METHODS FOR DECISION MAKING WITH MULTIPLE OBJECTIVES

AND

THEIR APPLICATIONS TO A HEAT EXCHANGER NETWORK SYNTHESIS

by

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INTRODUCTION

INTRODUCTION

In synthesizing a system, the objective function or functions which are criteria for its performance are minimized (or maximized) subject to a variety of equality and/or inequality constraints. The majority of the system synthesis techniques are predicated on the assumption that a system has only one objective or all of its objectives can be subsumed under one scalar function; the optimal solution minimizes (or maximizes) only one objective function. However, a real system frequently involves multiple conflicting or non-commensurable objectives. This is especially true for a large scale system. Several methods and their variations have been proposed for solving problems with more than one objective function and have been applied mainly to economic and resource management systems.

Conventionally, a chemical and/or industrial process system has been designed optimally by considering only one objective which is an economic efficiency (profit or cost), although it is often difficult or even impossible to express variables and parameters associated with such a system in a monetary unit. For example, the available energy is invaluable to a nation where the energy resources are seriously depleted, since the available energy once lost in a process cannot be recovered by any means; yet the cost of energy resources can be extremely low because of the artificial or manipulated world market condition. Furthermore, the excessive loss of available energy may result in severe pollution, which in turn, may lead to destruction of the environment or the human life. This again cannot be monetarily taken into account. It is natural that the concept of multi-objective analysis should be introduced in synthesizing a chemical and/or industrial process system.

This thesis consists basically of two portions. One is devoted to the interpretations of the basic concepts and terminologies and the comprehensive review of methods for a multi-objective problem (Chapter II). The other is devoted to applications of the methods to the optimal synthesis of a large heat exchanger system where two objectives are simultaneously taken into consideration (Chapter III).

CHAPTER II

ON METHODS FOR DECISION MAKING WITH MULTIPLE OBJECTIVES

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1. INTRODUCTION

Numerous papers have been published during the last fifty years on systems synthesis or optimization techniques. These techniques have been applied to many problems. The optimal solutions obtained, however, have seldom been implemented for real systems. One of the major reasons is the unrealistic nature of the solution of any of the problems which optimize a single objective, as pointed out by Zadeh (1963):

"One of the most serious weaknesses of the current theories of optimal control is that they are predicated on the assumption that the performance of a system can be measured by a single number. The trouble is that, in general, there is more than one consideration that enters into the assessment of performance of a system and in most cases these considerations can not be subsumed under a single scalar valued criterion."

Any real system to be synthesized is usually of multiple objectives or goals, all of which cannot be attained completely because of a variety of constraints which may be technical, economical or ecological. The decision maker must trade off one objective against others in synthesizing a system.

The subject of a multi-objective system is not new. The concept of non-inferiority, which plays a key role in analyzing the system, has been well known as the concept of the Pareto optimality in welfare economics (see, e.g., Henderson and Quandt, 1971). Nevertheless, this subject has not become a major concern of system scientists and engineers. It may be due to the fact that a multi-objective problem does not always give rise to a unique decision (solution) and often requires more or less subjective evaluation by the decision maker. Only lately have system scientists and engineers become active in developing methods to systematically trade off more than one objective and in applying the methods. General discussion and reviews of the literature on multi-objective problems have been presented by Mayor (1969), Freeman and Haveman (1970), Roy (1971), Cohon and Marks (1975), Etoh (1976), and Nakayama and Sawaragi (1976).

In the present work, decision making methods for a multi-objective problem are reviewed comprehensively and critically from the viewpoint of system engineering. In addition, the basic concepts and terminologies involved in such methods are explained.

2. INFERIOR, NON-INFERIOR, PREFERRED AND SUPERIOR DECISIONS

A multi-objective optimization (minimization) problem is generally formulated as

Minimize

$$f_i(\underline{x}), \qquad i = 1, 2, \ldots, n$$

subject to

$$g_i(\underline{x}) \leq 0, \qquad i = 1, 2, \ldots, k$$

where $\underline{x}$ is an m-dimensional decision vector, and all functions, $f_i(\underline{x})$ and $g_i(\underline{x})$, are assumed to be non-linear. For simplicity, this is rewritten in the vector form as

Minimize

$$\underline{f}(\underline{x}) \tag{1}$$

subject to

$$\underline{g}(\underline{x}) \leq 0 \tag{2}$$

where $\underline{f}(\underline{x})$ is an n-dimensional objective function vector, and $\underline{g}(\underline{x})$ is a k-dimensional constraint function vector. Note that the objective function vector, $\underline{f}(\underline{x})$, can be mathematically viewed as a mapping from the decision vector space to the objective vector space. These two vector spaces are manipulated simultaneously in a multi-objective problem. The region defined by the constraint set in the m-dimensional vector space

$$X = \{\underline{x} \mid \underline{g}(\underline{x}) \leq 0\} \tag{3}$$

is referred to here as the feasible region in the decision space. Each vector $\underline{x} \in X$ determines a unique objective vector $\underline{f}(\underline{x})$ in the n-dimensional vector space, and the feasible region in the objective space, F, is defined as

$$F = \{\underline{f}(\underline{x}) \mid \underline{x} \in X\} \tag{4}$$

Distinction between these two feasible regions, X and F, must be clearly kept in mind in the ensuing discussion.

Definition: Inferior Decision

Decision $\underline{x} \in X$ is an inferior decision (solution) if and only if there exists at least one decision $\underline{x}' \in X$ such that

$$\begin{aligned} &\underline{f}(\underline{x}') \leq \underline{f}(\underline{x}) \\ &f_i(\underline{x}') < f_i(\underline{x}) \end{aligned} \tag{5a}$$

for some i = 1, 2,...,n. This can be stated equivalently as

$$\begin{aligned} &\underline{f}(\underline{x}') \leq \underline{f}(\underline{x}) \\ &\underline{f}(\underline{x}') \neq \underline{f}(\underline{x}) \end{aligned} \tag{5b}$$

Let us consider the following example involving two objectives and two decision variables:

Minimize

$$f_1(x_1, x_2) = \cos x_1 + |x_2 - 1| + 1$$

$$f_2(x_1, x_2) = \sin x_1 + 1$$

subject to

$$0 \leq x_1 \leq 2\pi$$

$$0 \leq x_2 \leq 3$$

Figures 1-a and 1-b show the feasible regions in the decision and objective spaces, respectively. Decision $\underline{a} = (\pi, 3)$ is an inferior decision, because there is another decision $\underline{b} = (7\pi/6, 1)$ which satisfies

$$\underline{f}(\underline{b}) \leq \underline{f}(\underline{a})$$

$$\underline{f}(\underline{b}) \neq \underline{f}(\underline{a})$$

We can verify that, from the sufficient condition, Eq. (5a), for a solution to be an inferior decision, all decisions corresponding to the interior points of the feasible region, F, are inferior decisions in any multi-objective problem.

Definition: Non-inferior Decision

Any decision $\underline{x} \in X$ other than the inferior decisions (solutions) is defined as a non-inferior decision. From this definition, we know that the sets of the inferior and non-inferior decisions complement each other.

Suppose that $\underline{x}'$ is a non-inferior decision. Then there is no other decision $\underline{x} \in X$ which fulfills the condition that at least one component of $\underline{f}(\underline{x})$ is less than and the remaining components are equal to the corresponding components of $\underline{f}(\underline{x}')$. In other words, any objective function $f_i(\underline{x}')$ can not be improved (decreased) without simultanously degrading (increasing) at least one of the other objective functions. The non-inferior decision defined here is also well known as the Pareto optimum (Henderson and Quandt, 1971) or the efficient decision (Geoffrion, 1967) in economics.

The set of non-inferior decisions for a minimization problem with two objectives is indicated by the heavy solid lines in Figs. 2-a and 2-b. Two important properties of the non-inferior decisions can be observed from these figures. All non-inferior points must lie on the boundary of the feasible region in the objective space (see Fig. 2-b) but not necessarily in the decision space (see Fig. 2-a). Though the non-inferior set is continuous for a convex problem, it may not be for a non-convex problem as shown in the figures.

Since the determination of the non-inferior set is insufficient in most cases to uniquely synthesize a system optimally, the decision maker must select one of the non-inferior decisions, which is considered to be the best in some sense.

Definition: Preferred Decision

A preferred decision is a non-inferior decision which is chosen as the final decision (solution) based on some additional criteria. As the preferred decision is the best fit for the criteria introduced by the decision maker, some authors call it the optimal decision (solution) or the best-compromise decision (Belenson and Kapur, 1973). Others (Zadeh, 1963; Haimes et al., 1975) have defined the optimal decision differently. The term, a preferrred decision, will be used hereafter in the present work.

Definition: Superior Decision

Decision $\underline{x}' \varepsilon X$ is a superior decision if and only if

$$\underline{f}(\underline{x}') \leq \underline{f}(\underline{x}) \qquad \text{for any } \underline{x} \varepsilon X \tag{6}$$

This definition is identical to the definition of an optimal decision used by some authors (Zadeh, 1963; Reid and Citron, 1971; Vemuri, 1974; Haimes et al., 1975). A superior decision is a special case of a non-inferior decision because it does not satisfy the definition of the inferior decision. Figures 3-a and 3-b show an example of a superior decision. Note that most multi-objective problems have no superior decisions in the feasible region. If there exist superior decisions, all of them correspond to a unique point in the objective space, and the set of the non-inferior objectives contains the superior point only. Inversely, if the non-inferior set in the objective sapce consists of only one point, it is a superior point and the associated decision vector or vectors are superior decisions.

3. GENERATION OF THE NON-INFERIOR SET

Since no superior solution exist for many of the problems, the decision maker is required to specify the preferred decision based on the available information. The preferred decision must be selected from the non-inferior set as mentioned previously. The knowledge of the set of the non-inferior decisions in its entirety is particularly useful for the selection of the preferred decision. This section is devoted to the discussion of the techniques generating the non-inferior set.

3.1 Weighting Method (Parametric Method)

Kuhn and Tucker (1951) presented their well-known condition for the optimality of single-objective problems, and extended their work to multi-objective problems to identify the non-inferiority condition (also see Cohon and Marks, 1975). The condition states:

If decision $\underline{x} \in X$ is non-inferior to a minimization problem with multiple objectives, there exist an n-dimensional vector $\underline{w}$ and a k-dimensional vector $\underline{\lambda}$ such that

$$
\begin{aligned}
&\left(\frac{\partial \underline{f}}{\partial \underline{x}}\right)^T \underline{w} + \left(\frac{\partial \underline{g}}{\partial \underline{x}}\right)^T \underline{\lambda} = 0 \\
&\underline{w} \geq 0, \qquad \underline{w} \neq 0 \\
&\underline{g}(\underline{x}) \leq 0 \qquad (\text{or } \underline{x} \in X) \\
&\lambda_i g_i(\underline{x}) = 0, \qquad i = 1,2,\ldots,k \\
&\underline{\lambda} \geq 0
\end{aligned}
\tag{7}
$$

This is a necessary condition for the optimality of a non-convex problem and a necessary and sufficient condition for the optimality of a convex problem. It is very difficult to solve these equations for decision vector $\underline{x}$ either analytically or numerically, if $\underline{f}(\underline{x})$ and $\underline{g}(\underline{x})$ are in the general form.

On the assumption that the feasible region is convex in the objective space, Zadeh (1963) has shown that the Kuhn-Tucker condition for a non-inferior decision for a multi-objective problem can be proved to be the optimal condition for the following single-objective problem:

Minimize

$$J = \underline{w}^T \underline{f}(\underline{x}) \tag{8}$$

subject to

$$\underline{x} \in X$$

It follows that the entire non-inferior set of a multi-objective problem can be obtained by finding the optimal solution to the problem having the single objective function, J, with varying weight $\underline{w}$ in the non-negative vector space except $\underline{w} = 0$. This optimization problem is numerically much easier to solve than the Kuhn-Tucker condition. The primary drawback of this weighting method is the assumption of the convexity of feasible region F.

Figure 4 provides the geometrical interpretation of this approach for a three-objective problem. All planes with the nomal vector, $\underline{w}$, can be expressed by

$$\underline{w}^T \cdot \underline{f}(\underline{x}) = \text{constant} \tag{9}$$

The smaller the constant in this equation, the closer to the origin the plane. This method searches the plane which satisfies Eq. (9) and which supports the feasible region, F, because the objective vector, $\underline{f}$, corresponding to the point of contact between the plane and the feasible region yields the least value of the objective function given by Eq. (8).
For a problem with more than three objectives, Eq. (9) represents a set of hyperplanes, and the feasible region, F, forms a hypervolume.

3.2 ε-constraint Method

This method is also based on the Kuhn-Tucker condition for non-inferior decision (Cohon and Marks, 1975). The first equation of the Kuhn-Tucker condition can be rewritten as

$$\left(\frac{\partial f_1}{\partial \underline{x}}\right)^T w_1 + \sum_{i=2}^{n} \left(\frac{\partial f_i}{\partial \underline{x}}\right)^T w_i + \left(\frac{\partial \underline{g}}{\partial \underline{x}}\right)^T \underline{\lambda} = 0$$

Since only relative values of the weights are significant, we can assume that w_1 is 1 without loss of generality. Thus, the equation becomes

$$\left(\frac{\partial f_1}{\partial \underline{x}}\right)^T + \left(\frac{\partial \tilde{\underline{f}}}{\partial \underline{x}}\right)^T \tilde{\underline{w}} + \left(\frac{\partial \underline{g}}{\partial \underline{x}}\right)^T \underline{\lambda} = 0 \qquad (10)$$

where

$$\tilde{\underline{f}} = (f_2(\underline{x}),\ f_3(\underline{x}), \ldots, f_n(\underline{x}))^T$$

$$\tilde{\underline{w}} = (w_2, w_3, \ldots, w_n)^T$$

This equation allows us to interpret $\tilde{\underline{w}}$ in the second term as a Lagrangian multiplier vector. This interpretation implies that a non-inferior decision satisfying the above equation can be obtained by solving the optimization problem:

Minimize

$$J = f_1(\underline{x}) \qquad (11)$$

subject to

$$\tilde{\underline{f}}(\underline{x}) \le \underline{\varepsilon}$$

$$\underline{g}(\underline{x}) \le 0 \qquad (\text{or } \underline{x} \in X) \qquad (12)$$

where ε is an (n-1)-dimensional constant vector. $\underline{\varepsilon}$ varies parametrically to yield the set of non-inferior decisions. Note that each ε_i must not be smaller than a certain value in order to render the feasbie decision set defined by constraint (12) non-empty. To identify the minimum value of ε_i, the following auxiliary problem with a single objective must be solved.

Minimize

$$J_i = f_i(\underline{x})$$

subject to

$$\underline{x} \in X$$

where other objectives $f_j(\underline{x})$, $j \neq i$, are entirely neglected. The optimal value of J_i is the minimum value of ε_i. Though the ε-constraint method is somewhat intricate compared with the weighting method, it is widely used because of its applicability to non-convex problems.

3.3 Analytical Approach

For a certain class of problems, the non-inferior value of each objective function can be expressed as an explicit function of the weights on the objective functions (Reid and Vemuri 1971; Vemuri, 1974). This method is limited to problems fulfilling the following two assumptions.

First, each objective $f_i(\underline{x})$ is of the Cobb-Douglas type:

$$f_i = \prod_{j=1}^{m} (x_j)^{a_{ij}}, \qquad i = 1, 2, \ldots, n \tag{13}$$

where a_{ij} are real numbers. Second, no constraints are imposed on the decision variables except that $x_i > 0$.

As pointed out by Zadeh (1963), a non-inferior decision can be generated by solving the minimization problem with the single objective function:

$$J = \sum_{i=1}^{n} w_i f_i(\underline{x})$$

Applying the first assumption for $f_i(\underline{x})$ to this equation yields

$$J = \sum_{i=1}^{n} w_i \prod_{j=1}^{m} (x_i)^{a_{ij}} \tag{14}$$

This is the so-called posynomial function because all x_i are positive. Then, the geometric programming (Beveridge and Schechter, 1970) is applicable in solving this problem. The optimal objective function, J^o, can be evaluated from

$$J^o = \prod_{i=1}^{n} \left(\frac{w_i}{\alpha_i}\right)^{\alpha_i} \tag{15}$$

where

$$\begin{aligned} &\sum_{i=1}^{n} \alpha_i = 1 \\ &\sum_{i=1}^{n} \alpha_i a_{ij} = 0, \qquad j = 1, 2, \ldots, m \end{aligned} \tag{16}$$

Also, the following equations are valid for optimal decision $\underline{x}^o$.

$$J^o = \frac{w_i}{\alpha_i} f_i\ (\underline{x}^o), \qquad i = 1, 2, \ldots, n$$

Therefore, we obtain the relationship between the optimal objective functions and the weighting coefficients as

$$f_i(\underline{x}^o) = \left(\frac{\alpha_i}{w_i}\right) \prod_{i=1}^{n} \left(\frac{w_i}{\alpha_i}\right)^{\alpha_i}, \qquad i = 1, 2, \ldots, n \tag{17}$$

α_i should be evaluated from the (M + 1) linear equations given by equality constraint (16). If $n = m + 1$, all α_i can be determined uniquely. Otherwise, we encounter an additional difficulty (Beveridge and Schechter, 1970). When $n > m + 1$, there are more unknowns than equations, and equation (15) must be minimized with respect to $\alpha_1, \alpha_2, \ldots, \alpha_n$ subject to equality constraint (16).

Once all α_i are determined, it is easy to calculate, by means of Eq. (17), the optimal objective functions which correspond to the components of the non-inferior objective function vector for the multi-objective problem. This method is powerful and requires less numerical effort than the other methods, but its applicability is very limited because of the imposed assumptions mentioned previously.

3.4 Other Methods

Beeson (1971) has presented the adaptive search approach which incorporates the weighting method with the gradient searching method. The weighting coefficients do not change systematically but randomly at each step. The disadvantages of this method are the enormous computational effort required and the inapplicability to the selection of the preferred decision.

The goal programming and the goal attainment method were originally developed for the selection of the preferred decision of a multi-objective problem (Haimes et al., 1975). These methods are also applicable to generating the entire set of non-inferior decisions by parametrically changing the weighting factors which are involved in their formulations. We shall discuss the two methods in detail later.

4. TRADE-OFF SURFACE (CURVE) AND TRADE-OFF RATIO

Given an optimization problem with n objectives, the set of non-inferior objective vectors forms a (n-1)-dimensional manifold in a n-dimensional objective space. The manifold is called a trade-off surface. (Note that our concern in this section is limited to the problem which does not involve the superior decision.) The trade-off surface can be graphically shown only in the case of n = 2 or 3, and it can provide the significant information to the decision maker selecting the preferred decision. For a problem with more than three objectives, it is not easy to visualize the trade-off surface, and, therefore, we should resort to mathematical approaches.

Suppose that the trade-off surface for a problem with n objective functions is mathematically expressed as

$$G(\underline{f}) = 0 \qquad (18)$$

where G is a scalar function. Since any non-inferior objective, $\underline{f}$, must satisfy this equation, one component of $\underline{f}$ is dependent on the others and determined by means of Eq. (18). Let us denote the dependent component by f_i. Then, instead of Eq. (18), the trade-off surface may be expressed as

$$f_i = f_i\ (f_j;\ j \neq i) \qquad (19)$$

The trade-off ratio between the i-th and j-th objectives is denoted by T_{ij} which is defined as

$$T_{ij} = -\frac{\partial f_i}{\partial f_j} \qquad (20)$$

where f_i is assumed to be differentiable with respect to f_j. The differentiation of Eq. (13) with respect to f_j yields

$$\frac{\partial G}{\partial f_j} + \frac{\partial G}{\partial f_i} \cdot \frac{\partial f_i}{\partial f_j} = 0$$

or

$$-\frac{\partial f_i}{\partial f_j} = \left(\frac{\partial G}{\partial f_j}\right) / \left(\frac{\partial G}{\partial f_i}\right) \tag{21}$$

Then another form of the definition of the trade-off ratio is written as

$$T_{ij} = \left(\frac{\partial G}{\partial f_j}\right) / \left(\frac{\partial G}{\partial f_i}\right) \tag{22}$$

As illustrated schematically in Fig. 5, the negative of the trade-off ratio is the slope of the tangent to the trade-off curve between f_i and f_j which is the intersection between the trade-off surface and the f_i-f_j plane. Thus, an increase of one unit in the j-th objective results in a decrease of T_{ij} units in the i-th objective, if all other objectives remain at their current values. It has been shown that T_{ij} has the properties that (Haimes et. al., 1975)

$$\begin{aligned} T_{ij} &= 1/T_{ji} \\ T_{ij} &= T_{ik}T_{kj} \end{aligned} \tag{23}$$

These properties allow us to calculate any T_{ij} from the set, $\{T_{12}, T_{13}, \ldots, T_{1n}\}$. Furthermore, there is at least one positive trade-off ratio, i.e., there is at least one $\partial f_i/\partial f_j$ which is negative. The positive trade-off ratios are of special interest, since the two objectives related to the positive trade-off ratio are in conflict with each other, that is, to improve one of them, we must sacrifice the other.

It is well-known that the trade-off ratio, T_{ij}, is closely related with the generalized Lagrangian multiplier (Luenberger, 1973; Haimes and Hall, 1974). As indicated in the section on the ε-constraint method, the generation of non-inferior decisions for a minimization problem with multiple objectives is essentially equivalent to solving the problem:

Minimize

$$J = f_1(\underline{x})$$

subject to

$$f_i(\underline{x}) - \varepsilon_i \leq 0, \qquad i = 2, 3, \ldots, n$$

$$\underline{x} \in X \text{ (or } \underline{g}(\underline{x}) \leq 0)$$

The generalized Lagrangian, L, is formed as:

$$L = f_1(\underline{x}) + \sum_{i=2}^{n} \lambda_{1i} \, (f_i(\underline{x}) - \varepsilon_i) + \sum_{i=1}^{k} \mu_i g_i(\underline{x}) \tag{24}$$

Kuhn and Tucker (1951) have derived the necessary conditions for the optimality of this problem. λ_{1i} must satisfy the conditions:

$$\begin{aligned} &\lambda_{1i} \, (f_i(\underline{x}) - \varepsilon_i) = 0, \qquad i = 2, 3, \ldots, n \\ &\lambda_{1i} \geq 0, \qquad i = 2, 3, \ldots, n \end{aligned} \tag{25}$$

We are interested only in a problem where every objective conflicts with at least one of the others. Then, there is at least one active constraint which prevents the minimum value of L to be lower than a certain level. Suppose that the active constraint is identified by subscript j. Then, we have

$$\begin{aligned} &f_j(\underline{x}) = \varepsilon_j \\ &\lambda_{1j} > 0 \end{aligned} \tag{26}$$

at the optimum. On the other hand, by differentiating Lagrangian L with respect to ε_j, we obtain

$$\lambda_{1j} = - \frac{\partial L}{\partial \varepsilon_j} \tag{27}$$

By virtue of Eq. (26) and the fact that

$$L = f_1(\underline{x})$$

for the optimal solution, Eq. (27) becomes

$$\lambda_{1j} = - \frac{\partial f_1}{\partial f_j} \tag{28}$$

The right hand side of this equation is identical to the definition of the trade-off ratio between the first and j-th objectives [see Eq. 20]. Furthermore, replacing subscript 1 by i yields an important relationship:

$$\lambda_{ij} = T_{ij} \tag{29}$$

Table 1 summarizes the relationship between the trade-off ratio, T_{ij}, and Lagrangian multiplier, λ_{ij}. The surrogate worth trade-off method, which is discussed later, is based on this relationship (Haimes and Hall, 1974).

5. SELECTION OF THE PREFERRED DECISION

Since the objective function is a vector value function in a multi-objective problem all feasible decisions cannot be ordered in sequence by the use of the objective function as performed in the case of a single objective. Thus, we need a scalar index representing the decision maker's preference in finally deciding the preferred decision. Alternatively stated, we have to introduce a scalar function, $v(\underline{f})$, mapping from the objective space to the preference space. Fingure 6 shows the relationship among the decision, objective and preference spaces. The scalar function, $v(\underline{f})$, is termed as a value function. Its construction is the same as that of a utility function, ordinal utility function, preference function or worth function in the literature (Keeney and Raiffa, 1976).

The form of a value function is chosen subjectively by the decision-maker, but it should be a monotoneous function with respect to each objective, f_i. In a minimization problem with multiple objectives, any objective vector, $\underline{f}$, is preferred or indifferent to the objective vector, $\underline{f} + \underline{\delta}$, for any non-negative vector $\underline{\delta}$. Therefore, it it is assumed that the smaller the magnitude of the value function the more preferred it is by the decision maker, the value function, $v(\underline{f})$, must satisfy

$$v(\underline{f}) \leq v\ (\underline{f} + \underline{\delta}) \qquad \text{for } \underline{\delta} \geq 0$$

This condition implies that $v(\underline{f})$ is monotonically increasing with respect to any component of $\underline{f}$.

Any two objectives $\underline{f}$ and $\underline{f}'$, are comparable through a value function and satisfy one and only one of the following relationships (Keeney and Raiffa, 1976):

(1) $\underline{f}$ is preferred to $\underline{f}'$ (written $\underline{f} \succ \underline{f}'$), i.e., $v(\underline{f}) < v(\underline{f}')$

(2) $\underline{f}$ is indifferent to $\underline{f}'$ (written $\underline{f} \sim \underline{f}'$), i.e., $v(\underline{f}) = v(\underline{f}')$

(3) $\underline{f}$ is less preferred than $\underline{f}'$ (written $\underline{f} \prec \underline{f}'$), i.e., $v(\underline{f}) > v(\underline{f}')$

Thus, given the value function, $v(\underline{f})$, the preferred decision can be obtained from the following optimization problem with a single objective:

Minimize

$$J = v(\underline{f}(\underline{x}))$$

subject to

$$\underline{x} \in X$$

Geoffrion (1967) have shown that the preferred decision for the above optimization problem is one of the inferior decisions of the original multi-objective problem.

In most cases the decision maker does not know the explicit functional form of $v(\underline{f})$. Methods for treating the unknown value function, $v(\underline{f})$, and deciding the preferred decision are reviewed.

5.1 Utility Function Approach

The utility function, $u(\underline{f})$, commonly used in the field of economics, indicates the level of satisfaction which the decision maker derives from a condition characterized by the objective function, $\underline{f}(\underline{x})$ (Henderson and Quandt, 1971). Thus,

$$u(\underline{f}) \gtreqless u(\underline{f}')$$

implies

$$\underline{f} \succsim \underline{f}'$$

or

$$v(\underline{f}) \lesseqgtr v(\underline{f}')$$

The preferred objective, $\underline{f}^*$, yields the maximum vlaue of $u(\underline{f}^*)$ or, equivalently, the minimum value of $v(\underline{f}^*)$. The utility function approach structures and assesses the utility function, $u(\underline{f})$. Instead of minimizing the value function, $v(\underline{f})$, the utility function, $u(\underline{f})$, is maximized, i.e.,

Maximize

$$J = u(\underline{f}(\underline{x}))$$

subject to

$$\underline{x} \in X$$

Any otpimization technique for a single-objective problem is applicable to solving this problem.

The most difficult task in this approach is to determine the utility function, $u(\underline{f})$, with a satisfactory accuracy. To circumvent this difficulty, a variety of forms has been assumed for the utility function. One of the assumptions which are frequently imposed is the utility independence assumption, that is,

$$u(\underline{f}) = \sum_{i=1}^{n} k_i u_i(f_i) \qquad (30)$$

or

$$u(\underline{f}) = \prod_{i=1}^{n} [a_i + b_i u_i(f_i)] \tag{31}$$

where $u_i(f_i)$ is the utility function attributed to the i-th objective, and k_i, a_i, and b_i are constant (Keeney and Raiffa, 1976).

5.2 Indifference Function Approach

There are many different conditions, any two of which can not be distinguished by the preference criterion of the decision maker. The locus of such conditions in the objective space is an indifference sufrace (Henderson and Quandt, 1971; Keeney and Raiffa, 1976). In terms of the value function, $v(\underline{f})$, an indifference surface is defined as the set,

$$\{\underline{f} \mid v(\underline{f}) = \text{constant}\}.$$

A different constant value yields a different indifference surface. These indifference surfaces do not intersect each other, and, therefore, every point in the objective space lies on one and only one indifference surface. The trade-off surface is tangent to one of the indifference surfaces at the preferred point.

The curvature of an indifference curve, $v(\underline{f}) = $ constant, is determined by a set of the derivatives:

$$\frac{\partial f_i}{\partial f_j}, \qquad j = 1, 2, \ldots, i-1, i+1, \ldots, n$$

The negative of the derivative is termed as the marginal rate of substitution of f_i for f_j and denoted by M_{ij}, i.e.,

$$M_{ij} = -\frac{\partial f_i}{\partial f_j} = \left(\frac{\partial v}{\partial f_j}\right) / \left(\frac{\partial v}{\partial f_i}\right) \tag{32}$$

The marginal rate of substitition, M_{ij}, corresponds to the trade-off ratio, T_{ij}, of the trade-off surface. Since the trade-off and indifference surfaces are tangent to each other at the preferred point, $\underline{f}^*$, we have

$$T_{ij}(\underline{f}^*) = M_{ij}(\underline{f}^*) \tag{33}$$

Note that indifference surfaces are obtainable without knowing the functional form of a value function, $v(\underline{f})$. They are usually determined by directly comparing many sampled points in the objective space based on the decision maker's preference.

The indifference surfaces of a two-objective problem yield a set of contours, and the preferred point is readily located as shown in Fig. 7. This graphical approach is not applicable to a problem with more than two objectives, in which the preferred point is numerically searched to satisfy Eq. (33). As the numer of objectives increases, the difficulty of searching the preferred point increases drastically in this approach.

5.3 Surrogate Worth Trade-off Method

The algorithm of this method (Haimes and Hall, 1974; Haimes et al. 1975) consists of two parts. One is the generation of the non-inferior set which forms the trade-off surface in the objective space. The other is the search for the preferred decision in the non-inferior set. The feature of this method is that the preferred decision is located by the use of the surrogate worth function newly introduced by Haimes and Hall (1974).

A surrogate worth function, W_{ij}, estimates the desirability of the trade between a decrease of T_{ij} units in the i-th objective and an increase of one unit in the j-th objective; the other objectives remain at their current values. Thus, W_{ij} is a function of the trade-off ratio, T_{ij}, and the non-inferior objective, $\underline{f}(\underline{x})$. Haimes and Hall (1974) have defined W_{ij} in such a way that

$$W_{ij} > 0 \tag{34}$$

when the trade is desirable, i.e., T_{ij} units of $f_i(\underline{x})$ are preferred over one unit of $f_j(\underline{x})$ for a given $\underline{f}(\underline{x})$;

$$W_{ij} = 0 \qquad (35)$$

when the trade is even; and

$$W_{ij} < 0 \qquad (36)$$

when the trade is undesirable. The larger the absolute value of W_{ij}, the greater the desirability or undesirability of the trade. The numerical value of W_{ij} will depend on the decision maker's response to the question:

Is it desirable to reduce $f_i(\underline{x})$ by T_{ij} units when $f_j(\underline{x})$ is increased by one unit and other objectives are maintained at their current levels? It should be relatively simple to answer this question, since the attained levels of all objectives are known (Haimes et al., 1975).

The surrogate worth function, W_{ij}, can be more easily understood in terms of the marginal rate of substitution, M_{ij}. Recall that M_{ij} is a slope of the indifference curve, i.e., M_{ij} units of $f_i(\underline{x})$ is equivalent to one unit of $f_j(\underline{x})$ according to the decision maker's preference. If the difference, $T_{ij} - M_{ij}$, is positive for a non-inferior decision, an increase of one unit in $f_j(\underline{x})$ will result in a further decrease in $f_i(\underline{x})$ than that which is required to maintain the decision at the same value of $v(\underline{f})$. Thus, such a change is desirable, and W_{ij} must be positive at the decision. Consequently, W_{ij} is essentially identical to the difference, $T_{ij} - M_{ij}$, which is illustrated in Fig. 8 (Nakayama and Sawaragi, 1976). Obviously,

$$T_{ij} - M_{ij} = 0, \qquad j=1,2,\ldots,i-1,i+1,\ldots,n \qquad (37)$$

at the preferred decision and then,

$$W_{ij} = 0, \qquad j=1,2,\ldots,i-1,i+1,\ldots,n$$

The computational scheme for this method is as follows:

(1) Obtain every non-inferior objective, $\underline{f}(\underline{x})$, and trade-off ratio, T_{ij}(or λ_{ij}), corresponding to each of the non-inferior objectives by soliving the Kuhn-Tucker condition for the generalized Lagrangian formed for the ε-constraint method [see Eq. (24)].

(2) Estimate W_{ij} from the interaction with the decision maker.

(3) Search the preferred decision which satisfies

$$W_{ij} = 0, \qquad j=1,2,\ldots,i-1,i+1,\ldots,n$$

Advantageously, this algorithm does not require us to simultaneously take more than two objectives into consideration, even if there are many objectives involved in a problem. One of the difficulties is to assign a numerical magnitude to W_{ij}. Nakayama and Sawaragi (1976) have presented the modified algorithm using only the sign of W_{ij} for searching the preferred decision.

5.4 Interactive Approach

Geoffrion (1970) proposed a man-machine interactive mathematical programming approach to a multi-objective problem and applied it to an aggregated operating problem of an academic department (Geoffrion et al., 1972). In this approach, the value function, $v(\underline{f})$, reflecting the decision maker's preference is minimized by iterative calculations. The decision maker is not required to identify an explicit form of $v(\underline{f})$ but is required to provide the marginal rate of substitution, $M_{ij}(\underline{f})$, at each iteration. Since the marginal rate of substitution is a local information of $v(\underline{f})$, it is generally easier to specify than the functional form of $v(\underline{f})$.

The gradient of the value function, $v(\underline{f}(\underline{x}))$, is given by the equation:

$$\left(\frac{\partial v}{\partial \underline{x}}\right)^T = \left(\frac{\partial \underline{f}}{\partial \underline{x}}\right)^T \left(\frac{\partial v}{\partial \underline{f}}\right)^T$$

or

$$\left(\frac{\partial v}{\partial \underline{x}}\right)^T = \sum_{j=1}^{n} \left(\frac{\partial v}{\partial f_j}\right)\left(\frac{\partial f_j}{\partial \underline{x}}\right)^T$$

where superscript T stands for a transpose of a vector or matrix. The right hand side of this equation can be rearranged as follows:

$$\left(\frac{\partial v}{\partial \underline{x}}\right)^T = \left(\frac{\partial v}{\partial f_i}\right) \sum_{j=1}^{n} M_{ij} \left(\frac{\partial f_j}{\partial \underline{x}}\right)^T \tag{38}$$

where M_{ij} is the marginal rate of substitution between the i-th and j-th objectives defined by Eq. (32), i.e.,

$$M_{ij} = \left(\frac{\partial v}{\partial f_j}\right) \Big/ \left(\frac{\partial v}{\partial f_i}\right) = - \frac{\partial f_i}{\partial f_j}$$

$(\partial v/\partial f_i)$ in Eq. (38) is a scalar multiplier. Then, the direction of gradient is determined by

$$\underline{g}_s = \sum_{j=1}^{n} M_{ij} \left(\frac{\partial f_j}{\partial \underline{x}}\right)^T \tag{39}$$

$\underline{g}_s$ can be evaluated if the marginal rates of substitution can be obtained iteratively with the decision maker. Thus, we can employ any gradient method which is for minimizing a multi-variable function. For instance, the steepest descent method (Luenberger, 1973) is defined by the iterative algorithm

$$\underline{x}^{i+1} = \underline{x}^{i} - \alpha \underline{g}_s \tag{40}$$

where α is a positive scalar representing a step size. This iterative scheme yields the minimum of the value function, $v(\underline{f}(\underline{x}))$, if it converges to a decision which is the preferred decision, $\underline{x}^*$.

5.5 Lexicographic Approach

This approach was first proposed by Georgescu-Roegen (1954) for modeling the human behavior in the decision process. The basic idea underlining this algorithm is to simultaneously minimize as many of the objectives as possible; starting with the most important and going down the hierarchy (Haimes et al., 1975). Thus, the decision maker is required to give a rank in order of importance to each objective prior to solving the problem.

Suppose that the subscripts of the objectives indicate not only the components of the objective vector, $\underline{f}(\underline{x})$, but also the priorities of the objectives, i.e., $f_1(\underline{x})$ is the first and most important component of $\underline{f}(\underline{x})$, $f_2(\underline{x})$ is the second and second most important component of $\underline{f}(\underline{x})$, and so on. Thus, the first subproblem to be solved is:

Minimize

$$J_1 = f_1(\underline{x})$$

subject to

$$\underline{x} \in X$$

If this gives rise to a unique $\underline{x}$ for the optimal $f_1(\underline{x})$, the solution is considered as the preferred decision. Otherwise, the second subproblem is imposed as:

Minimize

$$J_2 = f_2(\underline{x})$$

subject to

$$\underline{x} \in X$$

$$f_1(\underline{x}) = f_1^o$$

where f_1^o is the optimal value of $f_1(\underline{x})$ attained in the first subproblem. This procedure is repeated to obtain a unique solution which is the preferred decision. In general, the i-th subproblem is formulated as:

Minimize

$$J_i = f_i(\underline{x})$$

subject to

$$\underline{x} \in X$$

$$f_j(\underline{x}) = f_j^{\,o}, \quad j=1,2,\ldots,i-1$$

Since the algorithm is terminated when a unique solution is reached, some lower ranked objectives might be ignored and do not contribute to the determination of the preferred decision at all.

Consider a problem with two objectives and one decision variable as illustrated in Fig. 9. By solving the first subproblem, we have the following feasible region for the second subproblem.

$$x_b \leq x \leq x_c$$

$f_2(x)$ is minimum at $x = x_c$ in this region. Therefore, x_c is the preferred decision given by this method.

The algorithm explained above minimizes the value function, $v(\underline{f})$, which has the following property.

Let $\underline{f}$ and $\underline{f}'$ by different objective vectors in the feasible region, F. Then,

$$v(\underline{f}) < v(\underline{f}') \tag{41}$$

if and only if the first non-zero component of the difference, $\underline{f} - \underline{f}'$, is negative. $v(\underline{f})$ can not be written in an explicit form.

Note that the preferred decision is very sensitive to the ranking of the objectives. For instance, if the priorities of $f_1(x)$ and $f_2(x)$ are changed in the example problem (Fig. 9), x_e becomes the preferred decision instead of x_c. The preferred decision can not lie in the region:

$$x_c < x < x_e$$

All decisions in this region as well as x_c and x_e are non-inferior. Care must be exercised to apply this method to a problem where more than one objective are of nearly equal importance (Haimes et al., 1975).

Waltz's method (Waltz, 1967; Seinfeld, 1970) reduces the sensitivity of the preferred decision to the priorities of the objectives by modifying all subproblems except the first as follows;

Minimize

$$J_i = f_i(\underline{x})$$

subject to

$$\underline{x} \in X$$

$$f_j(\underline{x}) \leq f_j^o + \delta_j, \qquad j=1,2,\ldots,i-1 \tag{42}$$

where δ_j's are tolerances determined by the decision maker. This modification expands the feasible region for the second subproblem in the example (Fig. 9) as

$$x_a \leq x \leq x_d$$

x_d yields the minimum value of $f_2(x)$ in this region and becomes the preferred decision.

Waltz (1967) remarked that we generally have no idea what constitutes a reasonable numerical value for the constraint, $f_j^o + \delta_j$, until we have worked the previous subproblem. This is the essential difference of this method from the ε-constraint approach or the Ignizio's goal programming which will be mentioned in later sections.

5.6 Weighting Method

Suppose that a value function, $v(\underline{f})$, is a linear combination of all objectives, $f_i(\underline{x})$, that is,

$$v(\underline{f}) = \sum_{i=1}^{n} w_i f_i(\underline{x}) \tag{43}$$

where w_i's are weighting coefficients representing the relative importance of the objectives. w_i's are non-negative and at least one of them is positive. The preferred decision, $\underline{x}^*$, can be found by solving the following problem:

Minimize

$$J = v(\underline{f}) = \sum_{i=1}^{n} w_i f_i(\underline{x})$$

subject to

$$\underline{x} \in X$$

This formulation yields a non-inferior decision which is the preferred decision (Kuhn and Tucker, 1951; Zadeh, 1963).

The ensuing analysis for the weighting coefficients, w_i, is carried out in the objective space, and the objective vector, $\underline{f}$, is considered as an independent variable instead of the decision vector, $\underline{x}$. The preferred objective, $\underline{f}^*$, defined as

$$\underline{f}^* = \underline{f}(\underline{x}^*) \tag{44}$$

is, in fact, on the trade-off surface. Thus, $\underline{f}^*$ must solve the optimization problem:

Minimize

$$J = \sum_{i=1}^{n} w_i f_i$$

subject to

$$G(\underline{f}) = 0 \tag{45}$$

where constraint (45) constitutes the trade-off surface in the objective space. The Lagrangian, L, is formed as

$$L = \sum_{i=1}^{n} w_i f_i + \lambda G(\underline{f}) \tag{46}$$

where λ is a Lagrangian multiplier. The optimality conditions,

$$\left(\frac{\partial L}{\partial f_i}\right)^* = 0, \qquad i=1,2,\ldots,n \tag{47}$$

yield

$$w_i + \lambda \left(\frac{\partial G}{\partial f_i}\right)^* = 0, \qquad i=1,2,\ldots,n \tag{48}$$

where the asterisk denotes the evaluation at the preferred objective vector, $\underline{f}^*$. By eliminating λ from these equations, we have

$$\frac{w_j}{w_i} = \left(\frac{\partial G}{\partial f_j}\right)^* \Big/ \left(\frac{\partial G}{\partial f_i}\right)^* , \qquad j=1,2,\ldots,i-1,i+1,\ldots,n \tag{49}$$

or

$$\frac{w_j}{w_i} = -\left(\frac{\partial f_i}{\partial f_j}\right)^* , \qquad j = 1,2,\ldots,i-1,i+1,\ldots,n \tag{50}$$

The right hand side of the above equation is identical to the definition of the trade-off ratio at the preferred objective vector, T_{ij}^* [see eq. (21)]. Since only the relative magnitudes of w_i's are significant, we can assume that $w_i = 1$ without any loss of generality. Finally, the following relationship is obtained.

$$w_j = T_{ij}^* , \qquad j=1,2,\ldots,i-1,i+1,\ldots,n \tag{51}$$

where $w_i = 1$. This implies that, in order to apply the weighting method, we must know the trade-off ratio at the preferred objective vector to specify w_i, without knowing the preferred objective vector. Most works using this method, however, assume that each weighting coefficient is constant over the entire objective space.

The weighting method is applicable to the generation of the entire set of non-inferior decisions, which has been explained in the corresponding section (Zadeh, 1963; Everett, 1963).

5.7 ε-constraint Method

If the maximum allowable levels ($\varepsilon_1, \varepsilon_2, \ldots, \varepsilon_{i-1}, \varepsilon_{i+1}, \ldots, \varepsilon_n$) for n-1 objectives ($f_1, f_2, \ldots, f_{i-1}, f_{i+1}, \ldots, f_n$) can be specified in advance, a non-inferior decision can be derived by solving the problem (Cohon and Marks, 1975):

Minimize

$$J = f_i(\underline{x})$$

subject to

$$f_j(\underline{x}) \leq \varepsilon_j \quad , \qquad j=1,2,\ldots,j-1,j+1,\ldots,n$$

$$\underline{x} \in X$$

This solution (non-inferior decision) is interpreted as the preferred decision. The same preferred decision is obtainable by using the penalty method (Luenberger, 1973) as follows:

Minimize

$$J = f_i(\underline{x}) + M \cdot \{ \sum_{j \neq i} U(f_j - \varepsilon_j)\}$$

subject to

$$\underline{x} \in X$$

where

$$U(f_j - \varepsilon_j) = \begin{cases} 1, & f_j(\underline{x}) > \varepsilon_j \\ 0, & f_j(\underline{x}) \leq \varepsilon_j \end{cases}$$

M = large positive number

Thus, the ε-constraint method assumes that the value function, $v(\underline{f})$, is in the form of

$$v(\underline{f}) = f_i(\underline{x}) + M \cdot \{ \sum_{j \neq i} U(f_j - \varepsilon_j)\} \tag{52}$$

If a unique decision cannot be attained in the first (n-1) iterations in the lexicographic approach, the n-th iteration carried out in the manner described above. $f_i(\underline{x})$ in this method corresponds to the least important objective at the n-th iteration of the lexicographic approach. The difference between these two methods is that in the lexicographic approach the allowable levels, $f_j^o + \delta_j$, are described one by one in each iteration, while in the ε-constraint method all ε_j's are determined at once in advance. The preferred decision is sensitive to the choice of the objective function, $f_i(\underline{x})$, in this method as well as in the lexicographic approach.

The graphical explanations for this method are provided in Figs. 10-a and 10-b for the following problem:

Minimize

$$J = f_2(\underline{x})$$

subject to

$$f_1(\underline{x}) \leq \varepsilon_1$$

$$\underline{x} \in X \quad (\text{or} \quad \underline{f}(x) \in F)$$

The entire non-inferior set is indicated by the heavy solid lines.

5.8 Goal Programming

The goal programming was originally developed by Charnes et al., (Charnes et al., 1955; Charnes and Cooper, 1961; Lee, 1972) for a linear model. The goal vector, $\underline{f}_g$, is defined as a target in the objective space which the decision maker wishes to reach but is unable to do so because $\underline{x} \in X$. Thus, the goal vector is an infeasible objective vector. Charnes et al. (1955) has assumed that the value function, $v(\underline{f})$, to be minimized is

$$v(\underline{f}) = \sum_{i=1}^{n} |f_i - f_{gi}|$$

or

$$v(\underline{f}) = \sum_{i=1}^{n} (z_i^+ + z_i^-) \tag{53}$$

where z_i^+ and z_i^- are the absolute values of the positive and negative deviations, respectively, from the goal of the i-th objective, i.e.,

$$\left.\begin{aligned} & f_i - f_{gi} = z_i^+ - z_i^- \\ & z_i^+ ,\ z_i^- \geq 0 \\ & z_i^+ \cdot z_i^- = 0 \end{aligned}\right\} \tag{54}$$

The goal programming by Charnes et al. (1955), therefore, can be stated as

Minimize

$$J = \sum_{i=1}^{n} (z_i^+ + z_i^-)$$

subject to

$$\underline{x} \varepsilon X \text{ (or } \underline{g}(\underline{x}) = 0)$$

Note that we do not have to pay attention to the third condition of Eq. (54) which is automatically satisfied at the optimum. It follows that this optimization can be carried out by any technique for the linear programming as long as $\underline{f}(\underline{x})$ and $\underline{g}(\underline{x})$ are linear.

While Charnes et al. (1955) have introduced a linear value function given by Eq. (53) to render their problem linear, it is possible to employ other value functions for a nonlinear model. The general goal programming is formulated as follows:

Minimize

$$J = d(\underline{f}(\underline{x}), \underline{f}_g) \tag{55}$$

subject to

$$\underline{x} \varepsilon X$$

where $d(\underline{f}, \underline{f}_g)$ stands for a distance between the two vectors. One of the well-known distance expressions in a finite-dimensional space is (Nakayama and Sawaragi, 1976):

$$d(\underline{f}, \underline{f}_g) = \{ \sum_{i=1}^{n} w_i |f_i - f_{gi}|^p \}^{\frac{1}{p}} \tag{56}$$

which is called an L_p-distance with weighting coefficients. The sum of the absolute values corresponding to $p = 1$, the Euclidean distance corresponding to $p = 2$ and the Chebyshev distance corresponding to $p = \infty$ can be represented by this equation. An example of the Euclidean distance ($w_i = 1$) is shown in

Fig. 11. The goal programming is capable of determining the non-inferior decision set by parametrically changing the weighting coefficients, w_i.

Ignizio (1976) has incorporated a lexicographic approach into the goal programming. Let $f_1(\underline{x})$ be the most important objective. The first subproblem to be solved is, then:

Minimize

$$J_1 = z_1^+ = \max\{0,\ f_1 - f_{g1}\} \tag{57}$$

subject to

$$\underline{x} \in X$$

As can be seen from the form of J_1, the negative deviation from the goal does not come in to play in evaluation of the decision vector, $\underline{x}$, in this method Since z_1^+ is non-negative, the optimal value of J_1 is zero or positive. If the optimal J_1 is zero, the goal, f_{g1}, has been attained, and the solutions of this subproblem satisfy

$$f_1(\underline{x}) \leq f_{g1} \tag{58}$$

If the first subproblem does not give a unique solution, the following second subproblem is solved:

Minimize

$$J_2 = z_2^+ = \max\{0,\ f_2 - f_{g2}\} \tag{59}$$

subject to

$$\underline{x} \in X$$

$$f_1(\underline{x}) \leq f_{g1}$$

where $f_2(\underline{x})$ is the second most important objective. As performed in the lexicographic approach, these procedures are repeated to obtain a unique solution which is the preferred decision (See Fig. 12).

Gembichi (1973) has proposed the goal attainment method (also see Haimes et al., 1975). In this method, the value function, $v(\underline{f})$, is defined as

$$v(\underline{f}) \equiv \alpha \equiv \max_{i} \left\{ \frac{f_i(\underline{x}) - f_{gi}}{w_i'} \right\} \tag{60}$$

where f_{gi} is the i-th component of the goal vector, $\underline{f}_g$, and w_i' is the relative over or under attainment of the goal for the i-th objective.

Suppose that f_1 is more important than f_2 in a two-objective problem (see Figs. 13-a and 13-b). Then, a difference between f_1 and f_{g1} should affect the value function more than that between f_2 and f_{g2}. Thus w_1 should be smaller than w_2'. It follows that the smaller weighting coefficient is associated with the more important objective.

From Eq. (60), we have the following inequality:

$$\alpha \underline{w}' \geq \underline{f}(\underline{x}) - \underline{f}_g$$

or

$$f(\underline{x}) \leq \underline{f}_g + \alpha \underline{w}' \tag{61}$$

where $\underline{w}'$ is a vector consisting of components w_i'. The preferred decision is obtained from solving the following problem:

Minimize

$$J = \alpha$$

subject to

$$\underline{f}(\underline{x}) \leq \underline{f}_g + \alpha \underline{w}'$$

$$\underline{x} \in X$$

This approach is illustrated for the case of two objectives in Figs. 13-a and 13-b. Note that the value function given by Eq. (60) is a special case

This approach is illustrated for the case of two objectives in Figs. 13-a and 13-b. Note that the value function given by Eq. (60) is a special case of Eq. (56) with $p \to \infty$ and $w_i = (w_i')^{-1}$, and, therefore, this method is included in the general goal programming.

6. CONCLUDING REMARKS

The majority of available techniques for solving a multi-objective problem has been discussed. Some of the techniques are capable of generating a set of non-inferior decisions, any of which is not inferior to any feasible decision. However, no technique can provide a unique optimal solution without resorting to the decision maker's preferential selection. To eventually reach a unique decision, namely, the preferred decision, a value function reflecting the decision maker's preference is introduced, which is ultimately optimized.

Techniques for solving a multi-objective problem can be classified basically into two categories depending on a manner of assessing the value function. The techniques in one of the categories do not assume any functional form of the value function and locate the preferred decision by utilizing information generated from the interaction with the decision maker. The indifference function approach, the surrogate worth trade-off method and the interactive approach belong to this category. The techniques in the other category include the utility function approach, the lexicographic approach, the weighting method, the ε-constraint method and the goal programming, each of which assumes a specific functional form or a key property of the value function.

In spite of their great applicability, multi-objective optimization system synthesis techniques have been employed only in a limited number of fields such as welfare economics, management science and water resource research. Additional effort need to be expended to explore applicability in many other fields.

7. SUMMARY

All feasible decisions in a multi-objective problem can be divided into two sets. One is a set of inferior decisions which are inferior to a feasible decision or decisions with respect to all objectives. The other is a set of non-inferior decisions defined as the complementary set of the inferior decision set. A preferred decision is a non-inferior decision chosen as the final decision (solution) in the light of an additional criterion introduced by the decision maker. If the non-inferior decision set consists of only one decision, it is superior to any feasible decision with respect to some objectives and equal to any feasible decision with respect to the remaining objectives. Such a decision is called the superior decision or optimal decision.

There are several methods which can generate the entire set of non-inferior decisions. Among them, the weighting method and the ε-constraint method are most commonly used because of the simplicity of calculation schemes involved in their applications.

The trade-off surface is formed by all non-inferior points in the objective space. The intersection between the trade-off surface and a plane formed by two objective functions, say f_i and f_j, gives rise to the trade-off curve between the two objectives. The negative of the slope of the trade-off curve is termed the trade-off ratio denoted by T_{ij}. Thus, an increase of one unit in one of the objectives, f_j, results in a decrease of T_{ij} units in f_i, if all other objectives remain at their current values. The trade-off ratio is closely related with the generalized Lagrangian multiplier.

The criterion for selecting the preferred decision from the non-inferior set is generally called the value function, $v(\underline{f})$, which is chosen somewhat subjectively but based on the decision maker's preference, and which should be a monotonous function with respect to each objective. The indifference surface is the locus of the points of the constant value function in the objective space. Thus, any two conditions on the same indifference surface can not be distinguished by the decision maker based on the preference criterion. The indifference curve is defined similarly to the trade-off curve and the marginal rate of substitution for the indifference surface is defined similarly to the trade-off ratio for the trade-off surface.

Techniques for determining the preferred decision can be classified basically into two categories. A technique in one of the categories assumes a specific functional form or a key property of the value function, and a technique in the other does not. The utility function approach, the lexicographic approach, the weighting method, the ε-constraint method and the goal programming belong to the former, and the indifference function approach and the surrogate worth trade-off method belong to the latter.

NOMENCLATURE

$\underline{a}$ = decision vector

a_i = constant value

a_{ij} = exponent of the Cobb-Douglas equation

$\underline{b}$ = decision vector

b_i = constant value

d = distance between two vectors

$\underline{f}$ = objective function vector

$\underline{f}'$ = objective function vector

$\tilde{\underline{f}}$ = objective function vector defined as $\tilde{\underline{f}} = (f_2, f_3, \ldots, f_n)^T$

$\underline{f}^*$ = preferred objective function vector

F = set of feasible objective function vectors

f_i = i-th component of the objective function vector, $\underline{f}$

f_j^o = optimal value of f_j in the j-th subproblem

$\underline{f}_g$ = goal vector

f_{gi} = i-th component of the goal vector, $\underline{f}_g$

$\underline{g}$ = constraint function vector

G = scalar function representing the trade-off surface

g_i = i-th component of the constraint function vector, $\underline{g}$

$\underline{g}_s$ = gradient vector in the steepest ascent direction

J = scalar objective function

J^o = optimal value of the objective function, J

J_i = scalar objective function associated with the i-th subproblem

k = dimension of the constraint function vector, $\underline{g}$

k_i = constant value

L = Lagrangian

M = large scalar number

M_{ij} = marginal rate of substitution of f_i for f_j

M_{ij}^* = marginal rate of substitution of f_i for f_j at the preferred objective vector, $\underline{f}^*$

n = dimension of the objective function vector, $\underline{f}$

p = exponent in Eq. (56)

T_{ij} = trade-off ratio between the i-th and j-th objectives

T_{ij}^* = trade-off ratio between the i-th and j-th objectives at the preferred objective vector, $\underline{f}^*$

u = utility function

U = unit step function

v = value function

$\underline{w}$ = weighting coefficient vector

$\tilde{\underline{w}}$ = weighting coefficient vector defined as $\tilde{\underline{w}} = (w_2, w_3, \ldots, w_n)^T$

w_i = i-th component of the weighting coefficient vector, $\underline{w}$

W_{ij} = surrogate worth function associated with the i-th and j-th objectives

$\underline{x}$ = decision vector

$\underline{x}'$ = decision vector

$\underline{x}^*$ = preferred decision vector

$\underline{x}^o$ = optimal solution of a single objective problem

$\underline{x}^i$ = i-th approximation of the preferred decision vector $\underline{x}^*$

X = set of feasible decision vectors

x_a, x_b, x_c, x_d, x_e = decisions in the example of the lexicographic approach

z_i^+ = positive deviation from the goal of the i-th objective

z_i^- = negative deviation from the goal of the i-th objective

Greek Symbols

α = step size in iterative calculation for searching the preferred decision, $\underline{x}^*$

α = scalar variable in the goal attainment method

α_i = scalar value defined by Eq. (16)

$\underline{\delta}$ = vector whose components are small non-negatives

δ_i = small positive scalar

$\underline{\varepsilon}$ = allowable level vector for the objective function vector, $\underline{f}$

ε_i = allowable level of the i-th objective function, f_i

λ = Lagrangian multiplier

$\underline{\lambda}$ = Lagrangian multiplier vector

λ_{ij} = Lagrangian multiplier

μ_i = Lagrangian multiplier

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Table 1. Relationship between Trade-off Ratio T_{ij} and Lagrangian Multiplier λ_{ij}

T_{ij}	λ_{ij}
+ ($T_{ij}=\lambda_{ij}$)	+ ($T_{ij}=\lambda_{ij}$)
0	0
-	0

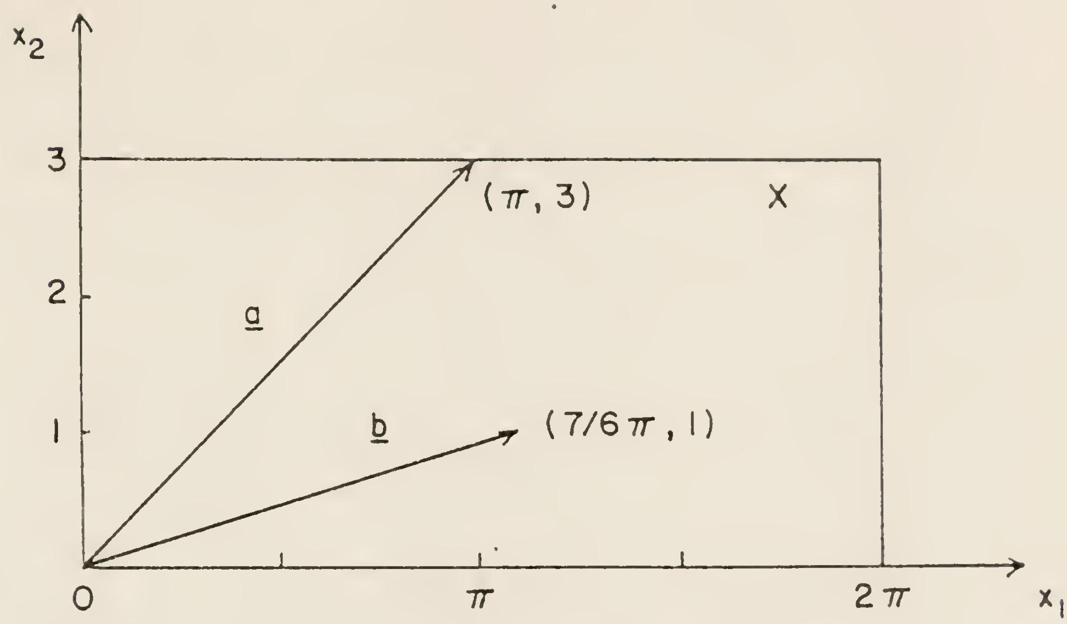


Fig. 1-a. Inferior decision a and non-inferior decision b in the decision space.

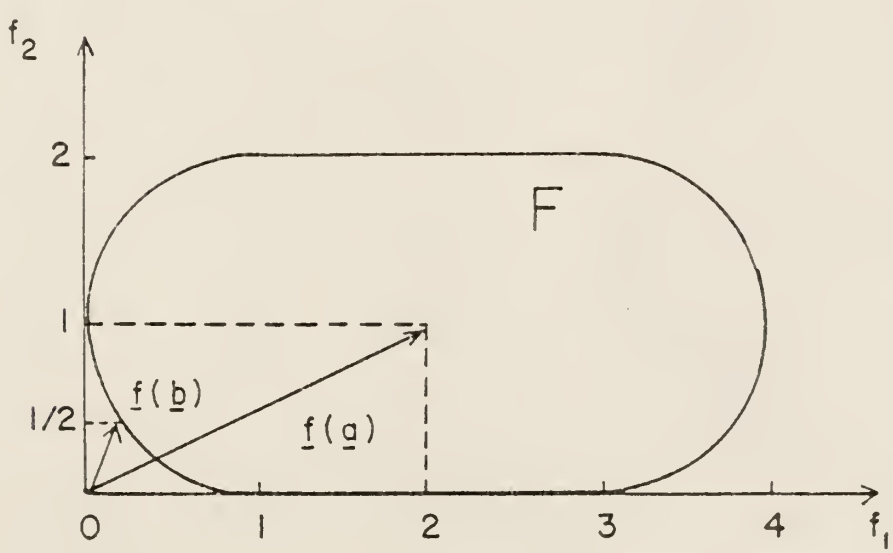


Fig. 1-b. Inferior objective f(a) and non-inferior objective f(b) in the objective space.

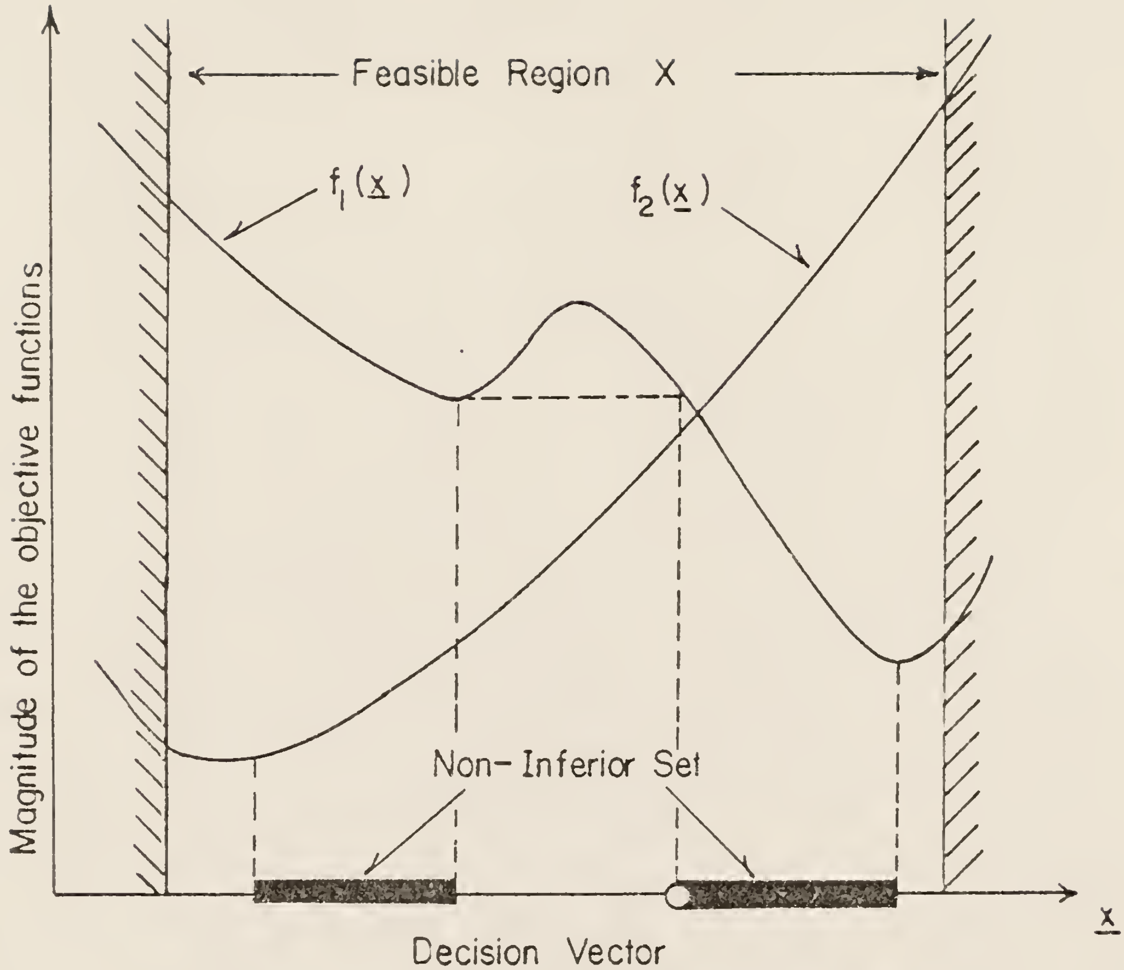


Fig. 2-a. Non-inferior set for a two-objective minimization problem in the decision space.

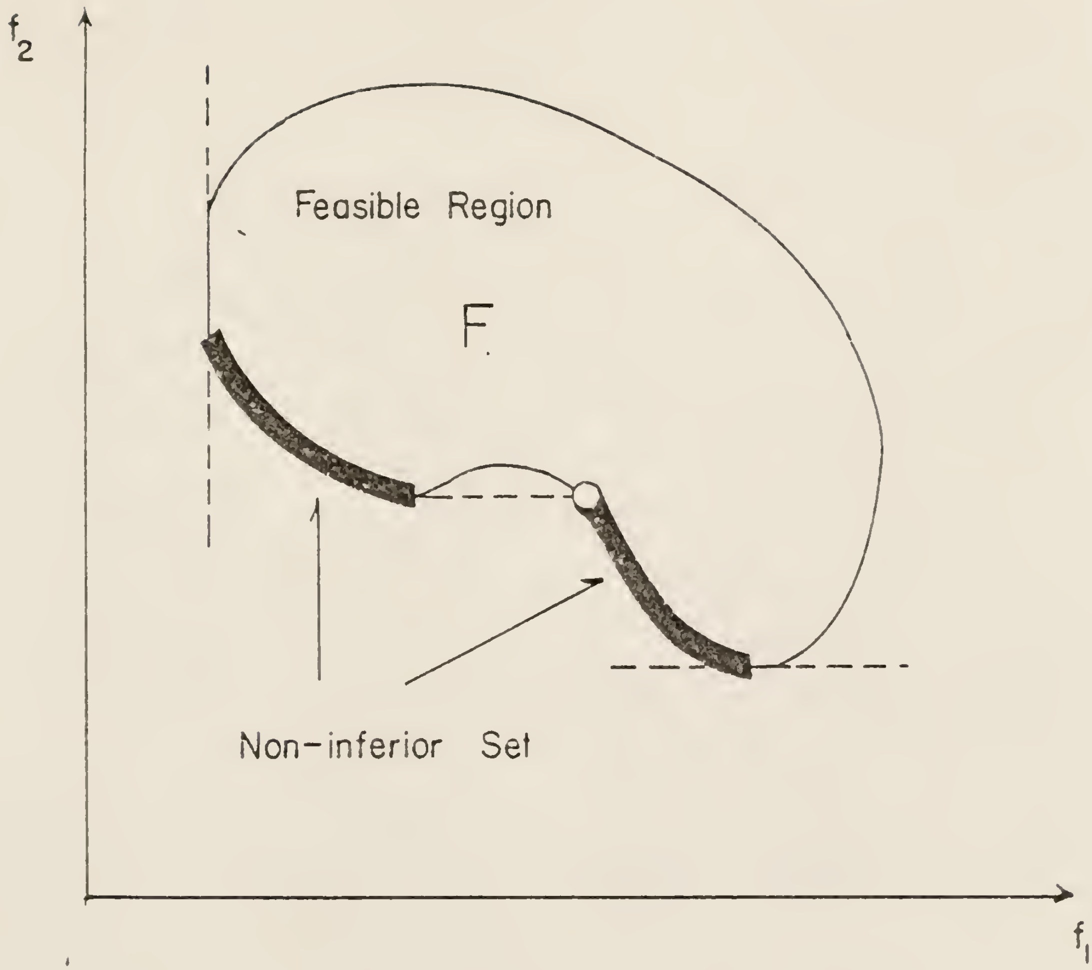


Fig. 2-b. Non-inferior set for a two-objective minimization problem in the objective space.

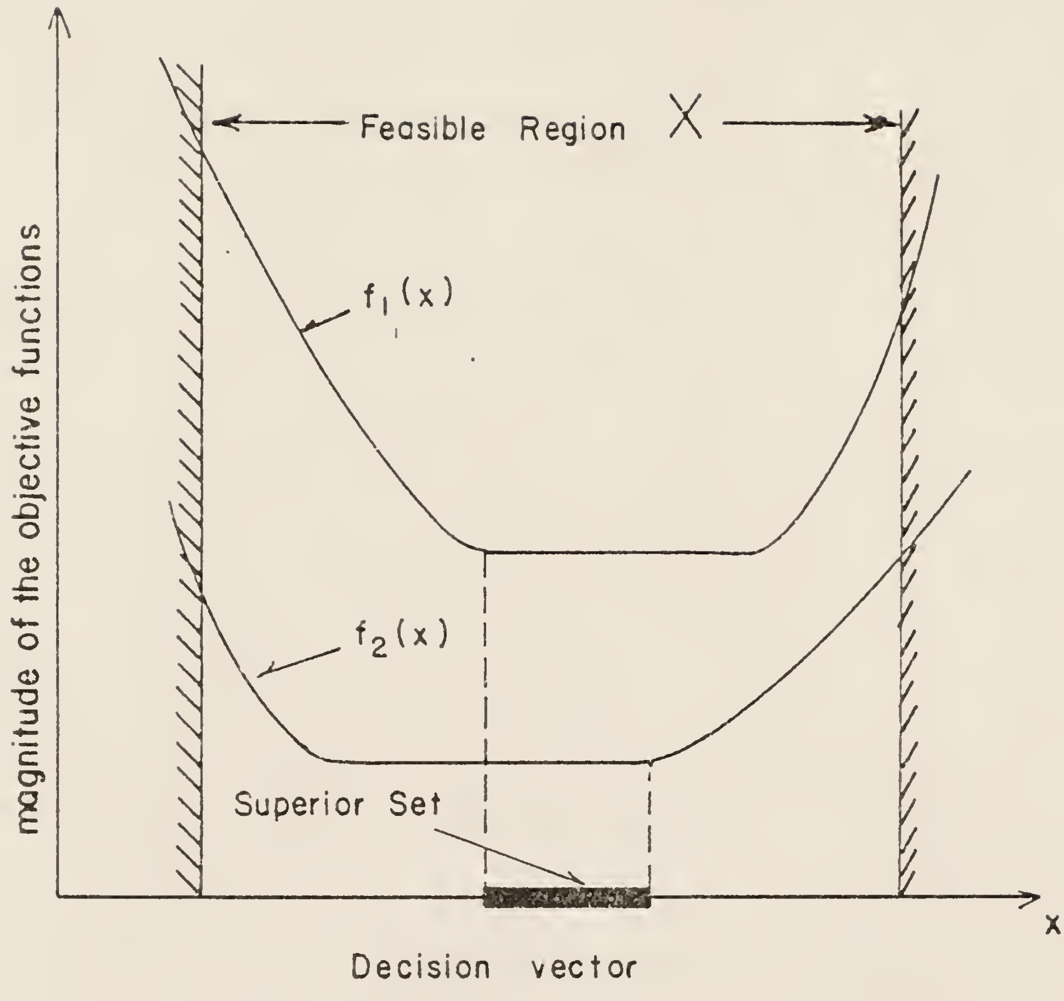


Fig. 3-a. Superior decisions for a two-objective minimization problem in the decision space.

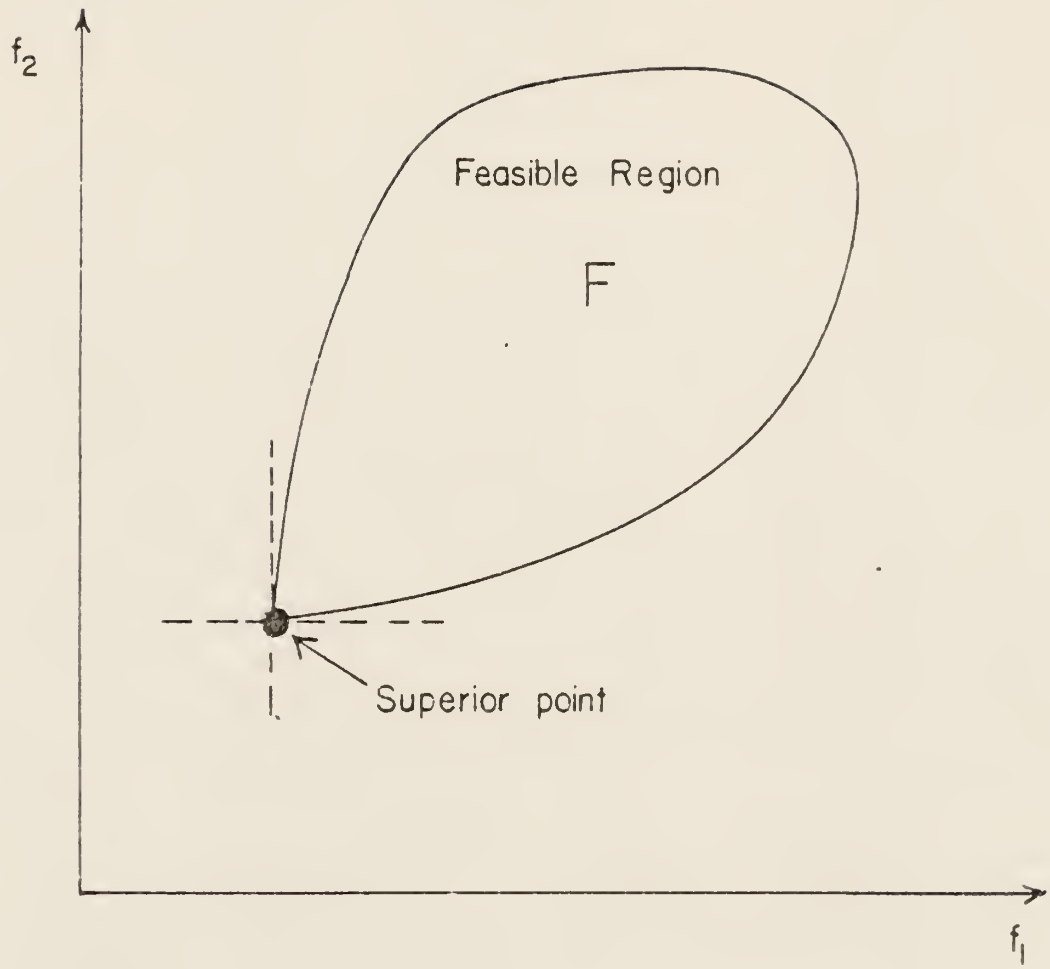


Fig. 3-b. Superior point for a two-objective minimization problem in the objective space.

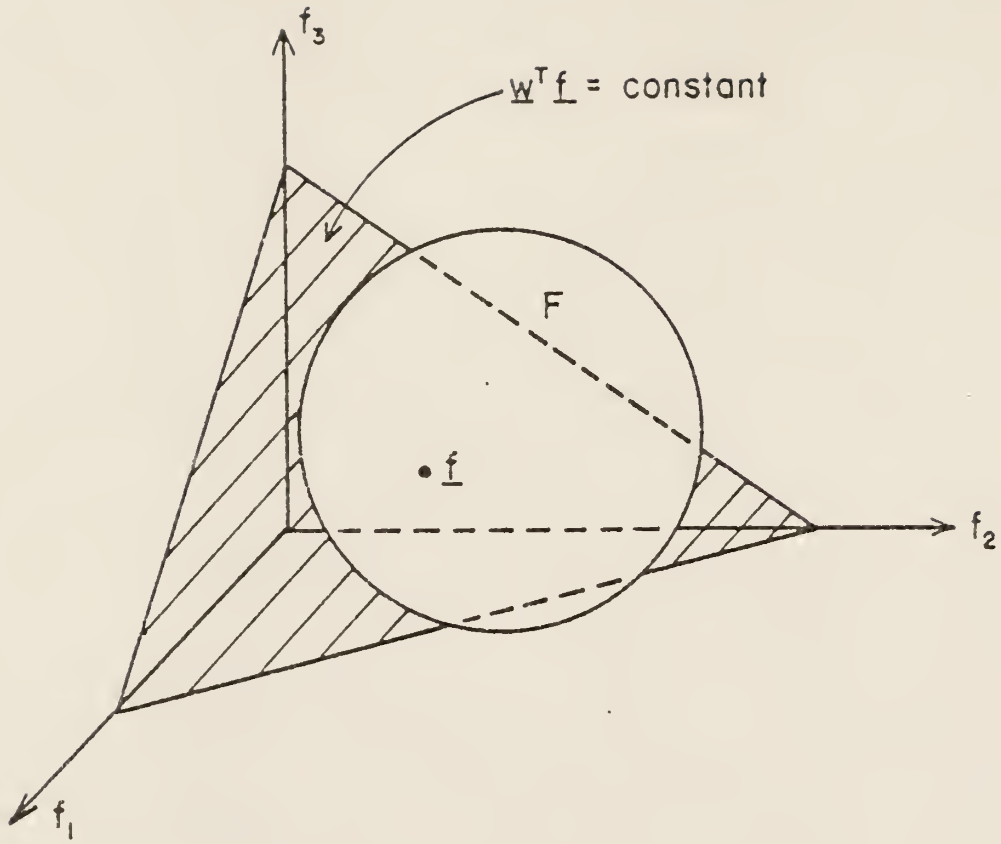


Fig. 4. Geometrical explanation of the weighing method.

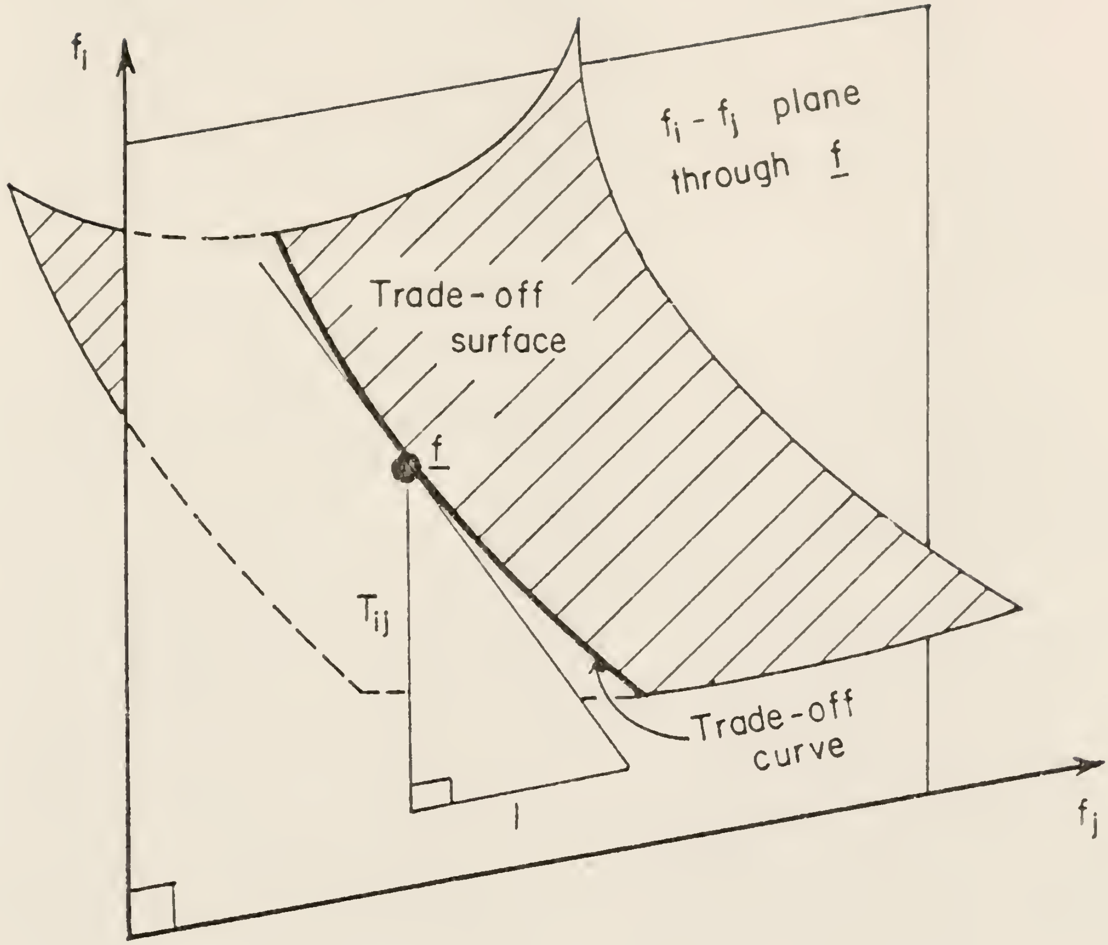
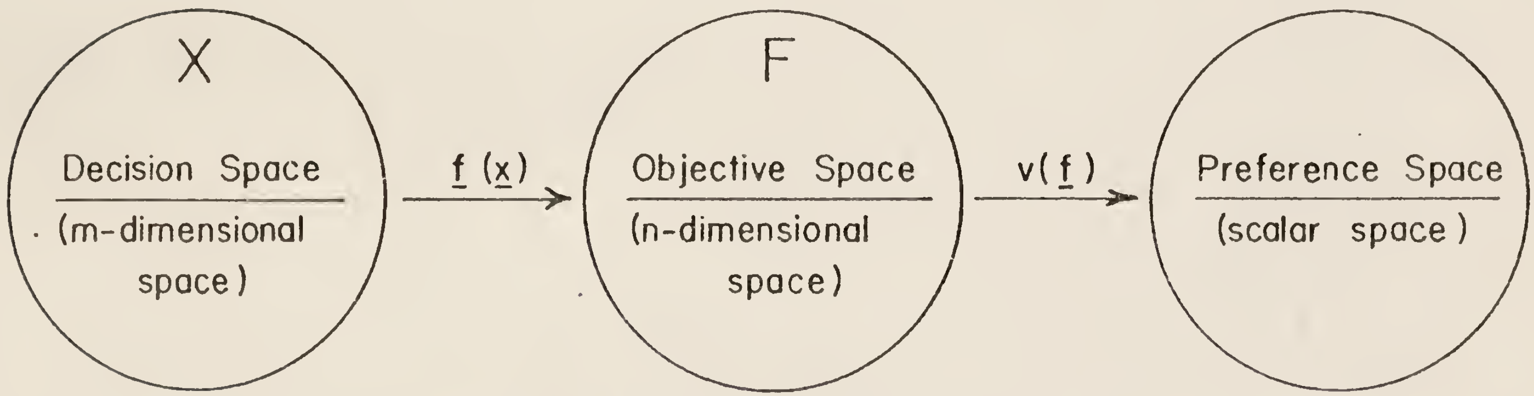


Fig. 5. Geometrical explanation of the trade-off ratio, T_{ij}.



$\underline{f}(\underline{x})$: objective function

$v(\underline{f})$: value function

Fig. 6. Relationship among the decision, objective and preference spaces.

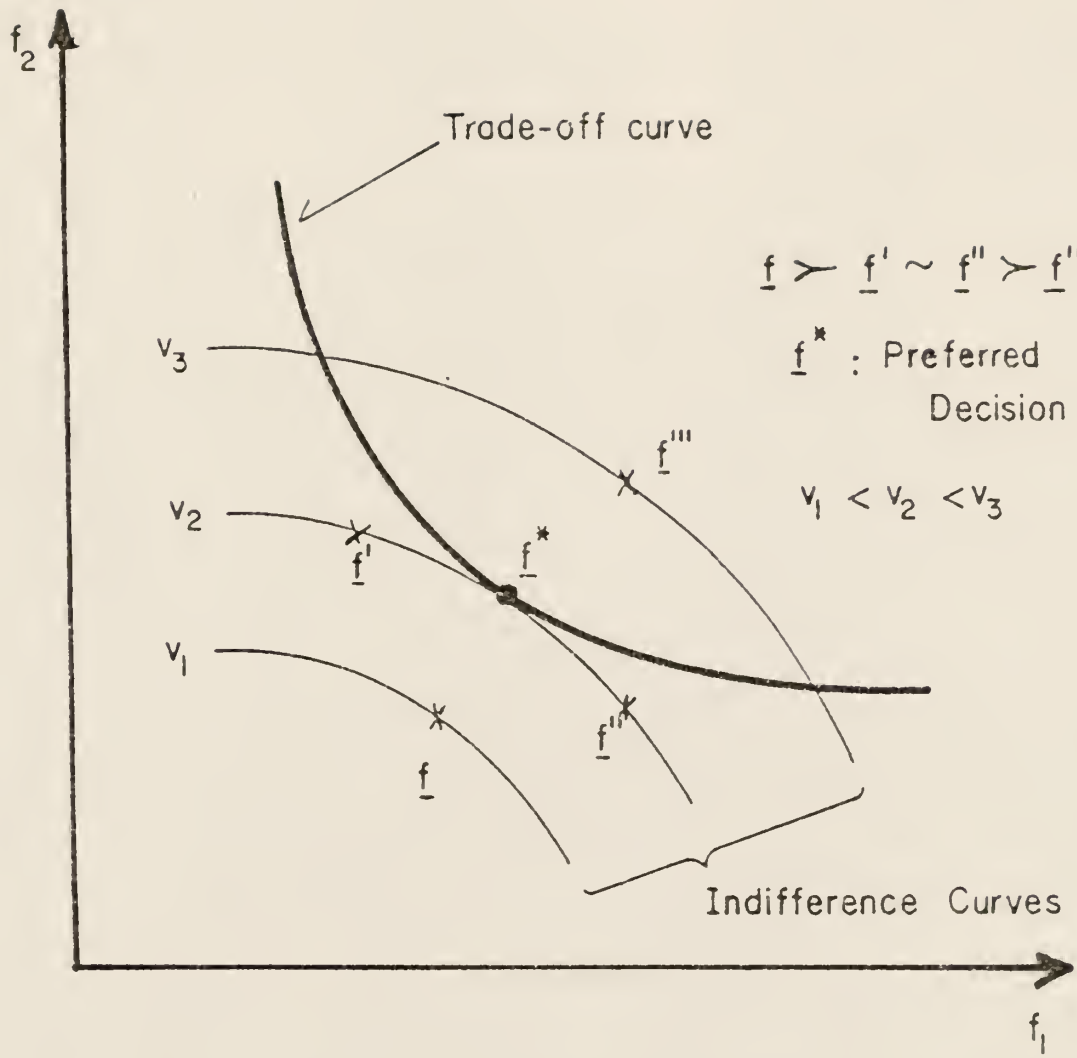


Fig. 7. Graphical approach of the indifference function method (Keeney and Raiffa, 1976).

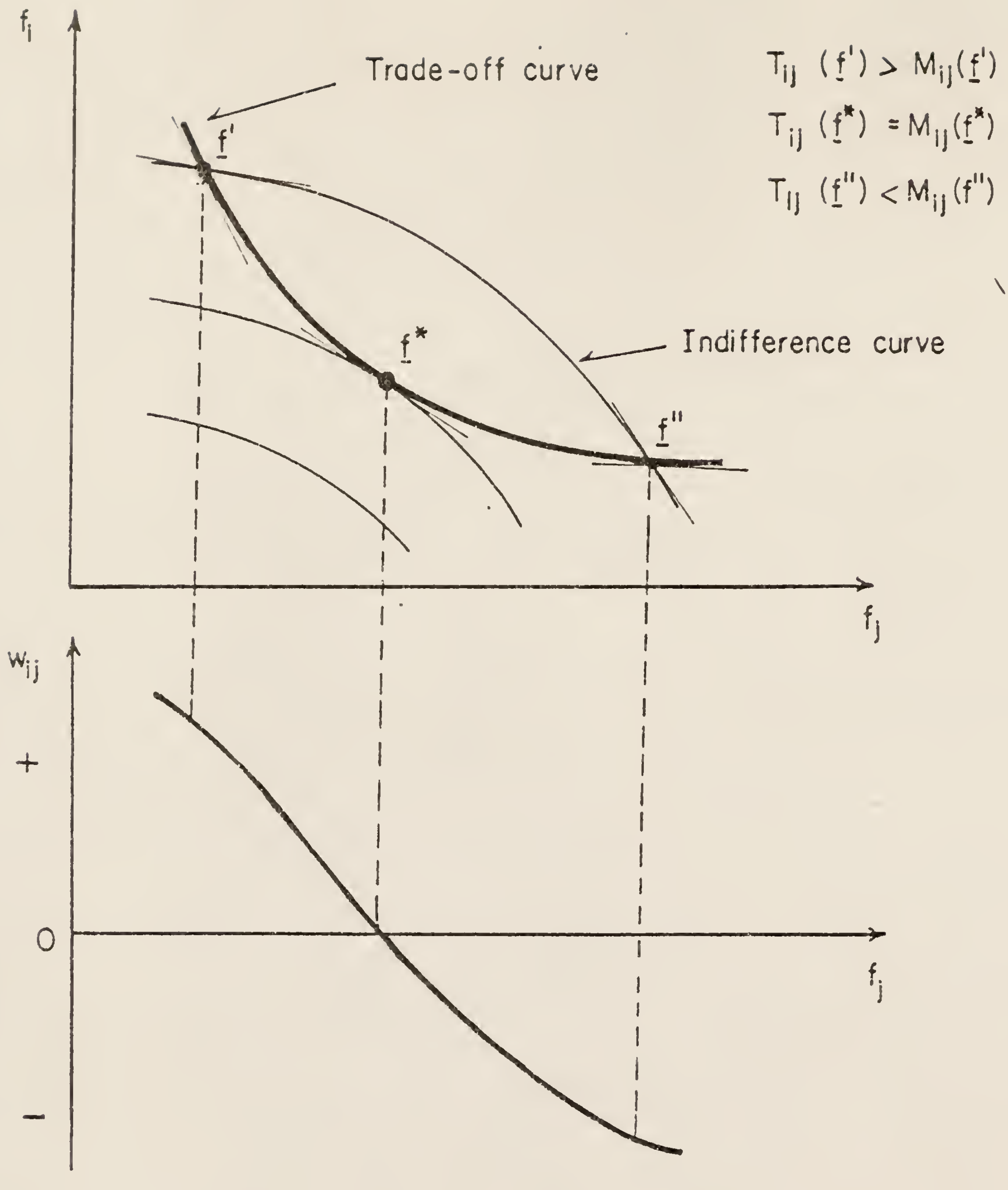


Fig. 8. Surrogate worth function W_{ij} (Nakayama and Sawaragi, 1976).

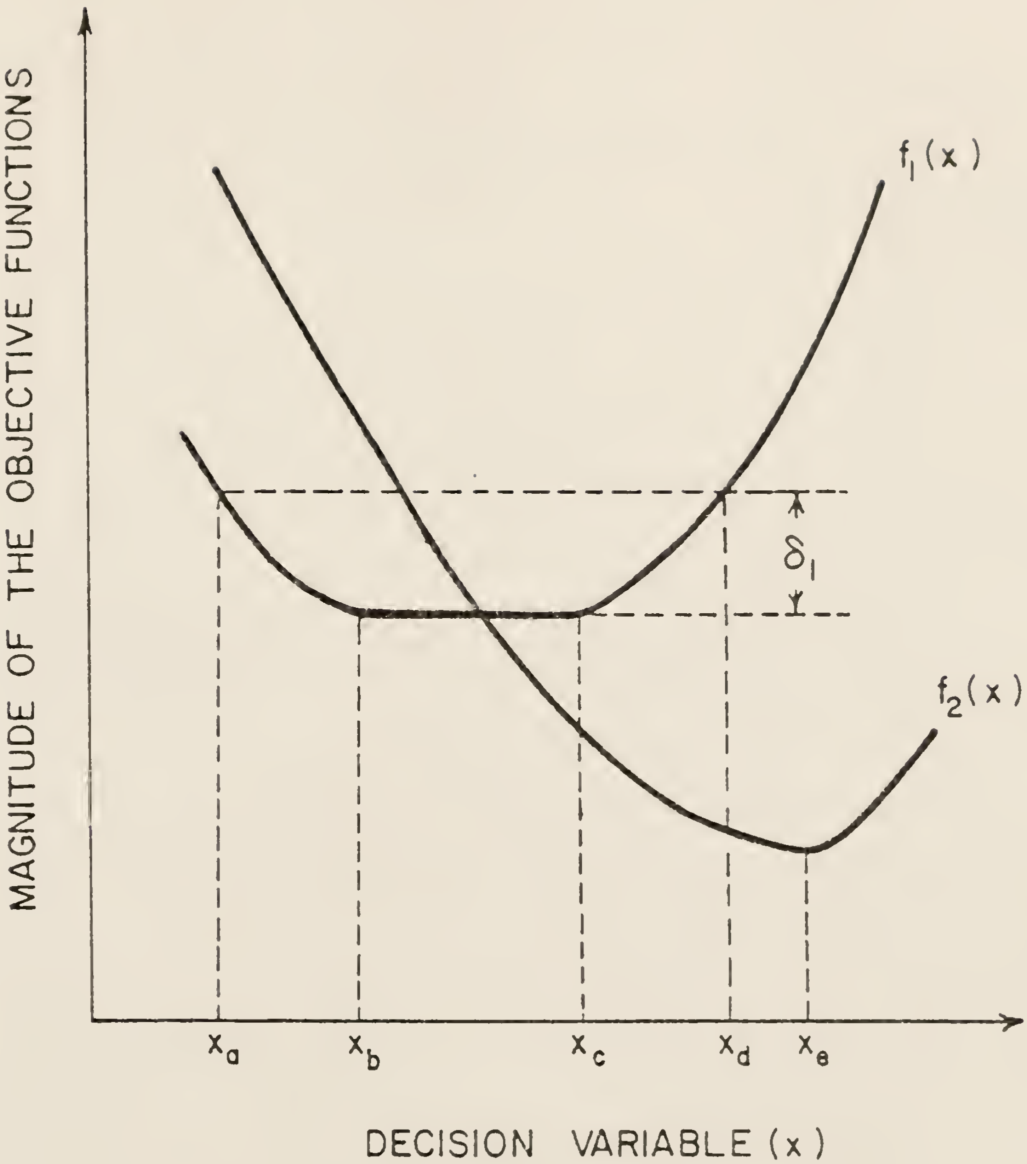


Fig. 9. Lexicographic approach for a problem with two objectives and one decision variable.

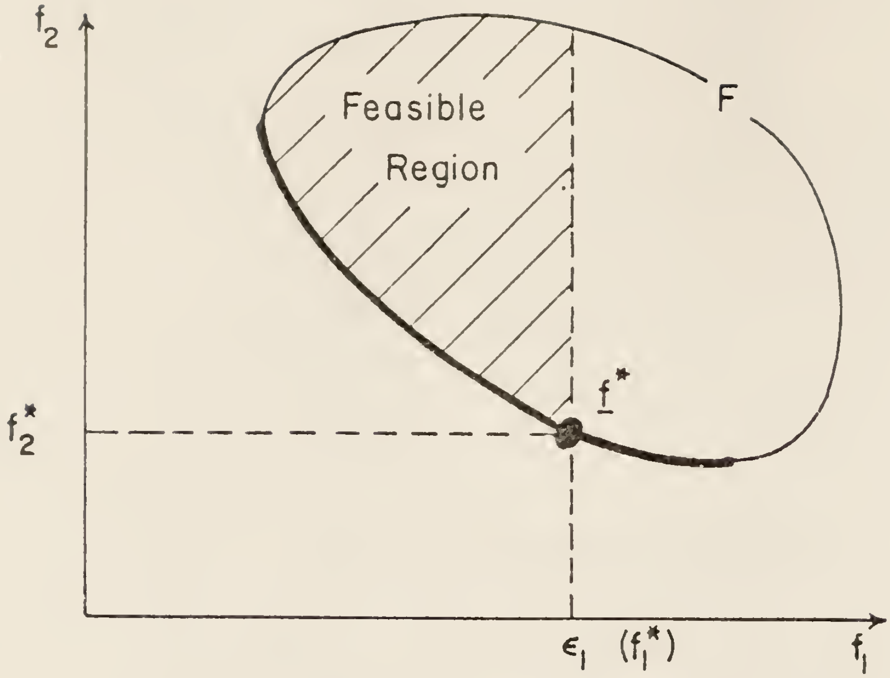


Fig. 10-a. ε-constraint method for a convex problem.

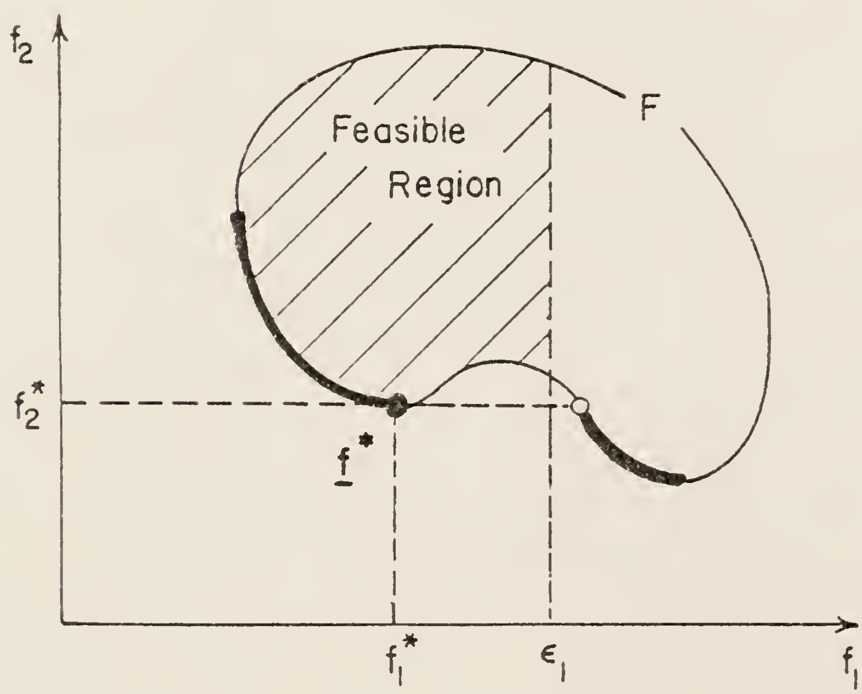


Fig. 10-b. ε-constraint method for a non-convex problem.

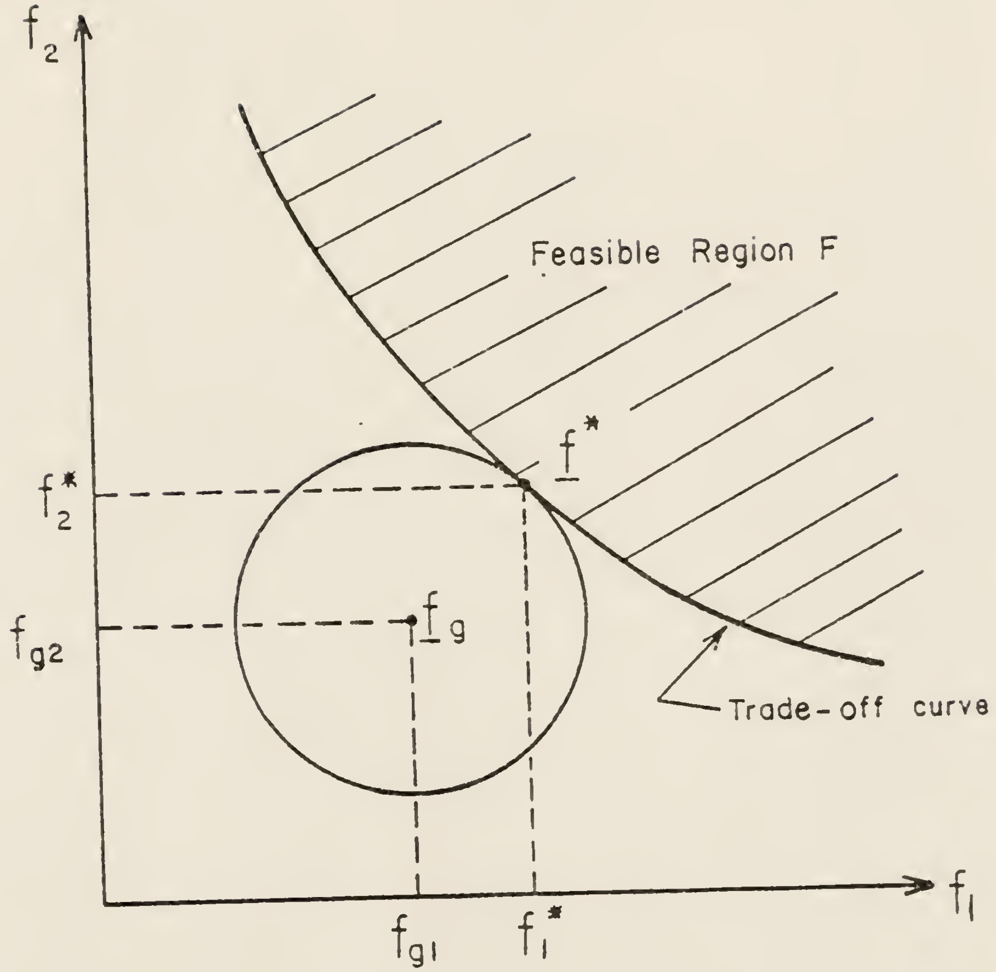


Fig. 11. Goal programming with $d(\underline{f}, \underline{f}g) = \{\sum_{i=1}^{2} (f_i - f_{gi})^2\}^{1/2}$

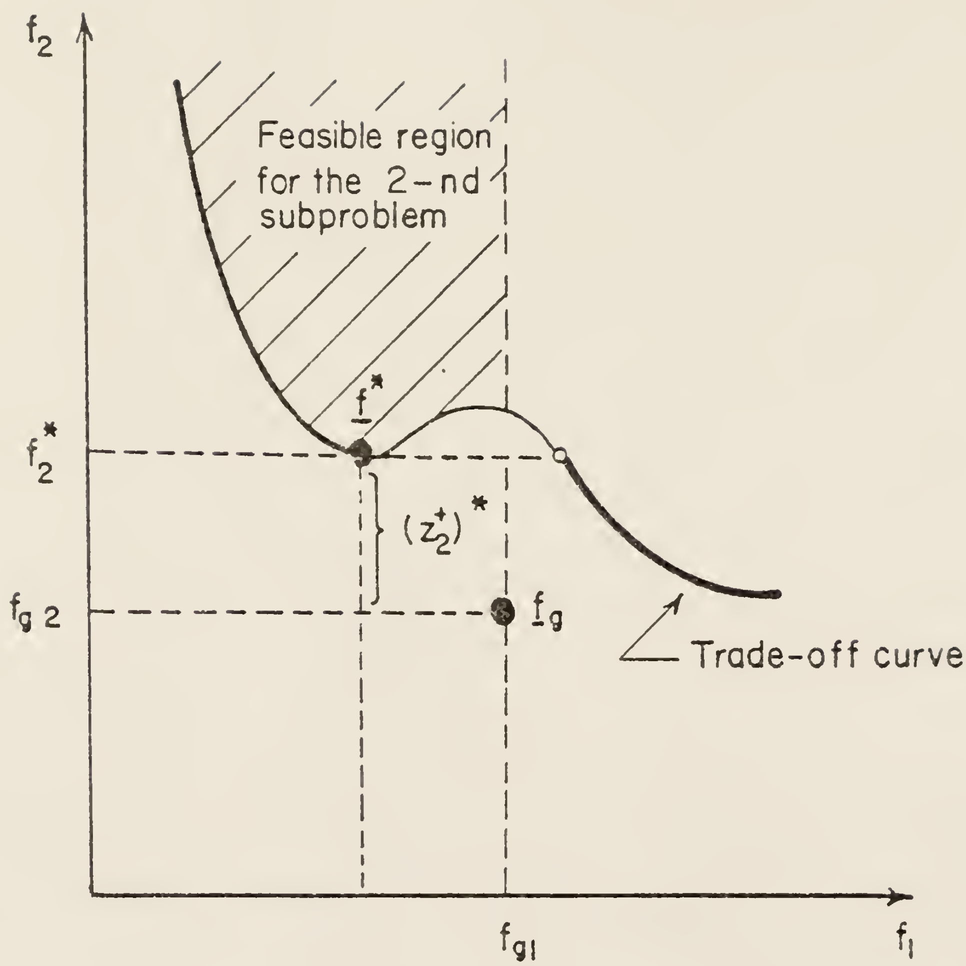


Fig. 12. Ignizio's goal programming.

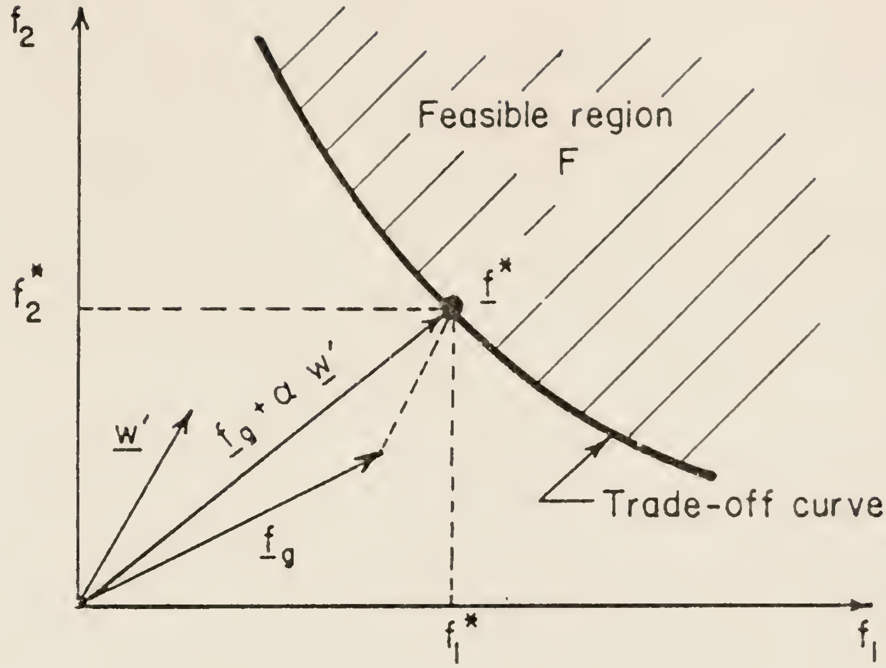


Fig. 13-a. Goal attainment method for a convex problem.

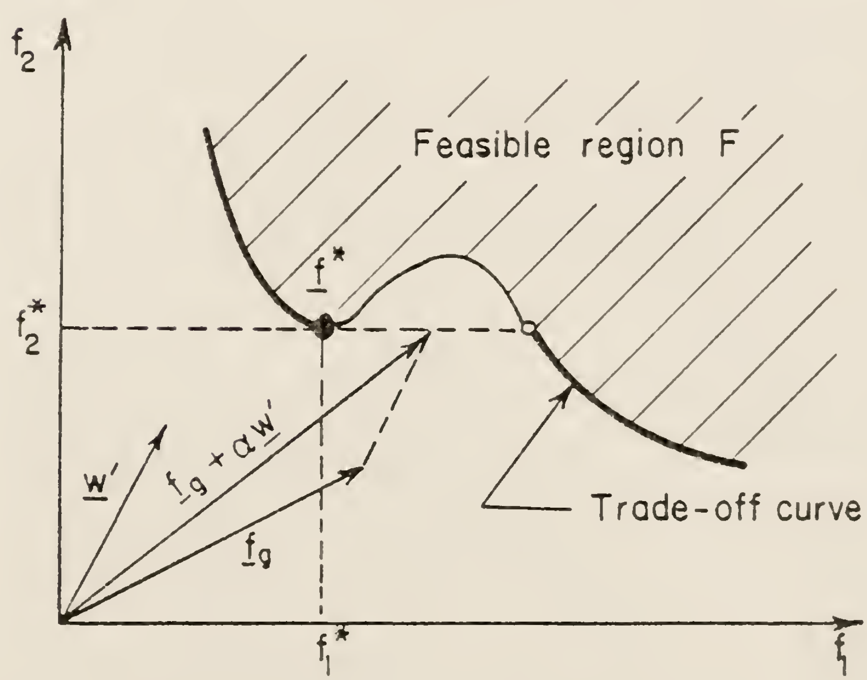


Fig. 13-b. Goal attainment method for a non-convex problem.

CHAPTER III

MULTI-OBJECTIVE OPTIMIZATION OF A HEAT EXCHANGE NETWORK SYSTEM

MULTI-OBJECTIVE OPTIMIZATION OF A HEAT EXCHANGE NETWORK SYSTEM

1. INTRODUCTION

The optimization of a heat exchanger system for energy recovery has been attracting increasing attention because of the public's concern over the energy shortage and thermal pollution of the environment. This fact is reflected in the recent increase in the number of published papers on this subject (see, e.g., Hendry et al., 1973; Hoffman, 1974; Chen, 1977).

Each of the early works on the optimization of heat exchanger systems has taken into account only a single objective such as the capital cost (e.g., Rudd, 1968; Henley and Williams, 1973; Takamatsu et al., 1976; Chen, 1977) or a combination of the capital and running costs (e.g., Hwa, 1965; Masso and Rudd, 1969; Lee et al., 1970; Kobayashi et al., 1971; Pho and Lapidus, 1973; Rathore and Powers, 1975; Nishida et al., 1976). Minimization of the cost function does not necessarily lead to an effective design of the heat exchanger system from the standpoint of energy conservation. Because of the artificial manipulation of the market or the arbitrarily pricing of an energy resource, its cost in a monetary unit does not always reflect its availability or usefulness.

Thermodynamically, the energy conservation is equivalent to the minimization of the loss of available energy in a system. The available energy, which can be converted into mechanical energy, cannot be recovered by any means, once it is lost. It appears that the thermodynamics has been totally neglected in studies of heat exchanger system synthesis except in one case (Umeda et al., 1977).

In addition to the conventional objective function, namely, the cost, the present work introduces the rate of available energy loss as the second objective function to be minimized in optimizing a heat exchanger system. Since minimizing the available energy loss requires the maximum heat transfer area which renders the capital cost maximum, the added objective function is in conflict with the conventional one. This work shows that the techniques for solving a multi-objective problem (see Chapter II) can be applied to the analysis and synthesis of the heat exchanger network system.

2. LARGE HEAT EXCHANGER SYSTEM

The system considered here consists of nine heat exchangers of the counter-current type and involves three cold streams and four hot streams (see Fig. 1). The configuration of the system is similar to those analyzed by Hwa (1965), Takamatsu et al. (1970, 1976), Henley and Williams (1973), and Chen (1977).

2.1 Process Equations for a Heat Exchanger of the Counter-Current Type

In this section, we derive the basic equations governing each heat exchanger included in the system, i.e., the relationships among the inlet and outlet temperatures of the hot and cold streams through the heat exchanger. Figure 2 shows briefly the temperature distributions of both streams in the heat exchanger. T_i and T_o denote, respectively, the inlet and outlet temperatures for the cold stream; similarly, t_i and t_o denote, respectively, the inlet and outlet temperatures for the hot stream. The heat transfer rate and heat balance equations are, respectively,

$$\frac{dT}{da} = - \frac{U}{WC_p} (t - T) \tag{1}$$

$$wc_p \frac{dt}{da} = WC_p \frac{dT}{da} \tag{2}$$

where

T = temperature of the cold stream, $^{\circ}R$

t = temperature of the hot stream, $^{\circ}R$

a = heat transfer area, ft^2

U = overall heat transfer coefficient, Btu/hr-ft^2-$^{\circ}R$

WC_p = heat capacity flow rate of the cold stream, Btu/hr-$^{\circ}R$

wc_p = heat capacity flow rate of the hot stream, Btu/hr-$^{\circ}R$

Eq. (2) can be rewritten as

$$\frac{dt}{dT} = \frac{WC_p}{wc_p} \equiv R \tag{3}$$

Here, the ratio of heat capacity flow rates, R, is assumed invariant with respect to temperature variation. Integrating Eq. (3) subject to the boundary condition at a = 0 (see Fig. 2) yields

$$t - t_i = R(T - T_o) \tag{4}$$

This equation must satisfy the condition at a = A also, i.e.,

$$t_o - t_i = R(T_i - T_o) \tag{5}$$

which is the overall heat balance around the heat exchanger. Substitution of Eq. (4) into Eq. (1) gives rise to

$$\frac{dT}{da} = \frac{U}{WC_p} \{-(R-1)T + RT_o - t_i\} \tag{6}$$

If $R \neq 1$, by applying the boundary condition at a = A, we have

$$T = \frac{t_i - T_o}{R-1} \exp\left\{ - \frac{Ua}{WC_p} (R - 1) \right\} + \frac{RT_o - t_i}{R - 1}$$

Since $T = T_i$ at a = A, this equation becomes

$$T_i = \frac{t_i - T_o}{k(R - 1)} + \frac{RT_o - t_i}{R - 1} \tag{7}$$

where

$$k = \exp\left\{ \frac{UA}{WC_p} (R - 1) \right\} \tag{8}$$

The process performance of the heat exchanger is expressed by Eqs. (5) and (7). These process equations can be rearranged in the following linear forms (Yagi and Nishimura, 1969; Takamatsu et al., 1976; also see Appendix A):

$$\begin{pmatrix} T_o \\ t_o \end{pmatrix} = \begin{pmatrix} 1 - \alpha & \alpha \\ \alpha R & 1 - \alpha R \end{pmatrix} \begin{pmatrix} T_i \\ t_i \end{pmatrix} \tag{9a}$$

$$\begin{pmatrix} T_i \\ t_i \end{pmatrix} = \frac{1}{1 - R - \alpha R} \begin{pmatrix} 1 - \alpha R & -\alpha \\ -\alpha R & 1 - \alpha \end{pmatrix} \begin{pmatrix} T_o \\ t_o \end{pmatrix} \tag{9b}$$

$$\begin{pmatrix} T_i \\ t_o \end{pmatrix} = \frac{1}{1 - \alpha} \begin{pmatrix} 1 & -\alpha \\ \alpha R & 1 - \alpha - \alpha R \end{pmatrix} \begin{pmatrix} T_o \\ t_i \end{pmatrix} \tag{9c}$$

$$\begin{pmatrix} T_o \\ t_i \end{pmatrix} = \frac{1}{1 - \alpha R} \begin{pmatrix} 1 - \alpha - \alpha R & \alpha \\ -\alpha R & 1 \end{pmatrix} \begin{pmatrix} T_i \\ t_o \end{pmatrix} \tag{9d}$$

where

$$\alpha = \frac{k - 1}{Rk - 1} \tag{10}$$

Any one of Eqs. (9a) through (9d) is identical to the combination of Eqs. (5) and (7); however, it is easier to use the former than the latter if A and any two of t_i, t_o, T_i and T_o are known. Furthermore, by eliminating R from Eqs. (5) and (7), we have

$$k = \frac{t_i - T_o}{t_o - T_i}$$

or

$$\exp\left\{ \frac{UA}{WC_p} (R - 1) \right\} = \frac{t_i - T_o}{t_o - T_i}$$

Therefore, we have another pair of the process equations:

$$\left. \begin{aligned} & t_i - t_o = R(T_o - T_i) \\ & A = \frac{WC_p}{U(R - 1)} \ln \left(\frac{t_i - T_o}{t_o - T_i} \right) \end{aligned} \right\} \tag{9e}$$

which is a convenient form to evaluate the heat transfer area from any three known temperatures among t_i, t_o, T_i and T_o. A process equation is selected from Eqs. (9a) through (9e), depending on the problem to which it is applied. Note that Eq. (9e) can be transformed into the commonly used form for the counter-current heat exchanger;

$$Q = WC_p(T_o - T_i) = wc_p(t_i - t_o) = UA\Delta t_m$$

where

$$\Delta t_m = \frac{(t_i - T_o) - (t_o - T_i)}{\ln\left(\frac{t_i - T_o}{t_o - T_i}\right)}$$

In case both of the hot and cold streams have the same heat capacity flow rate, i.e., R = 1, the equations corresponding to Eqs. (9a) through (9e) can be obtained by letting R approach unity as shown below.

The constant, α, in Eqs. (9a) through (9e) is a function of R, and

$$\begin{aligned}\lim_{R \to 1} \alpha &= \lim_{R \to 1} \frac{k - 1}{Rk - 1} \\ &= \lim_{R \to 1} \frac{\left\{\frac{dk}{dR}\right\}}{\left\{k + R\,\frac{dk}{dR}\right\}} \\ &= \frac{UA/WC_p}{1 + UA/WC_p} \qquad (11) \\ &\equiv \beta\end{aligned}$$

where

$$k = \exp\left\{\frac{UA}{WC_p}(R - 1)\right\}$$

Substitution of Eq. (11) and R = 1 into Eqs. (9a) through (9e) yields respectively,

$$\begin{pmatrix} T_o \\ t_o \end{pmatrix} = \begin{pmatrix} 1-\beta & \beta \\ \beta & 1-\beta \end{pmatrix} \begin{pmatrix} T_i \\ t_i \end{pmatrix} \tag{12a}$$

$$\begin{pmatrix} T_i \\ t_i \end{pmatrix} = \frac{1}{1-2\beta} \begin{pmatrix} 1-\beta & -\beta \\ -\beta & 1-\beta \end{pmatrix} \begin{pmatrix} T_o \\ t_o \end{pmatrix} \tag{12b}$$

$$\begin{pmatrix} T_i \\ t_o \end{pmatrix} = \frac{1}{1-\beta} \begin{pmatrix} 1 & -\beta \\ \beta & 1-2\beta \end{pmatrix} \begin{pmatrix} T_o \\ t_i \end{pmatrix} \tag{12c}$$

$$\begin{pmatrix} T_o \\ t_i \end{pmatrix} = \frac{1}{1-\beta} \begin{pmatrix} 1-2\beta & \beta \\ -\beta & 1 \end{pmatrix} \begin{pmatrix} T_i \\ t_o \end{pmatrix} \tag{12d}$$

$$\left.\begin{aligned} & t_i - t_o = T_o - T_i \\ & A = \frac{WC_p}{U} \, \frac{T_o - T_i}{t_o - T_i} \end{aligned}\right\} \tag{12e}$$

Equations (12a) through (12e) can also be obtained directly by integrating Eq. (6) with R = 1.

2.2 Design Objectives and Constraints

The heat exchanger network system shown in Fig. 1 is to be optimally designed. Table 1 summarizes all specified conditions including the initial and terminal temperatures, the heat capacity flow rates, and the overall heat transfer coefficients. There are 24 design variables involved in the system; 6 cold stream temperatures (T_{A2}, T_{B2}, T_{B3}, T_{C2}, T_{C3}, T_{C4}), 9 hot stream temperatures (t_{A2}, t_{A3}, t_{B2}, t_{B3}, t_{C2}, t_{C3}, t_{C4}, t_{D2}, t_{D3}), and 9 heat transfer areas (A_j, j = 1, 2, ..., 9). These design variables must be subject to 18 equality constraints which are process equations for 9 heat exchangers, yielding 6 degrees of freedom for optimization. The nine inequality constraints imposed on the system are

$$A_j \geq 0, \qquad j = 1, 2, \ldots, 9$$

These constraints prevent us from considering an unrealizable design.

Two objective functions to be minimized are considered here. One is the total heat transfer area, namely,

$$f_1 = \sum_{j=1}^{9} A_j \tag{13}$$

which is the objective function adopted by Takamatsu et. al. (1970, 1976), Henley and Williams (1972), and Chen (1977). While some investigators (Pho and Lapidus, 1973; Ponton and Donaldson, 1974; Kelahan and Gaddy, 1976) have employed the sum of $(A_j)^{0.6}$ rather than Eq. (13) to take into account the economies of scale, the ensuing discussion is essentially valid, even if A_j in Eq. (13) is replaced by $(A_j)^{0.6}$.

The other objective function is a thermodynamical index; the rate of available energy loss (Keenan, 1941, 1951);

$$f_2 = T_{sur}\dot{\sigma} \tag{14}$$

where T_{sur} is a temperature of the surroundings, namely, $537^{\circ}R$ ($298^{\circ}K$), and $\dot{\sigma}$ is the overall rate of entropy creation. The reduction in the available energy loss is desirable from the standpoint of energy conservation. Suppose that the process is operated in a steady state fashion without heat exchange between it and the surroundings. Eq. (14), then, becomes (see Appendix B)

$$f_2 = T_{sur} \{ \sum_{j=A}^{C} (WC_p)_j \ln \frac{T_{jo}}{T_{ji}} + \sum_{j=A}^{D} (wc_p)_j \ln \frac{t_{jo}}{t_{ji}} \} \qquad (15)$$

The first term in the square bracket is constant because the initial and terminal temperatures of the cold stream are fixed. Hence, the value of Eq. (15) is affected only by the second term associated with the hot streams.

The two objectives, Eqs. (13) and (15), to be minimized are in conflict with each other. Note that minimizing the available energy loss, e.g., minimizing $\dot{\sigma}$, requires the maximum heat transfer area. Thus, the techniques for a multi-objective problem are applicable to this problem.

3. TRADE-OFF CURVE FOR THE HEAT EXCHANGE SYSTEM

The trade-off curve, which is the locus of non-inferior decisions, provides useful information for the designer to select the preferred design of the system with multiple objectives. There are several techniques for generating the trade-off curve, which are described in the previous chapter. Since the objective functions given by Eqs. (13) and (15) are not of the Cobb-Douglas type, the functional relationship for the trade-off curve (see section 3.3, Chapter II) can not be identified explicitly, and thus the analytical approach is not useful for the present heat exchanger system. However, all other approaches are applicable to this problem.

We resort to the most commonly used ε-constraint method (see section 3.2, Chapter II) to generate the trade-off curve. The ε-constraint method for a problem with two objectives is formulated as:

Minimize

$$J = f_2 \tag{16}$$

subject to

$$\underline{g} \geq 0 \tag{17}$$

$$f_1 \leq \varepsilon \tag{18}$$

where Eq. (17) represents the entire set of equality and inequality constraints. The trade-off curve is obtained by solving this optimization problem by parametrically changing ε. Note that ε cannot be less than a certain value which is the minimum value of f_1.

3.1 Minimum Total Heat Transfer Area

The minimum total heat tranfer area for the heat exchanger system can be attained by solving the following optimization problem:

Minimize

$$J = f_1 = \sum_{j=1}^{9} A_j \tag{19}$$

subject to

$$\underline{h} = 0 \tag{20}$$

$$A_j \geq 0, \qquad j = 1, 2, \ldots, 9 \tag{21}$$

where Eq. (20) stands for 18 process equations for 9 heat exchangers involved in the system. Chen (1977) has demonstrated that the adaptive random search technique developed by Fan et al. (1975) is effective for this problem. We reoptimize the system by the random search technique by selecting a different set of independent variables from Chen's selections

As explained in the previous section, there are 6 degrees of freedom in this problem. A_1, A_2, A_4, A_5, A_6, and A_7 are chosen as independent variables, which allow us to solve readily the process equations and to determine the other dependent design variables. The calculation scheme is given below.

Step 1. Assign positive numbers to A_1, A_2, A_4, A_5, A_6, and A_7 at random, and calculate α for each heat exchange by Eqs. (8) and (10).

Step 2. To evaluate all design variables, solve the 18 process equations in seriatim, starting with heat exchanger 1, as

Heat exchanger 1 [see Eq. (9a)],

$$\begin{pmatrix} T_{B2} \\ t_{A2} \end{pmatrix} = \begin{pmatrix} 1 - \alpha_1 & \alpha_1 \\ \alpha_1 R_1 & 1 - \alpha_1 R_1 \end{pmatrix} \begin{pmatrix} T_{B1} \\ t_{A1} \end{pmatrix}$$

Heat exchanger 2 [see Eq. (9a)],

$$\begin{pmatrix} T_{A2} \\ t_{A3} \end{pmatrix} = \begin{pmatrix} 1 - \alpha_2 & \alpha_2 \\ \alpha_2 R_2 & 1 - \alpha_2 R_2 \end{pmatrix} \begin{pmatrix} T_{A1} \\ t_{A2} \end{pmatrix}$$

Heat exchanger 3 [see Eq. (9e)],

$$t_{D2} = R_3 (T_{A2} - T_{A3}) + t_{D1}$$

$$A_3 = \frac{(WC_p)_A}{U_3(R_3 - 1)} \ln \frac{t_{D1} - T_{A3}}{t_{D2} - T_{A2}}$$

Heat exchanger 4 [see Eq. (9a)],

$$\begin{pmatrix} T_{C2} \\ t_{D3} \end{pmatrix} = \begin{pmatrix} 1 - \alpha_4 & \alpha_4 \\ \alpha_4 R_4 & 1 - \alpha_4 R_4 \end{pmatrix} \begin{pmatrix} T_{C1} \\ t_{D2} \end{pmatrix}$$

Heat exchanger 5 [see Eq. (9c)],

$$\begin{pmatrix} T_{C4} \\ t_{C2} \end{pmatrix} = \frac{1}{1 - \alpha_5} \begin{pmatrix} 1 & -\alpha_5 \\ \alpha_5 R_5 & 1 - \alpha_5 - \alpha_5 R_5 \end{pmatrix} \begin{pmatrix} T_{C5} \\ t_{C1} \end{pmatrix}$$

Heat exchanger 6 [see Eq. (9a)],

$$\begin{pmatrix} T_{C3} \\ t_{C3} \end{pmatrix} = \begin{pmatrix} 1 - \alpha_6 & \alpha_6 \\ \alpha_6 R_6 & 1 - \alpha_6 R_6 \end{pmatrix} \begin{pmatrix} T_{C2} \\ t_{C2} \end{pmatrix}$$

Heat exchanger 7 [see Eq. (9a)],

$$\begin{pmatrix} T_{B3} \\ t_{C4} \end{pmatrix} = \begin{pmatrix} 1 - \alpha_7 & \alpha_7 \\ \alpha_7 R_7 & 1 - \alpha_7 R_7 \end{pmatrix} \begin{pmatrix} T_{B2} \\ t_{C3} \end{pmatrix}$$

Heat exchanger 8 [see Eq. (9e)],

$$t_{B2} = R_8(T_{B3} - T_{B4}) + t_{B1}$$

$$A_8 = \frac{(WC_p)_B}{U_8(R_8 - 1)} \ln \frac{t_{B1} - T_{B4}}{t_{B2} - T_{B3}}$$

Heat exchanger 9 [see Eq. (9e)],

$$t_{B3} = R_9(T_{C3} - T_{C4}) + t_{B2}$$

$$A_9 = \frac{(WC_p)_C}{U_9(R_9 - 1)} \ln \frac{t_{B2} - T_{C4}}{t_{B3} - T_{C3}}$$

Since six out of nine A_j's are selected as independent variables and only positive numbers are assigned to them in Step 1, six inequality constraints can be eliminated from Eq. (21). It is worth noting that there are only 3 inequality constraints ($A_j \geq 0$, j = 3, 8, 9) that need to be checked here, as compared to 36 inequality constraints imposed by Chen (1977).

Table 2 summarizes the optimal results by this method along with Chen's results. The designs of the system are slightly different, but the total heat transfer areas are essentially identical. The minimum value of f_1 which is 70,071 yields the lower limit of ε in Eq. (18). The optimal design derived here is depicted in Fig. 3.

3.2 Trade-Off Curve

Figure 4 displays the trade-off curve for the heat exchanger system under consideration. The curve has been constructed from the solutions of the following problem by varying the parameter, ε, from 70,071 to 140,000:

Minimize

$$J = f_2 = T_{sur} \{ \sum_{j=A}^{C} (WC_p)_j \ln \frac{T_{jo}}{T_{ji}} + \sum_{j=A}^{D} (wc_p)_j \ln \frac{t_{jo}}{t_{ji}} \} \qquad (22)$$

subject to

$$\underline{h} = 0 \qquad (23)$$

$$A_j \geq 0, \qquad j = 1, 2, \ldots, 9 \tag{24}$$

$$f_1 = \sum_{j=1}^{9} A_j \leq \varepsilon \tag{25}$$

The independent variables selected and the scheme for solving Eq. (23) are the same as those employed in evaluating the minimum total heat transfer area in the previous section.

As can be seen in Fig. 4, this problem is a typical convex problem. As the total heat transfer area increases, both the rate of available energy loss and trade-off ratio between the two objectives decrease gradually. The rate of available energy loss can be reduced to approximately half if its maximum value, if the total heat transfer area is allowed to be roughly twice its minimum value, i.e., $f_1 = 14 \times 10^4$ ft^2 (1.3×10^4 m^2).

Figure 5 shows the change in the optimal design of the system as ε increases. The sizes of most heat exchangers change abruptly around $f_1 = 113{,}000$. This is probably due to the fact that the configuration of the system is changed by the appearance of heat exchanger 6. Table 3 provides the numerical values of 9 non-inferior designs selected along the trade-off curve.

4. SELECTION OF THE PREFERRED DECISION

This section is devoted to the extraction of the preferred designs of the heat exchanger system by applying the three different methods. The author is the decision maker here and, therefore, his preference is reflected in the allowable limit of f_1 in the lexicographic approach, the weighting coefficient in the weighting method and the surrogate worth function in the surrogate worth trade-off method.

4.1 Lexicographic Approach

The lexicographic approach proposed by Waltz (1967) is utilized to seek the preferred design of the system (see section 5.5, Chapter II). The first objective, namely, the total heat transfer area is assumed to be more important than the second objective, namely, the rate of available energy loss. Thus, the first subproblem to be solved is as follows:

Minimize

$$J_1 = f_1 = \sum_{j=1}^{9} A_j$$

subject to

$$\underline{h} = 0$$

$$A_j \geq 0 , \qquad j = 1,2,\ldots,9$$

This is identical to the problem of the minimum total heat transfer area solved in subsection 3.1. The minimum value J_1 is already known to be 70,071.

Since the J_1-minimum is 70,071, we allow the total heat transfer area to be as large as 80,000. Then, the following second subproblem yields the preferred design.

Minimize

$$J_2 = f_2 = T_{sur} \left\{ \sum_{j=A}^{C} (WC_p)_j \ln \frac{T_{jo}}{T_{ji}} + \sum_{j=A}^{D} (wc_p)_j \ln \frac{t_{jo}}{t_{ji}} \right\}$$

subject to

$$\underline{h} = 0$$

$$A_j \geq 0 , \qquad j = 1,2,\ldots,9$$

$$f_1 = \sum_{j=1}^{9} A_j \leq 80,000$$

The independent variables selected and the scheme for evaluating all dependent variables are the same as those employed in section 3. The results of this optimization are listed in the first columns of Table 4 and also depicted in Fig. 6.

Compared to the single-objective design given in Table 2, the preferred two-objective design contains an additional exchanger (exchanger 1), has a larger exchanger 4 and somewhat smaller exchangers 7 and 9. These observations show that exchangers 7 and 9 are especially effective for reducing the system size, while exchangers 1 and 4 are especially effective for conserving the available energy. Overall, the increase of approximately 10,000 ft^2 in the heat transfer area results in the decrease of 0.17×10^8 Btu/hr in the loss of available energy.

The formulation of the second subproblem is the same as those of the ε-constraint method with $\varepsilon_1 = 80,000$ (see section 5.7, Chapter II) and the Ignizio's goal programming with $f_{g1} = 80,000$ and $f_{g2} = 0$ (see section 5.8, Chapter II). The differences among these methods, which are discussed in section 5.5 of Chapter II, are significant only for a problem with more than two objectives.

4.2 Weighting Method

Suppose that the value function of the heat exchanger system can be expressed as a linear combination of the two objectives, f_1 and f_2, whose weighting coefficients are 1,000 and 1, respectively, i.e.,

$$v(\underline{f}) = (1{,}000)\ f_1 + (1)\ f_2 \tag{26}$$

Since the trade-off curve of this problem is convex, the preferred design can be uniquely obtained by the weighting method stated as (see section 5.6, Chapter II)

Minimize

$$J = v(\underline{f})$$

$$= (1{,}000) \sum_{j=1}^{9} A_j + T_{sur}\left\{ \sum_{j=A}^{C} (WC_p)_j \ln \frac{T_{jo}}{T_{ji}} + \sum_{j=A}^{D} (wc_p)_j \ln \frac{t_{jo}}{t_{ji}} \right\} \tag{27}$$

subject to

$$\underline{h} = 0$$

$$A_j \geq 0 \ , \qquad j = 1,2,\ldots,9$$

The second column in Table 4 shows the preferred design obtained by this method, which is comparable with that obtained by the lexicographic approach in the first column. Although the sizes of the exchangers are slightly different, the structures of the networks are essentially identical in both designs (see Fig. 7). Note that the trade-off ratio, T_{21}, must be 1,000 for this preferred design (see section 5.6, Chapter II).

4.3 Surrogate Worth Trade-off Method

To apply the surrogate worth trade-off method (see section 5.3, Chapter II), 15 non-interior points on the trade-off curve in Fig. 4

have been selected, and the objective values of these non-interior points have been calculated by the ε-constraint method (see section 3.3, Chapter II). Table 5 lists these results. The trade-off ratio, T_{21}, is essentially the generalized Lagrangian multiplier, λ_{21}, which satisfies the following Kuhn-Tucker condition (see section 4, Chapter II):

$$\left(\frac{\partial f_2}{\partial \underline{x}}\right)^T + \left(\frac{\partial f_1}{\partial \underline{x}}\right)^T \lambda_{21} + \left(\frac{\partial \underline{h}}{\partial \underline{x}}\right)^T \underline{\mu} = 0 \tag{28}$$

$$\lambda_{21}(f_1 - \varepsilon) = 0$$

$$\lambda_{21} > 0$$

$$\underline{h} = 0$$

$$\underline{\mu} > 0$$

where $\underline{\mu}$ is a Lagrangian multiplier vector and $\underline{x}$ is the independent design variable vector, i.e.,

$$\underline{x} = (A_1, A_2, A_4, A_5, A_6, A_7)^T$$

It is difficult to solve this condition analytically or numerically because of the complexity of the forms of f_2 and $\underline{h}$. Since the trade-off ratio is defined as

$$T_{21} = -\frac{\partial f_2}{\partial f_1}, \tag{29}$$

it can be approximately estimated as

$$T_{21} \simeq -\frac{f_2(f_1+\Delta f_1) - f_2(f_1-\Delta f_1)}{2\Delta f_1} \tag{30}$$

The trade-off ratios listed in Table 5 have been obtained by this approximation.

The fourth column of Table 5 shows the values of the surrogate worth function, W_{21}, for each non-inferior point, which are the responses of the decision maker to the question: Is it desirable to reduce f_2 by T_{21}

units by increasing f_1 by a single unit? The decision maker has assigned an integer between -10 and 10 to each trade. A positive integer indicates a desirable trade and vice versa. The larger the absolute value of W_{21}, the greater the desirability or undesirability of the trade. Since a numerical value of zero is assigned to the non-inferior point, $(f_1,f_2) = (85{,}000,\ 0.892 \times 10^8)$, it is the preferred point. The preferred design corresponding to this point is shown in Fig. 8 and tabulated in Table 5 along with those obtained by the lexicographic approach and the weighting method. The preferred design obtained by this method lays the most stress on the reduction of the available energy loss among these preferred designs, but the differences among the designs are not appreciable.

5. CONCLUDING REMARKS

A large heat exchanger system has been optimized by the use of techniques for a multi-objective optimization coupled with a random search technique. The set of independent variables selected here leads to a simple calculation scheme for solving the equality constraints and reduces drastically the number of equality constraints to be computed.

The trade-off curve between the total heat transfer area and the rate of available energy loss shows that the feasible region of this problem is convex in the objective space. The preferred designs have been obtained by three different methods. These designs are essentially identical; the heat exchanger sizes are slightly different, but the differences are not significant when compared to the possible errors involved in modeling such a large system. The comparison between the design minimizing the total heat transfer area only and the preferred designs obtained here indicates that heat exchangers 7 and 9 are effective for economizing the system size, while heat exchangers 1 and 4 are effective for minimizing the available energy loss.

The weighting method is the simplest among the three methods for selecting the preferred design. However, it is not easy to specify in advance the reasonable weighting coefficients for a problem with more than several objectives. The surrogate worth trade-off method is time-consuming and laborious, but it does not require any subjective judgment of the decision maker until the calculation is completed. The lexicographic approach is situated between the previous two methods, and the allowable levels of each objective are successively determined by the decision maker in the course of the hierarchial calculation. Since the lexicographic approach is fairly simple conceptually and numerically, it may be applicable to many engineering problems with multiple objectives.

6. SUMMARY

The process equations have been derived for a heat exchanger of the counter-current type, and they are rearranged in linear forms. A large heat exchanger system consisting of 9 heat exchangers, 3 cold streams and 4 hot streams have been optimized. The equality constraints imposed are given by 9 sets of the process equations for the 9 heat exchangers. The inequality constraints express the fact that none of the heat transfer areas can be negative. The total heat transfer area and the rate of available energy loss have been employed as objective functions to be minimized.

The trade-off curve between the two objectives has been constructed by means of the ε-constraint method. The curve has shown that this is a typical convex problem. Three different methods have been applied to the determination of the preferred design. The preferred designs obtained by the three methods are almost identical. The comparison between the single-objective and two-objective designs indicates that two of the heat exchangers are especially effective for minimizing the system size and two others are especially effective for minimizing the loss of available energy.

NOMENCLATURE

A = heat transfer area, ft^2

A_j = heat transfer area of the j-th heat exchanger, ft^2

a = heat transfer area, ft^2

C_{pj} = specific heat capacity of the j-th cold stream, Btu/lb_m-°R

c_{pj} = specific heat capacity of the j-th hot stream, Btu/lb_m-°R

$\underline{f}$ = objective function vector

f_1 = first objective function which represents the total heat transfer area, ft^2

Δf_1 = small deviation in f_1, ft^2

f_2 = second objective function which represents the rate of available energy loss, Btu/hr

$\underline{g}$ = constraint function vector

$\Delta\dot{H}$ = difference between the enthalpy flow rate at the inlet and that at the outlet of the system, Btu/hr

$\underline{h}$ = equality constraint function vector

h_{cj} = specific enthalpy of the j-th cold stream, Btu/lb_m

J = scalar objective function

J_j = scalar objective function of the j-th subproblem

k = parameter defined by Eq. (8)

P = pressure, atm

Q = rate of heat transfer from the hot stream to the cold stream, Btu/hr

$\dot{Q}$ = rate of heat transfer from the surroundings to the system, Btu/hr

R = ratio of heat capacity flow rates of the cold and hot streams

R_j = ratio of heat capacity flow rates of the cold and hot streams through the j-th heat exchanger

$\Delta\dot{S}$ = difference between the entropy flow rate at the inlet and that at the outlet of the system, Btu/hr-°R

$\dot{S}_{sur}$ = rate of entropy change of the surroundings, Btu/hr-°R

s_{cj} = specific entropy of the j-th cold stream, Btu/lb_m-°R

Δs_{cj} = difference between the specific entropy of the j-th cold stream at the inlet and that at the outlet of the system, Btu/lb_m-°R

Δs_{hj} = difference between the specific entropy of the j-th hot stream at the inlet and that at the outlet of the system, Btu/lb_m-°R

T = temperature of the cold stream, °R

T_{21} = trade-off ratio between f_2 and f_1

T_i = temperature of the cold stream at the inlet of the heat exchanger, °R

T_o = temperature of the cold stream at the outlet of the heat exchanger, °R

T_{Aj}, T_{Bj}, T_{Cj} = intermediate temperatures of the cold streams, °R

T_{ji} = temperature of the j-th cold stream at the inlet of the system,°R

T_{jo} = temperature of the j-th cold stream at the outlet of the system, °R

T_{sur} = temperature of the surroundings, namely, 537°R

t = temperature of the hot stream, °R

t_i = temperature of the hot stream at the inlet of the heat exchanger, °R

t_o = temperature of the hot stream at the outlet of the heat exchanger, °R

t_{Aj}, t_{Bj}, t_{Cj}, t_{Dj} = intermediate temperatures of the hot streams, °R

t_{ji} = temperature of the j-th hot stream at the inlet of the system, °R

t_{jo} = temperature of the j-th hot stream at the outlet of the system, °R

Δt_m = log-mean temperature difference, °R

U = overall heat transfer coefficient, Btu/hr-ft^2-°R

U_j = overall heat transfer coefficient of the j-th heat exchanger, Btu/hr-ft^2-°R

v = value function

v_{cj} = specific volume of the j-th cold stream, ft^3/lb_m

$\dot{W}$ = rate of the shaft work done by the system, Btu/hr

W_{21} = surrogate worth function associated with f_2 and f_1

W_j = mass flow rate of the j-th cold stream, lb_m/hr

WC_p = heat capacity flow rate of the cold stream, Btu/hr-°R

$(WC_p)_j$ = heat capacity flow rate of the j-th cold stream, Btu/hr-°R

$\dot{W}_{max}$ = maximum rate of the shaft work done by the system, Btu/hr

w_j = mass flow rate of the j-th hot stream, lb_m/hr

wc_p = heat capacity flow rate of the hot stream, Btu/hr-°R

$(wc_p)_j$ = heat capacity flow rate of the j-th hot stream, Btu/hr-°R

$\underline{x}$ = independent design variable vector

Greek Symbols

α = parameter defined by Eq. (10)

α_j = parameter associated with the j-th heat exchanger

β = parameter defined by Eq. (11)

ε = maximum allowable level of f_1

λ_{21} = generalized Lagrangian multiplier

$\underline{\mu}$ = Lagrangian multiplier vector

$\dot{\sigma}$ = rate of the total entropy creation, Btu/hr-°R

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Table 1. Fixed Conditions for Optimization

I. Initial temperatures of the hot streams

$t_{A1} = 860\ ^{\circ}R = 478\ ^{\circ}K$

$t_{B1} = 1060 = 589$

$t_{C1} = 990 = 550$

$t_{D1} = 960 = 533$

II. Initial and terminal temperatures of the cold streams

$T_{A1} = 590\ ^{\circ}R = 328\ ^{\circ}K$ $\qquad T_{A3} = 860\ ^{\circ}R = 478\ ^{\circ}K$

$T_{B1} = 640 = 350$ $\qquad T_{B4} = 910 = 506$

$T_{C1} = 610 = 339$ $\qquad T_{C5} = 960 = 533$

III. Heat capacity flow rates

$(wc_p)_A = 1.5 \times 10^6$ Btu/hr-$^{\circ}$R $= 6.81 \times 10^8$ cal/hr-$^{\circ}$K

$(wc_p)_B = 1.5 \times 10^6 = 6.81 \times 10^8$

$(wc_p)_C = 1.5 \times 10^6 = 6.81 \times 10^8$

$(wc_p)_D = 1.5 \times 10^6 = 6.81 \times 10^8$

$(WC_p)_A = 1.0 \times 10^6 = 4.54 \times 10^8$

$(WC_p)_B = 1.0 \times 10^6 = 4.54 \times 10^8$

$(WC_p)_C = 1.35 \times 10^6 = 6.13 \times 10^8$

IV. Overall heat transfer coefficients

$U_j = 100$ Btu/hr-ft^2-$^{\circ}$R $= 4.88 \times 10^5$ cal/hr-m^2-$^{\circ}$K, $\quad j = 1,2,3,7,8$

$U_j = 135 = 6.59 \times 10^5$, $\quad j = 4,5,6,9$

Table 2. Comparison of results for the minimum total heat exchanger area problem.

Heat Exchanger area	Chen (1977)	Present work
A_1 (ft^2)	0	0
A_2	5,635	5,528
A_3	13,256	13,261
A_4	4,181	4,184
A_5	0	0
A_6	0	0
A_7	20,622	20,628
A_8	545	544
A_9	25,832	25,826
$J = \Sigma A_j$	70,071	70,071

Table 3. Selected Non-Inferior Designs of the Heat Exchanger System

A_1 (ft^2)	0	9252	14148	18344	20099	17246	19859	22857
A_2	5628	9116	11296	14702	19501	23114	24869	26943
A_3	13261	13474	13370	12955	12280	11488	11587	11673
A_4	4184	14712	21731	27220	33154	37608	43172	46766
A_5	0	0	0	0	0	0	0	0
A_6	0	0	0	0	0	8682	9668	11946
A_7	20628	12891	9804	8855	8190	10732	11200	11534
A_8	544	675	1079	967	1067	2948	2706	2692
A_9	25826	19880	18572	16957	15709	8182	6939	5589
$f_1 = \sum_{j=1}^{9} A_j$	70071	80000	90000	100000	110000	120000	130000	140000
f_2	1.113×10^8	0.943×10^8	0.851×10^8	0.784×10^8	0.733×10^8	0.692×10^8	0.654×10^8	0.622×10^8

Table 4. Preferred Designs of the Heat Exchanger System Determined by the Three Different Methods

	Lexicographic Approach	Weighting method	Sarrogate Worth Trade-off Method
A_1	9,251	10,691	12,069
A_2	9,115	10,215	10,598
A_3	13,474	13,280	13,337
A_4	14,712	16,683	17,891
A_5	0	0	0
A_6	0	0	0
A_7	12,892	11,601	11,232
A_8	675	897	822
A_9	19,881	19,537	19,053
f_1	80,000	82,904	85,000
f_2	0.943×10^8	0.912×10^8	0.892×10^8

Table 5. Non-Inferior Points and the Decision Maker's Responses for the Heat Exchanger Problem

f_1 (ft^2)	f_2 (Btu/hr)	T_{21} (Btu/hr-m^2)	W_{21}
70,071	1.1127×10^8	2096	10
75,000	1.0094	1700	5
80,000	0.9433	1173	1
85,000	0.8921	927	0
90,000	0.8506	778	-2
95,000	0.8142	664	-3
100,000	0.7842	577	-4
105,000	0.7565	515	-5
110,000	0.7327	430	-6
115,000	0.7135	410	-8
120,000	0.6917	417	-10
125,000	0.6719	378	-10
130,000	0.6539	346	-10
135,000	0.6373	319	-10
140,000	0.6221	305	-10

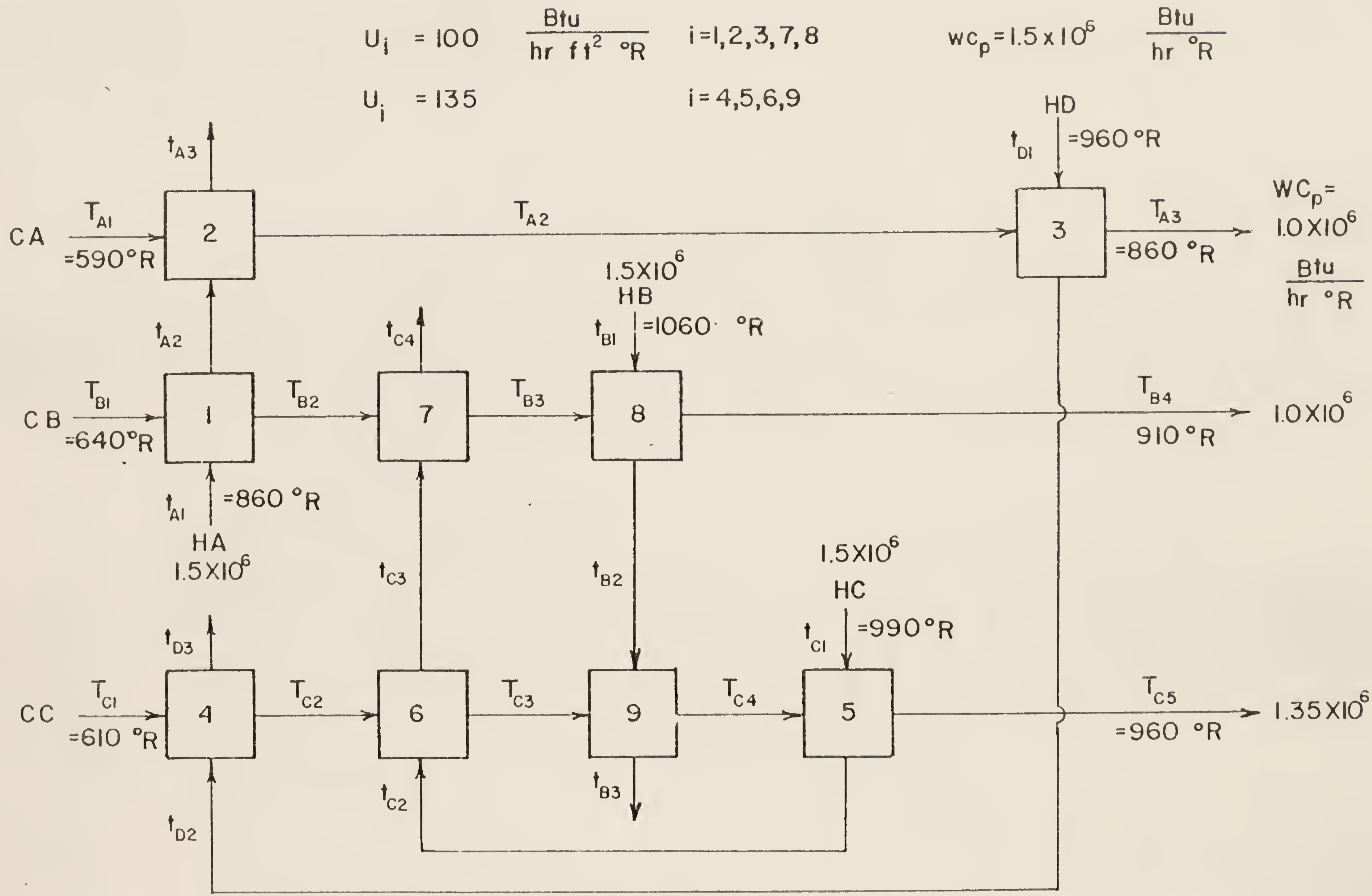

Fig. 1. A composite network involving nine heat exchangers (Hwa, 1965; Takamatsu et al., 1976). (CA, CB, CC = cold streams; HA, HB, HC, HD = hot streams)

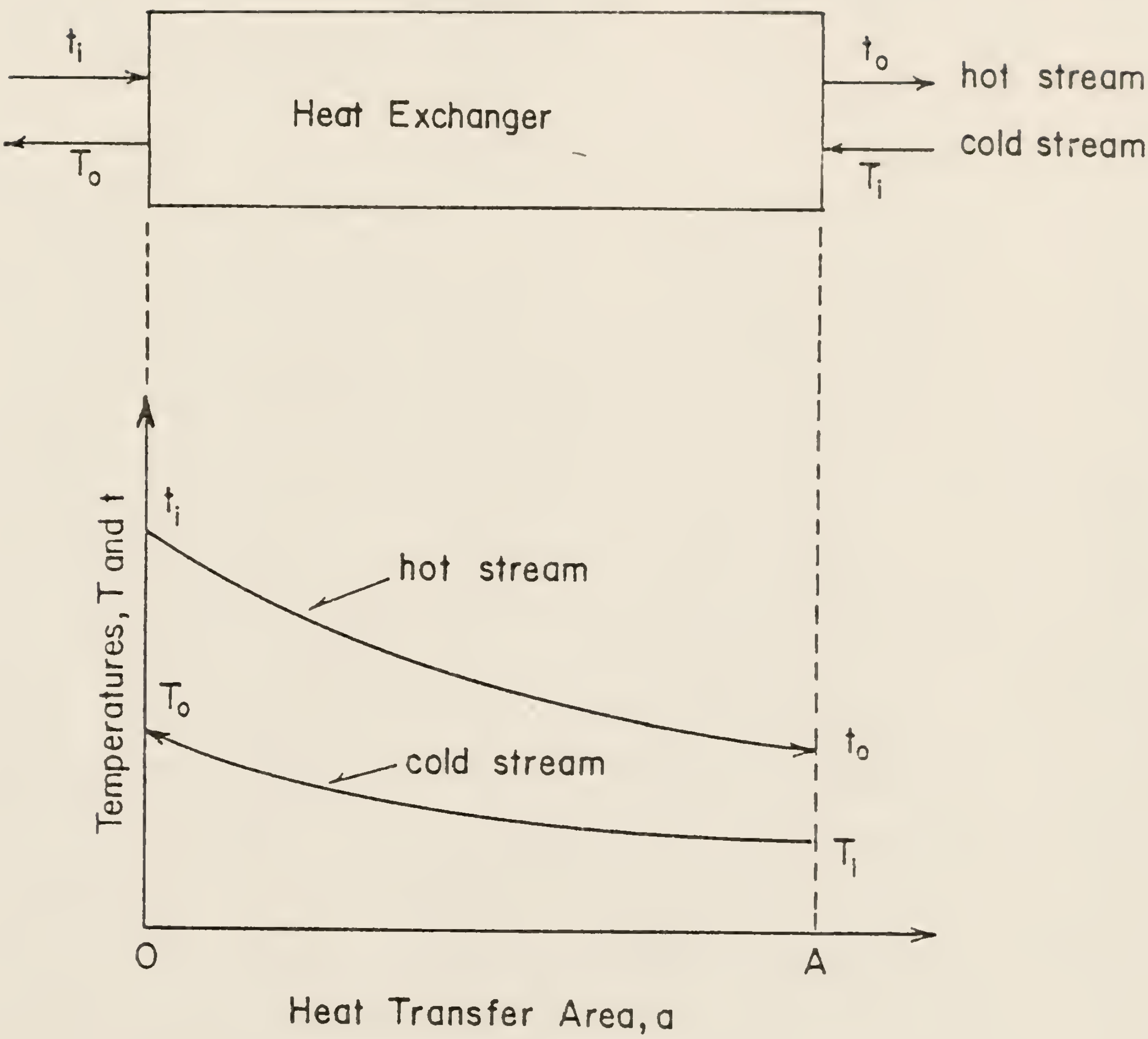

Fig. 2. Temperature distributions in a heat exchanger of counter-current type.

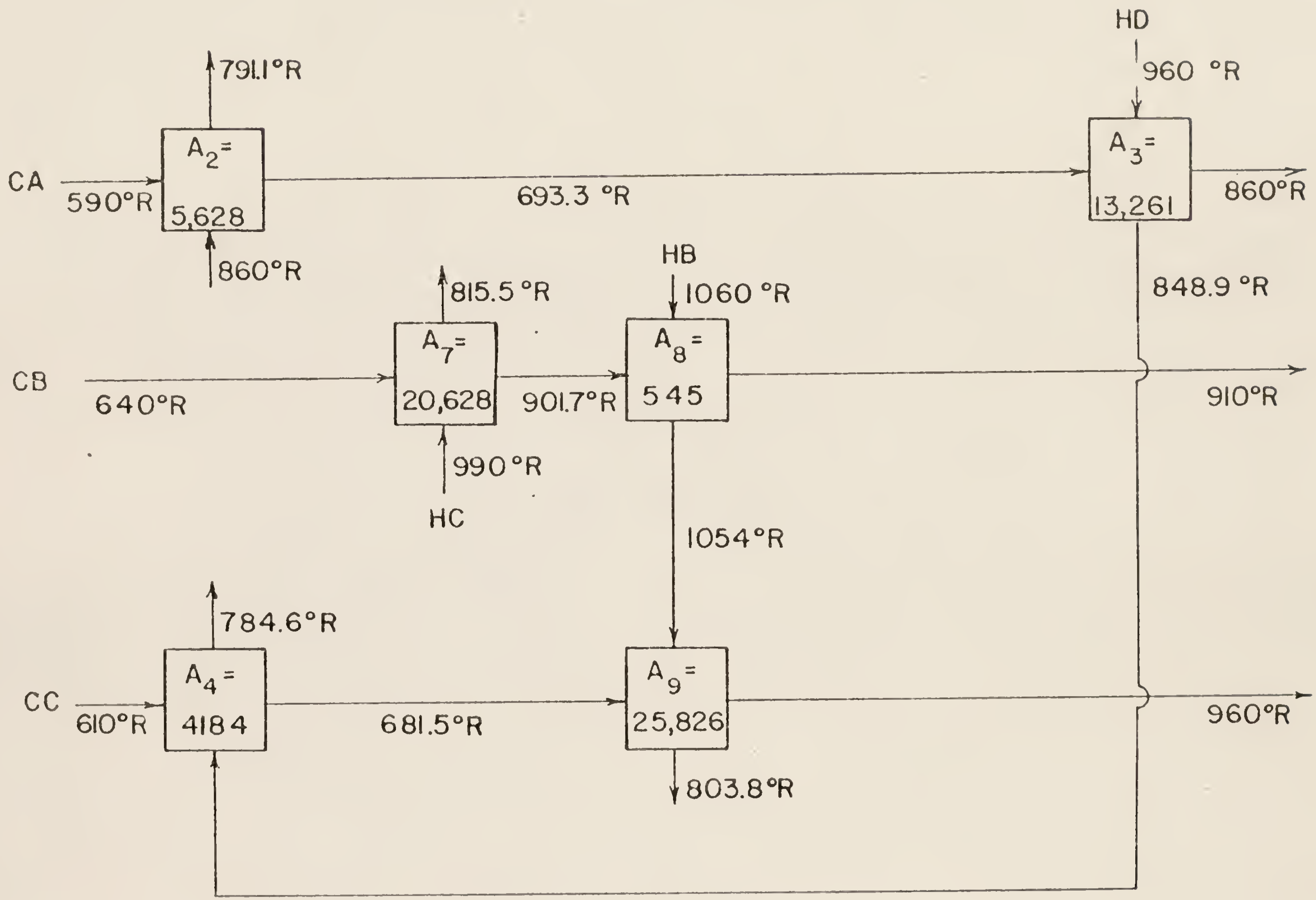

Fig. 3. The minimum-area network ($f_1 = \Sigma A_i = 70,071$).

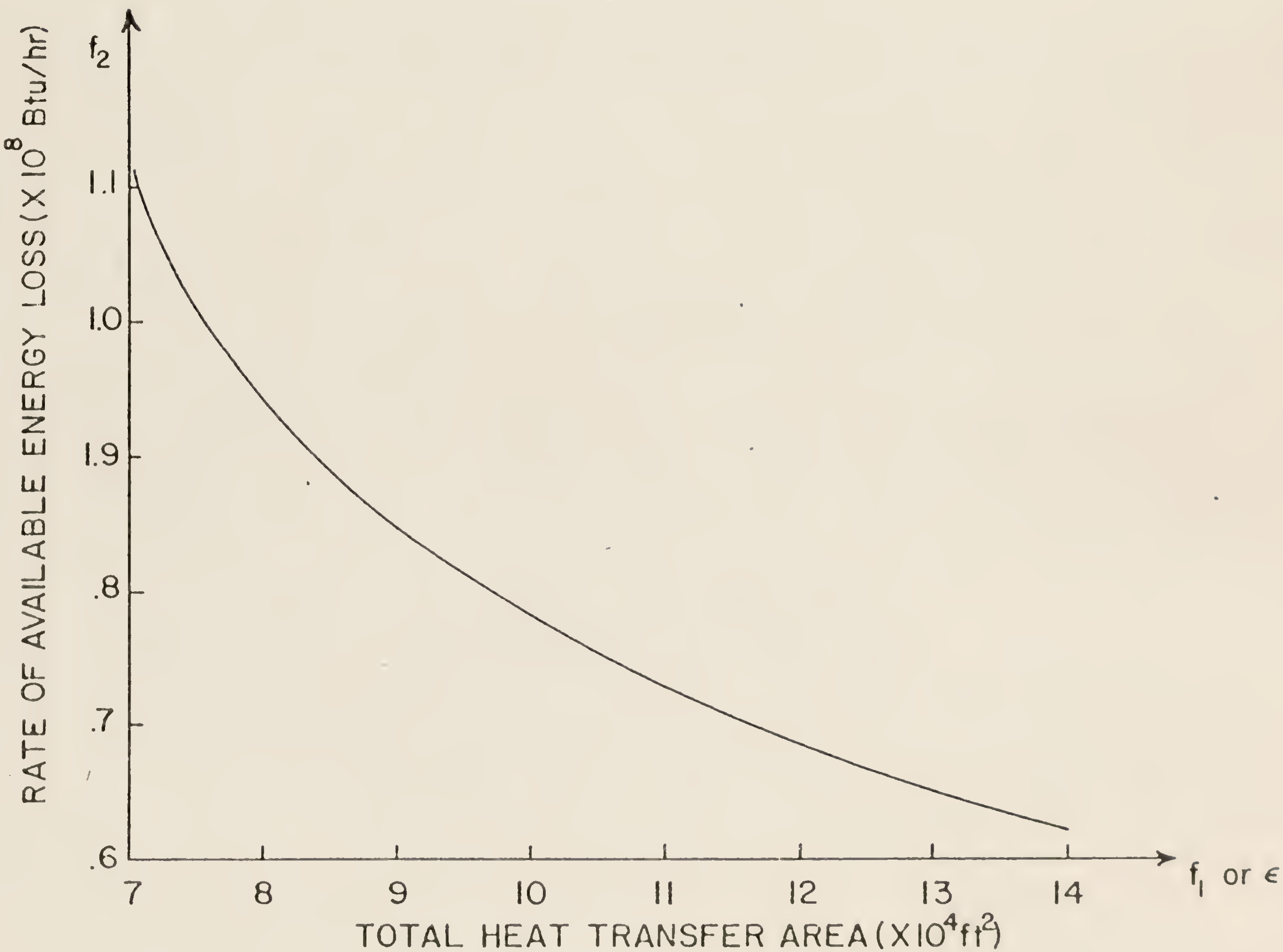

Fig. 4. Trade-off curve for the composite heat exchanger system.

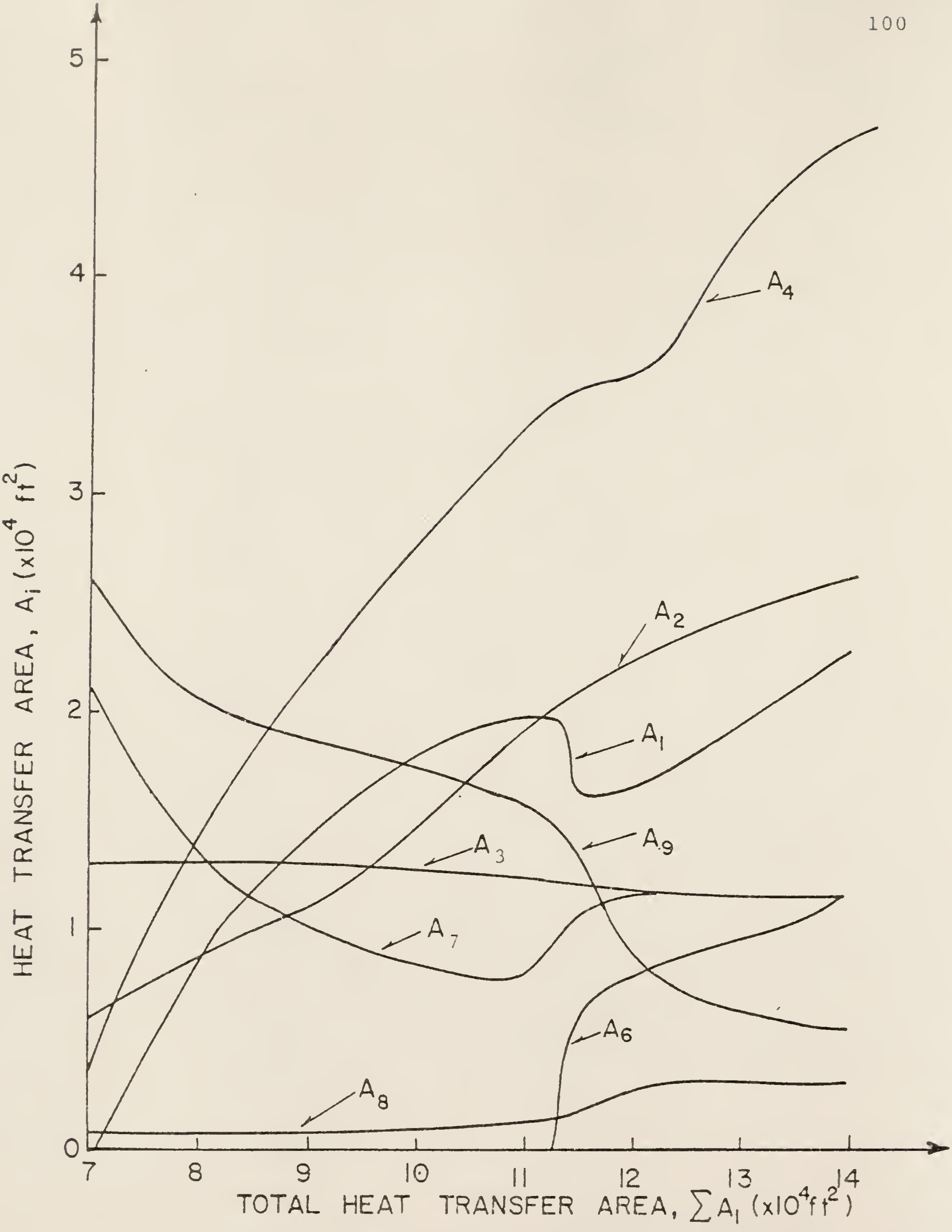

Fig. 5. Change of the optimal design with the increase of the total heat transfer area.

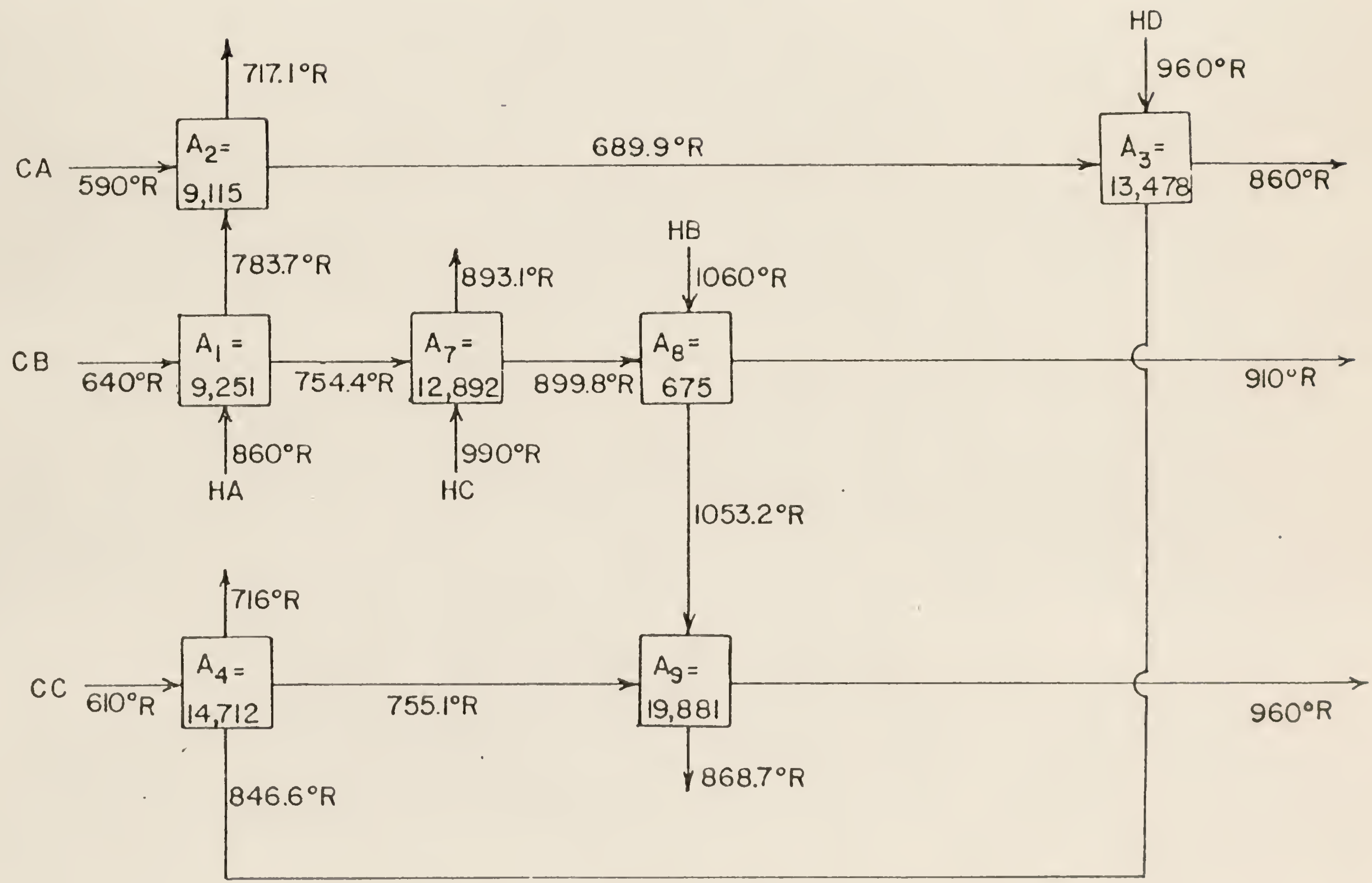

Fig. 6. Preferred network determined by the lexicographic approach.

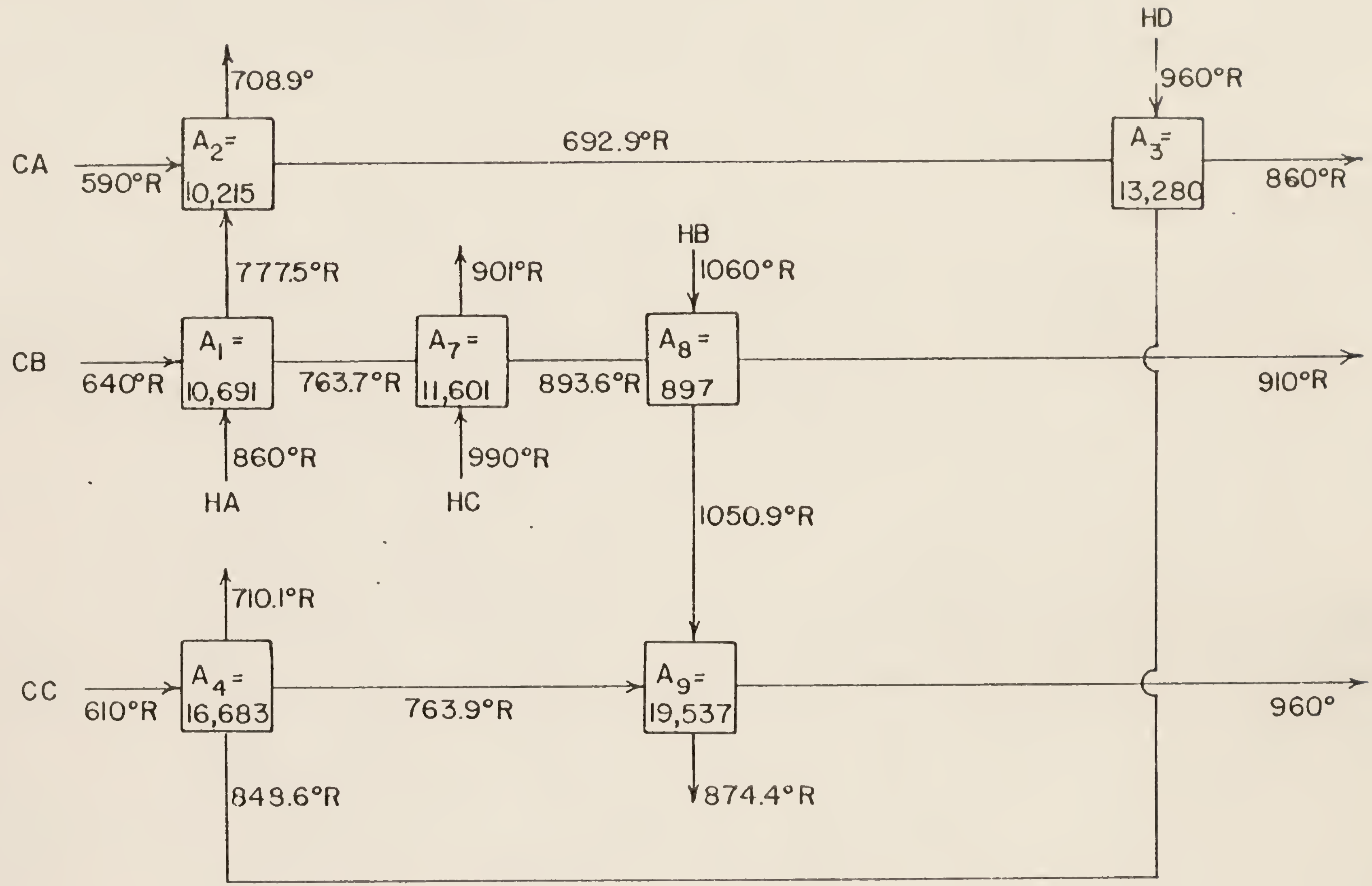

Fig. 7. Preferred network determined by the weighting method.

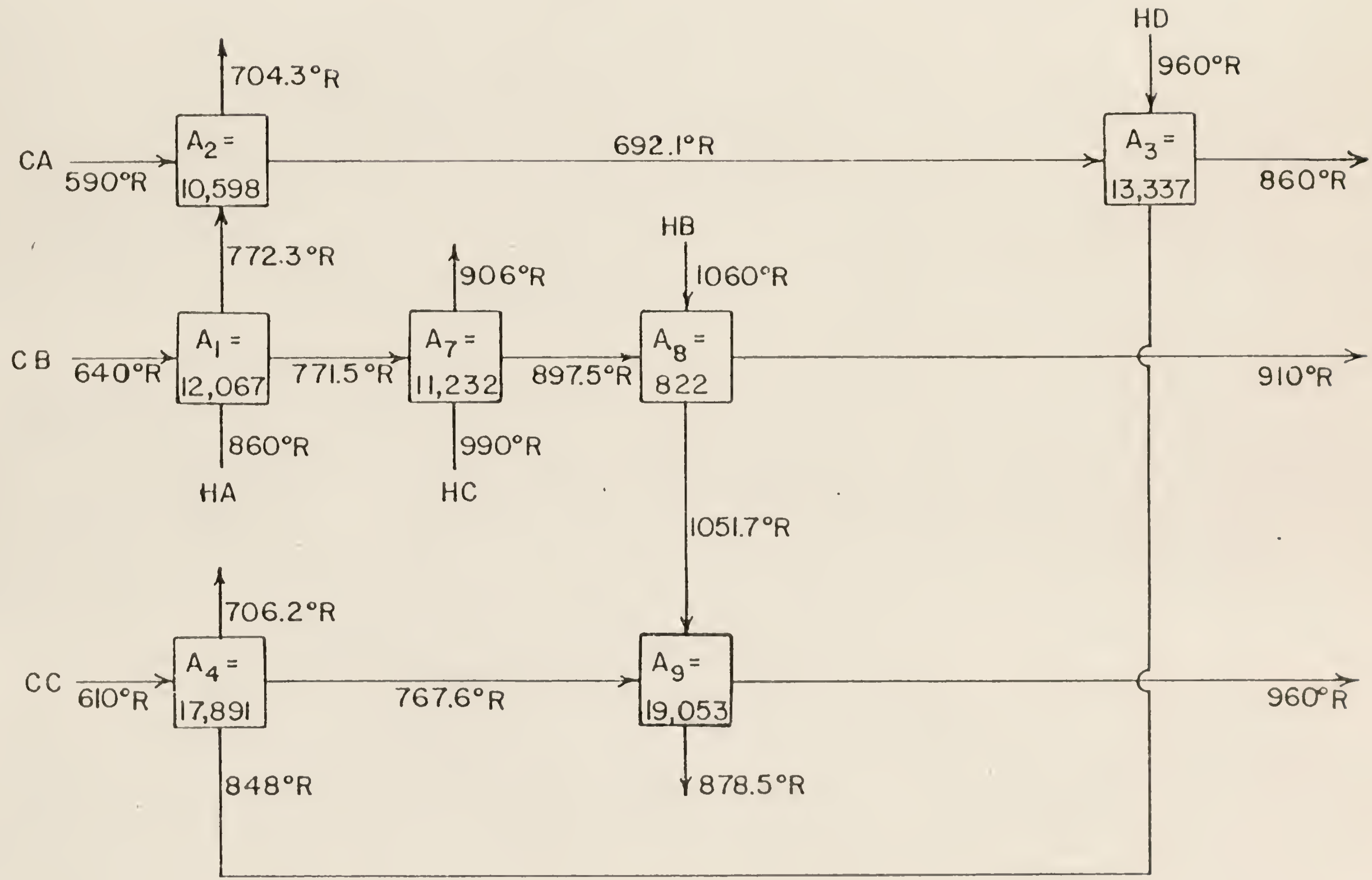

Fig. 8. Preferred network determined by the surrogate worth trade-off method.

APPENDIX A: DERIVATION OF THE PERFORMANCE EQUATION FOR A HEAT EXCHANGER IN A LINEAR FORM

Suppose that a heat exchanger is of the plug flow and counter-current type. Then, the temperatures of both the cold and hot streams in the exchanger change with the position. Figure 2 depicts the temperature distributions of both streams in the heat exchanger. T_i and T_o denote, respectively, the inlet and outlet temperatures of the cold stream; similarly, t_i and t_o denote, respectively, the inlet and outlet temperatures of the hot stream. The differential energy balance gives:

$$dq = WC_p\ dT = wc_p dt = -U(t - T)da \tag{1}$$

or

$$wc_p \frac{dt}{da} = WC_p \frac{dT}{da} = -U(t - T) \tag{2}$$

Equation (2) can be rearranged as

$$\frac{dt}{dT} = \frac{WC_p}{wc_p} = R \tag{3}$$

Here, the ratio of heat capacity flow rates, R, is assumed invariant with respect to temperature variation. Integrating Eq. (3) subject to the boundary condition at a = 0 yields

$$t - t_i = R(T - T_o) \tag{4}$$

Substitution of Eq. (4) into Eq. (2) gives

$$\frac{dT}{da} = -\frac{U}{WC_p}\left[(R-1)T - RT_o + t_i\right) \tag{5}$$

If $R \neq 1$, integration of this equation from a = 0 to a = A yields

$$T_i = \frac{t_i - T_o}{R - 1} \exp\ -\frac{UA}{WC_p}(R - 1)\ + \frac{RT_o - t_i}{R - 1} \tag{6}$$

Note that $T = T_i$ at a = A. By letting

$$k = \exp\ \left\{\frac{-UA(R - 1)}{WC_p}\right\}$$

Eq. (6) becomes

$$T_i = \frac{t_i - T_o}{k(R - 1)} + \frac{RT_o - t_i}{R - 1} \tag{7}$$

which can be rewritten as

$$T_o = \frac{k(R - 1)}{Rk - 1} T_i + \frac{k - 1}{RK - 1} t_i \tag{8}$$

Integrating Eq. (3) subject to the boundary condition at a = A yields

$$t - t_o = R(T - T_i) \tag{9}$$

or

$$T = \frac{t - t_o}{R} + T_i \tag{10}$$

By substituting this equation into eq. (2), we have

$$\frac{dt}{da} = -\frac{U}{wc_p} [\frac{t(R - 1)}{R} + \frac{t_o}{R} - T_i] \tag{11}$$

If $R \neq 1$, integration of this equation from a = 0 to a = A, where $t = t_o$, gives

$$\frac{R(t_o - T_i)}{t_i(R - 1) + t_o - RT_i} = \exp[-\frac{UA}{wc_p}\frac{R - 1}{R}]$$

$$= \exp[-\frac{UA}{WC_p}(R - 1)]$$

$$= \frac{1}{k}$$

Rearrange this equation gives

$$t_o = \frac{R - 1}{Rk - 1} t_i + \frac{R(k - 1)}{R k - 1} T_i \tag{12}$$

Equations (8) and (12) which are in the linear form can be expressed as

$$\begin{pmatrix} T_o \\ t_o \end{pmatrix} = \begin{pmatrix} \frac{k(R-1)}{R k-1} & \frac{(k-1)}{R k-1} \\ \frac{R(k-1)}{R k-1} & \frac{R-1}{R k-1} \end{pmatrix} \begin{pmatrix} T_i \\ t_i \end{pmatrix} \tag{13}$$

By letting

$$\alpha = \frac{k - 1}{R k - 1},$$

Eq. (19) becomes

$$\begin{pmatrix} T_o \\ t_o \end{pmatrix} = \begin{pmatrix} 1-\alpha & \alpha \\ \alpha R & 1-\alpha R \end{pmatrix} \begin{pmatrix} T_i \\ t_i \end{pmatrix} \tag{14}$$

This is Eq. (9a) in Chapter III. Simple algebraic transformation of Eqs. (8) and (12) leads to Eqs. (9b), (9c), and (9d) in the same chapter.

Suppose that a heat exchanger is of the complete mixing type. Then, the temperature distribution of the cold stream in the exchanger is uniform and is the same as that at the outlet. This is also true for the hot stream. The overall heat balance equation and the heat transfer rate equation for the exchanger are expressed, respectively, as

$$Q = wc_p\,(t_i - t_o) = WC_p\,(T_o - T_i) \tag{15}$$

$$Q = UA\,(t_o - T_o) \tag{16}$$

Equation (15) can be rearranged as

$$t_o = -R\,(T_o - T_i) + t_i \tag{17}$$

where

$$R = \frac{WC_p}{wc_p}$$

Equating Eq. (16) and the right hand side of Eq. (15) gives rise to

$$t_o - T_o = \frac{WC_p}{UA}\,(T_o - T_i) \tag{18}$$

Substitution of Eq. (17) into Eq. (18) and rearrangement of the equation result in

$$T_o = \left(1 - \frac{1}{1 + R + \frac{WC_p}{UA}}\right) T_i + \frac{1}{1 + R + \frac{WC_p}{UA}}\, t_i \tag{19}$$

By substituting this equation back into Eq. (17), we have

$$t_o = \frac{R}{1 + R + \frac{WC_p}{UA}} T_i + \left(1 - \frac{R}{1 + R + \frac{WC_p}{UA}}\right) t_i \tag{20}$$

Equations (19) and (20) can be written in the matrix form as

$$\begin{pmatrix} T_o \\ t_o \end{pmatrix} = \begin{pmatrix} 1 - \alpha & \alpha \\ \alpha R & 1 - \alpha R \end{pmatrix} \begin{pmatrix} T_i \\ t_i \end{pmatrix} \tag{21}$$

where

$$\alpha = \frac{1}{1 + R + \frac{WC_p}{UA}}$$

This is the same form as that for a heat exchanger of the plug flow type except for the definition of the parameter, α.

Combinations of linear forms of the plug flow types and those of the complete mixing types can approximate performance equations of a variety of process equipment with wide ranges of mixing conditions.

APPENDIX B: DERIVATION OF THE RATE OF AVAILABLE ENERGY LOSS (Keenan, 1941, 1951; Reistad, 1970)

Suppose that an open system is operated under the steady state condition. For this system, the first law of thermodynamics is written as

$$\Delta\dot{H} = \dot{Q} - \dot{W} \tag{1}$$

where $\Delta\dot{H}$ is the difference between the enthalphy flow rate at the inlet and that at the outlet of the system, $\dot{Q}$ is the rate of heat transfer from the surroundings to the system, and $\dot{W}$ is the rate of shaft work done by the system. The rate of total entropy creation, $\dot{\sigma}$, is given by

$$\dot{\sigma} = \Delta\dot{S} + \dot{S}_{sur} \tag{2}$$

where $\Delta\dot{S}$ stands for the difference between the rate of entropy flow at the inlet and that at the outlet of the system, and $\dot{S}_{sur}$ stands for the rate of entropy change of the surroundings.

Since we know that

$$\dot{S}_{sur} = -\frac{\dot{Q}}{T_{sur}} \tag{3}$$

where T_{sur} is the surrounding temperature, elimination of $\dot{Q}$ and $\dot{S}_{sur}$ from Eqs. (1) and (2) gives rise to

$$\dot{W} = T_{sur}\,\Delta\dot{S} - \Delta\dot{H} - T_{sur}\,\dot{\sigma} \tag{4}$$

Because of the second law of thermodynamics ($\dot{\sigma} \geq 0$), the maximum rate of the shaft work done by the system is

$$\dot{W}_{max} = T_{sur}\,\Delta\dot{S} - \Delta\dot{H} \tag{5}$$

The rate of available energy loss is defined as the difference between $\dot{W}_{max}$ and $\dot{W}$, i.e.,

$$\dot{W}_{max} - \dot{W} = T_{sur}\,\dot{\sigma} \tag{6}$$

Suppose that a heat exchanger system involving m cold streams and n hot streams is isolated from the surroundings ($\dot{Q} = 0$).
Then, Eq. (2) becomes

$$\dot{\sigma} = \sum_{j=1}^{m} W_j\,\Delta s_{cj} + \sum_{j=1}^{n} w_j\,\Delta s_{hj} \tag{7}$$

where

Δs_{cj} = difference between the specific entropy of the j-th cold stream at the inlet and that at the outlet of the system

Δs_{hj} = difference between the specific entropy of the j-th hot stream of the inlet and that at the outlet of the system

W_j = mass flow rate of the j-th cold stream

w_j = mass flow rate of the j-th hot stream

On the other hand, we have

$$ds_{cj} = \left(\frac{\partial s_{cj}}{\partial T}\right)_P dT + \left(\frac{\partial s_{cj}}{\partial P}\right)_T dP \tag{8}$$

The following relationships exist among the thermodynamical functions:

$$\left(\frac{\partial s_{cj}}{\partial T}\right)_P = \frac{1}{T}\left(\frac{\partial h_{cj}}{\partial T}\right)_P = \frac{C_{pj}}{T}$$

$$\left(\frac{\partial s_{cj}}{\partial P}\right)_T = -\left(\frac{\partial v_{cj}}{\partial T}\right)_P$$

where h_{cj}, v_{cj} and C_{pj} are, respectively, the specific enthalpy, specific volume and specific heat capacity of the j-th cold stream. Applying these relationships to Eq. (8), we obtain

$$ds_{cj} = \frac{C_{pj}}{T}dT - \left(\frac{\partial v_{cj}}{\partial T}\right)_P dP \tag{9}$$

Since all streams are liquid, the volume change with respect to the temperature is negligible. Therefore,

$$ds_{cj} = \frac{C_{pj}}{T}\, dT \tag{10}$$

If C_{pj} is constant, this equation becomes

$$\Delta s_{cj} = C_{pj} \ln \frac{T_{jo}}{T_{ji}} \tag{11}$$

where T_{ji} and T_{jo} denote the temperature of the j-th cold stream at the inlet and that at the outlet, respectively. Similarly, we have

$$\Delta s_{hj} = c_{pj} \ln \frac{t_{jo}}{t_{ji}} \tag{12}$$

for the j-th hot stream. Thus, Eq. (7) becomes

$$\dot{\sigma} = \sum_{j=1}^{m} (WC_p)_j \ln \frac{T_{jo}}{T_{ji}} + \sum_{j=1}^{n} (wc_p)_j \ln \frac{t_{jo}}{t_{ji}} \tag{13}$$

From Eqs. (6) and (13), the rate of available energy can be written in the form of

$$\dot{W}_{max} - \dot{W} = T_{sur}\left[\sum_{j=1}^{m} (WC_p)_j \ln \frac{T_{jo}}{T_{ji}} + \sum_{j=1}^{n} (wc_p)_j \ln \frac{t_{jo}}{t_{ji}}\right] \tag{14}$$

which is Eq. (15) in section 2.2.

CHAPTER IV

CONCLUSION AND RECOMMENDATION

CONCLUSION AND RECOMMENDATION

Optimization techniques for a multi-objective system, which have been reviewed and applied in this study, should enable us to take into account more than one objective function in synthesizing an optimal but realizable system. Although most of the basic concepts and terminologies involved in these techniques have been developed originally in the fields of economics, management science and mathematics, these techniques are readily applicable to various engineering problems because of their general nature. The heat exchanger network system studied in Chapter II is such an example.

The following two possible extensions concerning the heat exchanger system are recommended for future work.

1. Optimization of the network structure. The network optimized here was fixed in advance. It may be possible to improve the system performance by optimizing the network structure as well as the sizes of each heat exchangers. The structual parameter method may be applicable to this extension.
2. Extension of the system boundary. A heat exchanger system is usually built as an auxiliary system of a main plant. Thus, the optimization should be carried out simultaneously for both the heat exchanger system and the main plant.

There are many other chemical and/or industrial process systems which have been optimized conventionally by minimizing a single cost function for each system. In view of the growing public concern over the energy depletion and the conservation of the environment, these systems should be reanalyzed and resythesized by explicitly taking into account other objective functions that characterize the thermodynamic efficiencies of the systems and the esthetic values. The techniques for solving a multi-objective problem will be effective for these future works.

METHODS FOR DECISION MAKING WITH MULTIPLE OBJECTIVES

AND

THEIR APPLICATIONS TO A HEAT EXCHANGER NETWORK SYNTHESIS

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ABSTRACT

The basic concepts and terminologies employed in multi-objective system analysis and synthesis have been interpreted, and the techniques for solving a multi-objective optimization problem have been comprehensively and critically reviewed. A large heat recovery system consisting of nine heat exchangers has been optimized to demonstrate the applicability of the techniques. Two objective functions considered are the total heat transfer area and the loss of available energy. As indicated by the trade-off curve between them, these two objectives are in conflict with each other, and this problem is a typical convex problem. The preferred designs of the heat exchanger system have been determined by three different methods.