OPTIMAL SCHEDULED MAINTENANCE POLICY BASED ON MULTIPLE CRITERIA DECISION MAKING

by

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CHAPTER 1. INTRODUCTION

This study is to determine the optimal scheduled maintenance policy that performing a preventive replacement once the critical item has reached a specified age in addition to failure replacement when necessary. The replacement action returns the item to the as new condition, thus continuing to provide the same service as the item just been replaced. This assumption implies that various costs, failure distribution etc. used in the analysis are always the same.

A variety of criteria have been used for finding the optimal replacement age. In the conventionally approaches, usually only one criterion was considered. However, in a real life, the decision maker always wants to attain more than one objective or goal in selecting the course of action. Then, the problem becomes how to determine the preventive replacement age of a critical item subject to multiple criteria.

The objectives of this study, therefore are: (i) to formulate the model of a replacement problem subject to multiple criteria. (ii) to solve this problem by using several multiple objective decision making methods.

In Chapter 2, the concepts of planned replacement are introduced. Among the topics discussed are: the classification of maintenance models, the conditions required to make preventive replacement worthwhile, factors influence the preventive replacement age, the needs of multiple objective decision making.

In Chapter 3, taking aircraft as an example, the problem is to determine the preventive replacement age of the critical item (e.g. an

engine). Mathematical models have been developed for four criteria: (1) the replacement cost per unit time, (2) the availability of the critical item, (3) the mission reliability, and (4) the expected cost of failure during the mission.

In Chapter 4, solutions are found by using four methods for multiple objective decision making: (1) the stricted selection method, (2) the lexicographic method, (3) Waltz's lexicographic method, and (4) the sequential multiple objective problem solving technique (SEMOPS). The decision process, the implications, the advantages and disadvantages of each method are discussed in the chapter.

In Chapter 5, the present study is briefly summarized. The extensions of the present study to the multiple criteria problem with multiple decision variables and to the group replacement problem are proposed.

CHAPTER 2. CONCEPTS OF PLANNED REPLACEMENT

2.1 Planned replacement

According to the surveys made by Jorgenson and McCall [9] and McCall [14], models of stochastically failing equipment fall into two classes. In 'preparedness' models the equipment is subject to random failures and the actual state of the equipment — good or failed — is known only with some uncertainty. The corresponding physical situation is that equipment is required for an emergency, but the equipment is not operated until the emergency occurs. In 'preventive maintenance' models, the equipment is subject to random failures, but the equipment is operated continuously so that the actual state of the equipment is always known with certainty.

Under different knowledge about the distribution of times to failure (the failure distribution is known, the form of the distribution is known but the failure rate is unknown, and the failure distribution is unknown etc.) the models can be further classified by the maintenance procedures implemented into periodic policy, sequential policy and opportunistic policy.

The periodic policy in 'preventive maintenance' models with failure distribution known has been studied extensively [1,9]. It is noted that the maintenance actions such as repair and overhaul can be considered to be equivalent to replacement provided that it is reasonable to assume that such actions also return the equipments to the as new condition.

Since the earliest study on the planned-replacement problem was made by Campbell [5] in 1941, many have published on this topic. The basic problems which were discussed and analyzed are: (1) what is a valid and good criterion to find the optimal replacement age? (2) when is the planned replacement?

Several investigators have demonstrated [1,4,6,16] (Some with more and some with less mathematical rigor) that when a measure is selected as a criterion, two conditions are required to make planned replacement potentially worthwhile. The first is that the preventive replacement of a item must 'cost' less, in some sense, than an failure replacement. The second condition is that the failure characteristic of the item must display 'wearout'; i.e. the failure rate must increase with age.

2.2 Factors influence the replacement age

The following two factors make the replacement models different:

(1) the measure selected as a criterion, and

(2) the failure characteristic of the item.

2.2.1 Criterion for assessing the replacement age

The following typical criteria have been used for finding the optimal replacement age.

(1) Expected cost per unit of time

The cost of a preventive replacement and the cost of a failure replacement are considered. The objective is to determine the optimal replacement age of the equipment to minimize the total expected replacement cost per unit time.

(2) Maintenance downtime

Barlow, Hunter, and Proschan [3] first considered maintenance downtime, and derived an integral equation leading to the planned maintenance interval which minimizes this time.

(3) Availability

Availability is defined [13] as the fraction of the total desired operating time that the system or item is actually operable. It is the principal measure of the effectiveness of maintained systems. The objective is to determine the replacement age of the equipment to maximize the availability or to maintain above a specific level.

(4) Mission reliability

Mission reliability is the probability that the system or item will operate properly without a failure for a given interval of mission duration, given that it was opererable at the beginning of the mission. In other words, it represents the conditional probability of nonfailure of the system for a period of time required to complete a mission. Ladany and Aharoni [12] consider it as a criterion to determine the optimal replacement age of a item that there is a requirement for a standard minimal value of mission reliability to be fulfilled by the item in a mission of specific length.

(5) Average effective earning

In conditions of industrial production each equipment needs to earn a revenue. The net revenue rate is measured as the value of output per time unit less the cost of producing that output, exclusive of maintenance cost. Kay [10] maximizes the average effective earning to determine the optimal schedule period.

(6) Expected maximum failure cost during the mission

Some complex systems are so expensive or a mission is so important or a system failure will involve so many lives that we cannot afford to have a major failure during a mission. Although the probability of occurring a major accident is very low, minimizing the expected failure cost during the mission or being below a specific tolerable value is usually desired. Ladany and Aharoni [12] defined the expected inflight cost for an item with specific mission length is the product of the inflight failure cost (including loss of lives, damage to the whole aircraft, etc.) and the probability that the item will fail during the mission.

2.2.2 Failure characteristic

One of the prerequisites for a planned-replacement program is that the failure distribution has a wearout characteristic, or equivalently, that the failure rate increases with age.

A bulk of available failure analysis indicates that the exponential failure distribution is by far the most common of all distributions. Consequently, we know that a large class of devices is unsuitable for planned replacement.

However, many researches [4,11,17] disclosed that a significant fraction of electrical and mechanical parts, especially that of commercial or military aircraft, displayed a normal, logarithmic-normal or Weibull types of failure distribution. With increased discrepancy of actual failure

distribution from the exponential failure distribution and using maintenance policy based on exponential failure distribution, the results of these kind of parts failure become more critical. Therefore, in this study, Weibull distribution will be used in constructing replacement models.

2.3 The need of multiple objective decision making

In the previous section, several typical criteria have been described. Although they are not exhaustive, they demonstrate that there are many criteria that can be used to determine the replacement age. Traditionally, only one criterion has been considered. The optimal replacement age for one criterion is surely not an optimal solution for other criterion. At most of the cases, the solution is even not a satisfactory solution if other criteria were also considered. However, in a real life, the decision maker (DM) always wants to attain more than one objective or goal in selecting the course of action. One of the major reasons for the scarcity of multiple objective formulation and consideration in literature is that until recently, almost all the solution strategies developed involved a single objective function.

The growing tendency to incorporate more and diverse criteria in a system design and the resulting difficulty in consolidating such criteria have provided an impetus to the development of multiple criteria methodology in various forms.

In recent years, multiple objective analysis has been applied to a wide variety of problems [7] including water resources management, econometrics and development planning, academic planning, capital budgeting, financial

planning, land use planning, manpower planning, media planning, public administration, transportation planning, and many others. However, no paper has been found applying multiple-objective decision making methods in the field of reliability analysis and maintenance policy making.

In this study, a replacement age problem subject to multiple criteria is developed and four different kind of multiple objective decision making methods are introduced and applied to solve the problem.

CHAPTER 3. STATEMENT OF THE PROBLEM AND MODEL DEVELOPMENT

3.1 Statement of the problem

For every piece of military or commercial equipment, there exists critical items whose failure could result in the shutdown of the whole equipment or the hazardous conditions for individuals using the equipment or the unaccomplishment of the tactical functions of the equipments. As a consequence, it is desirable to use multiple criteria in determining the replacement age of such critical items. The following four criteria are most likely to be considered:

(1) the replacement cost per unit of time

(2) the availability of the equipment

(3) the reliability of mission accomplishment

(4) the expected cost of mission failure

Reliance on a single criterion might cause heavy damage or high opportunity cost.

In determining the maintenance policy of the critical item of a aircraft, Ladany and Aharoni [12] selected the strictest result (i.e. the lowest replacement age) as the optimal replacement age by considering three criteria: (1) the replacement cost per unit of time, (2) the mission reliability of the equipment, and (3) the inflight failure cost. However, the following deficiencies exists in their model and the decision making method:

(1) The principal measure of the effectiveness of a maintained system, availability, is not considered as a criterion in the model.

(2) The equation of the replacement cost per unit of time did not take account of replacement times of the preventive and the failure replacements.

(3) Using the strictest selection method, the decision maker is essentially detached from the decision process.

In this study, in addition to the strictest selection method, several methods for multiple objective decision making are used for obtaining the optimal scheduled maintenance policy based on four criteria.

3.2 Model development

The problem is to determine the preventive replacement age for a critical item of a complex system (e.g. an aircraft) based on the following multiple criteria:

(1) the replacement cost per unit time

(2) the mission reliability of a complex system

(3) the expected mission failure cost

(4) the availability of the item

Notations:

$A(t_p)$ = the availability of the critical item

C_1 = the cost of a preventive replacement

C_2 = the cost of a failure replacement

C_3 = the cost of a mission failure (including loss of lives, damage to whole system etc.)

$C(t_p)$ = the total expected replacement cost per unit of time

E_f = the expected mission failure cost

$f(X)$ = the failure density function of the critical item

$F(X)$ = the unreliability, $F(X) = 1 - R(X)$

$F(X,H)$ = the probability of failure between X and X+H, given it is not failed at X

H = the mission length

$R(X)$ = the reliability, $R(X) = \int_X^{\infty} f(X)dt$.

$R(X,H)$ = the mission reliability, $R(X,H) = 1 - F(X,H)$

t_1 = the mean replacement time for the preventive replacement

t_2 = the mean replacement time for the failure replacement

t_p = the preventive replacement age

Assuming that the planning horizon is very long, and that an item which is replaced behaves like a new item with zero length of life, the above mentioned four criteria are evaluated as follow:

(1) Replacement cost per unit of time

The replacement policy is to perform a preventive replacement once the critical item has reached a specified age t_p, and a failure replacement when it is necessary. This policy is illustrated in Fig. 3.1.

There are two possible cycles of operation:

One cycle being determined by the critical item reaching its preventive replacement age t_p, the other being determined by the failure occuring before the preventive replacement age. These two possible cycles are illustrated in Fig. 3.2. The total expected replacement cost per unit time, $C(t_p)$, is:

$$C(t_p) = \frac{\text{total expected replacement cost per cycle}}{\text{expected cycle length}}$$

Total expected replacement cost per cycle

= cost of a preventive replacement cycle · probability of a preventive replacement cycle + cost of a failure replacement cycle · probability of a failure replacement cycle

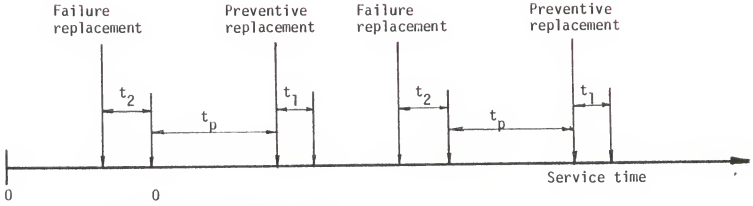
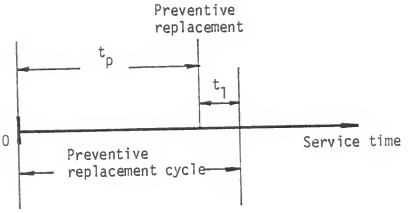


Fig. 3.1 Replacement policy



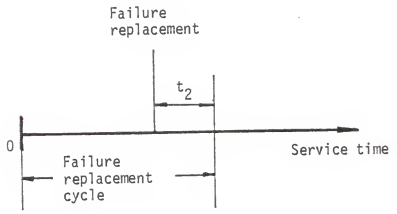


Fig. 3.2 Preventive and failure replacement cycle

$$= C_1 R(t_p) + C_2 F(t_p)$$

Expected cycle length

= length of a preventive replacement cycle . probability of a preventive replacement cycle + expected length of a failure replacement cycle . probability of a failure replacement cycle

$$= (t_p + t_1)R(t_p) + (\text{expected length of a failure replacement cycle})\ F(t_p)$$

If a preventive replacement occurs at time t_p, then the mean time to failure is the mean of the shaded portion of Fig. 3.3. Since the unshaded area is an impossible region for failure, the mean of the shaded area is:

$$\frac{\int_0^{t_p} tf(t)dt}{1 - R(t_p)}$$

Then:

the expected length of a failure replacement cycle

$$= \frac{\int_0^{t_p} tf(t)dt}{1 - R(t_p)} + t_2 = \frac{\int_0^{t_p} tf(t)dt}{F(t_p)} + t_2$$

the expected cycle length

$$= (t_p + t_1)\ R(t_p) + \left(\frac{\int_0^{t_p} tf(t)dt}{F(t_p)} + t_2 \right) F(t_p)$$

$$= t_p R(t_p) + t_1 \cdot R(t_p) + \int_0^{t_p} tf(t)dt + t_2 F(t_p)$$

Since

$$\int_0^t R(x)dx = t \cdot R(t) + \int_0^t tf(t)dt$$

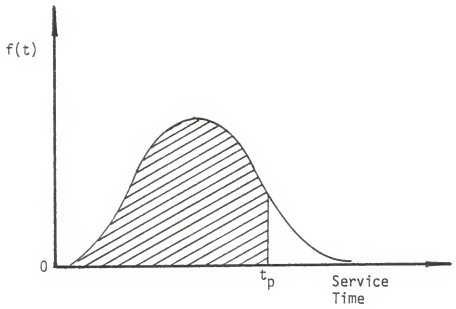


Fig. 3.3. Mean time to failure

The expected cycle length = $\int_0^{t_p} R(t)dt + t_1R(t_p) + t_2F(t_p)$

Then

$$C(t_p) = \frac{C_1R(t_p) + C_2F(t_p)}{\int_0^{t_p} R(t)dt + t_1R(t_p) + t_2F(t_p)} \tag{3.1}$$

This relates the preventive replacement age, t_p, to the total expected replacement cost per unit time.

(2) Availability of the critical item

Availability is a principal measure of the effectiveness of a maintained system. Availability is defined [13] as the fraction of the total desired operating time that the system is actually operable, or it can be defined as the ratio of uptime to total time.

When the service time of the critical item is considered, the availability of the item, $A(t_p)$, is:

$$A(t_p) = \frac{\text{expected operating length in a cycle}}{\text{expected cycle length}}$$

The expected operating length in a cycle

= operating length in a preventive cycle · probability of a preventive cycle + operating length in a failure cycle · probability of a failure cycle

$= t_pR(t_p) + \int_0^{t_p} tf(t)dt$

$= \int_0^{t_p} R(t)dt$

as the expected cycle length

$$= \int_0^{t_p} R(t)dt + t_1 R(t_p) + t_2 F(t_p)$$

The availability is:

$$A(t_p) = \frac{\int_0^{t_p} R(t)dt}{\int_0^{t_p} R(t)dt + t_1 R(t_p) + t_2 F(t_p)} \qquad (3.2)$$

This is a measure of the critical item's availability because it gives the probability the system will be available for operation if it is in an ideal support environment without consideration for downtime other than t_1 and t_2, such as administrative time, supply time etc.

(3) Mission reliability

A mission is an interval of activity of length H which starts at age X of the item and ends at age X+H. During the interval of length H, the equipment, say aircraft, may even perform more than one flight, but all flights belong to the same mission (e.g., a commercial flight with landings for refuelling).

The conditional probability of a failure during a mission of length H of a critical item given it is operable at age X, F(X,H), is the probability that it will fail between X and X+H, divided by the probability it does not fail until X, i.e.,

$$F(X,H) = \frac{\int_0^{X+H} f(t)dt}{R(X)} = \frac{F(X+H) - F(X)}{R(X)} = \frac{1-R(X+H) - 1+R(X)}{R(X)}$$

$$= 1 - \frac{R(X+H)}{R(X)}$$

Then, the mission reliability, which is the probability for the item to succeed during the mission of length H, if it started at age X is

$$R(X,H) = 1-F(X,H) = \frac{R(X+H)}{R(X)} \tag{3.3}$$

Reliability is a monotonic decreasing function of time. The mission reliability, R(X,H), will be the minimum at the preventive replacement age, that is $R(t_p,H)$.

(4) Expected mission failure cost

When a system is very expensive, or a mission is very important, or a system failure will involve many lives, it cannot afford to have a major failure during a mission. The cost of a mission failure includes the loss of lives, the damage to whole system etc. The expected mission failure cost during a mission of length H, E_f, is the product of the mission failure cost and the probability that the item starting the mission at age X will fail during the mission; i.e.

$$E_f = C_3 \cdot F(X,H) = C_3\left[1 - \frac{R(X+H)}{R(X)}\right] \tag{3.4}$$

Unreliability is a monotonic increasing function of time. The mission unreliability, F(X,H), will be the maximum at the preventive age, that is $F(t_p,H)$.

3.3 Case of Weibull distribution

When the failure of an item is distributed according to the Weibull distribution, the failure density function, the failure function, the

reliability function and the hazard function, respectively, as found in Von Alven [18] are:

$$f(t) = \frac{B}{A} t^{B-1} \exp(-\frac{1}{A} t^{B}) \tag{3.5}$$

$$F(t) = 1 - \exp(-\frac{1}{A} t^{B}) \tag{3.6}$$

$$R(t) = \exp(-\frac{1}{A} t^{B}) \tag{3.7}$$

and

$$Z(t) = \frac{B}{A} t^{B-1} \tag{3.8}$$

where A and B are parameters of the distribution which has a mean of

$$E(t) = A^{\frac{1}{B}} \Gamma(\frac{1}{B} + 1)$$

and a variance of

$$V(t) = A^{\frac{2}{B}} \left\{ \Gamma(1 + \frac{2}{B}) - [\Gamma(\frac{1}{B} + 1)]^2 \right\}$$

Substituting eqs. (3.6), (3.7) into eq. (3.1), we obtain the replacement cost per unit of time as

$$C(t_p) = \frac{C_1 \exp(-\frac{1}{A} t_p^B) + C_2[1 - \exp(-\frac{1}{A} t_p^B)]}{\int_0^{t_p} \exp(-\frac{1}{A} t_p^B) dt_p + t_1 \exp(-\frac{1}{A} t_p^B) + t_2[1-\exp(1 - \frac{1}{A} t_p^B)]} \tag{3.9}$$

Substituting eqs. (3.6) and (3.7) into eq. (3.2) for the availability model, we get

$$A(t_p) = \frac{\int_0^{t_p} \exp(-\frac{1}{A} t_p^B) dt_p}{\int_0^{t_p} \exp(-\frac{1}{A} t_p^B) dt_p + t_1 \exp(-\frac{1}{A} t_p^B) + t_2[1 - \exp(-\frac{1}{A} t_p^B)]} \tag{3.10}$$

Substituting eq. (3-7) into eq. (3.3) for the case of the mission reliability model, we get

$$R(t_p,H) = \frac{\exp[-\frac{1}{A}(t_p + H)^B]}{\exp(-\frac{1}{A} t_p^B)} \tag{3.11}$$

Substituting eq. (3.7) into eq. (3.4) for the case of the expected mission dissaster cost case, we get

$$E_{t_p} = C_3 \cdot F(t_p\ H) = C_3\left\{1 - \frac{\exp[-\frac{1}{A}(t_p + H)^B]}{\exp(-\frac{1}{A} t_p^B)}\right\} \tag{3.12}$$

Numerical example

A realistic numerical example is taken from Bell, Kamins & McCall [3]. An aircraft engine is found to have failure characteristics which are closely represented by a Weilbull distribution with $A = 2.6954 \times 10^9$, $B = 3.0$.

It is also reasonable to assume the following numerical values:

C_1 = \$25,000

C_2 = \$37,500

C_3 = \$2,500,000

t_1 = 8 hrs

t_2 = 16 hrs

H = 16 hrs

The figures of failure density function, the failure function, the reliability function and the hazard function are shown in Figs. 3.4, 3.5, 3.6 and 3.7, respectively.

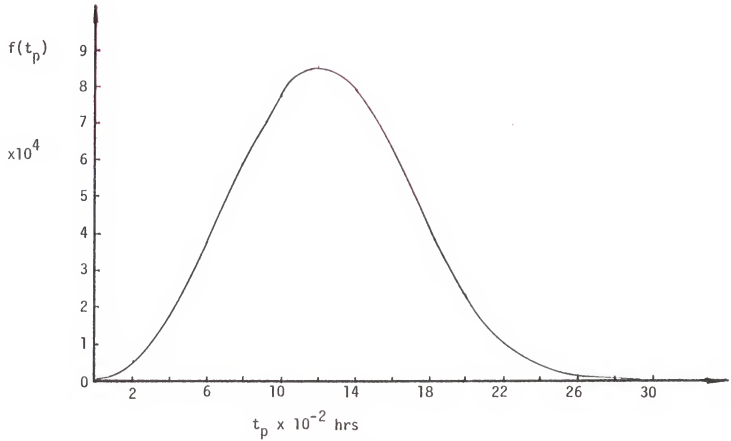


Fig. 3.4 The failure density function

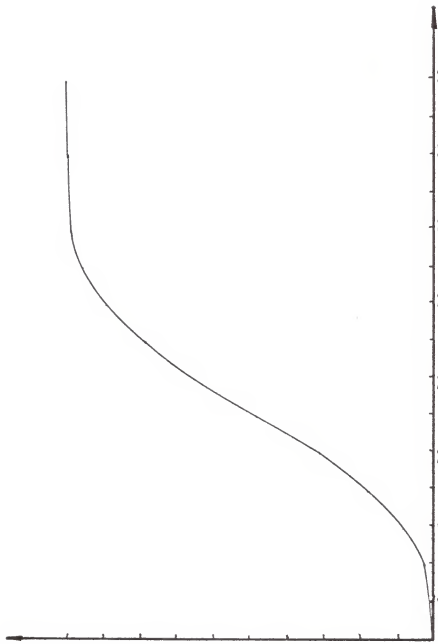


Fig. 3.5 The failure function

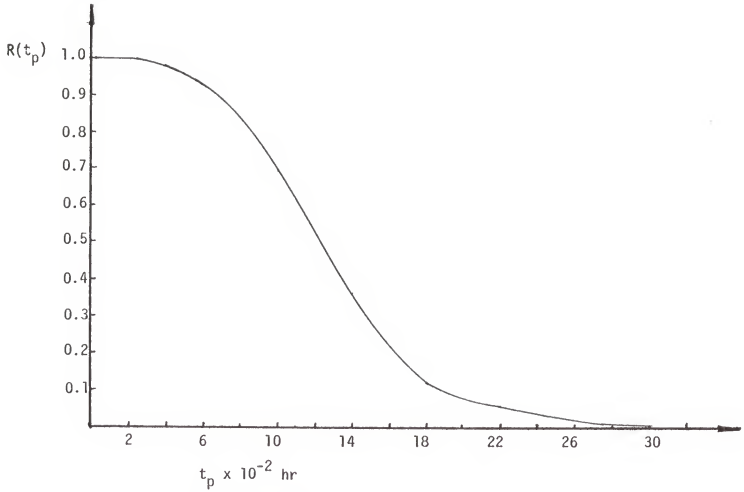


Fig. 3.6 The reliability function

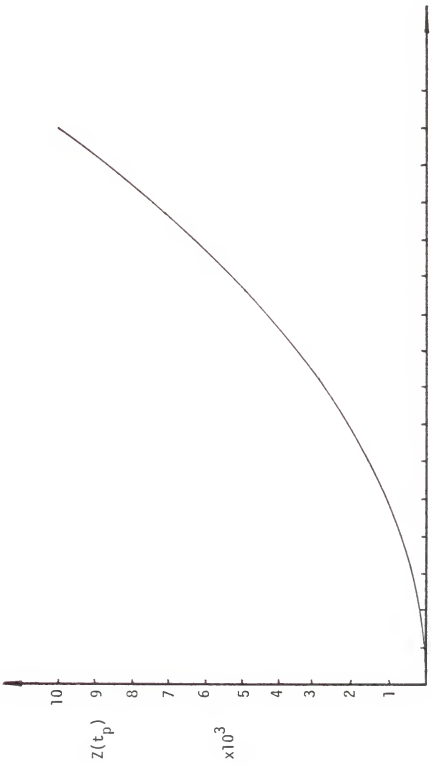


Fig. 3.7 The hazard function

CHAPTER 4. REPLACEMENT AGE OF THE CRITICAL ITEM SUBJECT TO MULTIPLE CRITERIA

In this chapter, the optimum replacement ages of a critical item for an aircraft subject to multiple criteria are obtained by several methods. One method (the strictest selection method) was proposed by Ladany and Aharoni [12]. The other newly developed methods for multiple objective decision making are applied to this field for the first time.

The numerical example shown in section 3.3 is repeatly used to demonstrate the decision making methods. The results obtained from these methods are compared and discussed.

4.1 The strictest selection method

With multiple criteria, it will provide the critical item with different preventive replacement ages for each criterion. Ladany and Aharoni [12] proposed that the optimality of maintaining an aircraft requires the application of the strictest maintenance policy, i.e., the criterion of the shortest preventive replacement age will be applied.

To apply the strictest selection method, the optimal preventive replacement age for each criterion has to be found out first. By the models developed in chapter 3, the optimal preventive replacement ages for the four criteria are found as follow:

(1) Replacement cost per unit of time

The minimal replacement cost per unit of time is obtained by equalling the first derivative of eq (3.1) to zero. The resulting condition for the optimal preventive replacement time, t_p^*, is

$$Z(t_p^*) \int_0^{t_p^*} R(t)dt - F(t_p^*) = \frac{C_1}{C_2-C_1} - Z(t_p^*) \left[\frac{C_2t_1-C_1t_2}{C_2-C_1}\right] \tag{4.1}$$

where $Z(t_p)$ is the hazard function. In the case of Weibull distribution, t_p^* is obtained when the following equation is fulfilled.

$$\frac{B}{A} t_p^{*B-1} \left(\int_0^{t_p^*} \exp(-\frac{1}{A} t_p^B)dt_p + \frac{C_2t_1-C_1t_2}{C_2-C_1} \right) - 1 + \exp(-\frac{1}{A} t_p^{*B}) - \frac{C_1}{C_2-C_1} = 0 \tag{4.2}$$

With the numerical example given in section 3.3, the curve of replacement cost per unit of time $C(t_p)$ is shown in Fig. 4.1. The optimal solution is found by using a numerical integration and the golden section method. The optimal preventive replacement age of this specific numerical example is 1455 hours.

(2) Availability of the critical item

Using availability as a criterion for deciding the preventive replacement age of the item, either requires to find a t_p value that maximize the availability or to find a heighest t_p value satisfying a standard minimum value $A(X)_{min}$; i.e.,

$$A(X)_{min} \leq A(t_p)$$

In the case of maximizing the availability, the optimal preventive replacement time t_p^* is obtained again by equating the first derivative of eq. (3.2) to zero. The resulting condition for t_p^* is

$$Z(t_p^*) \int_0^{t_p^*} R(t)dt = \frac{t_2}{t_2-t_1} - R(t_p^*) \tag{4.3}$$

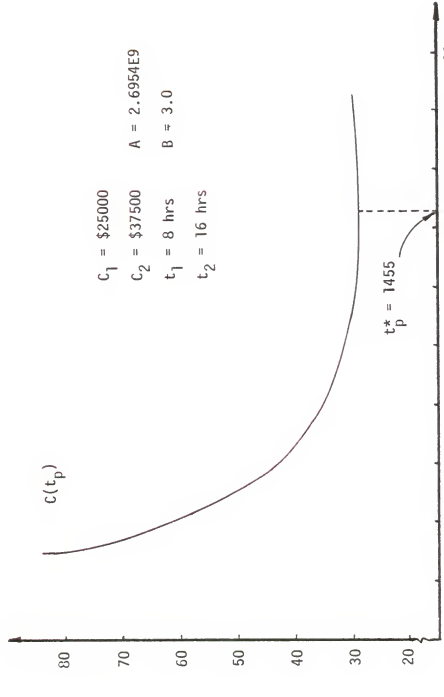


Fig. 4.1 The curve of replacement cost per unit of time

where $Z(t_p)$ is the hazard function.

For Weibull distribution, the optimal preventive replacement age t_p^* is obtained when the following equation is fufilled:

$$\frac{B}{A} t_p^{*B-1} \int_0^{t_p^*} \exp(-\frac{1}{A} t_p^B) dt_p = \frac{t_2}{t_2-t_1} - \exp(-\frac{1}{A} t_p^{*B}) \tag{4.4}$$

With the numerical example given in section 3.3, the curve of availability of the item is shown in Fig. 4.2. The optimal preventive replacement age of this specific numerical example is 1129 hours.

(3) Mission reliability

It is desired that the probability of failure of the critical item will not exceed the maximum acceptable risk. So as a criterion for selecting the preventive replacement age of the item, it usually requires the item to fulfill a standard minimal value $R(X,H)_{min}$ in a mission of length H. Then $R(X,H)_{min}$ becomes the limiting condition for determining t_p^* for the critical item. It is required that

$$R(X,H)_{min} \leq \frac{R(t_p^*+H)}{R(t_p^*)} \tag{4.5}$$

and the highest value of t_p^* fulfilling about equation should be chosen. For the numerical example with Weibull distribution given in section 3.3, the curve of mission reliability of the item is shown in Fig. 4.3. Setting $R(X,H)_{min}$ at 0.985, the optimal preventive replacement age of this specific numerical example is 913 hours.

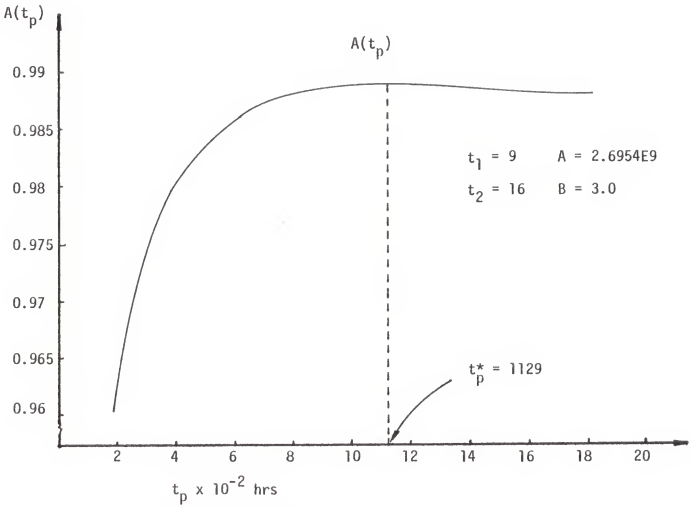


Fig. 4.2 The curve of availability of the equipment

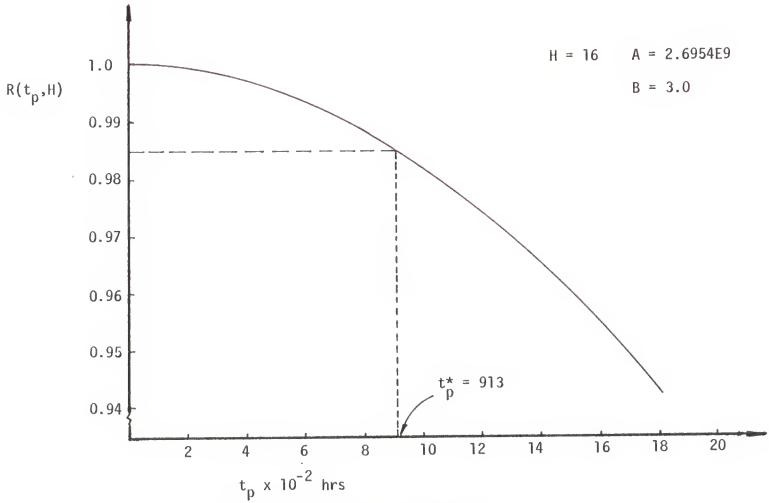


Fig. 4.3 The curve of mission reliability

(4) Expected inflight failure cost

It is obvious that if the critical item is replaced with a preventive replacement just before the mission, the new item will have an age of zero. The probability of failure during the mission will be minimal so is the expected mission failure cost. So the condition for finding t_p^* for the critical item is obtained from the requirement that the expected mission failure cost should equal the preventive replacement cost of the item just prior to the mission plus the expected mission failure cost of newly replaced critical item, i.e.,

$$C_3F(t_p^*,H) = C_3F(0,H) + C_1$$

that is

$$\frac{R(t_p^* + H)}{R(t_p^*)} = R(H) - \frac{C_1}{C_3} \tag{4.6}$$

For the numerical example with Weibull distribution given in section 3.3, the curve of expected mission failure cost $C_3F(t_p,H)$ is shown in Fig. 4.4. By eq. (4.6) we can find the optimal preventive replacement age at 743 hours with minimal expected mission failure cost \$25,005 which is equal to the cost that can occur by replacing the critical item just before the mission.

The following results in hours for the optimal preventive replacement age, t_p^*, were found for the critical item according to the four criteria:

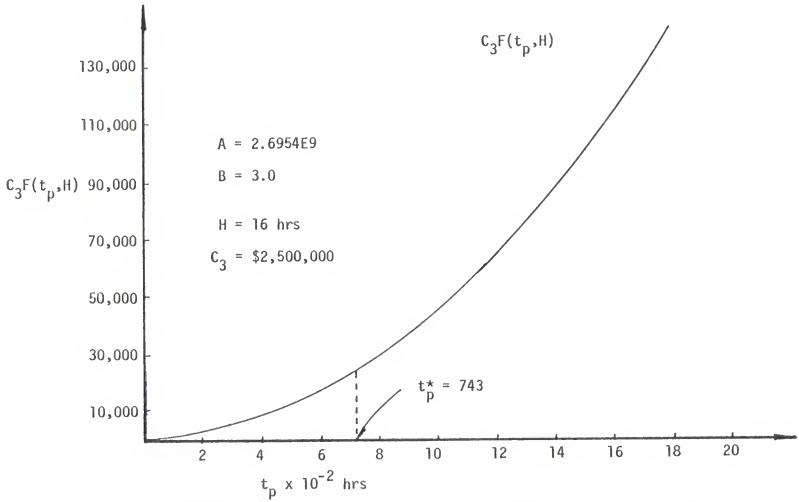


Fig. 4.4 The curve of expected cost of failure during the mission

Criteria	t_p^* (hr.)
replacement cost per unit of time	1,455
availability	1,129
mission reliability	913
mission failure cost	743

The lowest replacement age, 743 hours, has been chosen in order to assure that (a) at least the minimal mission reliability is fulfilled, (b) the expected mission failure cost is not higher than the total cost consisting of the expected mission failure cost of a new item, and the preventive replacement cost, (c) the lowest replacement cost per unit of time and the highest availability are obtained, given that (a) and (b) are fulfilled.

The optimal replacement age obtained by the strictest selection method also demonstrated in Fig. 4.5.

4.2 Lexicographic method

This method requires that the objectives are ranked in order of importance by the decision maker (DM). The satisfying solution obtained by this method is one which maximizes the objectives starting with the most important and going down according to the order of the importance of the objectives [7].

Assume that a multiple objective decision problem consists of n decision variables, m constraints and K objectives. Let the subscripts of the objectives indicate not only the components of the objective vector, $\underline{f}(\underline{X})$, but also the priorities of the objectives, i.e., $f_1(\underline{X})$ is the first component of $\underline{f}(\underline{X})$ and the most important objective, $f_2(\underline{X})$ is the second component and the second most important objective, and so on. Then the first problem to be solved is:

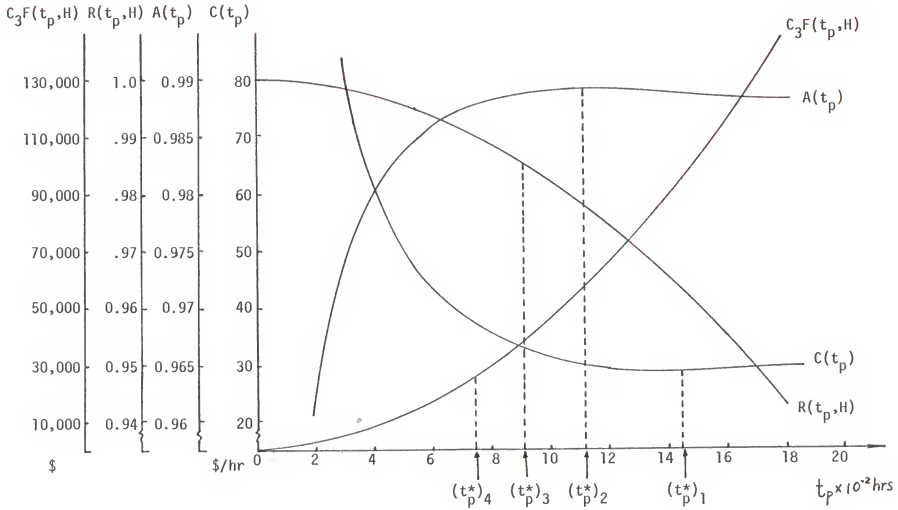


Fig. 4.5 The optimal replacement age by strictest selection method

$$\left.\begin{array}{ll} \text{Max} & f_1(\underline{X}) \\ \text{s.t.} & g_j(\underline{X}) \leq 0 \qquad j = 1,2, \ldots, m \end{array}\right\} \tag{4.7}$$

Let f_1^* be the solution to the above problem. If this problem gives a unique $\underline{X}$ for f_1^*, the solution is considered as the satisfying solution to the entire problem. Otherwise, the second problem is to be solved, i.e.,

$$\left.\begin{array}{ll} \text{Max} & f_2(\underline{X}) \\ \text{s.t.} & g_j(\underline{X}) \leq 0 \quad j = 1, 2, \ldots, m \\ & f_1(\underline{X}) = f_1^* \end{array}\right\} \tag{4.8}$$

Let f_2^* be the solution to (4.8). If (4.8) gives a unique $\underline{X}$ for f_2^*, the solution is the satisfying solution to the entire problem. Otherwise the procedure is repeated until all K objectives have been considered. In general, the ith problem is:

$$\left.\begin{array}{ll} \text{Max} & f_i(\underline{X}) \\ \text{s.t.} & g_j(\underline{X}) \leq 0 \quad j = 1,2, \ldots, m \\ & f_\ell(\underline{X}) = f_\ell^* \quad \ell = 1,2, \ldots, i\text{-}1 \end{array}\right\} \tag{4.9}$$

Since the procedure is terminated when a unique solution is obtained at the ith problem; the solution will be the satisfying solution to the entire problem; the objectives ranked less important than $f_i(\underline{X})$ are ignored by this method. The rationale for this method is that individuals tend to make decisions in this manner.

The Replacement Age Example

Assume the DM ranks the importance of four criteria as: (1) mission reliability (2) expected cost of inflight failure cost (3) replacement cost per unit of time (4) the availability of the equipment. Then the first problem to be solved is:

$$\left.\begin{array}{ll} \text{Find} & t_p \\ \text{s.t.} & R(t_p,H) \geq R(t_p,H)_{min} \\ & t_p \geq 0 \end{array}\right\} \qquad (4.10)$$

where $R(t_p,H)_{min} = 0.985$.

The solution is $t_p \leq 913$ hrs (see Fig. 4.6.) Since this is not a unique solution, the second problem to be solved is:

$$\left.\begin{array}{ll} \text{Find} & t_p \\ \text{s.t.} & C_3F(t_p,H) \leq C_3F(0,H) + C_1 \\ & R(t_p,H) \geq 0.985 \\ & t_p \geq 0 \end{array}\right\} \qquad (4.11)$$

The solution to (4.11) is $t_p \leq 743$ (see Fig. 4.6). Since this is still not a unique solution, the third problem to be solved is:

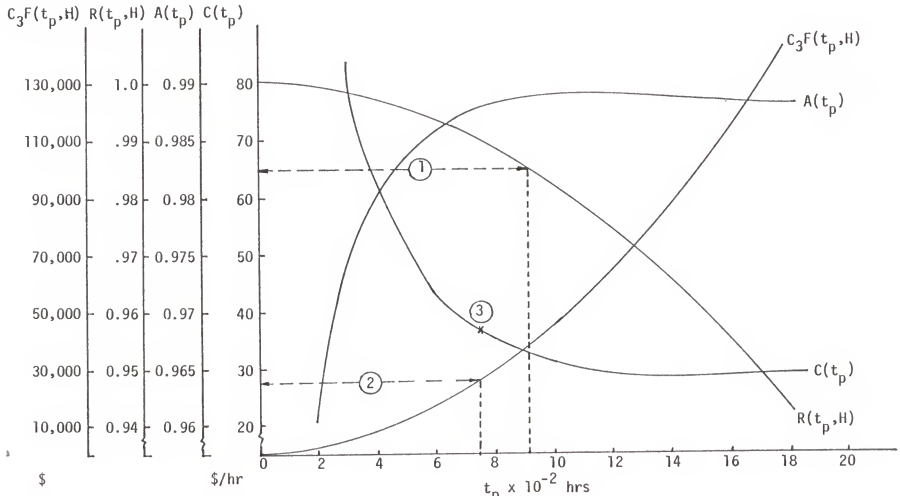


Fig. 4.6 Finding replacement age by lexicographic method

$$\left.\begin{array}{ll} \underset{\{t_p\}}{\text{Max}} & C(t_p) \\ \text{s.t.} & R(t_p, H) \geq 0.987 \\ & C_3F(t_p,H) \leq C_3F(0,H) + C_1 \\ & t_p \geq 0 \end{array}\right\} \tag{4.12}$$

The solution to (4.12) is t_p = 743 hrs. For this is a unique solution, the procedure is terminated. The t_p = 743 hr is the satisfying solution to the entire problem. The less important criterion, availability of the equipment, is ignored by this method. The optimal replacement age of this specific example obtained by the lexicographic method is illustrated in Fig. 4.6. Since there are four criteria in the problem, the DM may rank their importance in 4! = 24 different ways. The results for these 24 different ways of rankings are shown at Table 4.1. Results in Table 4.1 indicate that for this specific numerical example, the satisfying solution is dominated by the first priority or the first and second priorities. It is noted that the satisfying solution will be different if the priorities of the four criteria are changed. Since the solution is very sensitive to the ranking of the objectives given by the DM; the analyst should exercise caution in applying this method when some objectives are of nearly equal importance.

4.3 Waltz's lexicographic method

A variation of the method proposed by Waltz [19] may reduce the sensitivity of the DM's priority of criteria. After the first objective is

Table 4.1 The results of different rankings of criterion importance by lexicographic method

Priority 1	Priority 2	Priority 3	Priority 4	t_p^*(hr)
$C(t_p)$	Those criteria in this area will not affect the solution			1455
$A(t_p)$				1129
$R(t_p,H)$	$A(t_p)$			913
$R(t_p,H)$	$C(t_p)$			913
$R(t_p,H)$	$C_3,F(t_p,H)$			743
$C_3F(t_p,H)$				743

maximized, the second objective is maximized subjected to keeping the first objective within a certain percentage of its optimum. The third objective is then maximized keeping the first two within a certain percentage of the optimum values found in the previous step. The ith problem then is:

$$\left.\begin{array}{ll} \text{Max} & f_i(\underline{X}) \\ \text{s.t.} & g_j(\underline{X}) \leq 0 \qquad j = 1,2, \ldots, m \\ & f_\ell(\underline{X}) = f_\ell^* - \delta_\ell, \quad \ell = 1,2, \ldots, j-1 \end{array}\right\} \tag{4.13}$$

where δ_ℓ's are tolerances determined by the DM.

The Replacement Age Example:

Now, if the DM decides that the importance of four criteria are ranked as (1) the replacement cost per unit of time, (2) the availability of the equipment, (3) the mission reliability, and (4) the expected inflight failure cost, then the first solution to be solved is:

$$\left.\begin{array}{ll} \underset{\{t_p\}}{\text{Min}} & C(t_p) \\ \text{s.t.} & t_p \geq 0 \end{array}\right\} \tag{4.14}$$

The solution is $t_p^* = 1455$ hrs, and $C(t_p) = 28.92$ \$/hr (See Fig. 4.7). However, the DM decides the replacement cost per unit time, $C(t_p)$, is satisfactory when it is less than 30.5 \$/hr. Then the second problem to be solved is:

$$\left.\begin{array}{ll} \underset{\{t_p\}}{\text{Max.}} & A(t_p) \\ \text{s.t.} & C(t_p) \leq 30.5 \\ & t_p \geq 0 \end{array}\right\} \tag{4.15}$$

The solution to (4.15) is $t_p^* = 1129$ hrs. and $A(t_p^*) = 0.9888$ (see Fig. 4.7). The DM thinks that it is acceptable when the availability of the item is higher than 0.9875. Then the third problem to be solved is:

$$\left.\begin{array}{ll} \underset{\{t_p\}}{\text{Max.}} & R(t_p,H) \\ \text{s.t.} & C(t_p) \leq 30.5 \\ & A(t_p) \geq 0.9875 \\ & t_p \geq 0 \end{array}\right\} \tag{4.16}$$

The solution to (4.16) is $t_p^* = 1057$ hrs, $R(t_p^*,H) = 0.98$ (See Fig. 4.7). At this point, the DM find that the value of mission reliability can not be relaxed. So a unique solution is obtained and the procedure is terminated. The satisfying solution is:

$t_p^* = 1057$ hrs. $\quad C(t_p) = 30.5$ \$/hr

$A(t_p) = 0.98877$

$R(t_p,H) = 0.98$

$C_3F(t_p,H) = \$50000$

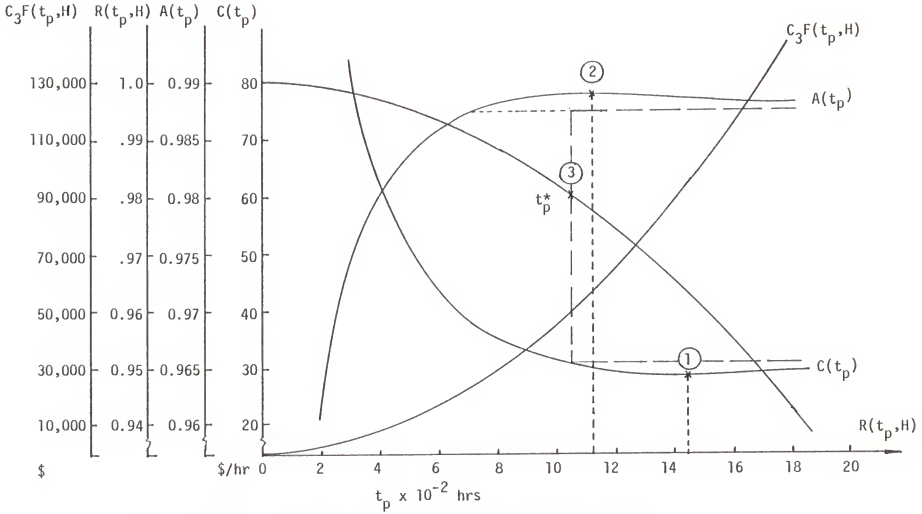


Fig. 4.7 Finding replacement age by Waltz's lexicographic method

It is noted that the sensitivity of the priority ranking to the solution is reduced. The optimal replacement age of this specific example obtained by the Waltz's lexicographic method is illustrated in Fig. 4.7.

4.4 SEMOPS - An Interactive Method

One class of multiple objective decision method is generally referred as 'Interactive Methods'. It is rely on the progressive definition of the DM's preferences along with the exploration of the criterion space. Much work has been done recently on this class of methods [7]. The progressive definition takes place through a DM-Analyst or DM-Machine dialogue at each iteration. At each such dialogue, the DM is asked about some trade-off or preference information based upon the current solution (or the set of current solutions) in order to determine a new solution. These methods assume that the DM is unable to indicate 'a priori' preference information due to the complexity of the problems. However, the DM is able to give a preference information on a local level to a particular solution. As the solution process progresses, the DM not only indicates his preferences but also learns about the problem.

A Sequential Multiobjective Problem Solving technique (SEMPOS) was proposed by Monarchi, Kisiel and Duckstein [15]. The DM is allowed to trade off one objective vs another in an interactive manner. Some implicit trade-off information in the form that the DM be able to indicate whether he is satisfied at the current achievement level are generated by the mechanism of the method. SEMOPS cyclically uses a surrogate objective function based on goals and the DM's aspirations about achieving the objective.

The goal levels are conditions imposed on the DM by external forces and the aspiration levels are the attainment levels of the objectives that the DM personally desires to achieve. (Goals do not change, but aspiration levels change as each iterative cycle goes.)

Let $\underline{AL} = (AL_1, \ldots, AL_k)$ be the DM's aspirations levels, and $\underline{f}(\underline{X}) = \{f_1(\underline{X}), \ldots, f_k(\underline{X})\}$ be the multiple objectige functions. First, a 'relevant range' of $\underline{f}(\underline{X})$ for each goal is chosen as $[f_i(\underline{X})_L, f_i(\underline{X})_U]$. The 'relevant range' is the interval that the DM thinks the $f_i(\underline{X})$ will be in of the problem he is considering. By transforming the original response surface $f_i(\underline{X})$ to $Y_i(\underline{X})$ where $Y_i(\underline{X})$ is defined on the interval (0,1]. Then for any goal:

$$Y_i(\underline{X}) = \frac{f_i(\underline{X}) - f_i(\underline{X})_L}{f_i(\underline{X})_U - f_i(\underline{X})_L}$$

$Y_i(\underline{X})$ is in the interval (0,1]. In examining the results of each iteration, it will be necessary to confirm that $f_i(\underline{X})$ has indeed remained in the specified interval.

The $\underline{AL}$ may be transformed into $\underline{A}$ by the same transformation

$$A_i = \frac{AL_i - f_i(\underline{X})_L}{f_i(\underline{X})_U - f_i(\underline{X})_L}$$

Then $\underline{A}$ is in [0,1]

The dimensionless indicator of achievement, d, is defined for five types of objectives:

(1) at most

$$f_i(\underline{X}) \leq AL_i; \qquad d_i = f_i(\underline{X})/AL_i$$

(2) at least

$$f_i(\underline{X}) \geq AL_i; \qquad d_i = AL_i / f_i(\underline{X})$$

(3) equals

$$f_i(\underline{X}) = AL_i; \qquad d_i = \frac{1}{2}\left(\frac{AL_i}{f_i(\underline{X})} + \frac{f_i(\underline{X})}{AL_i}\right)$$

(4) within an interval

$$AL_{iL} \leq f_i(\underline{x}) \leq AL_{iU}; \qquad d_i = \left(\frac{AL_{iU}}{AL_{iL} + AL_{iU}}\right)\left(\frac{AL_{iL}}{f_i(\underline{X})} + \frac{f_i(\underline{X})}{AL_{iU}}\right)$$

(5) outside an interval

$$f_i(\underline{X}) \leq AL_{iL}$$

or ; $$d_i = \left(\frac{AL_{iL} + AL_{iU}}{AL_{iU}}\right)\left(\frac{1}{\frac{AL_{iL}}{f_i(\underline{X})} + \frac{f_i(\underline{X})}{AL_{iU}}}\right)$$

$$f_i(\underline{X}) \geq AL_{iU}$$

Types (1), (2) and (4) are the most common. In each instance, values of $d_i \leq 1$ imply that the goal is satisfied.

The algorithm generates information under the guidance of the DM so that he can make a decision. Information concerning the interrelationships between objectives is in terms of how achievement or non-achievement of one objective affects the aspiration levels of other objectives.

The mechanism whereby information is generated for the DM to evaluate is the cyclical optimization of a surrogate objective function s.

$$s = \sum_{t \in T'} d_t$$

Where T' is the subset of the set of T objectives as those objectives making up s at a given iteration of the decision-making process. It is important to note that this optimization only provides information to the DM; it does not solve the decision problem. The word surrogate is used in recognition of the fact that the true preference function of the DM is unknown. The value of each d_t in s reflects whether the t-th objective has been satisfied; unsatisfied objectives have values $d_i > 1$. Of course, the nonlinearity prevents direct comparison among the values of d.

Operationally, SEMOPS is a three-step algorithm involving setup, iteration and termination. Setup involves transforming the original problem into a principal surrogate objective function problem and a set of auxiliary problems involving surrogate objective functions. The iteration step is the interactive segment of the algorithm and involves a cycling between a optimization phase (by analyst) and an evaluation phase (by the DM) until a satisfaction is reached, which terminates the algorithm.

The first iteration, i = 1, solves the principle problem, and a set of T auxiliary problems formed as follows, where the aspiration level of each objective is given as the goal of each objective, i.e., $AL_i = b_i$, i = 1,2, ..., T.

The principle problem:

$$\left.\begin{array}{ll} \min & s_1 = \sum_{t=1}^{T} dt \\ \text{s.t.} & \underline{x} \in X \end{array}\right\} \qquad (4.17)$$

The set of auxiliary problem, $\ell = 1,2, \ldots, T$.

$$\left.\begin{array}{ll} \min & s_{1\ell} = \sum_{\substack{t=1 \\ t\neq\ell}}^{T} d_t \\ \text{s.t.} & \underline{x} \in X \\ & f_\ell(\underline{x}) \geq AL_\ell \end{array}\right\} \qquad (4.18)$$

Solving (4.17) and (4.18) forms the optimization phase. The resulting policy vector and objectives for the principal problem and the set of auxiliary problems are presented to, and are used in the evaluation phase by the DM. This information provides the effects of changing the aspiration level for any of the goals. Some simple graphical representation of the information can provide the DM with rough estimates of the interaction among the goals that takes place through the constraint set as his aspiration levels changes.

In general, the ith iteration solves the following principle problem, and a set of auxiliary problems.

The principle problem:

$$\left.\begin{array}{ll} \min & s_i = \sum_{t\in T} d_t \\ \text{s.t.} & \underline{x} \in X \\ & f_j(x) \geq AL_j, \quad j \in (T\text{-}T') \end{array}\right\} \qquad (4.19)$$

The set of auxiliary problems, $\ell \in T'$ (the number of $T' = T - i + 1$):

$$
\left.
\begin{aligned}
&\min \quad s_{i\ell} = \sum_{\substack{t \in T' \\ t \neq \ell}} d_t \\
&\text{s.t.} \quad \underline{X} \in X \\
&\qquad f_j(\underline{X}) \geq AL_j \qquad \text{for } \forall j,\ j \in (T-T') \\
&\qquad f_\ell(\underline{X}) \geq AL_\ell, \qquad \text{for one } \ell, \quad \ell \in T'
\end{aligned}
\right\} \tag{4.20}
$$

The optimization phase solves (4.19) and (4.20). The resulting solutions are used in the evaluation phase and a guidance is given by the DM for the next iteration cycle. The search continues until a satisfactory solution is found.

The Replacement Age Example:

In this study of finding the replacement age involves four goals and one non-negative decision variable. Their various formulations are expressed below:

Goals and criterion functions

$$
Z_1 = C(t_p) = \frac{C_1 R(t_p) + C_2 F(t_p)}{\int_0^{t_p} R(t)dt + t_1 R(t_p) + t_2 F(t_p)} \leq AL_1
$$

$$
Z_2 = A(t_p) = \frac{\int_0^{t_p} R(t)dt}{\int_0^{t_p} R(t)dt + t_1 R(t_p) + t_2 F(t_p)} \geq AL_2
$$

$$Z_3 = R(t_p,H) = \frac{R(t_p+H)}{R(t_p)} \geq AL_3$$

$$Z_4 = C_3F(t_p,H) = C_3\left[1 - \frac{R(t_p+H)}{R(t_p)}\right] \leq AL_4$$

Goal level (initial aspiration levels)

$GL_1 = 30.5$ \$/hr, $GL_2 = 0.9885$, $GL_3 = 0.99$, $GL_4 = \$30{,}000$

Relevant range of Z_i

$r(Z_1) = [0, 80]$ $r(Z_2) = [0.9, 1]$

$r(Z_3) = [0.9, 1]$ $r(Z_4) = [0, 200{,}000]$

Setup procedure:

We begin by transforming the criterion functions:

$$Y_1 = \frac{Z_1}{80}, \qquad Y_2 = \frac{Z_2 - 0.9}{1 - 0.9}$$

$$Y_3 = \frac{Z_3 - 0.9}{1 - 0.9}, \qquad Y_4 = \frac{Z_4}{200{,}000}$$

The initial aspiration levels are assumed to be equal to the goal levels, $AL_i = GL_i$, $i = 1,2,3,4$ and so the values of A_i are:

$$A_1 = \frac{30.5}{80} = 0.38125 \qquad A_2 = \frac{0.9885 - 0.9}{1 - 0.9} = 0.885$$

$$A_3 = \frac{0.99 - 0.9}{1 - 0.9} = 0.9 \qquad A_4 = \frac{30{,}000}{200{,}000} = 0.15$$

The elements of $\underline{d}$ are formulated from the structure of the goals:

$$d_1 = Y_1/A_1, \quad d_2 = A_2/Y_2, \quad d_3 = A_3/Y_3, \quad d_4 = Y_4/A_4$$

First Cycle:

The principal problem to be solved on the first cycle is:

$$\begin{aligned} \min \quad & S_1 = d_1 + d_2 + d_3 + d_4 \\ \text{s.t.} \quad & t_p > 0 \end{aligned}$$

Also as part of the first cycle, we construct four auxiliary problems which attempt to satisfy each of the goals in turn. If $Z_1 \leq 30.5$ is entered as a constraint, d_1 is deleted from the surrogate objective function giving:

$$\left.\begin{aligned} \min \quad & s_{1.1} = d_2 + d_3 + d_4 \\ \text{s.t.} \quad & Z_1 \leq 30.5 \\ & t_p > 0 \end{aligned}\right\}$$

Similarly, the second, third and fourth auxiliary problems are:

$$\left.\begin{aligned} \min \quad & s_{1.2} = d_1 + d_3 + d_4 \\ \text{s.t.} \quad & Z_2 \geq 0.9885 \\ & t_p > 0 \end{aligned}\right\}$$

$$\left.\begin{array}{ll} \min & s_{1.3} = d_1 + d_2 + d_4 \\ \text{s.t.} & Z_3 \geq 0.99 \\ & t_p > 0 \end{array}\right\}$$

$$\begin{array}{ll} \min & s_{1.4} = d_1 + d_2 + d_3 \\ \text{s.t.} & Z_4 \leq 30000 \\ & t_p > 0 \end{array}$$

The optimum results for the five problems are tabulated in Table 4.2.

The information from these five problem can be presented to the DM in the graphical form. Figs 4.8 to 4.11 show the effects of imposing goal attainment at the current aspiration level compared to solution of the principle problem. Let us compare $\underline{Z_1}$ and $\underline{Z_{1.1}}$ in Fig. 4.8. The shaded portion of each axis indicates goal attainment relative to $\underline{AL}$. Assuming Z_i are linearly related, then dZ_2/dZ_1, dZ_3/dZ_1, dZ_4/dZ_1 are constant. The DM uses this information to predict the approximate levels of Z_2, Z_3 and Z_4 for a given aspiration level with Z_1 entered as a constraint. Denote a desired level of attainment on goal 1 by $\hat{Z}_1$. Then we can calculate the effect of this estimate entered as a constraint. We have:

$$\Delta Z_2 = (Z_{1.12} - Z_{1.2}) \cdot (\hat{Z}_1 - Z_{1.1}) / (Z_{1.11} - Z_{1.1})$$

$$\Delta \hat{Z}_3 = (Z_{1.13} - Z_{1.3}) \cdot (\hat{Z}_1 - Z_{1.1}) / (Z_{1.11} - Z_{1.1})$$

Table 4.2 Results of the first cycle (SEMOPS)

$\underline{AL}$ = $\underline{QL}$ = (30.5, 0.9885, 0.99, 30,000)

s_1	t_p	$\underline{d}$	$\underline{Z}$
s_1 = 3.9696	613	$\underline{d_1}$ = (1.4010, 1.0316, 0.9662, 0.5707)	$\underline{Z_1}$ = (42.73, 0.9858, 0.9932, 17,120)
$s_{1.1}$ = 3.7903	1058	$\underline{d_{1.1}}$ = (1.0, 0.9969, 1.1252, 1.6681)	$\underline{Z_{1.1}}$ = (30.49, 0.9888, 0.9800, 50,043)
$s_{1.2}$ = 3.4051	925	$\underline{d_{1.2}}$ = (1.0588, 1.0, 1.0638, 1.2831)	$\underline{Z_{1.2}}$ = (32.29, 0.9885, 0.9846, 38,493)
$s_{1.3}$ = 3.0010	635	$\underline{d_{1.3}}$ = (1.3615, 1.0275, 0.9713, 0.6120)	$\underline{Z_{1.3}}$ = (41.53, 0.9861, 0.9927, 18,360)
$s_{1.4}$ = 3.1662	816	$\underline{d_{1.4}}$ = (1.1373, 1.0060, 1.0229, 1.0)	$\underline{Z_{1.4}}$ = (34.69, 0.9880, 0.9880, 30,000)

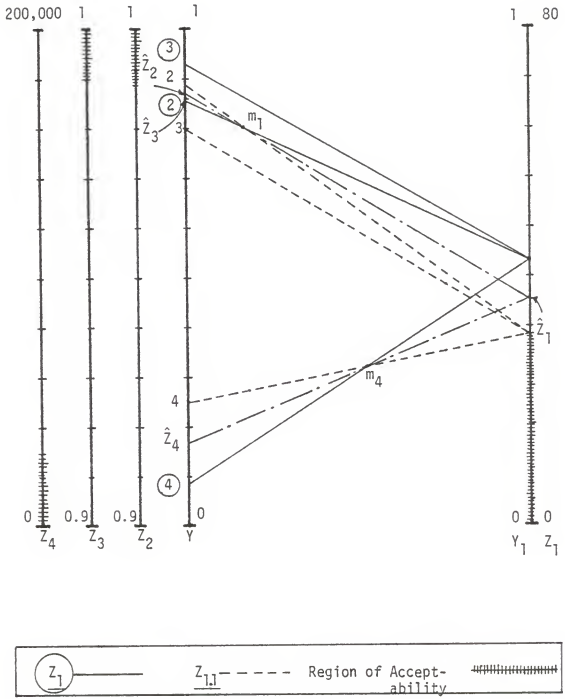


Fig. 4.8 First cycle: goal 1 satisfied at $AL_1 = 30.5$

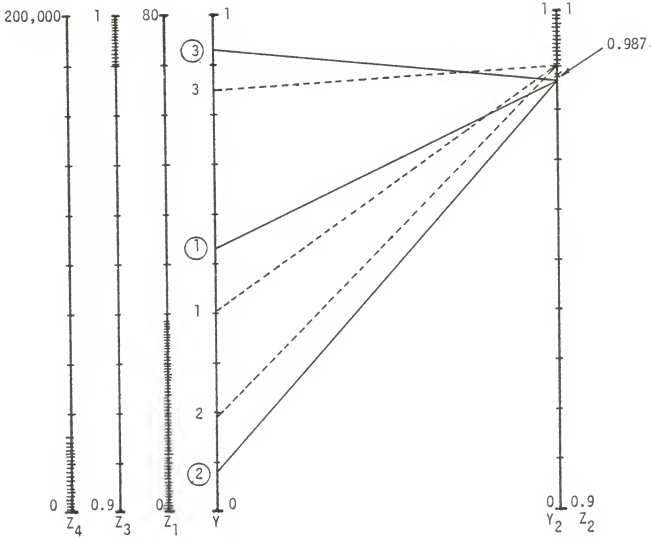
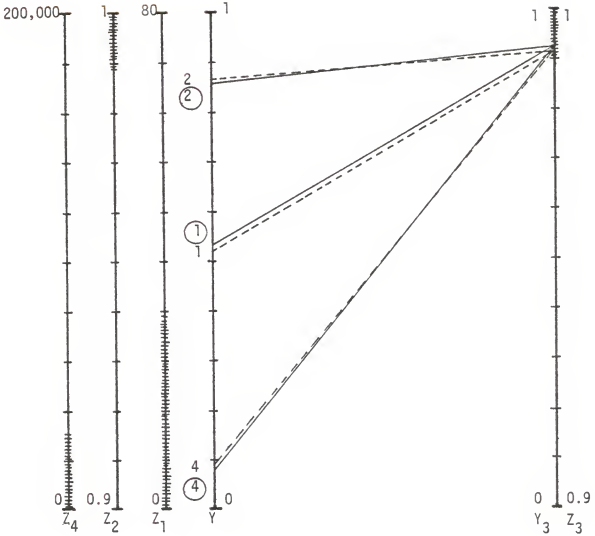


Fig. 4.9 First cycle: goal 2 satisfied at $AL_2^1 = 0.9885$



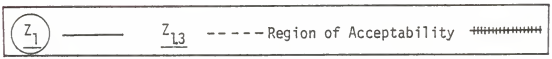
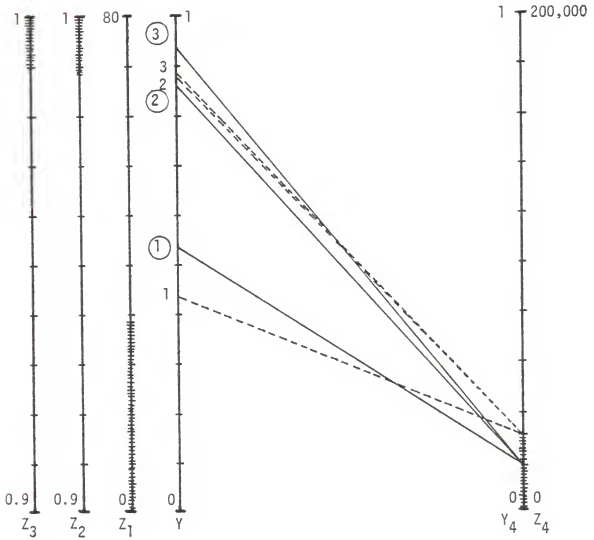


Fig. 4.10 First cycle: goal 3 satisfied at $AL_3^1 = 0.99$



Z_1 ——— $Z_{1,4}$ ----- Region of Acceptability

Fig. 4.11 First cycle: goal 4 satisfied at $AL_4^1 = 30{,}000$

$$\Delta\hat{Z}_4 = (Z_{1.14} - Z_{1.4}) \cdot (\hat{Z}_1 - Z_{1.1}) / (Z_{1.11} - Z_{1.1})$$

where $Z_{a.bc}$ = the cth element of $Z_{\underline{a.b}}$:

Rather than ask the DM to solve these equations for trial values of $\hat{Z}_1$, we can solve the equations graphically by Fig. 4.8. Given $\hat{Z}_1$, we observe that lines passing through m_2 and m_4 find the $\hat{Z}_4$ and $\hat{Z}_2$. (The m_2 and m_4 are the points of intersection of $\underline{Z_1}$ and $Z_{\underline{1.1}}$ in Fig. 4.8). The $\hat{Z}_3$ can be found by proportionality. (i.e. if $\hat{Z}_1$ is the midpoint of $(Z_{1.11} - Z_{1.1})$, then $\hat{Z}_3$ will be the midpoint of $(Z_{1.13} - Z_{1.3})$. So the DM can use a ruler to 'try out' different $\hat{Z}_1$ to see how they affect the other goals.

Examining these results shown in Table 4.2, it is apparent that the change in availability is relatively independent of attainment or non-attainment of the other goals. It seems reasonable to choose an aspiration level for this goal and enter it as a constraint. From Fig. 4-9, the DM assesses the impact of such an action and sets a new aspiration level of 0.987 for goal 2. DM considers that such a decrease is not a really different from the original goal of 0.9885.

Second Cycle:

The aspiration levels are now $\underline{AL}^1$ = (30.5, 0.987, 0.99, 30,000) and achievement of goal 2 has been entered as a constraint. The principal problem to be solved on this cycle is:

$$\min \quad s_{2.2} = d_1 + d_3 + d_4$$

$$\text{s.t.} \quad Z_2 \geq 0.987$$

$$t_p > 0$$

and the three auxiliary problems are:

$$\min \quad s_{2.21} = d_3 + d_4$$

$$\text{s.t.} \quad Z_2 \geq 0.987$$

$$Z_1 \leq 30.5$$

$$t_p > 0$$

$$\min \quad s_{2.23} = d_1 + d_4$$

$$\text{s.t.} \quad Z_2 \geq 0.987$$

$$Z_3 \geq 0.99$$

$$t_p > 0$$

and

$$\min \quad s_{2.24} = d_1 + d_3$$

$$\text{s.t.} \quad Z_2 \geq 0.987$$

$$Z_4 \leq 30000$$

$$t_p > 0$$

The optimum results are presented in Table 4.3 and their effects on goals 1, 3 and 4 are illustrated in Figs. 4.12, 4.13 and 4.14.

Inspection of Table 4.3 reveals that if $Z_1 \leq 38.33$ can be contented then the DM will reach a satisfactory solution. However the DM thinks the replacement cost per unit of time is too high. From Fig. 4.14, the reduction of replacement cost can be achieved by increasing the expected failure cost (goal 4). We assume that the DM has modified his aspiration on goal 4 as $Z_4 \leq 50{,}000$.

Third Cycle:

The aspiration level for this cycle are $\underline{AL}_2$ = (30.5, 0.987, 0.99, 50,000) and both goals 2 and 3 are entered as constraints. The principal problem is:

$$\min \quad s_{3.24} = d_1 + d_3$$

$$\text{s.t.} \quad Z_2 \geq 0.987$$

$$Z_4 \leq 50000$$

$$t_p > 0$$

and the two auxiliary problems are:

$$\min \quad s_{3.243} = d_3$$

$$\text{s.t.} \quad Z_2 \geq 0.987$$

$$Z_4 \leq 50000$$

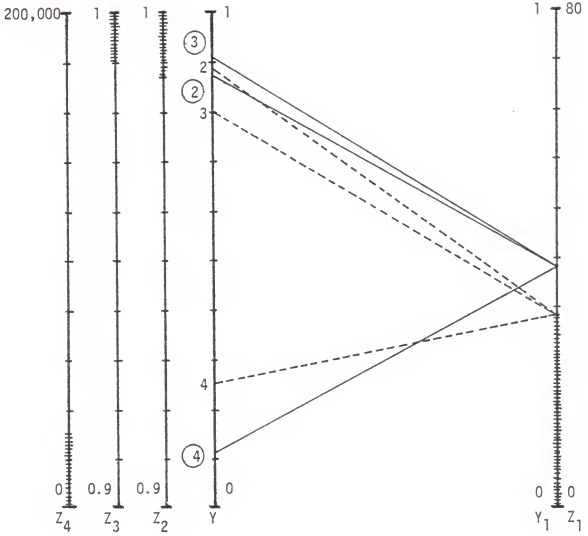
$$Z_1 \leq 30.5$$

$$t_p > 0$$

Table 4.3 Results of the second cycle (SEMOPS)

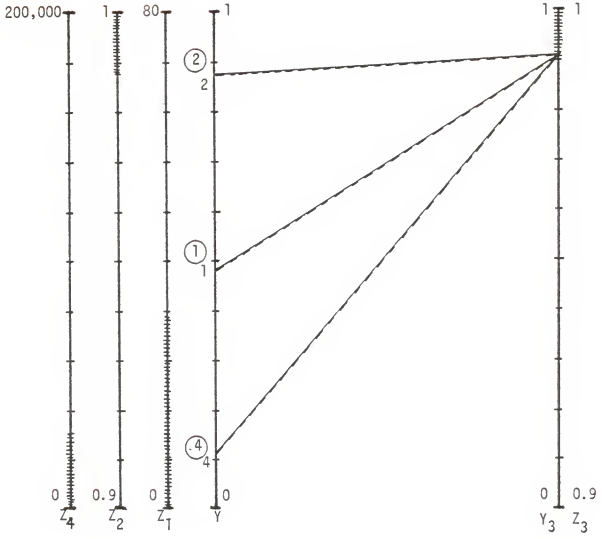
$\underline{AL_1}$ = (30.5, 0.987, 0.99, 30,000)

s_2	t_p	$\underline{d}$	$\underline{Z}$
s_{22} = 2.9975	705	$\underline{d_{22}}$ = (1.2567, 1.0, 0.9892, 0.7514)	$\underline{Z_{22}}$ = (38.33, 0.9870, 0.9910, 22,542)
s_{221} = 2.7934	1058	$\underline{d_{221}}$ = (1.0, 0.9800, 1.1252, 1.6681)	$\underline{Z_{221}}$ = (30.49, 0.9888, 0.9800, 50,043)
s_{223} = 2.0081	705	$\underline{d_{223}}$ = (1,2567, 1.0, 0.9892, 0.7514)	$\underline{Z_{223}}$ = (38.33, 0.9870, 0.9910, 22,540)
s_{224} = 2.1602	816	$\underline{d_{224}}$ = (1.1374, 0.9890, 1.0229, 1.0)	$\underline{Z_{224}}$ = (34.69, 0.9880, 0.9880, 30,000)



Z_{22} ——— Z_{221} -----Region of Acceptability ++++++++++

Fig. 4.12 Second cycle: goal 1 satisfied at $AL_1^2 = 30.5$



Z_{22} ——— Z_{223} ----- Region of Acceptability

Fig. 4.13 Second cycle: goal 3 satisfied at $AL_3^2 = 0.99$

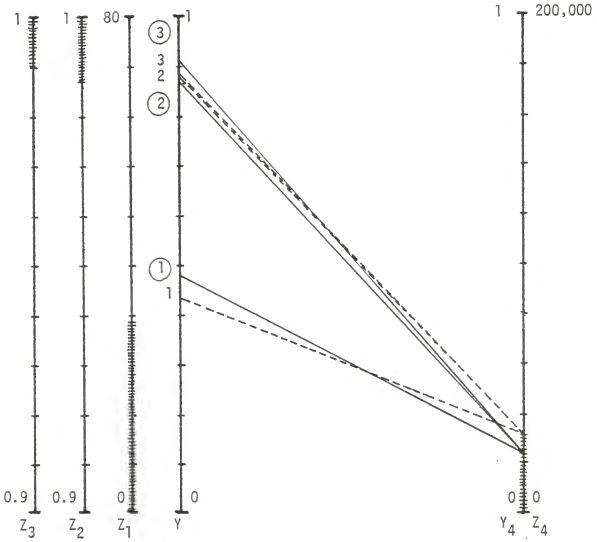


Fig. 4.14 Second cycle: goal 4 satisfied at $AL_1^2 = 30{,}000$

and

$$\min \quad s_{3.243} = d_1$$

$$\text{s.t.} \quad Z_2 \geq 0.987$$

$$Z_4 \leq 50000$$

$$Z_3 \geq 0.99$$

$$t_p > 0$$

The results are tabulated in Table 4.4. In examining Figs. 4.15 and 4.16, the DM learns that goal 1 and goal 3 cannot satisfied simultaneously at present aspiration level. From Fig 4.16, the DM can make a trade-off between the replacement cost per unit of time (goal 1) and the mission reliability (goal 3). Assume that the DM has modified his aspiration on goal 3 so that he will be contented if $Z_3 \geq 0.985$. He arrived at this decision by making the trade-off between goal 1 and goal 3 on Fig. 4.16 and can roughly predict that the value of Z_1 for $Z_3 = 0.985$ is 33.2 \$/hr.

Fourth Cycle and Termination:

The principal problem to be solved on this cycle is

$$\min \quad s_{4.243} = d_1$$

$$\text{s.t.} \quad Z_2 \geq 0.987$$

$$Z_4 \leq 500000$$

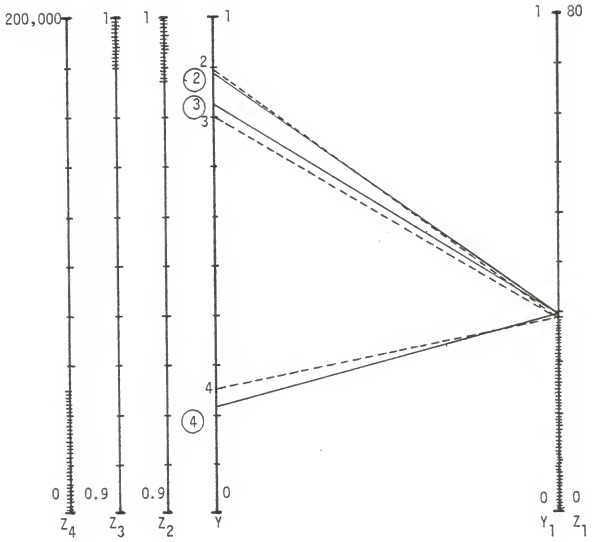
$$Z_3 \geq 0.987$$

$$t_p > 0$$

Table 4.4 Results of the third cycle (SEMOPS)

$\underline{AL_2}$ = (30.5, 0.987, 0.99, 50,000)

s_3	t_p	$\underline{d}$	$\underline{Z}$
s_{324} = 2.1178	982	$\underline{d_{324}}$ = (1.0293, 0.9813, 1.0885, 08659)	$\underline{Z_{324}}$ = (31.39, 0.9887, 0.9827, 43,294)
s_{3241}= 1.1247	1057	$\underline{d_{3241}}$= (1.0, 0.9800, 1.1247, 0.9990)	$\underline{Z_{3241}}$= (30.50, 0.9888, 0.9800, 49,950)
s_{3243}= 1.2105	743	$\underline{d_{3243}}$= (1.2105, 0.9953, 0.9999, 0.4995)	$\underline{Z_{3243}}$= (36.92, 0.9874, 0.9900, 24,975)



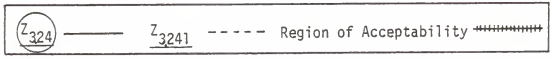
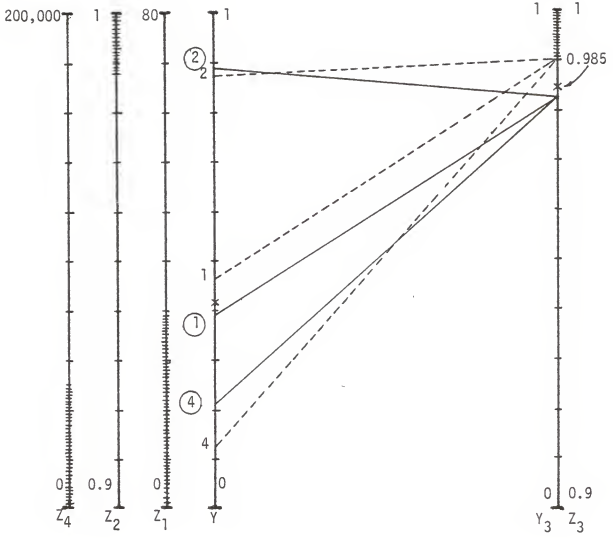


Fig. 4.15 Third cycle: goal 1 satisfied at $AL_1^3 = 30.5$



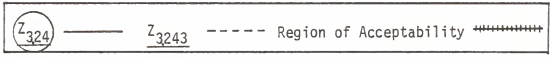


Fig. 4.16 Third cycle: goal 3 satisfied at $AL_3^3 = 0.99$

and there is no auxiliary problem. The results are tabulated in Table 4.5. From this Table, the DM is content with this result with t_p = 912.9 hrs.

$$Z_1 = C(t_p) = 32.52 \text{ \$/hr}$$

$$Z_2 = A(t_p) = 0.9885$$

$$Z_3 = R(t_p H) = 0.985$$

$$Z_4 = C_3 F(t_p, H) = \$37,470$$

4.5 Discussions

In section 4.1, the strictest selection method was applied to the sample problem. By selecting the shortest preventive replacement age, the minimal requirement values set for some criteria were all satisfied. However, if there were the cases that maximal values were set to be not exceeded by some criteria, then the strictest selection method would mean selecting the longest preventive replacement age. It is the intention of the strictest selection method to assure that all the minimal (or maximal) values set for some criteria are satisfied. In the case, where the minimal requirement values are set for some criteria is mixed with the maximal values are set for other criteria in a problem, the strictest selection will be infeasible. The D.M. is least disturbed by the analyst in the strictest selection method. Once the D.M. decides the minimal (or maximal) values for some criteria, the solution is essentially determined. Change the requirement values (minimal or maximal) for some criteria is the only way the D.M. can affect the solution of the problem.

Table 4.5 Results of the fourth cycle (SEMOPS)

$\underline{AL}_3$ = (30.5, 0.987, 0.985, 50,000)

s_4	t_p	$\underline{d}$	$\underline{Z}$
s_{4243} = 1.0662	913	d_{4243} = (1.0662, 0.9836, 1.0, 0.7494)	Z_{4243} = (32.52, 0.9885, 0.9850, 37,470)

For the lexicographic method, the results are influenced by the DM's ranking of the importance of each criterion. The solution is very sensitive to the ranking of each criterion. Some requirement values (minimal or maximal) for lower ranking criteria are usually not satisfied.

For the DM's point of view, the Waltz's lexicographic method is very different from the above two methods. There is a feedback of information to the D.M. for determining the allowable relaxation at a specific value of each criterion. Satisfying solution instead of optimal solution of each criterion is pursued, and the sensitivity of the ranking to the solution is reduced.

The optimization technique used in SEMOPS does not solve problems; it generates information so that the DM can detect the inconsistent constraint set and select an acceptable alternative. The key feature of the SEMOPS is its interactive nature. This avoids the problem of specifying the DM's preference structure by allowing him to keep within himself the transformations that he makes to convert numbers into value judgments. The DM can develop a ranking of goals as he receives information concerning the feasible alternatives. He may also revise his preferences during the course of the interactions. For the numerical example, it also shows that SEMOPS prompts the modification of conflicting aspirations so that an acceptable solution can be determined. SEMOPS accomplishes this by revealing to the DM the extent to which his aspirations will have to be modified to achieve a feasible alternative.

These four methods show the different degree of participation of DM in solving the multiple criteria decision making problem. The more participation of the DM, the more time needed in analyzing. Rather to find which one is the best of the four methods, it is better to equip with them all and to apply them in following manner:

Solve the problem by the strictest selection method first. If the DM satisfies with the solution obtained, the whole problem is solved. Otherwise the analyst should ask the DM to supply more information, such as the ranking of the importance of each criterion, and solve it by the lexicographic method. If the DM still not satisfies with the second solution, the DM should be asked to participate more in the decision making process for determining the allowable relaxation at a specific value of each criterion. If the third solution is still considered not satisfying, then the interactive method, SEMOPS, is employed to generate information to the DM. By which he can detect the inconsistant constraint set and modify the conflicting aspirations so that an acceptable solution can be determined.

It becomes obvious, whether the more complex method should be selected to solve the problem depend on the DM's satisfaction with the solutions found by simpler method and the information the DM can supply.

CHAPTER 5. CONCLUSIONS AND EXTENSIONS FOR FUTURE STUDY

This study is to determine the age at which an operating critical item should be replaced with a new item. The replacement policy is to perform a replacement once the critical item has reached a specified age (preventive replacement) or a failure occurs (corrective replacement). A variety of criteria has been used for finding the optimal replacement age. However, in the conventional approaches, usually only one criterion was considered. It is obvious that the replacement age will depend on what criterion is used to evaluate the decision. The optimal replacement age for one criterion is surely not a optimal solution for other criterion.

This study employes several method for multiple objective decision making to determine the replacement age of the critical item of a maintained system. The four critera considered are: (1) the replacement cost per unit time, (2) the availability of the critical item, (3) the mission reliability, and (4) the expected cost of failure during the mission. The solutions are found by using the following four methods for MODM problems:

(1) strictest selection method

(2) lexicographic method

(3) Waltz's lexicographic method

(4) sequential multiple objective problem solving techniques (SEMOPS)

Each method has been described and demonstrated in detail. The results obtained by each method are directly related to the assumptions and implications of each method.

The example developed in the present study is an one decision variable with multiple criteria problem. The multiple criteria problem with multiple decision variables can be approached by the same way. In addition, the models for the replacement of a single item can be extended into group replacement models. They are presented below for the future study.

1. Multiple Criteria Problem with Multiple Decision Variables.

For the problems studied, the only decision variable is the preventive replacement age, t_p. The cost of a preventive replacement, C_1, the cost of a failure replacement, C_2, the mean replacement time for the preventive replacement, t_1 and the mean replacement time for the failure replacement, t_2, are all assumed to be independent and constant. However, in a real life, the replacement cost is always related to the mean replacement time. If shorter mean replacement time is desired, it requires to allocate more personnel and facilities to the replacement work. Therefore, with a shorter mean replacement time, it requires a high replacement cost. That is, the costs are functions of replacement time such as

$$C_1 = F(t_1) \tag{5.1}$$

$$C_2 = F(t_2) \tag{5.2}$$

With the modification of (5.1) and (5.2), the sample problem in this study becomes a multiple criteria problem with three decision variables : t_p, t_1 and t_2. All the four methods demonstrated in this study is applicable to the multiple decision variables problem. The decision process

is unchanged. Only the optimization technique should use optimization methods for multiple variables.

2. Scheduled Group Replacement Maintenance Policy Based on Multiple Objective Decision Making

For every piece of equipment such as aircraft, there exists critical item whose defect could result in hazardous or unsafe conditions for individuals using the equipment or could prevent the successful accomplishment of the equipment's tactical function. As a consequence, such critical item needs to be replaced before the failure.

In the previous study, the replacement age of a single critical item has been determined subject to multiple criteria. Several multiple objective decision making methods were applied to determine the optimal preventive replacement age. The solutions and implications of the different multiple objective decision making methods were compared and explained in detail.

It is often worthwhile to replace similar items in groups rather than singly, since the cost of replacing an item under group replacement conditions may be lower, i.e. there are economies of scale. The determination of the group replacement age for the similar critical items or a group of non-similar critical items should be also based on multiple criteria.

The proposed problem is to determine the optimal group preventive replacement age for a critical item of a equipment based on the following multiple criteria:

(1) the replacement cost per unit of time,

(2) the availability of the equipment,

(3) the mission reliability of the equipment, and

(4) the cost of failure during mission.

Assuming there are several similiar items which are subject to failure. Whenever an item fails it is replaced by a new item. There is also the possibility that the group replacement can be performed at fixed interval of time. The item replaced will behave like a new item with zero length of life. Then the above four criteria can be evaluated as follows:

(1) The replacement cost per unit of time

If $f(t)$ denotes the failure density function of an item, then the reliability, $R(x)$, and the unreliability, $F(x)$, of an item at age x are respectively

$$R(x) = \int_x^\infty f(t)dt$$

$$F(x) = 1 - R(x)$$

Let C_g is the cost of replacing one item under conditions of group replacement, C_f is the cost of a failure replacement, and K is the total number of items in the group, then the replacement cost per unit of time $C(t_p)$ is

$$C(t_p) = \frac{KC_g + KM(t_p)C_f}{t_p + Kt_g}$$

where t_p = the group replacement age

t_g = the replacement time of one item under conditions of group replacement

$M(t_p)$ = the expected number of times one item fails in interval $(0, t_p)$.

$M(t)$ can be determined by renewal theory [8] as follows.

Let $N(t)$ = number of failures in interval $(0,t)$

$t_1, t_2, \ldots, t_r$ = intervals between failures

S_r = time up to the rth failure = $r_1 + r_2 + \ldots + r_r$

$F_r(t)$ = cumulative distribution function of S_r.

Now the probability of $N(t) = r$ is the probability that t lies between the rth and the (r+1)th failure. This is obtained as follows:

$$P[N(t) < r] = 1 - F_r(t)$$

$$P[N(t) > r] = F_{r+1}(t)$$

since

$$P[N(t) < r] + P[N(t) = r] + P[N(t) > r] = 1$$

$$P[N(t) = r] = F_r(t) - F_{r+1}(t)$$

$$M(t) = \sum_{r=0}^{\infty} rP[N(t) = r] = \sum_{r=0}^{\infty} r[F_r(t) - F_{r+1}(t)]$$

$$M(t) = \sum_{r=1}^{\infty} F_r(t)$$

On taking Laplace transforms of both sides, we get

$$M^*(s) = \frac{f^*(s)}{s[1-f^*(s)]}$$

The problem is then to determine $M(t)$ from $M^*(s)$. This can be done by determining $f(t)$ from $f^*(s)$, a process termed inversion Laplace transforms.

The optimal group replacement age, t_p^*, that minimizes the replacement cost per unit of time is obtained by solving

$$\frac{dC(t_p)}{dt_p} = 0$$

(2) Availability

The availability of the equipment when all the critical items considered are operating, $A(t_p)$, is [13]

$$A(t_p) = \frac{t_p - KM(t_p)t_f}{t_p + Kt_g}$$

where t_p is the failure replacement time of one item.

The optimal group replacement age, t_p^*, that maximizes the availability is obtained by solving

$$\frac{dA(t_p)}{dt_p} = 0$$

If there is requirement for a standard minimum value, $A(t_p)_{min}$, to be fulfilled by the item, then it is required that

$$A(t_p)_{min} \leq A(t_p^*)$$

and the highest value of t_p^* satisfying above equation should be selected.

(3) Mission reliability

The probability, that at a specified time T the unit is operating and will continue to operate for a interval of time H, is defined as "interval reliability RI(H,X)" which is given by [2]

$$RI(H,X) = R(X+H) + \int_0^X R(X-y+H)dM_{11}(y)$$

where $dM_{11}(y)$ is the probability of a completed repair occurring at time y, $M_{ij}(t) = E[N_{ij}(t)]$, the expected number of visits to state j in (0,t) if at time 0 the unit enter state i. The operating state is labeled by 1 and the failed state by 0.

$$M_{11}(t) = \int_0^t M_{01}(t-z)dF(z)$$

$$M_{01}(t) = \int_0^t [1 + M_{11}(t-z)]dG(z)$$

where F(t) is the failure time distribution and G(t) is the repair time distribution. If F(t) and G(t) are known, then $M_{11}(t)$ and $M_{01}(t)$ can be determined. The mission reliability, $R(H|X)$, is the probability thatall the critical items are operating at X and will continue to operate for a interval of length H.

$$R(H|X) = R(H,X)^K$$

If there is a minimum mission reliability requirement, $R(H|X)_{min}$, then we require the lowest value, i.e., $R(H|t_p)$ still higher than $R(H|X)_{min}$

$$R(H|X)_{min} \leq R(H|t_p^*)$$

The highest value of t_p^* satisfying the above inequality should be selected.

(4) Cost of failure during mission

The cost of failure of any critical item during the mission, C, includes the cost of damage to the whole equipment, possible loss of lives, etc. It has a higher value than C_g and C_f. The expected cost of failure during the mission of length H, E_f, is

$$E_f = C[1 - R(H|t_p)]$$

$$= C[1 - RI(H,t_p)^K].$$

The condition for finding the optimal group replacement age t_p^* is obtained from the requirement that the expected cost of failure during the mission should not greater than the group replacement cost of the items just prior to the mission, C_g, plus the expected cost of failure of the newly replaced item during the mission:

$$C[1-R(H|t_p)] = C[1-R(H|0)] + C_g$$

Since each criterion is based on a different objective function and considers different factors, this problem should be solved by methods for multiple criteria decision making [7].

REFERENCES

1. Barlow, R. E., and F. Proschan, "Planned Replacement," in K. J. Arrow, S. Karlin and H. Scarf (eds.), Studies in Applied Probability and Management Science, Standford University Press, Standford, pp. 63-87, 1962.

2. Barlow, R. E., and L. C. Hunter, "Reliability Analysis of a One Unit System", Operations Research, Vol. 9, No. 2, pp. 200-208, 1961.

3. Barlow, R. E., L. C. Hunter and F. Proschan, "Optimum Checking Procedures" 7th Nat. Sym. on Rel. and Q.C., pp. 485-495, Philadelphia, Pa., Jan. 1961.

4. Bell, C. F., M. Kamins, and J. J. McCall, "Some Elements of Planned Replacement Theory", Proc. 1966 Annual Symposium on Reliability, San Francisco, Jan. 1966.

5. Campbell, N. R., "The Replacement of Perishable Members of a Continually Operating System", Supplement, Proc. of the Royal Stat. Soc., Vol. 7, 1941.

6. Cho, H. H., "On the Proper Preventive Maintenance", Proc. 9th Nat. Sym. on Rel. and Q.C., San Francisco, Jan. 1963.

7. Hwang, C. L., and A. S. M. Masud, "Multiple Objective Decision Making Methods and Applications, A State-of-the-Art Survey", Presented at the Joint National Meeting of the Institute of Management Science and Operations Research Society of America, New York, May 1-3, 1978.

8. Jardine, A. K. S., Maintenance, Replacement and Reliability, Pitman Publishing, 1973.

9. Jorgenson, D. W. and McCall, J. J., "Optimal Scheduling of Replacement and Inspection", Operations Research, Vol. 11, No. 5, pp. 732-746, Sept - Oct. 1963.

10. Kay, E., "The Effectiveness of Preventive Maintenance", Int. J. Prod. Res., Vol. 14, No. 3, pp. 329-344, 1976.

11. Kolner, T. K., "An Aircraft Engine Overhaul Model and its Application in Airline Operation", Presented at the Twelfth National Meeting of the Operation Research Society of America, Pittsburgh, Pa., Nov. 14-15, 1957.

12. Ladany, S. P., and M. Aharoni, "Maintenance Policy of Aircraft According to Multiple Criteria", Int. J. Systems Sci., Vol. 6, No. 11, pp. 1093-1101, 1975.

13. Lie, C. H., C. L. Hwang, and F. A. Tillman, "Availability of Maintained Systems: A State-of-the-Art Survey", AIIE Trans, Vol. 9, No. 3, pp. 247-259, Sept. 1977.

14. McCall, J. J., "Maintenance Policies for Stochastically Failing Equipment: A Survey", Management Science, Vol. 11, No. 5, pp. 493-524, 1965.

15. Monarchi, D. E., C. C. Kisiel, and L. Duckstein, "Interactive Multi-objective Programming in Water Resources: A Case Study", Water Resources Research, Vol. 9, No. 4, pp. 837-850, 1973.

16. Morse, P. M., Queues, Inventories, and Maintenance, John Wiley and Sons, Inc., New York, 1958.

17. United States Air Force, "Procedure for Determining Aircraft Engine (Propulsion Unit) Failure Rates, Actuarial Engine Life and Forecasting Monthly Engine Changes by the Actuarial Method", United States Air Force Technical Order T.O. 00-25-128, October 20, 1959.

18. Von Alven, W. H. (ed.), Reliability Engineering, ARING Research Corporation, Pretice Hall, Englewood Cliffs, New Jersey, 1964.

19. Waltz, F. M., "An Engineering Approach: Hierachical Optimization Criteria", IEEE Trnas. on Automatic Control, Vol. AC-12, No. 2, pp. 179-180, 1967.

20. Welker, E. L. "Relationship Between Equipment Reliability, Preventive Maintenance Policy and Operating Costs" ARING Research Corporation, Publication No. 101-9-135, Feb. 1959.

OPTIMAL SCHEDULED MAINTENANCE POLICY BASED ON MULTIPLE CRITERIA DECISION MAKING

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Usually, several criteria instead of one should be considered so as to determine the replacement age of a critical item of a maintained system. In this study, mathematical models have been developed for four criteria: (1) the replacement cost per unit of time, (2) the availability of the critical item, (3) the mission reliability, and (4) the expected cost of failure during the mission. The solutions are obtained using four methods for multiple objective decision making: (1) the strictest selection method, (2) the lexicographic method, (3) the Waltz's lexicographic method, and (4) the sequential multiple objective problem solving technique (SEMOPS).

Using the aircraft engine as an example, the optimal replacement age has been found by the four different methods. The optimum results and the implication of the methods are discussed. The extensions of the present study to the multiple criteria problem with multiple decision variables and to the group replacement problem are proposed.