

Essays on optimal extraction of groundwater in Western Kansas

by

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AN ABSTRACT OF A DISSERTATION

submitted in partial fulfillment of the  
requirements for the degree

DOCTOR OF PHILOSOPHY

Department of Agricultural Economics  
College of Agriculture

KANSAS STATE UNIVERSITY  
Manhattan, Kansas

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# Abstract

The two studies presented in this dissertation examine incentives for groundwater extraction and their resulting effect on aquifer depletion. Both studies apply dynamic optimization methods in a context of irrigated agriculture in arid and semi-arid regions such as in western Kansas. The first study examines the effects of capital subsidies aimed at increasing irrigation application efficiency. The second study examines the effects of changing incentives posed by changes in climatic patterns and by technical progress in the form of increasing crop water productivity. Both studies have significant policy and groundwater management implications.

Subsidies for the adoption of (more) efficient irrigation technologies are commonly proposed and enacted with the goal of achieving water conservation. These subsidies are more politically feasible than water taxes or water use restrictions. The reasoning behind this type of policy is that increased application efficiency makes it possible to sustain a given level of crop production per acre with lower levels of groundwater pumping, all else equal.

Previous literature argues that adoption of more efficient irrigation systems may not reduce groundwater extraction. Rewarding the acquisition of more efficient –and capital intensive– irrigation equipment affects the incentives farmers have to pump groundwater. For instance, the farmer may choose to produce more valuable and water intensive crops or to expand the irrigated acreage after adopting the more efficient irrigation system. Hence, the actual impact of the policy on overall groundwater extraction and related aquifer depletion is unclear.

The first chapter examines the effects of such irrigation technology subsidies using a model of inter-temporal common pool groundwater use with substitutable technology and declining well-yields from groundwater stocks, where pumping cost and stock externalities

arise from the common property problem. An optimal control analytical model is developed and simulated with parameters from Sheridan County, Kansas— a representative region overlying the Ogallala aquifer. The study contrasts competitive and optimal allocations and accounts for endogenous and time-varying irrigation capital on water use and groundwater stock. The analysis is the first to account for the labor savings from improved irrigation technologies.

The results show that in the absence of policy intervention, the competitive solution yields an early period with underinvestment in efficiency-improving irrigation technology relative to the socially efficient solution, followed by a period of over-investment. This suggests a potential role for irrigation capital subsidies to improve welfare over certain ranges of the state variables. In contrast to previous work, the findings are evidence that significant returns may be achieved from irrigation capital subsidies. Finally, a policy scenario is simulated where an irrigation technology subsidy is implemented to explore whether such a program can capture significant portions of the potential welfare gain. Results indicate that the technology subsidy can improve welfare, but it captures a relatively small portion of the potential gains in welfare.

The second chapter presents a dynamic model of groundwater extraction for irrigation where climate change and technical progress are included as exogenous state variables— in addition to the usual state variable of the stock of groundwater. The key contributions of this study are (i) an intuitive description of the conditions under which groundwater extraction can be non-monotonic, (ii) a numerical demonstration that extraction is non-monotonic in an important region overlying the Ogallala Aquifer, and (iii) the predicted gains from management are substantially larger after accounting for climate and technical change.

Intuitively, optimal extraction is increasing in early periods when the marginal benefits of extraction are increasing sufficiently fast due to climate and technical change compared to the increase in the marginal cost of extraction. In contrast, most previous studies include the stock of groundwater as the only state variable and, consequently, recommend a monotonically decreasing extraction path.

In this study, the numerical simulations for a region in Kansas overlying the Ogallala

Aquifer indicate that optimal groundwater extraction peaks 23 years in the future and the gains from management are large (29.5%). Consistent with previous literature, the predicted gains from management are relatively small (6.1%) when ignoring climate and technical change. The realized gains from management are not substantially impacted by incorrect assumptions of climate and technical change when formulating the optimal plan.

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# Dedication

I dedicate this dissertation to my grandparents. To Abuela Chola, who taught me to enjoy life and to love unconditionally; and to Lala Kitu, who encouraged my curiosity and taught me to learn, and, of course, also shows me unconditional love. In loving memory of Abuelo Pichi, who taught me to work hard and to care for loved ones above all; and to Lolo Arnulfo, who showed me the beauty of intellectual work and the strength of living a principled life. Their presence, strong as ever, carried me through innumerable struggles.



# Abbreviations and Definitions

<b>AF</b>	Acre-foot: the volume of water to cover one acre of surface to a depth of one foot.
<b>AI</b>	Acre-inch: the volume of water to cover one acre of surface to a depth of one inch. available in wells.
<b>CC</b>	Climate change.
<b>CMIP5</b>	5th Climate Model Intercomparison Program .
<b>EQIP</b>	Environmental Quality Incentives Program.
<b>ET</b>	Evapotranspiration: the process by which water is transferred from the land to the atmosphere by evaporation from the soil and other surfaces and by transpiration from plants.
<b>FWY</b>	Fully watered yield. Crop yield achieved when crop's water requirement is met.
<b>GMD</b>	Groundwater management districts: local units of government that provide water-use administration, planning, and information. Five groundwater management districts were created in the 1970s in the western and central parts of Kansas.
<b>GSE</b>	The Gisser-Sanchez Effect: the results of the seminal study by Micha Gisser and David Sanchez that suggested the benefits of optimally managing an aquifer are only marginally superior to the myopic outcomes.
<b>KGS</b>	Kansas Geological Survey.
<b>LEMA</b>	Local Enhanced Management Area is the authority granted to groundwater management districts (GMDs) through K.S.A. 82a-1041, to consider a specific conservation plan to meet local goals.
<b>NCCV</b>	National Climate Change Viewer.
<b>NIR</b>	Net irrigation requirement: the amount of irrigation water that must be made available for crop evapotranspiration to achieve crop fully-watered yield.
<b>No CC</b>	Planing scenario in which climate change is not considered.
<b>No CC or TC</b>	Planing scenario in which neither climate change nor technical change are considered.
<b>No TC</b>	Planing scenario in which technical change is not considered.
<b>NPV</b>	Net present value of farmer benefits: the sum of the stream of farmer net benefits discounted to reflect the time-value of money.
<b>TC</b>	Technical change/progress.
<b>USDA</b>	United States Department of Agriculture.
<b>USGS</b>	United States Geological Survey.
<b>VMP</b>	Value of the marginal product of groundwater applied to irrigation. In this context, the (inverse) demand function for groundwater.
<b>WIMAS</b>	Water Information Management and Analysis System.
<b>WRIS</b>	Water Rights Information System.

# Chapter 1

## The Impact of Irrigation Capital Subsidies on Common-pool Groundwater Use and Depletion: Results for Western Kansas

### 1.1 Introduction

Growing concerns about competing demands and heightened scarcity of water resources have prompted a renewed interest in water allocation and policy. In North America and many other agricultural regions worldwide, extreme weather events have created short-term stresses on depleting water supplies. To address the perceived scarcity problem, policies are often proposed to achieve water conservation, often with the goal of improving irrigation efficiency. Subsidies for the adoption of efficient irrigation technology are commonly proposed

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and enacted, in part because they are more politically feasible than water taxes or water use restrictions. This paper examines the effects of irrigation technology subsidies using a model of inter-temporal common pool groundwater use with substitutable technology and declining well yields dependent on groundwater stocks, where pumping cost and stock externalities arise from the common property problem. The stock externality arises because pumping is constrained by the stock of groundwater and the pumping externality arises because changes in groundwater stock affect the cost of pumping at the margin (Provencher and Burt, 1993). The effects of the common-pool externalities are found by comparing the optimal control solution to the trajectory of water use under competitive pumping. The model is most closely related to that of Burness and Brill (2001). Like Burness and Brill, this study contrasts competitive and optimal allocations and account for endogenous and time-varying irrigation capital on water use and stock. However, the policy analysis accounts for the labor savings from improved irrigation technologies, which is an often overlooked reduction in adoption costs.

The potential efficacy of the policy instrument is illustrated via a numerical simulation based on agronomic and hydrologic parameters from Sheridan County, KS, where irrigated farming depends on groundwater pumping from the Ogallala aquifer. The study region is representative of places with low urbanization and industrialization pressure, slow natural recharge rates, and few remaining hydrologic connections between the aquifer and surface water bodies that provide ecological services. This setting is descriptive of significant portions of the 174,000 square miles overlying the Ogallala aquifer<sup>1</sup> as well as a number of other agricultural regions worldwide, where the principal trade-off is between current or future water-use to produce food.

Efficient irrigation is often advocated as a valid way to achieve water savings because

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<sup>1</sup>The aquifer and the areas it underlies are not uniform. The northern section of the aquifer (Nebraska in particular) is thought of exhibiting larger saturated thickness and higher recharge rates as well as more extensive interconnectivity with surface systems. However, variation exists even within states. For instance, net recharge in Kansas vary from 0.05 acre-inches per year to 6 acre-inches per year resulting in extremes of rapid aquifer depletion and places of positive changes in saturated thickness (Gutentag, 1984). Parameters for Sheridan County, KS, are close to what can be considered average levels of aquifer parameters, a fact that partially motivates its selection as area of study.

more efficient irrigation systems make it possible for a farmer to deliver the same amount of effective water with a lower volume of pumped water. However, the higher efficiency modifies the underlying economic incentives that influence farmer behavior. Irrigation is a land-augmenting input capable of transforming low quality land at the margin into higher quality land, where all other factors of production become more productive resulting in more land being brought into irrigated production (Caswell and Zilberman, 1983). Adoption of a more efficient irrigation system may influence yields and affect revenues, impose different pumping requirements and affect costs, or both. The adoption of a given irrigation technology may be influenced by exogenous factors such as aquifer and land conditions (Caswell and Zilberman, 1986) or endogenous factors such as crop choice when a crop rotation is already in place and farmers evaluate an irrigation system upgrade. The direction of causality with respect to crop choice is not clear, though. A number of studies model farmer decisions as the combination of crop choice, acreage, and irrigation system – e.g., Ellis et al. (1985) and Scheierling et al. (2006)– yet others argue that the irrigation system choice drive crop choice so that more efficient irrigation systems make it more profitable to produce more water intensive crops; e.g., Pfeiffer and Lin (2014). A recent study by Pfeiffer and Lin (2014) shows that increased irrigation efficiency always results in higher groundwater use when groundwater demand is relatively elastic.

Previous work on river systems shows that under certain circumstances adopting more efficient irrigation technologies results in higher water use and faster resource depletion – e.g., Ward and Pulido-Velazquez (2008), Scheierling et al. (2006), and Huffaker (2008). This result is driven by the presumed reduction in return flows as irrigation technology becomes more efficient and consumptive water use increases. In this case, subsidizing the adoption of more efficient technology generates higher farm returns but reduces the availability of water to downstream users. Even absent downstream use effects, higher water use may result from associated increases in application intensity, irrigated acreage expansion, or both (Ellis et al. (1985), Scheierling et al. (2006), and Pfeiffer and Lin (2014)).

A separate body of literature addresses the common-pool externalities in groundwater use in a dynamic context– e.g., Gisser and Sanchez (1980); Shah et al. (1995); Burness and Brill

(2001); Wang and Segarra (2011). The possibility of time-varying and endogenous irrigation capital and application efficiency is rarely incorporated in these models; exceptions include Burness and Brill (2001) and Shah et al. (1995). The role of irrigation capital subsidies - in isolation or in combination with other policy instruments - to correct the common pool externalities has not been fully explored. Ding and Peterson (2012) studied the cost-effectiveness of two water-conservation programs in Kansas. One of the programs they considered is very similar to an irrigation capital subsidy where the optimization occurs over a discrete choice of irrigation technology. Their study focused on comparing the cost of achieving a water conservation goal under each of the analyzed policies and under different hydrologic conditions but it does not compare competitive and optimal cases, nor does it quantify potential welfare gains from management.

## 1.2 Model

As the main trade-off analyzed in this research is between current versus future agricultural irrigation for food production, net farm benefits are an appropriate metric for social welfare. The optimal control model employed maximizes the present value of net farm benefits over a time horizon by choosing optimal amounts of irrigation capital and water pumped, where the state variable is aquifer water table height and the dynamic constraint is the change in water table height. Because water is a “weakly essential” input for farming in the area, the revenue function is the area beneath the inverse demand curve of effective water, where effective water is defined as pumped water times an efficiency factor that depends on irrigation capital, and where the evapotranspiration requirements are determined by the typical crop mix in the region. The cost function is linear in applied water and inversely related to water table height and well yield. The model incorporates maintenance and operation cost of irrigation capital as well as a labor-saving feature accounting for labor savings from efficient irrigation technologies.

### 1.2.1 Hydrologic model

A very simple hydrologic model of an unconfined aquifer is employed. Sensitivity analyses by [Burness and Brill \(1992\)](#) indicate that including further hydrologic details has little quantitative effects on results. Furthermore, [Brozović et al. \(2010\)](#) indicate that the use of single-cell models may be adequate for small aquifers or a relatively small area within a much larger aquifer, as is the case here. Since the aquifer model is employed to provide the state variable only, the hydrological model is kept as simple as possible. The state variable for the optimization problem is the elevation in feet above sea level of the water table. The evolution of the water table elevation (or height) is determined by:

$$\begin{aligned}\dot{H} &= \frac{1}{A_S} [N - (1 - \alpha)w], \\ H(0) &= H_0 \quad , \\ H(t) &\geq H_c\end{aligned}\tag{1.1}$$

where  $H(t)$  is water table height at time  $t$ ,  $H_0$  is initial water table height,  $H_c$  is the elevation of the aquifer bottom,  $A_S$  is the acreage overlying the aquifer times the specific yield,  $N$  is the (exogenous) volume of natural recharge per period,  $\alpha \in (0, 1)$  is the fraction of applied irrigation water that becomes return flow, and  $w(t)$  is the total volume of irrigation water pumped at time  $t$ . The return flow fraction is endogenous in this setting and is specified as a function of the per-acre level investment in irrigation capital,  $k$  (in \$/acre). The return flow fraction is assumed to be decreasing in capital ( $\partial\alpha/\partial k < 0$ ), because more advanced irrigation technology implies that a larger share of delivered water is consumed by crops.

At the most basic level, the relationship is rather simple: the more water is consumed for irrigation, the faster the aquifer declines. However, a coupled system such as this involves feedback loops: extraction of water for irrigation affects the aquifer but the state of the aquifer also affects irrigation costs for farmers. In this formulation, farmers not only choose water extraction, but also the level of investment in irrigation capital, which then affects the proportion of applied water returning to the aquifer,  $\alpha$ . [Figure 1.1](#) is a simple representation

of the physical relationships in the irrigation-aquifer coupled system.

The effects of aquifer depletion are felt inter-temporally via the decreasing water table height due to pumping, which affects pumping cost due to both increased pumping lifts and reduced well yields. The declining well-yield function, in acre-feet per hour, follows [Sloggett and Mapp \(1984\)](#):  $Y = 2Q_0d[H(t) - H_C - d/2]$ , where  $d$  is the drawdown of the water table elevation at the pump relative to the pre-pumping elevation and  $Q_0$  is a constant that depends on hydrologic properties. Clearly, well yields decline as the water table elevation declines.

### 1.2.2 Pumping costs, agronomy and application efficiency

The marginal cost of pumping water in ( $\$/acre - feet$ ) at time  $t$  is

$$C(H) = \frac{C_0}{Y} \left[ \frac{(S_L - H(t))}{(S_L - H_0)} \right] \quad (1.2)$$

where  $S_L$  is the surface level elevation and  $C_0$  is the cost of pumping water for an hour at the initial lift ( $S_L - H_0$ ). The ratio  $C_0/Y$  is the cost of pumping per acre foot ( $AF$ ) at the initial lift, and the term in brackets scales this cost by the lift at time  $t$  in proportion to initial lift. Water pumping decreases the water table height, which has a compounded impact on the cost of pumping water. A decrease in  $H(t)$  increases marginal cost directly and via reduced well yields, causing pumps to work harder and use more energy, which results in higher irrigation costs per acre-foot. This formulation imposes pumping to become unprofitable before the aquifer is depleted.

The crop water requirements  $C_R$ , in acre-feet per acre, are assumed fixed for a given crop mix at a level in which each crop in the mix achieves fully-watered-yield ( $FWY$ ). The amount of water required to meet  $FWY$  in the area of study is  $C_RA$ , where  $A$  is irrigated area in acres. The water accounting identity that defines application efficiency is  $e(k) = C_RA/w$ , where  $e(k) \in (0, 1)$  is application efficiency. Application efficiency is an increasing function of capital:  $e'(k) > 0$ . With increased investments in irrigation technology, application efficiency

increases and water extracted decreases, all else equal.

Part of the pumped water is evapotranspired (consumptive use), part of it is evaporated to the atmosphere, and part of it returns to the aquifer via return flows,  $\alpha(k)$ . All three of these proportions depend on the amount of irrigation technology capital per acre,  $k$ . Efficient technologies allow for lower amounts of pumped water, only small fractions of which evaporate or become return flows, creating the incentive to invest in irrigation capital.

### 1.2.3 Capital costs

Realistic models of irrigation technology adoption consider the discrete choice among commercially available irrigation technologies, as in [Caswell and Zilberman \(1985\)](#) or as in [Ding and Peterson \(2012\)](#), where the choice is determined by the levels of expected profits under different irrigation technologies given current aquifer conditions. [Caswell and Zilberman \(1985\)](#) considers both water and non water costs associated with each irrigation system but omit the upfront investment level, while [Ding and Peterson \(2012\)](#) explicitly include it and compare it to the net present value of expected benefits in their irrigation technology choice model.

This study, however, considers a setting in which irrigation capital is continuously malleable as in [Burness and Brill \(2001\)](#), the cost of capital is annualized, and capital investments reduce labor costs. This assumption captures the idea that there is a range of efficiency levels and capital costs within each irrigation technology type, and with a large number of possible choices this relationship is usefully approximated with a continuous function. Measuring capital costs as an annualized expense merely amortizes investment costs into annual rental payments on capital. To calibrate the application efficiency function  $e(k)$ , a function is calibrated to data points representing the three most common irrigations systems: furrow, center pivot, and subsurface drip irrigation (see table [1.1](#) for benchmark values).

Figure [1.2](#) illustrates the procedure followed to calibrate irrigation efficiency as a function of capital per acre. The typical procedure consists of assigning a pre-specified irrigation efficiency level to a given irrigation system. However, it is possible to achieve a range of



irrigation efficiency levels with a given irrigation system depending on field conditions. Furthermore, the amount of capital investment required for the installation of a given irrigation system also varies. The efficiency function covers the overlaps over the levels of irrigation efficiency and capital investment requirements for different irrigation systems.

The optimal level of capital is derived from both the cost of pumping water and the financial and operational cost for a given level of capital per acre,  $k$ . The total cost of capital depends on the total stock of capital,  $K = kA$ . Following [Burness and Brill \(2001\)](#), operation and maintenance costs are assumed to be a proportion  $\delta \in (0, 1)$  of the capital stock,  $K$ , and that the rental rate of capital is fixed at  $r \in (0, 1)$ , implying a total cost of capital of  $(\delta + r)K$ . Solving the water accounting identity for acreage, results in  $A = \frac{e(k)w}{C_R}$ , implying that total capital can be written  $K = ke(k)w/C_R$  and the cost of capital becomes  $(\delta + r)K = \eta ke(k)w$ , where  $\eta = \frac{r+\delta}{C_R}$ .

#### 1.2.4 Labor saving effects of capital

[Bernardo et al. \(1987\)](#) explore the role of labor-intensive irrigation practices as an application efficiency augmenting factor given an irrigation system. In their setting, the presence of water supply limits may force a farmer to increase the application efficiency of his existing irrigation system by using more labor-intensive practices. However, it is also clear that highly efficient irrigation systems have lower baseline labor requirements. The latter relationship is modeled as a decreasing function of irrigation capital investment so the higher the investment, the lower the cost of labor to manage the system. The starting point is a baseline labor cost per acre  $\theta$  and apply a labor-saving factor  $L(k)$ ,  $L'(k) < 0$  such that labor cost per acre is expressed as  $\theta L(k)$ . In this formulation labor costs act as a component of the cost of capital, falling as  $k$  rises.

#### 1.2.5 Farm benefits

Net farm benefits at any given time are defined as the area under the value of marginal product (VMP) curves minus pumping and capital costs. The inverse factor demands for

water  $p^w(w, k)$  and capital  $p^k(w, k)$  may be obtained from static profit maximization. Farm output is a function of effective water  $e(k)w$  and is defined as  $Q = F(e(k)w)$  such that  $F()$  is monotonic, increasing and concave. Furthermore, water is assumed to be a weakly essential input so that the VMP of capital is zero when water input is zero, i.e. there are no gains from more efficient irrigation systems when there is no irrigation. Farm quasi-revenues are:

$$R(e(k)w) = \int_Z [p^w(w, k)dw + p^k(w, k)dk] = \int_0^{w^*} p^w(w, k^*)dw$$

since the first integral is independent of path,  $Z$  is any path from  $(0, 0)$  to  $(w^*, k^*)$ , and water is a weakly essential input as described above. The net farm benefits at any given period is:

$$B = R(e(k)w) - C(H)w - \eta e(k)wk - \theta L(k) \quad (1.3)$$

### 1.2.6 Solving the optimization

Two types of solutions are considered which correspond to two types of farmer behavior: myopic and planning solutions.

In the myopic scenario, the farmer maximizes (1.3) in each period given aquifer conditions. This myopic behavior describes a competitive setting in which the farmer does not consider the future consequences of his present decisions which is exactly the common pool resource problem. The first order conditions for the myopic solution are

$$\begin{aligned} R'(e(k)w)e(k) - C(H) - \eta e(k)k &= 0, \text{ and} \\ e'(k)R'(e(k)w)w - \eta w[e(k) + e'(k)k] - \theta L'(k) &= 0. \end{aligned}$$

The planning solution (dynamic optimization) consists of maximizing the net present value of farm benefits:

$$V = \int_0^{t^*} e^{-rt} [R(e(k)w) - C(H)w - \eta e(k)wk - \theta L(k)] dt \quad (1.4)$$

subject to (1.2), where future net benefits are assumed to be discounted at the cost of capital,  $r$ . The current value Hamiltonian is

$$\tilde{H} = R(e(k)w) - C(H)w - \eta e(k)wk - \theta L(k) + \mu \frac{1}{A_S} [N + (\alpha(k) - 1)w],$$

yielding the optimality conditions

$$\begin{aligned} R'(e(k)w)e(k) - C(H) - \eta e(k)k + \mu \frac{1}{A_S} [N - (1 - \alpha(k))] &= 0 \\ e'(k)R'(e(k)w)w - \eta w [e(k) + e'(k)k] - \theta L'(k) + \mu \frac{1}{A_S} \alpha'(k) &= 0 \\ \dot{\mu} - r\mu &= C'(H)w, \end{aligned}$$

where the primes indicate first derivatives and  $\mu$  is the current value co-state variable (marginal user cost) of water, which represents the value per unit of water conserved at a point in time.

The planning solution is an appropriate proxy for the Social Planners allocation in situations where social welfare is defined by the benefits obtained by farmers, i.e. where higher-value uses of groundwater such as urban or industrial use are negligible. Such circumstances describe large spans of arid and semi-arid regions in the United States and the world.

### 1.3 Case Study: Sheridan County, Kansas.

The setting and assumptions of the model specified above closely describe the circumstances faced by the region in western Kansas overlying the Ogallala aquifer. Figures 1.3 and 1.4 show the aquifer and aquifer conditions in the area. Hydrological and extraction conditions are not uniform in the region. However, the choice of Sheridan County is appropriate on three counts. Firstly, there is near uniformity within the county with respect to the agronomic and hydrologic variables at levels that make the area representative of the average irrigated farm in western Kansas. Second, the depletion of the aquifer has reached levels in which farmers are concerned with the continuity of their operations and are demanding institutional solu-

tions to the problem. Finally, the recent implementation of a Local Enhanced Management Area (LEMA) in the county has brought much attention from groundwater management authorities and could become a framework upon which future policies are based.

### 1.3.1 Model parameterization and initial values

Parameter and aquifer initial values for Sheridan County are presented in table 1.1. Aquifer parameters were obtained from the Kansas Geological Survey (KGS), the Water Rights Information System (WRIS), and the Water Information Management and Analysis System (WIMAS). Labor saving is calibrated using a baseline labor requirement of 0.8 hours per acre for surface irrigation versus 0.05 hours per acre associated with center pivot irrigation (Bernardo et al., 1987) and wage rates from the Bureau of Labor Statistics Occupational Handbook (median agricultural wage rate). The interest rate on loans to farmers was obtained from the Kansas City Federal Reserve Bank (November 2011). Maintenance and operation costs,  $\delta$ , were set at 10 percent since the U.S. Master Depreciation Guide states that irrigation systems are 7 to 15-years properties.

To establish the crop water requirement  $C_R$  the main crops under irrigation in Kansas are considered. Rather than specifying FWY as the maximum crop evapotranspiration (ET), the Net Irrigation Requirement (NIR) or maximum ET less effective rainfall (Clark, 2009) is employed. The NIR for each crop was obtained from the National Engineering Handbook. The weights (acreage shares) assigned to each crop were obtained from Clark (2009). Table 1.1 summarizes the calculation of the crop water requirement. The functional forms and fitted parameters for the application efficiency, return flows, and labor savings functions are summarized in Table 1.3. The choice of functional forms ensures tractability and the required (0,1) range for any possible value of  $k$ .

The pumping cost from 1.2 is calibrated by applying the well-yield formula and the parameter values:  $C_0 = 0.975$ ,  $S_L = 2,755$ ,  $H_0 = 2,644.2ft$ ,  $Q_0 = 3.48E - 07$ , and  $H_c = 2,583.2$ . The calculation of pumping cost at initial lift follows Rogers and Alam (2006) for an initial lift of 111.5ft, and an electric motor driven pump with electricity cost of 0.0834

per kW/h.

The parameterization of the revenue function  $R(e(k)w)$  requires the estimation of the water (inverse) demand function. [Hendricks and Peterson \(2012\)](#) present an estimation of water demand elasticity using field-level data from Kansas over a period of 16 years and controlling for field-farmer and year-fixed effects. Their estimated total elasticity of demand is employed (-0.1) and a linear water demand function is recovered using the mean values of water cost, quantity of water demanded, and application efficiency in their study. The resulting inverse demand function is  $p^w(w, k) = 286.19e(k) - 0.00377e(k)^2w$ .

### 1.3.2 Simulation results

Numerical solutions are obtained for the baseline comparison in the absence of any policy for the planning and the myopic solutions. [Figures 1.5 –1.10](#) illustrate the difference between the myopic and planning solutions over time. The planning solution yields consistently higher water-table elevation in comparison to the myopic solution implying lower pumping costs once the steady state is reached. The implication is, all else equal, lower production costs in the future, which may allow for cheaper food, are consistent with the planning solution in comparison to the myopic case.

In the earlier periods, there is excessive pumping, which drives the rapid depletion of the aquifer. However, rapidly increasing pumping costs result in reduced levels of groundwater extraction in later periods, which are below the levels under the planning solution. The relatively low cost of pumping in earlier periods encourages underinvestment under the myopic solution, but as the aquifer depletes and water becomes more expensive, the levels of investment on irrigation capital increase in order to gain application efficiency. The result is underinvestment in the earlier periods but eventual overinvestment relative to the planning solution. As the model uses a malleable definition of irrigation capital, these results cannot be related directly to the rates of investment in particular irrigation systems over time. However, the results can be put in some perspective by assuming a negligible share of farmers invest in the most expensive subsurface drop system, and that that system

efficiencies reported in Table 1.1 remain constant over time. Under these assumptions, the simulated gap at the end of the planning horizon implies that about 5 percent more farmers would have upgraded from flood to center-pivot systems in the myopic solution than in the planners solution. From Figure 1.9 it is evident that myopic pumping leads to more acreage irrigated early on, but irrigated acreage also declines more rapidly over time, leaving less acreage irrigated than optimal in the long-run.

An interesting result is that relatively early in the simulation, the planning solution dominates the myopic outcomes with respect to overall Net Private Benefits. This is evidence that significant returns may be achieved from policy intervention. The next section presents the simulated effectiveness of alternative policy instruments in capturing this potential gain.

### 1.3.3 Policy alternative: irrigation capital subsidies.

An irrigation efficiency-improving subsidy in the form of a matching funds program is considered. For every dollar per acre invested in irrigation capital, the agent receives a matching amount to be added to the investment. The problem is now modified so that the application efficiency, return flow, and labor load functions become  $e(k) = 1 - \hat{e}_1 \exp[-\hat{e}_2 2k]$ ,  $\alpha(k) = \hat{\alpha}_1 \exp[-\hat{\alpha}_2 2k]$ , and  $L(k) = \hat{L}_1 \exp[-\hat{L}_2 2k]$ , where the values for the fitted parameters remain the same as in the baseline.

Finally, since the farmers are receiving the matching funds to implement the application efficiency-improving irrigation technology, they are responsible for the operation and maintenance costs of the overall irrigation capital investment per acre,  $2k$ , and only pay the financial cost on their own portion,  $k$ . The adjusted capital costs entering the optimization then become  $\eta e(k)w2k - \frac{r}{C_R} e(k)wk$ , which to some extent prevents the abuse of the subsidy on the part of the agents.

This type of subsidy policy is akin to that offered under the Environmental Quality Incentives Program (EQIP) which was reauthorized in the Food, Conservation, and Energy Act of 2008 (2008 Farm Bill). A feature of EQIP contracts is a clause requiring that the agent does not extend the area of cropland under irrigation. In this formulation, a simplified

version of the policy is modeled consisting of matching funds and no limitations with respect to the acreage resulting from the optimized irrigation efficiency and groundwater extraction in the simulation.

### 1.3.4 Policy Analysis

Figure 1.11 illustrates the impact of the policy being considered with respect to the water table elevation, Figure 1.12 shows the time path of water extraction levels, figures 1.13 and 1.14 do so for application efficiency and investment in irrigation capital, respectively. With respect to the saturation of the aquifer (water table height), the policy scenario results in aquifer levels below the planning optimum in the long run.

With respect to water extraction, the irrigation capital subsidy is an improvement on the myopic solution in about the first two decades but subsequently becomes indistinguishable from the myopic solution. It can also be seen that irrigation capital and application efficiency is consistently above the myopic solution such that it helps bridge the difference between the myopic and the planning solutions in the first 40 or so years but it exacerbates the overinvestment in the long-run. The clear implication is that any policy of this type would have to be periodically revised and eventually eliminated, perhaps even replaced by an irrigation capital tax for the later periods.

Figures 1.15 and 1.16 illustrate the impact of the policy with respect to irrigated acreage and total Net Private Benefits received by irrigators respectively. As expected, the irrigation capital subsidy results in consistently higher net private benefits than the myopic solution.

From a social efficiency point of view, though, such a subsidy imposes a burden on society in general and the taxpayer in particular that needs to be accounted for. Consequently, the total amount of (additional) subsidies paid to farmers is subtracted from the net private benefits received by the farmers to approximate the Net Social Benefits under each scenario. These results are illustrated in Figure 1.17 and summarized in Table 1.4.

From Table 1.4 it can be seen that with respect to Net Social Benefits the application efficiency-improving subsidy policy is welfare improving. Additionally, from a local rather

than societal point of view, a subsidy policy would be more attractive to the group that might pose the biggest resistance to policy intervention, namely the farmers, since the burden of the cost of the subsidy will be spread outside the region and will affect taxpayers everywhere but the bulk of the benefits will accrue regionally. In fact, the EQIP programs are funded by state and federal governments so all the benefits accrue to the region while the costs are partially incurred by the whole country.

With respect to irrigated acreage, the irrigation capital subsidy results in consistently higher irrigated acreage than the myopic case. This is an indication that irrigation capital subsidies result in the incorporation of otherwise unfit land into crop production, corroborating previous findings by various authors – e.g., [Ward and Pulido-Velazquez \(2008\)](#); [Scheierling et al. \(2006\)](#). However, in contrast to that literature, it seems that in the absence of the surface water-groundwater interactions, the actual use of water is decreased with the irrigation capital subsidy despite the increased irrigated acreage.

In the long run, though, the planning solution yields significantly higher net private benefits than the rest of the cases.

## 1.4 Conclusions

Similar to the results derived from surface water models, the preliminary results suggest that excessively efficient irrigation technologies may lead to increased or inefficient use, rather than conservation, of water, at least in certain periods of the resource’s life cycle. Like [Burness and Brill \(2001\)](#) and [Shah et al. \(1995\)](#), the results indicate that in the absence of policy intervention, the open access solution yields an early period with underinvestment in efficiency-improving irrigation technology relative to the socially efficient solution, which is followed by a period of overinvestment. This suggests a potential role for irrigation capital subsidies to improve welfare over certain ranges of the state variables. In contrast to previous work, there is evidence that significant returns may be achieved from policy intervention. Despite exacerbating the overinvestment problem in the long run, irrigation technology subsidy programs may capture significant portions of the potential welfare gain in present value



terms.

The focus of this paper is to assess the impact of irrigation capital subsidies on common-pool groundwater use and depletion in a dynamic setting and contrast outcomes between the competitive and planning scenarios. To isolate the effect of endogenous irrigation capital and to relate these findings to previous literature, other model parameters including prices, crop water requirements, and crop yields, were assumed to remain constant over the planning horizon. To the extent that future long-term trends, such as climate change, enhance the incentives for myopic farmers to invest in efficient technologies in later periods when precipitation is more scarce, the role of irrigation capital and policies affecting it may be even more important. Modeling these effects through time-varying policy instruments is a potentially fruitful topic for further research.

## 1.5 Figures

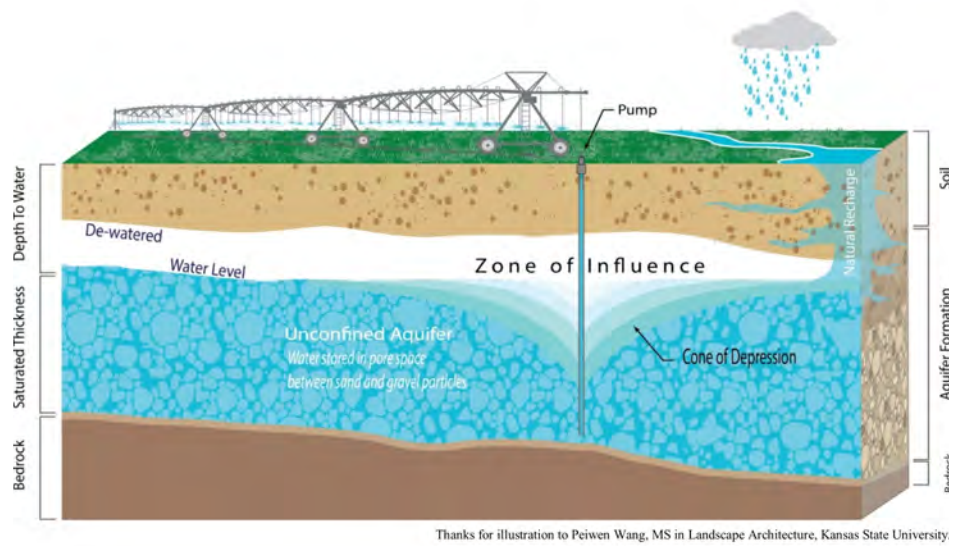


Figure 1.1: Typical Context of Irrigated Agriculture.

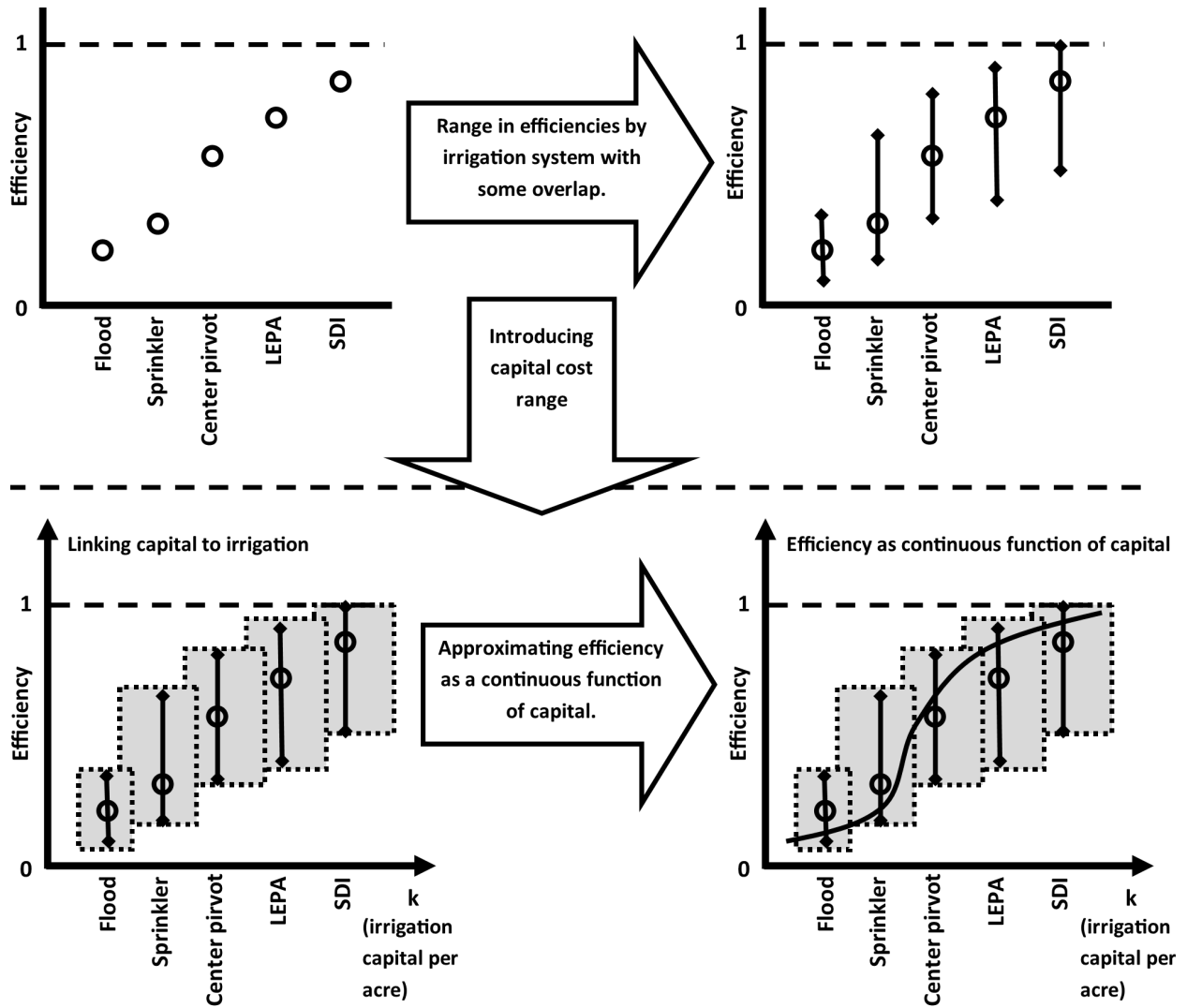


Figure 1.2: Irrigation efficiency as a function of capital.

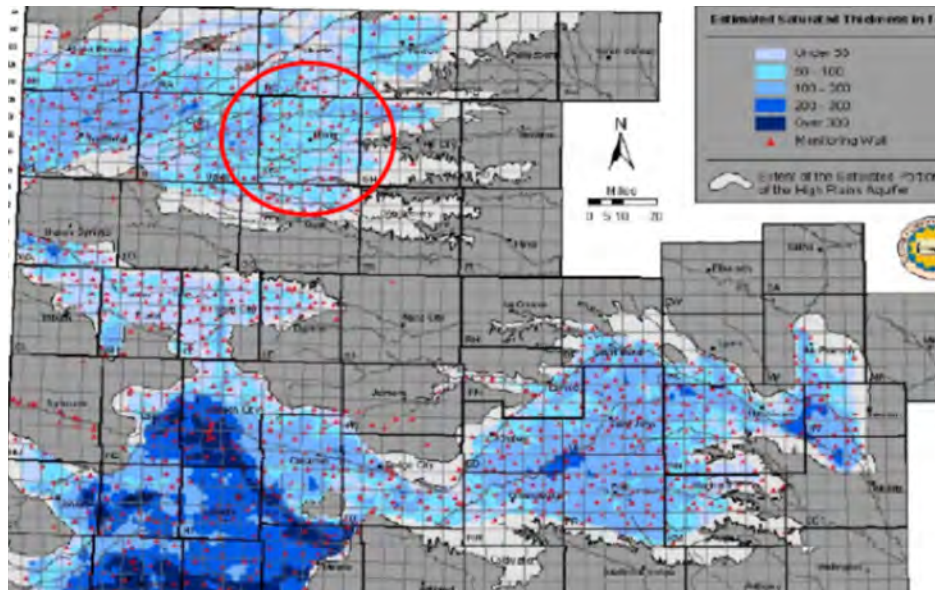


Figure 1.3: Saturated Thickness of the Ogallala in western Kansas. Source: Kansas Geological Survey.

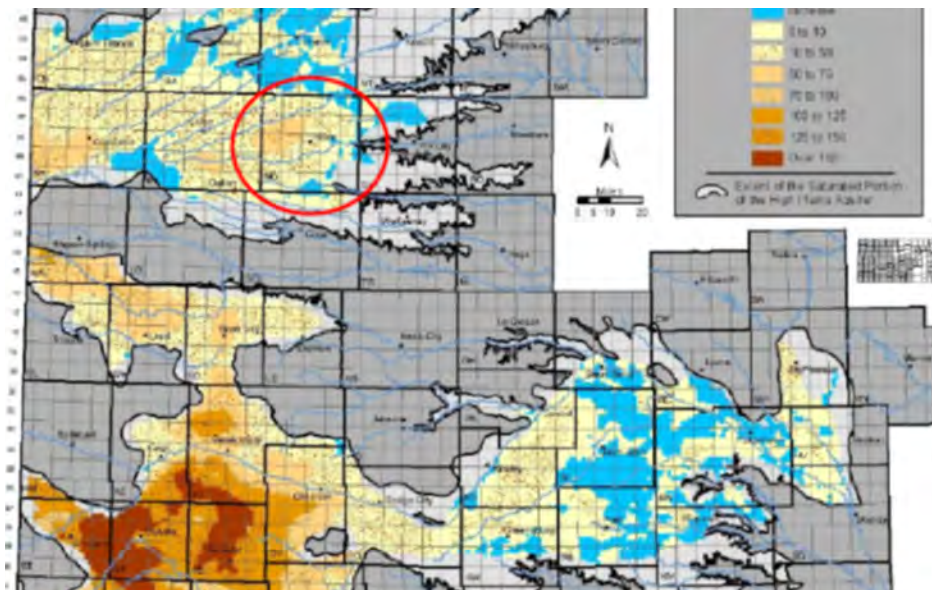


Figure 1.4: Change in water levels from pre-development to 2009-2011 average. Source: Kansas Geological Survey.

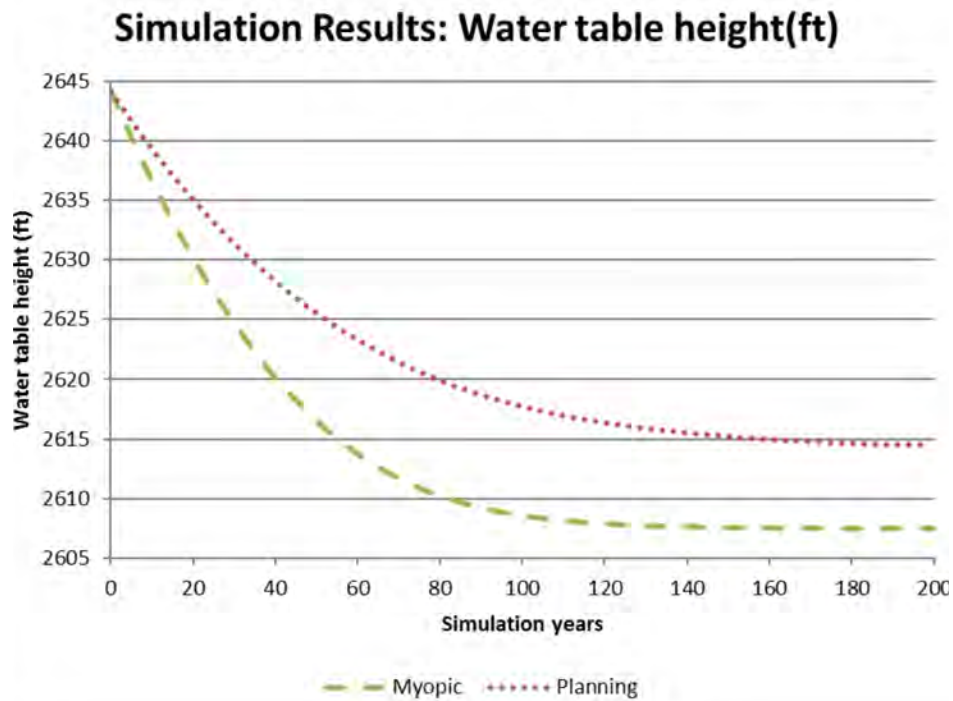


Figure 1.5: Simulated Aquifer Water Table Elevation.

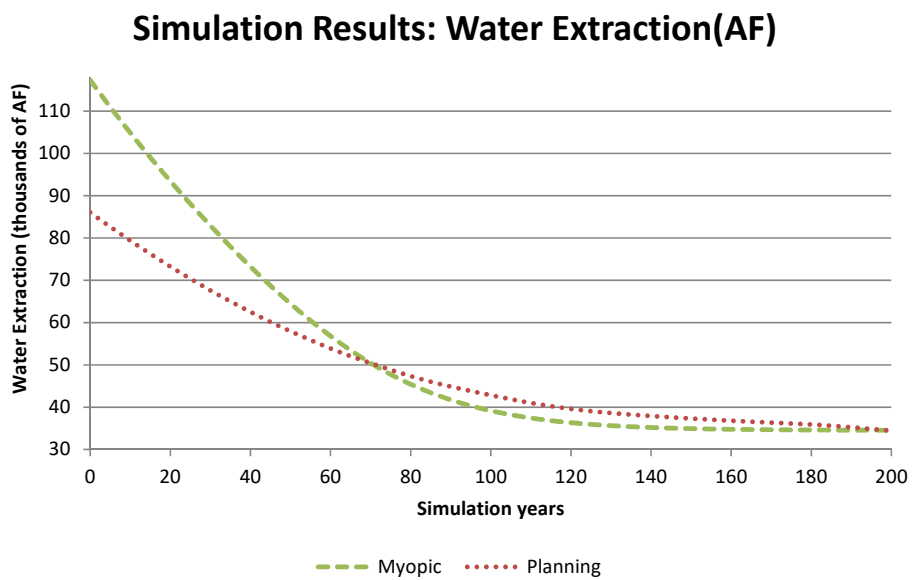


Figure 1.6: Simulated Total Water Pumped.

### Simulation Results: Application Efficiency

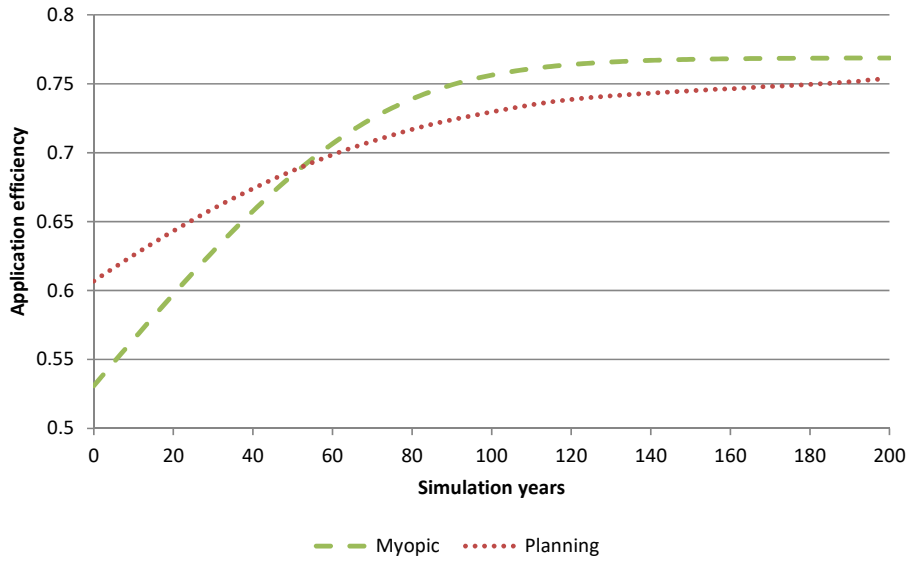


Figure 1.7: Simulated Application Efficiency.

### Simulation Results: Irr. Capital(\$)

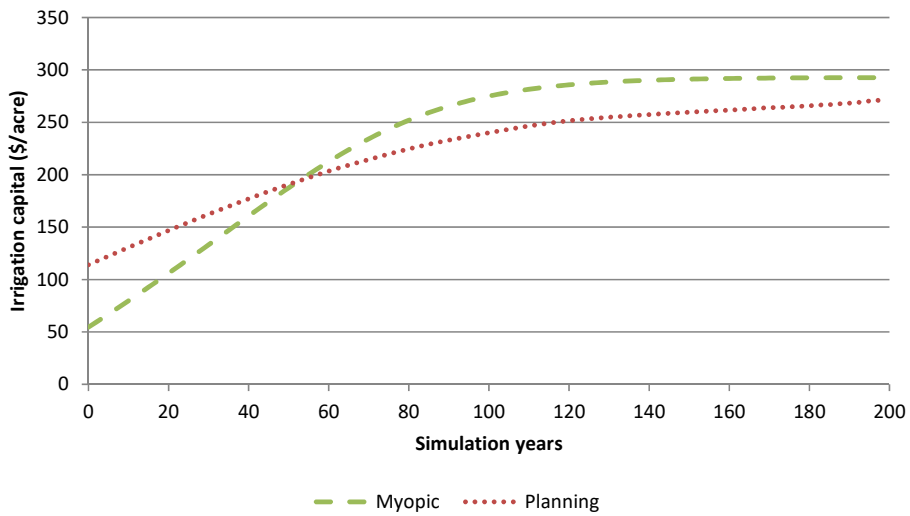


Figure 1.8: Simulated Irrigation Capital per Acre.

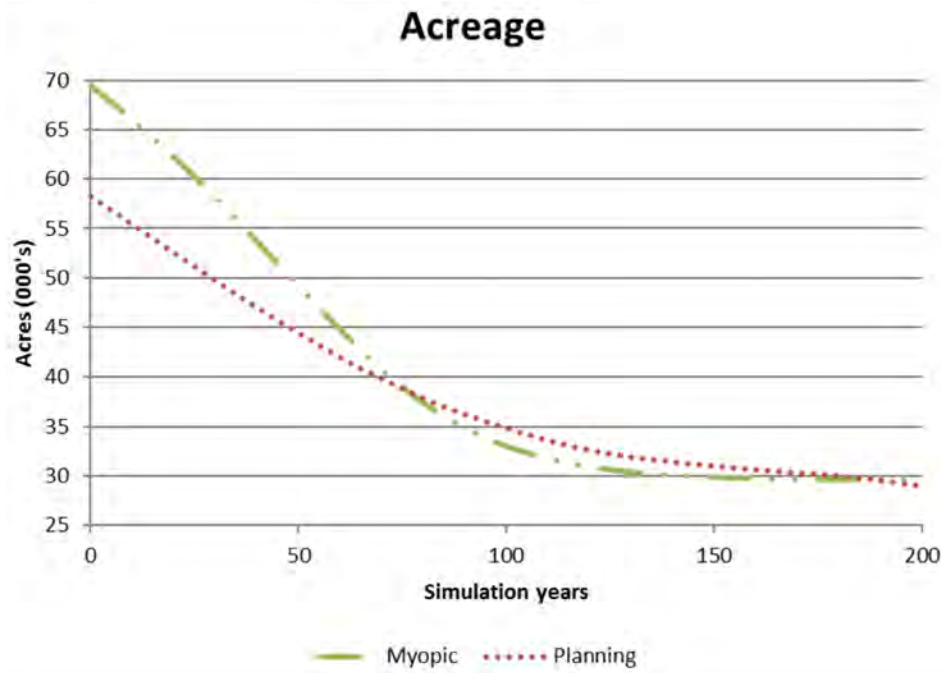


Figure 1.9: Simulated Irrigated Acreage.

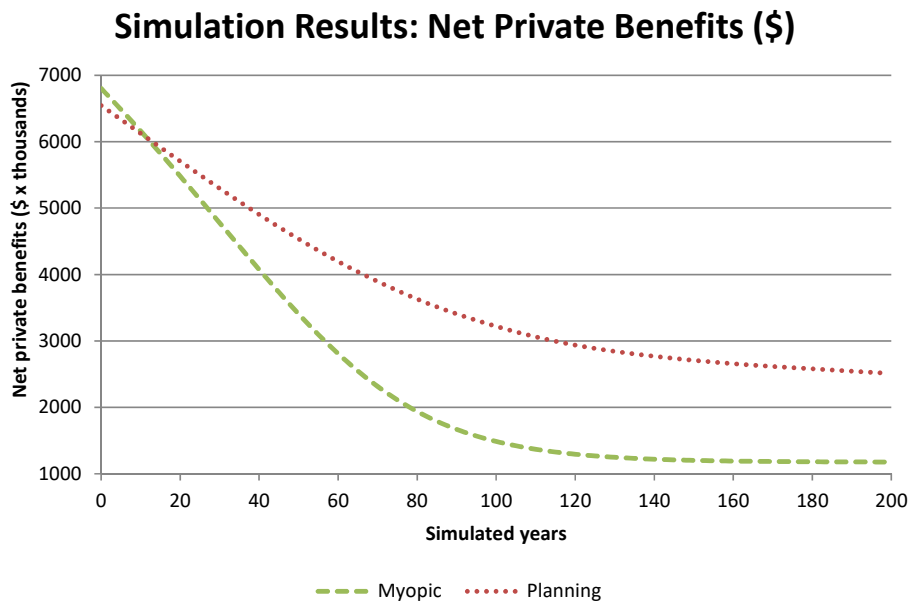


Figure 1.10: Simulated Net Private Benefits.

### Simulation Results: Water table elevation (ft)

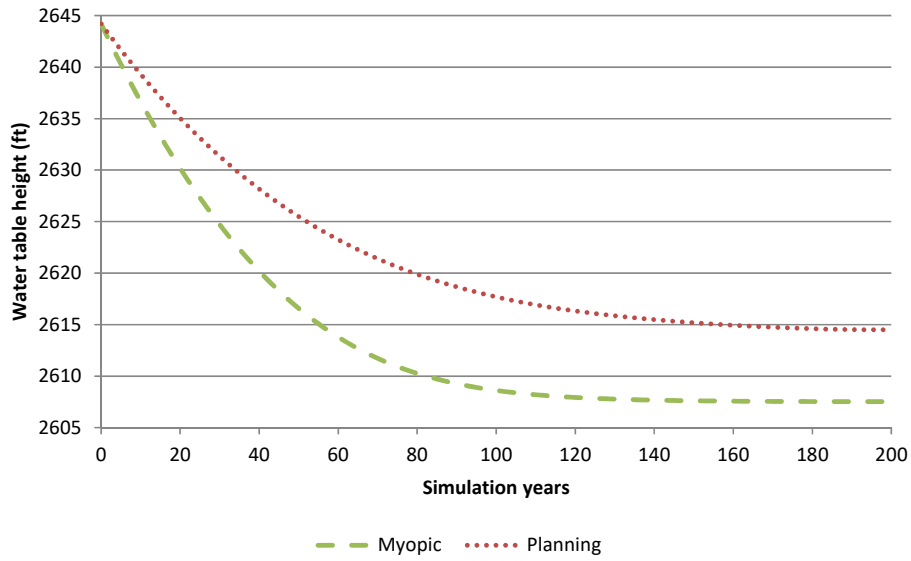


Figure 1.11: Water table height with irrigation capital subsidy.

### Simulation Results: Water Extraction(AF)

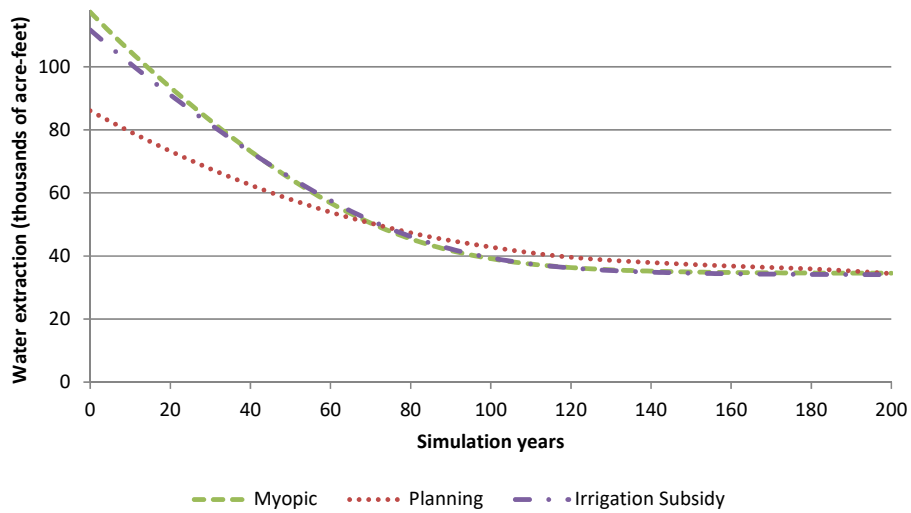


Figure 1.12: Water Pumping with Irrigation Capital Subsidies.



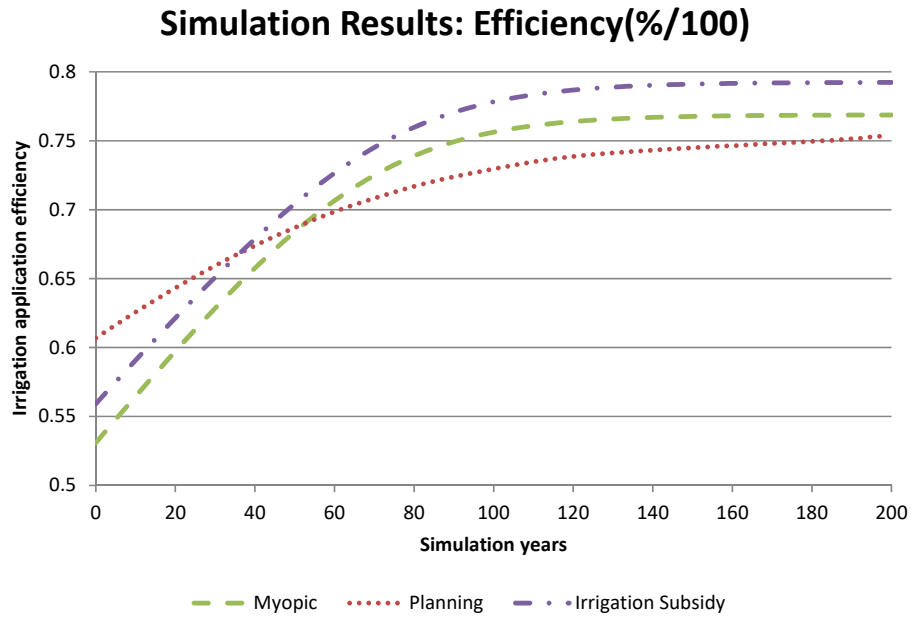


Figure 1.13: Application Efficiency with Irrigation Capital Subsidy.

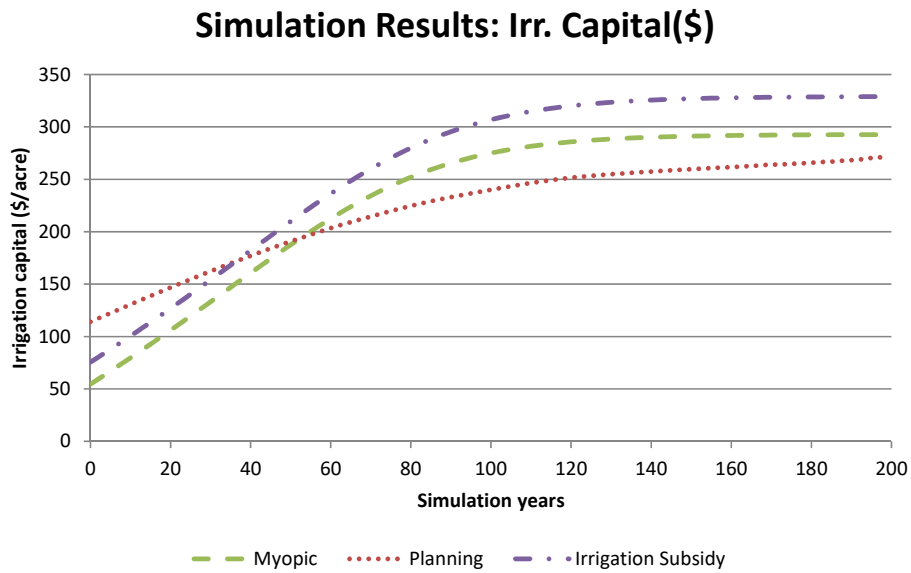


Figure 1.14: Irrigation Capital per Acre with Capital Subsidy.

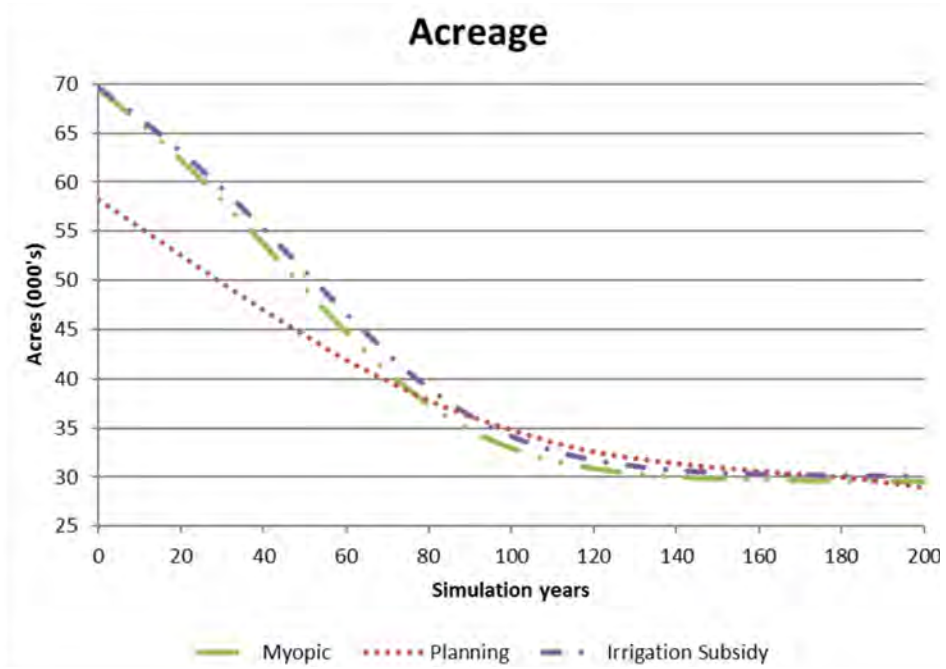


Figure 1.15: Irrigated Acreage With An Irrigation Capital Subsidy.

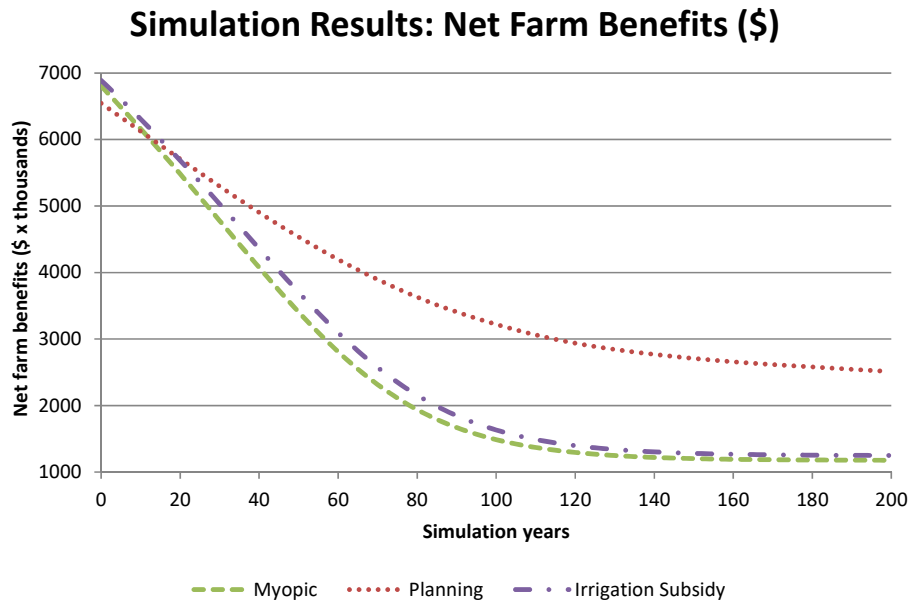


Figure 1.16: Net Private Benefits with Irrigation Capital Subsidy.

### Simulation Results: Net Social Benefits (\$)

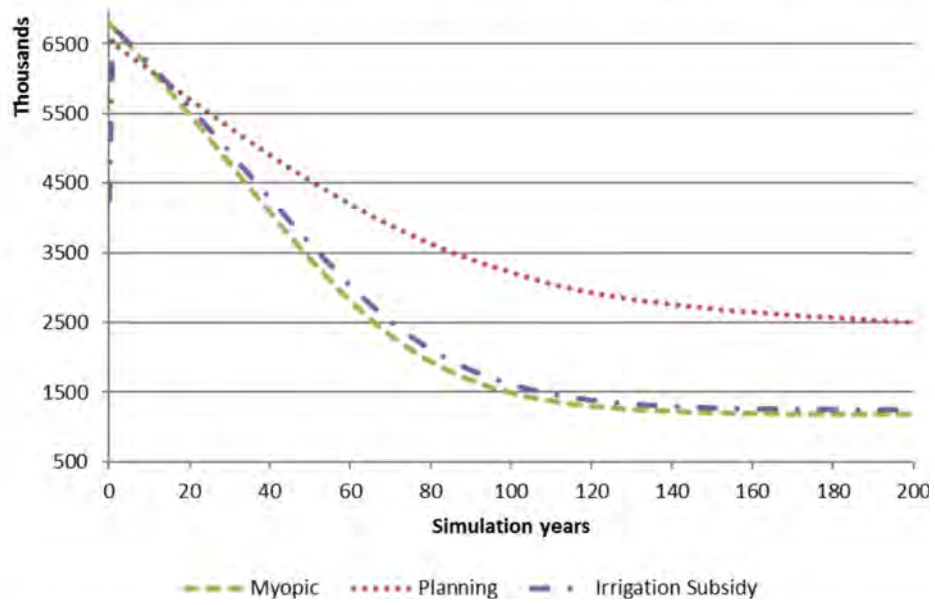


Figure 1.17: Net Social Benefits with Irrigation Capital Subsidy.

## 1.6 Tables

Table 1.1: Parameter and aquifer initial values for Sheridan, KS.

Parameter	Value
Area overlying the aquifer	4155,620.5 acres
Irrigated area	77,745 acres
Specific yield	0.1725
Depth to water	111.5 ft.
Saturated thickness	61.03 ft.
Drawdown	20 ft.
Rate of natural recharge	28,747.08 AF/year
Efficiency	
Flood irrigation	50%
Center pivot	90%
Subsurface drip	98%
Capital costs	
Flood irrigation	\$33/acre
Center pivot	\$575/acre
Subsurface drip	\$1,188/acre
Discount rate	3.89%
Depreciation rate	10%
Baseline labor requirement	0.8 hs/acre
Wage rate	\$9.12/hr

Table 1.2: Crop Water requirement per acre for Sheridan, KS.

Crop	Area Covered	NIR(AI)	NIR(AF)	Weighed NIR(AF)
Corn	86.9%	10.9	0.91	0.79
Soybeans	4.8%	10.1	0.84	0.04
Alfalfa	4.8%	11.8	0.98	0.05
Wheat	2.8%	6.5	0.54	0.01
Sorghum	0.7%	8.6	0.72	0.005
$C_R$	0.897204			

Table 1.3: Fitting of efficiency, return flow, and labor loading functions

Function	Form	Fitted function
Application efficiency	$e(k) = 1 - \hat{e}_1 \exp[-\hat{e}_2 k]$	$e(k) = 1 - 0.551e^{-0.00297k}$
Return flows	$\alpha(k) = \hat{\alpha}_1 \exp[-\hat{\alpha}_2 k]$	$\alpha(k) = 0.29257e^{-0.00192k}$
Labor requirement	$L(k) = \hat{L}_1 \exp^{-\hat{L}_2 k}$	$L(k) = 1.1839e^{-0.00512k}$

Table 1.4: Net present value of rents from irrigation.

	Planning	Myopic	Subsidy
Net Farmer Benefits			
NPV (\$ millions)	142.5	133.1	138
Gain (\$ millions)	9.4		6.1
	7.04%		3.64%
Net Social Benefits			
NPV (\$ millions)	142.5	133.1	135.3
Gain (\$ millions)	9.4		2.2
	7.04%		1.67%

# Chapter 2

## Optimal Groundwater Management under Climate Change and Technical Progress

### 2.1 Introduction

The economic dependency on irrigation of large agricultural regions such as the Great Plains in the United States makes aquifer depletion a much-discussed policy and research issue. Premature aquifer depletion can be costly. Temporal misallocation of the resource results in suboptimal levels of social welfare derived from mining the resource over time. Furthermore, premature depletion results in a diminished ability to cope with the added stress of higher evapotranspirative needs associated with climate change.

Despite the likely shift in groundwater demand over time, it is seldom accounted for in the groundwater management literature. The shifts in temperature levels and seasonal distribution of precipitation associated with climate change are expected to increase the demand for irrigation groundwater over time. Technical progress in the form of improvements in crop varieties that result in increased evapotranspiration productivity are similarly associated with shifts in the demand for irrigation groundwater. In this paper, climate change

and technical progress are exogenous state variables that modify the periodic value marginal product (VMP) of groundwater in an optimal control problem. The gains from management are found to be orders-of-magnitude larger than the case with static groundwater demand. Also, optimal extraction reaches a peak in the future whereas extraction is monotonically decreasing when groundwater demand is static.

There is a long history of literature studying groundwater as a common pool resource, in which a socially optimal extraction path is compared to the competitive, or rather non-intervention, extraction path. The implicit argument is that policy intervention is worthwhile if there is a significant difference between optimal and competitive paths in terms of social welfare. In a seminal work, [Gisser and Sanchez \(1980\)](#) found that the quantitative difference between competitive extraction and a socially optimal groundwater extraction rule was negligible. These results, referred to as the “Gisser Sanchez Effect” (GSE), have provided an economic rationale for opposing interventions that conserve groundwater for future use and focus on allocation of groundwater among different uses rather than over time ([Gisser and Sanchez, 1980](#)). The policy implications are important. For instance, the High Plains Ogallala Regional Aquifer Study commissioned by the Department of Commerce and the US Congress in 1982 predicted little to no difference in outcomes between a non-intervention projection and a management scenario; no significant management initiatives were implemented but the predictions in the study failed because the assumed dynamics of some factors were incorrect ([Peterson and Bernardo, 2003](#)).

Although the GSE persisted in the dynamic solutions in numerous studies since the 1980s, it has been increasingly clear that the GSE resulted from rather stringent and unrealistic assumptions. See [Koundouri \(2004\)](#), for a critical survey. One of the key assumptions in [Gisser and Sanchez \(1980\)](#), and most of the models that followed, was that of a static demand for extracted groundwater. [Brill and Burness \(1994\)](#) found that the GSE is not robust to the assumption of static demand and that growing demand will lead to an optimal extraction path with periods of both increasing and decreasing rates of pumping. Models with static demand involve only one dynamic state variable – namely the water table height – and impose a monotonic extraction path that asymptotically reaches a steady state. However,

historic data are generally consistent with non-monotonic extraction paths. [Steward and Allen \(2016\)](#) show that groundwater extraction paths follow curves similar to the Hubert curve in oil extraction studies. They estimate that peak “groundwater depletion caused by overtapping”, i.e. extraction rates beyond the rate of available recharge, has already occurred in many areas of the high Plains Aquifer while other areas are predicted to face peak extraction levels in the future.

Most groundwater economics studies assume static demand for groundwater – an incomplete list includes [Gisser and Sanchez \(1980\)](#); [Feinerman and Knapp \(1983\)](#); [Nieswiadomy \(1985\)](#); [Negri \(1989\)](#); [Pulido-Velazquez et al. \(2008\)](#); [Esteban and Albiac \(2011\)](#); [de Frutos Cachorro et al. \(2014\)](#); and [Esteban and Dinar \(2016\)](#). Few exceptions have incorporated non-static demand induced by additional control variables. [Kim et al. \(1989\)](#) presented an  $n$ -stage optimal control problem that incorporates separate groundwater demand curves for a set of crops over which the planner optimized intraseasonally resulting in a possibly shifting aggregate groundwater demand curve. Another exception is the approach by [Burness and Brill \(2001\)](#) and [Quintana-Ashwell and Peterson \(2016\)](#) which employs a model of substitutable irrigation capital in which investments in irrigation capital resulted in changing value marginal product of pumped groundwater over time.

This paper relaxes the monotonicity imposed by single-state models<sup>1</sup>, allowing for the possibility of non-monotonic paths. The formulation incorporates time-varying groundwater demand, explicitly linking the demand shifts to climate change and technical progress. Increases in precipitation result in inward shifts of the demand curve while increasing potential evapotranspiration result in outward shifts of demand. Technical progress causes the marginal value product of groundwater to increase over time in a manner consistent with increasing water productivity of irrigated crops. Optimal extraction is increasing when marginal benefits are increasing faster than marginal costs, as in early periods, then decreasing once marginal costs increase faster than marginal benefits. It is shown that it may be optimal for a manager to allow higher rates of extraction in the near future; for instance, results from Sheridan County, KS, indicate that peak groundwater demand occurs 23 years

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<sup>1</sup>See theorem 9.5.1 in [Leonard and Van Long \(1992\)](#), for an intuitive proof of this result.



into the future.

While climate change and technical progress make groundwater for irrigation more valuable and productive, the decline in the stocks of groundwater results in increasing marginal pumping costs due to increasing pumping lifts and decreasing groundwater well yields. The net effect of these counteracting forces on the optimal extraction path is ambiguous. Furthermore, when climate, technology, or both are assumed static, the plans that are optimal under those assumptions are not optimal when both climate change and technical progress are realized.

This paper also explores the nature of the “information effects” of prescribing extraction plans that assume climate, technology, or both are static. The “law of unintended consequences” is typically cited in political and economic policy circles to highlight the potential of well-intended policies to result in undesired and undesirable outcomes. However, [Merton \(1936\)](#) points-out that undesired is not equal to undesirable. Management plans that are optimal under a specific scenario but sub-optimal in any other realization of the future may still be desirable vis-a-vis non-intervention outcomes. To have a sense of how desirable the unintended consequences of each plan are, the difference between forward looking plans and myopic outcomes is contrasted under different realized scenarios.

## 2.2 Conceptual Framework

In this section, a stylized dynamic model of groundwater use is developed. The model in this section is simplified as a linear-quadratic formulation in order to obtain analytical solutions that develop intuition. In the next section, a more realistic formulation that requires a numerical solution is introduced.

The analytical and numerical models are based on the single-cell framework, which has been a workhorse of the groundwater management literature since its inception ([Feinerman and Knapp, 1983](#), [Gisser and Sanchez, 1980](#)). The single-cell model considers an aquifer underlying a flat land surface with vertical sides and holding water that flows laterally at an instantaneous rate so that withdrawals affect the water-table elevation equally in all

locations throughout the aquifer regardless of where it is pumped. A large number of users of water are assumed to be distributed across the land surface, with identical technology and exogenous prices so that a representative, competitive user can be aggregated to reflect basin-level outcomes.

The single-cell model can be criticized for its strong assumptions about hydrology, which do not accord with the spatial heterogeneity and the slow rates of lateral flow observed in many aquifers (Saak and Peterson, 2012). Recent literature has relaxed the assumptions of instantaneous lateral flow and spatial uniformity (Brozović et al., 2010, Gaudet et al., 2001, Guilfoos et al., 2013, Peterson et al., 2013, Pfeiffer and Lin, 2012, Saak and Peterson, 2007, Suter et al., 2012, Xabadia et al., 2004) to study spatially varying common-pool impacts. However, Brozović et al. (2010) indicate that the more parsimonious single-cell model may be adequate for small aquifers or a relatively small area within a much larger aquifer, as is the case of interest here. Moreover, the focus is on region-level outcomes as opposed to spatial patterns within the region.

A state variable of the model is the water table elevation,  $H$ , typically measured in feet above sea level<sup>2</sup>. The aquifer saturated thickness and well-pumping lift can be formulated from the water table elevation as

$$SaT = H - H_c,$$

$$Lift = S_L - H,$$

where  $SaT$  is saturated thickness,  $H_c$  is the elevation of the bottom of the aquifer and  $S_L$  is the elevation at the surface (i.e., top of the well).

As the aquifer depletes, groundwater is pumped from deeper underground and the value of  $H$  decreases. The more groundwater extraction exceeds the net recharge of the aquifer, the larger the decrease in water table elevation. The equation of motion for the water table elevation is

$$\dot{H} = \frac{dH}{dt} = \frac{1}{A_S} [r - (1 - \alpha) w], \quad (2.1)$$

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<sup>2</sup>For simpler notation,  $t$ -subscripts are generally omitted from dynamic variables, i.e. ( $H(t) = H_t = H$ ), but included when needed for clarity.

where  $A_S$  is the number of acres overlying the aquifer times the specific yield<sup>3</sup>,  $r$  is the instantaneous net rate of natural recharge of the aquifer,  $\alpha$  is the portion of water applied that returns to the aquifer, and  $w$  is the instantaneous rate of groundwater extracted. The initial water table elevation is  $H(0) = H_0$  and  $S_L > H > H_c$ .

The regional net benefits or rents from irrigation are represented by a reward function:

$$R(w; \beta, H) = B(w; \beta) - C(H)w; \quad (2.2)$$

where  $w$  is extracted groundwater in acre-feet,  $\beta$  is a parameter that determines the marginal value of groundwater, and  $H$  is the water-table elevation of the aquifer and represents the amount of groundwater in the aquifer which declines with  $H$ .  $B(w; \beta)$  is the benefit from extracting  $w$  units of water and  $C(H)$  is the (linear in  $H$ ) cost of pumping each acre-foot (AF) of groundwater, where  $B'(w; \beta) \geq 0$ ,  $B''(w; \beta) \leq 0$ , and  $C'(H) \leq 0$ . It is assumed that there are no benefits when there is no irrigation (i.e.,  $B(0; \beta) = 0$ ).

The analysis in this section assumes the reward function is quadratic so that the benefits from irrigation are the area under a linear VMP schedule,

$$B(w; \beta) = \int_0^w (\beta + \gamma u) du = \beta w + \frac{\gamma}{2} w^2, \quad (2.3)$$

and that the marginal pumping cost function is a linear function of pumping lift,  $C(H) = c(S_L - H)$ . The possibility that the marginal value of extracted groundwater may shift over time is also considered such that

$$\dot{\beta} = b_0 + b_1 \beta. \quad (2.4)$$

The special case of static VMP, which is implicit in most previous studies, occurs when  $b_0 = b_1 = 0$ .

First, consider the outcome of competitive pumping, in which farmers maximize the periodic rents from irrigation in a myopic fashion. The solution to this optimization results

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<sup>3</sup>The quantity of water which a unit volume of aquifer, after being saturated, will yield by gravity; it is expressed either as a ratio or as a percentage of the volume of the aquifer; specific yield is a measure of the water available to wells.

in groundwater extraction in each period such that the value of the marginal product (VMP) of groundwater equals its marginal cost of extraction:  $VMP = B'(w; \beta) = C(H)$ . When the VMP schedule of groundwater is assumed fixed over time (i.e., fixed  $\beta$ ), only the marginal cost of extraction,  $C(H)$ , changes over time. As the aquifer declines,  $H$  decreases and  $C(H)$  increases, resulting in a decreasing groundwater extraction path over time. However, when the VMP schedule is allowed to change over time, for instance due to improved technologies or changing climate, both the cost and value of groundwater extraction vary over time and myopic extraction may not be monotonically decreasing over time.

Figure 2.1a illustrates cases where myopic extraction is increasing between two time periods and Figure 2.1b shows the case in which it is decreasing between two time periods. For simplicity, the case of a linear VMP schedule is illustrated. Increasing extraction over time occurs when the increase in the value of groundwater is large relative to the increase in extraction costs at the margin. Decreasing extraction over time occurs when the increase in extraction cost is large relative to the increase in the value of groundwater.

Next, consider the dynamically optimal solution. A planner choosing an extraction path to maximize the net present value of the stream of rents from irrigation would solve:

$$\max \int_0^{\infty} e^{-\rho t} [B(w; \beta) - C(H)w] dt \quad (2.5)$$

subject to (1) and (4). In this specification, the problem is a linear-quadratic control problem with one control variable,  $w$ , the aquifer state variable  $H$ , and an exogenous state variable  $\beta$ . The full problem and its analytical solution are presented in the supplementary appendix. The optimal solution is a linear feedback rule of the form:

$$w = V + W_1\beta + W_2H, \quad (2.6)$$

where  $V$ ,  $W_1$ , and  $W_2$  are coefficients that depend on model parameters. The change in extraction over time may be written as

$$\dot{w} = W_1\dot{\beta} + W_2\dot{H}. \quad (2.7)$$

In the appendix, it is shown that the sign of  $W_2$  is positive. The sign of  $W_1$  is not unequivocally positive, however. In the supplementary appendix it is shown that  $W_1$  is positive for the range of plausible parameters for an aquifer. Only in the cases in which there is both a high discount rate (10 percent or more) and very low expected productivity gains (no more than 10 percent in productivity gains throughout the planning horizon) is the sign of  $W_1$  negative. The two exceptional conditions of poor gains in productivity and the heavy discounting of future benefits means that groundwater is not (more) valuable in the future. At the extreme of no productivity gains and completely discounted future benefits, the optimal solution would be myopic implying no gains from management are possible.

There is no authoritative reference with respect to potential gains in agricultural productivity into the long-term, but working assumptions employed by USDA (Sands, 2014) foresee increases in productivity between nearly 50 percent (oilseeds) to nearly doubling (coarse grains) by year 2050. The discount rate condition is even more arbitrary; but the discourse in academic and policy circles seems to be more for de-penalizing future benefits by employing lower social discount rates (even zero or negative, in some cases; e.g., Hellweg et al., 2003) than for heavier discounting. Consequently, the available evidence points to scenarios in which groundwater will be more valuable in the future, in which case the sign of  $W_1$  would be positive.

The positive sign of  $W_2$  indicates that decreased levels of groundwater stock create an incentive for smaller amounts of groundwater extraction. Similarly, the positive sign of  $W_1$  indicates that increases in the value of water create an incentive for greater amounts of groundwater extraction.

A key insight from equation (2.7) is that extraction may not be monotonically decreasing over time. Intuitively, the term  $W_1\dot{\beta}$  represents the impact of changes in marginal benefits over time on extraction and the term  $W_2\dot{H}$  represents the impact of changes in marginal costs over time. If the benefits from irrigation are increasing sufficiently fast, extraction

increases over time. Of course, the benefits from irrigation are not likely to continue a rapid increase over an infinite horizon. If marginal benefits increased rapidly until infinity, then extraction is monotonically increasing in the unrealistic case of a bottomless aquifer. In the case of an aquifer with a bottom, then the solution is unstable because there is an incentive to always conserve the water to some future period with greater benefits. Therefore, two extraction paths are most likely .

The first likely path is when marginal benefits increase slowly enough in all periods such that equation (2.7) is negative and extraction is monotonically decreasing. In the special case of static marginal benefits ( $\dot{\beta} = 0$ ), the extraction path is guaranteed to be monotonically decreasing as in much of the previous literature (e.g. Gisser and Sanchez, 1980; Feinerman and Knapp, 1983; Allen and Gisser, 1984; Pardo et al., 1998; Burness and Brill, 2001; Esteban and Albiac, 2011; Quintana Ashwell and Peterson, 2015; and Esteban and Dinar, 2016). The second likely path is when marginal benefits increase sufficiently fast in early periods and then begins to slow down relative to the increase in costs. In this second case, extraction is increasing in early periods then declines in later periods—effectively creating a peak in groundwater demand.

This stylized model illustrates that dynamically optimal groundwater extraction is not necessarily monotonically decreasing over time and provides an economic intuition on the conditions that result in increasing extraction. However, a limitation of this stylized model is that the linear pumping cost formulation approximates the marginal cost of pumping at the initial lift but it progressively underestimates the marginal cost of pumping over time. As illustrated in figure 2.1a, a marginal cost of pumping with shifts that are unrealistically slow over time may erroneously prescribe increases in extraction when a more realistic formulation results in lower extraction. Similarly, if an increasing path may be optimal over a time lapse, the increasingly underestimated marginal cost of pumping would result in increasing rates of extraction at larger magnitudes over a greater length of time than a more realistic formulation. Furthermore, the linear (in lift) marginal cost of pumping implies an unrealistic bottomless aquifer and a decreasing shadow value of groundwater— i.e., its value decreases as it becomes scarcer (Tomini, 2014).

A more realistic formulation would account for nonlinear (with respect to aquifer water levels) increases in pumping costs over time. Unfortunately, no closed-form solutions are possible for such a formulation of the pumping cost function: The model must be solved numerically. The next section describes the numerical solution methods including a nonlinear pumping cost function and details the decomposition of  $\beta$  as a function of climate (CC) and technical change (TC) variables to model the shifts in the VMP schedule due to these effects.

## 2.3 Numerical Simulation Model

Optimal control problems are analytically untractable, except under specific functional forms of the equations of motion and the reward function as in the previous section. In this section, declining well yields are incorporated into the model which makes the pumping cost function nonlinear (Brill and Burness, 1994). One important aspect of incorporating declining well yields is that it effectively places a bottom on the aquifer. Unfortunately, the optimal control problem with declining yields can not be formulated to give analytical solutions. Consequently, a discrete numerical simulation model is created with reasonable parameter values in order to examine the dynamically optimal path of extraction and compare myopic and planned solutions.

The parameter values in the numerical simulation model are obtained or calibrated for Sheridan County, KS, which is a particularly useful region to study. The hydrological and agricultural uniformity of the region make the assumptions of a representative user and a single-cell aquifer applicable. The region is also interesting from a policy perspective due to the recent implementation of a farmer-led initiative “Sheridan 6 Local Enhanced Management Area (LEMA6)”, which roughly establishes a 20 percent reduction from historical pumping across the area.<sup>4</sup> The planned allocations presented in this paper are informative for such policies. Another advantage of selecting this region in Kansas is the wealth of agricultural, agronomical, hydrological, and water use data.

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<sup>4</sup>All farmers were given an allocation of 55 acre-inches per irrigated acre for the period between 2013 and 2017, inclusive.

### 2.3.1 Hydrology and Pumping Costs

The hydrologic assumptions of the model are based on the single-cell aquifer framework as summarized by equation (2.1) in the previous section. However, the cost of pumping is modified here to capture the potentially nonlinear effects of declining groundwater stocks. The cost of pumping depends on the amount of energy used by pumps to deliver groundwater from the water table in the aquifer to the outlet in the irrigation system at a given pressure. The decline in the stock of groundwater available in the aquifer affects the cost of (amount of energy used in) pumping in at least two ways. First, more energy is required to pump each unit of water because it is transported from deeper in the aquifer (i.e., pumping lift ( $S_L - H$ ) increases). Second, more time is needed to extract a unit of groundwater as well yields—volume of extraction per unit of time—decline. For example, a 50 percent reduction in well yield requires 100 percent more time to pump a given amount of groundwater. The well yield function proposed by Sloggett and Mapp (1984) is used:  $Y = 2Q_0d(SaT - \frac{d}{2}) = 2Q_0d(H - H_c - \frac{d}{2})$ , where  $d$  is drawdown and  $Q_0$  is a constant calculated based on well site characteristics. This function has been previously applied in the economics literature (Brill and Burness, 1994; Burness and Brill, 2001 and Quintana and Peterson, 2015 among others). Combining these two effects, the marginal pumping costs in dollars per acre-foot is

$$C(H) = \frac{c_0}{Y} \left[ \frac{S_L - H}{S_L - H_0} \right] = C_0 \left[ \frac{S_L - H}{H - H_c - \frac{d}{2}} \right], \quad (2.8)$$

where  $C_0 = c_0 / (2Q_0d(S_L - H_0))$ . An important feature of equation (2.8) is that as the water table reaches the bottom of the aquifer ( $H \rightarrow H_c$ ), the denominator approaches zero and the marginal cost of pumping approaches infinity. Therefore, the water table never goes lower than the bottom of the aquifer after accounting for the drawdown caused by pumping.

The parameter values for Sheridan County, KS, are summarized in table 2.1. The parameters in (2.8) are calculated following Rogers and Alam (2006) such that  $C_0 = 0.975$ ,  $S_L = 2,755$ ,  $H_0 = 2,644.2$  ft.,  $Q_0 = 3.48E - 07$ , and  $H_c = 2,583.2$ . Aquifer initial conditions and parameters were obtained from the Kansas Geological Survey (KGS), the Water



Rights Information System (WRIS), and the Water Information Management and Analysis System (WIMAS). At initial conditions, the marginal cost of pumping is \$22 per acre-foot of groundwater.

### 2.3.2 Climate Change

Climate change affects both water availability and the demand for water (Doll, 2002). It is multifaceted and spatially heterogeneous: different climate variables change in different directions in different regions. Some regions of the world could face a decline in their water availability while others could see a surplus water supply (Elliott et al., 2014). In the Midwest US, irrigation requirements are expected to increase (Doll, 2002). Projections from USGS's National Climate Change Viewer show that little to no change in average annual precipitation over time. However, the temporal pattern of precipitation is expected to decrease at critical times in the growing season, making irrigation increasingly valuable.

To reflect the changes in the pattern as well as the levels of precipitation within the season, the climate change variables associated with the model are: average precipitation between January and April ( $J$ ), average precipitation between May and August ( $M$ ) precipitation, and average evapotranspiration from May to August ( $E$ ). Linear dynamics are devised for these variables following the equations of motion:

$$\dot{J} = a_0 - a_1 J \tag{2.9}$$

$$\dot{M} = a_2 - a_3 M \tag{2.10}$$

$$\dot{E} = a_4 - a_5 E. \tag{2.11}$$

The parameters  $a_0, a_1, \dots, a_5$  are calibrated using values for each of the climate variables at a starting point ( $t = 0$ ), at an intermediate point ( $t = 1$ ), and at the steady state (asymptotic value as  $t \rightarrow \infty$ ). Initial values for average January to April ( $J_0$ ) and May to August ( $M_0$ ) precipitation, and for May to August evapotranspiration ( $E_0$ ) are obtained from Hendricks and Peterson (2012). Terminal (asymptotic steady state) values for these

variables are the expected annual average levels projected for the entire period between the years 2075 and 2100 according to the ensemble average projection of the 5th Climate Model Intercomparison Program (CMIP5). Finally, the climate change variables are assumed to change at decreasing rates reaching steady states asymptotically, so the largest changes occur at the beginning. The largest changes in the United States Geological Survey's National Climate Change Viewer (USGS NCCV) are  $\dot{J} = 0.33$ ,  $\dot{M} = -0.13$ , and  $\dot{E} = 0.01$ . The parameters  $a_0$  thru  $a_5$  are found solving a system of 6 equations in 6 unknowns:

$$\begin{aligned}
 J_\infty &= \frac{a_0}{a_1} = \bar{J}_{(2075-2100)} \quad , \quad 0.33 = a_0 - a_1 J_0 \\
 M_\infty &= \frac{a_2}{a_3} = \bar{M}_{(2075-2100)} \quad , \quad -0.13 = a_2 - a_3 M_0 \\
 E_\infty &= \frac{a_4}{a_5} = \bar{E}_{(2075-2100)} \quad , \quad 0.01 = a_4 - a_5 E_0.
 \end{aligned}
 \tag{2.12}$$

The initial value for the variables and the calculated value for the parameters in equations (2.9) to (2.11) are reported in table 2.1. Notice that the value of these variables at time  $t$  are easily calculated as

$$\begin{aligned}
 J(t) &= \frac{a_0}{a_1} + \left( J_0 - \frac{a_0}{a_1} \right) e^{-a_1(t-1)} \\
 M(t) &= \frac{a_2}{a_3} + \left( M_0 - \frac{a_2}{a_3} \right) e^{-a_3(t-1)} \\
 E(t) &= \frac{a_4}{a_5} + \left( E_0 - \frac{a_4}{a_5} \right) e^{-a_5(t-1)};
 \end{aligned}
 \tag{2.13}$$

where  $t$  can be in continuous or discrete time.

### 2.3.3 Technical Change and Groundwater Demand

Although advances in agricultural biotechnology, equipment, and machinery may occur in response to market signals, these occur at aggregation levels that are distant from the relevant decision unit: the irrigator. Consequently, such technical changes are exogenous to farmers.

Technical change may occur in diverse ways. Advances in biotechnology may result in one or several of the following changes: (i) crop wilting points may be reduced; (ii) fully-

watered yields may be increased; (iii) potential evapotranspiration may decrease or increase; or (iv) the shape of the yield water response functions may change. Advances in equipment, machinery, and farming practices may result in improved precipitation effectiveness or improved application efficiency. All these changes modify the incentives of farmers to pump groundwater.

In this paper, technical progress is modeled as shifts in a linear groundwater (inverse) demand function that is conditional on climate conditions and where the intercept term represents the state of technology:

$$p^w(w; \beta_0, J, M, E) = \beta_0 - \beta_1 J - \beta_2 M + \beta_3 E - \beta_4 w, \quad (2.14)$$

where  $w$  is groundwater pumping,  $\beta_0$  is the intercept representing the state of technology,  $J$  is average daily precipitation between January and April,  $M$  is average daily precipitation between May and August, and  $E$  is evapotranspiration between May and August. The coefficients for  $J$ ,  $M$ , and  $E$  are calculated and rescaled from Hendricks and Peterson (2012). The demand function in equation (2.14) is consistent with a quadratic production function for a composite irrigated crop that depends on the volume of irrigation water applied. Quadratic crop yield response to irrigation is consistent with the agronomic literature that relates crop yields to irrigation application (Martin et al. 1984).

Climate change affects water demand by allowing  $J$ ,  $M$ , and  $E$  to enter as dynamic parameters that shift the (inverse) demand curve for groundwater. Technical change affects water demand through upward shifts in the VMP schedule by increasing  $\beta_0$  over time. The equation of motion for the intercept is specified as a linear approximation of the rate of technical progress:

$$\dot{\beta}_0 = b_0 - b_1 \beta_0. \quad (2.15)$$

The initial value for  $\beta_0$  is calculated residually from the intercept term in the (inverse) groundwater demand equation ( $\beta_0(0) = 232.67$ ). The terminal value is established to reflect

an assumption of water productivity asymptotically reaching a level twice<sup>5</sup> its initial value:

$$\beta_{\infty} = \frac{b_0}{b_1} = 2\beta_0.$$

Finally, to calculate the technical change parameter,  $b_0$  and  $b_1$ , a 1.1 percent increase in productivity is assumed on the initial period. This productivity increase is consistent with what Quintana and Featherstone (2015) found for a sample of Kansas farms over the period 1993 to 2011. The time path for the technical change parameter is described by:

$$\beta_0(t) = \frac{b_0}{b_1} + \left( \beta_0 - \frac{b_0}{b_1} \right) e^{-b_1(t-1)}. \quad (2.16)$$

### 2.3.4 Groundwater Extraction under Myopic and Alternative Planning Scenarios

The difference in periodic groundwater allocations between myopic and planned scenarios reflects the main societal trade-off between current versus future food production. In this context, net farm benefits are a good approximation for social welfare (Quintana and Peterson, 2015).

The rents from irrigation function is the area under the inverse groundwater demand curve minus the cost of extraction and represents the profits in addition to what can be achieved from dryland rather than actual farm profits. The periodic rent function for irrigation is expressed as

$$R(w; \beta_0, J, M, E, H) = \left[ \beta_0 - \beta_1 J - \beta_2 M + \beta_3 E - \frac{1}{2} \beta_4 w - C_0 \left[ \frac{S_L - H}{H - H_c - \frac{d}{2}} \right] \right] w. \quad (2.17)$$

Given the state of the climate, technology, and the aquifer  $(\beta_0, J, M, E, H)$ , the myopic

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<sup>5</sup>References on productivity ceilings are not easy to find in the literature. The productivity parameter ceiling of twice its initial value is somewhat arbitrary but based on Monsanto's stated goal of doubling yields of corn and other crops by year 2030 (Edgerton, 2009). Most studies found on the topic of productivity increase focus on yield trends suggesting average rates of productivity increases but few provide insight into the decreasing rates of the increases (Ewert et al., 2005, presents data showing the decreasing growth of productivity gains over time).

allocation is determined by first order conditions on the rent function:

$$\frac{\partial R(w; \cdot)}{\partial w} = \beta_0 - \beta_1 J - \beta_2 M + \beta_3 E - \beta_4 w - C_0 \left[ \frac{S_L - H}{H - H_c - \frac{d}{2}} \right] = 0. \quad (2.18)$$

In contrast, the optimal plan accounts for all state variables and maximizes the net present value of the stream of rents from irrigation

$$NPV = \max \sum_{t=0}^T \left( \frac{1}{1 + \rho} \right)^t R(w(t); \beta_0(t), J(t), M(t), E(t), H(t), t) \quad (2.19)$$

subject to the equations of motion (2.13),(2.16), and the discrete approximation of (2.1):

$$H_{t+1} - H_t = \frac{1}{A_S} [r - (1 - \alpha) w_t] \quad (2.20)$$

where  $w_t$  is the total amount of groundwater extracted in period  $t$ , as opposed to the instantaneous rate implied in equation (2.1). Similarly, the rate of recharge in this equation is in acre-feet per year (AF/yr, see table 2.1). The discount rate<sup>6</sup> considered,  $\rho = 0.0389$ , is the average interest rate on farm loans as reported from the Kansas City Federal Reserve Bank (November, 2011).

Four alternative plans of groundwater extraction are considered. Each of the plans achieves the highest gains from management for the scenario it assumes. The first plan accounts for changes in water table height, climate and technology and is labeled “Baseline.” The second plan, labeled “No TC”, assumes there is no technical change:  $\beta_0(t) = \beta_0(0)$ ,  $\forall t$ . The third plan assumes climate change does not realize (i.e.,  $J(t) = J_0$ ,  $M(t) = M_0$ ,  $E(t) = E_0$ ,  $\forall t$ ), and is labeled “No CC.” The last plan is labeled “No CC or TC” and considers only the aquifer dynamics with the assumption that neither climate nor technical change realize, i.e., the right-hand side of equations (2.9) through (2.11) and (2.15) equal zero.

By solving for different planned solutions, this paper is able to assess the “information

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<sup>6</sup>There is a growing literature that deals with the question of how future outcomes should be discounted. A discussion of the topic is beyond the scope of this paper but a good one is available in Gollier and Hammitt (2014). In any case the 3.89 percent rate used here is within the range employed in the literature.

effects” of prescribing a groundwater extraction plan without accounting for important dynamic factors that influence the incentives at play in irrigated agriculture. Each plan is the best for the state of the world it assumes. However, by comparing across the solutions it is possible to assess the information effects of plans that are formulated with assumptions that turn out to be incorrect. For example, it is possible to assess the risks, in welfare terms, of developing plans and implementing policies that assume no climate change or technical change, when in fact those changes are realized. Similarly, the risk of implementing policies can be assessed based on plans that do assume climate change or technical change, when in fact those changes do not occur as expected.

The NPV of the stream of benefits under each planning scenario under alternative realizations is calculated (i) when both technical change and climate change occur, (ii) when only technical change occurs, (iii) when only climate change occurs, and (iv) when demand is static. The outcomes indicate the loss in welfare from an incorrect plan that ignores a dynamic aspect of demand that should have been considered or accounts for a dynamic aspect of demand that does not actually occur.

The effects of climate change and technical change on the aquifer become apparent when comparing aquifer outcomes (water table elevation, pumping lift, saturated thickness) over time. Because conservation is not a goal in the optimization problem it is not necessarily expected that the optimal plan will result in an aquifer depleted to a lesser degree. Actually, because what is being maximized is the net present value of the rents from irrigation (NPVs), it is not even the case that periodic rents in the steady state would be higher under the optimal plan.

## 2.4 Results

The optimization problem in (2.19) is solved with a quasi-newton algorithm in MatLab<sup>®</sup>. The states of climate and technology are calculated from equations (2.13) and (2.16) for  $t = 0, 1, \dots, T$ . The control variable  $\mathbf{w}$  is a vector of size  $T + 1$  for which the entries correspond to periodic groundwater extractions. The elements of a vector  $\mathbf{h}$  represent the water table

elevation for each period and is calculated from each element in  $\mathbf{w}$ . With each trial value of  $\mathbf{w}$ , the values of  $\mathbf{h}$  and the NPV of stream of rents is updated. Outcomes under several planing horizons were compared ( $T = 200, 500, 700, 1000, 10000$ ) yielding insignificant differences in path and NPV amounts between horizons of more than 500 periods. Results from  $T = 500$  are reported unless otherwise indicated.

### 2.4.1 Groundwater extraction and depletion

Figure 2.2 shows the time path of groundwater extraction prescribed under each plan. The “*No CC or TC*” plan has a strictly decreasing trajectory because it considers only one state variable: the stock of groundwater, which drives pumping costs upward as the stock diminishes. All other plans and the myopic outcome exhibit periods of increasing rates of extraction induced by the increasing benefits of groundwater extraction from technical change or climate change. In particular, groundwater extraction reaches the highest peak in period 23 of the “*Baseline*” plan. The shape of the extraction path for all of these plans are consistent with equation (2.7) in section 2.2 and the intuition from figure 2.1: a monotonic path for plans considering a single state and a (possibly) non-monotonic path for plans incorporating more than one state variable.

Although periodic groundwater allocations differ across plans, all forward-looking plans prescribe lower rates of extraction than the myopic case for the first 37 periods of simulation. This translates into more saturated thickness under the forward-looking plans— i.e., groundwater conservation. Figure 2.3 shows the corresponding saturated thickness of the aquifer over time.

Notice in figure 2.2 that the “*Baseline*” plan prescribes the lowest rates of initial rates of extraction (i.e., groundwater conservation corresponding to a 20 percent reduction from the starting point of the myopic trajectory<sup>7</sup>) but also allows to reach the highest peak of extraction among the forward-looking plans. The differences in extraction between the

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<sup>7</sup>Prior to the establishment of Sheridan County’s Local Enhancement Management Area (LEMA), average groundwater use was 13.18 acre-inches (AI) per year. The LEMA established a limit of 55 AI over a five year period, i.e. an average of 11 AI per year; which is a reduction of approximately 17 percent in initial extraction rates.

“*Baseline*” and the other forward-looking plans is relatively large while the difference between the “*No TC*” and “*No CC*” plans is relatively small.

## 2.4.2 Gains under realized climate and technical change

Figure 2.4 depicts the path of realized rents from the different plans, assuming that climate and technology change over time as expected in the baseline case. The rents from irrigation are calculated from the periodic reward function and all the state variables are allowed to update even in the outcomes where one or more of the state variables were ignored in the planning process. That is, extraction occurs in the amount prescribed by each plan but the profits from that prescription are derived from updated technology and climate. The myopic outcomes are obtained from static periodic optimization considering the updated state variables for each period.

As expected, there are significant differences in the periodic rents between each of the plans and the myopic outcome (figure 2.4). The periodic rents from irrigation capture the “information effects” of prescribing allocations from plans that consider different information sets regarding state dynamics. Periodic rents under myopic pumping are slightly larger than any of the plans for the first 25 periods but decline precipitously thereafter and myopic rents are lower than any of the plans by period 30. The paths of groundwater extraction and rents from irrigation have similar patterns but the difference in relative magnitudes among the different plans is much larger with respect to groundwater extraction. Another interesting feature is that planned peak groundwater extraction (around period 25) precedes planned peak groundwater rents (near period 40).

Notice also in figure 2.4 that future rents from the alternative plans (not the “*Baseline*”) yield rents consistently higher than the “*Baseline*” and myopic plans from period 50 onward. However, if the goal is to maximize the stream of discounted benefits rather than to conserve groundwater, then the problem is to identify the best path for decline. In this paper the optimal path is determined by the maximization of the NPV of rents from irrigation over the life of the resource which in this formulation reaches a steady state within the first one



hundred periods.

A salient result from the simulations is that the myopic scenario reaches a steady state much quicker than any of the forward-looking management plans because aquifer depletion also occurs at a faster rate. The implication of this result is that the “option value” of having a reserve stock of water is also eliminated in a relatively early stage. This means that the ability to alleviate the effects of extreme weather or market conditions is essentially forfeited early on under the myopic regime.

Table 2.2 compiles the net present value of the rents from irrigation (NPV), the potential gains from management, the cumulative groundwater extraction, and the amounts of water potentially saved under each alternative plan for the first two hundred periods of simulation.

The “*Baseline*” extraction plan yields the highest NPV of cumulative rents at \$685 million while the myopic outcome accumulates the lowest NPV of rents at \$529 million. The alternative plans, i.e. “*No TC*”, “*No CC*” and “*No CC or TC*”; yield accumulated NPVs of \$681 million, \$679 million, and \$659 million respectively. All the forward-looking groundwater management plans yield significant potential gains from management for the first 200 periods of simulation. The optimal plan yields the highest gains at 29.5 percent larger NPV than the myopic case. The “*No TC*” plan yields 28.8 percent gains from management. The “*No CC*” plan yields 28.4 percent gains and the “*No CC or TC*” plan yields 24.6 percent in potential NPV of rent gains from managing the aquifer.

Cumulative extraction over the first 200 periods was as high as 10.16 million acre-feet for the myopic plan and as low as 9.37 million acre-feet (a 7.8 percent groundwater savings relative to the myopic outcome) for the “*No CC or TC*” plan. The “*Baseline*” plan prescribed the largest amount of groundwater extraction among the planned outcomes at 9.86 million acre-feet resulting in water savings of 2.9 percent compared to the myopic outcomes. The “*No TC*” and “*No CC*” plans extracted 9.78 and 9.66 million acre-feet representing savings of 3.8 and 4.9 percent from to the myopic case, respectively.

The contrast between cumulative NPVs and cumulative groundwater extraction highlights the implicit trade-off involved in the formulation of a groundwater management plan. If the extraction from the “*Baseline*” plan is implemented and becomes the basis of com-

parison, the results from table 2.2 can be conversely read as the costs in foregone rents to obtain savings in groundwater— i.e., conservation. For instance, following the “*No CC or TC*” plan would would save 494,600 AF(=  $9,863.5kAF - 9,368.9kAF$ ) or 5.014% of the “*Baseline*” extraction and cost \$25.79(=  $685.130 - 659.338$ ) millions or 3.7% of “*Baseline*” profits. Based on similar calculations, the “*No TC*” plan saves 0.88 percent of extracted groundwater with 0.5 percent foregone rents; and the “*No CC*” plan saves 2.03 percent of groundwater at a cost of 0.82 percent of foregone rents.

### 2.4.3 Cost of no management vs. cost of incomplete information

An extraction plan drawn with specific paths for technical and climate changes that do not realize would be costly. In that sense, the downside risk of prescribing an extraction plan expecting climate and technical change is the potential relative cost of a plan that is suboptimal in the alternative scenario that is realized.

Table 2.3 summarizes the gains from management for each plan evaluated under different realized scenarios. The first column replicates the gains from management in table 2.2. The remaining columns show results when technical change or climate change do not actually occur. The gains from management for each plan in each of the realized scenarios (the columns) is calculated in three steps. First, the periodic groundwater extraction under each plan is valued to the present using the value function and the periodic realizations of the different state variables. Second, myopic outcomes are calculated for each of the realized scenarios, including groundwater extraction and the associated present value of the net benefits. Finally, the value of each plan is compared to the value of the myopic outcome for each of the realized scenarios. In each case, the plan that correctly accounts for the realized scenario results in the highest gains from management.

The differences between best and worst plans under each realized scenario are a measure of the cost of implementing an imperfect plan. The greatest such difference is found in the last column of the table (when neither technical change nor climate change occurs), between the “*Baseline*” and the “*No CC or TC*” plans. The gains from these plans differ

by nearly \$ 18 million, or 5.2% ( $= 6.1\% - 0.9\%$ ) of myopic NPV. Thus, the worst outcome for an imperfect plan - i.e., the largest “downside” risk - results when extraction follows the “*Baseline*” plan but neither climate nor technical change actually occur.

In contrast, the smallest difference occurs when there is no climate change but technical progress actually occurs (next-to-last column), and is the 1.5% ( $= 21.5\% - 20.0\%$ ) difference between the “*Baseline*” and “*No CC*” plans. In the state of the world with technical change and no climate change, all plans perform similarly. The performance gaps are intermediate between these two extremes in the remaining columns.

Even in this worst case with the largest costs from an erroneous plan, the the “*Baseline*” plan is still preferred to the unregulated myopic outcome. In fact, gains of 0.9% are much larger than the typical GSE outcome. Nevertheless, if there are transaction costs to policy intervention, the gains from management may vanish. If the prevalent belief is that groundwater will not be more valuable in the future, it would be tempting to not advocate for improved allocation over time because little to no real gains may be achieved in practice and focus instead on developing institutions to optimize allocation among different -valuable-uses (Gisser, 1983).

In contrast, the gains from management when climate or technical change occur are much larger than when demand is static, achieving 20 to 30 percent gains depending on the plan implemented and the scenario realized. The first three columns in table 2.3 show that not having a management plan can cost about 30 percent in foregone profits when groundwater becomes more valuable over time, for instance due to climate or technical change as is the case in this example. Similarly, the cost of intervening but doing so incorrectly, i.e. having the “wrong plan” for the realized scenario, is modest with at most 5 percent in potential gains foregone vis-a-vis the optimal plan for that scenario. In sum, no-management is more costly than management with incomplete information.

## 2.4.4 Effects of alternative scenarios

As described in the previous sections, the model is parameterized to reflect initial conditions descriptive of Sheridan County, Kansas. This section presents the results of alternative initial aquifer conditions and alternative climate change scenarios. The exercise serves two purposes: it serves as a robustness check and it allows an assessment of how different changes in climate change scenarios and aquifer conditions – perhaps similar to conditions in other regions – affect the optimal paths and potential gains from management. Each of the alternative initial aquifer conditions scenarios are evaluated under the same climate and technical change as the “*Baseline*” case. By contrast, the alternative climate change scenarios “*Slower CC*”, “*Slower TC*”, and “*Slower CC & TC*” scenarios assume the same aquifer conditions as the “*Baseline*” but changes in climate and technology occur as described below. The shape of the resulting groundwater extraction paths shown in figure 2.5 are consistent with the intuition gained from section 2.2 under a variety of alternative parameter values and climate and technical change scenarios.

Figures 2.5 and 2.6 show how the “*Baseline*” extraction plan compares to the optimal extraction paths when alternative initial conditions are considered. Starting with figure 2.5, the first alteration in initial conditions, “+20% *Lift*” reflects an increase of 20 percent in the initial pumping energy cost (equivalent to a 20 percent increase in initial pumping lift), all else equal. The second alternative initial condition, “+20% *Sat.thick.*” illustrates the case in which 20 percent higher saturated thickness is initially observed. The “+20% *Net rech.*” is the optimal plan when the initial rate of natural recharge is 20 percent higher than observed.

The alternative climate and technical change scenarios consist in halving the rates<sup>8</sup> considered in the “*Baseline*” simulation. “*Slower CC*”, represents the optimal plan when climate change occurs at half the pace originally considered. “*Slower TC*”, represents the optimal

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<sup>8</sup>The periodic climate and technical parameters are given by:  $\beta_0(t) = \frac{b_0}{b_1} + \left(\beta_0(0) - \frac{b_0}{b_1}\right) e^{-0.5b_1(t-1)}$ ,  $J(t) = \frac{a_0}{a_1} + \left(J(0) - \frac{a_0}{a_1}\right) e^{-0.5a_1(t-1)}$ ,  $M(t) = \frac{a_2}{a_3} + \left(M(0) - \frac{a_2}{a_3}\right) e^{-0.5a_3(t-1)}$ , and  $E(t) = \frac{a_4}{a_5} + \left(E(0) - \frac{a_4}{a_5}\right) e^{-0.5a_5(t-1)}$ .

plan when technical change occurs at half the pace originally considered. “*Slower CC&TC*”, represents the optimal plan when both climate and technical change occurs at half the pace originally considered.

The outcomes with respect welfare gains, approximated by the net present value of the stream of irrigator rents, are presented in table 2.4. Increased lift, slower change, and high discount rates decrease the potential gains from management. The intuition behind the effect of high discount rates is straight forward: it vanishes any future gains by discounting future rents towards zero giving larger weight to rents earned earlier in the planning horizon. The effect of increased initial lift is that pumping cost increases quicker than in other scenarios thus drastically reducing the net present value of future rents. Finally, slower change implies that the value marginal product of groundwater increases at a lower pace which reduces value of future rents in a manner similar to higher discount rates but differing from it in that it also makes the paths flatter (recall that non-static demand drives the hump shape).

By contrast, a greater saturated thickness, higher net recharge, higher demand elasticity, and lower discount rate have the effect of increasing the potential gains from management. Lower discount rates assign higher importance to rents achieved in the future so earlier groundwater savings are not as costly in terms of NPV and can be translated into higher returns in the future when groundwater is more valuable. Higher demand elasticity makes the planner more sensitive to the nonlinear increases in pumping costs as the aquifer depletes, thus inducing larger earlier savings and extraction peaks. Greater saturated thickness and higher net recharge essentially allow for a larger amount of the resource to be managed and clearly contrasts what is observed with the higher lift scenario.

All plans, except the case with higher net recharge rate, reach the same steady state asymptotically. The case where a higher rate of recharge is considered allows for higher levels of sustainable groundwater pumping. A feature in the narrative opposing (regulated) groundwater management is that scarcity can be dealt with when it actually becomes a problem at the field level. However, the results indicate that waiting might be costly in terms of potential gains from management because the potential gains are reduced when lift increases and saturated thickness decreases.

The faster aquifer decline under the myopic outcomes reduces the stocks of groundwater to be allocated over time. The substantially lower initial levels of extraction in the plan that considers a higher lift scenario, in addition to a substantial reduction in the potential gains from management, indicates that a higher lift scenario results in greater reductions in benefits in the initial periods and less discounted net benefits over the long run.

## 2.5 Conclusions

This paper presents a framework to study the combined and individual effects of technical change and climate change on groundwater extraction, the resulting aquifer decline, and the expected rents from irrigation. The context of the study is a declining aquifer where groundwater well yields decrease with the amount of groundwater stored and groundwater demand is nonstatic due to climate and technical change. The problem is formulated as a nonlinear optimal control problem where groundwater extraction is the control variable and the elevation of the water table represents the state of the aquifer. The climate and technical variables evolve exogenously while the aquifer variable is periodically affected by groundwater extraction but not directly by the other state variables.

Four forward-looking extraction plans and one myopic extraction regime are simulated. The forward looking plans are computed by maximizing the net present value of the sum of the periodic rents from irrigation over the life of the aquifer while the myopic regime is computed as periodic rent optimization based on first order conditions and periodic realizations of the state variable. The *Baseline* plan has perfect foresight of the future realization of all state variables accounted for in that optimization. The remaining plans ignore or omit future realizations of climate change, technical progress, or both type of variables in prescribing the respective extraction paths.

The parameters in the model reflect agronomical and hydrological conditions in Sheridan County, KS and linear dynamics for technical and climate change are calibrated. Climate change variables include periodic average precipitation between January and April ( $J$ ), periodic average precipitation between May and August ( $M$ ), and periodic average evapotran-

spiration between May and August ( $E$ , alfalfa-based). A widely used formulation of the aquifer dynamics is employed to update the elevation of the water table in the aquifer.

The numerical results indicate that the predicted gains from management are only 6.1 percent of the discounted stream of rents from myopic extraction if the plan assumes that neither climate nor technical change occurs and these assumptions turn out to be correct. This result accords with much of the previous literature and could be cited as a rationale for opposing any intervention to manage groundwater extraction (Gisser, 1983). However, once the plausible changes in marginal benefits over time (through technical change and climate change) are accounted for, the predicted gains from management are between 20 and 30 percent of myopic rents when climate or technical change actually occur. This result provides a strong rationale for groundwater management. Furthermore, the results indicate that the gains from management are still large if technical change or climate change are realized even if the plan ignores the dynamics of these factors. In fact, any forward-looking plan fares better than the myopic regime in terms of cumulative net present value of rents from irrigation.

All regimes considering technical or climate change allow for periods in which groundwater pumping is increasing. However, all forward-looking plans suggest that an immediate decrease from the myopic levels of extraction is necessary. The *Baseline* plan mandates the greatest initial decrease in pumping rate or greatest initial groundwater savings. The periods of increasing pumping rates are driven by the increasing productivity of groundwater (technical progress) and the increasing net irrigation requirements induced by climate change (changing precipitation patterns and increasing evapotranspirative needs).

Increased rates of extraction correspond to higher levels of periodic rents from irrigation so that faster decline and higher profitability could be expected in the next several periods of plan implementation. However, disregarding the net present value logic of the optimization, the periodic rents in the long run are greatest for the most conservative plans because these have the smallest pumping lifts in the steady-state which result in the lowest pumping costs in the long run.

When the optimal path of extraction is determined by the maximization of the net present

value of the rents from irrigation, relatively large groundwater savings may be achieved at relatively small foregone profits. However, when limited alternative (valuable) uses are available for the resource, maximization of the NPV of cumulative rents seems to be an adequate plan evaluation metric and the question that remains is how to discount the future which is beyond the scope of this paper.

The optimality of a future peak groundwater extraction results from the non-static groundwater demand in the formulation. The shifts in groundwater demand are consistent with the notion that groundwater will be more valuable in the future. Two plausible avenues for how these shifts may occur are via changes in climate and technical progress. By including exogenous state variables (climate and technology) in addition to the aquifer state variable, this paper allows for a wider range of possible optimal paths for the aquifer eliminating the limitation of single-state formulation which force monotonic state paths. Because such demand shifts are almost certain to occur and because they induce paths that differ from the results of conventional one-state and static groundwater demand models, the inclusion of multiple state variables and non-static groundwater demand should be the norm, not the exception, in studies of optimal dynamic extraction of groundwater.

The rapid aquifer decline under the myopic outcomes and the results under alternative planning scenarios suggest that delaying the implementation of groundwater management plans may diminish the potential management gains achievable. Whether this should be an argument for conservation or not is not discussed in this paper, but savings may be achieved at the cost of relatively low foregone rents.

The contrast between the myopic and optimal extraction paths indicate that it is optimal to prescribe significant groundwater pumping restrictions – on the order of 20 percent – at the beginning of the planning horizon and to allow increased rates of groundwater extraction in later periods when groundwater is more valuable. Consequently, it should not be surprising if groundwater managers (of well managed resources) allow future increases in maximum groundwater extraction in areas where restrictive groundwater management policies exist, such as in Sheridan County, KS.

The formulation in this article employs private benefits as a proxy for social welfare.



This is an adequate formulation for cases in which there is little regional competition for the resource among other uses such as industrial or municipal. Furthermore, the formulation implicitly assumes that little interactions exist between the stocks of groundwater in the aquifer and surface waters and ecosystems. The formulation adequately describes the study area in Western Kansas to which it is applied. However, when circumstances merit significant impact on aquifer viability, ecosystems health, and availability to other competing uses, these aspects must be incorporated in the modeling. Given the strong rationale for conservation presented in this paper, it seems like the consideration of such environmental aspects would further strengthen, rather than negate, the main results from this paper.

Another caveat in our model is the assumption of a fixed rate of net natural recharge. Climate change can be expected to impact aquifer recharge. The growing literature on the subject would greatly benefit from increased attention from economists. The assumption of a fixed recharge rate in this paper is more palatable because it models the change in annual precipitation patterns in an area where annual mean precipitation is projected to have little to no change.

Finally, uncertainty or disbelief about climate change is an often raised objection to managing natural resources. The numerical analysis shows that the downside risk of accounting for changes in climate and technical progress that do not materialize is small. Although gains from management would be reduced, returns are still better than the myopic outcomes.

## 2.6 Figures

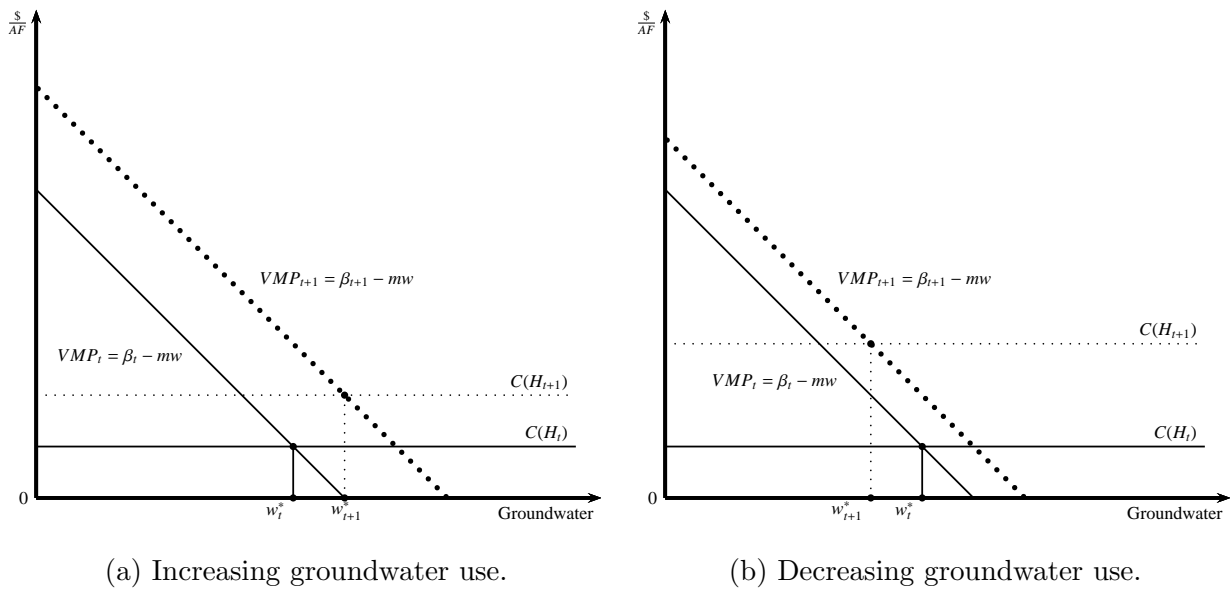


Figure 2.1: Marginal cost and value of groundwater changing over time.

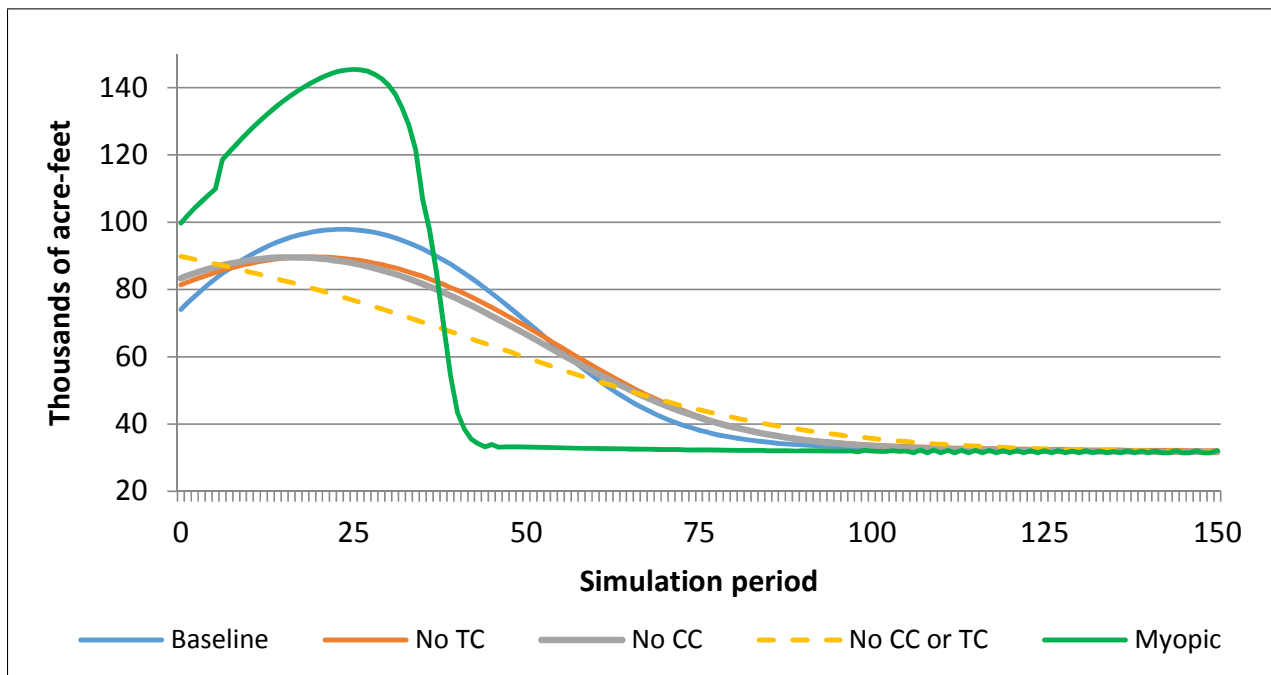


Figure 2.2: Periodic groundwater allocation under different planning scenarios versus the myopic scenario.

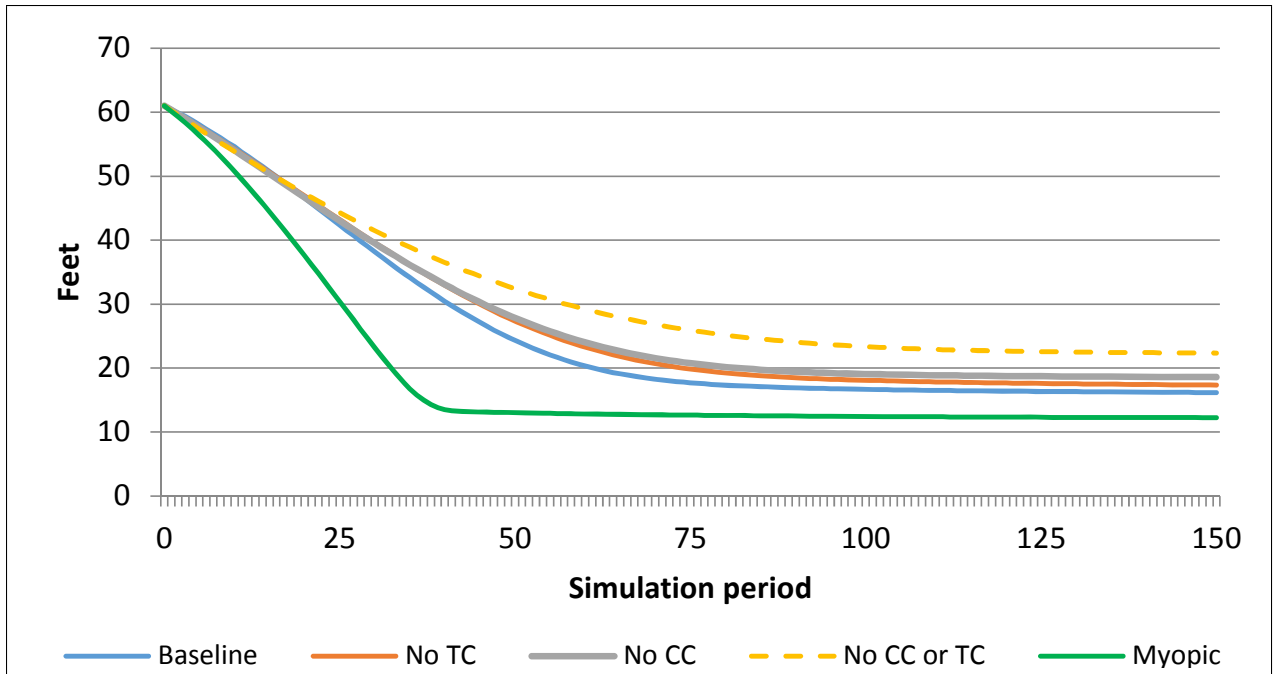


Figure 2.3: Aquifer saturated thickness under different planning scenarios versus the myopic scenario.

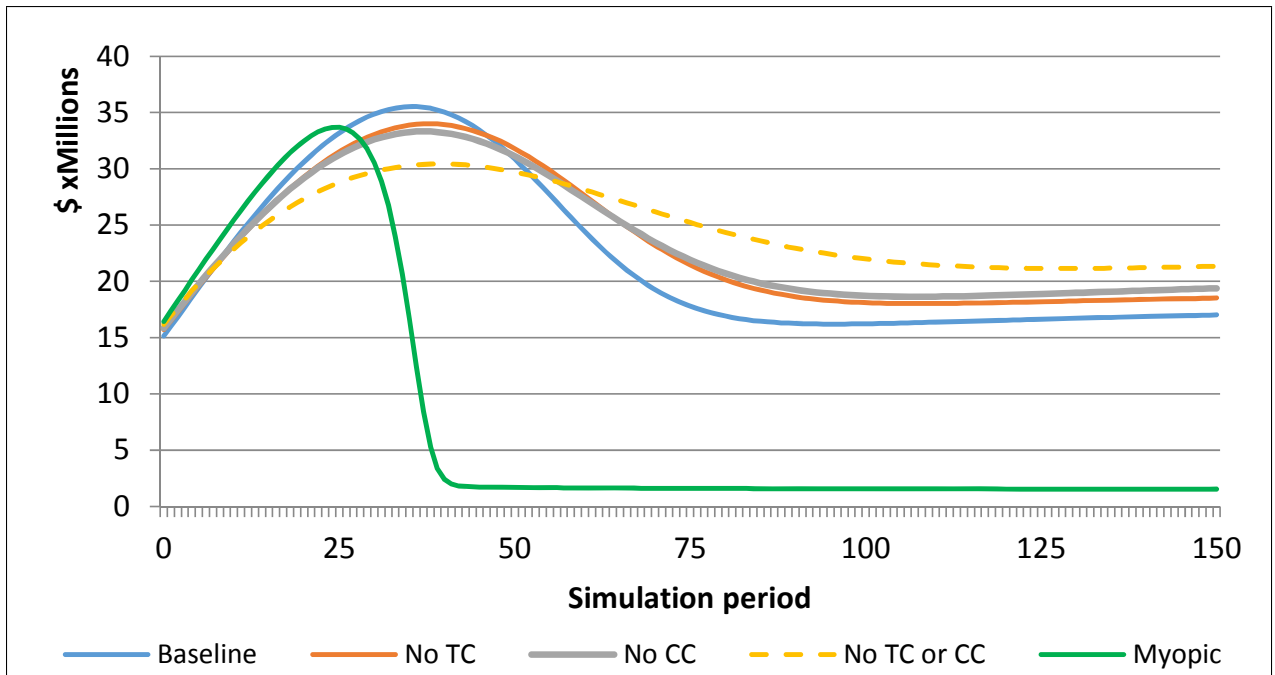


Figure 2.4: Periodic rents from irrigation under different planning scenarios versus the myopic scenario evaluated when both climate and technical change realize.

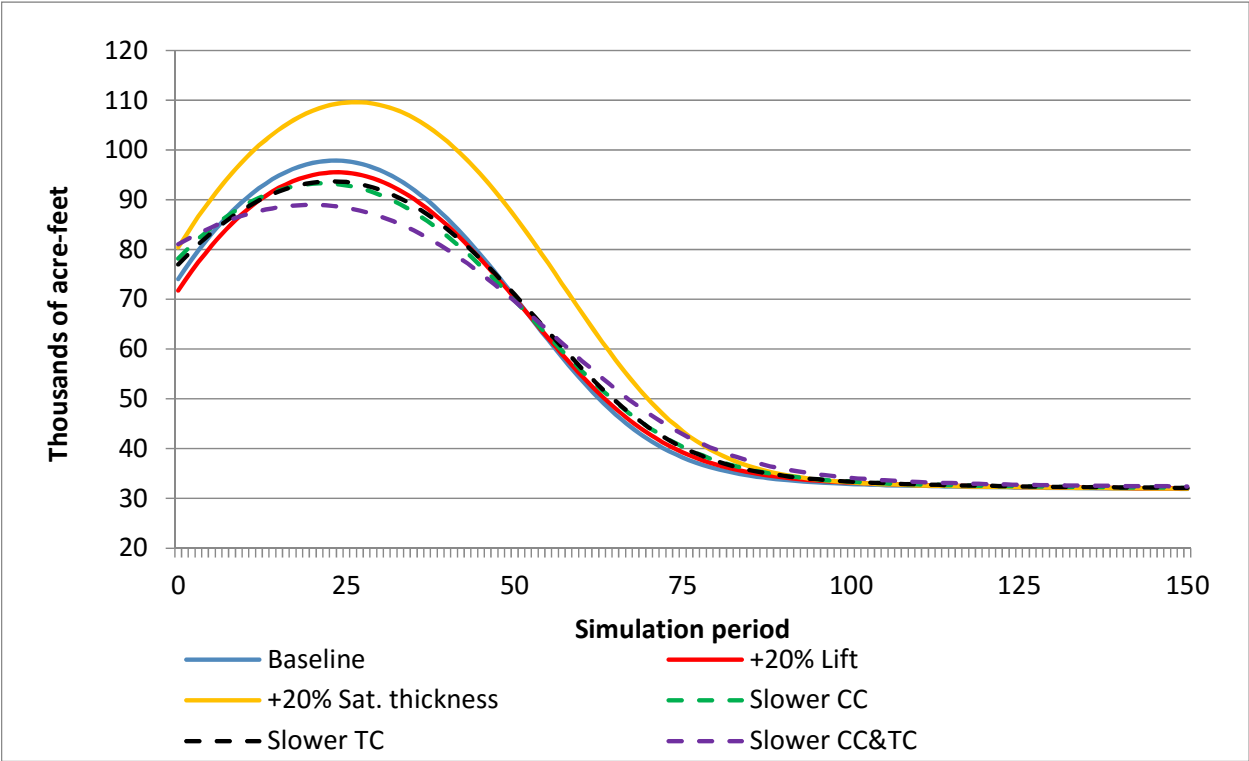


Figure 2.5: Optimal extraction paths under alternative initial conditions and parameters.

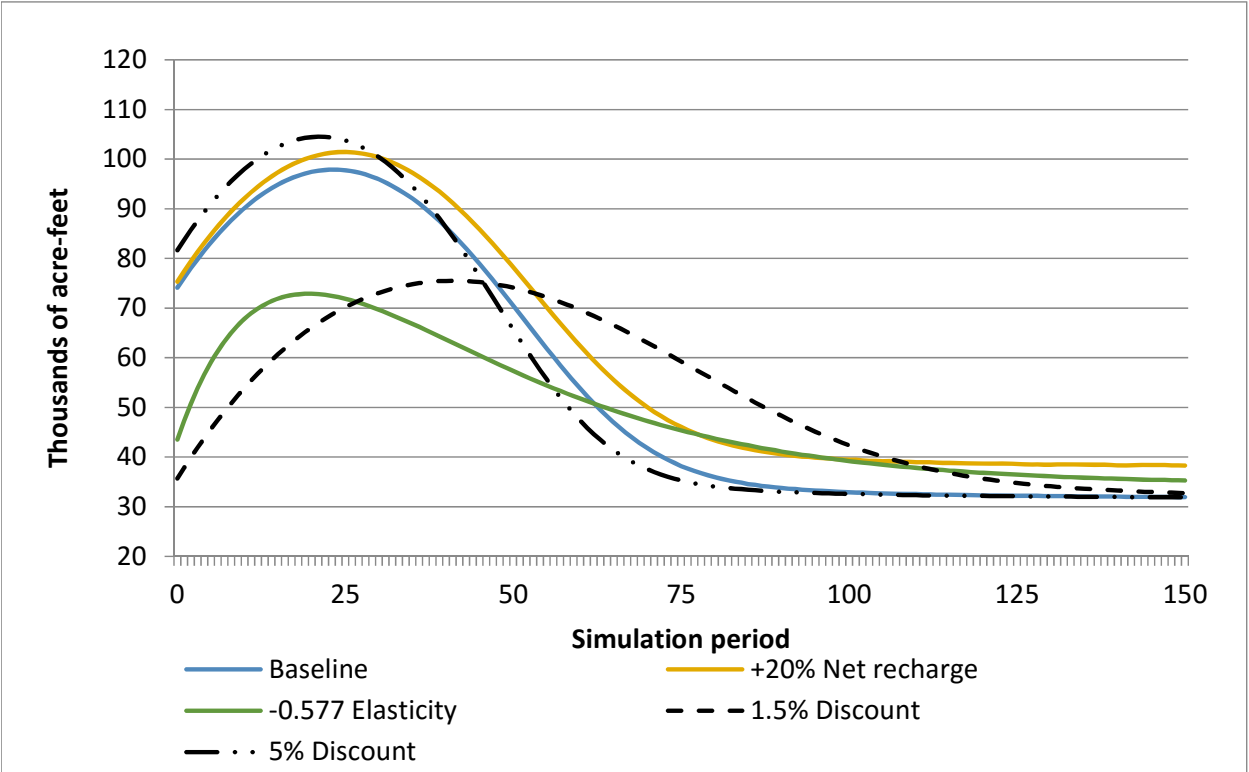


Figure 2.6: Optimal extraction paths under alternative initial conditions and parameters.

## 2.7 Tables

Parameter	Value
<b>Aquifer</b>	
Area over aquifer $\times$ specific yield ( $A_S$ )	716,844.54
Irrigated area	77,745 acres
Return flow ( $\alpha$ )	0.086795
Initial lift (depth to water)	111.5 ft.
Initial saturated thickness	61.03 ft.
Drawdown	20 ft.
Rate of natural recharge ( $r$ )	28,747.08 AF/yr
Discount rate ( $\rho$ )	0.0389
<b>Demand function</b>	
$\tilde{\beta}_0$	232.67
Coefficient on $J$ : $\beta_1$ .	44.548
Coefficient on $M$ : $\beta_2$ .	18.383
Coefficient on $E$ : $\beta_3$ .	15.055
Coefficient on $w$ : $\beta_4$ .	0.0031
<b>Cost function</b>	
$C_0 = 0.975$	$S_L = 2,755$
$Q_0 = 3.48E - 07$	$H_0 = 2,644.2$ ft.
	$H_c = 2,583.2$
<b>Technical change</b>	
$\dot{\beta}_0 = 10.134 - 0.024\beta_0$	$\beta_0(0) = 232.67$
<b>Climate change</b>	
$\dot{J} = 0.071833 - 0.01333J$	$J_0 = 4.31''$
$\dot{M} = 0.1484 - 0.01333M$	$M_0 = 12.37''$
$\dot{E} = 0.8199 - 0.01333E$	$E_0 = 35.14''$

Table 2.1: Parameters and aquifer initial values for Sheridan Co.,KS

Plan	NPV (\$ $\times$ millions)	Gains from Management	Total GW ( $AF \times 1,000$ )	GW Savings
Baseline	685.13	29.5%	9,863.5	2.9%
No TC	681.71	28.8%	9,776.6	3.8%
No CC	679.55	28.4%	9,663.0	4.9%
No CC or TC	659.34	24.6%	9,368.9	7.8%
Myopic	529.13		10,161.9	

Table 2.2: Net present value of rents from irrigation and accumulated groundwater extraction.

Plan	Realized Scenario			
	Climate Change Occurs		No Climate Change Occurs	
	TC Occurs	No TC Occurs	TC Occurs	No TC Occurs
Baseline	29.5%	30.1%	20.0%	0.9%
No TC	28.8%	30.9%	21.4%	4.0%
No CC	28.4%	30.8%	21.5%	4.6%
No CC or TC	24.6%	28.6%	20.0%	6.1%

Table 2.3: Evaluation of plans under different realized scenarios.

Plan	NPV (\$ $\times$ millions)	Gains from Management	Total GW ( $AF \times 1,000$ )	GW Savings	Effect on Welfare
Baseline	685.13	29.5%	9,863	2.9%	
+20% Lift	618.28	16.2%	9,243	10.0%	(-)
+20% Sat. thick.	710.72	15.1%	10,219	8.7%	(+)
+20% Net rech.	716.93	29.3%	11,088	3.0%	(++)
Slower TC	626.76	18.5%	9,857	3.0%	(--)
Slower CC	603.55	14.1%	9,832	3.1%	(--)
Slower CC&TC	546.77	3.3%	9,823	3.1%	(---)
-0.577 Elasticity	303.21	106.8%	19,536	$\approx 0$	(+++)
1.5% Discount	1,423.20	77.9%	6,241	10.8%	(+++)
5% Discount	532.80	18.8%	9,906	2.5%	(---)
(+),(-): Difference with baseline is less than 10%.					
(++),(--): Difference with baseline is between 10% and 50%.					
(+++),(---): Difference with baseline is more than 50%.					

Table 2.4: Net present value of benefits and groundwater extraction for first 200 years under alternative scenarios.

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# Appendix A

## Solution to the Linear-Quadratic Control Problem in Section 2.2.

The LQ problem has the form:

$$\begin{aligned} R(w; \beta, H) &= B(w; \beta) - C(H)w \\ &= \left( \beta - \frac{1}{2}\gamma w \right) w - \frac{1}{2}c_2 H^2 - (c_0 + c_1(S - H))w \\ &= (\beta - c_0 - c_1 S)w - \frac{1}{2}\gamma w^2 - \frac{1}{2}c_2 H^2 - c_1 Hw \end{aligned} \quad (\text{A.1})$$

where

$$\beta = \frac{b_0}{b_1} + \left( \tilde{\beta} - \frac{b_0}{b_1} \right) \exp[-b_1 t] \quad (\text{A.2})$$

$$\dot{H} = g(w, H) = n - aw \quad (\text{A.3})$$

where  $\tilde{\beta}$  is the initial value of  $\beta$ ,  $n = \frac{r}{As}$ ,  $a = \frac{1-\alpha}{As}$ ,  $r$  is rate of net recharge,  $\alpha$  is return flows, and  $As$  is area overlying the aquifer times specific yield. A well behaved reward function requires  $c_2 > \frac{c_1^2}{\gamma}$ .

Solving the control problem yields a  $3 \times 3$  linear dynamical system:

$$\dot{x} = \begin{bmatrix} \dot{\beta} \\ \dot{H} \\ \dot{w} \end{bmatrix} = \begin{bmatrix} -b_1 & 0 & 0 \\ 0 & 0 & -a \\ -(b_1 + \rho) & (\rho c_1 - a c_2) & \rho \gamma \end{bmatrix} \begin{bmatrix} \beta \\ H \\ w \end{bmatrix} + \begin{bmatrix} b_0 \\ n \\ \theta_1 \end{bmatrix} = Ax + b \quad (\text{A.4})$$

where  $\theta_1 = b_0 - c_1 n + \rho c_0 + \rho c_1 S$ .

The steady state is given by  $\dot{x} = 0$ .

$$x_{\infty}^* = -A^{-1}b = \begin{bmatrix} -b_1 & 0 & 0 \\ 0 & 0 & -a \\ -(b_1 + \rho) & (\rho c_1 - a c_2) & \rho \gamma \end{bmatrix}^{-1} \begin{bmatrix} b_0 \\ n \\ \theta_1 \end{bmatrix} \quad (\text{A.5})$$

$$= \begin{bmatrix} \frac{b_0}{b_1} \\ \frac{\theta_1}{a c_2 - \rho c_1} - \frac{b_0}{a b_1 c_2 - \rho b_1 c_1} (\rho + b_1) + n \gamma \frac{\rho}{a^2 c_2 - a \rho c_1} \\ \frac{n}{a} \end{bmatrix} \quad (\text{A.6})$$

And the trajectories over time depend on the (negative) eigenvalues and eigenvectors of matrix **A**. Eigenvalues:  $\lambda_1 = \frac{1}{2}\gamma\rho - \frac{1}{2}\sqrt{4c_2a^2 - 4c_1a\rho + \gamma^2\rho^2} < 0 \iff \rho c_1 < a c_2 c_2 < \frac{a}{\rho}$ ,

$$\lambda_2 = \frac{1}{2}\gamma\rho + \frac{1}{2}\sqrt{4c_2a^2 - 4c_1a\rho + \gamma^2\rho^2} > 0,$$

$$\lambda_3 = -b_1 < 0$$

with associated eigenvectors:

$$\left\{ \begin{bmatrix} v_{11} = 0 \\ v_{12} = \frac{\gamma\rho + \sqrt{4c_2a^2 - 4c_1a\rho + \gamma^2\rho^2}}{2(ac_2 - \rho c_1)} \\ v_{13} = 1 \end{bmatrix} \right\} \leftrightarrow \lambda_1 = \frac{1}{2}\gamma\rho - \frac{1}{2}\sqrt{4c_2a^2 - 4c_1a\rho + \gamma^2\rho^2} < 0,$$

$$\left\{ \begin{bmatrix} 0 \\ \frac{\gamma\rho - \sqrt{4c_2a^2 - 4c_1a\rho + \gamma^2\rho^2}}{2ac_2 - 2\rho c_1} \\ 1 \end{bmatrix} \right\} \leftrightarrow \lambda_2 = \frac{1}{2}\gamma\rho + \frac{1}{2}\sqrt{4c_2a^2 - 4c_1a\rho + \gamma^2\rho^2} > 0,$$

$$\left\{ \begin{array}{l} v_{31} = \frac{a(\rho c_1 - ac_2) + b_1(b_1 + \gamma\rho)}{b_1(b_1 + \rho)} \\ v_{32} = \frac{a}{b_1} \\ v_{33} = 1 \end{array} \right\} \leftrightarrow \lambda_3 = -b_1 < 0 \text{ so that the optimal paths are described by}$$

the eigenvectors associated with the negative eigenvalues:

$$\beta^*(t) = \beta_\infty^* + k_3 v_{31} e^{\lambda_3 t} + k_1 v_{11} e^{\lambda_1 t} \quad (\text{A.7})$$

$$H^*(t) = H_\infty^* + k_3 v_{32} e^{\lambda_3 t} + k_1 v_{12} e^{\lambda_1 t} \quad (\text{A.8})$$

$$u^*(t) = u_\infty^* + k_3 v_{33} e^{\lambda_3 t} + k_1 v_{13} e^{\lambda_1 t} \quad (\text{A.9})$$

where

$$k_1 = \frac{H_0 - H_\infty^*}{v_{12}} \quad (\text{A.10})$$

$$k_3 = \frac{\beta_0 - \beta_\infty^*}{v_{31}} \quad (\text{A.11})$$

The optimal paths are

$$\beta_t^* = \beta_\infty^* + (\beta_0 - \beta_\infty^*) e^{-b_1 t} \quad (\text{A.12})$$

$$\begin{aligned} H_t^* &= H_\infty^* + (\beta_0 - \beta_\infty^*) \left( \frac{a(b_1 + \rho)}{a(\rho c_1 - c_2 a) + b_1(b_1 + \gamma\rho)} \right) e^{-b_1 t} \\ &\quad + (H_0 - H_\infty^*) e^{\lambda_1 t} \end{aligned} \quad (\text{A.13})$$

$$\begin{aligned} u_t^* &= u_\infty^* + (\beta_0 - \beta_\infty^*) \left( \frac{b_1(b_1 + \rho)}{a(\rho c_1 - c_2 a) + b_1(b_1 + \gamma\rho)} \right) e^{-b_1 t} \\ &\quad + \frac{2(H_0 - H_\infty^*)(ac_2 - \rho c_1)}{\gamma\rho + \sqrt{4c_2 a^2 - 4c_1 a\rho + \gamma^2 \rho^2}} e^{\lambda_1 t} \end{aligned} \quad (\text{A.14})$$

As the LQ problem results in optimal control that is linear in the state variables:

$$u^* = V + W_1 \beta + W_2 H \quad (\text{A.15})$$

	$\rho$	$b_1$	$c_1$	$c_2$	$\gamma$	$a$
<b>Min</b>	0.000	0.016	0.1824	32.162	0.00015	0.0000000144
<b>Base</b>	0.0389	0.024	0.6689	64.324	0.0031	0.0000002198
<b>Max</b>	0.1000	0.044	1.3378	128.65	0.0301	0.0008506944

Table A.1: Range of plausible parameter values for signing  $W_1$ .

where

$$V = u_\infty^* - \left( \frac{v_{33}v_{12} - v_{32}v_{13}}{v_{31}v_{12}} \right) \beta_\infty^* - \frac{v_{13}}{v_{12}} H_\infty^* \quad (\text{A.16})$$

$$W_1 = \frac{v_{33}v_{12} - v_{13}v_{32}}{v_{31}v_{12}} = \frac{b_1(b_1 + \rho)}{a(\rho c_1 - ac_2) + b_1(b_1 + \gamma\rho)} \left( 1 - \frac{a}{b_1} \left( \frac{2a(ac_2 - \rho c_1)}{\gamma\rho + \sqrt{4a(ac_2 - \rho c_1) + \gamma^2\rho^2}} \right) \right) > 0 \quad (\text{A.17})$$

$$W_2 = \frac{v_{13}}{v_{12}} = \frac{2(ac_2 - \rho c_1)}{\gamma\rho + \sqrt{4c_2a^2 - 4c_1a\rho + \gamma^2\rho^2}} > 0 \quad (\text{A.18})$$

The sign of  $W_2$  follows from the condition on  $\lambda_1 < 0$ . The sign of  $W_1$  can not unequivocally be determined from necessary or sufficient conditions. Consequently a lower and upper limit for admissible parameter values (see table A.1) are established and used along with the baseline values to calculate the value of  $W_1$ .

The values in table A.1 were produced as follows:

- $\rho$  is the social discount rate conventionally set to account for the value of immediacy. In investment decisions it is set as to reflect the cost of capital. In our paper we use the average interest rate for farm loans as described (3.87 percent). The lower limit is set at 0 percent. The upper limit is set at 10 percent (the highest average farm mortgage rate is as high as 5.7 percent in the Dallas Fed area).
- $b_1 < 1$  is the coefficient that determines the rate of change of  $\beta$  and its steady state. This is a calibrated parameter that depends on initial conditions, assumption of initial level of productivity increase and steady state productivity level ( $\beta_\infty = b_0/b_1$ ), which this paper assumes to be twice that of the initial productivity ( $b_1 = 0.024$ ). Supposing the steady state could be as low as 10 percent higher than starting levels, we have that  $b_1 = 0.0436$ . If productivity is three times larger,  $b_1 = 0.016$ .

- $c_1$  is the cost of pumping one AF of groundwater at the initial lift. This parameter is calculated based on engineering formulas yielding an average of 0.6689 for a marginal cost of  $\$22/AF$  at the initial state. A lower-bound is set at  $\$6/AF$  yielding a parameter value of 0.1824. The upper bound is set at twice the base value at  $\$44/AF$  yielding a coefficient value of 1.3378.
- $c_2$  is a coefficient that accounts for the nonlinear impact of a decreasing aquifer. The value is calibrated based on nonlinear marginal pumping costs calculations based on a model of declining well yields at 64.324. The lower and upper limits are set at half and double those levels (32.162 and 128.648).
- $\gamma$  is the absolute value of the slope of the groundwater inverse demand curve. The base coefficient is 0.0031 ( $-0.11$  elasticity). An arbitrary upper limit is set at 0.01 which is equivalent to an elasticity of  $-0.03$ . Pfeiffer and Lin (2014) posit that elasticities may be underestimated in groundwater demand studies. Scheierling et al. (2006) presents a meta-analysis of irrigation water demand studies to that point with elasticity values as high as 1.86 for Howe et al. (1971). The lower limit is then set for an elasticity of  $-2$  which equates to a coefficient value of 0.00015.
- $a = (1 - \alpha)/A_S$  is an aquifer depletion coefficient that determines how much the water table elevation changes for every AF of groundwater extracted.  $\alpha$  is the portion of applied water that returns to the aquifer. Specific yield ranges from 5 to 25 percent. Consumptive use could be interpreted as the application efficiency which varies according to the irrigation system. Howell (2003) shows the range of application efficiency observable, which can be as low as 40 percent for flood irrigation (up to 70 percent) and as high as 98 percent for LEPA center pivot (92 percent average, 80 percent minimum) with low efficiency center pivots. The lower limit is established considering the area of the whole High Plains aquifer (174,000sq.miles), specific yield of 25 percent and application efficiency of 40 percent so that  $a = 0.0000000144$ . The upper limit is established for 98 percent efficiency on a 36sq.mile area (size of a township) with 5 percent specific yield so that  $a = 0.008506944$ . The used value is  $a = 0.0000002198$  for the average application efficiency and specific yield as well as the area for Sheridan county,



KS.

There are six parameters with three levels each, i.e. (min, base, max), so that there are  $3^6 = 729$  calculations for the coefficient. Every calculation yields  $W_1 > 0$ , even those violating  $c_2\gamma > c_1^2$  and  $ac_2 > \rho c_1$ . In only 23 admissible cases we see that  $a(ac_2 - \rho c_1) < b_1(b_1 + \gamma\rho)$  and  $\frac{a}{b_1} \left( \frac{2a(ac_2 - \rho c_1)}{\gamma\rho + \sqrt{4a(ac_2 - \rho c_1) + \gamma^2\rho^2}} \right) > 1$ ; all of which require the highest discount rates and the lowest terminal productivity (10 percent higher than in the present), failing any of these two conditions, the inequalities reverse.