Elementary preservice teachers' and elementary inservice teachers' knowledge of mathematical modeling

by

Sara Schwerdtfeger

B.S.E., Emporia State University, 1998
M.S., Emporia State University, 2006

AN ABSTRACT OF A DISSERTATION

submitted in partial fulfillment of the requirements for the degree

DOCTOR OF PHILOSOPHY

Department of Curriculum and Instruction
College of Education

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Manhattan, Kansas

2017

Abstract

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This survey research, tailored design method employed a brief online survey to a convenience sample of preservice and inservice elementary teachers to gain information about their knowledge of mathematical modeling in the elementary school classroom. For the purposes of this research, the definition of mathematical modeling was applying mathematics to real world problems with the purpose of understanding the problem. This study used non-experimental, survey research to determine if there was a statistical significant difference between preservice teachers' and inservice teachers' knowledge of mathematical modeling. Independent t-tests were used to determine there was no statistical significant difference in elementary preservice teachers and elementary inservice teachers knowledge of mathematical modeling. Another aspect of this

research was to determine if any variables were able to predict the preservice or inservice teachers' knowledge of mathematical modeling. Multiple regression was used to determine the variables of years of teaching experience, grade level currently taught, or type of school in which teaching occurs did not have any predictor aspects of knowledge of mathematical modeling. ANOVA was used to determine there was no relationship between preservice and inservice teachers' perceived knowledge of mathematical modeling and their actual knowledge of mathematical modeling.

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Dedication

This dissertation is dedicated to all the people who think they hate math, but then realize they just hate the way they were taught math as a student. May we, as teachers and educators, learn to alleviate this math anxiety and instill the love of learning mathematics in all students.

Chapter 1 - Introduction

Overview

Of all the levels of mathematics education, from elementary to graduate, the elementary level is the most difficult to teach. –Morris Kline, 1977

Teachers, both preservice and beginning inservice teachers, have a simplistic belief about what it takes to be a successful teacher (Stuart & Thurlow, 2000). Teachers often teach the way they were taught in elementary, high school and higher education (Tatto & Senk, 2011). Many teachers model their current mathematics instruction from previous mathematics teachers, even if the teachers themselves did not successfully learn in their own mathematics classes. The following scenario is a typical student experience in schools in the 1960s and 1970s. The student, Peter, shares experiences in elementary, high school, and university mathematics education experiences as described by Kline (1977). In a summary of the elementary mathematics experience, Peter determined the elementary school courses had been acceptable. However Peter decided some of the operations, such as addition, subtraction, multiplication and division, were baffling. It was not clear why the division of two fractions had to be performed by inverting the denominator and multiplying—but the teacher seemed to know the process was correct.

Later in high school, Kline (1977) describes a situation where a geometry teacher completed a proof using triangles. After a tedious effort of demonstration on the chalkboard, the proof was finally complete.

> "Peter dared to speak up: "but isn't it obvious? A triangle is a rigid figure. If you put three sticks together to form a triangle, you cannot change its size or shape." Peter had learned this at the age of five playing with Erector sets. The teacher's contempt was obvious. "Who's talking about sticks? We are concerned with triangles."

Frequently, preservice teachers report low levels of mathematical content knowledge due to lack of understanding of underlying concepts in mathematics. There may be a clear connection from the preservice teachers' understanding of mathematics and the mathematics that were taught in schools. Until the mathematics instruction in elementary classrooms is modified to address this issue, the next generation of elementary teachers may report the same findings.

Unlike the teacher in Peter's classroom, effective teachers design or select genuinely problematic tasks that integrate powerful concepts and processes that foster children's mathematics thinking and learning (English, Fox, & Watters, 2005). This process is the teacher's role in beginning to develop mathematical modeling tasks. To identify and use these problematic tasks, it takes time to develop truly thoughtful mathematical modeling questions. Friedrichsen, et al (2009) identified three potential sources of subject matter content for all teachers: teachers' own K-12 school experiences, teacher education and professional development programs, and teaching experience. In the scope of this study, the last two items in this list are explored including teacher education and teaching experience. Therefore, before the elementary teachers and elementary preservice teachers can develop understanding about mathematical modeling with enough depth to then facilitate the learning of students, it is helpful to assess the current knowledge of both groups of teachers.

Teaching elementary students mathematics is an important task. Teachers must have a firm foundation in elementary mathematics content as well as a foundation in the pedagogy to teach the math skills to elementary students from Kindergarten to 6^{th} grade. A true understanding of the definition of mathematical modeling is essential to facilitate mathematical modeling with students in the classroom. For the purposes of this research, the definition of mathematical modeling is applying mathematics to real world problems with the purpose of understanding the problem. Teachers who only know the content of the mathematics courses are unable to foster the understanding needed of students to use mathematical modeling to solve real world problems and create models to use in future situations.

Statement of the Problem

Mathematical modeling has recently come to the forefront of elementary mathematics classrooms because of the call to add mathematical modeling tasks in mathematics classes through the Common Core State Standards (NGACBP & CCSS, 2010). According to Ellis and Berry (2005), the recommendation for teachers to think differently about teaching mathematics includes more comprehensive knowledge of mathematics continuing beyond rote facts, skills, and procedures. This new focus on mathematical modeling requires a shift in thinking and practice for teachers in elementary, secondary, and teacher educators (Wolf, 2016). Although preservice teachers and inservice teachers vary in teaching experience, their knowledge in mathematical modeling may be similar as, quite possibly, neither had explicit instruction during their elementary education programs.

There is a proliferation of research related to mathematical modeling in high school and secondary schools. The CCSS (NGACBP & CCSS, 2010) specifically include commentary on how to teach mathematical modeling in high school and emphasize this concept throughout the high school standards. This is additionally developed through one of the Standards of Mathematical Practice (SMP), model with mathematics. These SMPs are to be used for all grade levels, Kindergarten through 12th grade. However, the literature and research including elementary teachers and elementary schools is lacking. Most recently, mathematical modeling in elementary schools with elementary teachers has begun to be reflected in the literature. Both mathematical modeling in elementary preparation programs as well as elementary teachers' understanding of mathematical modeling represent a gap in the research literature.

Research Purpose

The purpose of this survey research, tailored design method is to examine the relationship between elementary inservice and elementary preservice teachers' knowledge of mathematical modeling. This study used non-experimental, survey research to determine if there was a statistical significant difference between preservice teachers' and inservice teachers' knowledge of mathematical modeling. Secondary education and secondary mathematics teachers have been researched extensively in the area of mathematical modeling. This research conducted with elementary preservice and elementary inservice teachers leads researchers into a newer research area of mathematical modeling.

This study extends previous research in the following ways:

1. The researcher is using a mathematical modeling survey developed by Gould (2013). She developed this survey for use with secondary mathematics teachers. The survey is now being used for research with elementary preservice and inservice teachers as an extension of the original intent of the survey.
2. The relationship between elementary teachers and mathematical modeling has not been thoroughly researched. The elementary mathematics content specifically unique to grades Kindergarten through 6^{th} grade must be blended with the pedagogy of SMP's. This research aims to begin to add to the research and literature in this area.
3. Of the research that has been completed, the majority of mathematical modeling research is exclusively focused on either preservice (novice) or inservice (experienced) teachers but not both groups of teachers together. This research aims to

extend the thinking of this concept by comparing the knowledge of mathematical modeling in both groups of preservice and inservice teachers to determine if there is a difference in knowledge.

Research Questions

The research questions in this study aim to analyze the possible differences in preservice and inservice teachers mathematical modeling knowledge. The predictor variables of years of teaching experience, type of school in which teaching occurs, and grade level taught are examined in Including both inservice and preservice teachers in this study allows the researcher to ask the following questions about the knowledge of mathematical modeling:

1. Is there a statistical significant difference between inservice and preservice elementary teachers with regard to knowledge of mathematical modeling?
2. Is there is relationship among teachers' knowledge of elementary mathematical modeling and the number of years teaching experience, grade level taught, or type of school in which teaching occurs?
3. Is there a relationship between preservice and inservice teachers' perceived knowledge of mathematical modeling and their actual knowledge of mathematical modeling?

Design of the Study

The design of this study is survey research, tailored design method. The sample of participants for this study is a conveniences sample of elementary preservice teachers at one university and elementary inservice teachers in one school district. An online survey with the topic of mathematical modeling was emailed to 290 potential participants of this research. There were 146 participants that completed the online survey. This online survey included three sections including a demographics section, a section for mathematical models in elementary schools, and a section for mathematical modeling in elementary schools. Through this survey, data was collected about elementary preservice and inservice teachers' knowledge of mathematical models and mathematical modeling through a set of six and eight questions respectively. An average mathematical modeling score was also calculated from responses in sections two and three of the survey.

Brownell's Meaning Theory gives framework to this research study by describing how researchers ensure students have an opportunity to make sense of the mathematics presented. Schulman's Pedagogical Content Knowledge (PCK) theory aims to bridge the gap in teachers' learning of pure content and the pedagogical skills needed to teach students. Meaning Theory promotes understanding of mathematical procedures. Instruction in mathematics should start slowly by using a variety of concrete materials and move increasingly toward symbols or other abstractions. Teachers should also structure opportunities for students to apply mathematics concepts in real-world contexts (Brownell, 1935).

Limitations and Delimitations of the Study

There are several limitations and delimitations to this study. This research was conducted as part of the requirements of a doctoral dissertation and the researcher is a full-time instructor at the university where research was conducted with the elementary preservice teachers. The sample population was a convenience sample of preservice teachers at one university and inservice teachers in one district. Although the population of the university is quite diverse, little diversity exists in the population of the students in the teacher education program. The preservice teachers and inservice teachers are predominantly Caucasian females in this community. The elementary teachers selected to participate in this study was also a convenience sample and all participants were from the same district in one community. They may be limited in knowledge or training of mathematical modeling or may have extensive resources in mathematical modeling compared with other districts. Therefore, the generalizability of this study may be limited. Because of the limited population of possible participants that was dependent on the sample size of both the preservice and inservice respondents, there may be additional considerations for generalizability.

This online survey was conducted with elementary teachers and preservice elementary teachers. The online survey that was developed for this research study was modified from an online research study completed for another doctoral candidate in mathematics education. There was limited face-to-face contact with the participants in the study as the data collection was an online survey. The research of this study was framed around preservice and inservice teachers' knowledge of mathematical modeling. The researcher explained the research to the participants in one session. The survey was self-administered through an online format. Quantitative data was collected through this survey though qualitative data would allow for additional analysis.

Any professional development, advanced training, professional conferences that inservice teachers may have attended were not considered in this research. Professional development is somewhat difficult to define as typically it is assumed that professional development must occur throughout the year during teaching. This survey was designed to be a quick, online survey and the researcher did not want to require teachers to calculate professional development hours/days/sessions. However, looking at this aspect of inservice teachers would be interesting to add as factor for subsequent research.

For the scope of this study, a choice was made to exclude the analysis of race and ethnicity of the inservice and preservice teachers. There is little diversity and variance of the elementary inservice teachers in this district, as well as the population of elementary preservice teachers at the university.

Definitions of Terms

1. Preservice teacher: a teacher candidate enrolled in a teacher education preparation program at an accredited university
2. Inservice teacher: a licensed teacher currently teaching in the classroom or at an elementary school as an instructional coach, instructional strategist, or in some other specialized area of elementary education
3. Mathematical modeling: the definition used for this research is applying mathematics to a real world problem with the purpose of understanding the problem.
4. Modeling with mathematics: for purposes of this research, mathematical modeling and modeling with mathematics will be used interchangeably, with the same definition for each phrase.
5. Mathematical Model: a description of a system using mathematical concepts and language. This is not the same as the process of mathematical modeling.
6. Elementary classification of teachers and students: for this study, the elementary classification includes any teacher licensed to teach Kindergarten through 6^{th} grade, regardless of the type of school where the teaching occurs
7. Experience: refers to the years of elementary teaching experience of a teacher
8. Type of school: refers to the classification of the type of school in which the teaching occurs including, but not limited to, elementary school, middle school, junior high school, religious school, charter school, or any other type

9. Block 2: Term used to refer to the second to last semester of the elementary teacher education program of the preservice teachers. At the time of this survey, students enrolled in this semester of coursework had begun their elementary math methods course yet.
10. Block 3: Term used to refer to the last semester of the elementary teacher education program of the preservice teachers. At the time of this survey, students enrolled in this semester of coursework had completed their elementary math methods course but had only just begun their semester of student teaching.

Organization of the Study

This chapter has given a brief explanation of the overview, research problem, research purpose, research questions, limitations, and delimitations of the research, and definition of terms for this research conducted. The reasons for pursuing this research topic are summarized in this chapter. This research study addressed the differences in knowledge of mathematical modeling between preservice and inservice teachers. A significant number of participants agreed to participate in this study and enough data was collected to explore a possible relationship between current grade level taught, years experience teaching, and the type of school in which teaching occurs.

The subsequent chapters in this dissertation include the literature review in chapter 2. This chapter is organized by the explanation of mathematical modeling and misconceptions, examination of mathematical modeling and preservice teachers including elementary teacher education programs, and mathematical modeling with elementary teachers. The theoretical framework in which the study was framed, is also discussed in this chapter.

Chapter 3 describes the methodology of this research study organized by rationale for research design, participants, instrument, and procedures. The origination of the mathematical modeling survey is examined as well as the determination for the procedures of the implementation of this research study. The participants groups are examined at length in this chapter.

Chapter 4 presents the findings of this study using the data analysis of the ANOVA and a multiple regression model. This chapter examines the results of the statistical analyses used for this study. Findings revealed in the research analysis are presented and discussed in this chapter. Each of the research questions is discussed at length and the data analysis is presented with the significance noted.

Chapter 5 seeks to understand the findings in this research study. This chapter includes discussions, conclusions and implications about the research in this study. Recommendations for future research are given, including extending the research beyond the knowledge of mathematical modeling and stretching the research into the understanding of mathematical modeling.

Chapter 2 - Literature Review

Overview

The importance of mathematical modeling has been prevalent in secondary classrooms for many years. It has recently been brought to light in elementary classrooms through the adoption of the Common Core Standards in many states (Gould, 2016). It is unfamiliar territory for preservice teachers, many inservice teachers, as well as professors in education preparatory programs. Preservice and inservice teachers differ in content knowledge mastery and pedagogical knowledge, but these two groups may be similar in that most likely neither had explicit instruction in their undergraduate teacher training in mathematical modeling. The definition of what mathematical modeling is and what mathematical modeling is not must be considered for both groups of teachers. Therefore, research in this chapter will provide the theoretical background of mathematical modeling, mathematical modeling regarding elementary preservice teachers, and mathematical modeling regarding elementary inservice teachers.

To determine how preservice teachers and inservice teachers will understand mathematical modeling, it is important to clearly define and understand what mathematical modeling is and what mathematical modeling is not. Preservice teachers should be taught in the same manner in which they wish to teach their own students (Lowery, 2002). It is essential to review how elementary education programs instruct preservice teachers in mathematical modeling, as the preservice teachers will soon be in the elementary classrooms independently teaching mathematics to elementary students.

Common Core State Standards

The Common Core State Standards (CCSS) have been adopted in forty-two states and four territories. These relatively new standards call out to teachers, schools, professionals, and stakeholders to change the way they teach mathematics. Some teachers may be caught up in the traditions of mathematics including but not limited to drill and kill, embarrassment of students for lack of skill, and algorithm teaching with no understanding. (Kline,1977). Even as far back as 1910, John Dewey was asking questions about this issue of teaching mathematics without understanding. Dewey (1910) asked, “How many [students] acquired special skills by means of automatic drill so that their power of judgment and capacity to act intelligently in new situations was limited?” It is two different things to be good at math and to be good at teaching math to others.

According to Common Core State Standards Initiatives (NGACBP & CCSS, 2010), there are three key shifts in the implementation of mathematics curriculum in the Common Core State Standards. These three key shifts are focus, coherence, and rigor. Focus refers to altering the practice of covering as many topics as possible during one year in school to covering fewer topics but at a deeper level. Coherence refers to a set of standards that are connected from grade level to grade level. Topics and ideas in mathematics are not disconnected tricks and tips for students to memorized. Topics are connected and woven throughout many years of instruction. The final shift in instruction is rigor. Rigor refers to the development of conceptual understanding, procedural skills and fluency, and application in all areas of mathematics. These topics should have equal attention throughout the school year even though the standards that are covered may be different. These key shifts in mathematics are summarized in Figure 2-1.

Figure 2-1 Key Shifts in Mathematics

Key Shifts in Mathematics	
Shift 1: Focus	Rather than racing to cover many topics in a mile-wide, inch-deep curriculum, the standards ask math teachers to significantly narrow and deepen the way time and energy are spent in the classroom.
Shift 2: Coherence	Mathematics is not a list of disconnected topics, tricks, or mnemonics; it is a coherent body of knowledge made up of interconnected concepts. Therefore, the standards are designed around coherent progressions from grade to grade. Learning is carefully connected across grades so that students can build new understanding onto foundations built in previous years.
Shift 3: Rigor	Educators should pursue conceptual understanding, procedural skills and fluency, and application with equal intensity. Rigor refers to deep, authentic command of mathematical concepts, not making math harder or introducing topics at earlier grades. To help students meet the standards, educators will need to pursue, with equal intensity, three aspects of rigor in the major work of each grade: conceptual understanding, procedural skills and fluency, and application.

(CCSSM, 2010)

The CCSS are designed differently than the mathematics standards in place before them. These key shifts in teaching and learning are essential if students are to develop a full understanding of the mathematical concepts from elementary mathematics all the way through high school mathematics. In addition, the CCSS include content standards organized by grade in K-8 and by strand in high school mathematics. A focus on the foundational understanding in elementary mathematics will lead to richer concept understanding in the middle grades and into high school mathematics. Therefore, the authors of the CCSS also included the Standards of Mathematical practice to guide teachers in the implementation of the CCSS.

Standards of Mathematical Practice

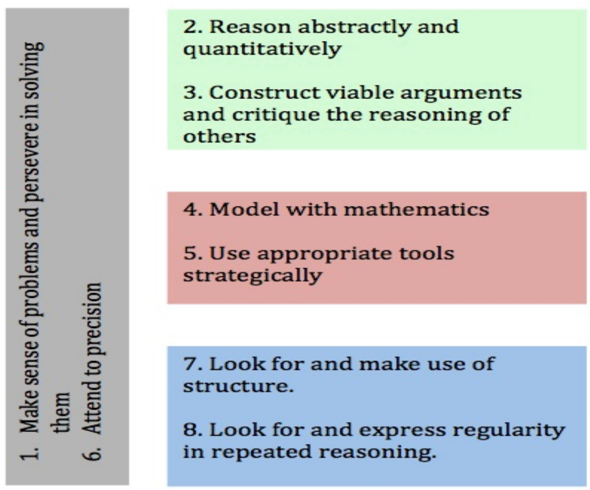
The Common Core State Standards in Mathematics (NGACBP & CCSS, 2010) include eight Standards for Mathematical Practice (SMP) that serve as teaching guidelines for curriculum in grades K through 12. The Practice Standards are: 1) Make sense of problems and persevere in solving them, 2) Reason abstractly and quantitatively, 3) Construct viable arguments and critique the reasoning of others, 4) Model with mathematics, 5) Use appropriate tools strategically, 6) Attend to precision, 7) Look for and make use of structure, 8) Look for and express regularity in repeated reasoning (CCSS, 2010). These practice standards are discussed at length in the CCSS document and include recommendations for teachers at all grade levels.

These SMP's should be used in conjunction with one another, not treated as separate items for discussion. Mathematical content should always be the vehicle in which to deliver instruction through the Standards for Mathematical Practice. The SMP's include eight standards that mathematics educators in Kindergarten thought grade 12 should seek to develop in the students they teach (CCSS, 2010). According to the Common Core State Standards,

> The first of these are the NCTM [National Council of Teachers of Mathematics] process standards of problem solving, reasoning and proof, communication, representation, and connections. The second of the strands of mathematical proficiency specified in the nations Research Council report Adding It Up: adaptive reasoning, strategic competence, conceptual understanding (comprehension of mathematical concepts, operations and relations), procedural fluency (skills in carrying out procedures flexibly, accurately, efficiently and appropriately), and productive disposition (habitual inclination to see mathematics as sensible, useful, and worthwhile, coupled with belief in diligence and one's own efficacy).

There are different ways to group the Mathematical Practices during instruction. Within a single lesson or grouped together for instruction during a unit or topic of study, the SMP's are an important part of lesson delivery. Identifying SMPs to group together for instruction through content delivery is essential. SMP1 and SMP6 could be a consistent theme throughout the unit. Then within more focused lessons, the following SMPs could be grouped together: SMP2 and SMP3, SMP4 and SMP5, SMP7 and SMP8. In Figure 2-2, this one possible grouping of SMPs for instruction is visually represented.

Figure 2-2 Standards of Mathematical Practice Grouping



The CCSS includes a major emphasis on the concept of mathematical modeling. The Standards of Mathematical Practice describe the expertise mathematical educators should seek to develop in their students with model with mathematics as one of these eight SMPs. O'Connell and SanGiovanni (2013) describe how a mathematical modeling problem can also lead to students developing other SMPs. In Figure 2-3, the part-part-whole model that could be used to begin to develop a mathematical modeling process in Kindergarten is represented. Using this part-part-whole model mat to find unknown information in kindergarten also supports:

- SMP1: Visual for students to construct meaning of the problem

- SMP3: Compose arguments about why their solution makes sense
- SMP6: Use precise calculations when solving problems
- SMP7: Discover commutative property using counters
- SMP8: Notice repeated actions and gain insight into operations

Figure 2-3 Part-Part-Whole Model Mat

Part	Part
Whole	

Mathematical Modeling

In the last two decades, mathematical modeling has been increasingly viewed as an educational approach to mathematics education from elementary levels to higher education (Erbas & et al, 2014). Historically, mathematical modeling has been a topic for secondary schools only (English, Fox & Watters, 2005). This trend is shifting to include all levels of educational study including elementary schools. Studies of mathematical modeling across all levels of education from elementary to secondary to higher education have developed definitions of mathematical modeling. Mathematical modeling has been defined in many similar ways across the research.

- According to the CCSS (NGACBP & CCSS, 2010), the application of mathematical modeling identifies mathematically proficient students as those who can apply what they know and are comfortable making assumptions and approximations to simplify a complicated situation, realizing that these may need revision later.

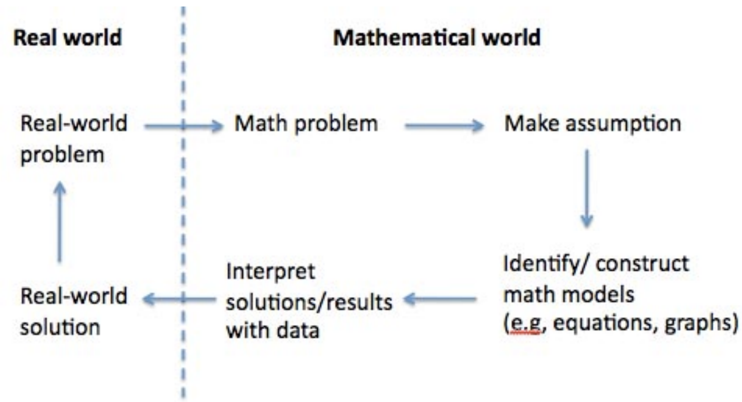
- Doerr and English (2003) define mathematical modeling as the system of elements, operations, relationships, and rules that can be used to describe, explain, or predict the behavior of some other familiar system.
- The practice of creating and analyzing a simplified and idealized mathematical representation of a real-world process or phenomenon is mathematical modeling (Teague, Levy, & Fowler, 2016).
- Mathematical modeling explicitly uses meaningful contexts that elicit the creation of useful systems or models (English, 2006).

Teague, Levy, and Fowler (2016) synthesize the work of the American Statistical Society in the *Guidelines for Assessment and Instruction in Mathematical Modeling Education* (GAIMME) report. The GAIMME report defines mathematical modeling in seven ideas, identified as steps in the report, that are not linear in order. These seven steps include: identify the problem, make assumptions, create a model, solve the model, analyze/assess solution, iterate the model, and implement the model (Teague, Levy, & Fowler, 2016). Teacher and student actions are also defined in this process. The teacher actions include: organize, monitor, and regroup. The student roles include: pose questions, validate conclusions, and build solutions. It is important to note that both teachers and students play important roles in the mathematical modeling process. Without each of these roles in the mathematical modeling process, the true intent of mathematical modeling is not realized.

In learning and teaching mathematics, the modeling approach can be useful by directing the focus on creating generalizable and reusable relations rather than solving a particular problem (Doerr & English, 2003). Problems should include multiple entry points and should include opportunities for the students to model the work rather than the teachers to model the work. This is a shift in thinking from the traditional mathematical instruction in elementary classrooms where all students are required to learn at the same rate and in the same way. For instance, even Marilyn Burns (1998), describes her own schools as sitting in her desk, doing her own math work on her own paper, not being allowed to talk to anyone and using her hand to shield her work from the eyes of her classmates. Even Brownell (1947) begins to define meaningful arithmetic as deliberately planned instruction designed to help students make sense of mathematics through mathematical relationships.

The National Council of Teachers of Mathematics (NCTM) has identified mathematical modeling as one of the major focal points in algebra standards. All students, pre-K through grade 12 are expected to use mathematical models and represent and understand quantitative relationships (Wolf, 2015). This is a call to all teachers, elementary and secondary, to use mathematical modeling tasks in math class. According to math4teaching.com, some steps a typical mathematical modeling may include real-world problem, math problem, make assumptions, identify and/or construct math models, interpret solutions and/or results, real-world solutions, and then back to real-world problems. The steps in the chart are circular in process, however there is not particular starting point as a mathematical modeling problem may begin at any one of these steps. Figure 2-4 shows this typical mathematical modeling process in a chart format.

Figure 2-4 Steps in Typical Mathematical Modeling Process



Mathematical modeling is the heart of this flow chart. Using content problems, students apply these flow chart steps to achieve a possible solution through mathematical modeling. Burns (1998) describes elementary students doing mathematics by involving them in activities, explorations, and experiments. Students must learn mathematical concepts and skills in the context of thinking, reasoning, and solving problems (Burns, 1998).

In all areas of mathematics, creating models strengthens students' understanding of math concepts and allows teachers' to assess that understanding (O'Connell & SanGiovanni, 2013; English, 2006; English, Fox & Watters, 2005). O'Connell and SanGiovanni (2013) explain how creating models of math problems is an invaluable skill for success in problem solving. When students are able to represent abstract thoughts through the creation of models, solutions are often not far behind. Students need to be given multiple experiences constructing math models (O'Connell & SanGiovanni, 2013). They need to be challenged to think about the math and determine a way to represent it (O'Connell & SanGiovanni, 2013).

True mathematical modeling problems are messy. Mathematical modeling problems should enable many approaches to the solution and mathematical ideas should be accessible at several levels (English, 2006). Students can analyze these relationships mathematically to draw conclusions and routinely interpret their mathematical results in the context of the situation and reflect on whether the results make sense, possibly improving the model if it has not served its purpose (NGACBP & CCSS, 2010). Applying the modeling in another context or situation may also serve to help students develop the reasons for using mathematical modeling.

Yackel, Cobb, and Wood (1990) concluded that children are not only capable of developing their own methods for completing school mathematics tasks, but each child has to construct his or her own mathematical knowledge. Children develop mathematical concepts as they engage in mathematical modeling by making sense of methods and explanations they see and hear from others. Additionally, Cobb, Wood, Yackel and McNeal (1992) describe how the teacher plays an important role in formalizing the mathematical discourse around problem solving in a classroom. If the teacher has set the norms in the classroom for students to determine that mathematics consists of fixed rules, then the discourse between students could be hindered to led to students to only follow such rules. However, when the teacher uses questioning techniques through mathematical discourse, learning of mathematical relationships was facilitated among students (Cobb & et al, 1992, Cobb & Yackel, 1996).

Mathematical Modeling Misconceptions

There are many misunderstandings about mathematical modeling with current teachers, professionals, and preservice teachers in elementary classrooms. According to CCSSM (NGACBP &CCSS, 2010), creating a math model and modeling with mathematics are two distinct constructs, and it is unfortunate the same root word of model appears in both (Cirillo, Pelesko, Felton-Koestler, & Rubel, 2016). The word model can be used as a noun, verb, or adjective. The use of the word model as an adjective such as a model citizen; and as a noun, such as creating a prototype model, is not the intent of this research. The use of the word model as a verb, such as modeling with mathematics to explain the amount of time needed to fill a tank of water using exponential growth, is the intent here. Elementary school teachers have unrepresented the term *model* to mean simply the use of manipulatives, a misunderstanding that is causing students to miss the mark when it comes to modeling with mathematics (Fletcher, 2016). Concrete objects or math manipulatives such as fraction strips, base 10 blocks, counting chips, or Geoboards are used frequently in Kindergarten through 8th grade classrooms to create a model in mathematics (Cirillo, Pelesko, Felton-Koestler, & Rubel, 2016). Mathematical modeling, although it may seem similar to creating a model in mathematics, is different.

Mathematical modeling is not an isolated task without social interactions. Mathematical modeling problems are intrinsically social experiences designed for small group work (English, Fox, & Watters, 2005). The role of a teacher in the elementary mathematics classroom during mathematical modeling should be that of a facilitator for students working in groups. Teachers should support children's mathematical development by encouraging students to lead the discussion, instead of the teacher dictating the direction of model development (English, Fox, & Watters, 2005).

Even published authors who have positive intentions for furthering mathematics education of inservice and preservice teachers seem to sometimes have a difficult time understanding the difference between mathematical modeling and modeling with mathematics. In her article, *Experiencing the Common Core State Standards for Mathematical Practices*, Johns (2016) describes how preservice teachers used the Math Practice #4: Model with Mathematics.

> "Model with mathematics: Preservice teachers used the base-ten blocks, drawings, and number names to show numbers in various ways. … Base-ten blocks helped them understand the exact meaning of the concepts of each place having a value of 10 times the place to its right and each place having a value of 1/10 to its left. They liked modeling

> with concrete materials before moving on to the number representation, which was more abstract."

The intent of the Common Core Standards for Mathematical Practice is for students to demonstrate mathematical modeling through creating models and solving real work world problems (CCSS, 2010). Although one cannot fault Johns (2016) for using this example to demonstrate mathematical modeling, it is a common mistake to identify these two separate constructs of creating a mathematics model and mathematical modeling as being the same.

Mathematical Modeling and Elementary Teacher Education

Teaching mathematics successfully is a complex task. Teacher education throughout the 20th century has consistently been structured across a divide between subject matter and pedagogy (Ball, 2000). A common approach is to require secondary teachers to major in the fields they will teach and then add knowledge of how children learn and classroom experience. However, in elementary education, the preservice teacher becomes, in some ways, a generalist. The elementary teacher must be able to teach all subjects. Hill and Ball (2009) state that the content knowledge teachers need is different from the knowledge needed by pure mathematicians or physicists.

Elementary teachers must be able to elicit conceptual understanding of topics in math from their students. Teachers require some specialized mathematical knowledge, such as being able to model integer arithmetic using different representations (Hill & Ball, 2009). Teaching mathematics requires specialized pedagogical knowledge about the subject, which pure mathematicians don't need (Ball, 2000). Meaningless mathematics will not penetrate any minds (Kline, 1977) and therefore the integration of content and pedagogy in elementary teaching is essential.

Typically, teachers teach the way they were taught in elementary, high school and higher education. Much of the lack of knowledge about teaching math comes from the way students learn math in elementary school (Tattoo & Senk, 2011). In the United States, a large number of elementary teachers are female. Female elementary teachers with math anxiety may pass this math anxiety on to their own students, especially the female students (Beilock, Gunderson, Ramirez, & Levine, 2010). Many times, these female students in elementary education programs lack the basic number sense and understanding to solve problems when the algorithm is not quickly remembered or understood. Mathematical modeling is one way to help preservice teachers begin to repair the damage of years of a lack of conceptual understanding.

Finding ways to integrate knowledge and practice is essential to help teachers develop the resources they need for their work. This is a call to preservice teacher education, as well as to professional development, where opportunities to study content are far more rare and the quality of mentor teacher feedback to student teachers should be valued (Jacobson, 2017). Future elementary teachers come to college with a set of previous knowledge and skill about mathematics. Many of these future teachers have had negative or even traumatic experiences during math class similar to the experiences described by Kline (1977) in his elementary school experiences.

Preservice teachers in teacher education programs should have many opportunities to participate in problem-based lessons. These problems should connect the National Council of Mathematics Teachers (NCTM) Content and Process Standards to problem solving if they are to become effective mathematics teachers in today's classrooms (Johns, 2016). Preservice teachers must begin to see themselves in some situations in the elementary math classroom as facilitator of knowledge and supporter of content.

According to Teague, Levy, and Fowler (2016), there are six modeling experiences preservice teachers, STEM students, and liberal arts majors should all be engaged in throughout their undergraduate experience. The experiences include open-ended projects, use of real data, collaboration, technology, technical writing, and common mathematical content. All of these experiences can be and should be incorporated into a mathematics methods course for preservice elementary teachers.

Bal and Doganay (2014) investigated different strategies and models used by teacher candidates when solving their real world problems. It was revealed that the knowledge of teacher candidates on the subject of mathematical modeling was insufficient, and teacher candidates used a typical strategy when solving real world problems. The preservice teachers need to understand mathematical modeling and then understand how to relay these concepts to students with understanding.

Based on the findings of Bal and Doganay (2014), the success of elementary school prospective teachers with regard to mathematical comprehension and modeling, was increased by means of the activities carried out during the mathematical modeling process. The education provided during this process was also effective. Within this context, it can be clearly understood that it is especially important for elementary school teacher candidates to gain experience with the mathematical modeling process to understand the mathematical concepts and to create modeling. One of the recommendations from Bal and Doganay (2014) was that the concept of mathematical modeling might be utilized in basic mathematics or math lessons and a course regarding mathematical modeling might also be included within the education programs.

Mathematical Modeling and Elementary Teachers

One problem with the organization of teachers' learning is that it tends to fragment practice and leave to individual teachers the challenge of integrating subject matter knowledge and pedagogy in the contexts of their work (Ball, 2000). Teachers assume the integration required to teach is simple and happens in the course of experience. However, this does not happen easily, and often does not happen at all (Ball, 2000). This means that elementary teachers may not have the knowledge of how to incorporate this content knowledge and pedagogy together for their students until they have many more years of teaching experience. The oversimplication or overcomplication of a mathematical modeling task is sometimes easy to do. Elementary teachers must be able to implement mathematical modeling in classrooms without falling to these two areas (Gould & Wasserman, 2014).

According to Meyer (2013), modeling with mathematics is one of the practice standards most in need of explication to teachers. Too many students learn mathematical procedures without any connections to meaning or the application that require these procedures. Models always have a purpose and they must be able to be reused; otherwise, there would be little reason to create a model. (Leinwand, Brahier, Huinker, 2014, English, Fox, & Watters, 20015). This is one of the areas that teachers could use additional explanation of mathematical modeling. Another benefit of the full understanding of mathematical modeling is the integration of mathematics and science modeling. Mathematical modeling is a powerful practice that can engage students and increase their understanding of mathematics (Wolf, 2015). According to Gould and Wasserman (2014), teachers need to find meaningful ways to incorporate mathematical modeling tasks into the curriculum adopted by the district.

Theoretical Framework

According to Yackel and Cobb (1990), teachers must use constructivism as a guiding framework from within to develop instructional situations that facilitate students' progressive construction of increasingly abstract mathematical conception and procedures. Instructional activities are not selected or designed to ensure that every student makes the same preselected mathematical constructions or the same relationships. This is the heart of mathematical modeling. Following the guiding work of theorists who focus on studying new ways of learning, the theoretical perspectives of Brownell's Meaning Theory and Schulman's Pedagogical Content Knowledge guide the examination of mathematical modeling.

Teaching by memorization, rote learning, and algorithm learning with no understanding does not lead to students developing the true meaning of mathematical modeling. During his research in educational setting, William Brownell developed the Meaning Theory of Mathematics, demonstrating a difference between applications and truly understanding the mathematics. Emphasis should be placed on place value and algorithms after the development of understanding of these topics. One of the basic tenets of the theory is "to make arithmetic less a challenge to the pupil's memory and more a challenge to his intelligence" (Brownell, 1944). Brownell's research has inspired generations of mathematics teachers and mathematics researchers to discover ways to teach meaning and understanding in mathematics. Brownell's "brilliant experimental work on the teaching of arithmetic reads as well today as it did forty years ago" (Kilpatrick, 1977). Learning without understanding does not promote mathematics.

Shulman (1987) describes pedagogical content knowledge as specialized knowledge distinguishing the teacher from the content specialist (Friedrichsen, et al, 2009). Pedagogical content knowledge includes useful representations, unifying ideas, clarifying examples and counterexamples, helpful analogies, important relationships, and connecting ideas. It must be connected to content knowledge or knowledge of the subject matter (Shulman, 1987). According to Shulman (1986), mere content knowledge is as useless pedagogically as contentfree skill. To blend the two aspects of teaching skills require that we pay as much attention to the content aspects of teaching as to the elements of the teaching process.

Modeling with Mathematics, the Standard of Mathematical Practice #4, supports both the content and pedagogy of mathematical modeling. Preservice elementary teachers and elementary inservice teachers must first develop personal knowledge of these concepts, so that they will be able to teach students to develop an understanding of mathematical modeling. Learning to support the mathematics content through the development of all of the SMPs should be an essential part of teacher preparatory programs as well as essential professional development for inservice teachers.

Summary

This chapter has given a detailed description of the literature reviewed for this research study. The literature reviewed for this research study examines several areas of mathematical modeling. The first area explored is that of the Common Core State Standards including the Standards of Mathematical Practice. Another area of literature reviewed was mathematical modeling including definitions of mathematical modeling, mathematical modeling misconceptions, mathematical modeling in teacher preparation programs, and mathematical modeling and inservice teachers. The final area explored in this literature review was the theoretical framework for which this research study was developed. Brownell's Meaning Theory and Schulman's Pedagogical Content Knowledge Theory were considered when designing the research study. The analysis of this research was also considered during the design of the survey and the research study.

The next chapter, Chapter 3, examines the methodology used for the survey research. The final two chapters, Chapter 4 and Chapter 5, examine the results of the data analysis as well as the implications and conclusions from this research.

Chapter 3 - Methodology

Rationale for Survey Research

This chapter explains methodological framework and research design that will be used in this research study. This research study will investigate inservice teachers' and preservice teachers' knowledge of mathematical modeling in the elementary classroom.

Survey research, tailored design method, was used for this study as the researcher will not manipulate or control the independent variables. Vogt, Gardner & Haeffele (2012) describe the use of survey research, tailored design studies in the case of real-world contexts when the researcher is able to satisfy five criteria including the following: data is best obtained directly from respondents, data can be obtained in brief answers with structured questions, respondents will give reliable answers, researcher knows he/she will use the data, and the researcher can expect an adequate response rate. All five of these criteria are met in this research. The data is obtained directly from respondents through the online survey. The survey includes questions that can be answered briefly through structured questions, including a scale response for fourteen questions. Brief answers to collect demographic information are part of the survey. Reliable answers are expected as the survey is confidential and is not tied to a grade or job performance including a job incentive or negative consequence. The researcher mapped out a plan to use and analyze the data. The last criterion of expecting a reasonable response rate is met through keeping the design of the survey short and easy to use.

The researcher compared preservice teachers knowledge of mathematical modeling with inservice teachers' knowledge of mathematical modeling using the causal-comparative structure as described by Gall, Gall, & Borg (2007). This involved the researcher studying the cognition of the research subjects without any intervention on the part of the researcher though survey research. The survey used for this research was an online survey implemented through an online link to the survey.

The independent variables in this study are the classification of the teacher, either preservice with no independent classroom teaching experience or inservice with classroom teaching experience; type of school in which teaching occurs; and number of years teaching experience. The dependent variable is mathematical modeling knowledge. The research design for this survey was dependent on two additional factors; the structure of the public school system and the structure of university student teaching experience.

The research questions for this study include:

1. Is there a statistical difference between inservice and preservice teachers with regard to knowledge of mathematical modeling?

   Null Hypothesis: There is no difference between inservice and preservice teachers with regards to knowledge of mathematical modeling.

2. Is there a relationship between the number of years teaching experience, grade level taught, or type of school in which teaching occurs in regard to knowledge of elementary mathematical modeling?

   Null Hypothesis: There is no relationship between completing grade level taught and knowledge of mathematical modeling.

   Null Hypothesis: There is no relationship between number of years teaching experience and knowledge of mathematical modeling.

   Null Hypothesis: There is no relationship between type of school in which teaching occurs and knowledge of mathematical modeling.

3. Is there a relationship between preservice and inservice teachers' perceived knowledge of mathematical modeling and their actual knowledge of mathematical modeling?
   Null Hypothesis: There is no relationship between preservice and inservice teachers' perceived knowledge of mathematical modeling and their actual knowledge of mathematical modeling.

Participants

Preservice teachers and inservice teachers participated in survey research to compare the difference in knowledge of modeling with mathematics between the two groups. Subjects are an accessible population of preservice and inservice teachers in a mid-western city. The participants come from a variety of local backgrounds, however the ethnicity and gender of the sample population is very similar. The demographics were self-reported by participants in the demographics section of the survey. Using the demographics reported from the participants, additional information about the groups was analyzed using the following criterion:

- Years of teaching experience
- Grade level taught
- Type of school

Eligibility to Participate in the Research

In order to be eligible to participant in this research study, participants had to meet some minimum qualifications. In the preservice teacher group, the preservice teachers had to be enrolled in one of their last semesters in the elementary education program at the university. These two semesters are referred to as Block 2 and Block 3. In Block 2, the preservice teachers are enrolled in educational methods courses as well as an observation and teaching course where they are in the elementary schools at least three day a week teaching lessons and observing teaching. In Block 3, the preservice teachers participate in what would be considered a traditional student teaching semester. Preservice teachers are at their public elementary school every workday and attend all professional development meetings with their mentor teacher. These preservice teachers also had to be enrolled in the elementary education program, which leads to K-6 teacher licensure upon meeting graduation requirements of the program.

For the inservice teacher participants to qualify to participate in this research study, the inservice teachers had to be employed in a Kindergarten through 6th grade elementary school.

Elementary Preservice Teachers

The participants were selected from two separate groups for this research study. One group of participants includes preservice teachers exclusively.

- A Midwestern division II university in East-Central Kansas was used for this study. The preservice teachers enrolled in elementary education in this university's school of education were asked to voluntarily participate in this research. The university's total enrollment for 2016-2017 was approximately 6,100 students.
- There were 136 preservice teachers that were asked to participate in this study. Depending on the number of students that agreed to voluntarily participant in this research, it was anticipated there could be approximately 90 preservice teachers in this study. There were 94 actual preservice teachers that completed the Mathematical Modeling survey.
- Participants must be enrolled in one of the last two semesters of their respective elementary teacher education program in order to participate in this research study.

Elementary Inservice Teachers

The second participant group was inservice teachers teaching elementary school in grades Kindergarten through 6^{th} grade.

- Elementary teachers who teach Kindergarten to 6^{th} grade mathematics were asked to participate in this research survey. The teachers were selected from the six elementary schools and one middle school in a district classified in the state as 5A public school. This school district serves approximately 4,600 students in the public schools. The local population of the city is approximately 25,000 people.
- K-6 teachers must currently teach elementary mathematics in their classroom or have taught elementary mathematics in K-6 in the last 2 years. Mathematics strategists or mathematics instructional coaches were also be included in the survey if they currently teach mathematics to elementary students or if they met the requirements of teaching mathematics in the last 2 years.
- There are approximately 160 K-6 elementary teachers, strategists, and/or instructional coaches in this district that were asked to participate in this study. Depending on the number of teachers that volunteer to participate, there could have been approximately 75 teachers participate in this study. There were 52 elementary inservice teachers that completed the Mathematical Modeling Survey.

Instruments

Development of Survey

In order to analyze the knowledge of elementary teachers about mathematical modeling, it was determined that an online survey would be implemented. Gould (2013) developed a mathematical modeling survey for her dissertation research and this survey was utilized for this research. The original survey was developed for public school mathematics teachers in grades 7-12. This survey was developed to be quickly and easily completed in an online environment (Gould, 2013). Participants answered questions about their knowledge of mathematical models in an elementary environment and their knowledge of applying mathematical modeling skills in the elementary classroom.

After correspondence with Gould (2013), Gould granted permission to the researcher to use this survey with this current research. Through the correspondence with Gould, it was communicated that even though the survey had three sections, each section was developed separately and independently and could be used separately for future research. For this survey, only two of the three sections will be used. One section of the survey, Mathematical Models and the Curriculum, contained questions specific for mathematics teachers to analyze the reasons mathematical modeling was included in the CCSS. This section was excluded from the survey, as it was outside the scope of this research.

While developing the original survey, it was given to mathematical modeling experts in the field and evaluated for validity. The mathematical modeling experts gave feedback to Gould (2013) and the survey was edited. After the edit, the survey was reevaluated and considered valid by the panel of mathematical modeling experts. In appendix B, the mathematical modeling experts' consensus is reported. Gould (2013) developed the survey to partially answer the questions that follow: 1) How do teachers describe a mathematical model? 2) How do teachers describe the mathematical modeling process? 3) What do teachers believe to be the purpose of mathematical modeling? Based on this information, the current research study will be used to answer the questions: 1) Is there a statistical difference between inservice and preservice teachers regarding the knowledge of mathematical modeling?, 2) Is there is relationship between completing the number of years teaching experience, grade level taught, or type of school in which teaching occurs in regard to knowledge of elementary mathematical modeling?, and 3) Is there a relationship between preservice and inservice teachers' perceived knowledge of mathematical modeling and their actual knowledge of mathematical modeling?

Using Dillman, Smyth, and Christian's (2009) recommendations for appropriate online survey formats, the format for the online survey was carefully considered. The number of questions on the screen at one time, the number of clicks or answers required by the participants, ease of use, and other formatting issues were considered. Another recommendation is to keep all email contacts to the participants short and to the point (Dillman, et al, 2009). This was strongly considered during the initial contact phase of the research and as well as throughout the survey development.

Mathematical Modeling Survey: Section 1-Demographics

This first section of the survey included the informed consent letter along with the informed consent digital form for participants to give consent or decline consent for the survey. Once informed consent was received, the participant continued on with the remainder of the survey. If informed consent was declined, the participant exited the survey automatically and a thank you message was displayed. Demographic information about the participant was also collected in the first section of the survey. This included grade level currently taught, years of experience teaching, identification of preservice or inservice teachers, type of school, and current enrollment in the teacher undergraduate preparatory program.

Next, participants rated their level of understanding about different elementary mathematical models questions and elementary mathematical modeling questions. There were nineteen questions in the entire survey for participants to complete. It was estimated by

Qualtrics that the survey should take approximately 10-15 minutes to complete the survey for this research. The entire survey is attached in Appendix A. The actual average time participants took to complete the complete the survey was 14.41 minutes.

An incentive for completing the survey was offered. There were two $25 Starbucks gift cards offered to participants. One card will be awarded to one participant in the preservice teacher group and one card will be awarded to one participant in the inservice teacher group. At the end of the survey, participants had the option to enter their email contact information if they wished to receive a chance to be awarded the incentive. This information was inputted through Qualtrics as separate from the rest of the survey data and did not lead to identifying the participants' answers in the survey. These gift cards were awarded after the survey was completed.

Mathematical Modeling Survey: Section 2-Mathematical Models

The second section of the survey included questions about mathematical models that students may develop in elementary classrooms. These questions were designed to determine the knowledge participants have about mathematical models. Participants were asked six questions about mathematical models and were asked to respond the questions on a five-point scale ranging from *completely disagree, somewhat disagree, neither agree nor disagree, somewhat agree, completely disagree, and I don't know*. *I don't know* was included as a choice in the responses to help ensure participants were not guessing on the answers. Any responses of *I don't know* were considered incorrect responses, as knowledge of that aspect of mathematical models was not known. A scale format was used but was assessed at the nominal level.

The six statements that were listed in this second section of the mathematical modeling survey follow:

Statement 1: Mathematical models can be physical manipulatives, for example, fraction tiles, pattern blocks or linking cubes.

Statement 2: Mathematical models can be equations or formulas, for example, base x height = area or the formula for perimeter.

Statement 3: Mathematical models can be visual representations such as a graph in the coordinate plane or a number line.

Statement 4: Mathematical models can be visual representations such as an elevation map of a mountain or a scientific scale drawing.

Statement 5: Mathematical models can be used to describe or summarize a given situations in compact form.

Statement 6: Mathematical models can be used to explain the underlying causes in a given situation.

Participants were asked to choose the option that best matched their opinion regarding the statement. Each statement was written as a simple description of what a mathematical model "can be." In some cases, examples were provided to clarify the intended meaning of the statement. Responses that indicated a correct understanding of the topic, those which agree with true statements and which disagree with false statements, were justified through a literature review and a consensus among experts in mathematical models and modeling (Gould, 2013).

Statement 1 in the Mathematical Models section refers to models that can be physical manipulatives, for example, fraction tiles, pattern blocks, or linking cubes. Pollak (2003) indicates that these may not be mathematical models because they are real-world objects that represent mathematical ones. This is the reverse of a mathematical model because these objects are only objects until a mathematical process has been attached to them. Any level of disagreement here was considered a correct conception.

Statement two in the Mathematical Models section refers to Mathematical Models section of the survey that can be equations or formulas. Examples listed in this question include formulas such as base x height = area or the formula for perimeter. Statement three in the mathematical models section refers to mathematical models that can be visual representations such as a graph in the coordinate plane or placing numbers on the number line. Within the CCSSM, examples of what are assumed to be mathematical models are listed. One such example is "formulas" and another is "graphs" (NGACBP, 2010). According to Wolf (2015), one stage of mathematical modeling is to form a mathematical solution using a mathematical model. Responses, which indicate agreement, were considered to be correct for both statements.

Statement four in the Mathematical Models section refers to mathematical models that can be visual representations such as a scaled map of the county or a scientific scale drawing. There is no single correct answer to this statement based on the literature. This is because the literature conflicts regarding these potential models. On the one hand, in Principles and Standards, mathematical models are defined strictly as "mathematical representations of the elements and relationships in an idealized version of a complex phenomenon" (NCTM, 2000, p. 70). Since the representations must be mathematical, maps or blueprints cannot qualify as mathematical models. Wolf (2015) gives the example of 8th graders using giant footprints from *Gulliver's Travels* to estimate the height of the giant. Therefore, responses to this question were unable to be deemed to be correct nor incorrect as even the mathematical experts could not agree on this answer. Instead, teachers' tendencies in responses were explored.

Statement five in the Mathematical Models section refers to mathematical models that can be used to describe or summarize a given situation in compact form. Statement six refers to mathematical models that can be used to explain the underlying causes in a given situation. In Principles and Standards, it is stated, "Mathematical models can be used to clarify and interpret [a] phenomenon" (NCTM, 2000, p. 70). This means that they can both describe a phenomenon or explain why the phenomenon is occurring. For both statements, agreement was considered correct.

Mathematical Modeling Survey: Section 3-Mathematical Modeling

The third and final section of this survey included questions about mathematical modeling. Participants were asked eight questions about mathematical modeling and were asked to respond the questions on a five-point scale ranging from completely disagree, somewhat disagree, neither agree nor disagree, somewhat agree, completely disagree, and I don't know. I don't know was included as a choice in the responses to help ensure participants were not guessing on the answers. Any responses of *I don't know* were considered an incorrect response, as knowledge of that aspect of mathematical modeling was not known. Scale format was used for the answers to these questions; however it was interpreted at the nominal level for this research.

The questions in this section of the survey refer to the participants' understanding of how often mathematical modeling characteristics occur in the elementary math classroom. Participants were asked to select which option best matches their personal estimation of how frequently a characteristic is part of the mathematical modeling process (Gould, 2013). The literature review was used to identify a consensus of correct answers on these mathematical modeling questions.

The following statements are the eight statements listed in the third section of the mathematical modeling survey:

Statement 1: Repeating steps is part of the mathematical modeling process

Statement 2: Mathematical modeling situations come from 'whimsical' or unrealistic scenarios

Statement 3: The mathematical modeling process involves making choices

Statement 4: The mathematical modeling process involves making assumptions

Statement 5: The mathematical modeling process involves determining if a solution makes sense in terms of the original situations

Statement 6: The mathematical modeling process involves making revisions

Statement 7: The mathematical modeling results in an exact answer or exact answers

Statement 8: A mathematical modeling situation can result in various, different mathematical models

Statements 1, 6, 7, and 8 refer to mathematical modeling as a process that includes repeating steps, making revisions, requiring exact answers, and creating various mathematical models. The Common Core State Standards (NGACBP, 2010) and Principles To Action (2014) each indicate various models that were created through revision and repeated step are necessary for mathematical modeling. Therefore, answers of completely disagree would be considered incorrect on statements 1 and 6. Doerr (2003) indicates that the mathematical reasoning through models is required in elementary schools. CCSSM (NGACBP, 2010) and Principles to Actions (2014) also indicate answers that are exact or approximate with justification could be acceptable for mathematical modeling problems through different representations. Wolf (2015) maintains that mathematical modeling is a rich math task that lends itself to variety of approaches and representations. Therefore, statements of *completely disagree* or *completely agree* would be considered incorrect on statement 7. Statement of *completely disagree* or *somewhat disagree* would be considered incorrect on statement 8.

Statement 2 refers to mathematical modeling as whimsical or unrealistic. English (2006) indicates mathematical modeling should come from real-life problems where students can apply created mathematical models in several areas of mathematics. Principle to Actions (2014) also indicates teachers should support the mathematical struggle through realistic problem solving. Therefore, the answer of *completely disagree* and *disagree* will be considered correct on statement 2.

Statement 3, 4, and 5 refer to mathematical modeling as making choices, making assumptions and reasonableness of solutions. CCSSM (NGACBP, 2010) and Principles to Actions (2014) both indicate mathematical modeling and problem solving must include the productive struggle in mathematics through assessing reasonableness of answers and making choices through modeling. Making assumptions and making sense of the mathematics must also be a part of the process (NGACBP, 2010, Doerr, 2003, English, 2006). Therefore, answers indicated by *completely agree* and *somewhat agree* would be considered correct.

Procedures

In Fall 2016, a brief summary of this research study was submitted to the research institution's Institutional Review Board (IRB) for approval. The informed consent document, mathematical modeling survey, and IRB form were submitted to the University Research Compliance Office. All required training for the researchers associated with this study was completed and up to date. The IRB tracking number for the university was 8561. The university approved the IRB on December 13, 2016.

Additionally, an IRB was submitted to another university for approval in Fall 2016. This university was where the researcher was employed and the research subjects were the preservice teachers. This IRB was approved on December 16, 2016. The Unified School district in which the research was conducted granted written approval for the researcher to conduct research with the elementary teachers in the district. This written approval was received in December 14, 2016.

An online survey was distributed to participants to gauge their level of knowledge regarding mathematical modeling. The procedures for the two different groups of participants are described in the next two sections.

Preservice Teachers

For the elementary preservice teachers, initial contact was made with the Director of Field Placement and Licensure and the Associate Dean of the College of Education to obtain permission to introduce this research during a meeting in which all student teachers were on campus. Written permission was granted to the researcher to complete this online survey with the preservice teachers and to schedule in fifteen minutes of time during the meeting to explain this research. During the introductory meeting about this research project, the participants were informed of the research and invited to voluntarily sign the digital informed consent form to participate. The link to the survey was uploaded to the university's learning management system for students enrolled in the last two semesters of the program to access. The voluntary aspect of the survey was reiterated at this face-to-face meeting. Preservice and inservice teachers that agreed to participate in the survey, were allotted time during this meeting to complete the survey. If participants were not able to start or complete the survey during this meeting, the link remained on the learning management system's website for two weeks for preservice teachers to access.

After the survey, a follow up email was automatically sent to all participants thanking them for completing the survey. The email also included information about whom to contact if there are any questions about the survey and whom to contact to confirm the selection of a participant to receive the incentive gift cards. The email correspondence sent to the preservice teachers is included in Appendix D.

Inservice Teachers

For group two, initial contact was made with the Associate Superintendent of Teaching and Learning at the school district where the research took place. Written permission to conduct this survey research was received from the Associate Superintendent of Teaching and Learning. Permission to contact elementary mathematics teachers in grades Kindergarten through 6^{th} to participate in this study was requested and granted. It was suggested by the school district, that the researcher first reach out to the six elementary school principals and the one middle school principal to explain this online survey research. The principals then forwarded the information in an email about the online survey to potential participants in the survey. Inservice teachers were asked to complete the survey within one week of receiving the email.

A reminder email was sent to the principals five days after the initial email and was asked to forward the email on to the teachers who received the original email. The survey window was then extended for an additional two weeks to give elementary inservice teachers additional time to complete the survey. Once the online survey was completed, a follow up email was automatically sent to participants, thanking them for their time and efforts in the survey. The email also included information about whom to contact if there were any questions about the survey and whom to contact to confirm the selection of the two participants that received the incentive gift cards. The emails sent to all research participants can be found in Appendices C through Appendix F.

Data Analysis

Conceptions about mathematical modeling have objective responses based on several reasons including teaching experience, undergraduate mathematical training, personal research, understanding of mathematical modeling, and personal opinion. The overall purpose of this research study was to determine if there are statistically significant differences between preservice teachers and inservice teachers in regard to their knowledge of mathematical modeling. Therefore, independent t-tests were run to determine if there were statistical differences between and within the two groups of participants.

Additionally, a multiple regression analysis was run to determine if any of the independent variables were able to predict or determine the value of the dependent variables. The amount of teaching experience, type of school, and current grade level taught were independent variables that could have an impact on the level of knowledge of mathematical modeling. Descriptive statistics were used to explain the data collected for this research study. For the quantitative analysis, all statistical analyses will be analyzed using the $p<.05$ level of significance. These tests were analyzed using the Statistical Package for the Social Sciences (SPSS) software system.

Summary

The specific methodology, procedures, and data analysis that were used in the research study were detailed in this chapter. The research utilized online survey research, tailored design method. The methodology for the design of the survey, the contact of the participants, and the procedures for this research are outlined in detail in this chapter. Information about the participants and selection of participants in this study was also detailed. This survey research, tailored design method examined the differences in knowledge of mathematical modeling between preservice teachers and current, inservice elementary teachers. This research study also aimed to examine any relationship between number of years teaching experience, current grade teaching occurs, or type of school in which teaching occurs in regard to knowledge of elementary mathematical modeling. The findings of the analysis of this study are reported in Chapter 4.

Chapter 4 - Results

Overview

The purpose of this survey research was to examine preservice and inservice teachers' knowledge about mathematical modeling. The researcher investigated if statistical differences in knowledge of preservice and inservice teachers regarding mathematical modeling existed. The independent variables of number of years teaching experience, grade level taught, or type of school in which teaching occurs were examined to determine any relationships regarding knowledge of mathematical modeling. This chapter provides statistical results for the specific research questions for this study:

1. Is there a statistical difference between inservice and preservice teachers with regard to knowledge of mathematical modeling?
2. Is there a relationship among the years of teaching experience, grade level taught, or type of school in which teaching occurs in regard to knowledge of elementary mathematical modeling?
3. Is there a relationship between preservice and inservice teachers' perceived knowledge of mathematical modeling and their actual knowledge of mathematical modeling?

This research study implemented an online survey distributed to elementary preservice teachers enrolled in an elementary education program at one university as well as elementary inservice teachers in one community. The total sample size, as well as the sample size of the actual participants in the survey, affected the statistical tests that were valid to analyze this set of data. The generalization of this study was also effected by the smaller sample size of the participant group.

This chapter continues with the analysis of the fourteen mathematical modeling questions as well as a total score for the knowledge of mathematical modeling in the online survey. For research question number one, the researcher conducted independent t-tests to determine if there was a statistical difference in the knowledge of mathematical modeling between preservice elementary teachers and inservice elementary teachers. The average mathematical modeling score for each participant group to determine if there was a statistical difference in the knowledge of mathematical model between the two groups. For the second research question, two different multiple regressions were used to determine if there were any factors such as years of teaching experience, grade level taught, or type of school were related to knowledge of mathematical modeling. For the third question, an ANOVA was run to determine if there was a relationship between perceived knowledge of mathematical modeling and actual knowledge of mathematical modeling in the entire sample of participants.

Sample Population

Participants

There were initially n=170 potential participants that opened the online survey. Of those participants, there were n=167 participants that agreed to informed consent of this survey. After participants agreed on the informed consent page, there were n=150 participants that met the qualification to complete the survey and n=148 participants input their demographics into the survey. In section 2 and section 3 of the survey, the options in the online survey required participants to answer each question before continuing. There were n=146 participants that completed sections 2 and 3. The summary of the number of participants that completed each part of this survey is described in the following table.

Table 4-1 Number of Participants Completing Survey Sections

	Number of Participants
Participants that clicked on survey	170
Participants that agreed to informed consent	167
Participants that met qualifications to complete survey	150
Participants that entered demographic information (Section 1)	148
Participants that completed Mathematical Models questions (Section 2)	146
Participants that completed Mathematical Modeling questions (Section 3)	146
Total preservice elementary teacher participants	94
Total inservice elementary teacher participants	52

The demographics section of the survey included some answer choices with qualitative explanation for additional information given by participants. The questions that had additional responses for demographics were type of school and grade level taught. There were five qualitative responses for the type of school question, which included two responses of teaching combined elementary and middle school and three responses indicating the participant was a preservice teacher. Table 4-2 summarizes the response to this demographics question.

Table 4-2 Responses to Type of School Demographic Question

Demographics Question	Number of Responses
In what type of school do you currently teach?	
Elementary	136
Middle	5
Other	5 included qualitative explanations: 3 preservice teachers 2 Combination of both elementary and middle school

Beyond the anticipated responses, there were eight additional qualitative responses for the demographics question that inquired about the grade level currently taught. These additional responses included intern (preservice teacher), multiple grade teacher, and instructional coach. Table 4-3 summarizes the response to this demographics question.

Table 4-3 Responses to Current Grade Level Taught Demographic Question

Demographics Question	Number of Responses
What grade level do you currently teach?	
Kindergarten	11
1st	9
2^{nd}	11
3^{rd}	15
4^{th}	14
5^{th}	14
6^{th}	3
Combination of several grades	9
Prospective elementary Teacher	52
Other	8 including qualitative explanations: 4 intern (preservice teachers) 1 5^{th}-8^{th} grade 1 6^{th}-8^{th} grade 1 Instructional coach K-5 1 Technology teacher

Response Rates

The specific number of participants outlined in the *Participants* section above represents a total response rate of 50.3%. There were 154 inservice teachers that were sent the online survey to complete. Out of those 154 teachers, 52 completed the survey, which represents a 33.76% response rate for the inservice teacher group. There were 136 preservice teachers that were sent the online survey. Out of those 136 preservice teachers, 94 completed the survey, which represents a 69.11% response rate for the preservice teacher group. The surveys that were not completed in full, including surveys with no answers or only demographic information, were not included in this analysis.

According to Sheenen (2001), the response rate expected for online surveys is 6%-75%. Therefore, the anticipated response rate of this survey was in the middle of this reported response rate because of the convenience sample selected for this survey. The actual response rate was a higher than expected for this online survey. This could be due to the local nature of the survey and that the participants were asked to complete the survey by someone who may have been familiar to them, either the researcher or their principal that forwarded the online survey to them through email.

Coding of Answers

The coding of the answers to the question was the next step of the analysis. Each of the 14 questions in Sections 2 and 3 were answered using a 5-point Likert scale ranging from *completely disagree* to *completely agree* with a choice for *I don't know*. For each of these questions, a scale format was used, but the answer to the question was assessed on a nominal level as either correct or incorrect according to the literature review. If participants answered, *I don't know*, that response was coded as incorrect. This response was included on the survey to increase the possibility that participants would answer honestly and not guess at answers if the answer was truly not known. A correct answer was coded as a 1 in the SPSS data analysis, while an incorrect answer was coded as a 2.

There were two sections of the Mathematical Modeling Survey that included coded answers. The section of the survey titled Mathematical Models included questions for teachers to rate statements that included what mathematical models could be or could not be. The participants choose one answer that best fits their knowledge of mathematical models. A mathematical model is a description of a system using mathematical concepts and language. The summary of the correct and incorrect answers for Section 2 Mathematical Models is included in the Table 4-4.

Table 4-4 Section 2: Mathematical Models Answers for Survey

	Correct Answers (Coded as 1)	Incorrect Answers (Coded as 2)
Q1	Neither Agree nor Disagree Somewhat Disagree Disagree	Completely Agree Somewhat Agree I Don't Know
Q2, Q3, Q5, Q6	Completely Agree Somewhat Agree	Neither Agree nor Disagree Somewhat Disagree Completely Disagree I Don't Know
Q4	Not included in quantitative analysis	Not included in quantitative analysis

The second of the two sections of the Mathematical Modeling Survey included a section of the survey titled Mathematical Modeling. This section included questions for teachers to rate statements that included what mathematical modeling could be or could not be. The participants choose one answer that best fits their knowledge of mathematical modeling. Mathematical modeling, for purposes of this research, is applying mathematics to a real world problem with the purpose of understanding the problem. The summary of the correct and incorrect answers for Section 3 Mathematical Modeling is included in the Table 4-5.

Table 4-5 Section 3: Mathematical Modeling Answers for Survey

	Correct Answers (Coded as 1)	Incorrect Answers (Coded as 2)
Q1, Q2, Q5,Q6, Q8	Completely Agree Somewhat Agree Neither Agree nor Disagree	Somewhat Disagree Completely Disagree I Don't Know
Q3, Q4	Completely Agree Somewhat Agree	Neither Agree nor Disagree Somewhat Disagree Completely Disagree I Don't Know
Q7	Neither Agree nor Disagree Somewhat Disagree I Don't Know Somewhat Agree	Completely Agree Completely Disagree

This coding of answers was completed in an Excel spreadsheet. All of the data from the Mathematical Modeling Survey in Qualtrics was uploaded into an Excel spreadsheet. Through Excel, formulas were written to code the answers according to the literature review and the mathematical modeling experts. In the Excel spreadsheet, a formula was also written to give a total average mathematical modeling score to each participant. This average score was calculated by averaging the coded score, one for correct and two for incorrect, on the thirteen questions in Section 2 and Section 3 of the survey. The answers to question 4 in the second section of the survey were not coded for correct or incorrect responses, as there was not a consensus from the mathematical modeling experts on an answer to that question. The answers to questions 4 were not considered in the qualitative analysis because a correct or incorrect answer to this question could not be determined.

Data Analysis

The data collected through the online Mathematical Modeling survey was analyzed in this section. Once the data was collected and organized, the answers to the questions were analyzed to answer the three research questions. Each of the research questions is addressed separately in the next three sections of Chapter 4.

Research Question 1

The first research question pertains to determining if there was statistical difference between inservice and preservice teachers with regard to knowledge of mathematical modeling. This question was explored by comparing the two groups of participants answers in three ways: the Mathematical Models section of the survey, the Mathematical Modeling section of the survey, and the total mathematical modeling score of the participants. The null hypothesis for this research question was that there is no difference between inservice and preservice teachers regarding knowledge of mathematical modeling. Descriptive statistics were calculated to explore the responses of the participants in these two sections of the survey.

The participants were divided into two different groups for this first analysis. One group contained the preservice teachers n=94 and the other group contained the inservice teachers n=52. The two groups contained unequal sample sizes, and this unequal group size was automatically corrected in SPSS (Field, 2012) in the statistical analysis of the t-test using the Welch t-test. A score of *one* is considered a correct response and a score of *two* is considered an incorrect response to the question. Therefore, a mean score closer to one represents an answer that is more correct than incorrect and a mean score closer to two represents an answer that is more incorrect than correct. The results of the Mathematical Models section of the survey include the mean in each category of only a difference of .06 at the most. Both groups scored were very similarly in that all five of the questions in the mathematical models section of the survey showed almost no difference in average scores. Table 4-6 shows the mean and the standard deviation of Section 2 of the online survey.

Table 4-6 Descriptive Statistics for Section 2 Mathematical Models with Preservice vs. Inservice Teachers

		Mean	Std. Deviation
Q1	Preservice	1.94	.246
	Inservice	1.92	.269
Q2	Preservice	1.22	.419
	Inservice	1.25	.437
Q3	Preservice	1.10	.296
	Inservice	1.13	.345
Q5	Preservice	1.33	.473
	Inservice	1.35	.480
Q6	Preservice	1.21	.411
	Inservice	1.27	.448

The test of homogeneity of variances was met using Lavene's statistic for the independent t-tests. Each of the five questions in Section 2 of the survey had a significance level greater than p=.05 indicating that the two groups have statistically equal variance. The degrees of freedom for each question are *df*=1, 144. However, none of the individual questions from the mathematical models section of the survey showed a statistical significance in the results. Each of the five questions indicated there was not a statistically significant difference between the preservice teachers answers and the inservice teachers answers on these questions. Both the preservice teacher and the inservice teachers answered the questions either consistently correct or incorrect. The results of these independent samples t-test are shown in Table 4-7.

Table 4-7 Independent samples t-test Summary of Section 2: Mathematical Models

	F	t	Sig.
Q1	.353	.298	.766
Q2	.509	-.362	.718
Q3	2.017	-.716	.475
Q5	.695	-.199	.842
Q6	.137	-.769	.443

For Section 3 of the survey, descriptive statistics were also reported for these eight questions. A score of one is considered a correct response and a score of two is considered an incorrect response to the question. Therefore, a mean score closer to *one* represents an answer that is more correct than incorrect, and a mean score closer to two represents an answer that is more incorrect than correct. The results of the Mathematical Modeling section of the survey include a slightly greater difference in the mean in each category with a difference of .16 at most. Both groups scored very similarly in that all eight of the questions in the mathematical modeling section of the survey showed almost no difference in scores. Table 4-8 shows the mean and the standard deviation of Section 3 of the online survey.

Table 4-8 Descriptive Statistics for Section 3: Mathematical Modeling with Preservice vs. Inservice Teachers

		Mean	Std. Deviation
Q1	Preservice	1.13	.335
	Inservice	1.29	.457
Q2	Preservice	1.70	.460
	Inservice	1.75	.437
Q3	Preservice	1.20	.404
	Inservice	1.25	.437
Q4	Preservice	1.66	.476
	Inservice	1.56	.502
Q5	Preservice	1.18	.387
	Inservice	1.15	.364
Q6	Preservice	1.12	.323
	Inservice	1.13	.345
Q7	Preservice	1.30	.460
	Inservice	1.35	.480
Q8	Preservice	1.47	.502
	Inservice	1.35	.480

The test of homogeneity of variances was met using Lavene's statistic for the independent t-tests. Each of the eight questions in Section 3 of the survey had a significance level greater than p=.05, indicating that the two groups have statistically equal variance. The degrees of freedom for each question are *df*=1, 144. Only one question in this section of the survey showed a statistical significant different between the two groups. There was a significant different in the scores in question 1 for preservice teachers (M=1.13, SD=.335) and inservice teachers (M=1.29, SD=.457) conditions: t(1,144)=5.897, p=.016. However, none of the other individual questions from the mathematical modeling section of the survey showed a statistical significance in the results. Each of the other seven questions indicated there was not a statistical significant different between the preservice teachers answers and the inservice teachers answers on these questions. Both the preservice teacher and the inservice teachers answered the questions either consistently correct or incorrect in questions 2-8. The results of these independent samples t-test are shown in Table 4-9.

Table 4-9 Independent samples t-test Summary of Section 3: Mathematical Modeling

	F	t	Sig.
Q1	22.033	-2.428	.016
Q2	1.599	-.613	.541
Q3	1.697	-.666	.506
Q4	4.072	1.214	.227
Q5	.698	.412	.681
Q6	.374	-.308	.759
Q7	1.316	-.598	.551
Q8	8.179	1.428	.156

The results of the individual questions showed only one question that was had a statistical difference between the preservice and inservice teachers. This information lead the researcher to continue to investigate this data. An average score for the Mathematical Modeling survey was calculated by averaging the answers of each of the questions on the survey, except question number four from Section 2 since it was excluded in the qualitative portion of the this research. A score closer to 1.0 represented answers that were more correct than incorrect. A score of closer to 2.0 represented answers of more incorrect than correct. The descriptive statistics for this analysis are summarized in Table 4-10.

Table 4-10 Descriptive Statistics for the Total Score on the Mathematical Modeling Survey with Preservice vs. Inservice Teachers

		Mean	Std. Deviation
Total score	Preservice	1.38	.170
	Inservice	1.40	.225

The test of homogeneity of variances was met using Lavene's statistic for the independent t-tests. The total score of the survey had a significance level greater than p=.05, indicating that the two groups have statistically equal variance. The degrees of freedom for each question are *df*=1, 144. When exploring the findings in the independent t-test results, it is found that this score does not have a statistical difference between the two groups of preservice teachers and inservice teachers. Both the preservice teacher and the inservice teachers answered the questions either consistently correct or incorrect in the average score. The results of these independent samples t-test are shown in Table 4-11.

Table 4-11 Independent samples t-test Summary for the Total Score on the Mathematical Modeling Survey with Preservice vs. Inservice Teachers

	F	t	Sig.
Total Score	3.847	-.623	.534

After analyzing the results of both the individual scores of the preservice and inservice teachers and the total average score of both groups on the Mathematical Modeling survey, it was determined that the null hypothesis was upheld. There is no difference on the knowledge of mathematical modeling between the preservice teachers and the inservice teachers in this research. Possible explanations of this conclusion will be explored in Chapter 5.

Research Question 2

The second research question focused on different variables in the participant group that included inservice teachers that may have a predictor effect on the knowledge of mathematical modeling. The three variables were grade level taught, years of experience teaching, and type of school in which teaching occurs. There are three null hypotheses for this question which include the following: there is no relationship between completing grade level taught and knowledge of mathematical modeling, there is no relationship between number of years teaching experience and knowledge of mathematical modeling, and there is no relationship between type of school in which teaching occurs and knowledge of mathematical modeling.

Of these original three predictor variables, one was deemed to be unsuitable for a multiple regression analysis because of the numbers of answers in some of the categories. The type of school variable could not be used for analysis, as there were only 7 responses out of 146 total responses selected that were anything other than elementary school. This response was expected because the scope of this research was focused on teachers with an elementary license, which includes grades Kindergarten through 6^{th}. Therefore the factor of type of school was excluded from this multiple regression and the null hypothesis was neither proved nor disproved in the scope of this study.

The remaining two variables were examined for factors within each category. It was determined that the years of teaching experience also had five distinct categories which include the following: teaching for less than 5 years, teaching for 5-10 years, teaching for more than 10 years, 0 years Block 3 and 0 years Block 2. For this research study, Block 3 indicates the preservice teachers' last semester in the elementary education program and Block 2 indicators the second to last semester in the elementary education program. There were originally six categories to select from for this factor which included teaching for less than 5 years, teaching for 5-10 years, teaching for more than 10 years, preservice teacher beginning teaching in the 2017-2018 school year, preservice teacher beginning teaching after the 2017-2018 school year, or currently not teaching elementary mathematics. Only the first 5 categories in the category years of teaching experience were used in this multiple regression, as anyone who selected the last category was exited from the survey as not an elementary teacher of mathematics. The assumptions for this multiple regression were met. The multiple regression analysis details are included in this section.

Years of Experience and Knowledge of Mathematical Modeling

The first multiple regression investigated the influence of the grade level currently taught and the knowledge of mathematical modeling. Five predictor variables were entered as categories into this multiple regression. More than 10 years of teaching experience, more than 5 but less than 10 years teaching experience, less than 5 years teaching experience, 0 years experience and in Block 3, and 0 years and in Block 2 were the five categories. The R^2=. 015 meant that years of teaching experience accounted for less than 1.5% of the variance of the total mathematical modeling scores. This percentage of predictor variable is non-significant. The predictor values in this model were all non-significant. The factor of 0 years-Block 3 was not included in the model. The Beta values, standard errors, and significance values are included in Table 4-12.

Table 4-12 Multiple Regression-Years of Teaching Experience

	B	SE B	Sig.
Constant	1.405	.029	
More than 10 years experience	-.065	.059	.277
Less than 10 but more than 5	.026	.050	.599
Less than 5	1.550E-5	.058	1.00
0 years-Block 2	-.019	.040	.631

Note: R^2= .015
*p<.05

Two Predictor Variables and Knowledge of Mathematical Modeling

The second multiple regression investigated the influence of the grade level currently taught and the years of teaching experience and the teachers' knowledge of mathematical modeling. The R^2= .005 meant that years of teaching experience and grade level currently taught accounted for less than 0.5% of the variance of the total mathematical modeling scores. This multiple regression was analyzed and the Beta values, standard errors, and significance values are included in Table 4-13.

Table 4-13 Multiple Regression-Years of Teaching Experience and Grade Currently Taught at Predictor Values for Knowledge of Mathematical Modeling

	B	SE B	Sig.
Constant	1.435	.052	
Years of Teaching Experience	-.008	.012	.514
Grade Currently Teaching	-.002	.006	.751

Note: R2= .005
*p<.05

After analyzing both of the multiple regression models, it was determined that the two null hypotheses were upheld and that there was no relationship between the number of years of teaching experience and the grade currently taught and knowledge of mathematical modeling.

Research Question 3

The third research question, is there a relationship between preservice and inservice teachers' perceived knowledge of mathematical modeling and their actual knowledge of mathematical modeling, was explored by comparing the participants answer to the question, "Do you understand the term 'Mathematical Modeling?'" with the participants average score on the Mathematical Modeling Survey. The null hypothesis for this question which follows: there is no relationship between preservice and inservice teachers' perceived knowledge of mathematical modeling and their actual knowledge of mathematical modeling. To determine any relationships between these answers, ANOVA was used to explore this data.

The researcher was interested to find out if teachers, both preservice and inservice, that initially identified they understood the term mathematical modeling, had scores to support this understanding. The opposite of this could also be true. Did the teachers, both preservice and inservice, that initially identified they did not understand the term mathematical modeling, have a score to support this lack of understanding? The descriptive statistics of this question show results that n=119 teachers reported that *yes*, they did understand the term mathematical modeling. There were n=4 teachers that answers they did not understanding the term mathematical modeling and n=23 teachers that answered *I don't know* to the question. Table 4-14 summarizes these descriptive statistics.

Table 4-14 Descriptive Statistics Comparing the Preservice and Inservice Perceived Knowledge and Actual Knowledge of Mathematical Modeling

Answer to the question: Do you understand the term mathematical modeling	N	Mean	Standard Deviation
Yes	119	1.400	.191
No	4	1.382	.217
I don't know	23	1.380	.198

The test of homogeneity of variances was met using Lavene's statistic for the independent t-tests. The total score of the survey had a significance level greater than p=.05

indicating that the three groups have statistically equal variance. The degrees of freedom for each question are *df*=2, 143. When exploring the findings in the ANOVA results, it is found that this score does not have a statistical difference between the three groups of answers of all the participants. All three groups of participants received an average score on the mathematical modeling survey from a 1.380-1.400, which is within a difference of 0.02. After analyzing the results of both the preservice and inservice teachers' perceived and actual knowledge of mathematical modeling on the Mathematical Modeling survey, it was determined that the null hypothesis was upheld. There is no different on the knowledge of mathematical modeling between teachers who identify that they do, do not, or do not know if they understand the term mathematical modeling. The results of the ANOVA statistics are shown in Table 4-15.

Table 4-15 ANOVA Results Comparing the Preservice and Inservice Perceived Knowledge and Actual Knowledge of Mathematical Modeling

	Sum of Squares	F	Sig.
Between Groups	.006	.086	.918
Within Groups	5.308		

Summary

This chapter examined the statistical analyses of the data collected in this survey research. After analyzing the independent t-test of the results of both the individual scores of the preservice and inservice teachers and the total average score of both groups on the Mathematical Modeling survey, it was determined that the null hypothesis was upheld. After analyzing both of the multiple regression models, it was determined that the two null hypotheses were upheld and that there was no relationship between the number of years of teaching experience and the grade currently taught and knowledge of mathematical modeling. After analyzing the ANOVA results of both the preservice and inservice teachers' perceived and actual knowledge of mathematical modeling on the Mathematical Modeling survey, it was determined that the null hypothesis was upheld. There is no different on the knowledge of mathematical modeling between teachers who identify that they do, do not, or do not know if they understand the term mathematical modeling. Possible explanations for each of these conclusions are explored in Chapter 5.

Chapter 5 - Conclusions and Implications

Research Summary

There is a proliferation of research related to mathematical modeling in high school and secondary schools, and researchers have a solid grasp on what mathematical modeling entails in the secondary mathematics classroom. When the CCSS (NGACBP, 2010) were published, specific commentary on how to teach mathematical modeling in high school and emphasize this concept throughout the high school standards was included. This is further developed through one of the Standards of Mathematical Practice (SMP), Model with Mathematics. However, the literature and research including elementary teachers and elementary schools is lacking. Most recently, mathematical modeling in elementary schools with elementary teachers has begun to be reflected in the literature (Gould, 2016). This research study will add to the literature on knowledge of mathematical modeling with elementary preservice and inservice teachers.

The purpose of this survey research, tailored design method was to examine the relationship between elementary inservice and elementary preservice teachers' knowledge of mathematical modeling. Additionally, relationships between some predictor variables were examined. This study used non-experimental, survey research to explore the following research questions:

1. Is there a statistical significant difference between inservice and preservice elementary teachers with regard to knowledge of mathematical modeling?
2. Is there is relationship among teachers' knowledge of elementary mathematical modeling and the number of years teaching experience, grade level taught, or type of school in which teaching occurs?

3. Is there a relationship between preservice and inservice teachers' perceived knowledge of mathematical modeling and their actual knowledge of mathematical modeling?

Summary of Findings

The data collected in this research enabled the researcher to answer these three research questions using inferential statistics tests and analysis. There were no statistical differences found between the group of preservice and inservice teachers in regard to knowledge of mathematical modeling. Variables examined to possibly predict the relationship of mathematical modeling knowledge and the predictor variable in this research were found to be non-significant. The amount of teaching experience and the grade level taught were not factors in predicting the knowledge of mathematical modeling by elementary preservice and elementary inservice teachers. There was also no significant relationship found between preservice and inservice teachers' perceived knowledge and actual knowledge of mathematical modeling.

Each of the three research questions and the findings of these research questions are discussed in this chapter. Recommendations for future research are based on the findings in this research study as well as the findings in the literature review. Finally, conclusions for this research study, as well as the conclusion to Chapter 5, are included in this chapter.

Discussion and Conclusions of Research Questions

Research Question #1: Preservice vs. Inservice Elementary Teachers Knowledge of Mathematical Modeling

The study examined if there was a statistical significant difference in the knowledge of mathematical modeling between elementary preservice teachers and elementary inservice teachers. The two groups of teachers, both preservice and inservice, were compared in their knowledge of mathematical modeling. The results indicted that there was not a statistical significant difference in these two groups within the sample of this population. Using the independent t-test, there was no significance in this analysis of the groups. Essentially both groups either answered as accurately as the other group or answered an inaccurately as the other group.

The original assumption of the research was that there would be a significant difference between these two groups. However, this was not the case. These results could be due to several factors. Many of the inservice teachers in this school district also graduated from the university in which the preservice teacher group was selected. This could show that the training in mathematical education has continued to be similar for many years. Another explanation for the similarity is that approximately one-third of these preservice teachers are also placed for their student teacher placement in the same school district that this survey research took place. The school district may offer professional development for both the inservice and preservice group that lead to knowledge or lack of knowledge about mathematical modeling.

A different conclusion could be drawn about the similarities in the preservice and inservice teachers' knowledge of mathematical modeling. According to Jacobson (2017), field experiences, which in this research study are referred to as Block 2 and Block 3, might have implications for how preservice teachers develop mathematical knowledge and beliefs. It may be that the very field experience the preservice teachers are experiencing is leading to the development of the knowledge of mathematical modeling. Further research is required to hone these distinctions in the field.

While it can be determined from this research study that preservice and inservice teachers may have no significant statistical differences between the two groups in terms of mathematical modeling, it would be interesting to determine how this knowledge was gained in both groups.

Does one group have theoretical knowledge while another group has pedagogical knowledge? Additional research studies and literature reviews need to be completed in order to fully examine this new question.

Research Question #2: Predictor Factors in Knowledge of Mathematical Modeling

In this research study, there were some factors identified that could be predictor factors of the knowledge of mathematical modeling. The factors of years of teaching experience were divided into categories and were examined to determine if years of teaching had any relationship on knowledge of mathematical modeling. These factors were examined and multiple regression statistical analysis was run to determine if the null hypotheses were proven or disproven. Next, the factors of both years of experience and current grade level taught were examined in a multiple regression. These were examined in order to determine if either one of these factors could be identified as predictor factors in terms of knowledge of mathematical modeling. The discussion of the two multiple regressions run are examined next.

Years of Teaching Experience

When determining if the years of teaching experience had any predictor factor on this knowledge of mathematical modeling in research question two, all years of experience were considered, even zero years experience. In order to examine this data, the research categorized the years of experience into five different categories. The five categories for years of experience included the following: more than 10 years experience, less than 10 but more than 5 years experience, less than 5 years experience, 0 years experience and enrolled in Block 3, and 0 years experience and enrolled in Block 2. This was purposeful on the researcher's part, as it may have shown that preservice teachers who recently had a mathematical modeling course may have greater knowledge of mathematical modeling. It may also have shown differences in teachers who were trained in their teacher preparation program before and after the CCSS were adopted by many states. However, again, there was not statistical significance in any number of years of experience teaching and knowledge of mathematical modeling.

While there were no significant statistical differences in the categories of years of experience in teaching in this convenience sample population, additional examination of these factors in a wide-scale population sample is warranted. The CCSS have only been adopted for the last seven years. It may be that differences in knowledge are slowly appearing, but in this limited survey research, there was not a large enough population to see any relationships.

Years of Teaching Experience and Current Grade Level Taught

When determining if the years of teaching experience and grade level taught had any predictor factor on this knowledge of mathematical modeling in research question number two, both of these factors were considered together. However, again, there was not a statistical significance difference in any number of years of experience teaching or in currently grade level taught and knowledge of mathematical modeling. For the scope of this study, since there was not statistically significant difference in the years of teacher experience, it seems logical that adding in one more factor does not affect the predictor variable. In this state, the licensure for an elementary teacher is Kindergarten through 6th grade. Therefore, all of these teachers would have had the same or similar training from their elementary preparatory programs.

Research Question #3: Overall Comparison of Perceived Knowledge of Mathematical Modeling

This study also examined if there was a relationship between perceived knowledge of mathematical modeling and actual knowledge of mathematical modeling. There is no statistically significant difference on the knowledge of mathematical modeling between teachers who identify that they do, do not, or do not know if they understand the term mathematical modeling. One of the reasons for that finding may be the size of the groups that answered this question. There were only four participants that answered *no*, they did not know what mathematical modeling was, and only twenty-three participants that answered *I don't' know.* There were 119 participants that answered *yes.* If the entire sample population had been much larger, a more representative sample of this question could have been taken.

Recommendations for Future Study

This study served as an addition to the current literature for mathematical modeling and elementary inservice and preservice teachers. There are a number of implications for future research that could come from this study. The implications for preservice teachers, inservice teachers, and teacher education programs are discussed in the following section.

Implications for Elementary Preservice and Inservice Teachers

Elementary preservice teachers must learn the content of their field and then learn to be able to teach that content during their undergraduate years in an elementary preparatory program. A mix of mathematics content and pedagogy is required when studying to be an elementary teacher. Examining all of the analysis of data from this research, it appears that elementary preservice teachers have similar knowledge of mathematical modeling that the inservice teachers also possess in the district studied. A possible next step in this research could be to determine how both the preservice teacher group and the inservice teacher group acquired this knowledge.

Elementary inservice teachers must constantly keep up with the changing educational landscape. With the CCSS adopted in most states, inservice teachers had to, in a way, relearn how to teach students mathematics. Elementary inservice teachers in this district have similar knowledge of mathematical modeling as the preservice teachers. This is important information for school districts to know as they begin to plan the professional development required throughout the year. Mathematical modeling would not be an area that would need differentiated information for new teachers and current teachers. Since this research determined that preservice teachers, which will soon be the new hires in a school district, have similar knowledge of mathematical modeling.

One topic of mathematical modeling that was identified as an area of need from the data in the survey, is knowledge of what a mathematical model is. Both groups, preservice and inservice teachers, had an average scored closer to incorrect than correct on this question. Additional professional development for inservice teachers and additional instruction for preservice teachers could aid in correcting this misconception.

Implications for Future Research

After examining the knowledge of mathematical modeling of elementary preservice and inservice teachers, more research ideas have come to light. Some of these future studies could include: expanding this study to include additional universities with elementary education programs, expanding the study to specifically look at the math methods course that preservice teachers complete in the undergraduate courses, expanding the study to include several districts in one state, or moving from examination of the knowledge of mathematical modeling to the pedagogically practice of mathematical modeling in the elementary classroom. Determining how this knowledge was acquired, through professional development or theory in their undergraduate coursework may be a next step.

Possible Threats to Validity

In the scope of the data collected in the survey, several independent t-tests were run in SPSS. According to Field (2012), when multiple independent t-tests are analyzed, the probability of finding a statistical difference is expected. Therefore, finding that one of the questions in Section 3 of the mathematical modeling survey had a statistical different is not surprising. These data must be considered in whole to truly understand the overall picture of knowledge of mathematical modeling.

Another possible threat to validity is a limitation of survey research. According to deMarrais and Lapan (2004), it is difficult to ensure through survey research that the respondents are answering truthfully. Respondents certainly could have completed this survey in consultation with other peers or researched answers before responding to the survey. However, through the purposeful design and implementation of this survey, those problems were minimal (deMarrais & Lapan, 2004). Within this survey research, another factor may be the sample size of this study. Although 170 participants clicked on the survey, only 146 completed the survey. For the scope of this survey, 146 participants was an excellent response rate. However, additional participants and an expanded population pool would be needed to truly begin to find statistical differences in the data set.

Conclusions

Knowledge of mathematical modeling by elementary preservice and elementary inservice teachers is an important concept for teachers to develop. This study examined the differences in knowledge of mathematical modeling between preservice and inservice elementary teachers as well as examined any predictor factors that may be present in the data. It was determined, in scope of this survey research study, that there were no significant statistical differences between these two groups. Through multiple regression analysis, it was determined that the identified factors collected though this research did not show a significant statistical differences between or among the factors.

Additional research on this topic could be dedicated to determining the next steps in this process, which would be teachers' understanding of mathematical modeling and teachers' pedagogical practices of mathematical modeling. For preservice teachers examining where the student develop their knowledge of mathematical modeling, either through their math methods course in the elementary preparatory program or through field experiences during student teaching could be an extension of this research. Examining elementary student's knowledge, understanding, and practice of mathematical modeling could continue further study of this topic.

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Appendix A - Mathematical Modeling Survey

Mathematical Modeling Survey

Section 1: Demographics

Mathematical Modeling Survey

Dear Educator,

I plan to conduct research for my Ph.D. on mathematical modeling in elementary classrooms. The title of the research is: Elementary Teachers and Elementary Preservice Teachers' Knowledge of Mathematical Modeling. The purpose of this study is to examine the relationship between inservice and preservice teachers understanding of mathematical modeling. In addition, it will inform the educational community of the importance of purposeful teacher education models at the university level as well as professional development in public schools regarding mathematical modeling. The study will begin December 2016 and end May 2017. It will involve an online survey that should take approximately 5-10 minutes of your time.

We are soliciting your participation because you are a teacher or preservice teacher in grades Kindergarten through 6th grade. Your experience could provide very valuable insight into mathematical modeling in elementary math classroom. Your participation is completely voluntary and you can withdraw at any time. There is no foreseeable risk or harm involved in this participation. Your time is valuable; therefore, the survey will be as brief as possible. The results of the study may be published, but your name will remain confidential and anonymous. If you are interested in participating in this study, please sign the informed consent form by typing your first and last name in the box provided and clicking "yes" to provide informed consent. These records will be kept completely separate from the survey results and will, in no way, be linked to your answers. If you to participate in the study, you may choose to have your name placed into a random drawing to receive one of two $25 Starbucks gift cards at the end of this survey. If you wish not to continue, please click "no".

If you have any questions regarding this study, please contact Sara Schwerdtfeger at 620-341-5409, or email sschwerd@emporia.edu. If you have any concerns regarding your rights as a participant in this study, you can contact the following individuals:

- Rick Scheidt, Chair, Committee on Research Involving Human Subjects, 203 Fairchild Hall, Kansas State University, Manhattan, KS 66506, (785) 532-3224.
- Jerry Jaax, Associate Vice Provost for Research Compliance and University Veterinarian, 203 Fairchild Hall, Kansas State University, Manhattan, KS 66506, (785) 532-3224.

Thank you very much for your consideration.

Best,
Sara Schwerdtfeger, Ph.D. candidate
Department of Curriculum & Instruction
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Department of Curriculum & Instruction
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785-532-8414

❍ Yes, I have read the informed consent and would like to continue to participate in the survey. Please type first and last name in the space provided. ____________________
❍ No, I do not wish to participate in this survey and will be exited from this survey.

Are you currently (within the last two years) a practicing or prospective teacher of mathematics in grade Kindergarten - 6th in the United States?

- ❍ Yes, practicing mathematics teacher for more than 10 years
- ❍ Yes, practicing mathematics teacher for at least 5 and less then 10 years
- ❍ Yes, practicing mathematics teacher for less than 5 years
- ❍ Yes, prospective mathematics teacher and planning to teach at the beginning of the 2017-2018 school year
- ❍ Yes, prospective mathematics teacher and planning to teach after the beginning of the 2017-2018 school year
- ❍ No, I am currently neither teaching mathematics nor planning to teach mathematics

What grade level do you currently teach?

- ❍ Kindergarten
- ❍ 1st
- ❍ 2nd
- ❍ 3rd
- ❍ 4th
- ❍ 5th
- ❍ 6th
- ❍ Combination of several grades
- ❍ Prospective elementary teacher (anticipating elementary teaching in the next year or two)
- ❍ Other: please describe ____________________

In what type of school do you teach?

- ❍ Elementary School
- ❍ Middle School
- ❍ Other: please describe ____________________

Do you understand the meaning of the term "mathematical modeling"?

- ❍ Yes
- ❍ No
- ❍ I don't know

Section 2: MATHEMATICAL MODELS

The questions in this section are intended to determine the level to which you agree with the statements about mathematical models. Mark the option that best matches your opinion. If you do not know, choose “Don’t Know.”

	Completely Disagree (1)	Somewhat Disagree (2)	Neither Agree Nor Disagree (3)	Somewhat Agree (4)	Agree (5)	I Don't Know (6)
Mathematical models can be physical manipulatives, for example, fraction tiles, pattern blocks, or linking cubes.	❍	❍	❍	❍	❍	❍
Mathematical models can be equations or formulas, for example, base x height = area, or the formula for perimeter.	❍	❍	❍	❍	❍	❍
Mathematical models can be visual representations such as a graph in the coordinate plane or a number line.	❍	❍	❍	❍	❍	❍
Mathematical models can be	❍	❍	❍	❍	❍	❍

visual representations such as an elevation map of a mountain or a scientific scale drawing.						
Mathematical models can be used to describe or summarize a given situation in compact form.	❍	❍	❍	❍	❍	❍
Mathematical models can be used to explain the underlying causes in a given situation.	❍	❍	❍	❍	❍	❍

Section 3: MATHEMATICAL MODELING

The questions in this section are intended to determine the frequency with which the given characteristic is part of the mathematical modeling process in your estimation. If you don't know, choose "Don't Know."

Options include: completely agree, somewhat agree, neither agree nor disagree, somewhat disagree, complete disagree, I don't know.

	Completely Disagree (1)	Somewhat Disagree (2)	Neither Disagree Nor Agree (3)	Somewhat Agree (4)	Completely Agree (5)	I Don't Know (6)
Repeating steps is part of the mathematical modeling process.	❍	❍	❍	❍	❍	❍
Mathematical modeling situations come from "whimsical" or unrealistic scenarios.	❍	❍	❍	❍	❍	❍
The mathematical modeling process involves making choices.	❍	❍	❍	❍	❍	❍
The mathematical modeling process involves making assumptions.	❍	❍	❍	❍	❍	❍
The mathematical modeling	❍	❍	❍	❍	❍	❍

process involves determining if a solution makes sense in terms of the original situation.						
The mathematical modeling process involves making revisions.	❍	❍	❍	❍	❍	❍
The mathematical modeling process results in an exact answer or exact answers.	❍	❍	❍	❍	❍	❍
A mathematical modeling situation can result in various, different mathematical models.	❍	❍	❍	❍	❍	❍

Thank you for participating in this survey. If you would like to be entered for a chance to win one of two $25 Starbucks gift cards, please click yes and enter your email address in the space provided. This information will not be connected in any way with your answers in the survey.

- ❍ Yes. Please enter your email address in which we can contact you to receive your gift card if you are selected. ____________________
- ❍ No, I do not wish to be entered into the drawing.

Appendix B - Summary of Expert Answers to Survey

Reported by Gould (2013)

Question	Summary of Expert Opinions
Mathematical models can be physical manipulatives, for example, fraction tiles, pattern blocks, or three-dimensional solids (like cubes, octahedra, and other polyhedra).	The majority of experts disagreed.
Mathematical models can be equations or formulas, for example, a quadratic equation or d = rt, the distance-rate formula.	The experts unanimously completely agreed.
Mathematical models can be visual representations such as a graph in the Cartesian plane or the real number line.	The experts unanimously completely agreed.
Mathematical models can be visual representations such as a scaled map of the county or an architectural blueprint.	The experts did not form a consensus opinion.
Mathematical models can be used to describe or summarize a given situation in a compact form.	The majority of experts completely agreed.
Mathematical models can be used to explain the underlying causes in a given situation.	The majority of experts somewhat agreed.
Repeating steps is part of the mathematical modeling process.	The majority of experts responded that this is usually the case.
Mathematical modeling situations come from "whimsical" or unrealistic scenarios.	The majority of experts responded that this is never the case.
The mathematical modeling process involves making choices.	The majority of experts responded that this is always the case.
The mathematical modeling process involves making assumptions.	The experts unanimously responded that this is always the case.
The mathematical modeling process involves determining if a solution makes sense in terms of the original situation.	The experts unanimously responded that this is always the case.
The mathematical modeling process involves making revisions.	The experts responded that this occurs at least half the time (between half the time and usually).
The mathematical modeling process results in an exact answer or exact answers.	The majority of experts responded that this occasionally the case.
A mathematical modeling situation can result in various, different mathematical models.	The majority of experts responded that this is usually the case.

Appendix C - Informed Consent Document

Informed Consent

December 2016

Dear Educator,

I plan to conduct research for my Ph.D. on the understanding of mathematical modeling in elementary classrooms. The title of the research is: *Elementary Teachers and Elementary Preservice Teachers' Understanding of Mathematical Modeling*. The purpose of this study is to examine the relationship between inservice and preservice teachers understanding of mathematical modeling. In addition, it will inform the educational community of the importance of purposeful teachers education models at the university level as well as professional development in public schools regarding mathematical modeling. The study will begin December 2016 and end May 2017. It will involve an online survey that should take approximately 10-15 minutes of your time.

We are soliciting your participation because you are a teacher or preservice teacher in grades Kindergarten through 6th grade. Your experience could provide very valuable insight into mathematical modeling in elementary math classroom. Your participation is completely voluntary and you can withdraw at any time. There is no foreseeable risk or harm involved in this participation. Your time is valuable; therefore, the survey will be as brief as possible. The results of the study may be published, but your name will remain confidential and anonymous. If you are interested in participating in this study, please sign the informed consent form by typing your first and last name in the box provided and clicking "yes" to provide informed consent. These records will be kept completely separate from the survey results and will, in no way, be linked to your answers. If you choose to participate in the study, your name will be placed into a random drawing to receive one of two $25 Starbucks gift cards. If you wish not to continue, please click "no".

If you have any questions regarding this study, please contact Sara Schwerdtfeger at 620-341-5409, or email sschwerd@emporia.edu. If you have any concerns regarding your rights as a participant in this study, you can contact the following individuals:

- Rick Scheidt, Chair, Committee on Research Involving Human Subjects, 203 Fairchild Hall, Kansas State University, Manhattan, KS 66506, (785) 532-3224.

- Jerry Jaax, Associate Vice Provost for Research Compliance and University Veterinarian, 203 Fairchild Hall, Kansas State University, Manhattan, KS 66506, (785) 532-3224.

Thank you very much for your consideration.

Best,

Sara Schwerdtfeger, Ph.D. candidate
Department of Curriculum & Instruction
College of Education
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Dr. Sherri Martinie, Assistant Professor
Department of Curriculum & Instruction
College of Education
Kansas State University

785-532-8414

Informed Consent Form

___Yes, I __________________ have read the informed consent and am interested in participating in the study entitled, *Elementary Teachers and Elementary Preservice Teachers' Understanding of Mathematical Modeling.*

___No, I do not wish to participant in the study entitled*, Elementary Teachers and Elementary Preservice Teachers' Understanding of Mathematical Modeling* and will be exited from this survey.

Appendix D - Initial Email to Participants

Thank you for considering participating in this important research on mathematical modeling in elementary classrooms. The title of the research is: *Elementary Teachers and Elementary Preservice Teachers' Understanding of Mathematical Modeling*. The purpose of this study is to examine the relationship between inservice and preservice teachers' understanding of mathematical modeling. This online survey should take approximately 10-15 minutes of your time.

Click on the following link to continue to the survey. At the end of the survey, you will be asked to enter your name and contact information if you wish to be included in a chance to received one of two $25 Starbucks gift cards as an incentive to complete this survey. This information will be kept separate from your survey answers and will in no way connect your identity to your survey answers.

Appendix E - Follow up Email to Participants

A few weeks ago, you received an email asking for your consideration to agree to participate in this important research on mathematical modeling in elementary classrooms. The title of the research is: *Elementary Teachers and Elementary Preservice Teachers' Understanding of Mathematical Modeling*. The purpose of this study is to examine the relationship between inservice and preservice teachers understanding of mathematical modeling. This online survey should take approximately 5-10 minutes of your time. If you already completed this survey, thank you so much for your time. If you have not had time to take the survey and would like to, please follow the directions at the end of this email.

Click on the following link to continue to the survey. At the end of the survey, you will be asked to enter your name and contact information if you wish to be included in a chance to received one of two $25 Starbucks gift cards as an incentive to complete this survey. This information will be kept separate from your survey answers and will in no way connect your identity to your survey answers.

Appendix F - Final Email to Participants

Thank you for taking the time to participate in this research survey on mathematical modeling in elementary schools.

Results of this study will be available from the researcher, Sara Schwerdtfeger, by August 2017. You may contact her at sschwerd@emporia.edu to receive a copy of the final report. All data collected, including participant names, will remain confidential.

You may contact my dissertation advisor, Dr. Sherri Martinie at martinie@ksu.edu for any additional questions about this research and to receive confirmation the incentives of two $25 Starbucks gift cards were awarded to two different participants. You may also contact the Kansas State University Institutional Review Board chair, Dr. Rick Scheidt or the Research Involving Human Subjects chair, Dr. Jerry Jax with questions about the process of this research. Both may be contacted at 203 Fairchild Hall, Kansas State University, Manhattan, KS 66506 and the phone number is 785-532-3224.