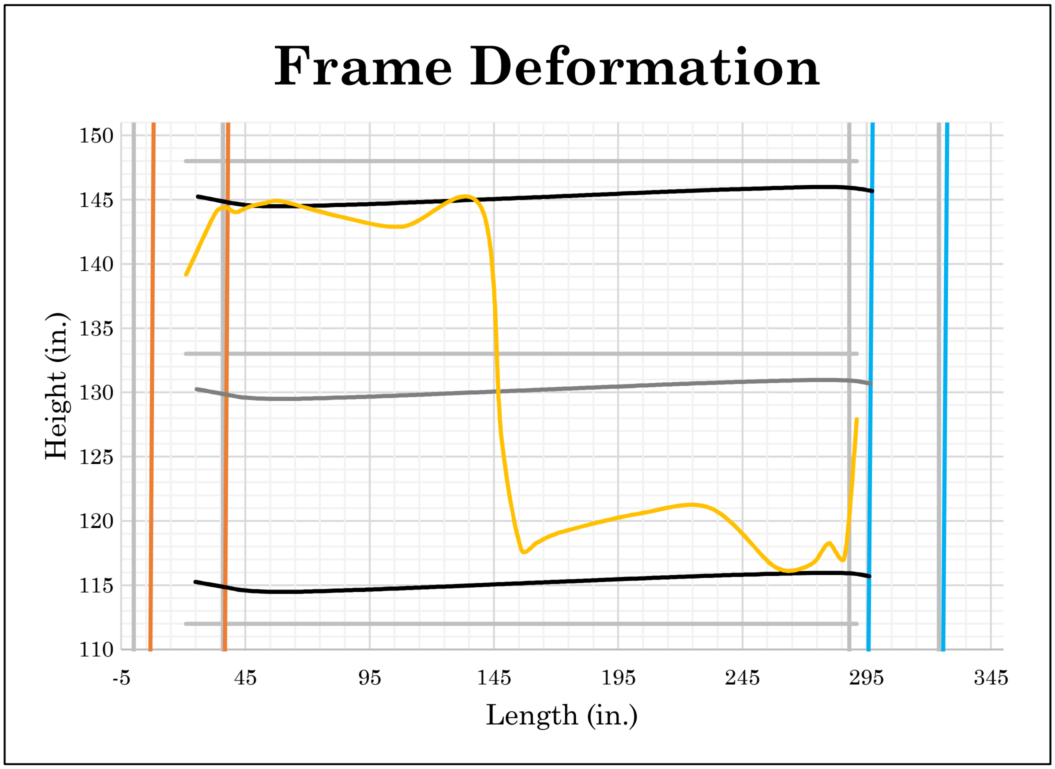


Figure 201: Frame Deformation

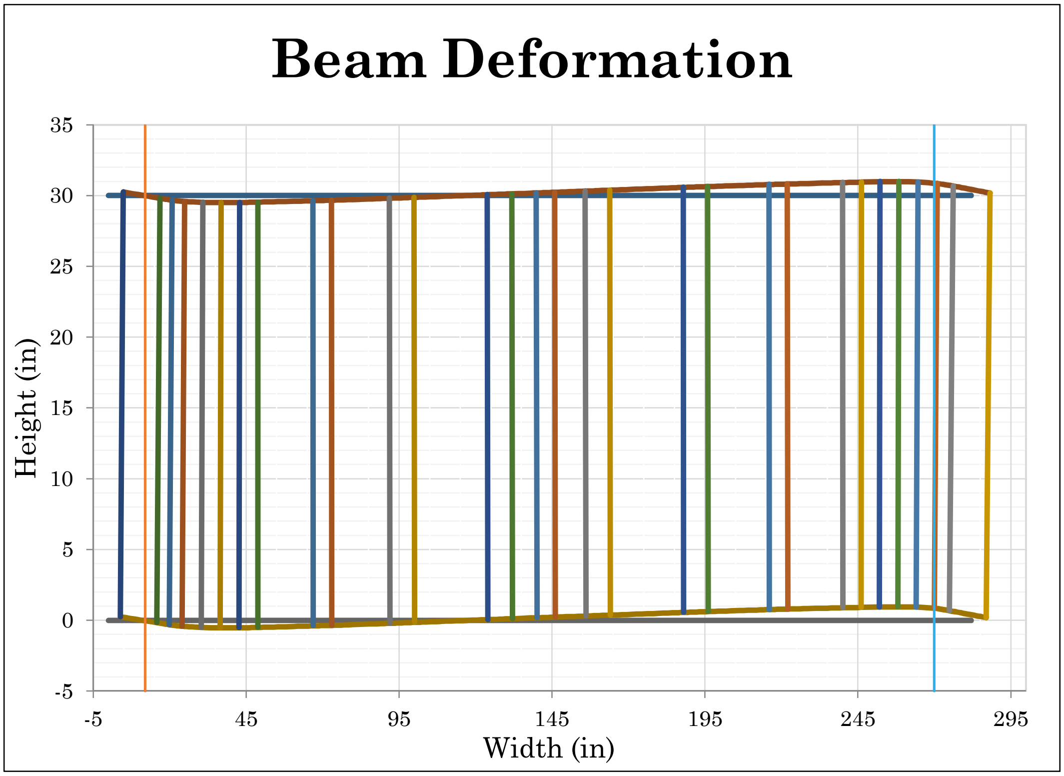


Figure 202: Beam Deformation

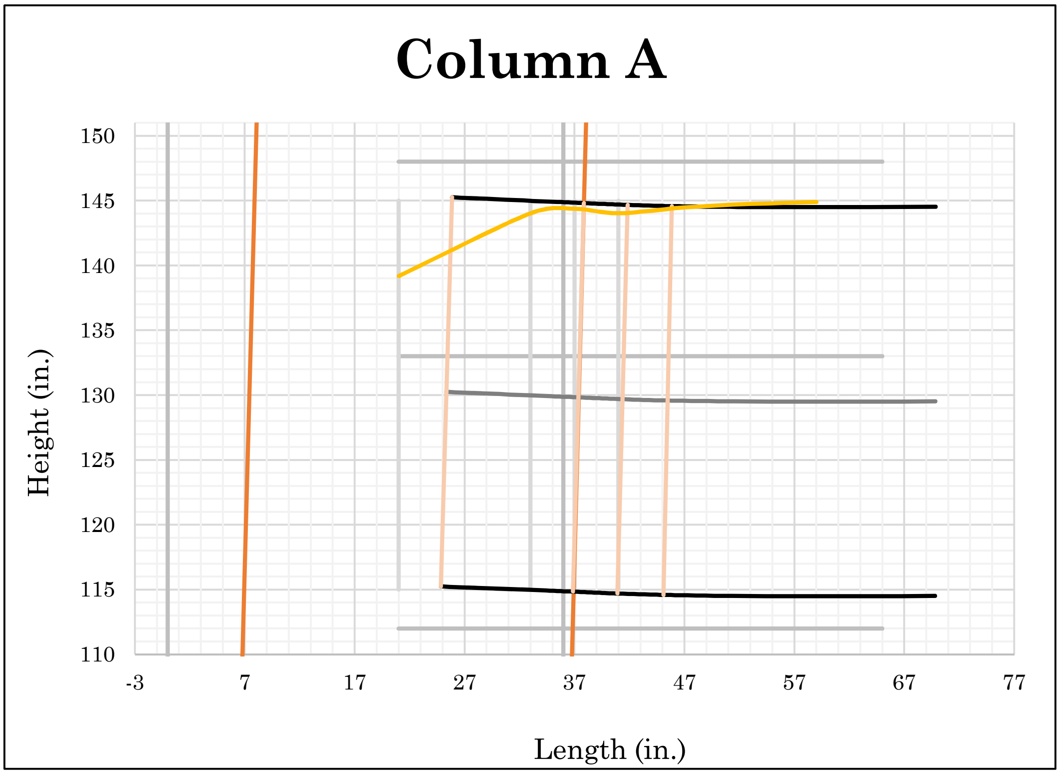


Figure 203: Joint at Column A

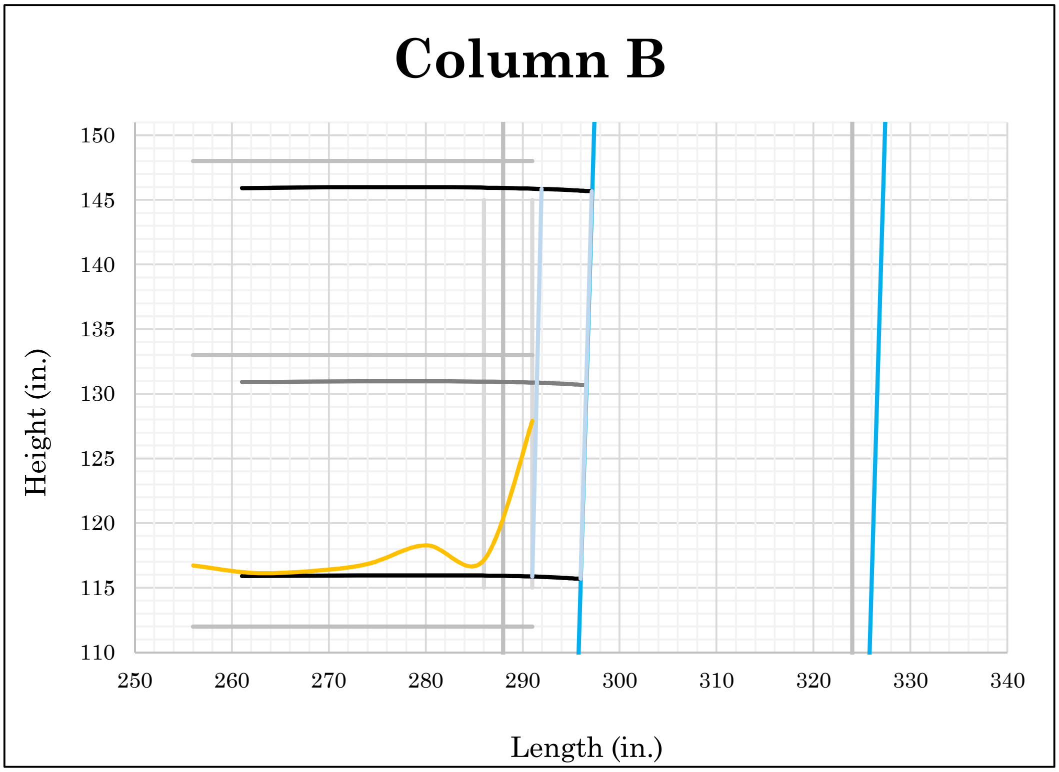


Figure 204: Joint at Column B

Frame 6.1.7

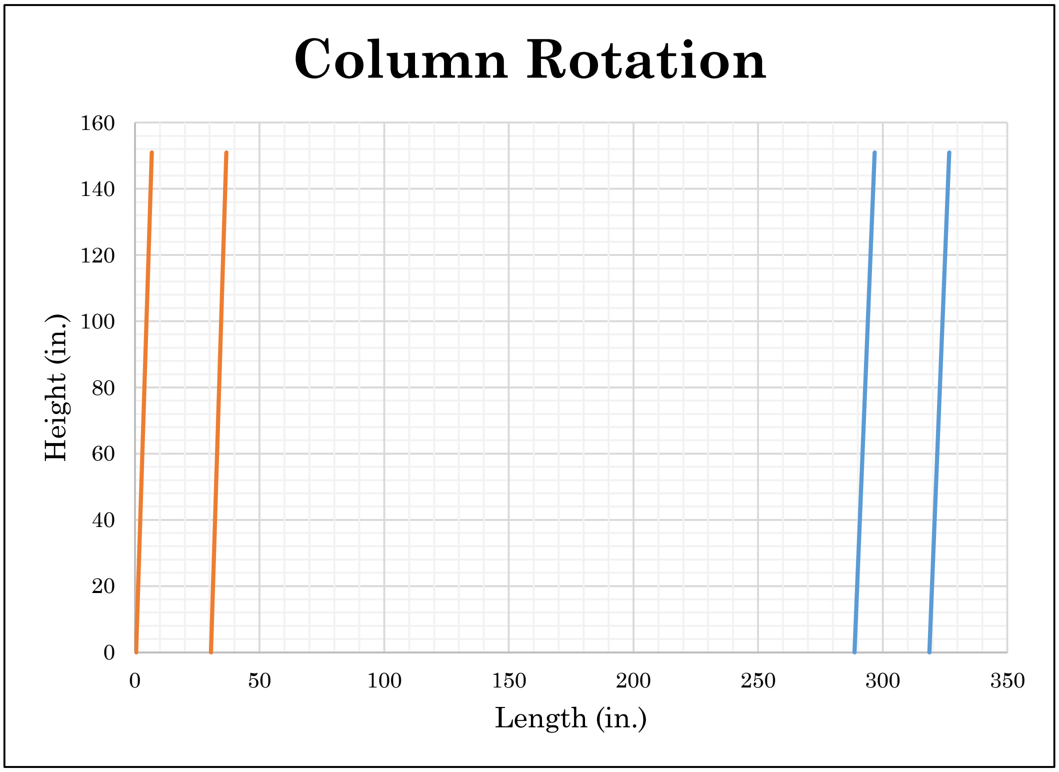


Figure 205: Column Rotation

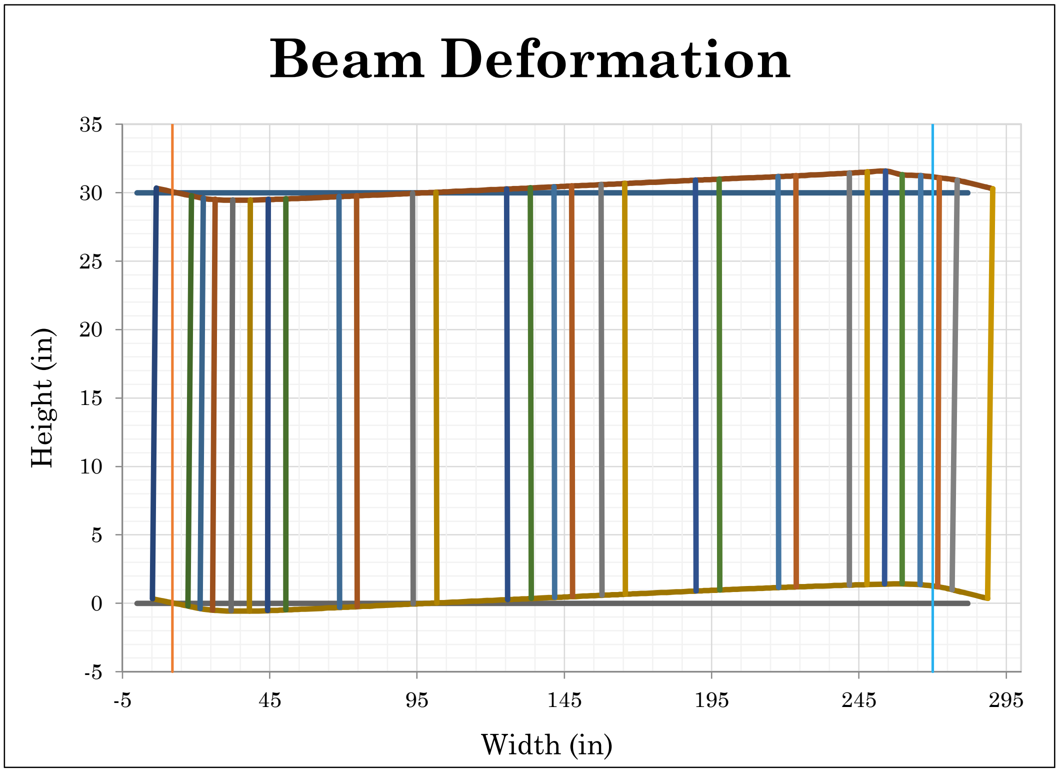


Figure 206: Beam Deformation

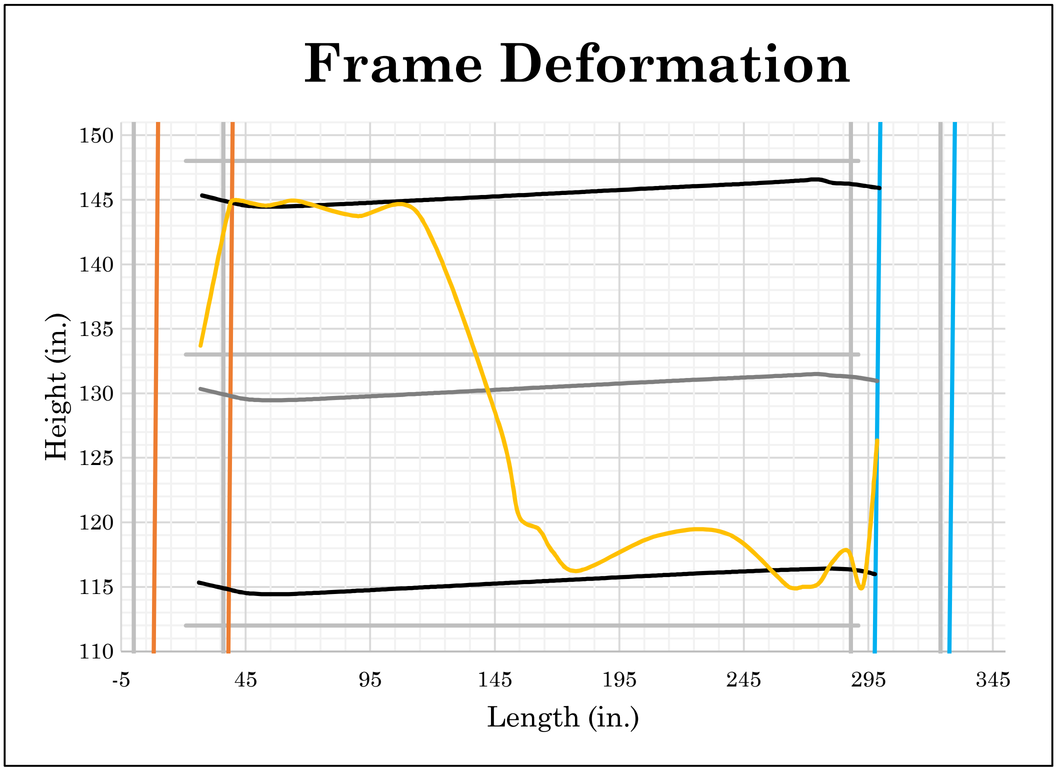


Figure 207: Frame Deformation

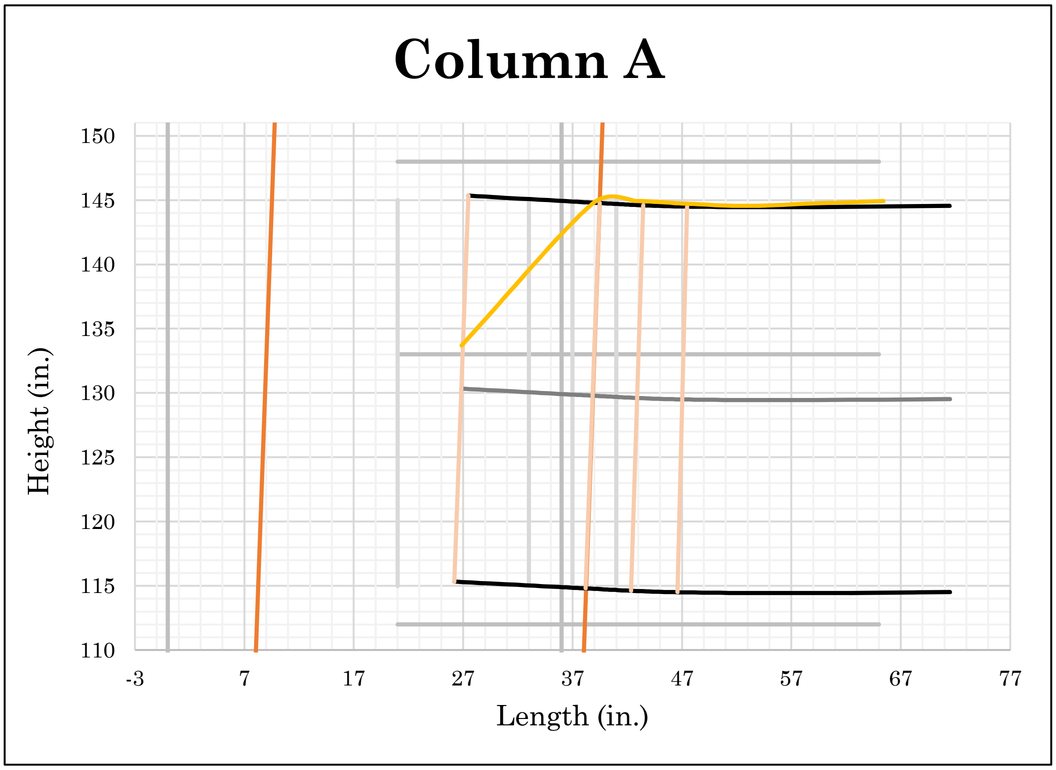


Figure 208: Joint at Column A

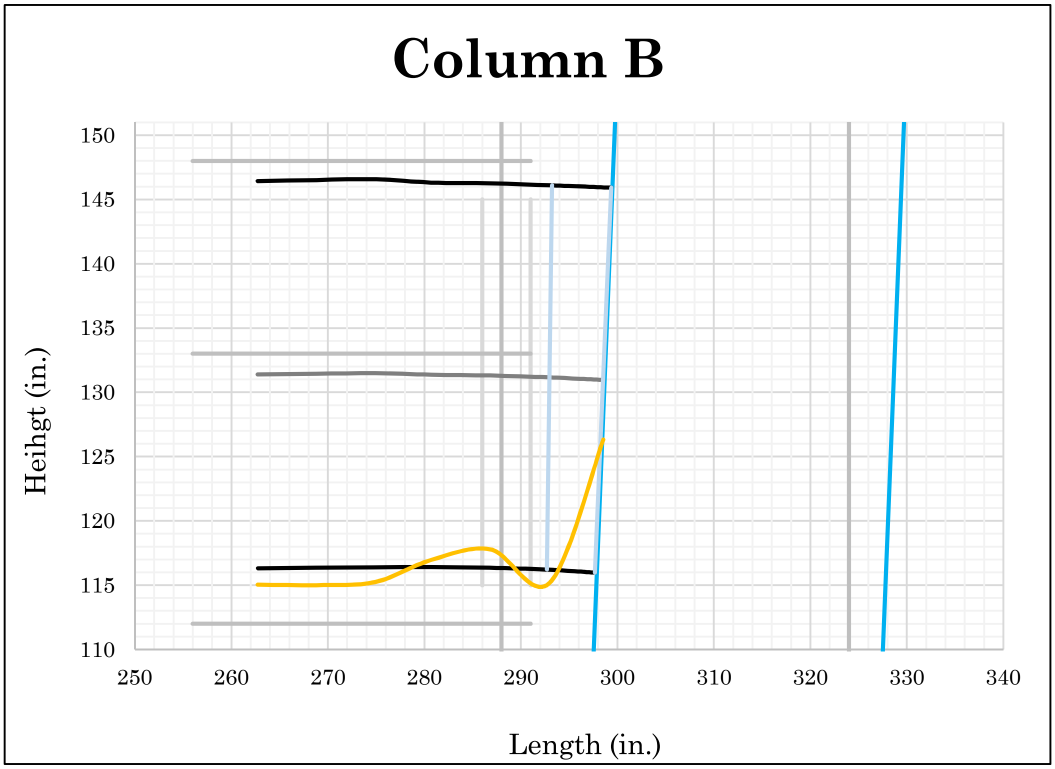


Figure 209: Joint at Column B

Frame 6.1.5

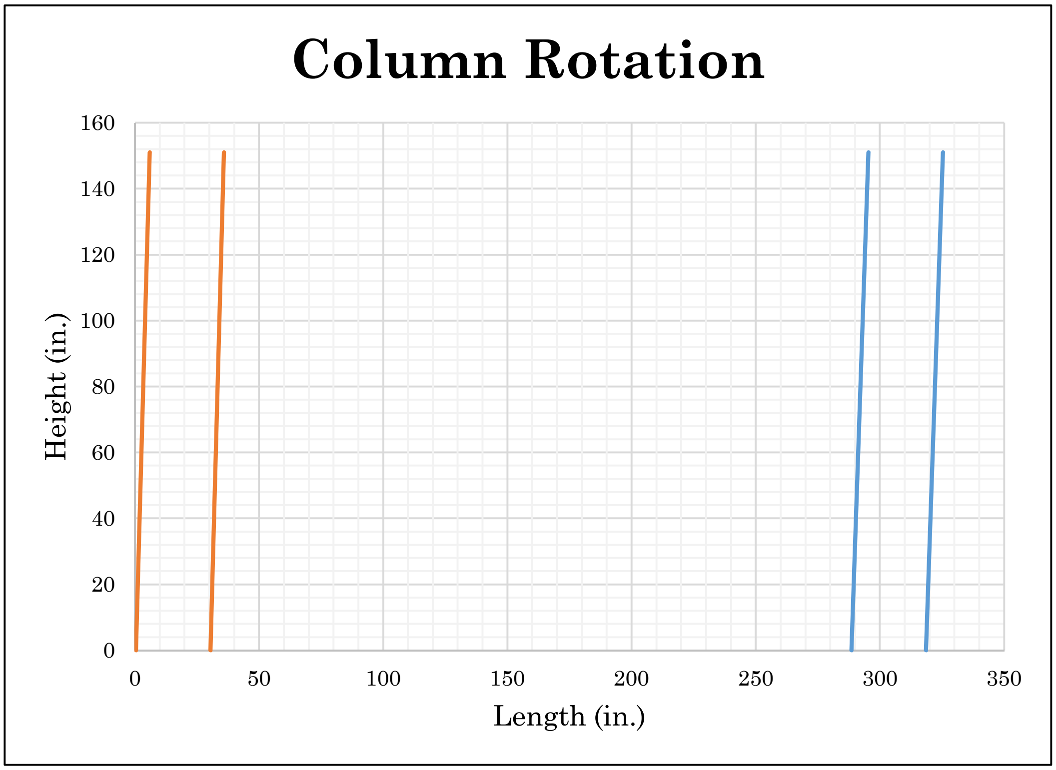


Figure 210: Column Rotation

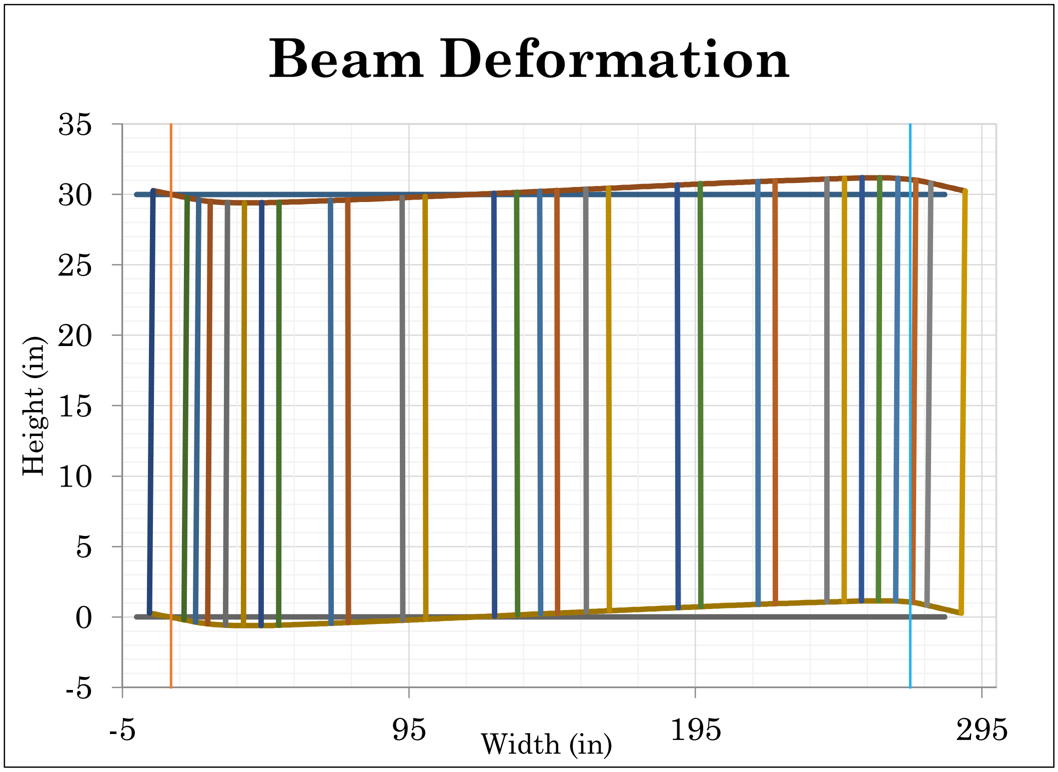


Figure 211: Beam Deformation

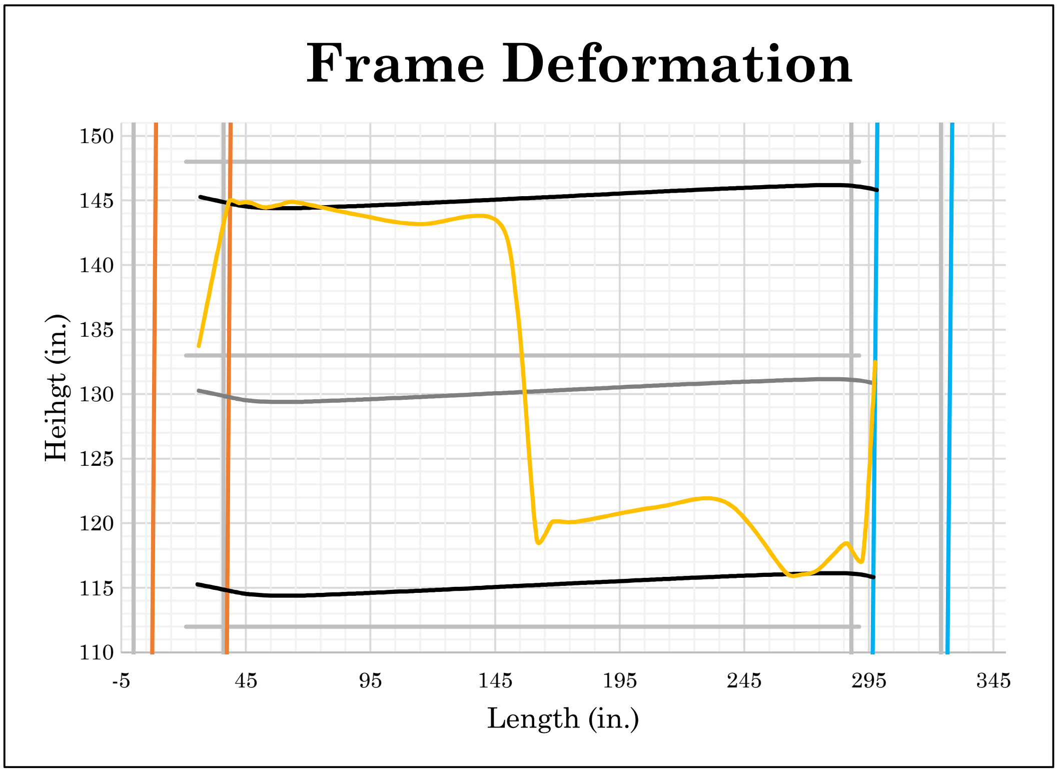


Figure 212: Frame Deformation

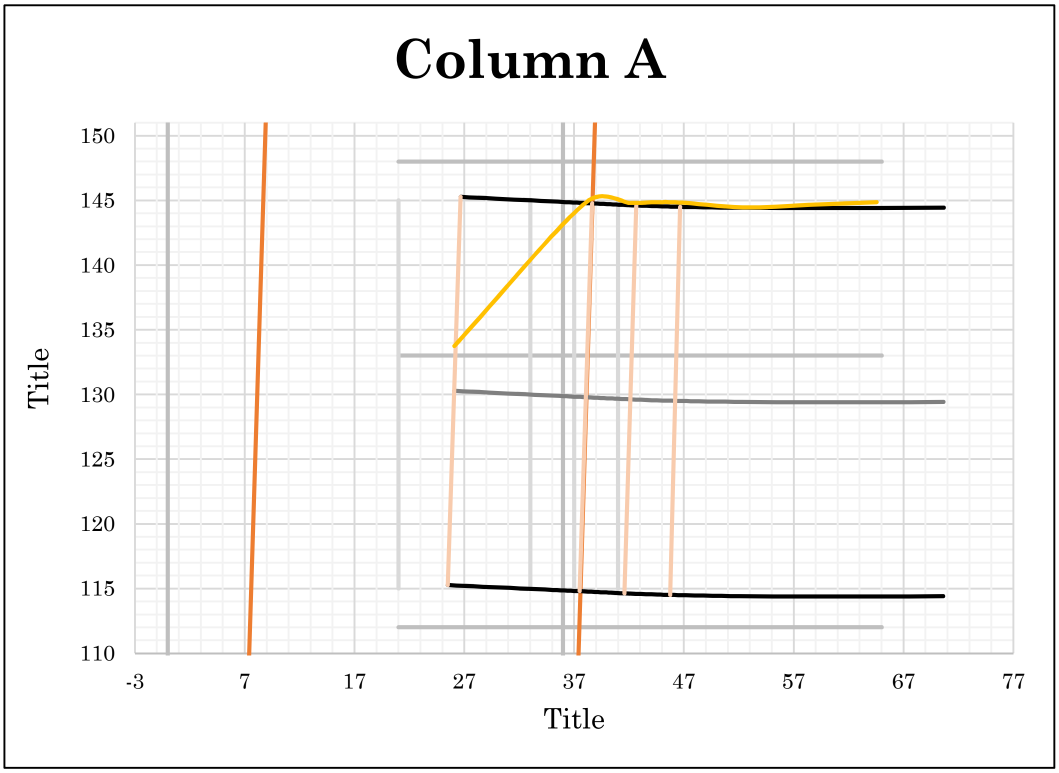


Figure 213: Joint at Column A

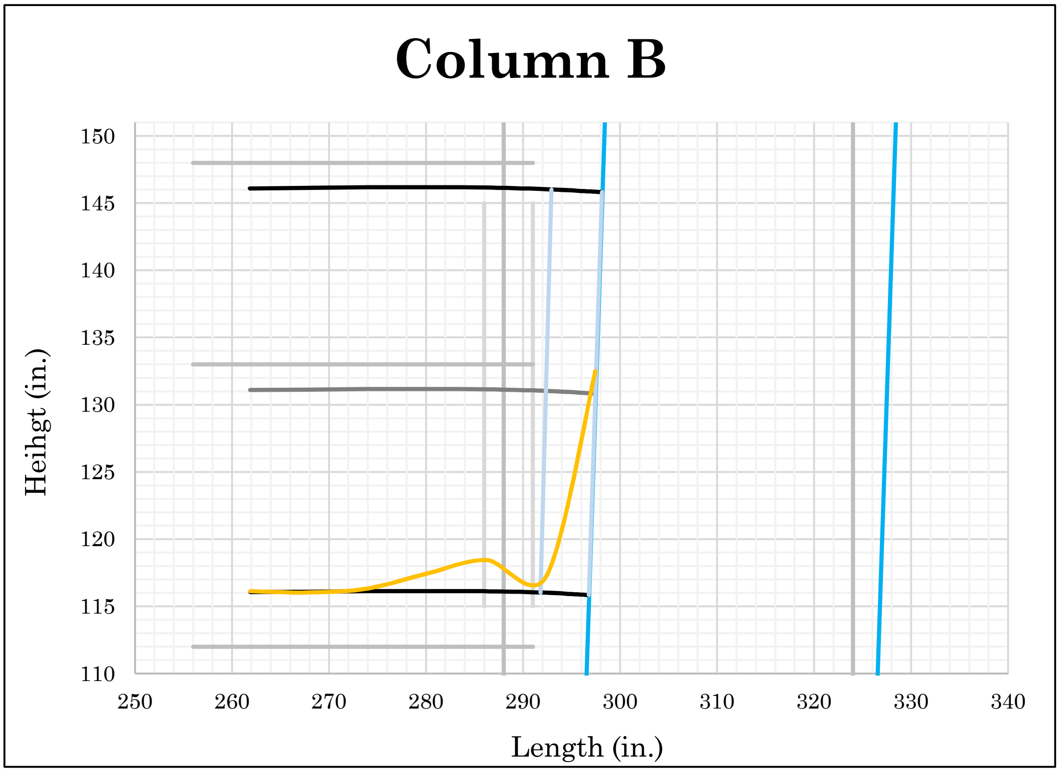


Figure 214: Joint at Column B

Appendix D - Elongation Calculations

Calculations for 5.3.2

One Second

Table 29: Elongation at one second for the five bay frames

Frame	7.5.7									
Location	Column A	Column B		Column C		Column D		Column E		Column F
	Right	Left	Right	Left	Right	Left	Right	Left	Right	Left
Time	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000
Drift (in.)	0.1911	0.2289	0.2289	0.2727	0.2727	0.3247	0.3247	0.3745	0.3745	0.4757
Rotation (°)	0.0725	0.0868	0.0868	0.1035	0.1035	0.1232	0.1232	0.1421	0.1421	0.1805
Neutral Axis	7.1735	28.7510	4.5108	28.5954	3.5769	26.0233	4.0656	28.2312	3.4369	32.3793
Top of Conc.	36.0000	36.0000	36.0000	36.0000	36.0000	36.0000	36.0000	36.0000	36.0000	36.0000
Centroid	15.0000	15.0000	15.0000	15.0000	15.0000	15.0000	15.0000	15.0000	15.0000	15.0000
Elongation	0.0099	0.0208	0.0159	0.0246	0.0206	0.0237	0.0235	0.0328	0.0287	0.0548
	0.0307		0.0404		0.0443		0.0563		0.0834	
	0.01%		0.02%		0.02%		0.02%		0.03%	
Total Beam									0.2553 in.	0.02%
Total									2.1229 in.	0.14%
Frame	7.5.5									
Location	Column A	Column B		Column C		Column D		Column E		Column F
	Right	Left	Right	Left	Right	Left	Right	Left	Right	Left
Time	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000
Drift	0.2447	0.3009	0.3009	0.3667	0.3667	0.4383	0.4383	0.5197	0.5197	0.6357
Rotation	0.0928	0.1142	0.1142	0.1392	0.1392	0.1663	0.1663	0.1972	0.1972	0.2412
Neutral Axis	4.8383	28.5679	4.6278	28.0802	4.9917	27.6668	4.9955	27.6731	4.1248	27.4313
Top of Conc.	36.0000	36.0000	36.0000	36.0000	36.0000	36.0000	36.0000	36.0000	36.0000	36.0000
Centroid	15.0000	15.0000	15.0000	15.0000	15.0000	15.0000	15.0000	15.0000	15.0000	15.0000
Elongation	0.0165	0.0270	0.0207	0.0318	0.0243	0.0368	0.0290	0.0436	0.0374	0.0523
	0.0435		0.0524		0.0611		0.0727		0.0898	
	0.02%		0.02%		0.02%		0.03%		0.03%	
Total Beam									0.3194 in.	0.02%
Total									2.8255 in.	0.19%
Frame	6.5.7									
Location	Column A	Column B		Column C		Column D		Column E		Column F
	Right	Left	Right	Left	Right	Left	Right	Left	Right	Left
Time	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000
Drift	0.2613	0.3183	0.3183	0.3725	0.3725	0.4326	0.4326	0.5022	0.5022	0.6211
Rotation	0.0991	0.1208	0.1208	0.1413	0.1413	0.1642	0.1642	0.1906	0.1906	0.2357
Neutral Axis	4.9624	29.7050	4.3413	28.4618	3.7919	28.9351	3.8155	29.4106	3.9078	29.3873
Top of Conc.	36.0000	36.0000	36.0000	36.0000	36.0000	36.0000	36.0000	36.0000	36.0000	36.0000
Centroid	15.0000	15.0000	15.0000	15.0000	15.0000	15.0000	15.0000	15.0000	15.0000	15.0000
Elongation	0.0174	0.0310	0.0225	0.0332	0.0276	0.0399	0.0320	0.0479	0.0369	0.0592
	0.0484		0.0557		0.0676		0.0800		0.0961	
	0.02%		0.02%		0.03%		0.03%		0.04%	
Total Beam									0.3476 in.	0.03%
Total									2.8556 in.	0.19%
Frame	6.5.5									
Location	Column A	Column B		Column C		Column D		Column E		Column F
	Right	Left	Right	Left	Right	Left	Right	Left	Right	Left
Time	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000
Drift	0.2447	0.2937	0.2937	0.3667	0.3667	0.4383	0.4383	0.5197	0.5197	0.6357
Rotation	0.0928	0.1114	0.1114	0.1392	0.1392	0.1663	0.1663	0.1972	0.1972	0.2412
Neutral Axis	5.0930	28.8730	3.5733	28.5529	3.9683	28.3045	4.0072	28.3683	3.7820	27.6857
Top of Conc.	36.0000	36.0000	36.0000	36.0000	36.0000	36.0000	36.0000	36.0000	36.0000	36.0000
Centroid	15.0000	15.0000	15.0000	15.0000	15.0000	15.0000	15.0000	15.0000	15.0000	15.0000
Elongation	0.0161	0.0270	0.0222	0.0329	0.0268	0.0386	0.0319	0.0460	0.0386	0.0534
	0.0430		0.0551		0.0654		0.0779		0.0920	
	0.02%		0.02%		0.03%		0.03%		0.04%	
Total Beam									0.3335 in.	0.03%
Total									2.8323 in.	0.19%

Table 30: Elongation at one second for the four bay frames

Frame	7.4.7							
Location	Column A	Column B		Column C		Column D		Column E
	Right	Left	Right	Left	Right	Left	Right	Left
Time	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000
Drift	0.2080	0.2501	0.2501	0.2955	0.2955	0.3461	0.3461	0.4234
Rotation	0.0789	0.0949	0.0949	0.1121	0.1121	0.1313	0.1313	0.1607
Neutral Axis	7.2043	28.5503	3.1570	28.3976	3.1692	28.7442	3.2239	27.4286
Top of Conc.	36.0000	36.0000	36.0000	36.0000	36.0000	36.0000	36.0000	36.0000
Centroid	15.0000	15.0000	15.0000	15.0000	15.0000	15.0000	15.0000	15.0000
Elongation	0.0107	0.0224	0.0196	0.0262	0.0232	0.0315	0.0270	0.0349
	0.0332		0.0458		0.0547		0.0618	
	0.01%		0.02%		0.02%		0.02%	
Total Beam	0.1955 in.							0.02%
Total	1.7186 in.							0.15%
Frame	7.4.5							
Location	Column A	Column B		Column C		Column D		Column E
	Right	Left	Right	Left	Right	Left	Right	Left
Time	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000
Drift	0.1972	0.2432	0.2432	0.2970	0.2970	0.3574	0.3574	0.4383
Rotation	0.0748	0.0923	0.0923	0.1127	0.1127	0.1356	0.1356	0.1663
Neutral Axis	4.7679	28.5396	4.7281	28.2091	4.7861	27.9914	4.6088	27.6519
Top of Conc.	36.0000	36.0000	36.0000	36.0000	36.0000	36.0000	36.0000	36.0000
Centroid	15.0000	15.0000	15.0000	15.0000	15.0000	15.0000	15.0000	15.0000
Elongation	0.0134	0.0218	0.0165	0.0260	0.0201	0.0308	0.0246	0.0367
	0.0352		0.0425		0.0508		0.0613	
	0.01%		0.02%		0.02%		0.02%	
Total Beam	0.1899 in.							0.02%
Total	1.7231 in.							0.15%
Frame	6.4.7							
Location	Column A	Column B		Column C		Column D		Column E
	Right	Left	Right	Left	Right	Left	Right	Left
Time	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000
Drift	0.2816	0.3499	0.3499	0.4090	0.4090	0.4724	0.4724	0.5789
Rotation	0.1069	0.1328	0.1328	0.1552	0.1552	0.1792	0.1792	0.2197
Neutral Axis	5.1656	28.4079	4.0483	29.4587	4.5237	30.0288	4.3442	27.7357
Top of Conc.	36.0000	36.0000	36.0000	36.0000	36.0000	36.0000	36.0000	36.0000
Centroid	15.0000	15.0000	15.0000	15.0000	15.0000	15.0000	15.0000	15.0000
Elongation	0.0183	0.0311	0.0254	0.0392	0.0284	0.0470	0.0333	0.0488
	0.0494		0.0645		0.0754		0.0822	
	0.02%		0.03%		0.03%		0.03%	
Total Beam	0.2715 in.							0.03%
Total	2.3633 in.							0.20%
Frame	6.4.5							
Location	Column A	Column B		Column C		Column D		Column E
	Right	Left	Right	Left	Right	Left	Right	Left
Time	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000
Drift	0.2697	0.3303	0.3303	0.4026	0.4026	0.4821	0.4821	0.5896
Rotation	0.1023	0.1253	0.1253	0.1528	0.1528	0.1829	0.1829	0.2237
Neutral Axis	3.5698	29.0071	3.6339	28.7581	3.8863	28.7387	3.3567	32.8726
Top of Conc.	36.0000	36.0000	36.0000	36.0000	36.0000	36.0000	36.0000	36.0000
Centroid	15.0000	15.0000	15.0000	15.0000	15.0000	15.0000	15.0000	15.0000
Elongation	0.0204	0.0306	0.0249	0.0367	0.0296	0.0439	0.0372	0.0698
	0.0511		0.0615		0.0735		0.1070	
	0.02%		0.02%		0.03%		0.04%	
Total Beam	0.2931 in.							0.03%
Total	2.3674 in.							0.20%

Table 31: Elongation at one second for the three bay frames

Frame	7.3.7					
Location	Column A	Column B		Column C		Column D
	Right	Left	Right	Left	Right	Left
Time	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000
Drift	0.2326	0.2774	0.2774	0.3252	0.3252	0.4005
Rotation	0.0883	0.1052	0.1052	0.1234	0.1234	0.1520
Neutral Axis	3.6294	27.6524	4.4815	28.8162	3.5717	27.6043
Top of Conc.	36.0000	36.0000	36.0000	36.0000	36.0000	36.0000
Centroid	15.0000	15.0000	15.0000	15.0000	15.0000	15.0000
Elongation	0.0175	0.0232	0.0193	0.0298	0.0246	0.0334
	0.0408		0.0491		0.0580	
	0.02%		0.02%		0.02%	
Total Beam					0.1479 in.	0.02%
Total					1.3836 in.	0.15%
Frame	7.3.5					
Location	Column A	Column B		Column C		Column D
	Right	Left	Right	Left	Right	Left
Time	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000
Drift	0.2217	0.2705	0.2705	0.3290	0.3290	0.4066
Rotation	0.0841	0.1027	0.1027	0.1248	0.1248	0.1543
Neutral Axis	4.4701	28.6200	4.5346	28.1757	4.6181	27.8791
Top of Conc.	36.0000	36.0000	36.0000	36.0000	36.0000	36.0000
Centroid	15.0000	15.0000	15.0000	15.0000	15.0000	15.0000
Elongation	0.0155	0.0244	0.0188	0.0287	0.0226	0.0347
	0.0399		0.0475		0.0573	
	0.02%		0.02%		0.02%	
Total Beam					0.1446 in.	0.02%
Total					1.3726 in.	0.15%
Frame	6.3.7					
Location	Column A	Column B		Column C		Column D
	Right	Left	Right	Left	Right	Left
Time	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000
Drift	0.2789	0.3292	0.3292	0.3916	0.3916	0.4796
Rotation	0.1058	0.1249	0.1249	0.1486	0.1486	0.1820
Neutral Axis	4.8799	28.9067	3.9429	28.7627	3.9822	27.9096
Top of Conc.	36.0000	36.0000	36.0000	36.0000	36.0000	36.0000
Centroid	15.0000	15.0000	15.0000	15.0000	15.0000	15.0000
Elongation	0.0187	0.0303	0.0241	0.0357	0.0286	0.0410
	0.0490		0.0598		0.0696	
	0.02%		0.02%		0.03%	
Total Beam					0.1784 in.	0.02%
Total					1.6576 in.	0.19%
Frame	6.3.5					
Location	Column A	Column B		Column C		Column D
	Right	Left	Right	Left	Right	Left
Time	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000
Drift	0.2938	0.3586	0.3586	0.4371	0.4371	0.5411
Rotation	0.1115	0.1361	0.1361	0.1659	0.1659	0.2053
Neutral Axis	3.6255	29.0280	3.4247	28.7606	3.3397	28.0098
Top of Conc.	36.0000	36.0000	36.0000	36.0000	36.0000	36.0000
Centroid	15.0000	15.0000	15.0000	15.0000	15.0000	15.0000
Elongation	0.0221	0.0333	0.0275	0.0398	0.0338	0.0466
	0.0554		0.0673		0.0804	
	0.02%		0.03%		0.03%	
Total Beam					0.2031 in.	0.03%
Total					1.8338 in.	0.21%

Table 33: Elongation at one second for the two bay frames

Frame	7.2.7			
Location	Column A	Column B		Column C
	Right	Left	Right	Left
Time	1.0000	1.0000	1.0000	1.0000
Drift	0.2486	0.3014	0.3014	0.3697
Rotation	0.0943	0.1144	0.1144	0.1403
Neutral Axis	5.4110	28.9459	4.0045	27.9218
Top of Conc.	36.0000	36.0000	36.0000	36.0000
Centroid	15.0000	15.0000	15.0000	15.0000
Elongation	0.0158	0.0278	0.0219	0.0316
	0.0436		0.0536	
	0.02%		0.02%	
Total Beam	0.0972 in.			0.02%
Total	1.0170 in.			0.17%
Frame	7.2.5			
Location	Column A	Column B		Column C
	Right	Left	Right	Left
Time	1.0000	1.0000	1.0000	1.0000
Drift	0.2440	0.2969	0.2969	0.3691
Rotation	0.0926	0.1126	0.1126	0.1401
Neutral Axis	4.4113	28.7824	4.1168	28.0367
Top of Conc.	36.0000	36.0000	36.0000	36.0000
Centroid	15.0000	15.0000	15.0000	15.0000
Elongation	0.0171	0.0271	0.0214	0.0319
	0.0442		0.0533	
	0.02%		0.02%	
Total Beam	0.0975 in.			0.02%
Total	1.0075 in.			0.17%
Frame	6.2.7			
Location	Column A	Column B		Column C
	Right	Left	Right	Left
Time	1.0000	1.0000	1.0000	1.0000
Drift	0.3452	0.4059	0.4059	0.5039
Rotation	0.1310	0.1540	0.1540	0.1912
Neutral Axis	4.9736	29.3530	4.4331	28.2368
Top of Conc.	36.0000	36.0000	36.0000	36.0000
Centroid	15.0000	15.0000	15.0000	15.0000
Elongation	0.0229	0.0386	0.0284	0.0442
	0.0615		0.0726	
	0.02%		0.03%	
Total Beam	0.1341 in.			0.03%
Total	1.3892 in.			0.23%
Frame	6.2.5			
Location	Column A	Column B		Column C
	Right	Left	Right	Left
Time	1.0000	1.0000	1.0000	1.0000
Drift	0.3268	0.3967	0.3967	0.4956
Rotation	0.1240	0.1505	0.1505	0.1880
Neutral Axis	3.7289	29.2869	3.2643	28.2392
Top of Conc.	36.0000	36.0000	36.0000	36.0000
Centroid	15.0000	15.0000	15.0000	15.0000
Elongation	0.0244	0.0375	0.0308	0.0435
	0.0619		0.0743	
	0.02%		0.03%	
Total Beam	0.1362 in.			0.03%
Total	1.3553 in.			0.22%

Table 32: Elongation at one second for the one bay frames

Frame	7.1.7	
Location	Column A	Column B
	Right	Left
Time	1.0000	1.0000
Drift	0.2584	0.3266
Rotation	0.0980	0.1239
Neutral Axis	6.6482	28.1707
Top of Conc.	36.0000	36.0000
Centroid	15.0000	15.0000
Elongation	0.0143	0.0285
	0.0428	
	0.02%	
Total Beam	0.0428 in.	0.02%
Total	0.6278 in.	0.20%
Frame	7.1.5	
Location	Column A	Column B
	Right	Left
Time	1.0000	1.0000
Drift	0.2538	0.3168
Rotation	0.0963	0.1202
Neutral Axis	4.2962	28.4121
Top of Conc.	36.0000	36.0000
Centroid	15.0000	15.0000
Elongation	0.0180	0.0281
	0.0461	
	0.02%	
Total Beam	0.0461 in.	0.02%
Total	0.6167 in.	0.19%
Frame	6.1.7	
Location	Column A	Column B
	Right	Left
Time	1.0000	1.0000
Drift	0.3369	0.4403
Rotation	0.1278	0.1671
Neutral Axis	7.6424	28.5951
Top of Conc.	36.0000	36.0000
Centroid	15.0000	15.0000
Elongation	0.0164	0.0396
	0.0561	
	0.02%	
Total Beam	0.0561 in.	0.02%
Total	0.8333 in.	0.26%
Frame	6.1.5	
Location	Column A	Column B
	Right	Left
Time	1.0000	1.0000
Drift	0.3513	0.4367
Rotation	0.1333	0.1657
Neutral Axis	3.5665	28.6495
Top of Conc.	36.0000	36.0000
Centroid	15.0000	15.0000
Elongation	0.0266	0.0395
	0.0661	
	0.03%	
Total Beam	0.0661 in.	0.03%
Total	0.8541 in.	0.27%

Two Seconds

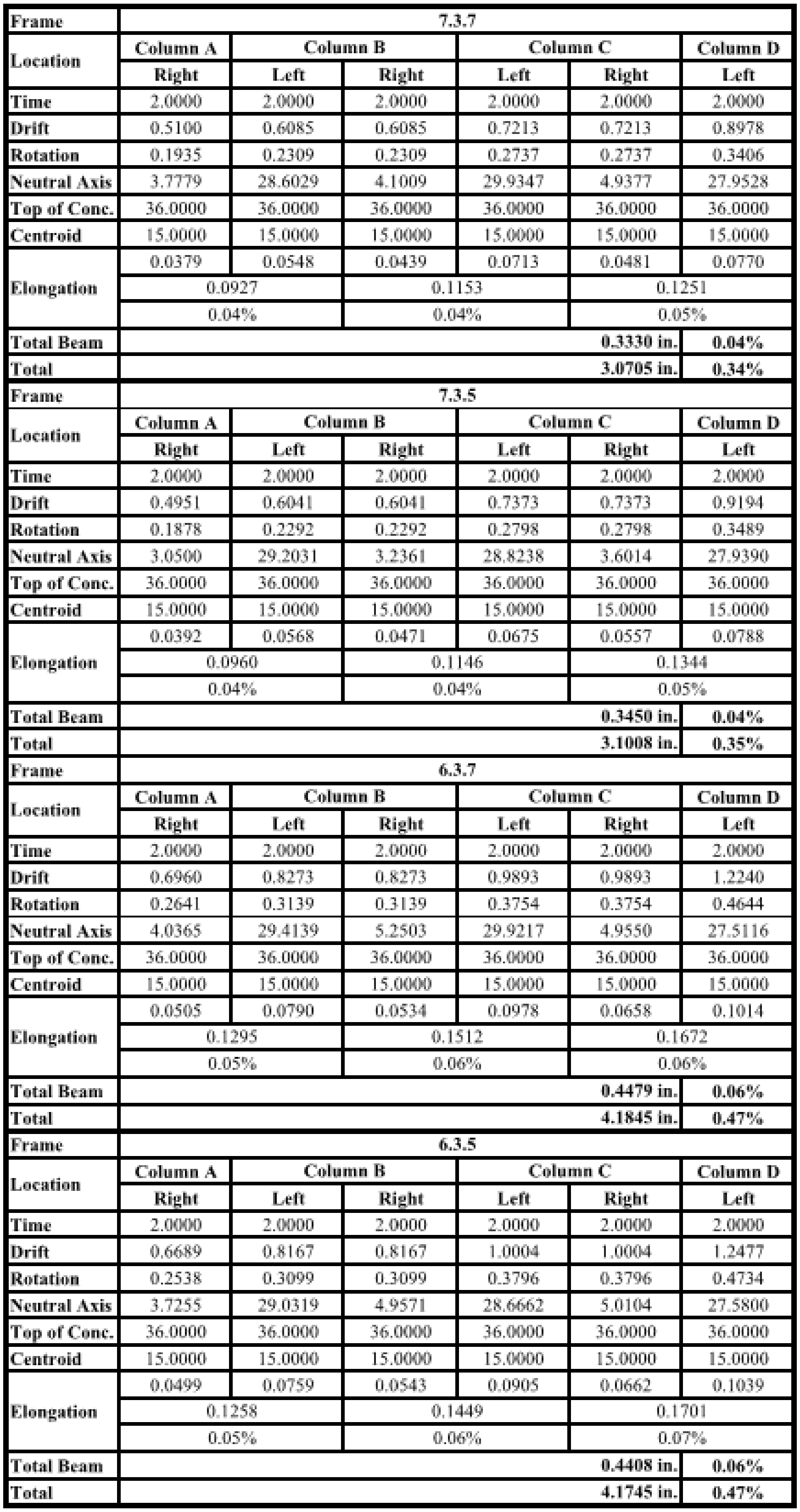
Table 34: Elongation at two seconds for the five bay frames

Frame	**7.5.7**									
Location	**Column A**	**Column B**		**Column C**		**Column D**		**Column E**		**Column F**
	Right	**Left**	**Right**	**Left**	**Right**	**Left**	**Right**	**Left**	**Right**	**Left**
Time	2.0000	2.0000	2.0000	2.0000	2.0000	2.0000	2.0000	2.0000	2.0000	2.0000
Drift (in.)	0.4288	0.5112	0.5112	0.6099	0.6099	0.7216	0.7216	0.8404	0.8404	1.0410
Rotation (°)	0.1627	0.1940	0.1940	0.2314	0.2314	0.2738	0.2738	0.3189	0.3189	0.3950
Neutral Axis	6.2078	28.9397	4.2721	28.7480	4.1630	28.2591	4.0569	30.4863	4.6834	28.8140
Top of Conc.	36.0000	36.0000	36.0000	36.0000	36.0000	36.0000	36.0000	36.0000	36.0000	36.0000
Centroid	15.0000	15.0000	15.0000	15.0000	15.0000	15.0000	15.0000	15.0000	15.0000	15.0000
Elongation	0.0250	0.0472	0.0363	0.0555	0.0438	0.0634	0.0523	0.0862	0.0574	0.0952
	0.0722		0.0919		0.1071		0.1385		0.1527	
	0.03%		0.04%		0.04%		0.05%		0.06%	
Total Beam	**0.5623 in.**									**0.04%**
Total	**4.7153 in.**									**0.32%**
Frame	**7.5.5**									
Location	**Column A**	**Column B**		**Column C**		**Column D**		**Column E**		**Column F**
	Right	**Left**	**Right**	**Left**	**Right**	**Left**	**Right**	**Left**	**Right**	**Left**
Time	2.0000	2.0000	2.0000	2.0000	2.0000	2.0000	2.0000	2.0000	2.0000	2.0000
Drift	0.5906	0.7241	0.7241	0.8848	0.8848	1.0640	1.0640	1.2697	1.2697	1.5514
Rotation	0.2241	0.2748	0.2748	0.3357	0.3357	0.4037	0.4037	0.4818	0.4818	0.5886
Neutral Axis	3.2335	29.1337	3.2241	28.6467	3.3985	28.2647	3.1210	28.2179	3.5928	26.9057
Top of Conc.	36.0000	36.0000	36.0000	36.0000	36.0000	36.0000	36.0000	36.0000	36.0000	36.0000
Centroid	15.0000	15.0000	15.0000	15.0000	15.0000	15.0000	15.0000	15.0000	15.0000	15.0000
Elongation	0.0460	0.0678	0.0565	0.0800	0.0680	0.0935	0.0837	0.1111	0.0959	0.1223
	0.1138		0.1364		0.1614		0.1949		0.2182	
	0.04%		0.05%		0.06%		0.08%		0.08%	
Total Beam	**0.8248 in.**									**0.06%**
Total	**6.9094 in.**									**0.47%**
Frame	**6.5.7**									
Location	**Column A**	**Column B**		**Column C**		**Column D**		**Column E**		**Column F**
	Right	**Left**	**Right**	**Left**	**Right**	**Left**	**Right**	**Left**	**Right**	**Left**
Time	2.0000	2.0000	2.0000	2.0000	2.0000	2.0000	2.0000	2.0000	2.0000	2.0000
Drift	0.5931	0.7137	0.7137	0.8445	0.8445	0.9873	0.9873	1.1494	1.1494	1.3994
Rotation	0.2251	0.2708	0.2708	0.3204	0.3204	0.3746	0.3746	0.4361	0.4361	0.5310
Neutral Axis	3.9930	29.5091	5.1387	29.1780	4.8884	29.5120	4.9779	30.5575	4.7753	27.0455
Top of Conc.	36.0000	36.0000	36.0000	36.0000	36.0000	36.0000	36.0000	36.0000	36.0000	36.0000
Centroid	15.0000	15.0000	15.0000	15.0000	15.0000	15.0000	15.0000	15.0000	15.0000	15.0000
Elongation	0.0432	0.0686	0.0466	0.0793	0.0566	0.0949	0.0655	0.1184	0.0778	0.1116
	0.1118		0.1259		0.1514		0.1839		0.1895	
	0.04%		0.05%		0.06%		0.07%		0.07%	
Total Beam	**0.7626 in.**									**0.06%**
Total	**6.4499 in.**									**0.44%**
Frame	**6.5.5**									
Location	**Column A**	**Column B**		**Column C**		**Column D**		**Column E**		**Column F**
	Right	**Left**	**Right**	**Left**	**Right**	**Left**	**Right**	**Left**	**Right**	**Left**
Time	2.0000	2.0000	2.0000	2.0000	2.0000	2.0000	2.0000	2.0000	2.0000	2.0000
Drift	0.5906	0.7057	0.7057	0.8848	0.8848	1.0640	1.0640	1.2697	1.2697	1.5514
Rotation	0.2241	0.2678	0.2678	0.3357	0.3357	0.4037	0.4037	0.4818	0.4818	0.5886
Neutral Axis	3.5172	29.0377	5.0860	28.3286	4.5187	28.0127	4.5386	28.1172	3.1949	26.6708
Top of Conc.	36.0000	36.0000	36.0000	36.0000	36.0000	36.0000	36.0000	36.0000	36.0000	36.0000
Centroid	15.0000	15.0000	15.0000	15.0000	15.0000	15.0000	15.0000	15.0000	15.0000	15.0000
Elongation	0.0449	0.0656	0.0463	0.0781	0.0614	0.0917	0.0737	0.1103	0.0993	0.1199
	0.1105		0.1244		0.1531		0.1840		0.2192	
	0.04%		0.05%		0.06%		0.07%		0.08%	
Total Beam	**0.7912 in.**									**0.06%**
Total	**6.8574 in.**									**0.47%**

Table 35: Elongation at two seconds for the four bay frames

Frame	7.4.7							
Location	Column A	Column B		Column C		Column D		Column E
	Right	Left	Right	Left	Right	Left	Right	Left
Time	2.0000	2.0000	2.0000	2.0000	2.0000	2.0000	2.0000	2.0000
Drift	0.4637	0.5560	0.5560	0.6618	0.6618	0.7790	0.7790	0.9613
Rotation	0.1760	0.2110	0.2110	0.2511	0.2511	0.2956	0.2956	0.3647
Neutral Axis	6.2957	29.0650	4.5043	28.9921	4.6361	29.7854	4.7739	28.0435
Top of Conc.	36.0000	36.0000	36.0000	36.0000	36.0000	36.0000	36.0000	36.0000
Centroid	15.0000	15.0000	15.0000	15.0000	15.0000	15.0000	15.0000	15.0000
Elongation	0.0267	0.0518	0.0386	0.0613	0.0454	0.0763	0.0528	0.0830
	0.0785		0.1000		0.1217		0.1358	
	0.03%		0.04%		0.05%		0.05%	
Total Beam							**0.4360 in.**	**0.04%**
Total							**3.8578 in.**	**0.33%**
Frame	7.4.5							
Location	Column A	Column B		Column C		Column D		Column E
	Right	Left	Right	Left	Right	Left	Right	Left
Time	2.0000	2.0000	2.0000	2.0000	2.0000	2.0000	2.0000	2.0000
Drift	0.4453	0.5469	0.5469	0.6676	0.6676	0.8045	0.8045	0.9950
Rotation	0.1690	0.2075	0.2075	0.2533	0.2533	0.3052	0.3052	0.3775
Neutral Axis	3.2484	29.1432	3.0429	28.7494	3.0464	28.5159	3.5728	27.6387
Top of Conc.	36.0000	36.0000	36.0000	36.0000	36.0000	36.0000	36.0000	36.0000
Centroid	15.0000	15.0000	15.0000	15.0000	15.0000	15.0000	15.0000	15.0000
Elongation	0.0347	0.0512	0.0433	0.0608	0.0529	0.0720	0.0609	0.0833
	0.0859		0.1041		0.1249		0.1442	
	0.03%		0.04%		0.05%		0.06%	
Total Beam							**0.4590 in.**	**0.04%**
Total							**3.9183 in.**	**0.33%**
Frame	6.4.7							
Location	Column A	Column B		Column C		Column D		Column E
	Right	Left	Right	Left	Right	Left	Right	Left
Time	2.0000	2.0000	2.0000	2.0000	2.0000	2.0000	2.0000	2.0000
Drift	0.6362	0.7726	0.7726	0.9140	0.9140	1.0693	1.0693	1.3157
Rotation	0.2414	0.2932	0.2932	0.3468	0.3468	0.4057	0.4057	0.4992
Neutral Axis	3.9394	29.0496	4.9055	29.8494	5.1759	30.6490	4.9809	27.3862
Top of Conc.	36.0000	36.0000	36.0000	36.0000	36.0000	36.0000	36.0000	36.0000
Centroid	15.0000	15.0000	15.0000	15.0000	15.0000	15.0000	15.0000	15.0000
Elongation	0.0466	0.0719	0.0517	0.0899	0.0595	0.1108	0.0709	0.1079
	0.1185		0.1415		0.1703		0.1789	
	0.05%		0.05%		0.07%		0.07%	
Total Beam							**0.6092 in.**	**0.06%**
Total							**5.3169 in.**	**0.45%**
Frame	6.4.5							
Location	Column A	Column B		Column C		Column D		Column E
	Right	Left	Right	Left	Right	Left	Right	Left
Time	2.0000	2.0000	2.0000	2.0000	2.0000	2.0000	2.0000	2.0000
Drift	0.6138	0.7497	0.7497	0.9152	0.9152	1.1028	1.1028	1.3584
Rotation	0.2329	0.2844	0.2844	0.3473	0.3473	0.4184	0.4184	0.5154
Neutral Axis	3.6341	29.0998	4.7504	28.6702	4.5934	28.4596	4.5608	32.9014
Top of Conc.	36.0000	36.0000	36.0000	36.0000	36.0000	36.0000	36.0000	36.0000
Centroid	15.0000	15.0000	15.0000	15.0000	15.0000	15.0000	15.0000	15.0000
Elongation	0.0462	0.0700	0.0509	0.0829	0.0631	0.0983	0.0762	0.1610
	0.1162		0.1337		0.1614		0.2373	
	0.05%		0.05%		0.06%		0.09%	
Total Beam							**0.6486 in.**	**0.06%**
Total							**5.3886 in.**	**0.46%**

Table 36: Elongation at two seconds for the three bay frames



Frame	7.3.7					
Location	Column A	Column B		Column C		Column D
	Right	Left	Right	Left	Right	Left
Time	2.0000	2.0000	2.0000	2.0000	2.0000	2.0000
Drift	0.5100	0.6085	0.6085	0.7213	0.7213	0.8978
Rotation	0.1935	0.2309	0.2309	0.2737	0.2737	0.3406
Neutral Axis	3.7779	28.6029	4.1009	29.9347	4.9377	27.9528
Top of Conc.	36.0000	36.0000	36.0000	36.0000	36.0000	36.0000
Centroid	15.0000	15.0000	15.0000	15.0000	15.0000	15.0000
Elongation	0.0379	0.0548	0.0439	0.0713	0.0481	0.0770
	0.0927		0.1153		0.1251	
	0.04%		0.04%		0.05%	
Total Beam					0.3330 in.	0.04%
Total					3.0705 in.	0.34%
Frame	7.3.5					
Location	Column A	Column B		Column C		Column D
	Right	Left	Right	Left	Right	Left
Time	2.0000	2.0000	2.0000	2.0000	2.0000	2.0000
Drift	0.4951	0.6041	0.6041	0.7373	0.7373	0.9194
Rotation	0.1878	0.2292	0.2292	0.2798	0.2798	0.3489
Neutral Axis	3.0500	29.2031	3.2361	28.8238	3.6014	27.9390
Top of Conc.	36.0000	36.0000	36.0000	36.0000	36.0000	36.0000
Centroid	15.0000	15.0000	15.0000	15.0000	15.0000	15.0000
Elongation	0.0392	0.0568	0.0471	0.0675	0.0557	0.0788
	0.0960		0.1146		0.1344	
	0.04%		0.04%		0.05%	
Total Beam					0.3450 in.	0.04%
Total					3.1008 in.	0.35%
Frame	6.3.7					
Location	Column A	Column B		Column C		Column D
	Right	Left	Right	Left	Right	Left
Time	2.0000	2.0000	2.0000	2.0000	2.0000	2.0000
Drift	0.6960	0.8273	0.8273	0.9893	0.9893	1.2240
Rotation	0.2641	0.3139	0.3139	0.3754	0.3754	0.4644
Neutral Axis	4.0365	29.4139	5.2503	29.9217	4.9550	27.5116
Top of Conc.	36.0000	36.0000	36.0000	36.0000	36.0000	36.0000
Centroid	15.0000	15.0000	15.0000	15.0000	15.0000	15.0000
Elongation	0.0505	0.0790	0.0534	0.0978	0.0658	0.1014
	0.1295		0.1512		0.1672	
	0.05%		0.06%		0.06%	
Total Beam					0.4479 in.	0.06%
Total					4.1845 in.	0.47%
Frame	6.3.5					
Location	Column A	Column B		Column C		Column D
	Right	Left	Right	Left	Right	Left
Time	2.0000	2.0000	2.0000	2.0000	2.0000	2.0000
Drift	0.6689	0.8167	0.8167	1.0004	1.0004	1.2477
Rotation	0.2538	0.3099	0.3099	0.3796	0.3796	0.4734
Neutral Axis	3.7255	29.0319	4.9571	28.6662	5.0104	27.5800
Top of Conc.	36.0000	36.0000	36.0000	36.0000	36.0000	36.0000
Centroid	15.0000	15.0000	15.0000	15.0000	15.0000	15.0000
Elongation	0.0499	0.0759	0.0543	0.0905	0.0662	0.1039
	0.1258		0.1449		0.1701	
	0.05%		0.06%		0.07%	
Total Beam					0.4408 in.	0.06%
Total					4.1745 in.	0.47%

Table 37: Elongation at two seconds for the two bay frames

Frame	7.2.7			
Location	Column A	Column B		Column C
	Right	Left	Right	Left
Time	2.0000	2.0000	2.0000	2.0000
Drift	0.5493	0.6592	0.6592	0.8230
Rotation	0.2084	0.2501	0.2501	0.3123
Neutral Axis	4.5159	29.7052	5.0389	28.2686
Top of Conc.	36.0000	36.0000	36.0000	36.0000
Centroid	15.0000	15.0000	15.0000	15.0000
Elongation	0.0381	0.0642	0.0435	0.0723
	0.1023		0.1158	
	0.04%		0.04%	
Total Beam	**0.2181 in.**			**0.04%**
Total	**2.2496 in.**			**0.37%**
Frame	7.2.5			
Location	Column A	Column B		Column C
	Right	Left	Right	Left
Time	2.0000	2.0000	2.0000	2.0000
Drift	0.5422	0.6617	0.6617	0.8323
Rotation	0.2057	0.2511	0.2511	0.3158
Neutral Axis	3.0885	29.2785	3.7267	28.2417
Top of Conc.	36.0000	36.0000	36.0000	36.0000
Centroid	15.0000	15.0000	15.0000	15.0000
Elongation	0.0428	0.0626	0.0494	0.0730
	0.1053		0.1224	
	0.04%		0.05%	
Total Beam	**0.2277 in.**			**0.04%**
Total	**2.2639 in.**			**0.37%**
Frame	6.2.7			
Location	Column A	Column B		Column C
	Right	Left	Right	Left
Time	2.0000	2.0000	2.0000	2.0000
Drift	0.7550	0.8963	0.8963	1.1251
Rotation	0.2865	0.3401	0.3401	0.4269
Neutral Axis	4.0052	29.9654	5.1971	27.9925
Top of Conc.	36.0000	36.0000	36.0000	36.0000
Centroid	15.0000	15.0000	15.0000	15.0000
Elongation	0.0550	0.0888	0.0582	0.0968
	0.1438		0.1550	
	0.06%		0.06%	
Total Beam	**0.2988 in.**			**0.06%**
Total	**3.0752 in.**			**0.51%**
Frame	6.2.5			
Location	Column A	Column B		Column C
	Right	Left	Right	Left
Time	2.0000	2.0000	2.0000	2.0000
Drift	0.7358	0.8978	0.8978	1.1336
Rotation	0.2792	0.3407	0.3407	0.4301
Neutral Axis	3.8408	29.1077	5.1718	27.9589
Top of Conc.	36.0000	36.0000	36.0000	36.0000
Centroid	15.0000	15.0000	15.0000	15.0000
Elongation	0.0544	0.0839	0.0584	0.0973
	0.1383		0.1557	
	0.05%		0.06%	
Total Beam	**0.2940 in.**			**0.06%**
Total	**3.0612 in.**			**0.51%**

Table 38: Elongation at two seconds for the one bay frames

Frame	7.1.7	
Location	Column A	Column B
	Right	Left
Time	2.0000	2.0000
Drift	0.5679	0.7187
Rotation	0.2155	0.2727
Neutral Axis	5.5483	28.5685
Top of Conc.	36.0000	36.0000
Centroid	15.0000	15.0000
Elongation	0.0355	0.0646
	0.1001	
	0.04%	
Total Beam	**0.1001 in.**	**0.04%**
Total	**1.3868 in.**	**0.44%**
Frame	7.1.5	
Location	Column A	Column B
	Right	Left
Time	2.0000	2.0000
Drift	0.5676	0.7159
Rotation	0.2154	0.2717
Neutral Axis	3.2704	28.7530
Top of Conc.	36.0000	36.0000
Centroid	15.0000	15.0000
Elongation	0.0441	0.0652
	0.1093	
	0.04%	
Total Beam	**0.1093 in.**	**0.04%**
Total	**1.3929 in.**	**0.44%**
Frame	6.1.7	
Location	Column A	Column B
	Right	Left
Time	2.0000	2.0000
Drift	0.7634	0.9795
Rotation	0.2896	0.3716
Neutral Axis	6.0829	28.3502
Top of Conc.	36.0000	36.0000
Centroid	15.0000	15.0000
Elongation	0.0451	0.0866
	0.1317	
	0.05%	
Total Beam	**0.1317 in.**	**0.05%**
Total	**1.8745 in.**	**0.59%**
Frame	6.1.5	
Location	Column A	Column B
	Right	Left
Time	2.0000	2.0000
Drift	0.7833	0.9868
Rotation	0.2972	0.3744
Neutral Axis	3.9020	28.3962
Top of Conc.	36.0000	36.0000
Centroid	15.0000	15.0000
Elongation	0.0576	0.0875
	0.1451	
	0.06%	
Total Beam	**0.1451 in.**	**0.06%**
Total	**1.9152 in.**	**0.60%**

Three Seconds

Table 39: Elongation at three seconds for the five bay frames

Frame	**7.5.7**									
Location	**Column A**	**Column B**		**Column C**		**Column D**		**Column E**		**Column F**
	Right	**Left**	**Right**	**Left**	**Right**	**Left**	**Right**	**Left**	**Right**	**Left**
Time	3.0000	3.0000	3.0000	3.0000	3.0000	3.0000	3.0000	3.0000	3.0000	3.0000
Drift (in.)	0.6748	0.8097	0.8097	0.9684	0.9684	1.1479	1.1479	1.3454	1.3454	1.6506
Rotation (°)	0.2560	0.3072	0.3072	0.3674	0.3674	0.4355	0.4355	0.5105	0.5105	0.6263
Neutral Axis	5.2213	29.0290	5.4760	29.0278	4.5640	28.9190	4.7602	29.6711	4.9835	27.3424
Top of Conc.	36.0000	36.0000	36.0000	36.0000	36.0000	36.0000	36.0000	36.0000	36.0000	36.0000
Centroid	15.0000	15.0000	15.0000	15.0000	15.0000	15.0000	15.0000	15.0000	15.0000	15.0000
Elongation	0.0437	0.0752	0.0511	0.0900	0.0669	0.1058	0.0778	0.1307	0.0892	0.1349
	0.1189		0.1410		0.1727		0.2086		0.2242	
	0.05%		0.05%		0.07%		0.08%		0.09%	
Total Beam									**0.8654 in.**	**0.07%**
Total									**7.4621 in.**	**0.51%**
Frame	**7.5.5**									
Location	**Column A**	**Column B**		**Column C**		**Column D**		**Column E**		**Column F**
	Right	**Left**	**Right**	**Left**	**Right**	**Left**	**Right**	**Left**	**Right**	**Left**
Time	3.0000	3.0000	3.0000	3.0000	3.0000	3.0000	3.0000	3.0000	3.0000	3.0000
Drift	0.8777	1.0815	1.0815	1.3293	1.3293	1.6062	1.6062	1.9254	1.9254	2.3466
Rotation	0.3330	0.4103	0.4103	0.5044	0.5044	0.6094	0.6094	0.7305	0.7305	0.8903
Neutral Axis	3.7448	29.0081	4.8687	28.3795	4.8702	27.9006	4.9985	27.6491	4.9528	26.1936
Top of Conc.	36.0000	36.0000	36.0000	36.0000	36.0000	36.0000	36.0000	36.0000	36.0000	36.0000
Centroid	15.0000	15.0000	15.0000	15.0000	15.0000	15.0000	15.0000	15.0000	15.0000	15.0000
Elongation	0.0654	0.1003	0.0726	0.1178	0.0892	0.1372	0.1064	0.1613	0.1281	0.1740
	0.1658		0.1903		0.2264		0.2677		0.3021	
	0.06%		0.07%		0.09%		0.10%		0.12%	
Total Beam									**1.1522 in.**	**0.09%**
Total									**10.3189 in.**	**0.70%**
Frame	**6.5.7**									
Location	**Column A**	**Column B**		**Column C**		**Column D**		**Column E**		**Column F**
	Right	**Left**	**Right**	**Left**	**Right**	**Left**	**Right**	**Left**	**Right**	**Left**
Time	3.0000	3.0000	3.0000	3.0000	3.0000	3.0000	3.0000	3.0000	3.0000	3.0000
Drift	0.9432	1.1419	1.1419	1.3656	1.3656	1.6087	1.6087	1.8862	1.8862	2.2736
Rotation	0.3579	0.4333	0.4333	0.5182	0.5182	0.6104	0.6104	0.7157	0.7157	0.8626
Neutral Axis	3.5440	29.0587	5.4442	28.6036	4.9907	28.5653	4.7854	29.6235	4.5457	25.7119
Top of Conc.	36.0000	36.0000	36.0000	36.0000	36.0000	36.0000	36.0000	36.0000	36.0000	36.0000
Centroid	15.0000	15.0000	15.0000	15.0000	15.0000	15.0000	15.0000	15.0000	15.0000	15.0000
Elongation	0.0716	0.1063	0.0723	0.1230	0.0905	0.1445	0.1088	0.1827	0.1306	0.1613
	0.1779		0.1953		0.2350		0.2915		0.2919	
	0.07%		0.08%		0.09%		0.11%		0.11%	
Total Beam									**1.1916 in.**	**0.09%**
Total									**10.4107 in.**	**0.71%**
Frame	**6.5.5**									
Location	**Column A**	**Column B**		**Column C**		**Column D**		**Column E**		**Column F**
	Right	**Left**	**Right**	**Left**	**Right**	**Left**	**Right**	**Left**	**Right**	**Left**
Time	3.0000	3.0000	3.0000	3.0000	3.0000	3.0000	3.0000	3.0000	3.0000	3.0000
Drift	0.8777	1.0539	1.0539	1.3293	1.3293	1.6062	1.6062	1.9254	1.9254	2.3466
Rotation	0.3330	0.3999	0.3999	0.5044	0.5044	0.6094	0.6094	0.7305	0.7305	0.8903
Neutral Axis	4.5385	28.4745	6.2855	27.7339	5.2750	27.3338	5.0836	27.3573	6.1140	25.6212
Top of Conc.	36.0000	36.0000	36.0000	36.0000	36.0000	36.0000	36.0000	36.0000	36.0000	36.0000
Centroid	15.0000	15.0000	15.0000	15.0000	15.0000	15.0000	15.0000	15.0000	15.0000	15.0000
Elongation	0.0608	0.0940	0.0608	0.1121	0.0856	0.1312	0.1055	0.1576	0.1133	0.1651
	0.1549		0.1729		0.2168		0.2630		0.2784	
	0.06%		0.07%		0.08%		0.10%		0.11%	
Total Beam									**1.0860 in.**	**0.08%**
Total									**10.2251 in.**	**0.70%**

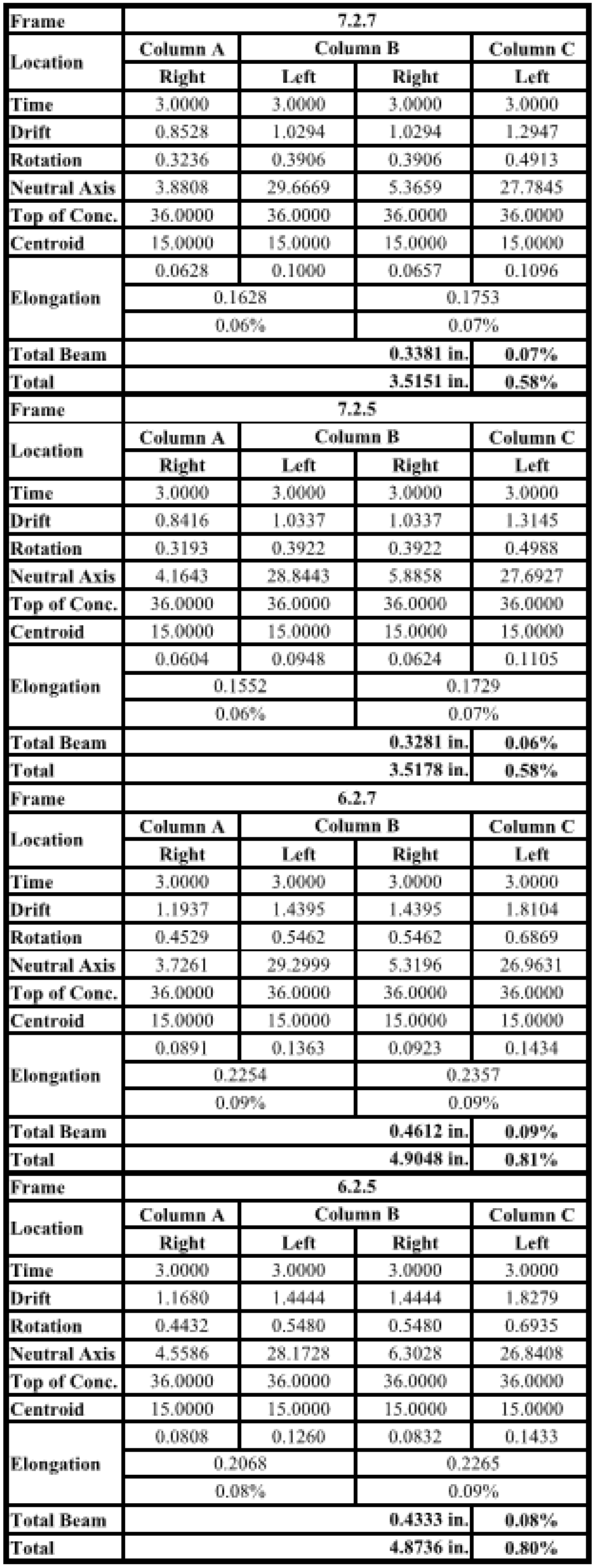
Table 40: Elongation at three seconds for the four bay frames

Frame	**7.4.7**							
Location	**Column A**	**Column B**		**Column C**		**Column D**		**Column E**
	Right	**Left**	**Right**	**Left**	**Right**	**Left**	**Right**	**Left**
Time	3.0000	3.0000	3.0000	3.0000	3.0000	3.0000	3.0000	3.0000
Drift	0.7260	0.8768	0.8768	1.0482	1.0482	1.2408	1.2408	1.5345
Rotation	0.2755	0.3327	0.3327	0.3977	0.3977	0.4708	0.4708	0.5822
Neutral Axis	5.5171	29.1200	5.4570	28.9637	5.1671	29.3835	5.1196	26.9490
Top of Conc.	36.0000	36.0000	36.0000	36.0000	36.0000	36.0000	36.0000	36.0000
Centroid	15.0000	15.0000	15.0000	15.0000	15.0000	15.0000	15.0000	15.0000
Elongation	0.0456	0.0820	0.0554	0.0969	0.0683	0.1182	0.0812	0.1214
	0.1276		0.1523		0.1865		0.2026	
	0.05%		0.06%		0.07%		0.08%	
Total Beam							**0.6690 in.**	**0.06%**
Total							**6.0953 in.**	**0.52%**
Frame	**7.4.5**							
Location	**Column A**	**Column B**		**Column C**		**Column D**		**Column E**
	Right	**Left**	**Right**	**Left**	**Right**	**Left**	**Right**	**Left**
Time	3.0000	3.0000	3.0000	3.0000	3.0000	3.0000	3.0000	3.0000
Drift	0.6996	0.8577	0.8577	1.0517	1.0517	1.2786	1.2786	1.5910
Rotation	0.2654	0.3254	0.3254	0.3991	0.3991	0.4851	0.4851	0.6037
Neutral Axis	3.8339	28.9704	5.3337	28.3595	5.3642	27.9901	5.2157	26.8884
Top of Conc.	36.0000	36.0000	36.0000	36.0000	36.0000	36.0000	36.0000	36.0000
Centroid	15.0000	15.0000	15.0000	15.0000	15.0000	15.0000	15.0000	15.0000
Elongation	0.0517	0.0794	0.0549	0.0930	0.0671	0.1100	0.0828	0.1253
	0.1311		0.1480		0.1771		0.2081	
	0.05%		0.06%		0.07%		0.08%	
Total Beam							**0.6642 in.**	**0.06%**
Total							**6.1427 in.**	**0.52%**
Frame	**6.4.7**							
Location	**Column A**	**Column B**		**Column C**		**Column D**		**Column E**
	Right	**Left**	**Right**	**Left**	**Right**	**Left**	**Right**	**Left**
Time	3.0000	3.0000	3.0000	3.0000	3.0000	3.0000	3.0000	3.0000
Drift	1.0094	1.2294	1.2294	1.4713	1.4713	1.7397	1.7397	2.1364
Rotation	0.3830	0.4665	0.4665	0.5583	0.5583	0.6601	0.6601	0.8106
Neutral Axis	3.3176	28.8010	5.2481	28.9237	5.1529	29.4706	4.8411	26.1605
Top of Conc.	36.0000	36.0000	36.0000	36.0000	36.0000	36.0000	36.0000	36.0000
Centroid	15.0000	15.0000	15.0000	15.0000	15.0000	15.0000	15.0000	15.0000
Elongation	0.0781	0.1124	0.0794	0.1357	0.0959	0.1667	0.1170	0.1579
	0.1905		0.2151		0.2627		0.2749	
	0.07%		0.08%		0.10%		0.11%	
Total Beam							**0.9431 in.**	**0.09%**
Total							**8.5294 in.**	**0.72%**
Frame	**6.4.5**							
Location	**Column A**	**Column B**		**Column C**		**Column D**		**Column E**
	Right	**Left**	**Right**	**Left**	**Right**	**Left**	**Right**	**Left**
Time	3.0000	3.0000	3.0000	3.0000	3.0000	3.0000	3.0000	3.0000
Drift	0.9762	1.2045	1.2045	1.4838	1.4838	1.8013	1.8013	2.2183
Rotation	0.3704	0.4570	0.4570	0.5630	0.5630	0.6835	0.6835	0.8416
Neutral Axis	4.4285	28.4916	6.2325	27.8411	5.7318	27.3788	4.9801	32.9203
Top of Conc.	36.0000	36.0000	36.0000	36.0000	36.0000	36.0000	36.0000	36.0000
Centroid	15.0000	15.0000	15.0000	15.0000	15.0000	15.0000	15.0000	15.0000
Elongation	0.0683	0.1076	0.0699	0.1262	0.0911	0.1477	0.1195	0.2633
	0.1760		0.1961		0.2387		0.3828	
	0.07%		0.08%		0.09%		0.15%	
Total Beam							**0.9936 in.**	**0.10%**
Total							**8.6777 in.**	**0.73%**

Table 41: Elongation at three seconds for the three bay frames

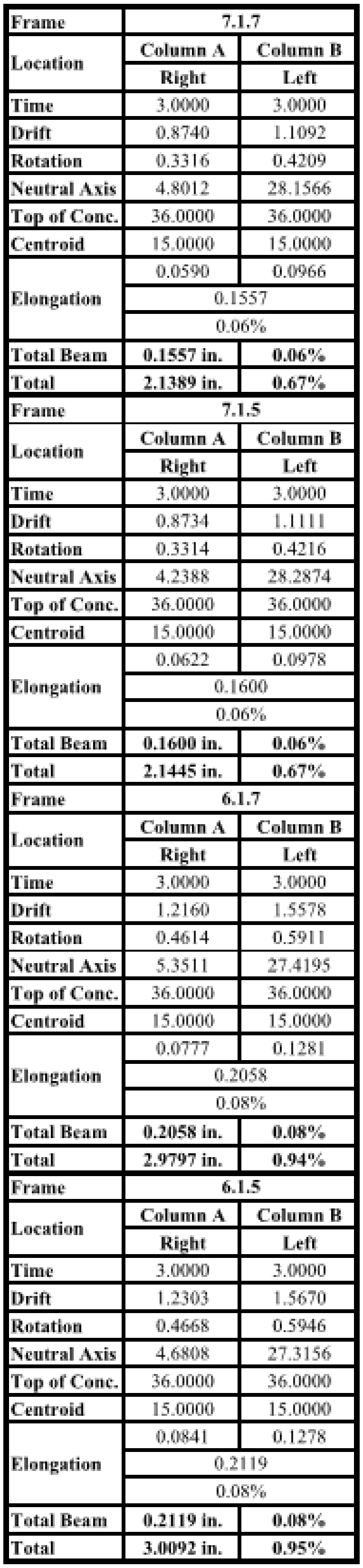
Frame	**7.3.7**					
Location	**Column A**	**Column B**		**Column C**		**Column D**
	Right	**Left**	**Right**	**Left**	**Right**	**Left**
Time	3.0000	3.0000	3.0000	3.0000	3.0000	3.0000
Drift	0.7913	0.9524	0.9524	1.1391	1.1391	1.4223
Rotation	0.3003	0.3614	0.3614	0.4322	0.4322	0.5397
Neutral Axis	3.8149	28.9787	5.6250	29.8837	5.2534	27.3338
Top of Conc.	36.0000	36.0000	36.0000	36.0000	36.0000	36.0000
Centroid	15.0000	15.0000	15.0000	15.0000	15.0000	15.0000
Elongation	0.0586	0.0882	0.0591	0.1123	0.0735	0.1162
	0.1468		0.1714		0.1897	
	0.06%		0.07%		0.07%	
Total Beam	**0.5079 in.**					**0.07%**
Total	**4.8130 in.**					**0.54%**
Frame	**7.3.5**					
Location	**Column A**	**Column B**		**Column C**		**Column D**
	Right	**Left**	**Right**	**Left**	**Right**	**Left**
Time	3.0000	3.0000	3.0000	3.0000	3.0000	3.0000
Drift	0.7720	0.9458	0.9458	1.1639	1.1639	1.4623
Rotation	0.2929	0.3589	0.3589	0.4416	0.4416	0.5548
Neutral Axis	4.0308	28.9184	5.6010	28.4277	5.5419	27.2096
Top of Conc.	36.0000	36.0000	36.0000	36.0000	36.0000	36.0000
Centroid	15.0000	15.0000	15.0000	15.0000	15.0000	15.0000
Elongation	0.0561	0.0872	0.0589	0.1035	0.0729	0.1182
	0.1433		0.1624		0.1911	
	0.06%		0.06%		0.07%	
Total Beam	**0.4968 in.**					**0.06%**
Total	**4.8408 in.**					**0.54%**
Frame	**6.3.7**					
Location	**Column A**	**Column B**		**Column C**		**Column D**
	Right	**Left**	**Right**	**Left**	**Right**	**Left**
Time	3.0000	3.0000	3.0000	3.0000	3.0000	3.0000
Drift	1.1023	1.3285	1.3285	1.6003	1.6003	1.9815
Rotation	0.4183	0.5041	0.5041	0.6072	0.6072	0.7518
Neutral Axis	3.7197	28.9297	5.5539	28.9761	5.0497	26.3370
Top of Conc.	36.0000	36.0000	36.0000	36.0000	36.0000	36.0000
Centroid	15.0000	15.0000	15.0000	15.0000	15.0000	15.0000
Elongation	0.0823	0.1226	0.0831	0.1481	0.1055	0.1488
	0.2049		0.2312		0.2542	
	0.08%		0.09%		0.10%	
Total Beam	**0.6904 in.**					**0.09%**
Total	**6.7031 in.**					**0.75%**
Frame	**6.3.5**					
Location	**Column A**	**Column B**		**Column C**		**Column D**
	Right	**Left**	**Right**	**Left**	**Right**	**Left**
Time	3.0000	3.0000	3.0000	3.0000	3.0000	3.0000
Drift	1.0681	1.3200	1.3200	1.6282	1.6282	2.0305
Rotation	0.4053	0.5009	0.5009	0.6178	0.6178	0.7704
Neutral Axis	4.5371	28.2867	6.2479	27.6107	5.7339	26.3313
Top of Conc.	36.0000	36.0000	36.0000	36.0000	36.0000	36.0000
Centroid	15.0000	15.0000	15.0000	15.0000	15.0000	15.0000
Elongation	0.0740	0.1161	0.0765	0.1360	0.0999	0.1524
	0.1902		0.2125		0.2523	
	0.07%		0.08%		0.10%	
Total Beam	**0.6549 in.**					**0.08%**
Total	**6.7017 in.**					**0.75%**

Table 42: Elongation at three seconds for the two bay frames



Frame	7.2.7			
Location	Column A	Column B		Column C
	Right	Left	Right	Left
Time	3.0000	3.0000	3.0000	3.0000
Drift	0.8528	1.0294	1.0294	1.2947
Rotation	0.3236	0.3906	0.3906	0.4913
Neutral Axis	3.8808	29.6669	5.3659	27.7845
Top of Conc.	36.0000	36.0000	36.0000	36.0000
Centroid	15.0000	15.0000	15.0000	15.0000
Elongation	0.0628	0.1000	0.0657	0.1096
	0.1628		0.1753	
	0.06%		0.07%	
Total Beam	**0.3381 in.**			**0.07%**
Total	**3.5151 in.**			**0.58%**
Frame	**7.2.5**			
Location	**Column A**	**Column B**		**Column C**
	Right	**Left**	**Right**	**Left**
Time	3.0000	3.0000	3.0000	3.0000
Drift	0.8416	1.0337	1.0337	1.3145
Rotation	0.3193	0.3922	0.3922	0.4988
Neutral Axis	4.1643	28.8443	5.8858	27.6927
Top of Conc.	36.0000	36.0000	36.0000	36.0000
Centroid	15.0000	15.0000	15.0000	15.0000
Elongation	0.0604	0.0948	0.0624	0.1105
	0.1552		0.1729	
	0.06%		0.07%	
Total Beam	**0.3281 in.**			**0.06%**
Total	**3.5178 in.**			**0.58%**
Frame	**6.2.7**			
Location	**Column A**	**Column B**		**Column C**
	Right	**Left**	**Right**	**Left**
Time	3.0000	3.0000	3.0000	3.0000
Drift	1.1937	1.4395	1.4395	1.8104
Rotation	0.4529	0.5462	0.5462	0.6869
Neutral Axis	3.7261	29.2999	5.3196	26.9631
Top of Conc.	36.0000	36.0000	36.0000	36.0000
Centroid	15.0000	15.0000	15.0000	15.0000
Elongation	0.0891	0.1363	0.0923	0.1434
	0.2254		0.2357	
	0.09%		0.09%	
Total Beam	**0.4612 in.**			**0.09%**
Total	**4.9048 in.**			**0.81%**
Frame	**6.2.5**			
Location	**Column A**	**Column B**		**Column C**
	Right	**Left**	**Right**	**Left**
Time	3.0000	3.0000	3.0000	3.0000
Drift	1.1680	1.4444	1.4444	1.8279
Rotation	0.4432	0.5480	0.5480	0.6935
Neutral Axis	4.5586	28.1728	6.3028	26.8408
Top of Conc.	36.0000	36.0000	36.0000	36.0000
Centroid	15.0000	15.0000	15.0000	15.0000
Elongation	0.0808	0.1260	0.0832	0.1433
	0.2068		0.2265	
	0.08%		0.09%	
Total Beam	**0.4333 in.**			**0.08%**
Total	**4.8736 in.**			**0.80%**

Table 43: Elongation at three seconds for the one bav frames



Frame	7.1.7	
Location	Column A	Column B
	Right	Left
Time	3.0000	3.0000
Drift	0.8740	1.1092
Rotation	0.3316	0.4209
Neutral Axis	4.8012	28.1566
Top of Conc.	36.0000	36.0000
Centroid	15.0000	15.0000
Elongation	0.0590	0.0966
	0.1557	
	0.06%	
Total Beam	**0.1557 in.**	**0.06%**
Total	**2.1389 in.**	**0.67%**
Frame	**7.1.5**	
Location	**Column A**	**Column B**
	Right	**Left**
Time	3.0000	3.0000
Drift	0.8734	1.1111
Rotation	0.3314	0.4216
Neutral Axis	4.2388	28.2874
Top of Conc.	36.0000	36.0000
Centroid	15.0000	15.0000
Elongation	0.0622	0.0978
	0.1600	
	0.06%	
Total Beam	**0.1600 in.**	**0.06%**
Total	**2.1445 in.**	**0.67%**
Frame	**6.1.7**	
Location	**Column A**	**Column B**
	Right	**Left**
Time	3.0000	3.0000
Drift	1.2160	1.5578
Rotation	0.4614	0.5911
Neutral Axis	5.3511	27.4195
Top of Conc.	36.0000	36.0000
Centroid	15.0000	15.0000
Elongation	0.0777	0.1281
	0.2058	
	0.08%	
Total Beam	**0.2058 in.**	**0.08%**
Total	**2.9797 in.**	**0.94%**
Frame	**6.1.5**	
Location	**Column A**	**Column B**
	Right	**Left**
Time	3.0000	3.0000
Drift	1.2303	1.5670
Rotation	0.4668	0.5946
Neutral Axis	4.6808	27.3156
Top of Conc.	36.0000	36.0000
Centroid	15.0000	15.0000
Elongation	0.0841	0.1278
	0.2119	
	0.08%	
Total Beam	**0.2119 in.**	**0.08%**
Total	**3.0092 in.**	**0.95%**

Four Seconds

Table 44: Elongation at four seconds for the five bay frames

Frame	**7.5.7**									
Location	**Column A**	**Column B**		**Column C**		**Column D**		**Column E**		**Column F**
	Right	**Left**	**Right**	**Left**	**Right**	**Left**	**Right**	**Left**	**Right**	**Left**
Time	4.0000	4.0000	4.0000	4.0000	4.0000	4.0000	4.0000	4.0000	4.0000	4.0000
Drift (in.)	0.9266	1.1225	1.1225	1.3539	1.3539	1.6141	1.6141	1.9109	1.9109	2.3281
Rotation (°)	0.3516	0.4259	0.4259	0.5137	0.5137	0.6124	0.6124	0.7250	0.7250	0.8833
Neutral Axis	4.9190	28.8287	6.3061	28.1607	5.0429	28.4172	4.8621	28.5871	4.7391	26.1206
Top of Conc.	36.0000	36.0000	36.0000	36.0000	36.0000	36.0000	36.0000	36.0000	36.0000	36.0000
Centroid	15.0000	15.0000	15.0000	15.0000	15.0000	15.0000	15.0000	15.0000	15.0000	15.0000
Elongation	0.0619	0.1028	0.0646	0.1180	0.0893	0.1434	0.1084	0.1719	0.1298	0.1715
	0.1647		0.1826		0.2327		0.2803		0.3013	
	0.06%		0.07%		0.09%		0.11%		0.12%	
Total Beam	**1.1616 in.**									**0.09%**
Total	**10.4177 in.**									**0.71%**
Frame	**7.5.5**									
Location	**Column A**	**Column B**		**Column C**		**Column D**		**Column E**		**Column F**
	Right	**Left**	**Right**	**Left**	**Right**	**Left**	**Right**	**Left**	**Right**	**Left**
Time	4.0000	4.0000	4.0000	4.0000	4.0000	4.0000	4.0000	4.0000	4.0000	4.0000
Drift	1.2523	1.5621	1.5621	1.9300	1.9300	2.3392	2.3392	2.8078	2.8078	3.3981
Rotation	0.4752	0.5927	0.5927	0.7323	0.7323	0.8875	0.8875	1.0653	1.0653	1.2892
Neutral Axis	4.3830	28.5422	6.1801	27.7194	5.7536	27.1147	5.2813	26.7974	5.0125	25.1482
Top of Conc.	36.0000	36.0000	36.0000	36.0000	36.0000	36.0000	36.0000	36.0000	36.0000	36.0000
Centroid	15.0000	15.0000	15.0000	15.0000	15.0000	15.0000	15.0000	15.0000	15.0000	15.0000
Elongation	0.0881	0.1401	0.0912	0.1626	0.1182	0.1877	0.1506	0.2194	0.1857	0.2284
	0.2281		0.2538		0.3059		0.3699		0.4141	
	0.09%		0.10%		0.12%		0.14%		0.16%	
Total Beam	**1.5718 in.**									**0.12%**
Total	**14.8612 in.**									**1.01%**
Frame	**6.5.7**									
Location	**Column A**	**Column B**		**Column C**		**Column D**		**Column E**		**Column F**
	Right	**Left**	**Right**	**Left**	**Right**	**Left**	**Right**	**Left**	**Right**	**Left**
Time	4.0000	4.0000	4.0000	4.0000	4.0000	4.0000	4.0000	4.0000	4.0000	4.0000
Drift	1.3218	1.6154	1.6154	1.9477	1.9477	2.3071	2.3071	2.7155	2.7155	3.2387
Rotation	0.5016	0.6129	0.6129	0.7390	0.7390	0.8753	0.8753	1.0302	1.0302	1.2287
Neutral Axis	3.4057	28.1501	5.3846	27.6111	4.3207	27.3252	3.9525	28.5464	3.8345	24.3761
Top of Conc.	36.0000	36.0000	36.0000	36.0000	36.0000	36.0000	36.0000	36.0000	36.0000	36.0000
Centroid	15.0000	15.0000	15.0000	15.0000	15.0000	15.0000	15.0000	15.0000	15.0000	15.0000
Elongation	0.1015	0.1407	0.1029	0.1627	0.1377	0.1883	0.1688	0.2436	0.2008	0.2011
	0.2422		0.2655		0.3261		0.4124		0.4019	
	0.09%		0.10%		0.13%		0.16%		0.16%	
Total Beam	**1.6481 in.**									**0.13%**
Total	**14.7943 in.**									**1.01%**
Frame	**6.5.5**									
Location	**Column A**	**Column B**		**Column C**		**Column D**		**Column E**		**Column F**
	Right	**Left**	**Right**	**Left**	**Right**	**Left**	**Right**	**Left**	**Right**	**Left**
Time	4.0000	4.0000	4.0000	4.0000	4.0000	4.0000	4.0000	4.0000	4.0000	4.0000
Drift	1.2523	1.5225	1.5225	1.9300	1.9300	2.3392	2.3392	2.8078	2.8078	3.3981
Rotation	0.4752	0.5777	0.5777	0.7323	0.7323	0.8875	0.8875	1.0653	1.0653	1.2892
Neutral Axis	4.9655	27.6857	6.6575	26.8712	5.0929	26.2553	4.2319	26.2218	3.8451	24.2212
Top of Conc.	36.0000	36.0000	36.0000	36.0000	36.0000	36.0000	36.0000	36.0000	36.0000	36.0000
Centroid	15.0000	15.0000	15.0000	15.0000	15.0000	15.0000	15.0000	15.0000	15.0000	15.0000
Elongation	0.0832	0.1279	0.0841	0.1517	0.1266	0.1744	0.1668	0.2087	0.2074	0.2075
	0.2111		0.2358		0.3010		0.3755		0.4149	
	0.08%		0.09%		0.12%		0.15%		0.16%	
Total Beam	**1.5384 in.**									**0.12%**
Total	**14.7882 in.**									**1.01%**

Table 45: Elongation at four seconds for the four bay frames

Frame	7.4.7							
Location	Column A	Column B		Column C		Column D		Column E
	Right	Left	Right	Left	Right	Left	Right	Left
Time	4.0000	4.0000	4.0000	4.0000	4.0000	4.0000	4.0000	4.0000
Drift	1.0000	1.2190	1.2190	1.4707	1.4707	1.7547	1.7547	2.1706
Rotation	0.3794	0.4625	0.4625	0.5580	0.5580	0.6658	0.6658	0.8236
Neutral Axis	5.2484	28.7604	5.8676	28.2319	5.2375	28.3330	4.9267	26.0070
Top of Conc.	36.0000	36.0000	36.0000	36.0000	36.0000	36.0000	36.0000	36.0000
Centroid	15.0000	15.0000	15.0000	15.0000	15.0000	15.0000	15.0000	15.0000
Elongation	0.0646	0.1111	0.0737	0.1289	0.0951	0.1549	0.1171	0.1582
	0.1757		0.2026		0.2500		0.2753	
	0.07%		0.08%		0.10%		0.11%	
Total Beam							**0.9036 in.**	**0.09%**
Total							**8.5186 in.**	**0.72%**
Frame	**7.4.5**							
Location	**Column A**	**Column B**		**Column C**		**Column D**		**Column E**
	Right	**Left**	**Right**	**Left**	**Right**	**Left**	**Right**	**Left**
Time	4.0000	4.0000	4.0000	4.0000	4.0000	4.0000	4.0000	4.0000
Drift	0.9660	1.1958	1.1958	1.4803	1.4803	1.8115	1.8115	2.2526
Rotation	0.3665	0.4537	0.4537	0.5617	0.5617	0.6873	0.6873	0.8547
Neutral Axis	4.5083	28.3984	6.4944	27.5610	5.9644	27.1122	5.4072	25.8560
Top of Conc.	36.0000	36.0000	36.0000	36.0000	36.0000	36.0000	36.0000	36.0000
Centroid	15.0000	15.0000	15.0000	15.0000	15.0000	15.0000	15.0000	15.0000
Elongation	0.0671	0.1061	0.0674	0.1231	0.0886	0.1453	0.1151	0.1620
	0.1732		0.1905		0.2339		0.2770	
	0.07%		0.07%		0.09%		0.11%	
Total Beam							**0.8746 in.**	**0.08%**
Total							**8.5808 in.**	**0.73%**
Frame	**6.4.7**							
Location	**Column A**	**Column B**		**Column C**		**Column D**		**Column E**
	Right	**Left**	**Right**	**Left**	**Right**	**Left**	**Right**	**Left**
Time	4.0000	4.0000	4.0000	4.0000	4.0000	4.0000	4.0000	4.0000
Drift	1.4182	1.7404	1.7404	2.0977	2.0977	2.4930	2.4930	3.0439
Rotation	0.5381	0.6604	0.6604	0.7959	0.7959	0.9459	0.9459	1.1548
Neutral Axis	3.1775	27.9214	5.1461	27.6435	4.5435	28.2446	4.0693	24.6716
Top of Conc.	36.0000	36.0000	36.0000	36.0000	36.0000	36.0000	36.0000	36.0000
Centroid	15.0000	15.0000	15.0000	15.0000	15.0000	15.0000	15.0000	15.0000
Elongation	0.1110	0.1489	0.1136	0.1756	0.1453	0.2187	0.1805	0.1950
	0.2600		0.2892		0.3639		0.3754	
	0.10%		0.11%		0.14%		0.15%	
Total Beam							**1.2885 in.**	**0.12%**
Total							**12.0817 in.**	**1.02%**
Frame	**6.4.5**							
Location	**Column A**	**Column B**		**Column C**		**Column D**		**Column E**
	Right	**Left**	**Right**	**Left**	**Right**	**Left**	**Right**	**Left**
Time	4.0000	4.0000	4.0000	4.0000	4.0000	4.0000	4.0000	4.0000
Drift	1.3762	1.7174	1.7174	2.1207	2.1207	2.5777	2.5777	3.1593
Rotation	0.5222	0.6516	0.6516	0.8046	0.8046	0.9780	0.9780	1.1986
Neutral Axis	4.9174	27.6783	6.2739	26.8959	5.3037	26.2304	4.4861	32.9294
Top of Conc.	36.0000	36.0000	36.0000	36.0000	36.0000	36.0000	36.0000	36.0000
Centroid	15.0000	15.0000	15.0000	15.0000	15.0000	15.0000	15.0000	15.0000
Elongation	0.0919	0.1442	0.0992	0.1671	0.1362	0.1917	0.1795	0.3751
	0.2361		0.2663		0.3279		0.5546	
	0.09%		0.10%		0.13%		0.21%	
Total Beam							**1.3849 in.**	**0.13%**
Total							**12.3362 in.**	**1.04%**

Table 46: Elongation at four seconds for the three bay frames

Frame	**7.3.7**					
Location	**Column A**	**Column B**		**Column C**		**Column D**
	Right	**Left**	**Right**	**Left**	**Right**	**Left**
Time	4.0000	4.0000	4.0000	4.0000	4.0000	4.0000
Drift	1.0884	1.3249	1.3249	1.6015	1.6015	2.0005
Rotation	0.4130	0.5027	0.5027	0.6077	0.6077	0.7590
Neutral Axis	3.7424	28.8153	6.1203	28.5569	5.2339	26.3199
Top of Conc.	36.0000	36.0000	36.0000	36.0000	36.0000	36.0000
Centroid	15.0000	15.0000	15.0000	15.0000	15.0000	15.0000
Elongation	0.0811	0.1212	0.0779	0.1438	0.1036	0.1500
	0.2024		0.2217		0.2536	
	0.08%		0.09%		0.10%	
Total Beam					**0.6776 in.**	**0.09%**
Total					**6.6931 in.**	**0.75%**
Frame	**7.3.5**					
Location	**Column A**	**Column B**		**Column C**		**Column D**
	Right	**Left**	**Right**	**Left**	**Right**	**Left**
Time	4.0000	4.0000	4.0000	4.0000	4.0000	4.0000
Drift	1.0661	1.3208	1.3208	1.6376	1.6376	2.0574
Rotation	0.4045	0.5012	0.5012	0.6213	0.6213	0.7806
Neutral Axis	4.6286	28.2248	6.5186	27.6580	5.9153	26.2153
Top of Conc.	36.0000	36.0000	36.0000	36.0000	36.0000	36.0000
Centroid	15.0000	15.0000	15.0000	15.0000	15.0000	15.0000
Elongation	0.0732	0.1157	0.0742	0.1373	0.0985	0.1528
	0.1889		0.2115		0.2513	
	0.07%		0.08%		0.10%	
Total Beam					**0.6517 in.**	**0.08%**
Total					**6.7336 in.**	**0.75%**
Frame	**6.3.7**					
Location	**Column A**	**Column B**		**Column C**		**Column D**
	Right	**Left**	**Right**	**Left**	**Right**	**Left**
Time	4.0000	4.0000	4.0000	4.0000	4.0000	4.0000
Drift	1.5539	1.8943	1.8943	2.2887	2.2887	2.8231
Rotation	0.5896	0.7187	0.7187	0.8684	0.8684	1.0711
Neutral Axis	3.6553	27.8473	5.2626	27.7807	4.5547	24.7672
Top of Conc.	36.0000	36.0000	36.0000	36.0000	36.0000	36.0000
Centroid	15.0000	15.0000	15.0000	15.0000	15.0000	15.0000
Elongation	0.1167	0.1612	0.1222	0.1937	0.1583	0.1826
	0.2779		0.3159		0.3409	
	0.11%		0.12%		0.13%	
Total Beam					**0.9347 in.**	**0.12%**
Total					**9.4947 in.**	**1.06%**
Frame	**6.3.5**					
Location	**Column A**	**Column B**		**Column C**		**Column D**
	Right	**Left**	**Right**	**Left**	**Right**	**Left**
Time	4.0000	4.0000	4.0000	4.0000	4.0000	4.0000
Drift	1.5163	1.8903	1.8903	2.3335	2.3335	2.8958
Rotation	0.5753	0.7172	0.7172	0.8854	0.8854	1.0986
Neutral Axis	4.8830	27.3448	6.0730	26.4299	5.3156	24.8094
Top of Conc.	36.0000	36.0000	36.0000	36.0000	36.0000	36.0000
Centroid	15.0000	15.0000	15.0000	15.0000	15.0000	15.0000
Elongation	0.1016	0.1545	0.1118	0.1766	0.1497	0.1881
	0.2561		0.2884		0.3378	
	0.10%		0.11%		0.13%	
Total Beam					**0.8823 in.**	**0.11%**
Total					**9.5181 in.**	**1.06%**

Table 47: Elongation at four seconds for the two bay frames

Frame	7.2.7			
Location	Column A	Column B		Column C
	Right	Left	Right	Left
Time	4.0000	4.0000	4.0000	4.0000
Drift	1.1762	1.4345	1.4345	1.8094
Rotation	0.4463	0.5443	0.5443	0.6865
Neutral Axis	3.6448	29.0344	5.5174	26.8474
Top of Conc.	36.0000	36.0000	36.0000	36.0000
Centroid	15.0000	15.0000	15.0000	15.0000
Elongation	0.0884	0.1333	0.0901	0.1420
	0.2218		0.2320	
	0.09%		0.09%	
Total Beam	0.4538 in.			0.09%
Total	4.8738 in.			0.80%
Frame	7.2.5			
Location	Column A	Column B		Column C
	Right	Left	Right	Left
Time	4.0000	4.0000	4.0000	4.0000
Drift	1.1571	1.4390	1.4390	1.8327
Rotation	0.4390	0.5460	0.5460	0.6954
Neutral Axis	4.6524	28.0130	6.5892	26.8256
Top of Conc.	36.0000	36.0000	36.0000	36.0000
Centroid	15.0000	15.0000	15.0000	15.0000
Elongation	0.0793	0.1240	0.0802	0.1435
	0.2033		0.2237	
	0.08%		0.09%	
Total Beam	0.4270 in.			0.08%
Total	4.8558 in.			0.80%
Frame	6.2.7			
Location	Column A	Column B		Column C
	Right	Left	Right	Left
Time	4.0000	4.0000	4.0000	4.0000
Drift	1.6880	2.0556	2.0556	2.5729
Rotation	0.6405	0.7799	0.7799	0.9762
Neutral Axis	3.6804	28.1262	5.0001	25.6889
Top of Conc.	36.0000	36.0000	36.0000	36.0000
Centroid	15.0000	15.0000	15.0000	15.0000
Elongation	0.1265	0.1787	0.1361	0.1821
	0.3052		0.3183	
	0.12%		0.12%	
Total Beam	0.6235 in.			0.12%
Total	6.9400 in.			1.15%
Frame	6.2.5			
Location	Column A	Column B		Column C
	Right	Left	Right	Left
Time	4.0000	4.0000	4.0000	4.0000
Drift	1.6586	2.0650	2.0650	2.6004
Rotation	0.6293	0.7835	0.7835	0.9866
Neutral Axis	4.7112	27.1001	6.0846	25.5354
Top of Conc.	36.0000	36.0000	36.0000	36.0000
Centroid	15.0000	15.0000	15.0000	15.0000
Elongation	0.1130	0.1655	0.1219	0.1814
	0.2785		0.3034	
	0.11%		0.12%	
Total Beam	0.5818 in.			0.11%
Total	6.9058 in.			1.14%

Table 48: Elongation at four seconds for the one bay frames

Frame	7.1.7	
Location	Column A	Column B
	Right	Left
Time	4.0000	4.0000
Drift	1.1999	1.5327
Rotation	0.4553	0.5816
Neutral Axis	4.5594	27.3065
Top of Conc.	36.0000	36.0000
Centroid	15.0000	15.0000
Elongation	0.0830	0.1249
	0.2079	
	0.08%	
Total Beam	0.2079 in.	0.08%
Total	2.9406 in.	0.92%
Frame	7.1.5	
Location	Column A	Column B
	Right	Left
Time	4.0000	4.0000
Drift	1.1928	1.5312
Rotation	0.4526	0.5810
Neutral Axis	4.6103	27.4539
Top of Conc.	36.0000	36.0000
Centroid	15.0000	15.0000
Elongation	0.0821	0.1263
	0.2084	
	0.08%	
Total Beam	0.2084 in.	0.08%
Total	2.9324 in.	0.92%
Frame	6.1.7	
Location	Column A	Column B
	Right	Left
Time	4.0000	4.0000
Drift	1.7304	2.2094
Rotation	0.6566	0.8383
Neutral Axis	5.0430	26.1761
Top of Conc.	36.0000	36.0000
Centroid	15.0000	15.0000
Elongation	0.1141	0.1635
	0.2776	
	0.11%	
Total Beam	0.2776 in.	0.11%
Total	4.2175 in.	1.33%
Frame	6.1.5	
Location	Column A	Column B
	Right	Left
Time	4.0000	4.0000
Drift	1.7369	2.2145
Rotation	0.6590	0.8402
Neutral Axis	4.8711	26.1420
Top of Conc.	36.0000	36.0000
Centroid	15.0000	15.0000
Elongation	0.1165	0.1634
	0.2799	
	0.11%	
Total Beam	0.2799 in.	0.11%
Total	4.2312 in.	1.33%

Five Seconds

Table 49: Elongation at five seconds for the five bay frames

Frame	**7.5.7**									
Location	**Column A**	**Column B**		**Column C**		**Column D**		**Column E**		**Column F**
	Right	**Left**	**Right**	**Left**	**Right**	**Left**	**Right**	**Left**	**Right**	**Left**
Time	5.0000	5.0000	5.0000	5.0000	5.0000	5.0000	5.0000	5.0000	5.0000	5.0000
Drift (in.)	1.1982	1.4657	1.4657	1.7787	1.7787	2.1298	2.1298	2.5310	2.5310	3.0623
Rotation (°)	0.4546	0.5561	0.5561	0.6749	0.6749	0.8081	0.8081	0.9603	0.9603	1.1618
Neutral Axis	4.9570	28.1789	6.0972	27.4937	4.8218	27.4760	4.3338	27.8139	4.1538	24.8182
Top of Conc.	36.0000	36.0000	36.0000	36.0000	36.0000	36.0000	36.0000	36.0000	36.0000	36.0000
Centroid	15.0000	15.0000	15.0000	15.0000	15.0000	15.0000	15.0000	15.0000	15.0000	15.0000
Elongation	0.0797	0.1279	0.0864	0.1472	0.1199	0.1760	0.1504	0.2148	0.1818	0.1991
	0.2076		0.2336		0.2959		0.3652		0.3809	
	0.08%		0.09%		0.11%		0.14%		0.15%	
Total Beam	**1.4832 in.**									**0.11%**
Total	**13.6489 in.**									**0.93%**
Frame	**7.5.5**									
Location	**Column A**	**Column B**		**Column C**		**Column D**		**Column E**		**Column F**
	Right	**Left**	**Right**	**Left**	**Right**	**Left**	**Right**	**Left**	**Right**	**Left**
Time	5.0000	5.0000	5.0000	5.0000	5.0000	5.0000	5.0000	5.0000	5.0000	5.0000
Drift	1.6464	2.0638	2.0638	2.5481	2.5481	3.0861	3.0861	3.6910	3.6910	4.4322
Rotation	0.6247	0.7830	0.7830	0.9668	0.9668	1.1708	1.1708	1.4002	1.4002	1.6813
Neutral Axis	4.7817	27.8978	6.5480	27.0943	5.6938	26.4238	4.7820	25.9124	4.3336	23.8178
Top of Conc.	36.0000	36.0000	36.0000	36.0000	36.0000	36.0000	36.0000	36.0000	36.0000	36.0000
Centroid	15.0000	15.0000	15.0000	15.0000	15.0000	15.0000	15.0000	15.0000	15.0000	15.0000
Elongation	0.1114	0.1763	0.1155	0.2041	0.1570	0.2335	0.2088	0.2667	0.2607	0.2588
	0.2877		0.3196		0.3905		0.4756		0.5195	
	0.11%		0.12%		0.15%		0.18%		0.20%	
Total Beam	**1.9929 in.**									**0.15%**
Total	**19.4604 in.**									**1.32%**
Frame	**6.5.7**									
Location	**Column A**	**Column B**		**Column C**		**Column D**		**Column E**		**Column F**
	Right	**Left**	**Right**	**Left**	**Right**	**Left**	**Right**	**Left**	**Right**	**Left**
Time	5.0000	5.0000	5.0000	5.0000	5.0000	5.0000	5.0000	5.0000	5.0000	5.0000
Drift	1.7269	2.1202	2.1202	2.5617	2.5617	3.0378	3.0378	3.5769	3.5769	4.2310
Rotation	0.6552	0.8044	0.8044	0.9719	0.9719	1.1525	1.1525	1.3570	1.3570	1.6050
Neutral Axis	3.3287	27.2064	4.7115	26.7763	3.5981	26.1397	3.3406	27.3267	3.3281	22.9042
Top of Conc.	36.0000	36.0000	36.0000	36.0000	36.0000	36.0000	36.0000	36.0000	36.0000	36.0000
Centroid	15.0000	15.0000	15.0000	15.0000	15.0000	15.0000	15.0000	15.0000	15.0000	15.0000
Elongation	0.1335	0.1714	0.1445	0.1998	0.1934	0.2241	0.2346	0.2920	0.2765	0.2215
	0.3049		0.3442		0.4175		0.5266		0.4980	
	0.12%		0.13%		0.16%		0.20%		0.19%	
Total Beam	**2.0912 in.**									**0.16%**
Total	**19.3457 in.**									**1.32%**
Frame	**6.5.5**									
Location	**Column A**	**Column B**		**Column C**		**Column D**		**Column E**		**Column F**
	Right	**Left**	**Right**	**Left**	**Right**	**Left**	**Right**	**Left**	**Right**	**Left**
Time	5.0000	5.0000	5.0000	5.0000	5.0000	5.0000	5.0000	5.0000	5.0000	5.0000
Drift	1.6464	2.0092	2.0092	2.5481	2.5481	3.0861	3.0861	3.6910	3.6910	4.4322
Rotation	0.6247	0.7623	0.7623	0.9668	0.9668	1.1708	1.1708	1.4002	1.4002	1.6813
Neutral Axis	4.9737	27.0324	6.3866	26.0848	3.9993	25.1793	3.1584	25.1785	3.1909	22.7318
Top of Conc.	36.0000	36.0000	36.0000	36.0000	36.0000	36.0000	36.0000	36.0000	36.0000	36.0000
Centroid	15.0000	15.0000	15.0000	15.0000	15.0000	15.0000	15.0000	15.0000	15.0000	15.0000
Elongation	0.1093	0.1601	0.1146	0.1871	0.1856	0.2080	0.2420	0.2488	0.2887	0.2269
	0.2694		0.3017		0.3937		0.4908		0.5156	
	0.10%		0.12%		0.15%		0.19%		0.20%	
Total Beam	**1.9712 in.**									**0.15%**
Total	**19.3841 in.**									**1.32%**

Table 50: Elongation at five seconds for the four bay frames

Frame	**7.4.7**							
Location	**Column A**	**Column B**		**Column C**		**Column D**		**Column E**
	Right	**Left**	**Right**	**Left**	**Right**	**Left**	**Right**	**Left**
Time	5.0000	5.0000	5.0000	5.0000	5.0000	5.0000	5.0000	5.0000
Drift	1.2990	1.5982	1.5982	1.9384	1.9384	2.3230	2.3230	2.8639
Rotation	0.4929	0.6064	0.6064	0.7355	0.7355	0.8814	0.8814	1.0865
Neutral Axis	5.1581	28.0470	5.7265	27.3691	4.8340	27.4507	4.3659	24.7466
Top of Conc.	36.0000	36.0000	36.0000	36.0000	36.0000	36.0000	36.0000	36.0000
Centroid	15.0000	15.0000	15.0000	15.0000	15.0000	15.0000	15.0000	15.0000
Elongation	0.0847	0.1381	0.0982	0.1588	0.1305	0.1915	0.1636	0.1849
	0.2228		0.2569		0.3220		0.3484	
	0.09%		0.10%		0.12%		0.14%	
Total Beam	**1.1502 in.**							**0.11%**
Total	**11.1726 in.**							**0.95%**
Frame	**7.4.5**							
Location	**Column A**	**Column B**		**Column C**		**Column D**		**Column E**
	Right	**Left**	**Right**	**Left**	**Right**	**Left**	**Right**	**Left**
Time	5.0000	5.0000	5.0000	5.0000	5.0000	5.0000	5.0000	5.0000
Drift	1.2604	1.5774	1.5774	1.9572	1.9572	2.3985	2.3985	2.9741
Rotation	0.4782	0.5985	0.5985	0.7426	0.7426	0.9100	0.9100	1.1284
Neutral Axis	4.8140	27.7207	6.5789	26.8824	5.7082	26.2553	5.0514	24.6635
Top of Conc.	36.0000	36.0000	36.0000	36.0000	36.0000	36.0000	36.0000	36.0000
Centroid	15.0000	15.0000	15.0000	15.0000	15.0000	15.0000	15.0000	15.0000
Elongation	0.0850	0.1329	0.0880	0.1540	0.1204	0.1788	0.1580	0.1903
	0.2179		0.2420		0.2992		0.3484	
	0.08%		0.09%		0.12%		0.14%	
Total Beam	**1.1075 in.**							**0.11%**
Total	**11.2751 in.**							**0.95%**
Frame	**6.4.7**							
Location	**Column A**	**Column B**		**Column C**		**Column D**		**Column E**
	Right	**Left**	**Right**	**Left**	**Right**	**Left**	**Right**	**Left**
Time	5.0000	5.0000	5.0000	5.0000	5.0000	5.0000	5.0000	5.0000
Drift	1.8659	2.2945	2.2945	2.7678	2.7678	3.2902	3.2902	3.9935
Rotation	0.7080	0.8706	0.8706	1.0501	1.0501	1.2482	1.2482	1.5149
Neutral Axis	3.0772	26.9958	4.5352	26.3953	3.7556	26.9762	3.4539	23.1407
Top of Conc.	36.0000	36.0000	36.0000	36.0000	36.0000	36.0000	36.0000	36.0000
Centroid	15.0000	15.0000	15.0000	15.0000	15.0000	15.0000	15.0000	15.0000
Elongation	0.1473	0.1823	0.1590	0.2089	0.2061	0.2610	0.2516	0.2153
	0.3296		0.3679		0.4671		0.4669	
	0.13%		0.14%		0.18%		0.18%	
Total Beam	**1.6314 in.**							**0.16%**
Total	**15.8433 in.**							**1.34%**
Frame	**6.4.5**							
Location	**Column A**	**Column B**		**Column C**		**Column D**		**Column E**
	Right	**Left**	**Right**	**Left**	**Right**	**Left**	**Right**	**Left**
Time	5.0000	5.0000	5.0000	5.0000	5.0000	5.0000	5.0000	5.0000
Drift	1.8118	2.2660	2.2660	2.7955	2.7955	3.3926	3.3926	4.1333
Rotation	0.6875	0.8598	0.8598	1.0606	1.0606	1.2871	1.2871	1.5680
Neutral Axis	4.9592	26.8861	5.9129	25.8708	4.2181	24.9683	3.6414	32.9338
Top of Conc.	36.0000	36.0000	36.0000	36.0000	36.0000	36.0000	36.0000	36.0000
Centroid	15.0000	15.0000	15.0000	15.0000	15.0000	15.0000	15.0000	15.0000
Elongation	0.1205	0.1784	0.1364	0.2013	0.1996	0.2240	0.2552	0.4909
	0.2989		0.3376		0.4236		0.7461	
	0.12%		0.13%		0.16%		0.29%	
Total Beam	**1.8061 in.**							**0.18%**
Total	**16.2054 in.**							**1.37%**

Table 51: Elongation at five seconds for the three bay frames

Frame	**7.3.7**					
Location	**Column A**	**Column B**		**Column C**		**Column D**
	Right	**Left**	**Right**	**Left**	**Right**	**Left**
Time	5.0000	5.0000	5.0000	5.0000	5.0000	5.0000
Drift	1.4230	1.7471	1.7471	2.1214	2.1214	2.6405
Rotation	0.5399	0.6629	0.6629	0.8049	0.8049	1.0018
Neutral Axis	3.5477	28.0678	5.6247	27.6554	4.8577	25.1757
Top of Conc.	36.0000	36.0000	36.0000	36.0000	36.0000	36.0000
Centroid	15.0000	15.0000	15.0000	15.0000	15.0000	15.0000
Elongation	0.1079	0.1512	0.1085	0.1778	0.1425	0.1779
	0.2591		0.2863		0.3204	
	0.10%		0.11%		0.12%	
Total Beam					**0.8658 in.**	**0.11%**
Total					**8.7978 in.**	**0.98%**
Frame	**7.3.5**					
Location	**Column A**	**Column B**		**Column C**		**Column D**
	Right	**Left**	**Right**	**Left**	**Right**	**Left**
Time	5.0000	5.0000	5.0000	5.0000	5.0000	5.0000
Drift	1.3991	1.7493	1.7493	2.1701	2.1701	2.7176
Rotation	0.5309	0.6637	0.6637	0.8234	0.8234	1.0311
Neutral Axis	4.8206	27.5035	6.4837	26.9605	5.6229	25.0932
Top of Conc.	36.0000	36.0000	36.0000	36.0000	36.0000	36.0000
Centroid	15.0000	15.0000	15.0000	15.0000	15.0000	15.0000
Elongation	0.0943	0.1448	0.0987	0.1719	0.1348	0.1817
	0.2392		0.2705		0.3164	
	0.09%		0.10%		0.12%	
Total Beam					**0.8261 in.**	**0.11%**
Total					**8.8623 in.**	**0.99%**
Frame	**6.3.7**					
Location	**Column A**	**Column B**		**Column C**		**Column D**
	Right	**Left**	**Right**	**Left**	**Right**	**Left**
Time	5.0000	5.0000	5.0000	5.0000	5.0000	5.0000
Drift	2.0485	2.5045	2.5045	3.0210	3.0210	3.7085
Rotation	0.7772	0.9502	0.9502	1.1461	1.1461	1.4069
Neutral Axis	3.5173	26.7579	4.5434	26.6137	3.8307	22.9365
Top of Conc.	36.0000	36.0000	36.0000	36.0000	36.0000	36.0000
Centroid	15.0000	15.0000	15.0000	15.0000	15.0000	15.0000
Elongation	0.1558	0.1950	0.1734	0.2324	0.2235	0.1949
	0.3508		0.4058		0.4184	
	0.14%		0.16%		0.16%	
Total Beam					**1.1750 in.**	**0.15%**
Total					**12.4574 in.**	**1.39%**
Frame	**6.3.5**					
Location	**Column A**	**Column B**		**Column C**		**Column D**
	Right	**Left**	**Right**	**Left**	**Right**	**Left**
Time	5.0000	5.0000	5.0000	5.0000	5.0000	5.0000
Drift	2.0064	2.5012	2.5012	3.0815	3.0815	3.8030
Rotation	0.7613	0.9490	0.9490	1.1691	1.1691	1.4427
Neutral Axis	4.8622	26.3474	5.5297	25.1074	4.5033	23.0235
Top of Conc.	36.0000	36.0000	36.0000	36.0000	36.0000	36.0000
Centroid	15.0000	15.0000	15.0000	15.0000	15.0000	15.0000
Elongation	0.1347	0.1880	0.1569	0.2063	0.2142	0.2021
	0.3227		0.3631		0.4163	
	0.13%		0.14%		0.16%	
Total Beam					**1.1021 in.**	**0.14%**
Total					**12.4941 in.**	**1.40%**

Table 52: Elongation at five seconds for the two bay frames

Frame	7.2.7			
Location	Column A	Column B		Column C
	Right	Left	Right	Left
Time	5.0000	5.0000	5.0000	5.0000
Drift	1.5461	1.8958	1.8958	2.3850
Rotation	0.5867	0.7193	0.7193	0.9049
Neutral Axis	3.6111	28.1810	5.3058	25.8930
Top of Conc.	36.0000	36.0000	36.0000	36.0000
Centroid	15.0000	15.0000	15.0000	15.0000
Elongation	0.1166	0.1655	0.1217	0.1720
	0.2821		0.2938	
	0.11%		0.11%	
Total Beam	0.5759 in.			0.11%
Total	6.4027 in.			1.06%
Frame	7.2.5			
Location	Column A	Column B		Column C
	Right	Left	Right	Left
Time	5.0000	5.0000	5.0000	5.0000
Drift	1.5224	1.9060	1.9060	2.4164
Rotation	0.5776	0.7232	0.7232	0.9168
Neutral Axis	4.7969	27.1758	6.3960	25.8929
Top of Conc.	36.0000	36.0000	36.0000	36.0000
Centroid	15.0000	15.0000	15.0000	15.0000
Elongation	0.1029	0.1537	0.1086	0.1743
	0.2566		0.2829	
	0.10%		0.11%	
Total Beam	0.5395 in.			0.10%
Total	6.3843 in.			1.05%
Frame	6.2.7			
Location	Column A	Column B		Column C
	Right	Left	Right	Left
Time	5.0000	5.0000	5.0000	5.0000
Drift	2.2367	2.7278	2.7278	3.3948
Rotation	0.8486	1.0349	1.0349	1.2879
Neutral Axis	3.5403	26.9379	4.4935	24.1960
Top of Conc.	36.0000	36.0000	36.0000	36.0000
Centroid	15.0000	15.0000	15.0000	15.0000
Elongation	0.1697	0.2157	0.1898	0.2067
	0.3854		0.3966	
	0.15%		0.15%	
Total Beam	0.7820 in.			0.15%
Total	9.1413 in.			1.51%
Frame	6.2.5			
Location	Column A	Column B		Column C
	Right	Left	Right	Left
Time	5.0000	5.0000	5.0000	5.0000
Drift	2.2072	2.7441	2.7441	3.4374
Rotation	0.8374	1.0411	1.0411	1.3041
Neutral Axis	4.6548	26.0321	5.4387	23.9555
Top of Conc.	36.0000	36.0000	36.0000	36.0000
Centroid	15.0000	15.0000	15.0000	15.0000
Elongation	0.1512	0.2005	0.1738	0.2039
	0.3517		0.3776	
	0.14%		0.15%	
Total Beam	0.7293 in.			0.14%
Total	9.1179 in.			1.50%

Table 53: Elongation at five seconds for the one bay frames

Frame	7.1.7	
Location	Column A	Column B
	Right	Left
Time	5.0000	5.0000
Drift	1.5678	2.0045
Rotation	0.5949	0.7606
Neutral Axis	4.4022	26.3293
Top of Conc.	36.0000	36.0000
Centroid	15.0000	15.0000
Elongation	0.1100	0.1504
	0.2604	
	0.10%	
Total Beam	0.2604 in.	0.10%
Total	3.8328 in.	1.21%
Frame	7.1.5	
Location	Column A	Column B
	Right	Left
Time	5.0000	5.0000
Drift	1.5552	2.0005
Rotation	0.5901	0.7590
Neutral Axis	4.7374	26.5807
Top of Conc.	36.0000	36.0000
Centroid	15.0000	15.0000
Elongation	0.1057	0.1534
	0.2591	
	0.10%	
Total Beam	0.2591 in.	0.10%
Total	3.8148 in.	1.20%
Frame	6.1.7	
Location	Column A	Column B
	Right	Left
Time	5.0000	5.0000
Drift	2.3207	2.9411
Rotation	0.8805	1.1158
Neutral Axis	4.7787	24.8332
Top of Conc.	36.0000	36.0000
Centroid	15.0000	15.0000
Elongation	0.1571	0.1915
	0.3486	
	0.14%	
Total Beam	0.3486 in.	0.14%
Total	5.6105 in.	1.76%
Frame	6.1.5	
Location	Column A	Column B
	Right	Left
Time	5.0000	5.0000
Drift	2.3291	2.9554
Rotation	0.8837	1.1213
Neutral Axis	4.7513	24.7573
Top of Conc.	36.0000	36.0000
Centroid	15.0000	15.0000
Elongation	0.1581	0.1910
	0.3491	
	0.14%	
Total Beam	0.3491 in.	0.14%
Total	5.6336 in.	1.77%

Six Seconds

Table 54: Elongation at six seconds for the five bay frames

Frame	7.5.7									
Location	Column A	Column B		Column C		Column D		Column E		Column F
	Right	Left	Right	Left	Right	Left	Right	Left	Right	Left
Time	6.0000	6.0000	6.0000	6.0000	6.0000	6.0000	6.0000	6.0000	6.0000	6.0000
Drift (in.)	1.4932	1.8400	1.8400	2.2368	2.2368	2.6798	2.6798	3.1849	3.1849	3.8276
Rotation (°)	0.5666	0.6981	0.6981	0.8487	0.8487	1.0167	1.0167	1.2083	1.2083	1.4521
Neutral Axis	4.8331	27.5731	5.7529	26.9440	3.9101	26.9167	3.4972	26.7367	3.5155	23.3268
Top of Conc.	36.0000	36.0000	36.0000	36.0000	36.0000	36.0000	36.0000	36.0000	36.0000	36.0000
Centroid	15.0000	15.0000	15.0000	15.0000	15.0000	15.0000	15.0000	15.0000	15.0000	15.0000
Elongation	0.1005	0.1532	0.1127	0.1769	0.1643	0.2115	0.2041	0.2476	0.2422	0.2111
	0.2537		0.2896		0.3758		0.4517		0.4533	
	0.10%		0.11%		0.15%		0.18%		0.18%	
Total Beam									1.8241 in.	0.14%
Total									17.0864 in.	1.16%
Frame	7.5.5									
Location	Column A	Column B		Column C		Column D		Column E		Column F
	Right	Left	Right	Left	Right	Left	Right	Left	Right	Left
Time	6.0000	6.0000	6.0000	6.0000	6.0000	6.0000	6.0000	6.0000	6.0000	6.0000
Drift	2.0686	2.5857	2.5857	3.1882	3.1882	3.8487	3.8487	4.5768	4.5768	5.4589
Rotation	0.7848	0.9810	0.9810	1.2095	1.2095	1.4601	1.4601	1.7361	1.7361	2.0704
Neutral Axis	4.8995	27.2749	6.6280	26.4680	5.0528	25.6626	3.8312	24.8612	3.3914	22.3580
Top of Conc.	36.0000	36.0000	36.0000	36.0000	36.0000	36.0000	36.0000	36.0000	36.0000	36.0000
Centroid	15.0000	15.0000	15.0000	15.0000	15.0000	15.0000	15.0000	15.0000	15.0000	15.0000
Elongation	0.1384	0.2102	0.1434	0.2421	0.2100	0.2718	0.2847	0.2989	0.3519	0.2660
	0.3486		0.3855		0.4818		0.5836		0.6179	
	0.14%		0.15%		0.19%		0.23%		0.24%	
Total Beam									2.4173 in.	0.19%
Total									24.1441 in.	1.64%
Frame	6.5.7									
Location	Column A	Column B		Column C		Column D		Column E		Column F
	Right	Left	Right	Left	Right	Left	Right	Left	Right	Left
Time	6.0000	6.0000	6.0000	6.0000	6.0000	6.0000	6.0000	6.0000	6.0000	6.0000
Drift	2.1515	2.6356	2.6356	3.1860	3.1860	3.7831	3.7831	4.4443	4.4443	5.2271
Rotation	0.8163	1.0000	1.0000	1.2087	1.2087	1.4352	1.4352	1.6859	1.6859	1.9826
Neutral Axis	3.2190	26.5345	4.0663	25.9343	3.3162	24.8395	3.0839	25.6423	3.0533	20.9987
Top of Conc.	36.0000	36.0000	36.0000	36.0000	36.0000	36.0000	36.0000	36.0000	36.0000	36.0000
Centroid	15.0000	15.0000	15.0000	15.0000	15.0000	15.0000	15.0000	15.0000	15.0000	15.0000
Elongation	0.1679	0.2013	0.1908	0.2307	0.2465	0.2465	0.2985	0.3132	0.3516	0.2077
	0.3692		0.4216		0.4930		0.6118		0.5593	
	0.14%		0.16%		0.19%		0.24%		0.22%	
Total Beam									2.4548 in.	0.19%
Total									23.8824 in.	1.62%
Frame	6.5.5									
Location	Column A	Column B		Column C		Column D		Column E		Column F
	Right	Left	Right	Left	Right	Left	Right	Left	Right	Left
Time	6.0000	6.0000	6.0000	6.0000	6.0000	6.0000	6.0000	6.0000	6.0000	6.0000
Drift	2.0686	2.5127	2.5127	3.1882	3.1882	3.8487	3.8487	4.5768	4.5768	5.4589
Rotation	0.7848	0.9533	0.9533	1.2095	1.2095	1.4601	1.4601	1.7361	1.7361	2.0704
Neutral Axis	4.8672	26.3197	5.2119	25.2483	3.0952	23.9407	3.3757	23.8611	3.2099	20.8554
Top of Conc.	36.0000	36.0000	36.0000	36.0000	36.0000	36.0000	36.0000	36.0000	36.0000	36.0000
Centroid	15.0000	15.0000	15.0000	15.0000	15.0000	15.0000	15.0000	15.0000	15.0000	15.0000
Elongation	0.1388	0.1884	0.1629	0.2164	0.2514	0.2279	0.2963	0.2686	0.3574	0.2117
	0.3272		0.3793		0.4792		0.5649		0.5690	
	0.13%		0.15%		0.19%		0.22%		0.22%	
Total Beam									2.3196 in.	0.18%
Total									23.9734 in.	1.63%

Table 55: Elongation at six seconds for the four bay frames

Frame	7.4.7							
Location	Column A	Column B		Column C		Column D		Column E
	Right	Left	Right	Left	Right	Left	Right	Left
Time	6.0000	6.0000	6.0000	6.0000	6.0000	6.0000	6.0000	6.0000
Drift	1.6255	2.0086	2.0086	2.4401	2.4401	2.9222	2.9222	3.5893
Rotation	0.6168	0.7621	0.7621	0.9258	0.9258	1.1087	1.1087	1.3617
Neutral Axis	4.9263	27.3518	5.2415	26.6035	4.1087	26.4249	3.6420	23.1701
Top of Conc.	36.0000	36.0000	36.0000	36.0000	36.0000	36.0000	36.0000	36.0000
Centroid	15.0000	15.0000	15.0000	15.0000	15.0000	15.0000	15.0000	15.0000
Elongation	0.1084	0.1643	0.1298	0.1875	0.1760	0.2211	0.2198	0.1942
	0.2728		0.3173		0.3971		0.4140	
	0.11%		0.12%		0.15%		0.16%	
Total Beam	**1.4012 in.**							**0.14%**
Total	**13.9869 in.**							**1.18%**
Frame	7.4.5							
Location	Column A	Column B		Column C		Column D		Column E
	Right	Left	Right	Left	Right	Left	Right	Left
Time	6.0000	6.0000	6.0000	6.0000	6.0000	6.0000	6.0000	6.0000
Drift	1.5810	1.9880	1.9880	2.4654	2.4654	3.0185	3.0185	3.7300
Rotation	0.5999	0.7543	0.7543	0.9354	0.9354	1.1452	1.1452	1.4150
Neutral Axis	4.8942	27.0696	6.4846	26.1621	5.1440	25.3272	4.3470	23.2831
Top of Conc.	36.0000	36.0000	36.0000	36.0000	36.0000	36.0000	36.0000	36.0000
Centroid	15.0000	15.0000	15.0000	15.0000	15.0000	15.0000	15.0000	15.0000
Elongation	0.1058	0.1589	0.1121	0.1822	0.1609	0.2064	0.2130	0.2046
	0.2647		0.2944		0.3674		0.4176	
	0.10%		0.11%		0.14%		0.16%	
Total Beam	**1.3440 in.**							**0.13%**
Total	**14.1270 in.**							**1.20%**
Frame	6.4.7							
Location	Column A	Column B		Column C		Column D		Column E
	Right	Left	Right	Left	Right	Left	Right	Left
Time	6.0000	6.0000	6.0000	6.0000	6.0000	6.0000	6.0000	6.0000
Drift	2.3343	2.8606	2.8606	3.4512	3.4512	4.1005	4.1005	4.9515
Rotation	0.8857	1.0853	1.0853	1.3093	1.3093	1.5555	1.5555	1.8781
Neutral Axis	3.0457	26.3008	3.9259	25.1658	3.3186	25.4444	3.1420	21.3545
Top of Conc.	36.0000	36.0000	36.0000	36.0000	36.0000	36.0000	36.0000	36.0000
Centroid	15.0000	15.0000	15.0000	15.0000	15.0000	15.0000	15.0000	15.0000
Elongation	0.1848	0.2141	0.2098	0.2323	0.2670	0.2836	0.3220	0.2084
	0.3989		0.4421		0.5506		0.5304	
	0.15%		0.17%		0.21%		0.21%	
Total Beam	**1.9220 in.**							**0.19%**
Total	**19.6200 in.**							**1.66%**
Frame	6.4.5							
Location	Column A	Column B		Column C		Column D		Column E
	Right	Left	Right	Left	Right	Left	Right	Left
Time	6.0000	6.0000	6.0000	6.0000	6.0000	6.0000	6.0000	6.0000
Drift	2.2760	2.8370	2.8370	3.4933	3.4933	4.2251	4.2251	5.1178
Rotation	0.8635	1.0764	1.0764	1.3253	1.3253	1.6028	1.6028	1.9412
Neutral Axis	4.7966	26.1292	4.6968	24.6943	3.1943	23.5874	3.0106	32.9351
Top of Conc.	36.0000	36.0000	36.0000	36.0000	36.0000	36.0000	36.0000	36.0000
Centroid	15.0000	15.0000	15.0000	15.0000	15.0000	15.0000	15.0000	15.0000
Elongation	0.1538	0.2091	0.1936	0.2243	0.2731	0.2403	0.3355	0.6079
	0.3629		0.4178		0.5134		0.9433	
	0.14%		0.16%		0.20%		0.37%	
Total Beam	**2.2375 in.**							**0.22%**
Total	**20.1867 in.**							**1.71%**

Table 56: Elongation at six seconds for the three bay frames

Frame	**7.3.7**					
Location	**Column A**	**Column B**		**Column C**		**Column D**
	Right	**Left**	**Right**	**Left**	**Right**	**Left**
Time	6.0000	6.0000	6.0000	6.0000	6.0000	6.0000
Drift	1.7914	2.2060	2.2060	2.6789	2.6789	3.3206
Rotation	0.6797	0.8370	0.8370	1.0164	1.0164	1.2598
Neutral Axis	3.1050	27.2403	5.1881	26.7193	4.3179	23.8573
Top of Conc.	36.0000	36.0000	36.0000	36.0000	36.0000	36.0000
Centroid	15.0000	15.0000	15.0000	15.0000	15.0000	15.0000
Elongation	0.1411	0.1788	0.1433	0.2079	0.1895	0.1948
	0.3199		0.3513		0.3843	
	0.12%		0.14%		0.15%	
Total Beam	**1.0555 in.**					**0.14%**
Total	**11.0524 in.**					**1.24%**
Frame	**7.3.5**					
Location	**Column A**	**Column B**		**Column C**		**Column D**
	Right	**Left**	**Right**	**Left**	**Right**	**Left**
Time	6.0000	6.0000	6.0000	6.0000	6.0000	6.0000
Drift	1.7654	2.2121	2.2121	2.7409	2.7409	3.4209
Rotation	0.6698	0.8393	0.8393	1.0399	1.0399	1.2978
Neutral Axis	4.8641	26.7957	6.2398	26.1404	4.9970	23.7273
Top of Conc.	36.0000	36.0000	36.0000	36.0000	36.0000	36.0000
Centroid	15.0000	15.0000	15.0000	15.0000	15.0000	15.0000
Elongation	0.1185	0.1728	0.1283	0.2022	0.1816	0.1977
	0.2913		0.3305		0.3793	
	0.11%		0.13%		0.15%	
Total Beam	**1.0011 in.**					**0.13%**
Total	**11.1404 in.**					**1.25%**
Frame	**6.3.7**					
Location	**Column A**	**Column B**		**Column C**		**Column D**
	Right	**Left**	**Right**	**Left**	**Right**	**Left**
Time	6.0000	6.0000	6.0000	6.0000	6.0000	6.0000
Drift	2.5710	3.1391	3.1391	3.7791	3.7791	4.6162
Rotation	0.9755	1.1909	1.1909	1.4336	1.4336	1.7510
Neutral Axis	3.2939	25.7636	3.8132	25.2109	3.3555	21.0677
Top of Conc.	36.0000	36.0000	36.0000	36.0000	36.0000	36.0000
Centroid	15.0000	15.0000	15.0000	15.0000	15.0000	15.0000
Elongation	0.1993	0.2238	0.2326	0.2555	0.2914	0.1855
	0.4231		0.4881		0.4769	
	0.16%		0.19%		0.18%	
Total Beam	**1.3881 in.**					**0.18%**
Total	**15.4934 in.**					**1.73%**
Frame	**6.3.5**					
Location	**Column A**	**Column B**		**Column C**		**Column D**
	Right	**Left**	**Right**	**Left**	**Right**	**Left**
Time	6.0000	6.0000	6.0000	6.0000	6.0000	6.0000
Drift	2.5186	3.1299	3.1299	3.8447	3.8447	4.7200
Rotation	0.9556	1.1874	1.1874	1.4585	1.4585	1.7904
Neutral Axis	4.6411	25.2913	4.4898	23.5968	3.6380	21.0016
Top of Conc.	36.0000	36.0000	36.0000	36.0000	36.0000	36.0000
Centroid	15.0000	15.0000	15.0000	15.0000	15.0000	15.0000
Elongation	0.1728	0.2133	0.2179	0.2189	0.2893	0.1876
	0.3861		0.4367		0.4769	
	0.15%		0.17%		0.18%	
Total Beam	**1.2997 in.**					**0.17%**
Total	**15.5129 in.**					**1.74%**

Table 57: Elongation at six seconds for the two bay frames

Frame	7.2.7			
Location	Column A	Column B		Column C
	Right	Left	Right	Left
Time	6.0000	6.0000	6.0000	6.0000
Drift	1.9612	2.4054	2.4054	3.0131
Rotation	0.7441	0.9126	0.9126	1.1431
Neutral Axis	3.5234	27.4416	4.9277	24.8359
Top of Conc.	36.0000	36.0000	36.0000	36.0000
Centroid	15.0000	15.0000	15.0000	15.0000
Elongation	0.1491	0.1982	0.1604	0.1963
	0.3472		0.3567	
	0.13%		0.14%	
Total Beam			**0.7040 in.**	**0.14%**
Total			**8.0837 in.**	**1.33%**
Frame	7.2.5			
Location	Column A	Column B		Column C
	Right	Left	Right	Left
Time	6.0000	6.0000	6.0000	6.0000
Drift	1.9381	2.4250	2.4250	3.0630
Rotation	0.7354	0.9201	0.9201	1.1621
Neutral Axis	4.7964	26.3417	5.9504	24.8051
Top of Conc.	36.0000	36.0000	36.0000	36.0000
Centroid	15.0000	15.0000	15.0000	15.0000
Elongation	0.1310	0.1821	0.1453	0.1989
	0.3131		0.3442	
	0.12%		0.13%	
Total Beam			**0.6573 in.**	**0.13%**
Total			**8.0834 in.**	**1.33%**
Frame	6.2.7			
Location	Column A	Column B		Column C
	Right	Left	Right	Left
Time	6.0000	6.0000	6.0000	6.0000
Drift	2.8140	3.4277	3.4277	4.2451
Rotation	1.0676	1.3004	1.3004	1.6103
Neutral Axis	3.2893	25.4878	3.8699	22.4040
Top of Conc.	36.0000	36.0000	36.0000	36.0000
Centroid	15.0000	15.0000	15.0000	15.0000
Elongation	0.2182	0.2381	0.2527	0.2082
	0.4563		0.4608	
	0.18%		0.18%	
Total Beam			**0.9171 in.**	**0.18%**
Total			**11.4038 in.**	**1.88%**
Frame	6.2.5			
Location	Column A	Column B		Column C
	Right	Left	Right	Left
Time	6.0000	6.0000	6.0000	6.0000
Drift	2.7829	3.4509	3.4509	4.3009
Rotation	1.0558	1.3092	1.3092	1.6315
Neutral Axis	4.4558	24.8107	4.6948	22.0596
Top of Conc.	36.0000	36.0000	36.0000	36.0000
Centroid	15.0000	15.0000	15.0000	15.0000
Elongation	0.1943	0.2242	0.2355	0.2011
	0.4185		0.4366	
	0.16%		0.17%	
Total Beam			**0.8551 in.**	**0.17%**
Total			**11.3898 in.**	**1.88%**

Table 58: Elongation at six seconds for the one bay frames

Frame	7.1.7	
Location	Column A	Column B
	Right	Left
Time	6.0000	6.0000
Drift	2.0033	2.5470
Rotation	0.7601	0.9663
Neutral Axis	4.2454	25.3824
Top of Conc.	36.0000	36.0000
Centroid	15.0000	15.0000
Elongation	0.1427	0.1751
	0.3178	
	0.12%	
Total Beam	**0.3178 in.**	**0.12%**
Total	**4.8681 in.**	**1.53%**
Frame	7.1.5	
Location	Column A	Column B
	Right	Left
Time	6.0000	6.0000
Drift	1.9900	2.5493
Rotation	0.7551	0.9672
Neutral Axis	4.7139	25.6801
Top of Conc.	36.0000	36.0000
Centroid	15.0000	15.0000
Elongation	0.1356	0.1803
	0.3159	
	0.12%	
Total Beam	**0.3159 in.**	**0.12%**
Total	**4.8553 in.**	**1.53%**
Frame	6.1.7	
Location	Column A	Column B
	Right	Left
Time	6.0000	6.0000
Drift	2.9580	3.7247
Rotation	1.1222	1.4130
Neutral Axis	4.5121	23.1629
Top of Conc.	36.0000	36.0000
Centroid	15.0000	15.0000
Elongation	0.2054	0.2014
	0.4068	
	0.16%	
Total Beam	**0.4068 in.**	**0.16%**
Total	**7.0895 in.**	**2.23%**
Frame	6.1.5	
Location	Column A	Column B
	Right	Left
Time	6.0000	6.0000
Drift	2.9604	3.7398
Rotation	1.1232	1.4187
Neutral Axis	4.5297	23.0141
Top of Conc.	36.0000	36.0000
Centroid	15.0000	15.0000
Elongation	0.2053	0.1985
	0.4038	
	0.16%	
Total Beam	**0.4038 in.**	**0.16%**
Total	**7.1040 in.**	**2.23%**

Calculations for Seven Seconds

7.5.7

Right End of Column A

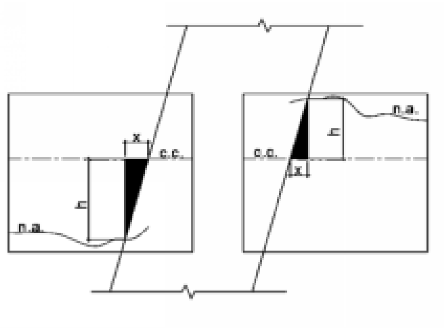
Column

time, t = 7.0000 sec
drift, Δ = 1.8080 in.
rotation, Θ = 0.6860 °

Beam

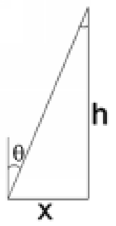
time, t = 7.0000 sec
neutral axis, n.a. = 4.5976 in.
centroid, c.c. = 15.0000 in.

Calculations



all measurements are taken from the top of the beam

q = rotation
h = distance between the n.a. and c.c.
x = elongation



$$\tan\theta = \frac{opposite}{adjacent}$$

$$\tan\theta = \frac{x}{h}$$

$$x = h\tan\theta$$ due to similar triangles

h = 10.4024 in.

x = 0.1246 in.

7.5.7

Left Side of Column B

Column

time, t = 7.0000 sec
drift, Δ = 2.2309 in.
rotation, Θ = 0.8464 °

Beam

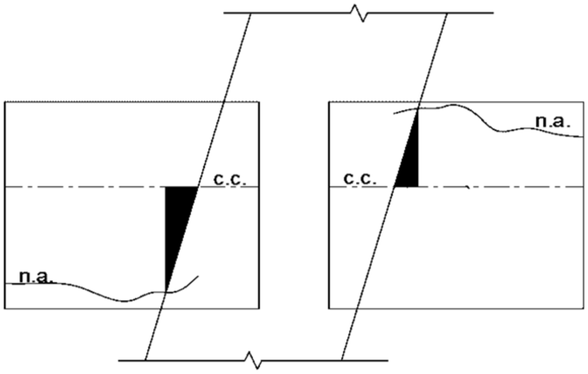
time, t = 7.0000 sec
neutral axis, n.a. = 27.1213 in.
centroid, c.c. = 15.0000 in.

Right Side of Column B

time, t = 7.0000 sec
drift, Δ = 2.2309 in.
rotation, Θ = 0.8464 °

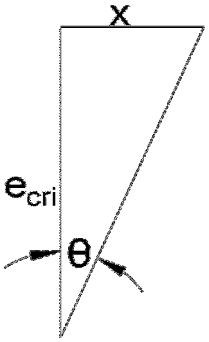
time, t = 7.0000 sec
neutral axis, n.a. = 4.6524 in.
centroid, c.c. = 15.0000 in.

Calculations



all measurements are taken from the top face of the beam

q = rotation
e_{cri} = distance between the n.a. and c.c.
x = elongation



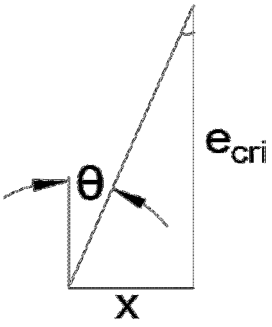
$$\tan\theta = \frac{opposite}{adjacent}$$

$$\tan\theta = \frac{x}{e_{cri}}$$

$$x = e_{cri}\tan\theta$$

e_{cri} = 12.1213 in.

x = 0.1791 in.



$$\tan\theta = \frac{opposite}{adjacent}$$

$$\tan\theta = \frac{x}{e_{cri}}$$

$$x = e_{cri}\tan\theta$$ due to similar triangles

e_{cri} = 10.3476 in.

x = 0.1529 in.

7.5.7

Left Side of Column C

Column

time, t = 7.0000 sec
drift, Δ = 2.7131 in.
rotation, Θ = 1.0293 °

Beam

time, t = 7.0000 sec
neutral axis, n.a. = 26.2572 in.
centroid, c.c. = 15.0000 in.

Right Side of Column C

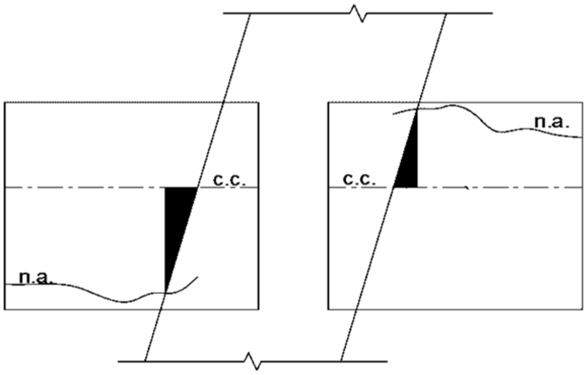
Column

time, t = 7.0000 sec
drift, Δ = 2.7131 in.
rotation, Θ = 1.0293 °

Beam

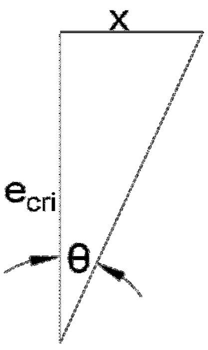
time, t = 7.0000 sec
neutral axis, n.a. = 3.5563 in.
centroid, c.c. = 15.0000 in.

Calculations



all measurements are taken from
the top of the beam

q = rotation
e_{cri} = distance between the n.a. and c.c.
x = elongation



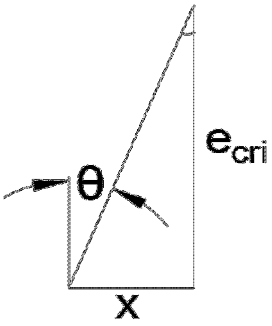
$$\tan\theta = \frac{opposite}{adjacent}$$

$$\tan\theta = \frac{x}{e_{cri}}$$

$$x = e_{cri}\tan\theta$$

e_{cri} = 11.2572 in.

x = 0.2023 in.



$$\tan\theta = \frac{opposite}{adjacent}$$

$$\tan\theta = \frac{x}{e_{cri}}$$

$$x = e_{cri}\tan\theta$$ due to similar triangles

e_{cri} = h = 11.4437 in.

x = 0.2056 in.

7.5.7

Left Side of Column D

Right Side of Column D

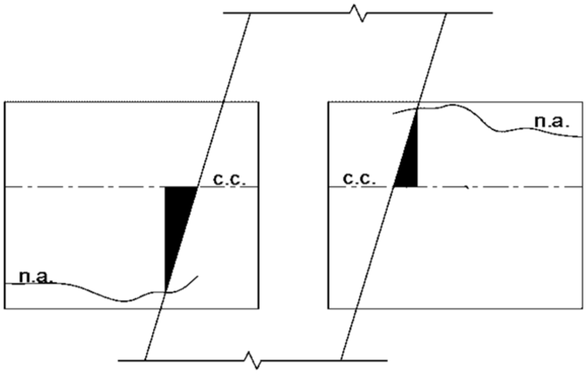
Column

	Left Side of Column D			Right Side of Column D	
time, t =	7.0000	sec	time, t =	7.0000	sec
drift, Δ =	3.2517	in.	drift, Δ =	3.2517	in.
rotation, Θ =	1.2336	°	rotation, Θ =	1.2336	°

Beam

	Left Side of Column D			Right Side of Column D	
time, t =	7.0000	sec	time, t =	7.0000	sec
neutral axis, n.a. =	25.9270	in.	neutral axis, n.a. =	3.2499	in.
centroid, c.c. =	15.0000	in.	centroid, c.c. =	15.0000	in.

Calculations

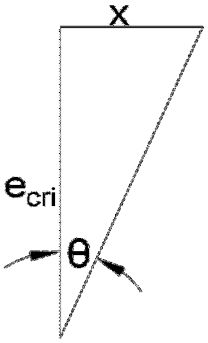


all measurements are taken from the top of the beam

q = rotation

e_{cri} = distance between the n.a. and c.c.

x = elongation



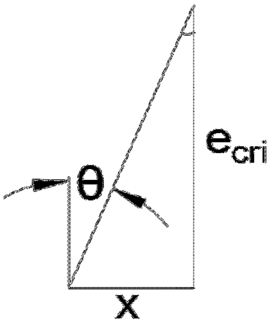
$$\tan\theta = \frac{opposite}{adjacent}$$

$$\tan\theta = \frac{x}{e_{cri}}$$

$$x = e_{cri}\tan\theta$$

e_{cri} = 10.9270 in.

x = 0.2353 in.



$$\tan\theta = \frac{opposite}{adjacent}$$

$$\tan\theta = \frac{x}{e_{cri}}$$

$x = e_{cri}\tan\theta$ due to similar triangles

e_{cri} = 11.7501 in.

x = 0.2530 in.

7.5.7

Left Side of Column D

Right Side of Column D

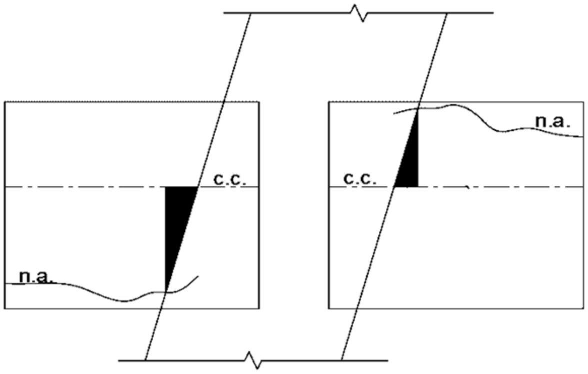
Column

time, t =	7.0000	sec	time, t =	7.0000	sec
drift, Δ =	3.8632	in.	drift, Δ =	3.8632	in.
rotation, Θ =	1.4655	°	rotation, Θ =	1.4655	°

Beam

time, t =	7.0000	sec	time, t =	7.0000	sec
neutral axis, n.a. =	25.6786	in.	neutral axis, n.a. =	3.2470	in.
centroid, c.c. =	15.0000	in.	centroid, c.c. =	15.0000	in.

Calculations

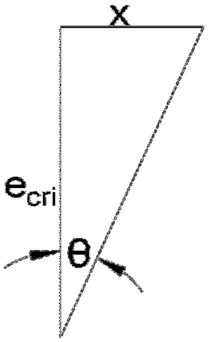


all measurements are taken from
the top of the beam

q = rotation

e_{cri} = distance between the n.a. and c.c.

x = elongation



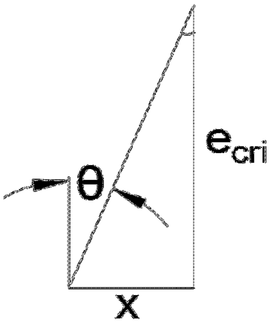
$$\tan\theta = \frac{opposite}{adjacent}$$

$$\tan\theta = \frac{x}{e_{cri}}$$

$$x = e_{cri}\tan\theta$$

e_{cri} = 10.6786 in.

x = 0.2732 in.



$$\tan\theta = \frac{opposite}{adjacent}$$

$$\tan\theta = \frac{x}{e_{cri}}$$

$x = e_{cri}\tan\theta$ due to similar triangles

e_{cri} = 11.7530 in.

x = 0.3007 in.

7.5.7

Left Side of Column F

Column

time, t = 7.0000 sec

drift, Δ = 4.6139 in.

rotation, ϴ = 1.7502 °

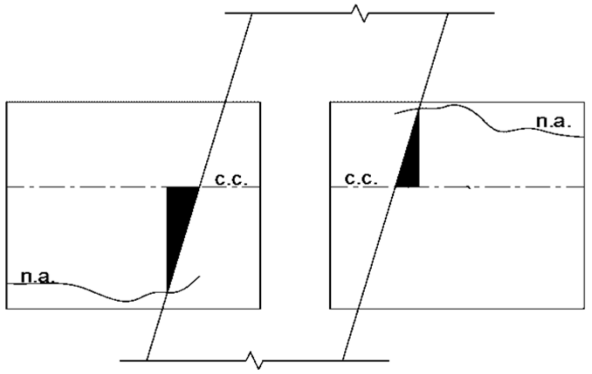
Beam

time, t = 7.0000 sec

neutral axis, n.a. = 21.9144 in.

centroid, c.c. = 15.0000 in.

Calculations

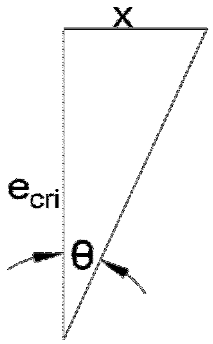


all measurements are taken from the top of the beam

q = rotation

e_{cri} = distance between the n.a. and c.c.

x = elongation



$$\tan\theta = \frac{opposite}{adjacent}$$

$$\tan\theta = \frac{x}{e_{cri}}$$

$$x = e_{cri}\tan\theta$$

e_{cri} = 6.9144 in.

x = 0.2113 in.

7.5.5

Right Side of Column A

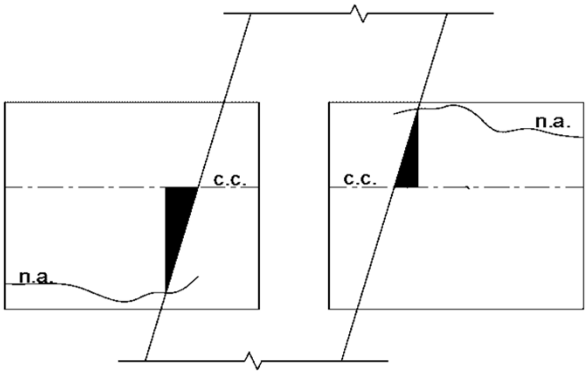
Column

time, t = 7.0000 sec
drift, Δ = 2.5016 in.
rotation, Θ = 0.9491 °

Beam

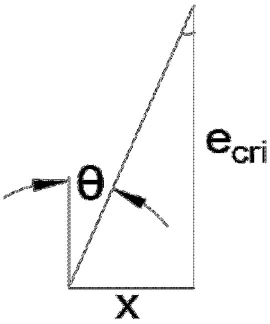
time, t = 7.0000 sec
neutral axis, n.a. = 4.8927 in.
centroid, c.c. = 15.0000 in.

Calculations



all measurements are taken from the top of the beam

q = rotation
e_{cri} = distance between the n.a. and c.c.
x = elongation



$$\tan\theta = \frac{opposite}{adjacent}$$

$$\tan\theta = \frac{x}{e_{cri}}$$

$x = e_{cri}\tan\theta$ due to similar triangles

e_{cri} = 10.1073 in.

x = 0.1674 in.

7.5.5

Left Side of Column B

Column

time, t = 7.0000 sec

drift, Δ = 3.1145 in.

rotation, Θ = 1.1816 °

Beam

time, t = 7.0000 sec

neutral axis, n.a. = 26.7116 in.

centroid, c.c. = 15.0000 in.

Right Side of Column B

time, t = 7.0000 sec

drift, Δ = 3.1145 in.

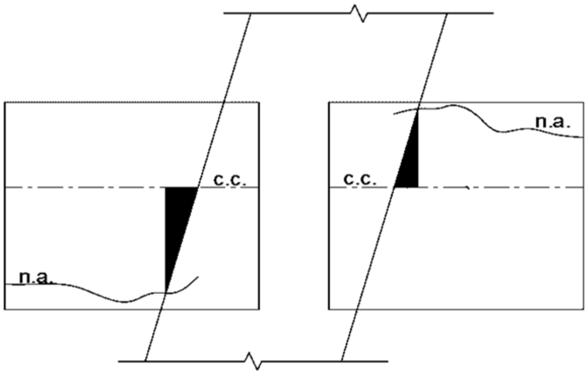
rotation, Θ = 1.1816 °

time, t = 7.0000 sec

neutral axis, n.a. = 6.3413 in.

centroid, c.c. = 15.0000 in.

Calculations

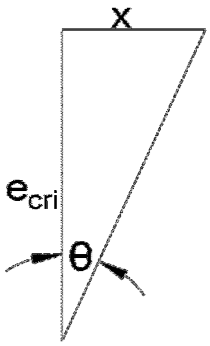


all measurements are taken from the top of the beam

q = rotation

e_{cri} = distance between the n.a. and c.c.

x = elongation



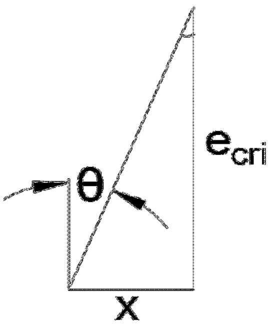
$$\tan\theta = \frac{opposite}{adjacent}$$

$$\tan\theta = \frac{x}{e_{cri}}$$

$$x = e_{cri}\tan\theta$$

e_{cri} = 11.7116 in.

x = 0.2416 in.



$$\tan\theta = \frac{opposite}{adjacent}$$

$$\tan\theta = \frac{x}{e_{cri}}$$

$x = e_{cri}\tan\theta$ due to similar triangles

e_{cri} = 8.6587 in.

x = 0.1786 in.

7.5.5

Left Side of Column C

Right Side of Column C

Column

time, t = 7.0000 sec
drift, Δ = 3.8350 in.
rotation, Θ = 1.4548 °

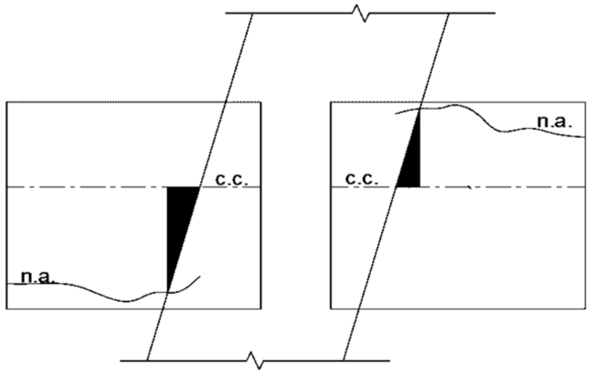
time, t = 7.0000 sec
drift, Δ = 3.8350 in.
rotation, Θ = 1.4548 °

Beam

time, t = 7.0000 sec
neutral axis, n.a. = 25.7774 in.
centroid, c.c. = 15.0000 in.

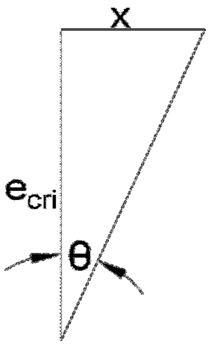
time, t = 7.0000 sec
neutral axis, n.a. = 3.8741 in.
centroid, c.c. = 15.0000 in.

Calculations



all measurements are taken from the top of the beam

q = rotation
e_{cri} = distance between the n.a. and c.c.
x = elongation



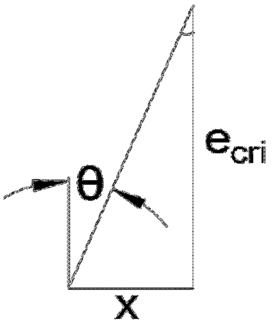
$$\tan\theta = \frac{opposite}{adjacent}$$

$$\tan\theta = \frac{x}{e_{cri}}$$

$$x = e_{cri}\tan\theta$$

e_{cri} = 10.7774 in.

x = 0.2737 in.



$$\tan\theta = \frac{opposite}{adjacent}$$

$$\tan\theta = \frac{x}{e_{cri}}$$

$x = e_{cri}\tan\theta$ due to similar triangles

e_{cri} = 11.1259 in.

x = 0.2826 in.

7.5.5

Left Side of Column D

Column

time, t = 7.0000 sec
drift, Δ = 4.6078 in.
rotation, Θ = 1.7479 °

Beam

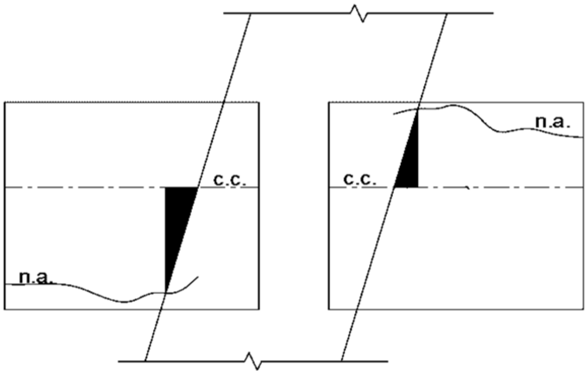
time, t = 7.0000 sec
neutral axis, n.a. = 24.7809 in.
centroid, c.c. = 15.0000 in.

Right Side of Column D

time, t = 7.0000 sec
drift, Δ = 4.6078 in.
rotation, Θ = 1.7479 °

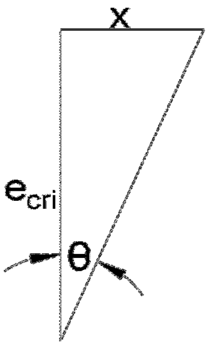
time, t = 7.0000 sec
neutral axis, n.a. = 3.0260 in.
centroid, c.c. = 15.0000 in.

Calculations



all measurements are taken from the top of the beam

q = rotation
e_{cri} = distance between the n.a. and c.c.
x = elongation



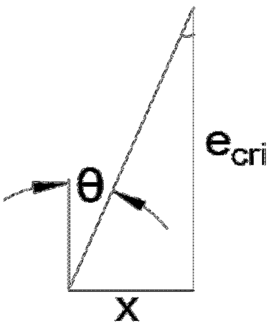
$$\tan\theta = \frac{opposite}{adjacent}$$

$$\tan\theta = \frac{x}{e_{cri}}$$

$$x = e_{cri}\tan\theta$$

e_{cri} = 9.7809 in.

x = 0.2985 in.



$$\tan\theta = \frac{opposite}{adjacent}$$

$$\tan\theta = \frac{x}{e_{cri}}$$

$x = e_{cri}\tan\theta$ due to similar triangles

e_{cri} = 11.9740 in.

x = 0.3654 in.

7.5.5

Left Side of Column D

Column

time, t = 7.0000 sec
drift, Δ = 5.4534 in.
rotation, Θ = 2.0683 °

Beam

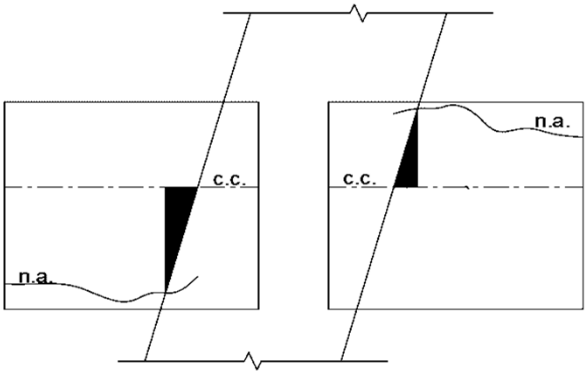
time, t = 7.0000 sec
neutral axis, n.a. = 23.8071 in.
centroid, c.c. = 15.0000 in.

Right Side of Column D

time, t = 7.0000 sec
drift, Δ = 5.4534 in.
rotation, Θ = 2.0683 °

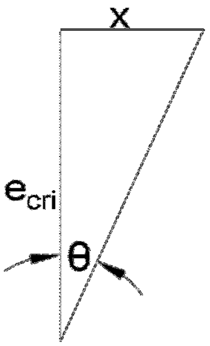
time, t = 7.0000 sec
neutral axis, n.a. = 3.1625 in.
centroid, c.c. = 15.0000 in.

Calculations



all measurements are taken from the top of the beam

q = rotation
e_{cri} = distance between the n.a. and c.c.
x = elongation



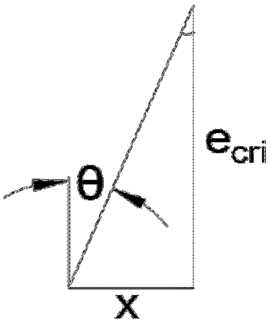
$$\tan\theta = \frac{opposite}{adjacent}$$

$$\tan\theta = \frac{x}{e_{cri}}$$

$$x = e_{cri}\tan\theta$$

e_{cri} = 8.8071 in.

x = 0.3181 in.



$$\tan\theta = \frac{opposite}{adjacent}$$

$$\tan\theta = \frac{x}{e_{cri}}$$

$x = e_{cri}\tan\theta$ due to similar triangles

e_{cri} = 11.8375 in.

x = 0.4275 in.

7.5.5

Left Side of Column E

Column

time, t = 7.0000 sec

drift, Δ = 6.4697 in.

rotation, Θ = 2.4534 °

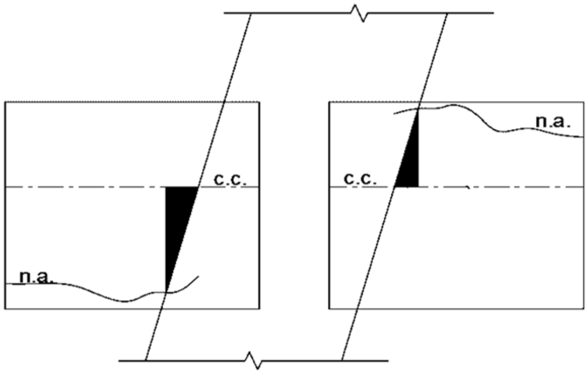
Beam

time, t = 7.0000 sec

neutral axis, n.a. = 21.1375 in.

centroid, c.c. = 15.0000 in.

Calculations

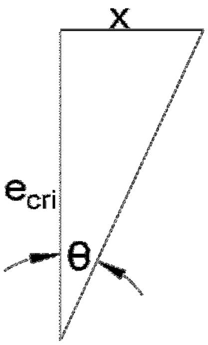


all measurements are taken from the top of the beam

q = rotation

e_{cri} = distance between the n.a. and c.c.

x = elongation



$$\tan\theta = \frac{opposite}{adjacent}$$

$$\tan\theta = \frac{x}{e_{cri}}$$

$$x = e_{cri}\tan\theta$$

e_{cri} = **6.1375** in.

x = **0.2630** in.

6.5.7

Right Side of Column A

Column

time, t = 7.0000 sec

drift, Δ = 2.5873 in.

rotation, Θ = 0.9816 °

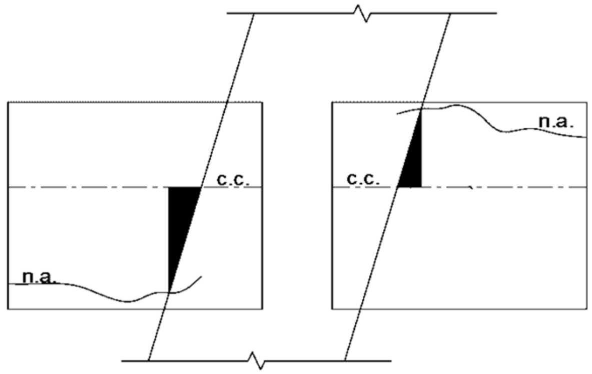
Beam

time, t = 7.0000 sec

neutral axis, n.a. = 3.0777 in.

centroid, c.c. = 15.0000 in.

Calculations

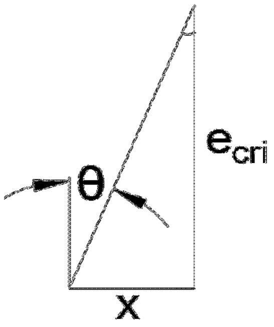


all measurements are taken from the top of the beam

q = rotation

e_{cri} = distance between the n.a. and c.c.

x = elongation



$$\tan\theta = \frac{opposite}{adjacent}$$

$$\tan\theta = \frac{x}{e_{cri}}$$

$$x = e_{cri}\tan\theta$$ due to similar triangles

e_{cri} = 11.9223 in.

x = 0.2043 in.

6.5.7

Left Side of Column B

Right Side of Column B

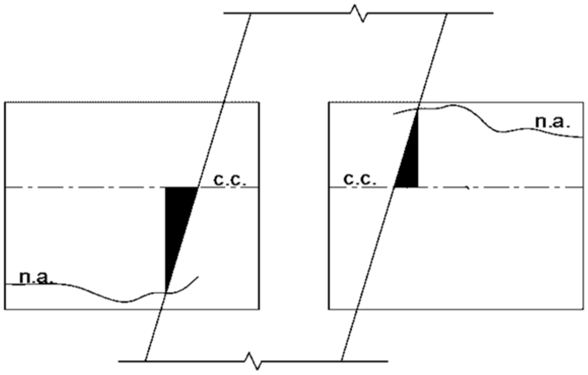
Column

	Left Side of Column B		Right Side of Column B	
time, t =	7.0000	sec	7.0000	sec
drift, Δ =	3.1586	in.	3.1586	in.
rotation, Θ =	1.1983	°	1.1983	°

Beam

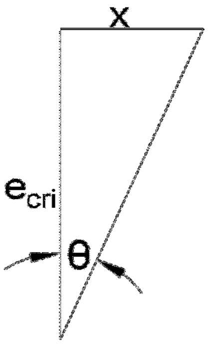
	Left Side of Column B		Right Side of Column B	
time, t =	7.0000	sec	7.0000	sec
neutral axis, n.a. =	26.0119	in.	3.8594	in.
centroid, c.c. =	15.0000	in.	15.0000	in.

Calculations



all measurements are taken from
the top of the beam

q = rotation
e_{cri} = distance between the n.a. and c.c.
x = elongation



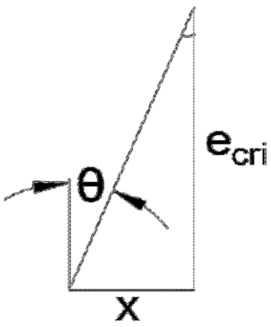
$$\tan\theta = \frac{opposite}{adjacent}$$

$$\tan\theta = \frac{x}{e_{cri}}$$

$$x = e_{cri}\tan\theta$$

e_{cri} = 11.0119 in.

x = 0.2303 in.



$$\tan\theta = \frac{opposite}{adjacent}$$

$$\tan\theta = \frac{x}{e_{cri}}$$

$x = e_{cri}\tan\theta$ due to similar triangles

e_{cri} = 11.1406 in.

x = 0.2330 in.

6.5.7

Left Side of Column C

Right Side of Column C

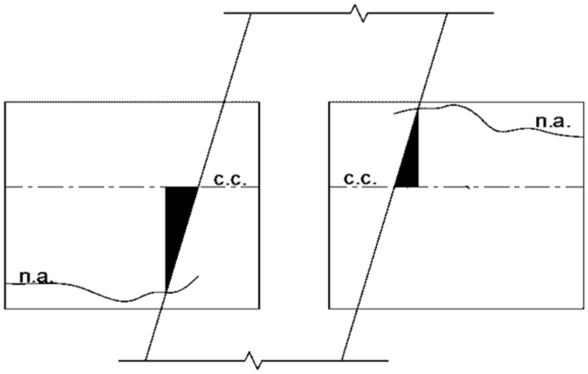
Column

Left Side of Column C			Right Side of Column C		
time, t =	7.0000	sec	time, t =	7.0000	sec
drift, Δ =	3.8234	in.	drift, Δ =	3.8234	in.
rotation, Θ =	1.4504	°	rotation, Θ =	1.4504	°

Beam

Left Side of Column C			Right Side of Column C		
time, t =	7.0000	sec	time, t =	7.0000	sec
neutral axis, n.a. =	25.1537	in.	neutral axis, n.a. =	3.1657	in.
centroid, c.c. =	15.0000	in.	centroid, c.c. =	15.0000	in.

Calculations

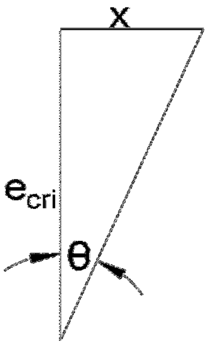


all measurements are taken from the top of the beam

q = rotation

e_{cri} = distance between the n.a. and c.c.

x = elongation



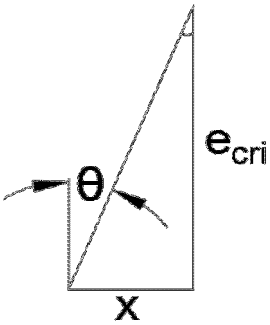
$$\tan\theta = \frac{opposite}{adjacent}$$

$$\tan\theta = \frac{x}{e_{cri}}$$

$$x = e_{cri}\tan\theta$$

e_{cri} = 10.1537 in.

x = 0.2571 in.



$$\tan\theta = \frac{opposite}{adjacent}$$

$$\tan\theta = \frac{x}{e_{cri}}$$

$x = e_{cri}\tan\theta$ due to similar triangles

e_{cri} = 11.8343 in.

x = 0.2996 in.

6.5.7

Left Side of Column D

Column

time, t = 7.0000 sec
drift, Δ = 4.5361 in.
rotation, Θ = 1.7207 °

Beam

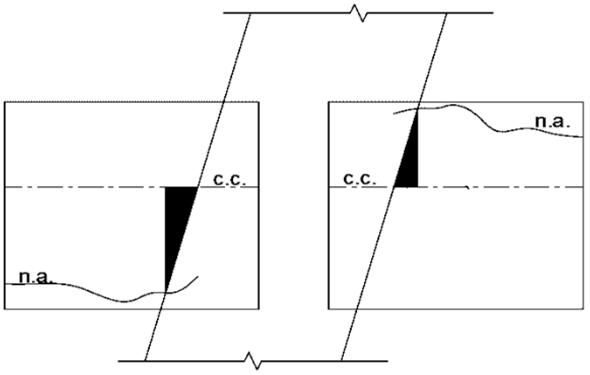
time, t = 7.0000 sec
neutral axis, n.a. = 23.5751 in.
centroid, c.c. = 15.0000 in.

Right Side of Column D

time, t = 7.0000 sec
drift, Δ = 4.5361 in.
rotation, Θ = 1.7207 °

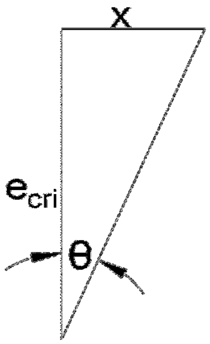
time, t = 7.0000 sec
neutral axis, n.a. = 3.0964 in.
centroid, c.c. = 15.0000 in.

Calculations



all measurements are taken from the top of the beam

q = rotation
e_{cri} = distance between the n.a. and c.c.
x = elongation



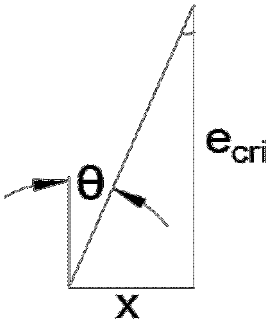
$$\tan\theta = \frac{opposite}{adjacent}$$

$$\tan\theta = \frac{x}{e_{cri}}$$

$$x = e_{cri}\tan\theta$$

e_{cri} = 8.5751 in.

x = 0.2576 in.



$$\tan\theta = \frac{opposite}{adjacent}$$

$$\tan\theta = \frac{x}{e_{cri}}$$

$x = e_{cri}\tan\theta$ due to similar triangles

e_{cri} = 11.9036 in.

x = 0.3576 in.

6.5.7

Left Side of Column D

Right Side of Column D

Column

time, t = 7.0000 sec
drift, Δ = 5.3068 in.
rotation, Θ = 2.0128 °

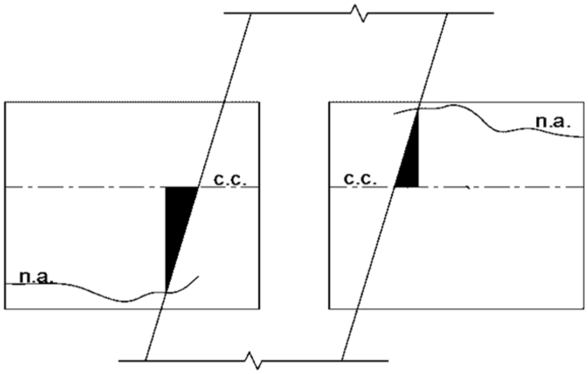
time, t = 7.0000 sec
drift, Δ = 5.3068 in.
rotation, Θ = 2.0128 °

Beam

time, t = 7.0000 sec
neutral axis, n.a. = 23.3637 in.
centroid, c.c. = 15.0000 in.

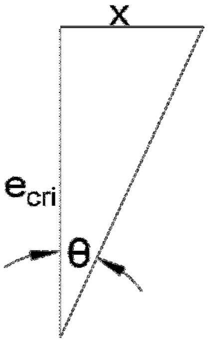
time, t = 7.0000 sec
neutral axis, n.a. = 3.1523 in.
centroid, c.c. = 15.0000 in.

Calculations



all measurements are taken from the top of the beam

q = rotation
e_{cri} = distance between the n.a. and c.c.
x = elongation



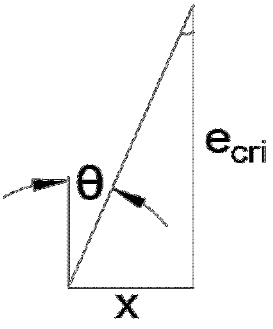
$$\tan\theta = \frac{opposite}{adjacent}$$

$$\tan\theta = \frac{x}{e_{cri}}$$

$$x = e_{cri}\tan\theta$$

e_{cri} = **8.3637** in.

x = **0.2939** in.



$$\tan\theta = \frac{opposite}{adjacent}$$

$$\tan\theta = \frac{x}{e_{cri}}$$

$x = e_{cri}\tan\theta$ due to similar triangles

e_{cri} = **11.8477** in.

x = **0.4164** in.

6.5.7

Left Side of Column E

Column

time, t = 7.0000 sec

drift, Δ = 6.2189 in.

rotation, Θ = 2.3584 °

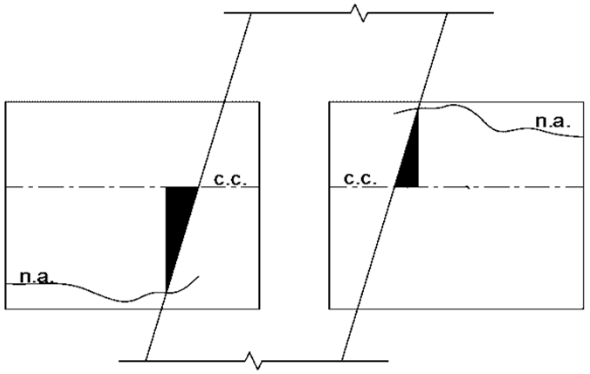
Beam

time, t = 7.0000 sec

neutral axis, n.a. = 18.7857 in.

centroid, c.c. = 15.0000 in.

Calculations

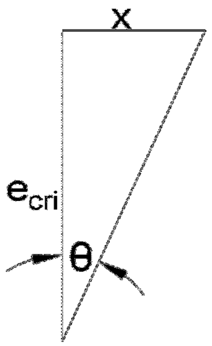


all measurements are taken from the top of the beam

q = rotation

e_{cri} = distance between the n.a. and c.c.

x = elongation



$$\tan\theta = \frac{opposite}{adjacent}$$

$$\tan\theta = \frac{x}{e_{cri}}$$

$$x = e_{cri}\tan\theta$$

e_{cri} = 3.7857 in.

x = 0.1559 in.

6.5.5

Right Side of Column A

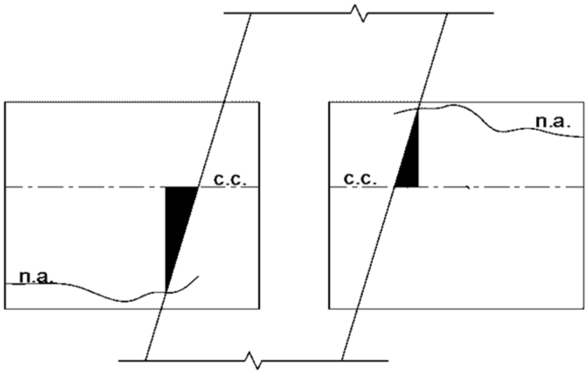
Column

time, t = 7.0000 sec
drift, Δ = 2.5016 in.
rotation, Θ = 0.9491 °

Beam

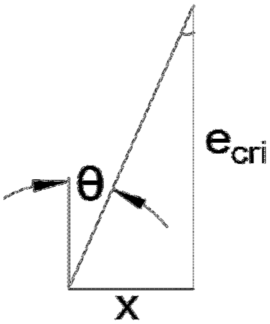
time, t = 7.0000 sec
neutral axis, n.a. = 4.5843 in.
centroid, c.c. = 15.0000 in.

Calculations



all measurements are taken from
the top of the beam

q = rotation
e_{cri} = distance between the n.a. and c.c.
x = elongation



$$\tan\theta = \frac{opposite}{adjacent}$$

$$\tan\theta = \frac{x}{e_{cri}}$$

$$x = e_{cri}\tan\theta$$ due to similar triangles

h = 10.4157 in.

x = 0.1726 in.

6.5.5

Left Side of Column B

Column

time, t = 7.0000 sec
drift, Δ = 3.0214 in.
rotation, Θ = 1.1463 °

Beam

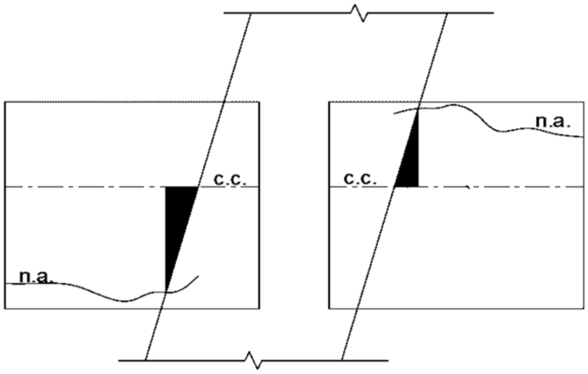
time, t = 7.0000 sec
neutral axis, n.a. = 25.9907 in.
centroid, c.c. = 15.0000 in.

Right Side of Column B

time, t = 7.0000 sec
drift, Δ = 3.0214 in.
rotation, Θ = 1.1463 °

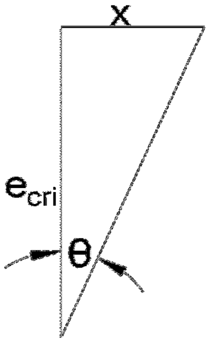
time, t = 7.0000 sec
neutral axis, n.a. = 4.1084 in.
centroid, c.c. = 15.0000 in.

Calculations



all measurements are taken from the top of the beam

q = rotation
e_{cri} = distance between the n.a. and c.c.
x = elongation



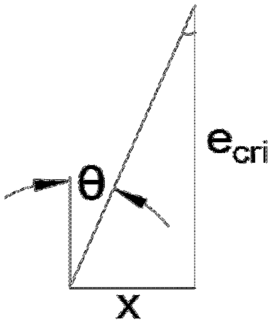
$$\tan\theta = \frac{opposite}{adjacent}$$

$$\tan\theta = \frac{x}{e_{cri}}$$

$$x = e_{cri}\tan\theta$$

h = 10.9907 in.

x = 0.2199 in.



$$\tan\theta = \frac{opposite}{adjacent}$$

$$\tan\theta = \frac{x}{e_{cri}}$$

$x = e_{cri}\tan\theta$ due to similar triangles

h = 10.8916 in.

x = 0.2179 in.

6.5.5

Left Side of Column C

Right Side of Column C

Column

time, t = 7.0000 sec
drift, Δ = 3.8350 in.
rotation, Θ = 1.4548 °

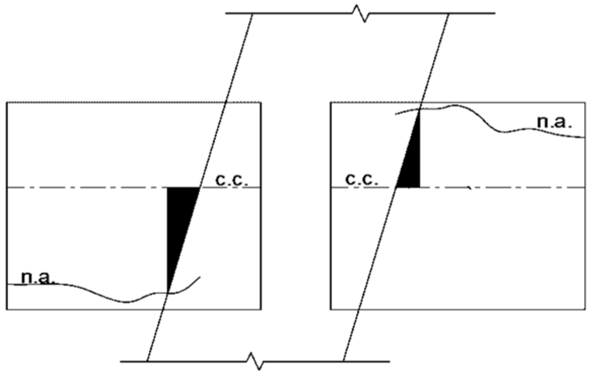
time, t = 7.0000 sec
drift, Δ = 3.8350 in.
rotation, Θ = 1.4548 °

Beam

time, t = 7.0000 sec
neutral axis, n.a. = 24.5828 in.
centroid, c.c. = 15.0000 in.

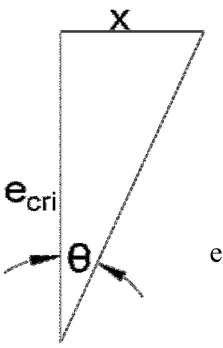
time, t = 7.0000 sec
neutral axis, n.a. = 3.2522 in.
centroid, c.c. = 15.0000 in.

Calculations



all measurements are taken from the top of the beam

q = rotation
h = distance between the n.a. and c.c.
x = elongation



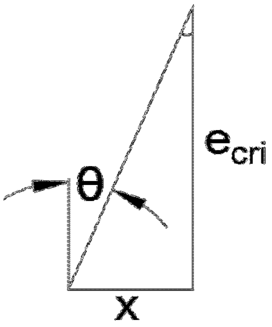
$$\tan\theta = \frac{opposite}{adjacent}$$

$$\tan\theta = \frac{x}{e_{cri}}$$

$$x = e_{cri}\tan\theta$$

e_{cri} = 9.5828 in.

x = 0.2434 in.



$$\tan\theta = \frac{opposite}{adjacent}$$

$$\tan\theta = \frac{x}{e_{cri}}$$

$x = e_{cri}\tan\theta$ due to similar triangles

h = 11.7478 in.

x = 0.2984 in.

6.5.5

Left Side of Column D

Column

time, t = 7.0000 sec

drift, Δ = 4.6078 in.

rotation, Θ = 1.7479 °

Beam

time, t = 7.0000 sec

neutral axis, n.a. = 22.5980 in.

centroid, c.c. = 15.0000 in.

Right Side of Column D

time, t = 7.0000 sec

drift, Δ = 4.6078 in.

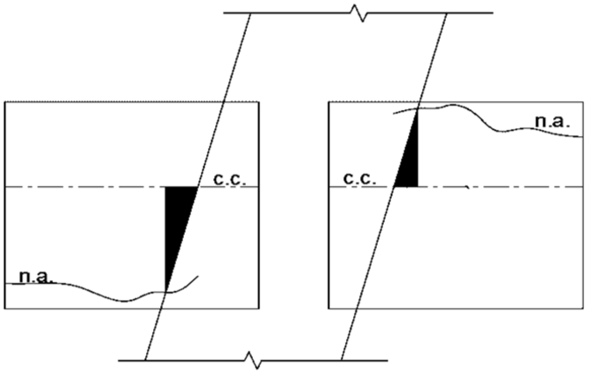
rotation, Θ = 1.7479 °

time, t = 7.0000 sec

neutral axis, n.a. = 3.6158 in.

centroid, c.c. = 15.0000 in.

Calculations

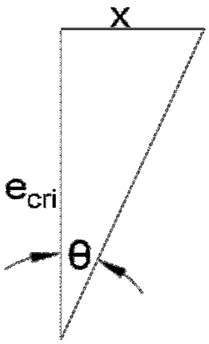


all measurements are taken from the top of the beam

q = rotation

h = distance between the n.a. and c.c.

x = elongation



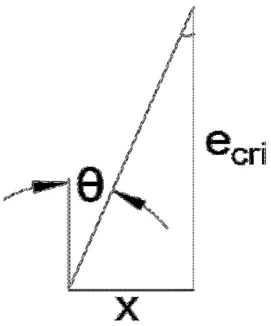
$$\tan\theta = \frac{opposite}{adjacent}$$

$$\tan\theta = \frac{x}{e_{cri}}$$

$$x = e_{cri}\tan\theta$$

e_{cri} = 7.5980 in.

x = 0.2319 in.



$$\tan\theta = \frac{opposite}{adjacent}$$

$$\tan\theta = \frac{x}{e_{cri}}$$

$x = e_{cri}\tan\theta$ due to similar triangles

e_{cri} = 11.3842 in.

x = 0.3474 in.

6.5.5

Left Side of Column D

Right Side of Column D

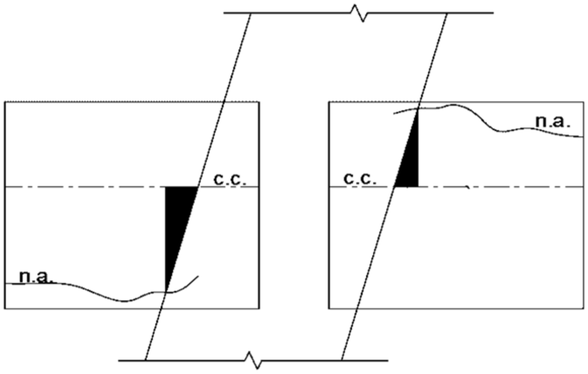
Column

	Left Side of Column D		Right Side of Column D	
time, t =	7.0000	sec	7.0000	sec
drift, Δ =	5.4534	in.	5.4534	in.
rotation, ϴ =	2.0683	°	2.0683	°

Beam

	Left Side of Column D		Right Side of Column D	
time, t =	7.0000	sec	7.0000	sec
neutral axis, n.a. =	22.0485	in.	3.4336	in.
centroid, c.c. =	15.0000	in.	15.0000	in.

Calculations

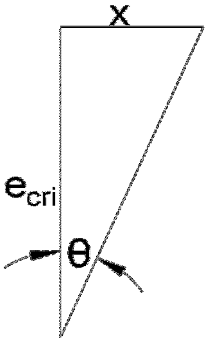


all measurements are taken from the top of the beam

q = rotation

e_{cri} = distance between the n.a. and c.c.

x = elongation



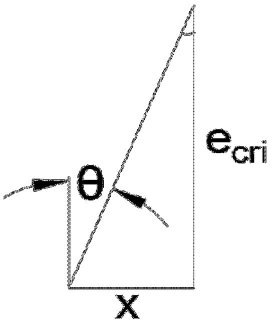
$$\tan\theta = \frac{opposite}{adjacent}$$

$$\tan\theta = \frac{x}{e_{cri}}$$

$$x = e_{cri}\tan\theta$$

e_{cri} = 7.0485 in.

x = 0.2546 in.



$$\tan\theta = \frac{opposite}{adjacent}$$

$$\tan\theta = \frac{x}{e_{cri}}$$

$x = e_{cri}\tan\theta$ due to similar triangles

e_{cri} = 11.5664 in.

x = 0.4177 in.

6.5.5

Left Side of Column E

Column

time, t = 7.0000 sec

drift, Δ = 6.4697 in.

rotation, Θ = 2.4534 °

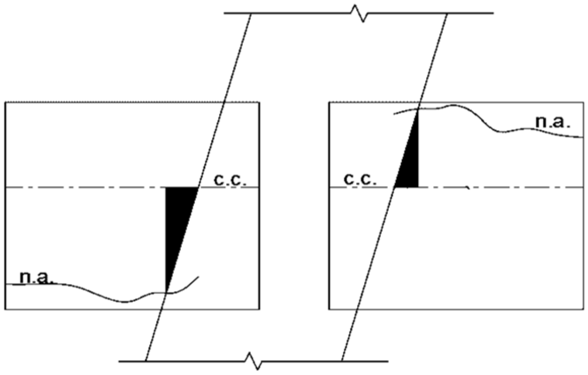
Beam

time, t = 7.0000 sec

neutral axis, n.a. = 18.7914 in.

centroid, c.c. = 15.0000 in.

Calculations

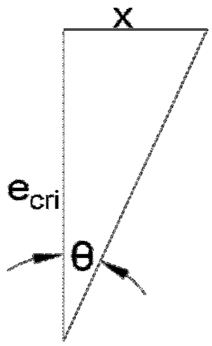


all measurements are taken from
the top of the beam

q = rotation

e_{cri} = distance between the n.a. and c.c.

x = elongation



$$\tan\theta = \frac{opposite}{adjacent}$$

$$\tan\theta = \frac{x}{e_{cri}}$$

$$x = e_{cri}\tan\theta$$

e_{cri} = 3.7914 in.

x = 0.1624 in.

7.4.7

Right Side of Column A

Column

time, t = 7.0000 sec

drift, Δ = 1.9719 in.

rotation, Θ = 0.7482 °

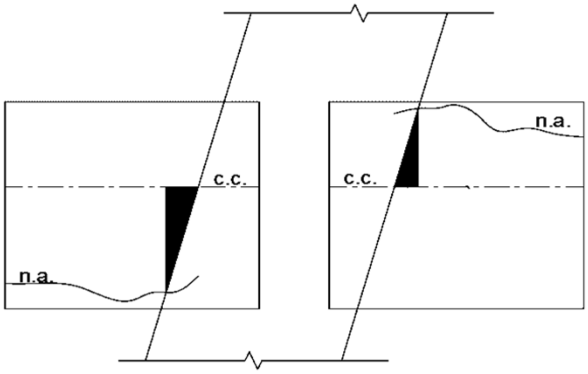
Beam

time, t = 7.0000 sec

neutral axis, n.a. = 4.6774 in.

centroid, c.c. = 15.0000 in.

Calculations

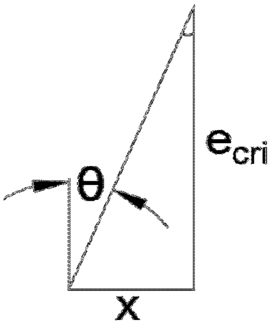


all measurements are taken from the top of the beam

q = rotation

e_{cri} = distance between the n.a. and c.c.

x = elongation



$$\tan\theta = \frac{opposite}{adjacent}$$

$$\tan\theta = \frac{x}{e_{cri}}$$

$x = e_{cri}\tan\theta$ due to similar triangles

e_{cri} = 10.3226 in.

x = 0.1348 in.

7.4.7

Left Side of Column B

Right Side of Column B

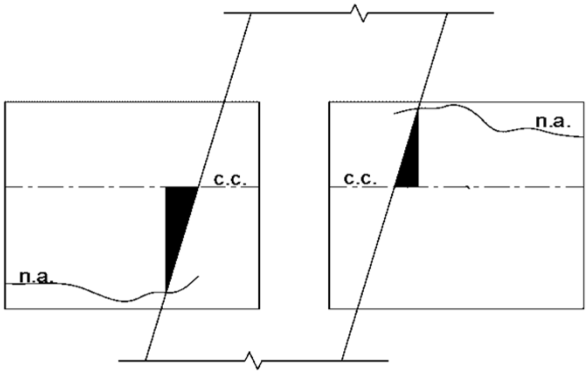
Column

	Left Side of Column B		Right Side of Column B	
time, t =	7.0000	sec	7.0000	sec
drift, Δ =	2.4352	in.	2.4352	in.
rotation, Θ =	0.9239	°	0.9239	°

Beam

	Left Side of Column B		Right Side of Column B	
time, t =	7.0000	sec	7.0000	sec
neutral axis, n.a. =	26.7809	in.	4.4574	in.
centroid, c.c. =	15.0000	in.	15.0000	in.

Calculations

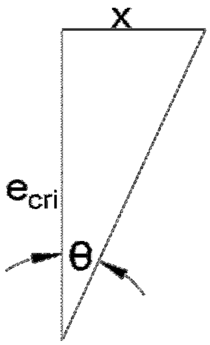


all measurements are taken from the top of the beam

q = rotation

e_{cri} = distance between the n.a. and c.c.

x = elongation



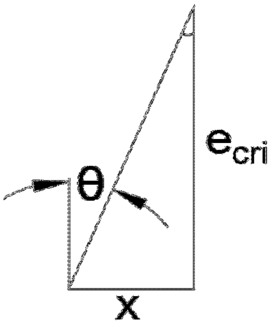
$$\tan\theta = \frac{opposite}{adjacent}$$

$$\tan\theta = \frac{x}{e_{cri}}$$

$$x = e_{cri}\tan\theta$$

e_{cri} = 11.7809 in.

x = 0.1900 in.



$$\tan\theta = \frac{opposite}{adjacent}$$

$$\tan\theta = \frac{x}{e_{cri}}$$

$x = e_{cri}\tan\theta$ due to similar triangles

e_{cri} = 10.5426 in.

x = 0.1700 in.

7.4.7

Left Side of Column C

Column

time, t = 7.0000 sec
drift, Δ = 2.9567 in.
rotation, Θ = 1.1217 °

Beam

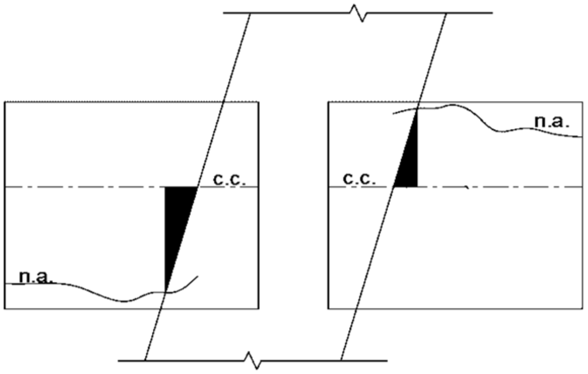
time, t = 7.0000 sec
neutral axis, n.a. = 25.8275 in.
centroid, c.c. = 15.0000 in.

Right Side of Column C

time, t = 7.0000 sec
drift, Δ = 2.9567 in.
rotation, Θ = 1.1217 °

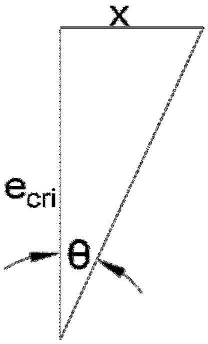
time, t = 7.0000 sec
neutral axis, n.a. = 3.5733 in.
centroid, c.c. = 15.0000 in.

Calculations



all measurements are taken from the top of the beam

q = rotation
e_{cri} = distance between the n.a. and c.c.
x = elongation



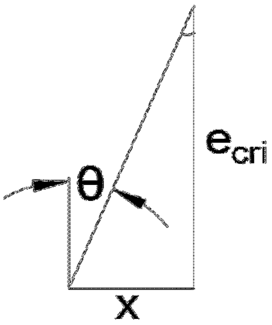
$$\tan\theta = \frac{opposite}{adjacent}$$

$$\tan\theta = \frac{x}{e_{cri}}$$

$$x = e_{cri}\tan\theta$$

e_{cri} = 10.8275 in.

x = 0.2120 in.



$$\tan\theta = \frac{opposite}{adjacent}$$

$$\tan\theta = \frac{x}{e_{cri}}$$

$x = e_{cri}\tan\theta$ due to similar triangles

e_{cri} = 11.4267 in.

x = 0.2237 in.

7.4.7

Left Side of Column D

Right Side of Column D

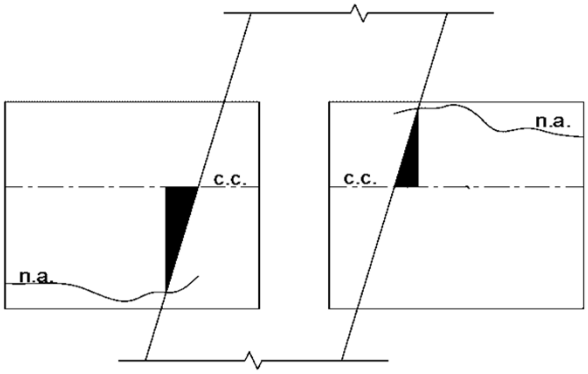
Column

time, t =	7.0000	sec	time, t =	7.0000	sec
drift, Δ =	3.5388	in.	drift, Δ =	3.5388	in.
rotation, Θ =	1.3425	°	rotation, Θ =	1.3425	°

Beam

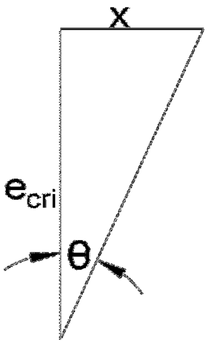
time, t =	7.0000	sec	time, t =	7.0000	sec
neutral axis, n.a. =	25.2582	in.	neutral axis, n.a. =	3.2996	in.
centroid, c.c. =	15.0000	in.	centroid, c.c. =	15.0000	in.

Calculations



all measurements are taken from the top of the beam

q = rotation
e_{cri} = distance between the n.a. and c.c.
x = elongation



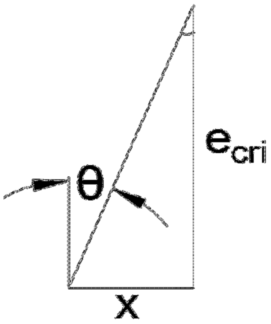
$$\tan\theta = \frac{opposite}{adjacent}$$

$$\tan\theta = \frac{x}{e_{cri}}$$

$$x = e_{cri}\tan\theta$$

e_{cri} = 10.2582 in.

x = 0.2404 in.



$$\tan\theta = \frac{opposite}{adjacent}$$

$$\tan\theta = \frac{x}{e_{cri}}$$

$x = e_{cri}\tan\theta$ due to similar triangles

e_{cri} = 11.7004 in.

x = 0.2742 in.

7.4.7

Left Side of Column D

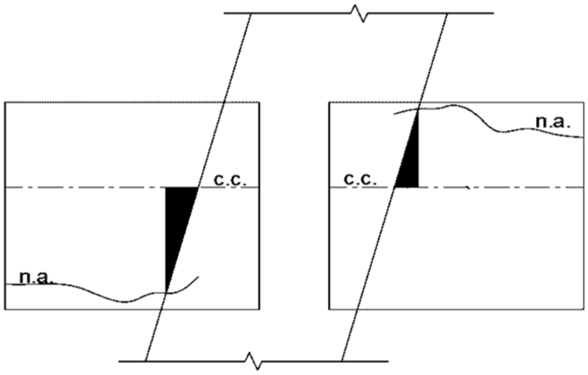
Column

time, t = 7.0000 sec
drift, Δ = 4.3296 in.
rotation, Θ = 1.6424 °

Beam

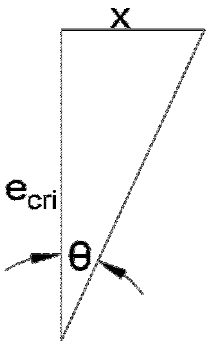
time, t = 7.0000 sec
neutral axis, n.a. = 21.4512 in.
centroid, c.c. = 15.0000 in.

Calculations



all measurements are taken from
the top of the beam

q = rotation
e_{cri} = distance between the n.a. and c.c.
x = elongation



$$\tan\theta = \frac{opposite}{adjacent}$$

$$\tan\theta = \frac{x}{e_{cri}}$$

$$x = e_{cri}\tan\theta$$

e_{cri} = **6.4512** in.

x = **0.1850** in.

7.4.5

Right Side of Column A

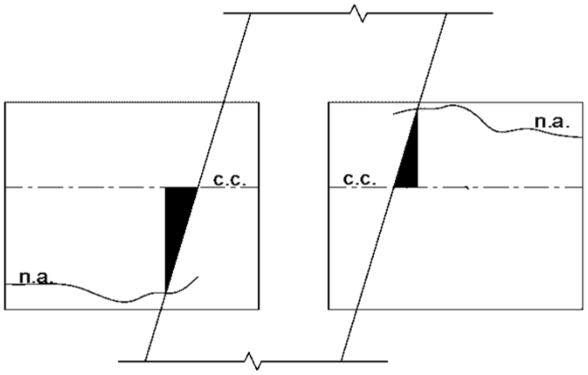
Column

time, t = 7.0000 sec
drift, Δ = 1.9253 in.
rotation, Θ = 0.7305 °

Beam

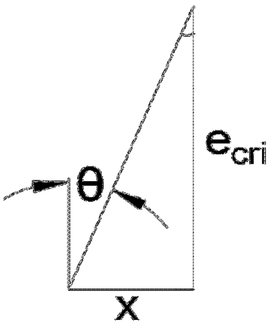
time, t = 7.0000 sec
neutral axis, n.a. = 4.8346 in.
centroid, c.c. = 15.0000 in.

Calculations



all measurements are taken from the top of the beam

q = rotation
e_{cri} = distance between the n.a. and c.c.
x = elongation



$$\tan\theta = \frac{opposite}{adjacent}$$

$$\tan\theta = \frac{x}{e_{cri}}$$

$x = e_{cri}\tan\theta$ due to similar triangles

e_{cri} = **10.1654** in.

x = **0.1296** in.

7.4.5

Left Side of Column B

Column

time, t = 7.0000 sec
drift, Δ = 2.4177 in.
rotation, Θ = 0.9173 °

Beam

time, t = 7.0000 sec
neutral axis, n.a. = 26.4464 in.
centroid, c.c. = 15.0000 in.

Right Side of Column B

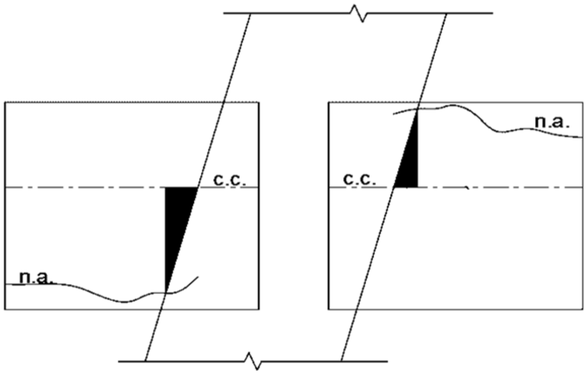
Column

time, t = 7.0000 sec
drift, Δ = 2.4177 in.
rotation, Θ = 0.9173 °

Beam

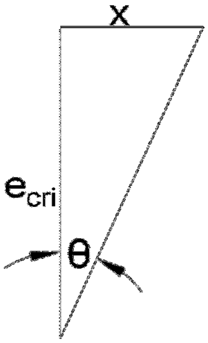
time, t = 7.0000 sec
neutral axis, n.a. = 5.9833 in.
centroid, c.c. = 15.0000 in.

Calculations



all measurements are taken from the top of the beam

q = rotation
e_{cri} = distance between the n.a. and c.c.
x = elongation



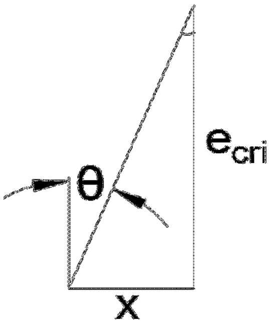
$$\tan\theta = \frac{opposite}{adjacent}$$

$$\tan\theta = \frac{x}{e_{cri}}$$

$$x = e_{cri}\tan\theta$$

e_{cri} = 11.4464 in.

x = 0.1833 in.



$$\tan\theta = \frac{opposite}{adjacent}$$

$$\tan\theta = \frac{x}{e_{cri}}$$

$x = e_{cri}\tan\theta$ due to similar triangles

e_{cri} = 9.0167 in.

x = 0.1444 in.

7.4.5

Left Side of Column C

Column

time, t = 7.0000 sec
drift, Δ = 2.9958 in.
rotation, Θ = 1.1366 °

Beam

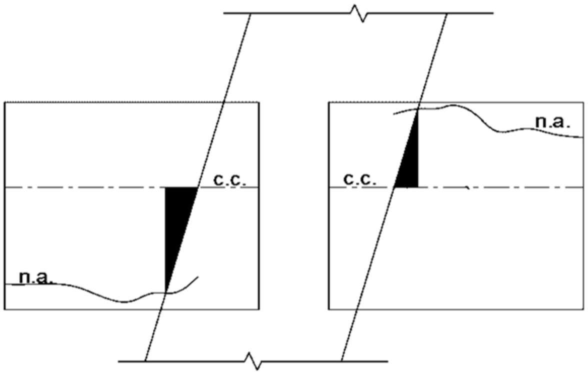
time, t = 7.0000 sec
neutral axis, n.a. = 25.3069 in.
centroid, c.c. = 15.0000 in.

Right Side of Column C

time, t = 7.0000 sec
drift, Δ = 2.9958 in.
rotation, Θ = 1.1366 °

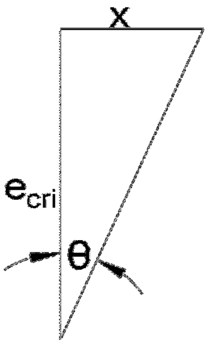
time, t = 7.0000 sec
neutral axis, n.a. = 4.1259 in.
centroid, c.c. = 15.0000 in.

Calculations



all measurements are taken from the top of the beam

q = rotation
e_{cri} = distance between the n.a. and c.c.
x = elongation



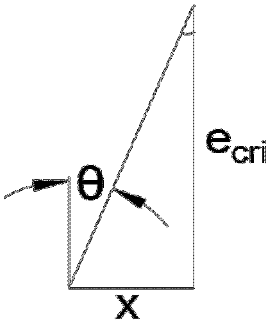
$$\tan\theta = \frac{opposite}{adjacent}$$

$$\tan\theta = \frac{x}{e_{cri}}$$

$$x = e_{cri}\tan\theta$$

e_{cri} = **10.3069** in.

x = **0.2045** in.



$$\tan\theta = \frac{opposite}{adjacent}$$

$$\tan\theta = \frac{x}{e_{cri}}$$

$x = e_{cri}\tan\theta$ due to similar triangles

e_{cri} = **10.8741** in.

x = **0.2157** in.

7.4.5

Left Side of Column D

Right Side of Column D

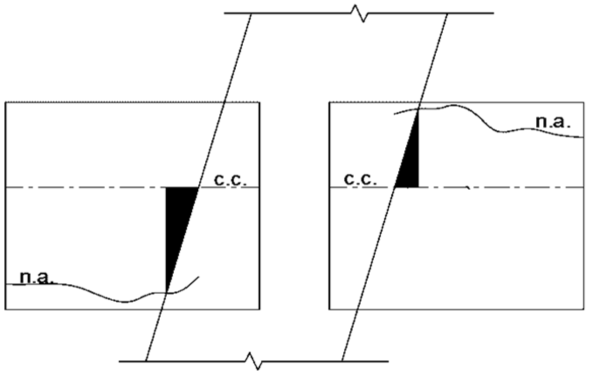
Column

time, t =	7.0000	sec	time, t =	7.0000	sec
drift, Δ =	3.6582	in.	drift, Δ =	3.6582	in.
rotation, Θ =	1.3878	°	rotation, Θ =	1.3878	°

Beam

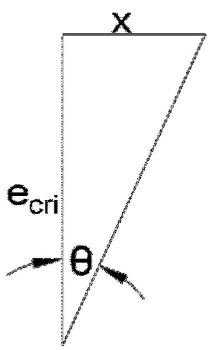
time, t =	7.0000	sec	time, t =	7.0000	sec
neutral axis, n.a. =	24.2720	in.	neutral axis, n.a. =	3.5917	in.
centroid, c.c. =	15.0000	in.	centroid, c.c. =	15.0000	in.

Calculations



all measurements are taken from the top of the beam

q = rotation
e_{cri} = distance between the n.a. and c.c.
x = elongation



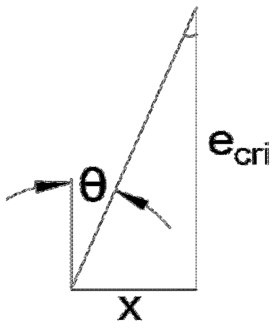
$$\tan\theta = \frac{opposite}{adjacent}$$

$$\tan\theta = \frac{x}{e_{cri}}$$

$$x = e_{cri}\tan\theta$$

e_{cri} = 9.2720 in.

x = 0.2246 in.



$$\tan\theta = \frac{opposite}{adjacent}$$

$$\tan\theta = \frac{x}{e_{cri}}$$

$x = e_{cri}\tan\theta$ due to similar triangles

e_{cri} = 11.4083 in.

x = 0.2764 in.

7.4.5

Left Side of Column D

Column

time, t = 7.0000 sec

drift, Δ = 4.5024 in.

rotation, Θ = 1.7079 °

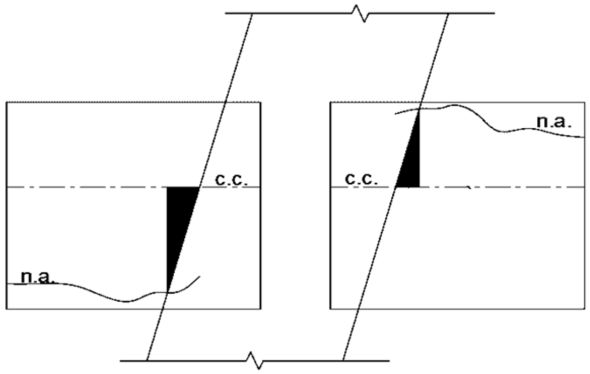
Beam

time, t = 7.0000 sec

neutral axis, n.a. = 21.7310 in.

centroid, c.c. = 15.0000 in.

Calculations

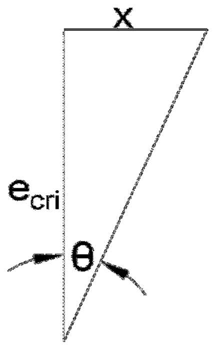


all measurements are taken from the top of the beam

q = rotation

e_{cri} = distance between the n.a. and c.c.

x = elongation



$$\tan\theta = \frac{opposite}{adjacent}$$

$$\tan\theta = \frac{x}{e_{cri}}$$

$$x = e_{cri}\tan\theta$$

e_{cri} = **6.7310** in.

x = **0.2007** in.

6.4.7

Right Side of Column A

Column

time, t = 7.0000 sec

drift, Δ = 2.8104 in.

rotation, Θ = 1.0663 °

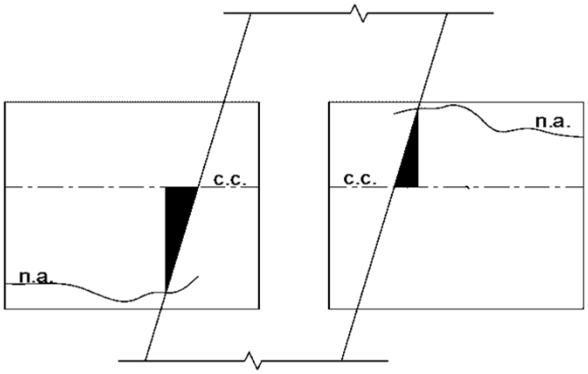
Beam

time, t = 7.0000 sec

neutral axis, n.a. = 3.1690 in.

centroid, c.c. = 15.0000 in.

Calculations

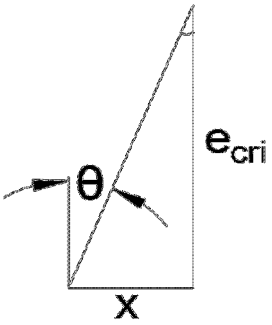


all measurements are taken from the top of the beam

q = rotation

e_{cri} = distance between the n.a. and c.c.

x = elongation



$$\tan\theta = \frac{opposite}{adjacent}$$

$$\tan\theta = \frac{x}{e_{cri}}$$

$$x = e_{cri}\tan\theta$$ due to similar triangles

e_{cri} = 11.8310 in.

x = 0.2202 in.

6.4.7

Left Side of Column B

Column

time, t = 7.0000 sec
drift, Δ = 3.4313 in.
rotation, Θ = 1.3018 °

Beam

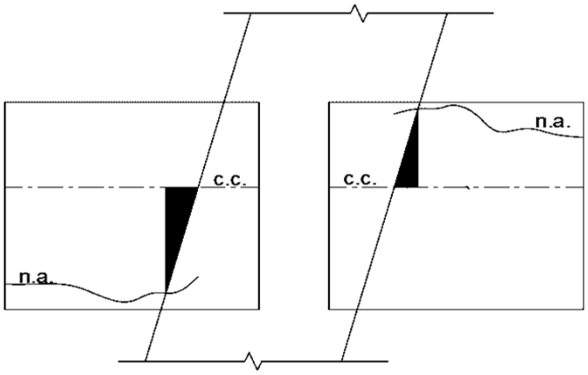
time, t = 7.0000 sec
neutral axis, n.a. = 25.7142 in.
centroid, c.c. = 15.0000 in.

Right Side of Column B

time, t = 7.0000 sec
drift, Δ = 3.4313 in.
rotation, Θ = 1.3018 °

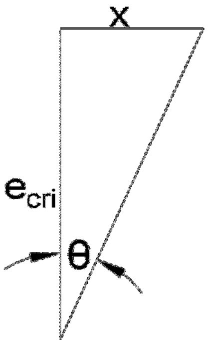
time, t = 7.0000 sec
neutral axis, n.a. = 3.6799 in.
centroid, c.c. = 15.0000 in.

Calculations



all measurements are taken from the top of the beam

q = rotation
e_{cri} = distance between the n.a. and c.c.
x = elongation



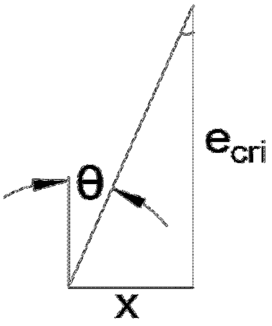
$$\tan\theta = \frac{opposite}{adjacent}$$

$$\tan\theta = \frac{x}{e_{cri}}$$

$$x = e_{cri}\tan\theta$$

e_{cri} = 10.7142 in.

x = 0.2435 in.



$$\tan\theta = \frac{opposite}{adjacent}$$

$$\tan\theta = \frac{x}{e_{cri}}$$

$$x = e_{cri}\tan\theta$$ due to similar triangles

e_{cri} = 11.3201 in.

x = 0.2572 in.

6.4.7

Left Side of Column C

Column

time, t = 7.0000 sec
drift, Δ = 4.1465 in.
rotation, Θ = 1.5730 °

Beam

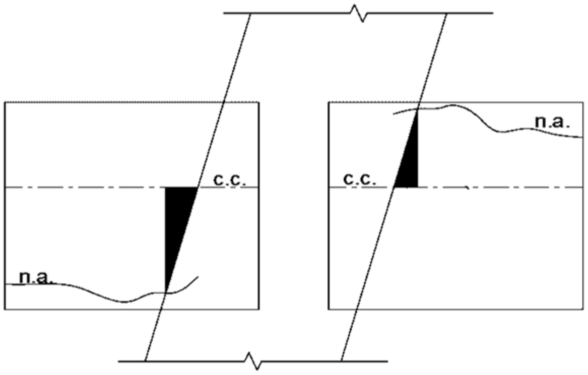
time, t = 7.0000 sec
neutral axis, n.a. = 24.1424 in.
centroid, c.c. = 15.0000 in.

Right Side of Column C

time, t = 7.0000 sec
drift, Δ = 4.1465 in.
rotation, Θ = 1.5730 °

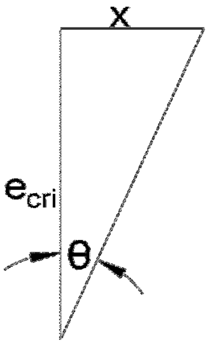
time, t = 7.0000 sec
neutral axis, n.a. = 3.1012 in.
centroid, c.c. = 15.0000 in.

Calculations



all measurements are taken from the top of the beam

q = rotation
e_{cri} = distance between the n.a. and c.c.
x = elongation



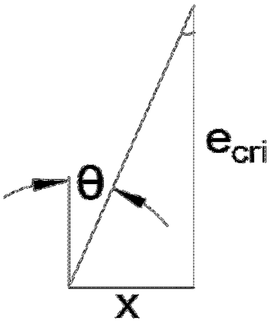
$$\tan\theta = \frac{opposite}{adjacent}$$

$$\tan\theta = \frac{x}{e_{cri}}$$

$$x = e_{cri}\tan\theta$$

e_{cri} = 9.1424 in.

x = 0.2511 in.



$$\tan\theta = \frac{opposite}{adjacent}$$

$$\tan\theta = \frac{x}{e_{cri}}$$

$x = e_{cri}\tan\theta$ due to similar triangles

e_{cri} = 11.8988 in.

x = 0.3267 in.

6.4.7

Left Side of Column D

Column

time, t = 7.0000 sec
drift, Δ = 4.9176 in.
rotation, Θ = 1.8653 °

Beam

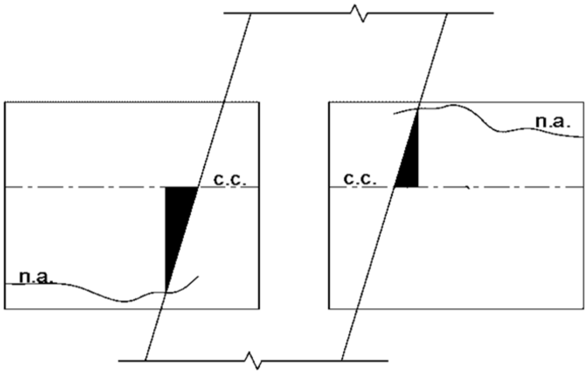
time, t = 7.0000 sec
neutral axis, n.a. = 23.4868 in.
centroid, c.c. = 15.0000 in.

Right Side of Column D

time, t = 7.0000 sec
drift, Δ = 4.9176 in.
rotation, Θ = 1.8653 °

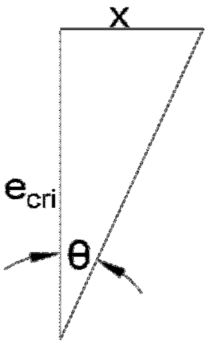
time, t = 7.0000 sec
neutral axis, n.a. = 3.0610 in.
centroid, c.c. = 15.0000 in.

Calculations



all measurements are taken from the top of the beam

q = rotation
e_{cri} = distance between the n.a. and c.c.
x = elongation



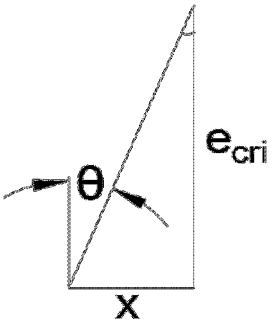
$$\tan\theta = \frac{opposite}{adjacent}$$

$$\tan\theta = \frac{x}{e_{cri}}$$

$$x = e_{cri}\tan\theta$$

e_{cri} = 8.4868 in.

x = 0.2764 in.



$$\tan\theta = \frac{opposite}{adjacent}$$

$$\tan\theta = \frac{x}{e_{cri}}$$

$$x = e_{cri}\tan\theta$$ due to similar triangles

e_{cri} = 11.9390 in.

x = 0.3888 in.

6.4.7

Left Side of Column D

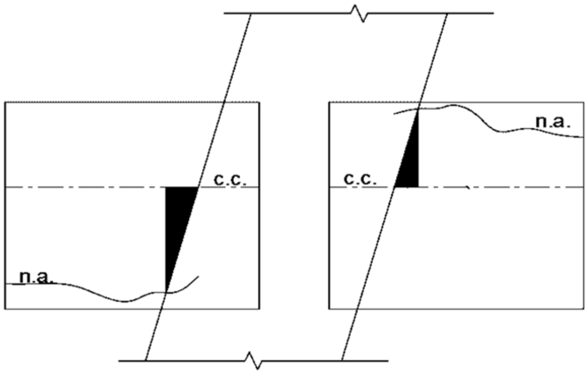
Column

time, t = 7.0000 sec
drift, Δ = 5.9061 in.
rotation, ϴ = 2.2399 °

Beam

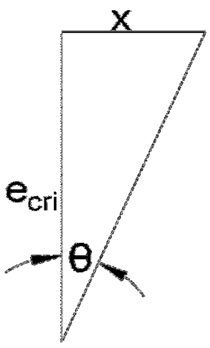
time, t = 7.0000 sec
neutral axis, n.a. = 19.2151 in.
centroid, c.c. = 15.0000 in.

Calculations



all measurements are taken from
the top of the beam

q = rotation
e_{cri} = distance between the n.a. and c.c.
x = elongation



$$\tan\theta = \frac{opposite}{adjacent}$$

$$\tan\theta = \frac{x}{e_{cri}}$$

$$x = e_{cri}\tan\theta$$

e_{cri} = **4.2151** in.

x = **0.1649** in.

6.4.5

Right Side of Column A

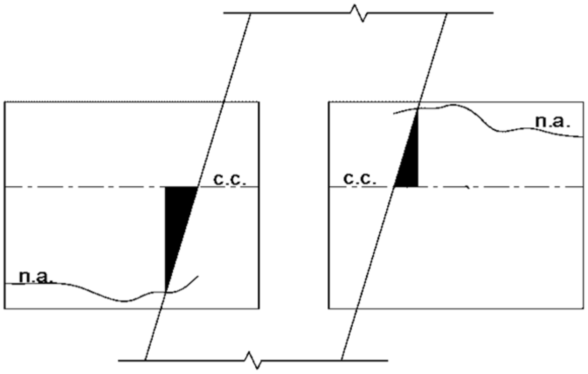
Column

time, t = 7.0000 sec
drift, Δ = 2.7541 in.
rotation, Θ = 1.0449 °

Beam

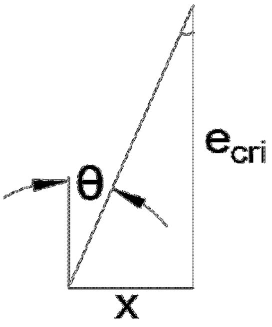
time, t = 7.0000 sec
neutral axis, n.a. = 4.5240 in.
centroid, c.c. = 15.0000 in.

Calculations



all measurements are taken from the top of the beam

q = rotation
e_{cri} = distance between the n.a. and c.c.
x = elongation



$$\tan\theta = \frac{opposite}{adjacent}$$

$$\tan\theta = \frac{x}{e_{cri}}$$

$$x = e_{cri}\tan\theta$$ due to similar triangles

e_{cri} = 10.4760 in.

x = 0.1911 in.

6.4.5

Left Side of Column B

Column

time, t = 7.0000 sec

drift, Δ = 3.4228 in.

rotation, ϴ = 1.2985 °

Beam

time, t = 7.0000 sec

neutral axis, n.a. = 25.5876 in.

centroid, c.c. = 15.0000 in.

Right Side of Column B

time, t = 7.0000 sec

drift, Δ = 3.4228 in.

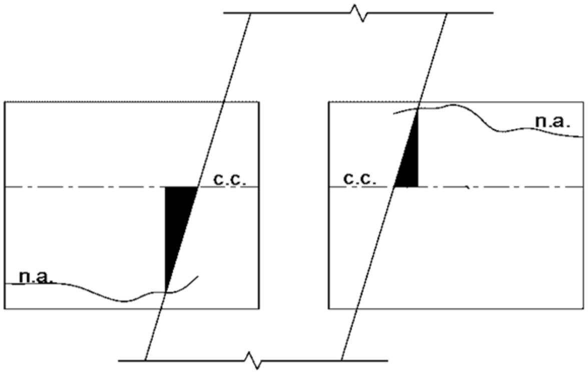
rotation, ϴ = 1.2985 °

time, t = 7.0000 sec

neutral axis, n.a. = 3.6834 in.

centroid, c.c. = 15.0000 in.

Calculations

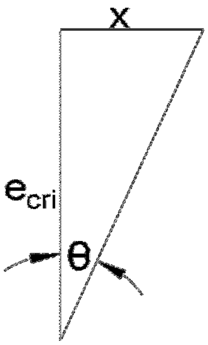


all measurements are taken from the top of the beam

q = rotation

e_{cri} = distance between the n.a. and c.c.

x = elongation



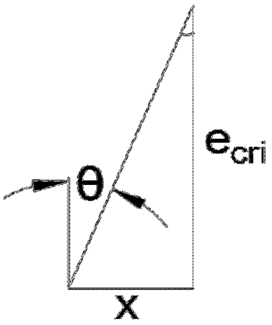
$$\tan\theta = \frac{opposite}{adjacent}$$

$$\tan\theta = \frac{x}{e_{cri}}$$

$$x = e_{cri}\tan\theta$$

e_{cri} = 10.5876 in.

x = 0.2400 in.



$$\tan\theta = \frac{opposite}{adjacent}$$

$$\tan\theta = \frac{x}{e_{cri}}$$

$x = e_{cri}\tan\theta$ due to similar triangles

e_{cri} = 11.3166 in.

x = 0.2565 in.

6.4.5

Left Side of Column C

Column

time, t = 7.0000 sec
drift, Δ = 4.2059 in.
rotation, Θ = 1.5955 °

Beam

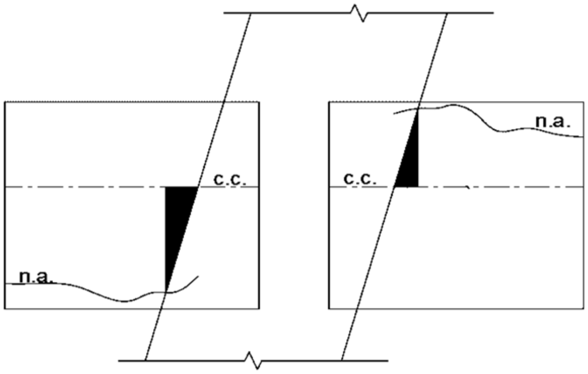
time, t = 7.0000 sec
neutral axis, n.a. = 23.4972 in.
centroid, c.c. = 15.0000 in.

Right Side of Column C

time, t = 7.0000 sec
drift, Δ = 4.2059 in.
rotation, Θ = 1.5955 °

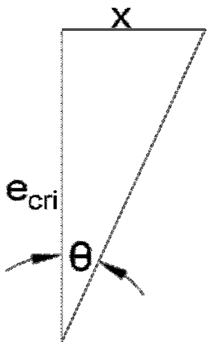
time, t = 7.0000 sec
neutral axis, n.a. = 3.2943 in.
centroid, c.c. = 15.0000 in.

Calculations



all measurements are taken from the top of the beam

q = rotation
e_{cri} = distance between the n.a. and c.c.
x = elongation



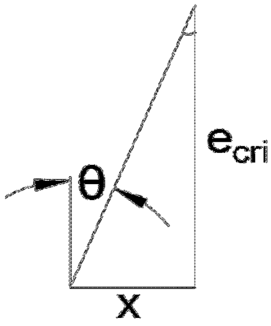
$$\tan\theta = \frac{opposite}{adjacent}$$

$$\tan\theta = \frac{x}{e_{cri}}$$

$$x = e_{cri}\tan\theta$$

e_{cri} = 8.4972 in.

x = 0.2367 in.



$$\tan\theta = \frac{opposite}{adjacent}$$

$$\tan\theta = \frac{x}{e_{cri}}$$

$x = e_{cri}\tan\theta$ due to similar triangles

e_{cri} = 11.7057 in.

x = 0.3260 in.

6.4.5

Left Side of Column D

Column

time, t = 7.0000 sec

drift, Δ = 5.0614 in.

rotation, Θ = 1.9198 °

Beam

time, t = 7.0000 sec

neutral axis, n.a. = 22.0068 in.

centroid, c.c. = 15.0000 in.

Right Side of Column D

time, t = 7.0000 sec

drift, Δ = 5.0614 in.

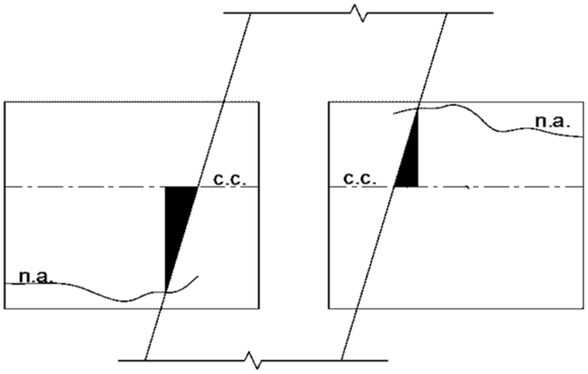
rotation, Θ = 1.9198 °

time, t = 7.0000 sec

neutral axis, n.a. = 3.2743 in.

centroid, c.c. = 15.0000 in.

Calculations

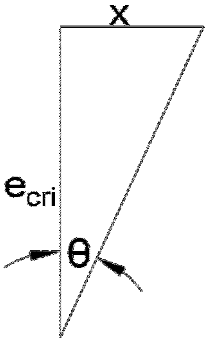


all measurements are taken from the top of the beam

q = rotation

e_{cri} = distance between the n.a. and c.c.

x = elongation



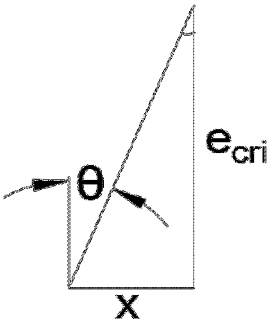
$$\tan\theta = \frac{opposite}{adjacent}$$

$$\tan\theta = \frac{x}{e_{cri}}$$

$$x = e_{cri}\tan\theta$$

h = 7.0068 in.

x = 0.2349 in.



$$\tan\theta = \frac{opposite}{adjacent}$$

$$\tan\theta = \frac{x}{e_{cri}}$$

$x = e_{cri}\tan\theta$ due to similar triangles

e_{cri} = 11.7257 in.

x = 0.3930 in.

6.4.5

Left Side of Column D

Column

time, t = 7.0000 sec

drift, Δ = 6.0929 in.

rotation, Θ = 2.3107 °

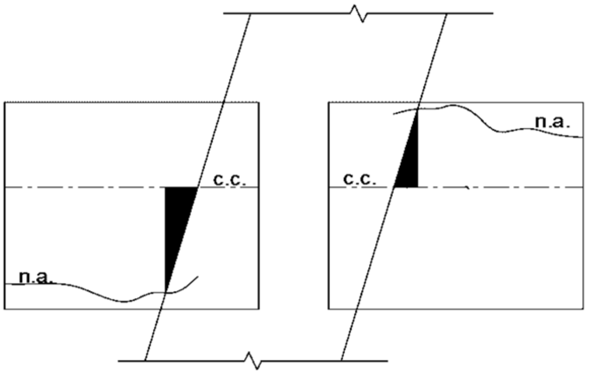
Beam

time, t = 7.0000 sec

neutral axis, n.a. = 32.9340 in.

centroid, c.c. = 15.0000 in.

Calculations

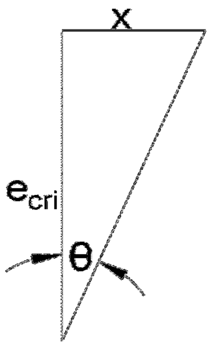


all measurements are taken from
the top of the beam

q = rotation

e_{cri} = distance between the n.a. and c.c.

x = elongation



$$\tan\theta = \frac{opposite}{adjacent}$$

$$\tan\theta = \frac{x}{e_{cri}}$$

$$x = e_{cri}\tan\theta$$

e_{cri} = **17.9340** in.

x = **0.7236** in.

7.3.7

Right Side of Column A

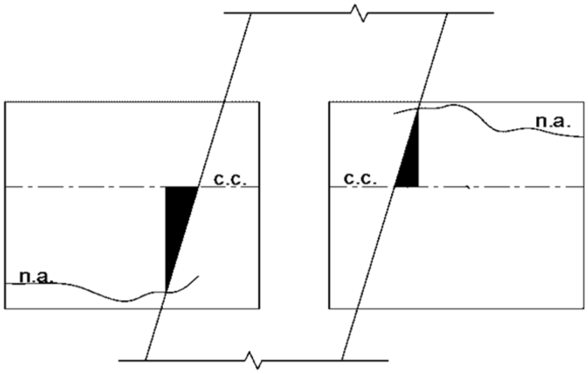
Column

time, t = 7.0000 sec
drift, Δ = 2.1891 in.
rotation, Θ = 0.8306 °

Beam

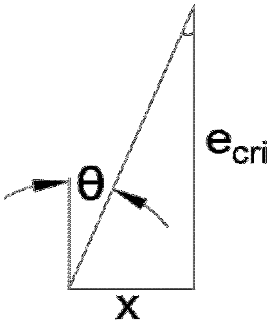
time, t = 7.0000 sec
neutral axis, n.a. = 3.5800 in.
centroid, c.c. = 15.0000 in.

Calculations



all measurements are taken from the top of the beam

q = rotation
e_{cri} = distance between the n.a. and c.c.
x = elongation



$$\tan\theta = \frac{opposite}{adjacent}$$

$$\tan\theta = \frac{x}{e_{cri}}$$

$x = e_{cri}\tan\theta$ due to similar triangles

e_{cri} = **11.4200** in.

x = **0.1656** in.

7.3.7

Left Side of Column B

Column

time, t = 7.0000 sec
drift, Δ = 2.6913 in.
rotation, ϴ = 1.0211 °

Beam

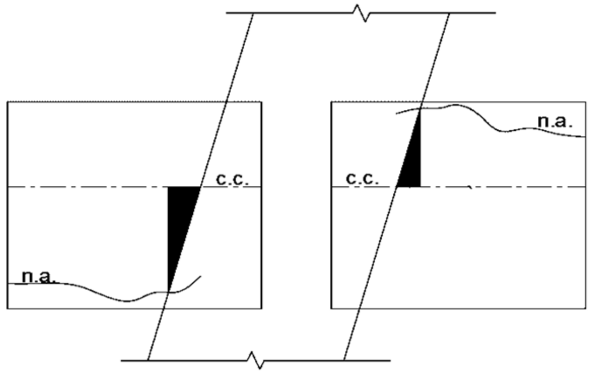
time, t = 7.0000 sec
neutral axis, n.a. = 26.4647 in.
centroid, c.c. = 15.0000 in.

Right Side of Column B

time, t = 7.0000 sec
drift, Δ = 2.6913 in.
rotation, ϴ = 1.0211 °

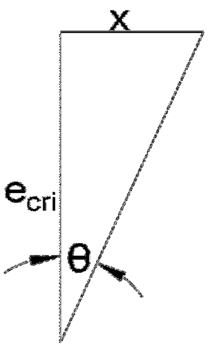
time, t = 7.0000 sec
neutral axis, n.a. = 4.5276 in.
centroid, c.c. = 15.0000 in.

Calculations



all measurements are taken from the top of the beam

q = rotation
e_{cri} = distance between the n.a. and c.c.
x = elongation



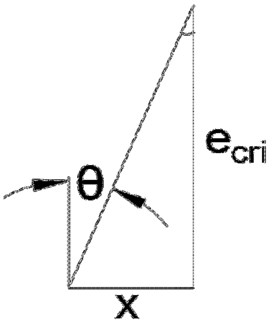
$$\tan\theta = \frac{opposite}{adjacent}$$

$$\tan\theta = \frac{x}{e_{cri}}$$

$$x = e_{cri}\tan\theta$$

e_{cri} = 11.4647 in.

x = 0.2043 in.



$$\tan\theta = \frac{opposite}{adjacent}$$

$$\tan\theta = \frac{x}{e_{cri}}$$

$x = e_{cri}\tan\theta$ due to similar triangles

e_{cri} = 10.4724 in.

x = 0.1867 in.

7.3.7

Left Side of Column C

Column

time, t = 7.0000 sec
drift, Δ = 3.2637 in.
rotation, Θ = 1.2382 °

Beam

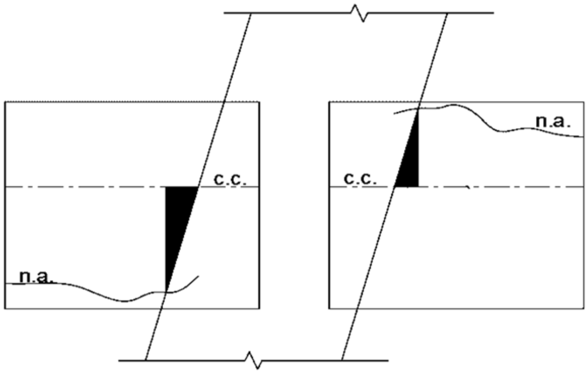
time, t = 7.0000 sec
neutral axis, n.a. = 25.5756 in.
centroid, c.c. = 15.0000 in.

Right Side of Column C

time, t = 7.0000 sec
drift, Δ = 3.2637 in.
rotation, Θ = 1.2382 °

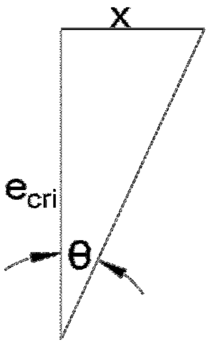
time, t = 7.0000 sec
neutral axis, n.a. = 3.7322 in.
centroid, c.c. = 15.0000 in.

Calculations



all measurements are taken from the top of the beam

q = rotation
e_{cri} = distance between the n.a. and c.c.
x = elongation



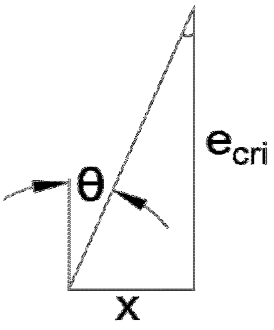
$$\tan\theta = \frac{opposite}{adjacent}$$

$$\tan\theta = \frac{x}{e_{cri}}$$

$$x = e_{cri}\tan\theta$$

e_{cri} = **10.5756** in.

x = **0.2286** in.



$$\tan\theta = \frac{opposite}{adjacent}$$

$$\tan\theta = \frac{x}{e_{cri}}$$

$x = e_{cri}\tan\theta$ due to similar triangles

e_{cri} = **11.2678** in.

x = **0.2435** in.

7.3.7

Left Side of Column D

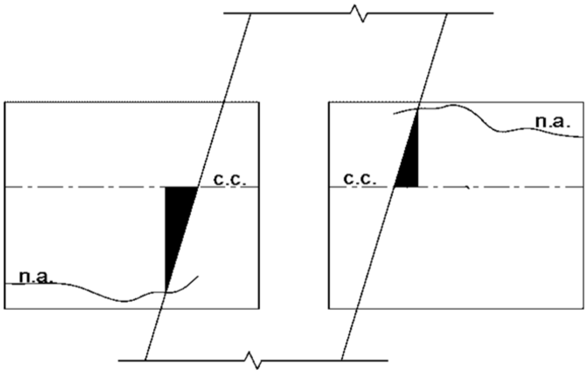
Column

time, t = 7.0000 sec
drift, Δ = 4.0290 in.
rotation, Θ = 1.5284 °

Beam

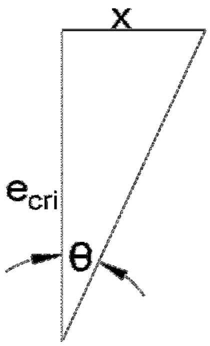
time, t = 7.0000 sec
neutral axis, n.a. = 22.3009 in.
centroid, c.c. = 15.0000 in.

Calculations



all measurements are taken from
the top of the beam

q = rotation
e_{cri} = distance between the n.a. and c.c.
x = elongation



$$\tan\theta = \frac{opposite}{adjacent}$$

$$\tan\theta = \frac{x}{e_{cri}}$$

$$x = e_{cri}\tan\theta$$

e_{cri} = 7.3009 in.

x = 0.1948 in.

7.3.5

Right Side of Column A

Column

time, t = 7.0000 sec

drift, Δ = 2.1573 in.

rotation, Θ = 0.8185 °

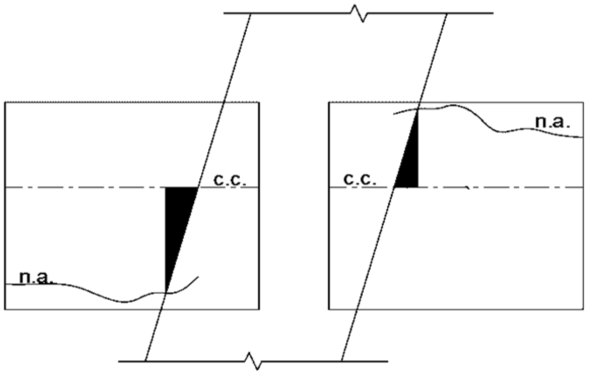
Beam

time, t = 7.0000 sec

neutral axis, n.a. = 4.7660 in.

centroid, c.c. = 15.0000 in.

Calculations

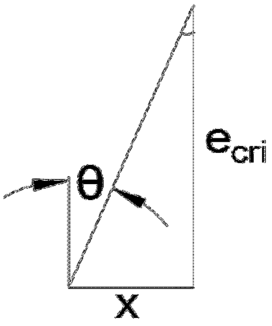


all measurements are taken from the top of the beam

q = rotation

e_{cri} = distance between the n.a. and c.c.

x = elongation



$$\tan\theta = \frac{opposite}{adjacent}$$

$$\tan\theta = \frac{x}{e_{cri}}$$

$x = e_{cri}\tan\theta$ due to similar triangles

e_{cri} = **10.2340** in.

x = **0.1462** in.

7.3.5

Left Side of Column B

Right Side of Column B

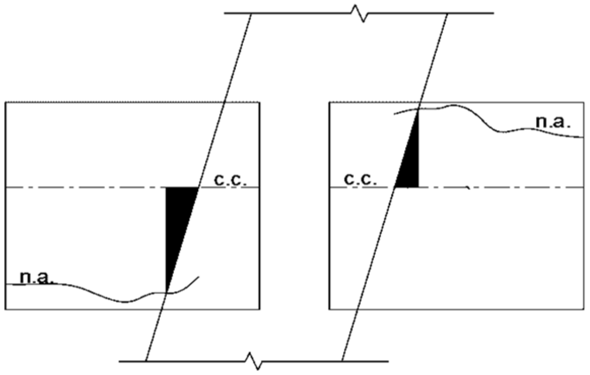
Column

time, t =	7.0000	sec	time, t =	7.0000	sec
drift, Δ =	2.6975	in.	drift, Δ =	2.6975	in.
rotation, Θ =	1.0234	°	rotation, Θ =	1.0234	°

Beam

time, t =	7.0000	sec	time, t =	7.0000	sec
neutral axis, n.a. =	26.0353	in.	neutral axis, n.a. =	5.6360	in.
centroid, c.c. =	15.0000	in.	centroid, c.c. =	15.0000	in.

Calculations

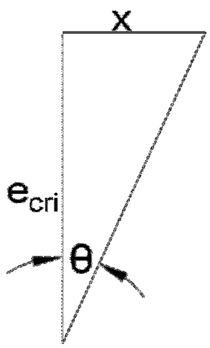


all measurements are taken from the top of the beam

q = rotation

e_{cri} = distance between the n.a. and c.c.

x = elongation



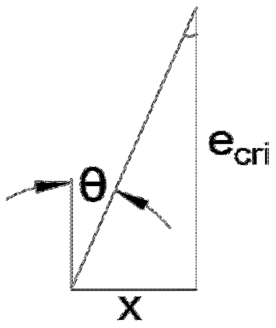
$$\tan\theta = \frac{opposite}{adjacent}$$

$$\tan\theta = \frac{x}{e_{cri}}$$

$$x = e_{cri}\tan\theta$$

e_{cri} = 11.0353 in.

x = 0.1971 in.



$$\tan\theta = \frac{opposite}{adjacent}$$

$$\tan\theta = \frac{x}{e_{cri}}$$

$x = e_{cri}\tan\theta$ due to similar triangles

e_{cri} = 9.3640 in.

x = 0.1673 in.

7.3.5

Left Side of Column C

Column

time, t = 7.0000 sec
drift, Δ = 3.3360 in.
rotation, Θ = 1.2656 °

Beam

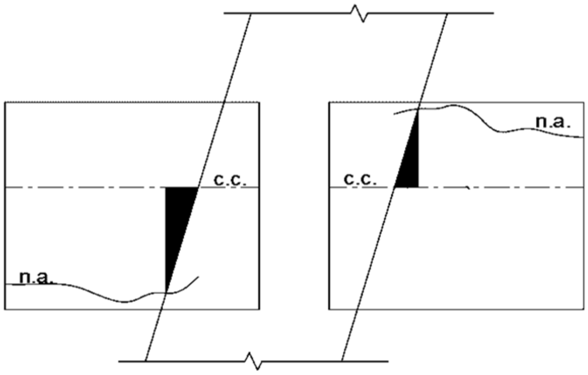
time, t = 7.0000 sec
neutral axis, n.a. = 25.1522 in.
centroid, c.c. = 15.0000 in.

Right Side of Column C

time, t = 7.0000 sec
drift, Δ = 3.3360 in.
rotation, Θ = 1.2656 °

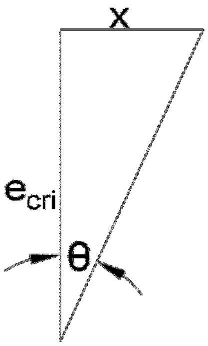
time, t = 7.0000 sec
neutral axis, n.a. = 4.3635 in.
centroid, c.c. = 15.0000 in.

Calculations



all measurements are taken from the top of the beam

q = rotation
e_{cri} = distance between the n.a. and c.c.
x = elongation



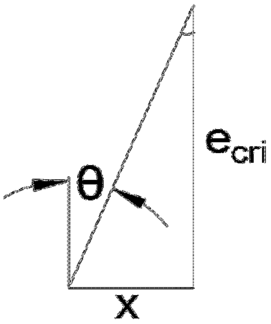
$$\tan\theta = \frac{opposite}{adjacent}$$

$$\tan\theta = \frac{x}{e_{cri}}$$

$$x = e_{cri}\tan\theta$$

e_{cri} = 10.1522 in.

x = 0.2243 in.



$$\tan\theta = \frac{opposite}{adjacent}$$

$$\tan\theta = \frac{x}{e_{cri}}$$

$x = e_{cri}\tan\theta$ due to similar triangles

e_{cri} = 10.6365 in.

x = 0.2350 in.

7.3.5

Left Side of Column D

Column

time, t = 7.0000 sec

drift, Δ = 4.1490 in.

rotation, Θ = 1.5739 °

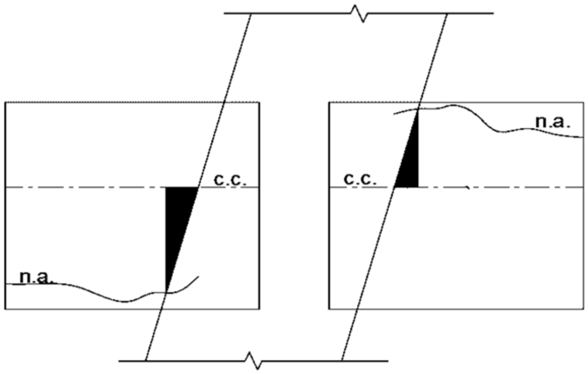
Beam

time, t = 7.0000 sec

neutral axis, n.a. = 22.1019 in.

centroid, c.c. = 15.0000 in.

Calculations

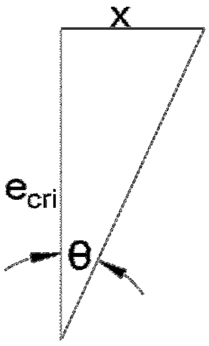


all measurements are taken from the top of the beam

q = rotation

e_{cri} = distance between the n.a. and c.c.

x = elongation



$$\tan\theta = \frac{opposite}{adjacent}$$

$$\tan\theta = \frac{x}{e_{cri}}$$

$$x = e_{cri}\tan\theta$$

e_{cri} = **7.1019** in.

x = **0.1951** in.

6.3.7

Right Side of Column A

Column

time, t = 7.0000 sec

drift, Δ = 3.0949 in.

rotation, Θ = 1.1742 °

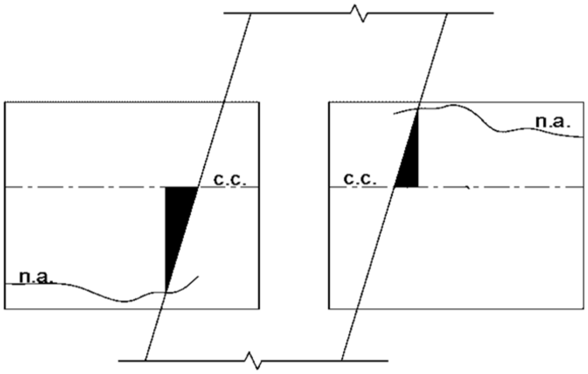
Beam

time, t = 7.0000 sec

neutral axis, n.a. = 3.0838 in.

centroid, c.c. = 15.0000 in.

Calculations

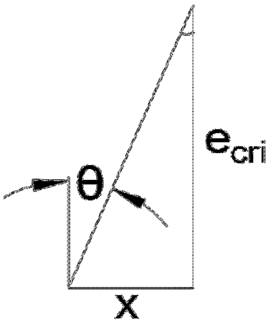


all measurements are taken from the top of the beam

q = rotation

e_{cri} = distance between the n.a. and c.c.

x = elongation



$$\tan\theta = \frac{opposite}{adjacent}$$

$$\tan\theta = \frac{x}{e_{cri}}$$

$x = e_{cri}\tan\theta$ due to similar triangles

e_{cri} = 11.9162 in.

x = 0.2442 in.

6.3.7

Left Side of Column B

Right Side of Column B

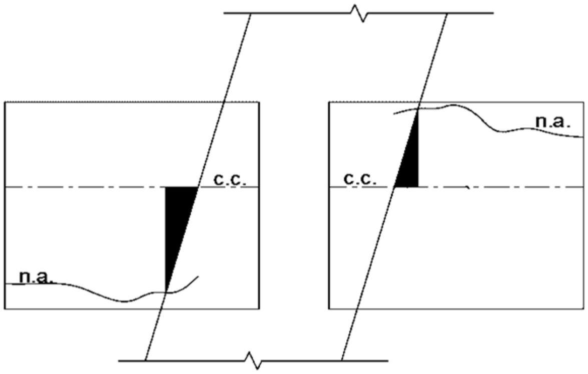
Column

time, t =	7.0000	sec	time, t =	7.0000	sec
drift, Δ =	3.7762	in.	drift, Δ =	3.7762	in.
rotation, Θ =	1.4325	°	rotation, Θ =	1.4325	°

Beam

time, t =	7.0000	sec	time, t =	7.0000	sec
neutral axis, n.a. =	25.0179	in.	neutral axis, n.a. =	3.4478	in.
centroid, c.c. =	15.0000	in.	centroid, c.c. =	15.0000	in.

Calculations

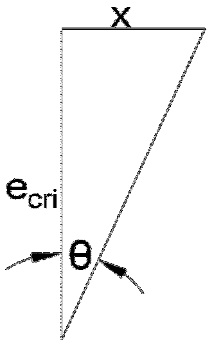


all measurements are taken from the top of the beam

q = rotation

e_{cri} = distance between the n.a. and c.c.

x = elongation



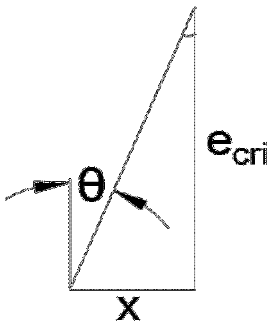
$$\tan\theta = \frac{opposite}{adjacent}$$

$$\tan\theta = \frac{x}{e_{cri}}$$

$$x = e_{cri}\tan\theta$$

e_{cri} = 10.0179 in.

x = 0.2505 in.



$$\tan\theta = \frac{opposite}{adjacent}$$

$$\tan\theta = \frac{x}{e_{cri}}$$

$x = e_{cri}\tan\theta$ due to similar triangles

e_{cri} = 11.5522 in.

x = 0.2889 in.

6.3.7

Left Side of Column C

Right Side of Column C

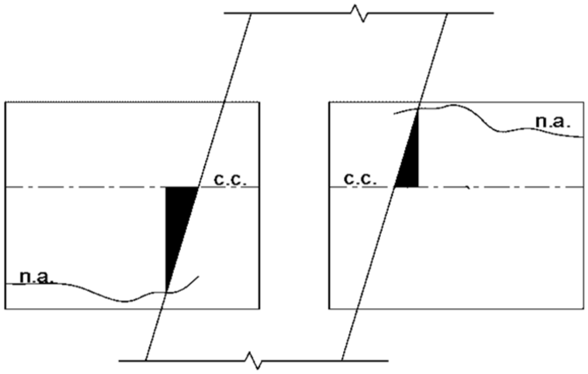
Column

time, t =	7.0000	sec	time, t =	7.0000	sec
drift, Δ =	4.5438	in.	drift, Δ =	4.5438	in.
rotation, Θ =	1.7236	°	rotation, Θ =	1.7236	°

Beam

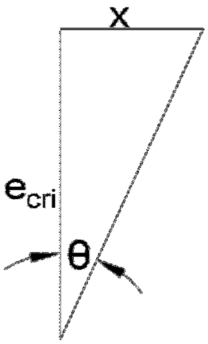
time, t =	7.0000	sec	time, t =	7.0000	sec
neutral axis, n.a. =	23.6544	in.	neutral axis, n.a. =	3.0915	in.
centroid, c.c. =	15.0000	in.	centroid, c.c. =	15.0000	in.

Calculations



all measurements are taken from
the top of the beam

q = rotation
e_{cri} = distance between the n.a. and c.c.
x = elongation



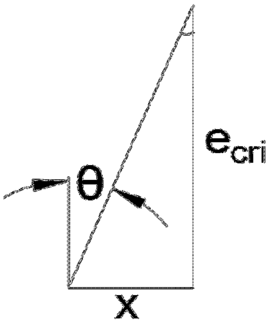
$$\tan\theta = \frac{opposite}{adjacent}$$

$$\tan\theta = \frac{x}{e_{cri}}$$

$$x = e_{cri}\tan\theta$$

e_{cri} = 8.6544 in.

x = 0.2604 in.



$$\tan\theta = \frac{opposite}{adjacent}$$

$$\tan\theta = \frac{x}{e_{cri}}$$

$x = e_{cri}\tan\theta$ due to similar triangles

e_{cri} = 11.9085 in.

x = 0.3583 in.

6.3.7

Left Side of Column D

Column

time, t = 7.0000 sec

drift, Δ = 5.5251 in.

rotation, Θ = 2.0955 °

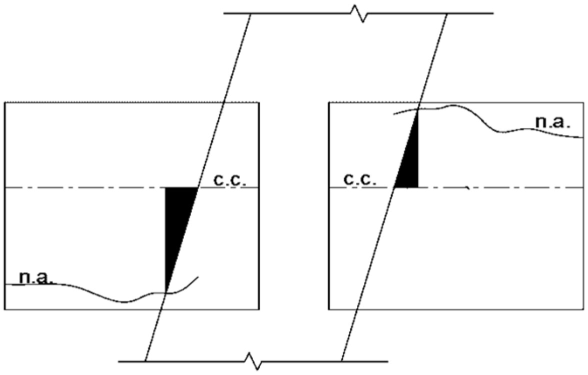
Beam

time, t = 7.0000 sec

neutral axis, n.a. = 19.1760 in.

centroid, c.c. = 15.0000 in.

Calculations

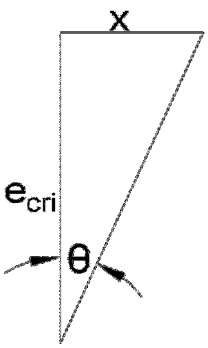


all measurements are taken from
the top of the beam

q = rotation

e_{cri} = distance between the n.a. and c.c.

x = elongation



$$\tan\theta = \frac{opposite}{adjacent}$$

$$\tan\theta = \frac{x}{e_{cri}}$$

$$x = e_{cri}\tan\theta$$

e_{cri} = **4.1760** in.

x = **0.1528** in.

6.3.5

Right Side of Column A

Column

time, t = 7.0000 sec

drift, Δ = 3.0417 in.

rotation, Θ = 1.1540 °

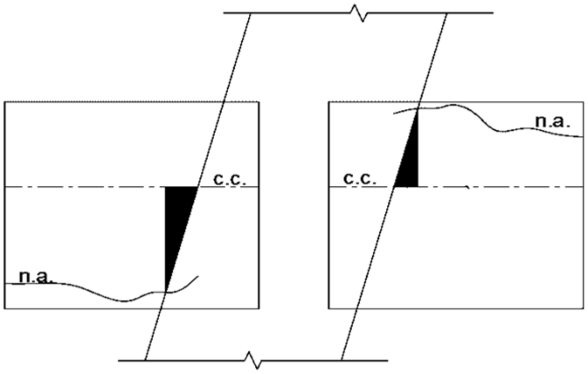
Beam

time, t = 7.0000 sec

neutral axis, n.a. = 4.2637 in.

centroid, c.c. = 15.0000 in.

Calculations

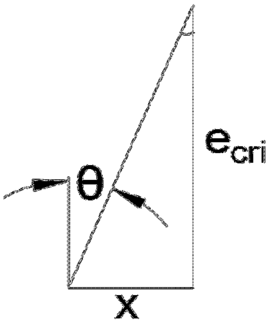


all measurements are taken from the top of the beam

q = rotation

e_{cri} = distance between the n.a. and c.c.

x = elongation



$$\tan\theta = \frac{opposite}{adjacent}$$

$$\tan\theta = \frac{x}{e_{cri}}$$

$$x = e_{cri}\tan\theta$$ due to similar triangles

e_{cri} = **10.7363** in.

x = **0.2163** in.

6.3.5

Left Side of Column B

Column

time, t = 7.0000 sec
drift, Δ = 3.7712 in.
rotation, ϴ = 1.4307 °

Beam

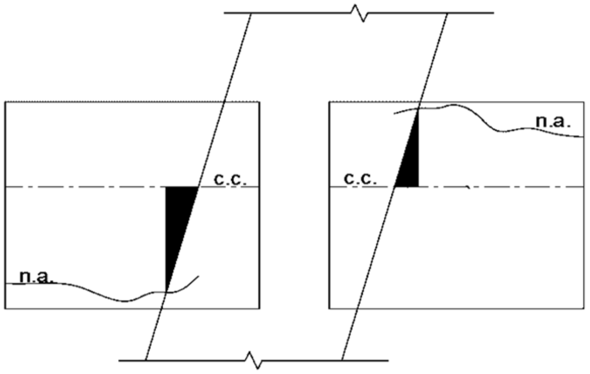
time, t = 7.0000 sec
neutral axis, n.a. = 24.4067 in.
centroid, c.c. = 15.0000 in.

Right Side of Column B

time, t = 7.0000 sec
drift, Δ = 3.7712 in.
rotation, ϴ = 1.4307 °

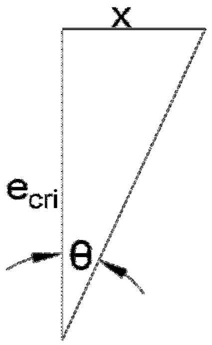
time, t = 7.0000 sec
neutral axis, n.a. = 3.4857 in.
centroid, c.c. = 15.0000 in.

Calculations



all measurements are taken from the top of the beam

q = rotation
e_{cri} = distance between the n.a. and c.c.
x = elongation



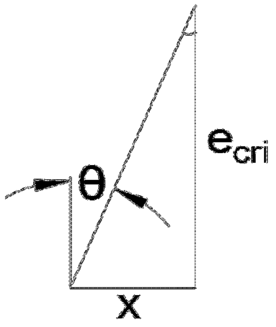
$$\tan\theta = \frac{opposite}{adjacent}$$

$$\tan\theta = \frac{x}{e_{cri}}$$

$$x = e_{cri}\tan\theta$$

e_{cri} = 9.4067 in.

x = 0.2349 in.



$$\tan\theta = \frac{opposite}{adjacent}$$

$$\tan\theta = \frac{x}{e_{cri}}$$

$x = e_{cri}\tan\theta$ due to similar triangles

e_{cri} = 11.5143 in.

x = 0.2876 in.

6.3.5

Left Side of Column C

Column

time, t = 7.0000 sec
drift, Δ = 4.6177 in.
rotation, Θ = 1.7516 °

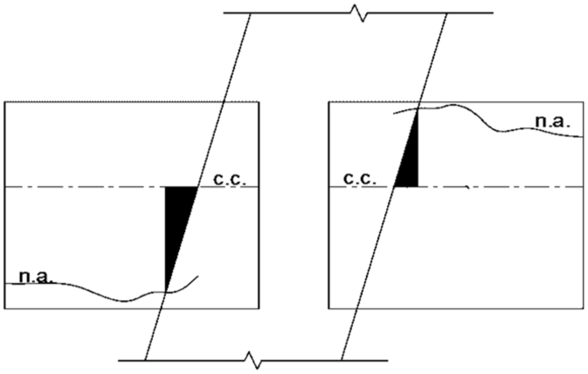
time, t = 7.0000 sec
drift, Δ = 4.6177 in.
rotation, Θ = 1.7516 °

Beam

time, t = 7.0000 sec
neutral axis, n.a. = 22.0350 in.
centroid, c.c. = 15.0000 in.

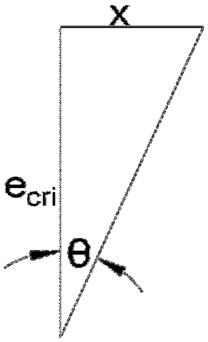
time, t = 7.0000 sec
neutral axis, n.a. = 3.0408 in.
centroid, c.c. = 15.0000 in.

Calculations



an
m

q = rotation
e_{cri} = distance between the n.a. and c.c.
x = elongation



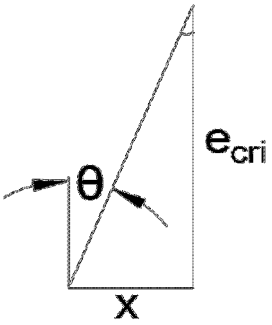
$$\tan\theta = \frac{opposite}{adjacent}$$

$$\tan\theta = \frac{x}{e_{cri}}$$

$$x = e_{cri}\tan\theta$$

e_{cri} = 7.0350 in.

x = 0.2151 in.



$$\tan\theta = \frac{opposite}{adjacent}$$

$$\tan\theta = \frac{x}{e_{cri}}$$

$x = e_{cri}\tan\theta$ due to similar triangles

e_{cri} = 11.9592 in.

x = 0.3657 in.

6.3.5

Left Side of Column D

Column

time, t = 7.0000 sec

drift, Δ = 5.6423 in.

rotation, Θ = 2.1399 °

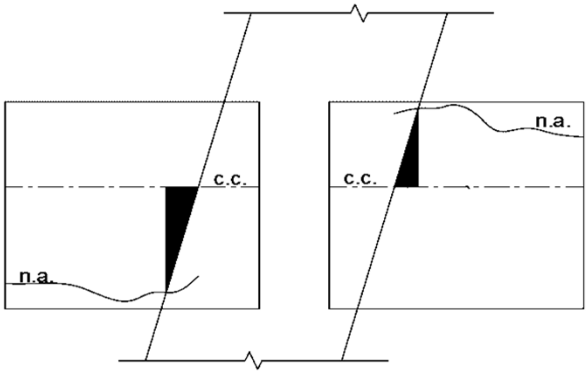
Beam

time, t = 7.0000 sec

neutral axis, n.a. = 19.0062 in.

centroid, c.c. = 15.0000 in.

Calculations

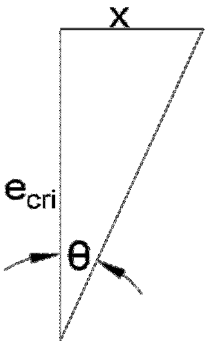


all measurements are taken from the top of the beam

q = rotation

e_{cri} = distance between the n.a. and c.c.

x = elongation



$$\tan\theta = \frac{opposite}{adjacent}$$

$$\tan\theta = \frac{x}{e_{cri}}$$

$$x = e_{cri}\tan\theta$$

e_{cri} = **4.0062** in.

x = **0.1497** in.

7.2.7

Right Side of Column A

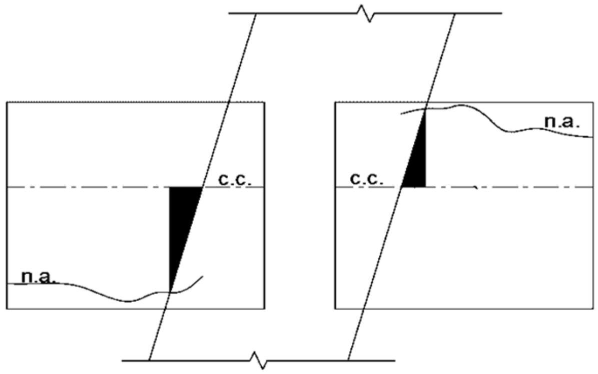
Column

time, t = 7.0000 sec
drift, Δ = 2.4145 in.
rotation, Θ = 0.9161 °

Beam

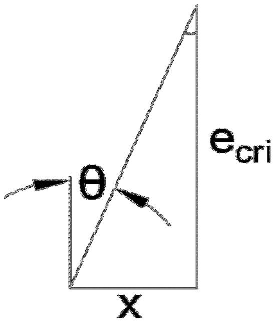
time, t = 7.0000 sec
neutral axis, n.a. = 3.3355 in.
centroid, c.c. = 15.0000 in.

Calculations



all measurements are taken from the top of the beam

q = rotation
e_{cri} = distance between the n.a. and c.c.
x = elongation



$$\tan\theta = \frac{opposite}{adjacent}$$

$$\tan\theta = \frac{x}{e_{cri}}$$

$x = e_{cri}\tan\theta$ due to similar triangles

e_{cri} = **11.6645** in.

x = **0.1865** in.

7.2.7

Left Side of Column B

Right Side of Column B

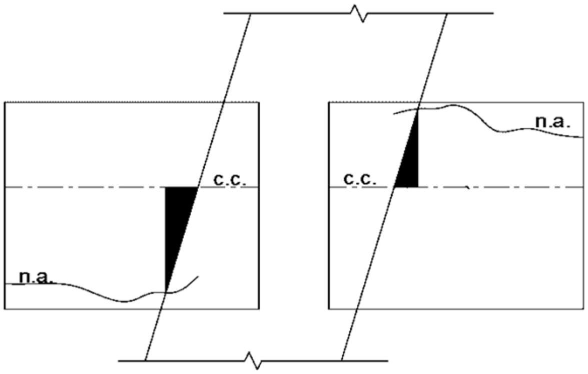
Column

time, t =	7.0000	sec	time, t =	7.0000	sec
drift, Δ =	2.9531	in.	drift, Δ =	2.9531	in.
rotation, Θ =	1.1204	°	rotation, Θ =	1.1204	°

Beam

time, t =	7.0000	sec	time, t =	7.0000	sec
neutral axis, n.a. =	26.5995	in.	neutral axis, n.a. =	4.4100	in.
centroid, c.c. =	15.0000	in.	centroid, c.c. =	15.0000	in.

Calculations

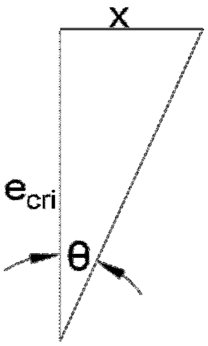


all measurements are taken from the top of the beam

q = rotation

e_{cri} = distance between the n.a. and c.c.

x = elongation



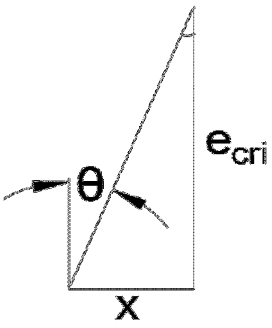
$$\tan\theta = \frac{opposite}{adjacent}$$

$$\tan\theta = \frac{x}{e_{cri}}$$

$$x = e_{cri}\tan\theta$$

e_{cri} = 11.5995 in.

x = 0.2269 in.



$$\tan\theta = \frac{opposite}{adjacent}$$

$$\tan\theta = \frac{x}{e_{cri}}$$

$x = e_{cri}\tan\theta$ due to similar triangles

e_{cri} = 10.5900 in.

x = 0.2071 in.

7.2.7

Left Side of Column C

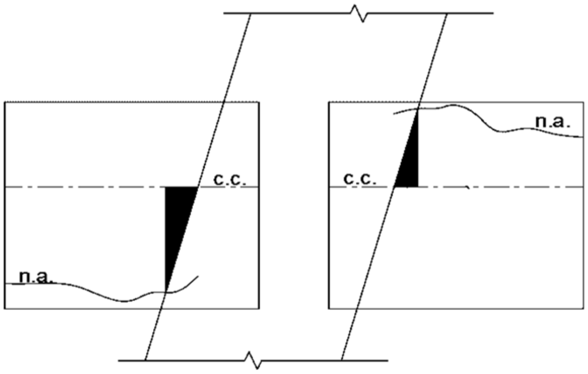
Column

time, t =	7.0000	sec
drift, Δ =	3.6836	in.
rotation, ϴ =	1.3974	°

Beam

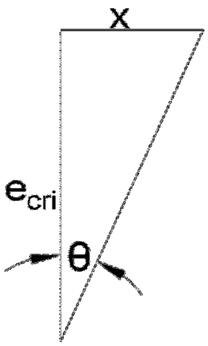
time, t =	7.0000	sec
neutral axis, n.a. =	23.5644	in.
centroid, c.c. =	15.0000	in.

Calculations



all measurements are taken from
the top of the beam

q = rotation
e_{cri} = distance between the n.a. and c.c.
x = elongation



$$\tan\theta = \frac{opposite}{adjacent}$$

$$\tan\theta = \frac{x}{e_{cri}}$$

$$x = e_{cri}\tan\theta$$

e_{cri} = **8.5644** in.

x = **0.2089** in.

7.2.5

Right Side of Column A

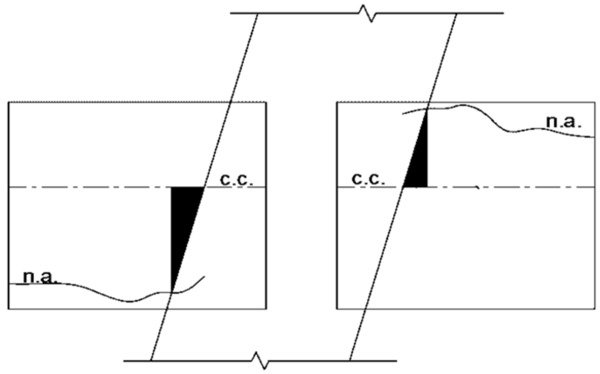
Column

time, t = 7.0000 sec
drift, Δ = 2.3825 in.
rotation, Θ = 0.9039 °

Beam

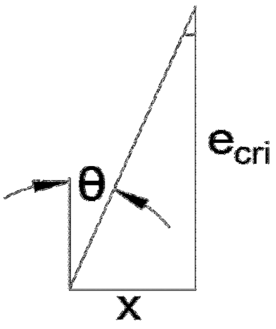
time, t = 7.0000 sec
neutral axis, n.a. = 4.6900 in.
centroid, c.c. = 15.0000 in.

Calculations



all measurements are taken from the top of the beam

q = rotation
e_{cri} = distance between the n.a. and c.c.
x = elongation



$$\tan\theta = \frac{opposite}{adjacent}$$

$$\tan\theta = \frac{x}{e_{cri}}$$

$$x = e_{cri}\tan\theta$$ due to similar triangles

e_{cri} = 10.3100 in.

x = 0.1627 in.

7.2.5

Left Side of Column B

Right Side of Column B

Column

time, t = 7.0000 sec
drift, Δ = 2.9738 in.
rotation, Θ = 1.1282 °

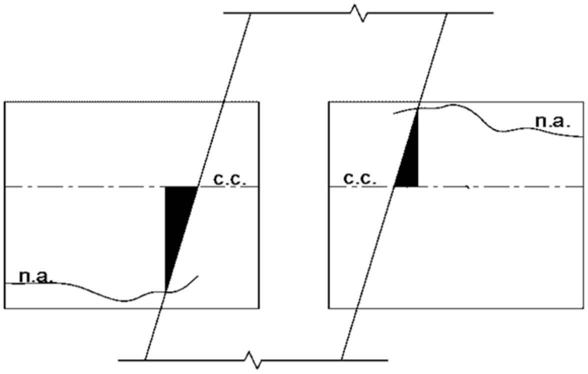
time, t = 7.0000 sec
drift, Δ = 2.9738 in.
rotation, Θ = 1.1282 °

Beam

time, t = 7.0000 sec
neutral axis, n.a. = 25.4003 in.
centroid, c.c. = 15.0000 in.

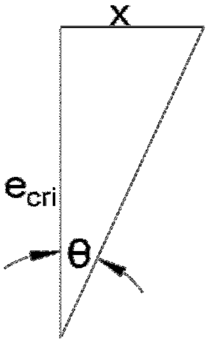
time, t = 7.0000 sec
neutral axis, n.a. = 5.3865 in.
centroid, c.c. = 15.0000 in.

Calculations



all measurements are taken from the top of the beam

q = rotation
e_{cri} = distance between the n.a. and c.c.
x = elongation



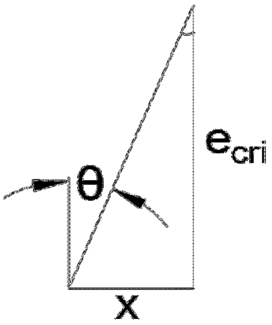
$$\tan\theta = \frac{opposite}{adjacent}$$

$$\tan\theta = \frac{x}{e_{cri}}$$

$$x = e_{cri}\tan\theta$$

e_{cri} = 10.4003 in.

x = 0.2048 in.



$$\tan\theta = \frac{opposite}{adjacent}$$

$$\tan\theta = \frac{x}{e_{cri}}$$

$x = e_{cri}\tan\theta$ due to similar triangles

e_{cri} = 9.6135 in.

x = 0.1893 in.

7.2.5

Left Side of Column C

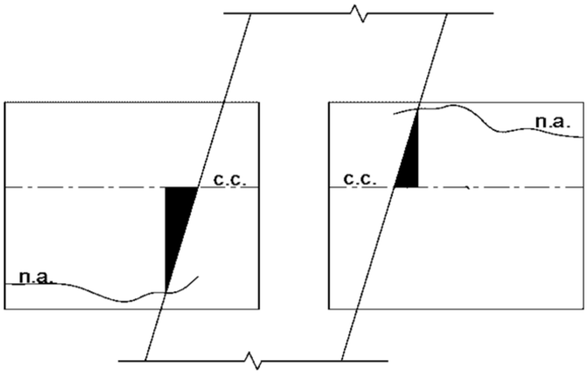
Column

time, t = 7.0000 sec
drift, Δ = 3.7446 in.
rotation, Θ = 1.4206 °

Beam

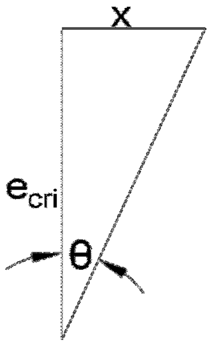
time, t = 7.0000 sec
neutral axis, n.a. = 23.5285 in.
centroid, c.c. = 15.0000 in.

Calculations



all measurements are taken from
the top of the beam

q = rotation
e_{cri} = distance between the n.a. and c.c.
x = elongation



$$\tan\theta = \frac{opposite}{adjacent}$$

$$\tan\theta = \frac{x}{e_{cri}}$$

$$x = e_{cri}\tan\theta$$

e_{cri} = 8.5285 in.

x = 0.2115 in.

6.2.7

Right Side of Column A

Column

time, t = 7.0000 sec

drift, Δ = 3.3952 in.

rotation, Θ = 1.2881 °

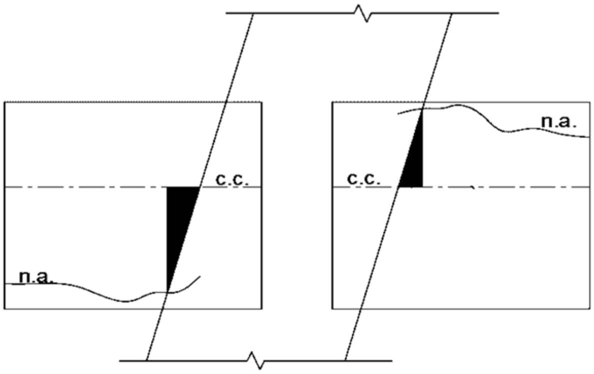
Beam

time, t = 7.0000 sec

neutral axis, n.a. = 3.0427 in.

centroid, c.c. = 15.0000 in.

Calculations

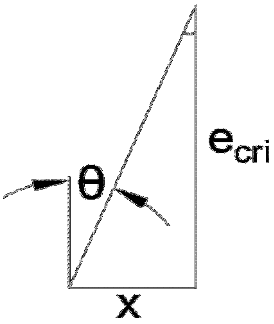


all measurements are taken from the top of the beam

q = rotation

e_{cri} = distance between the n.a. and c.c.

x = elongation



$$\tan\theta = \frac{opposite}{adjacent}$$

$$\tan\theta = \frac{x}{e_{cri}}$$

$$x = e_{cri}\tan\theta$$ due to similar triangles

e_{cri} = 11.9573 in.

x = 0.2689 in.

6.2.7

Left Side of Column B

Column

time, t = 7.0000 sec

drift, Δ = 4.1328 in.

rotation, Θ = 1.5678 °

Beam

time, t = 7.0000 sec

neutral axis, n.a. = 24.0376 in.

centroid, c.c. = 15.0000 in.

Right Side of Column B

Column

time, t = 7.0000 sec

drift, Δ = 4.1328 in.

rotation, Θ = 1.5678 °

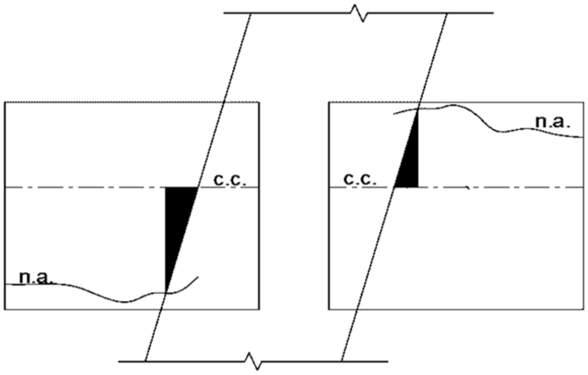
Beam

time, t = 7.0000 sec

neutral axis, n.a. = 3.3749 in.

centroid, c.c. = 15.0000 in.

Calculations

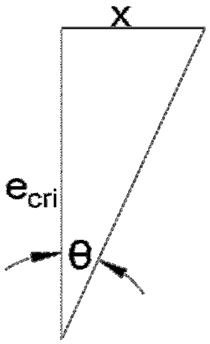


all measurements are taken from the top of the beam

q = rotation

e_{cri} = distance between the n.a. and c.c.

x = elongation



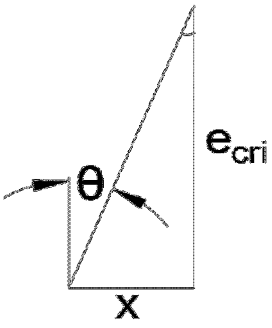
$$\tan\theta = \frac{opposite}{adjacent}$$

$$\tan\theta = \frac{x}{e_{cri}}$$

$$x = e_{cri}\tan\theta$$

e_{cri} = 9.0376 in.

x = 0.2474 in.



$$\tan\theta = \frac{opposite}{adjacent}$$

$$\tan\theta = \frac{x}{e_{cri}}$$

$x = e_{cri}\tan\theta$ due to similar triangles

e_{cri} = 11.6251 in.

x = 0.3182 in.

6.2.7

Left Side of Column C

Column

time, t = 7.0000 sec

drift, Δ = 5.0969 in.

rotation, Θ = 1.9333 °

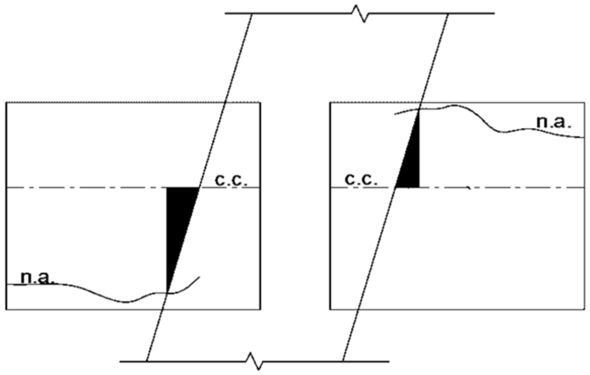
Beam

time, t = 7.0000 sec

neutral axis, n.a. = 20.3694 in.

centroid, c.c. = 15.0000 in.

Calculations

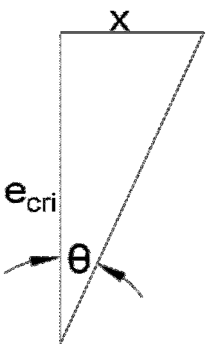


all measurements are taken from the top of the beam

q = rotation

e_{cri} = distance between the n.a. and c.c.

x = elongation



$$\tan\theta = \frac{opposite}{adjacent}$$

$$\tan\theta = \frac{x}{e_{cri}}$$

$$x = e_{cri}\tan\theta$$

e_{cri} = 5.3694 in.

x = 0.1812 in.

6.2.5

Right Side of Column A

Column

time, t = 7.0000 sec

drift, Δ = 3.3647 in.

rotation, Θ = 1.2765 °

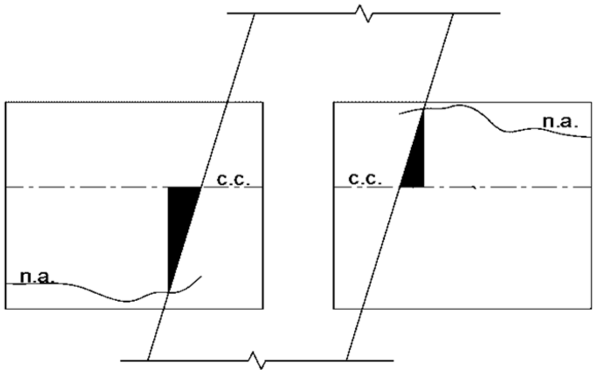
Beam

time, t = 7.0000 sec

neutral axis, n.a. = 4.1077 in.

centroid, c.c. = 15.0000 in.

Calculations

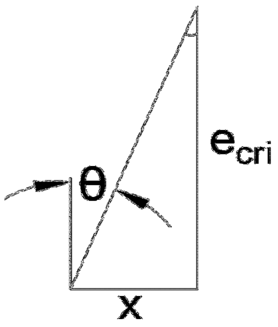


all measurements are taken from
the top of the beam

q = rotation

e_{cri} = distance between the n.a. and c.c.

x = elongation



$$\tan\theta = \frac{opposite}{adjacent}$$

$$\tan\theta = \frac{x}{e_{cri}}$$

$$x = e_{cri}\tan\theta$$ due to similar triangles

e_{cri} = 10.8923 in.

x = 0.2427 in.

6.2.5

Left Side of Column B

Column

time, t = 7.0000 sec

drift, Δ = 4.1602 in.

rotation, Θ = 1.5782 °

Beam

time, t = 7.0000 sec

neutral axis, n.a. = 23.4825 in.

centroid, c.c. = 15.0000 in.

Right Side of Column B

time, t = 7.0000 sec

drift, Δ = 4.1602 in.

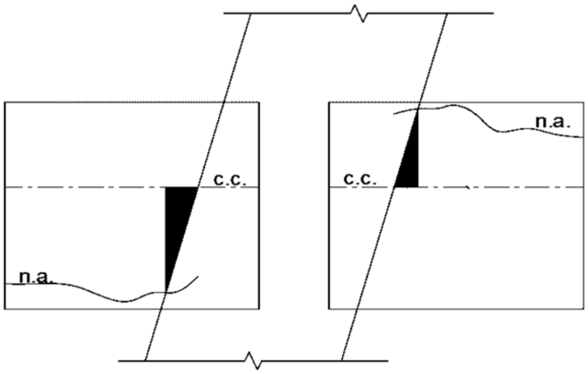
rotation, Θ = 1.5782 °

time, t = 7.0000 sec

neutral axis, n.a. = 3.9212 in.

centroid, c.c. = 15.0000 in.

Calculations

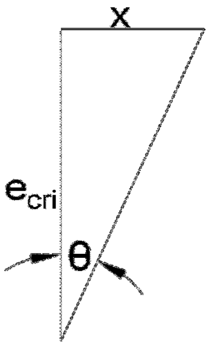


all measurements are taken from the top of the beam

q = rotation

e_{cri} = distance between the n.a. and c.c.

x = elongation



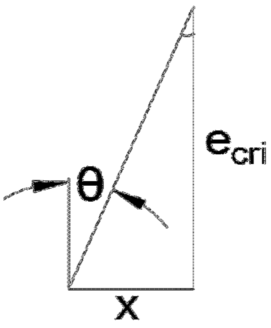
$$\tan\theta = \frac{opposite}{adjacent}$$

$$\tan\theta = \frac{x}{e_{cri}}$$

$$x = e_{cri}\tan\theta$$

e_{cri} = 8.4825 in.

x = 0.2337 in.



$$\tan\theta = \frac{opposite}{adjacent}$$

$$\tan\theta = \frac{x}{e_{cri}}$$

$x = e_{cri}\tan\theta$ due to similar triangles

e_{cri} = 11.0788 in.

x = 0.3052 in.

6.2.5

Left Side of Column C

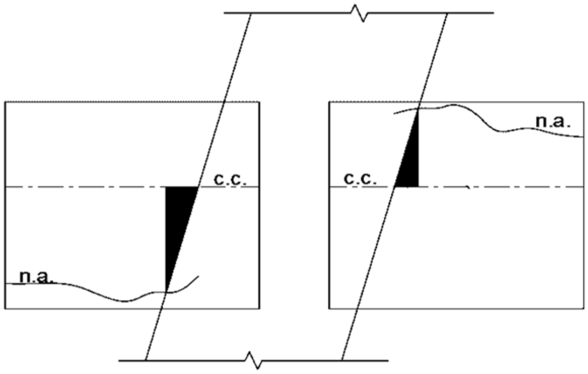
Column

time, t = 7.0000 sec
drift, Δ = 5.1618 in.
rotation, Θ = 1.9578 °

Beam

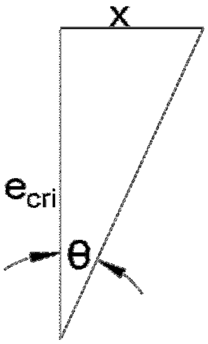
time, t = 7.0000 sec
neutral axis, n.a. = 19.9877 in.
centroid, c.c. = 15.0000 in.

Calculations



all measurements are taken from the top of the beam

q = rotation
e_{cri} = distance between the n.a. and c.c.
x = elongation



$$\tan\theta = \frac{opposite}{adjacent}$$

$$\tan\theta = \frac{x}{e_{cri}}$$

$$x = e_{cri}\tan\theta$$

e_{cri} = **4.9877** in.

x = **0.1705** in.

7.1.7

Right Side of Column A

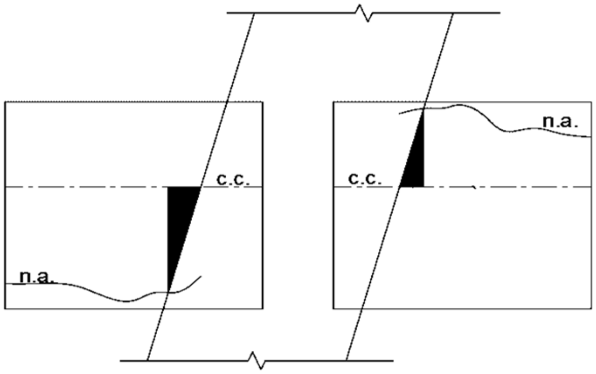
Column

time, t = 7.0000 sec
drift, Δ = 2.5001 in.
rotation, Θ = 0.9486 °

Beam

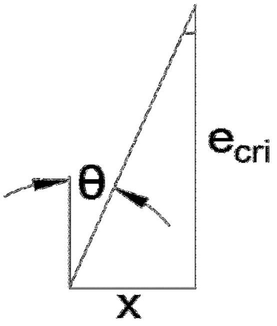
time, t = 7.0000 sec
neutral axis, n.a. = 4.0249 in.
centroid, c.c. = 15.0000 in.

Calculations



all measurements are taken from the top of the beam

q = rotation
e_{cri} = distance between the n.a. and c.c.
x = elongation



$$\tan\theta = \frac{opposite}{adjacent}$$

$$\tan\theta = \frac{x}{e_{cri}}$$

$x = e_{cri}\tan\theta$ due to similar triangles

e_{cri} = 10.9751 in.

x = 0.1817 in.

7.1.7

Left Side of Column B

Column

time, t = 7.0000 sec

drift, Δ = 3.1589 in.

rotation, Θ = 1.1984 °

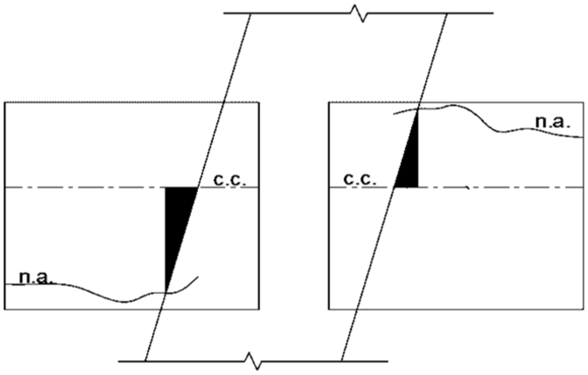
Beam

time, t = 7.0000 sec

neutral axis, n.a. = 24.3315 in.

centroid, c.c. = 15.0000 in.

Calculations

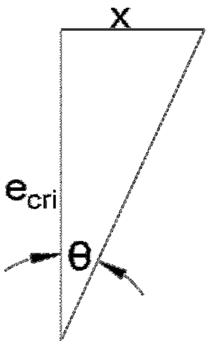


all measurements are taken from the top of the beam

q = rotation

e_{cri} = distance between the n.a. and c.c.

x = elongation



$$\tan\theta = \frac{opposite}{adjacent}$$

$$\tan\theta = \frac{x}{e_{cri}}$$

$$x = e_{cri}\tan\theta$$

e_{cri} = 9.3315 in.

x = 0.1952 in.

7.1.5

Right Side of Column A

Column

time, t = 7.0000 sec

drift, Δ = 2.4797 in.

rotation, Θ = 0.9408 °

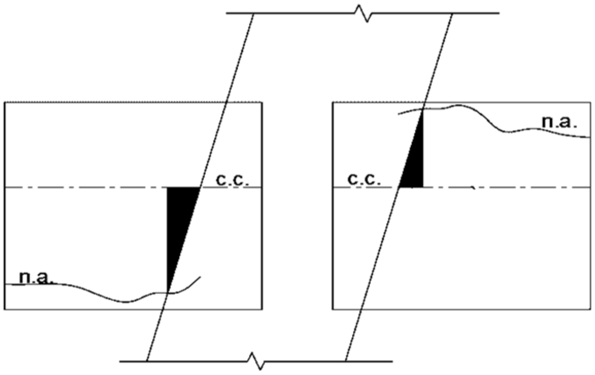
Beam

time, t = 7.0000 sec

neutral axis, n.a. = 4.6234 in.

centroid, c.c. = 15.0000 in.

Calculations

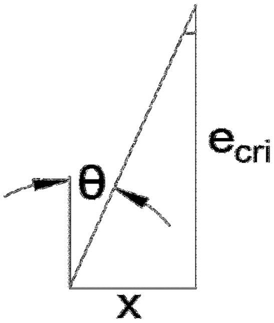


all measurements are taken from the top of the beam

q = rotation

e_{cri} = distance between the n.a. and c.c.

x = elongation



$$\tan\theta = \frac{opposite}{adjacent}$$

$$\tan\theta = \frac{x}{e_{cri}}$$

$$x = e_{cri}\tan\theta$$ due to similar triangles

e_{cri} = 10.3766 in.

x = 0.1704 in.

7.1.5

Left Side of Column B

Column

time, t = 7.0000 sec

drift, Δ = 3.1630 in.

rotation, Θ = 1.2000 °

1

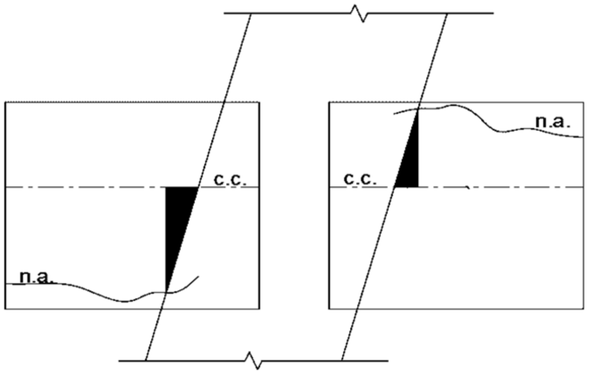
Beam

time, t = 7.0000 sec

neutral axis, n.a. = 24.5643 in.

centroid, c.c. = 15.0000 in.

Calculations

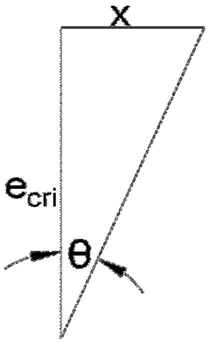


all measurements are taken from the top of the beam

q = rotation

e_{cri} = distance between the n.a. and c.c.

x = elongation



$$\tan\theta = \frac{opposite}{adjacent}$$

$$\tan\theta = \frac{x}{e_{cri}}$$

$$x = e_{cri}\tan\theta$$

e_{cri} = **9.5643** in.

x = **0.2003** in.

6.1.7

Right Side of Column A

Column

time, t = 7.0000 sec

drift, Δ = 3.6061 in.

rotation, Θ = 1.3681 °

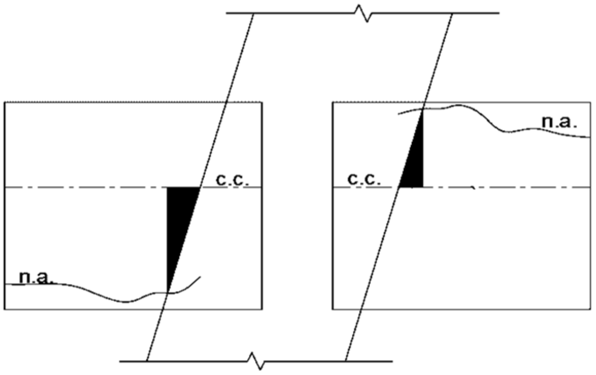
Beam

time, t = 7.0000 sec

neutral axis, n.a. = 4.2430 in.

centroid, c.c. = 15.0000 in.

Calculations

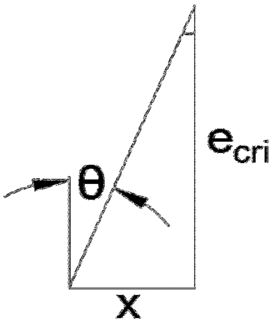


all measurements are taken from the top of the beam

q = rotation

e_{cri} = distance between the n.a. and c.c.

x = elongation



$$\tan\theta = \frac{opposite}{adjacent}$$

$$\tan\theta = \frac{x}{e_{cri}}$$

$$x = e_{cri}\tan\theta$$ due to similar triangles

e_{cri} = **10.7570** in.

x = **0.2569** in.

6.1.7

Left Side of Column B

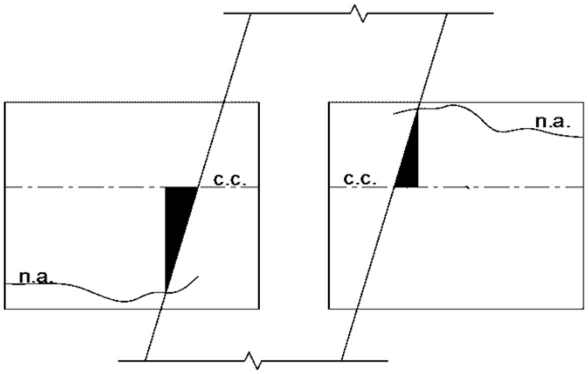
Column

time, t = 7.0000 sec
drift, Δ = 4.5220 in.
rotation, Θ = 1.7153 °

Beam

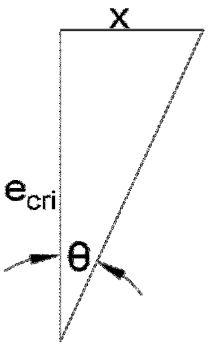
time, t = 7.0000 sec
neutral axis, n.a. = 21.1722 in.
centroid, c.c. = 15.0000 in.

Calculations



all measurements are taken from
the top of the beam

q = rotation
e_{cri} = distance between the n.a. and c.c.
x = elongation



$$\tan\theta = \frac{opposite}{adjacent}$$

$$\tan\theta = \frac{x}{e_{cri}}$$

$$x = e_{cri}\tan\theta$$

e_{cri} = **6.1722** in.

x = **0.1848** in.

6.1.5

Right Side of Column A

Column

time, t = 7.0000 sec

drift, Δ = 3.5996 in.

rotation, Θ = 1.3656 °

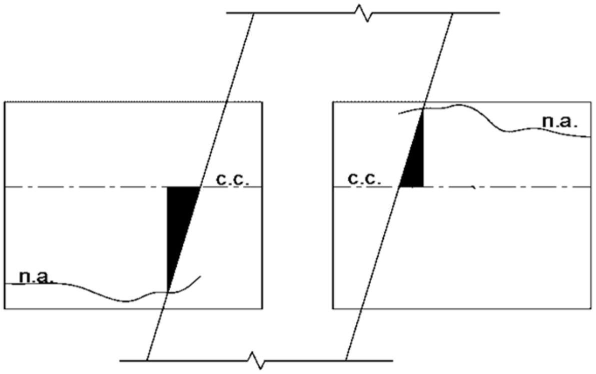
Beam

time, t = 7.0000 sec

neutral axis, n.a. = 4.2133 in.

centroid, c.c. = 15.0000 in.

Calculations

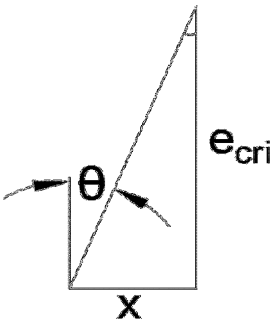


all measurements are taken from the top of the beam

q = rotation

e_{cri} = distance between the n.a. and c.c.

x = elongation



$$\tan\theta = \frac{opposite}{adjacent}$$

$$\tan\theta = \frac{x}{e_{cri}}$$

$x = e_{cri}\tan\theta$ due to similar triangles

e_{cri} = 10.7867 in.

x = 0.2571 in.

6.1.5

Left Side of Column B

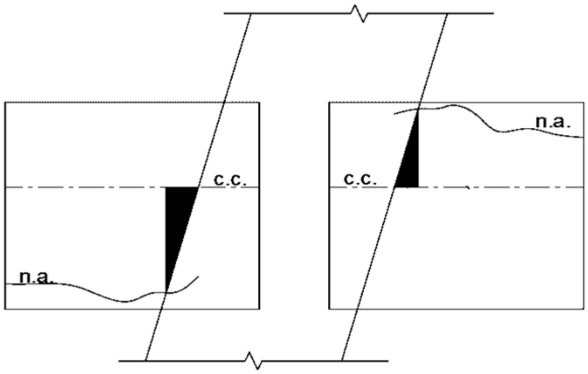
Column

time, t =	7.0000	sec
drift, Δ =	4.5296	in.
rotation, Θ =	1.7182	°

Beam

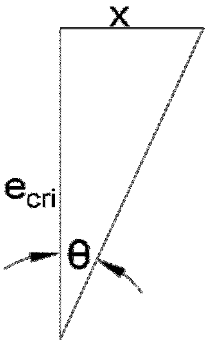
time, t =	7.0000	sec
neutral axis, n.a. =	21.0661	in.
centroid, c.c. =	15.0000	in.

Calculations



all measurements are taken from
the top of the beam

q = rotation
e_{cri} = distance between the n.a. and c.c.
x = elongation



$$\tan\theta = \frac{opposite}{adjacent}$$

$$\tan\theta = \frac{x}{e_{cri}}$$

$$x = e_{cri}\tan\theta$$

e_{cri} = 6.0661 in.

x = 0.1820 in.

Appendix E - Permission for Use

Re: 2006 IBC Structural/Seismic Design Manual - Request to Use

Don Schinske <dschinske@seaoc.org>

Mon 3/5/2018 11:20 AM

To: Gabrielle Liuzza <gfliuzza@ksu.edu>; info@seaoc.org <info@seaoc.org>;

Gabrielle,
You certainly may. Just reference the source, is all we ask.
Thanks for checking,
Don

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